Proceedings of the 5th International Conference on Nonlinear Dynamics
ND-KhPI2016
September 27-30, 2016, Kharkov, Ukraine

From Geometry, Kinematics and Dynamics of Billiards to the Extended Theory of Skew Collision between Two Rolling Bodies and Methodology of Vibro-Impact Dynamics

Katica R. (Stevanović) Hedrih[1]*

Abstract
Starting from explanation of geometry, kinematics and dynamics of game billiards, and phenomena of impact a rolling ball into different types of curved surfaces and direct and skew central collision of two rolling, same dimension, balls we open question of collision of two rolling axial symmetrically bodies with different dimensions and different forms. Use elementary approach and Petrović's theory presented in two books "Elements of mathematical phenomenology" and "Phenomenological mappings", extended theory of direct and skew central collision of two rolling, axially symmetric, but different dimensions and forms, bodies is formulated with all additional and new analytical expressions, theorems , to define all pre- and post- collision kinetic states. Use these new results complete methodology of vibro-impact system dynamics is formulated and applied for investigation kinetic parameters and phenomena in vibro-impact systems with successive collisions between two or a finite number of rolling bodies. Energy jumps in collisions between rolling bodies in vibro-impact system dynamics are indicated and analytically described in a number of these systems.

Keywords
Billiards, theory of rolling body collision, vibro-impact dynamics

[1] Mathematical Institute SANU Belgrade and Faculty of Mechanical Engineering, University of Niš, Serbia.
* **Corresponding author:** khedrih@sbb.rs,khedrih@eunet.rs

Introduction

"In connection with the game of billiards .... there are various dynamic tasks, whose solutions contain in this event. I think that people who know Theoretical mechanics and even students of polytechnics, with interest familiarize themselves with explanations of the entire original phenomenon that can be observed from the time of movement billiard balls".

Gaspar-Gistav de Koriolis, ***Mathematical theory of billiards game***

In the Reference [1] by *Gaspar-Gistav de Koriolis,* complexity and numerous various dynamic tasks, whose solutions contain in the game of billiards, are pointed out. Main are rolling of the rigid balls and impact of balls into vertical surface and central of skew collision between two or more rolling balls. Elements of geometry, kinematics and dynamics of billiards are presented in Reference [2]. In the Reference [3] dynamics of elliptic billiards and rolling ball impacts are presented. In Figure 1, a trajectory of billiards' rolling ball, along horizontal plane, bounded by boundary elliptic-cylindrical surface, with plan of the impact and outgoing angular velocities before and after impacts, are presented.

In Figure 2.a*, possible impact points at a rolling ball along horizontal trace to the boundary vertical cylindrical surface with plans of component and resultant impact translator velocities and angular velocity are presented. In same Figure 2, schematically presentation of kinematic state of the skew (b* and c*) impacts of a rolling ball to a boundary convex curvilinear vertical cylindrical surface with traces of ball rolling pre- and post- collision states are presented.

Let's expose, in short, the source of theory of impacts of rigid bodies. Beginning was with the competition and London Royal Society. Royal Scientific Society in London in 1668 announced a competition for the solution of problems of the dynamics of impact and on this competition have submitted their works, by now known scientists Vilis (Wallis, 1616-1703, Mechanica sive de mote-1688) and Huygens (Huygens – De motu corporum ex percusione). Using the results of the collision submitted to the Royal Scientific Society by Willis and Huygens, learned and giving their generalizations, Isaac Newton founded the fundamental basics of the theory of impacts. And before Newton and Huygens, and Willis, was exploring the dynamics of impacts. Thus, for example, collision problems are dealt with Galileo Galilei, who came to the conclusion that the impact force in relation to the pressure force infinitely large, but it came to the knowledge of the relationship of impact impulse and linear momentum.

It will be shown Carnot's theorem (Lazare Carnot 1753-1824., Principes fondamenteaux de l'équilibre et de movement - 1803), who says that "In a collision, the system inelastic material body's loss of kinetic energy is equal to the kinetic energy of the lost speed." The explanations on experimental method for obtaining coefficients of restitution of different types of impacts and collisions are defined.

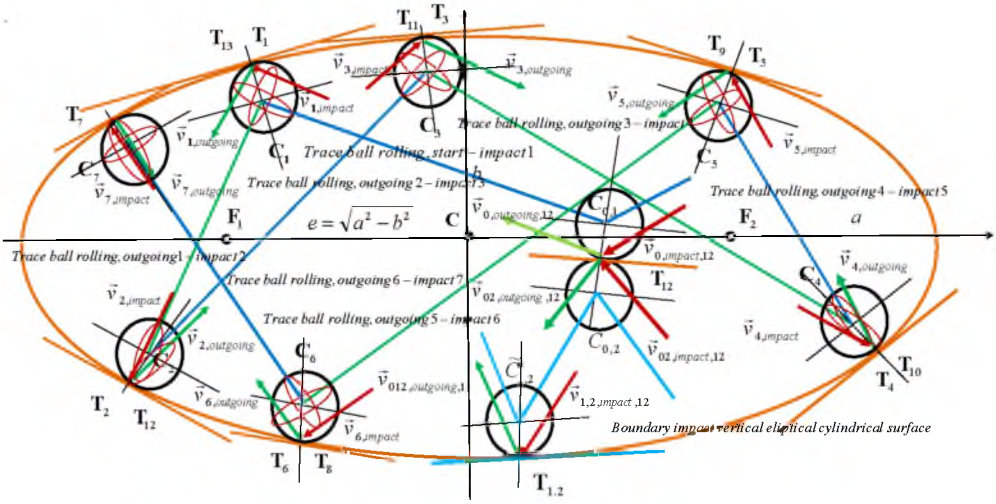
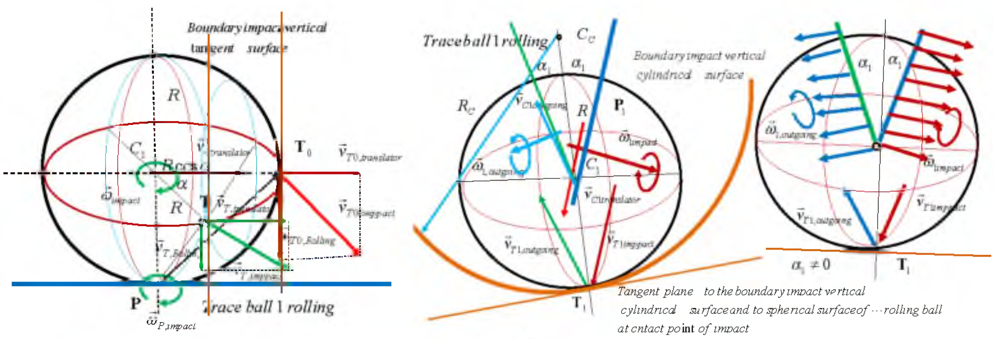


Figure 1. Trajectory of billiards ball rolling along horizontal plane bounded by boundary elliptic-cylindrical surface with plan of the impact and outgoing angular velocities before and after impacts.



a* b* c*

Figure 2. a* Possible impact points at rolling ball along horizontal trace to the boundary vertical cylindrical surface with plans of component and resultant impact translator velocities and angular velocity. Schematically presentation of kinematic state of the skew (b* and c*) impacts of a rolling ball to a boundary convex curvilinear vertical cylindrical surface with traces of ball rolling pre- and post- collision states

Starting from the geometric basis for switching to the impact theory, which is basically a theory of the dynamics of each ball of billiards. Shown are the plans of translational and angular velocities of rolling of one ball before and after the impact, the two rolling balls collision.

Let's expose, in short, the theory of the collision of two mass particles, as well as two balls or impact of a mass particle, as well as the impact of balls in the barrier. The output is hypotheses on collision and impact, define the various types of collisions and impacts. This problem is associated with the dynamics of the system with one side retaining constraints [12].

Numerous publications are dedicated under the keywords of the title of billiards, but words are about „mathematical billiards" [12] based on the Ponselle's theorems about polygonal lines introduced into some conical lines. In all these case word are about geometrical points and their geometric trace along polygonal line introduced into some conical line with corresponding principle. Mathematical billiards is useful only as very simple models of rough abstraction of real billiards ball traces, but without any model of real ball's dynamics and impacts.

For investigation of acceptable model of real billiards ball impact and collisions it is necessary to accept that balls are with no neglected dimensions, and that balls are in rolling motion along corresponding horizontal traces with corresponding angular velocities before and after impacts to the boundary vertical cylindrical surface or before and after collision between two balls. Then it is necessary to investigate kinematics and dynamics of billiards balls rolling, and to define corresponding kinetic parameters before and after impacts or collisions of two or more billiards balls.

Comparing the elements of mathematical phenomenology [13-15], [11], [7-9] and theory of impact and collision between rigid bodies [16] and identifying qualitative and mathematical analogy between geometry of moving geometric point in the plane with defined constraints propagation ray of light with the refusal of the obstacles and suggests that the trajectory of geometric point and ray of light analogue and can be used as a baseline determination of the trajectory of billiards ball.

But as billiard balls spherical rigid bodies orbit of their dynamics depend on the type of impact limiters in the form of the surface, and angles of impact velocity and outgoing velocities of mass center balls depend on the type of impact: whether the impact is skew or central!

Dynamics of impacts and collisions of the rolling balls contained in Reference [6] present new and original research results of theory of central collision of two rolling rigid bodies using as main kinematical elements angular velocities of each rolling ball pre- and post- collisions kinetic states. Using these advances in theory of central collisions obtained by author, some of vibro-impact system dynamics are investigated, solved and presented in published papers [4-5] and [10].

1. Kinematics of skew collision of two rolling rigid nalls

Let us to consider skew collision of two rolling rigid balls with different dimensions. For each of the balls $\mathbf{J}_{P1}$ and $\mathbf{J}_{P2}$ are known axial mass inertia moments for corresponding momentary axis of these rolling and $\vec{\omega}_{P1}(t_0)$ and $\vec{\omega}_{P2}(t_0)$ are known arrival (impact) angular velocities of these rolling balls at the moment, in kinetic state pre-collision. At the moment t_0, is start of collision between these arrival rolling balls and at this moment t_0, both bodies having a teaching point and common tangent plane. Then, we suppose that time duration $(t_0, t_0+\tau)$ of collision between rolling bodies is very short time τ (and tends to zero). After this very short time τ contact between rolling balls disappear and both balls separate and move away from one to another in rolling kinetic state with angular velocities $\vec{\omega}_{P1}(t_0+\tau)$ and $\vec{\omega}_{P2}(t_0+\tau)$, which we named as post collision outgoing angular velocities.

Let us to imagine that in teach (contact) point of rolling body in kinetic state of collision is present a tangent plane and it's normal. This tangent plane is plane of teach between rolling balls in collision. As mass center of both balls in collision are at this normal, if angular velocities are parallel and traces collinear, then this collision is centric central, and if not, then collision is eccentric or skew collision.

In the case that pre-collision arrival angular velocities of both rolling bodies are parallel with tangent plane through the contact point, then collision is direct central collision. In the case that pre-collision arrival angular velocities of both rolling bodies are not parallel with tangent plane through the contact point, then collision is skew collision between two rolling balls.

In Figure 3, geometric and kinematic parameters of two rolling balls in the skew collision kinetic stare. a* and b* possible impact points at two rolling balls along horizontal traces, a* ball with smaller radius and b* ball with bigger radius are presented. Angular velocities of each ball rolling are presented and also component velocities at each contact points - common point of collisions of the rolling balls. Tangent at contact point are also presented. c* and d* present vertical projection of the balls with corresponding trace of ball's rolling are shown.

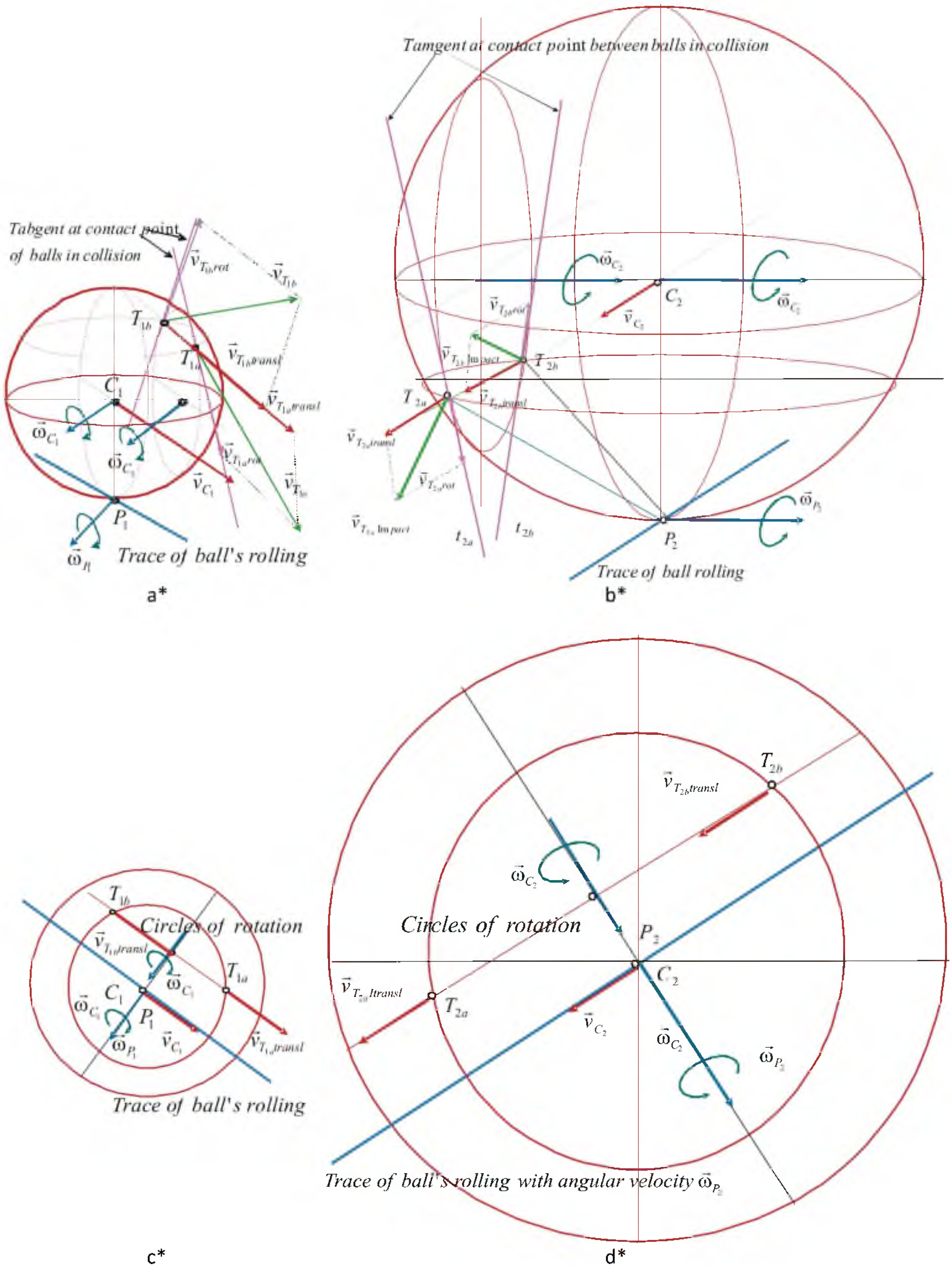


Figure 3. Geometric and kinematic parameters of two rolling balls in the skew collision kinetic stare. a* and b* possible impact points at two rolling balls along horizontal traces, a* ball with smaller radius and b* ball with bigger radius. Angular velocities of each ball rolling are presented and also component velocities at each contact points - common point of collisions of the rolling balls. Tangent at contact point are also presented. c* and d* present vertical projection of the balls with corresponding traces of ball's rolling.

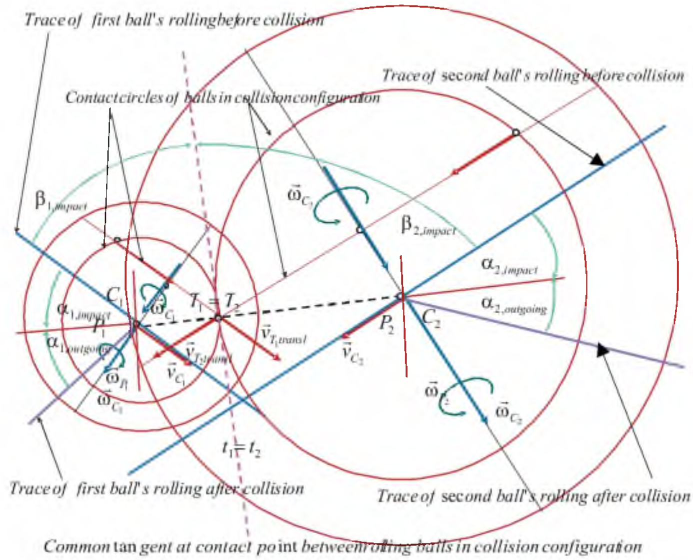


Figure 4. Schematically presentation of kinematic state of the skew collision of two rolling balls with traces of balls rolling pre- and post- collision states.

In Figure 4, schematic presentation of kinematic states of the skew collision of two rolling rigid balls, different radii, and with traces of balls rolling pre- and post- collision states is presented. In this Figure, a common tangent at the contact point between rolling balls is visible. This tangent is tangent of collision two rolling balls with different radii. This tangent of collision lies in the tangent plane of balls collision. Direction between centers of balls in state of collision configuration passes through contact point and is orthogonal to tangent and tangent plane of collision. Angles between this normal of the tangent plane of collision and corresponding trace of each ball's rolling present angle of arrival trace, and angle between trace of outgoing rolling ball after collision and this normal to the tangent plane of collision is equal to this angle of trace of arrival corresponding rolling ball.

If both angles of arrival rolling ball traces are equal zero and rolling trace collinear, then this collision is centric central. If only one angle of arrival trace in relation to the normal of tangent plane of collision of two rolling balls are different to zero, collision of two rolling balls is skew.

2. Dynamics of skew collision of two rolling rigid balls

At starting moment t_0 of collision two rolling bodies, each rolling body arrives with correspodnig pre collision arrival velocity, $\vec{\omega}_{P1}(t_0)$ and $\vec{\omega}_{P2}(t_0)$, and with starting of teach at common point $T_1 = T_2$, both bodies start to deformes in local area of contant point $T_1 = T_2$. Duration of this local deformation around of contant point P is up to moment when projection of the both body angular velocitues to the comomg tangent plane be equal fitst to other. At this kinetic state projection of relative angular velocities of the rolling bodies, first to other, about own momentary axis of rilling are equal to zero. Starting from this moment with projection of relative angular velocity one to other body, appear start of new kinetic state when bodies separated first from other, and in duration of this period, projection of relative angular velocity increaze up to moment when contact between bodies disapear, and bodies separate and ougoing with post collision autgoing angular velocities. During this period of time local deformation of the both bodies in collision disapear and returrn previous form. Starting from moment of rwo bodies separation we define as time period of postcollision of two rolling bodies.

Taking into acount that duration of collision between two rolling axial symetric rigid bodies is time interval $(t_0, t_0 + \tau)$ and that short time τ (and tends to zero) and with accordance of previous analysis of local deformation around contact point $T_1 = T_2$ between two rolling bodies, this time interval is possible separate in two sub-interval: τ' and τ''.

On the basis of the previous analysis of rates of changes of pre-collision arrival angular velocities of rolling bodies in periods compression and restitution in local contact area around contact point between two rolling bodies in collision we can define an analytic relation in the form:

$$k = \frac{\omega_r(t_0+\tau)}{\omega_r(t_0)} = \frac{\omega_{P2}(t_0+\tau)-\omega_{P1}(t_0+\tau)}{\omega_{P1}(t_0)-\omega_{P2}(t_0)} \tag{1}$$

Previous ratio k between post-collision outgoing relative angular velocities and pre-collision arrival relative angular velocities of the rolling bodies in collision is new definition and determination of the *coefficient of restitution* or coefficient of the collision of two rolling axial symmetric rigid bodies. This is new definition and generalization of coefficient of restitution of collision of two bodies in translator motions defined by Isaac Newton. This coefficient of restitution of collision of two rolling bodies, is real, dimension less number and can be determined experimentally, because depend of material of rolling bodies in impact.

Taking into account that coefficient of restitution is known from experimental data, then for obtaining post collision outgoing angular velocities of rolling bodies, we introduce theorem of the conservation angular momentum (kinetic momentum) for corresponding momentary axes of each of corresponding rolling body in collision for pre- and post-collision kinetic state in the following form:

$$\mathbf{J}_{P1}\vec{\omega}_{P1}(t_0)+\mathbf{J}_{P2}\vec{\omega}_{P2}(t_0)=\mathbf{J}_{P1}\vec{\omega}_{P1}(t_0+\tau)+\mathbf{J}_{P2}\vec{\omega}_{P2}(t_0+\tau) \tag{2}$$

Using the two previous expressions (1)-(2), it is not difficult to obtain the following expressions of post-collision outgoing angylar velocities of rolling bodies in the following forms:

$$\omega_{P1}(t_0+\tau)=\omega_{P1}(t_0)-\frac{1+k}{1+\frac{\mathbf{J}_{P1}}{\mathbf{J}_{P2}}}(\omega_{P1}(t_0)-\omega_{P2}(t_0)) \text{ and } \omega_{P2}(t_0+\tau)=\omega_{P2}(t_0)+\frac{1+k}{1+\frac{\mathbf{J}_{P2}}{\mathbf{J}_{P1}}}(\omega_{P1}(t_0)-\omega_{P2}(t_0)) \tag{3}$$

Obtained expressions (1), (2) and (3) are analogous to corresponding expressions in theory of collision between two bodies in translator motion pre- and post-collision kinetic states. It is possible to use Petrović's theory "Element of mathematical phenomenology" [13-15] to obtain same expressions (1), (2) and (3).

3. Methodology of investigation of vibro-impact system dynamics - An example

In Figure 5, phase trajectory branches (b*) in phase portraits, potential energy curves and total mechanical energy branches of two rolling disks for motion in interval between initial condition configuration and configurations of pre-first-collision and post-first-collision between two rolling disks (a*) with vibro-impact dynamics along a line are presented. Differential equations of two rolling disks along one staring line (trace) are in the form:

$$\ddot{x}_i+\omega_{0i}^2 x_i\left(\frac{\sqrt{b_{0i}^2+x_i^2}-l_{0i}}{\sqrt{b_{0i}^2+x_i^2}}\right)=0,\ i=1,2;\ \omega_{0i}^2=\frac{c_i r_i^2}{\mathbf{J}_{\mathbf{P}i}} \tag{4}$$

Precise equations of phase trajectory of previous conservative systems dynamics, described by two differential equations (4), are in the form:

$$\dot{x}_i^2=\dot{x}_{0i}^2-\omega_{0i}^2\left(x_i^2-x_{0i}^2+2l_{0i}\sqrt{\left(b_{0i}^2+x_i^2\right)}-2l_{0i}\sqrt{\left(b_{0i}^2+x_{0i}^2\right)}\right),\ i=1,2;\ \omega_{0i}^2=\frac{c_i r_i^2}{\mathbf{J}_{\mathbf{P}i}} \tag{5}$$

where known initial conditions, $x_{0i} = x_i(0)$ and $\dot{x}_{0i} = \dot{x}_i(0)$, $i = 1,2$, initial velocity and initial position of mass center of each of two rolling disks, or momentary initial angle coordinate and initial angular velocity around corresponding momentary axis for each of rolling disks:

$$\vartheta_{P0i} = \vartheta_{Pi}(0) = \frac{x_{0i}}{r_i} = \frac{x_i(0)}{r_i} \text{ and } \omega_{P0i} = \dot{\vartheta}_{P0i} = \dot{\vartheta}_{Pi}(0) = \frac{\dot{x}_{0i}}{r_i} = \frac{\dot{x}_i(0)}{r_i},\ i = 1,2\,.$$

Previous equations of phase trajectory of each rolling disks is, also, equation of curves of constant total mechanical energy of the system: $E_i = E_{k,i} + \mathbf{A}_{def,x,i} = E_{ki} + E_{pi} = E_{0i} = const_i$, $i = 1,2$.

Singular points are: **a*** only one singular stable center type point $x_{1,i} = 0$ for $l_{0i} \le b_{0i}$ with corresponding stable equilibrium position in which spring is: **a*1**. no stressed and in no deformed state with no extension, no compression for $l_{0i} = b_i$, $i = 1,2$, and spring is in natural state with deformation work equal to zero: $\mathbf{A}_{def,x,i=0} = E_{p,x,i=0} = 0$ and **a*2**. stressed and deformed by extension $\Delta l_{0i} = b_{0i} - l_{0i}$ for $l_{0i} < b_{0i}$, and deformation work in this state of each spring is in the form:

$$\mathbf{A}_{def,xi=0} = E_{p,xi=0} = \frac{1}{2}c_i\left(b_{0i} - l_{0i}\right)^2,\ i = 1,2\,.$$

b* Three singular points

$$x_{1i} = 0 \text{ and } x_{2/3,i} = \mp\sqrt{l_{0i}^2 - b_i^2} \text{ for } l_{0i} > b_i,\ i = 1,2$$

and present one trigger of three coupled three singular points, one singular no stable saddle type point $x_{1i} = 0$ for $l_{0i} > b_i$ and two singular stable center type points $x_{2/3,i} = \mp\sqrt{l_{0i}^2 - b_i^2}$ for $l_{0i} > b_i$.

Time of motion duration from initial position x_{0i} with initial velocity $\dot{x}_{0i}$ to the arbitrary position $x_i(t)$ in conservative system is in the form:

$$t_i = \int_{x_{0i}}^{x_i} \frac{dx_i}{\pm\omega_{0i}\sqrt{\frac{\dot{x}_{0i}^2}{\omega_{0i}^2} - \left(x_i^2 - x_{0i}^2 + 2l_{0i}\sqrt{\left(b_{0i}^2 + x_i^2\right)} - 2l_{0i}\sqrt{\left(b_{0i}^2 + x_0^2\right)}\right)}} \tag{6}$$

Taking into account that in position of collision exists the following relation: $x_{2,c1} = x_{1,c1} - 2\sqrt{r_1 r_2}$ for $r_2 < r_1$ as it is presented in Figure 5.a*, and after numerical determination of the coordinates $x_{1,c1}$ and $x_{2,c1} = x_{1,c1} - 2\sqrt{r_1 r_2}$ of balls' mass center position in state of collision, by using equations (6), and by use equations (5) of phase trajectory for corresponding ball in state of collision the angular velocities of rolling balls are determined and they present arrival angular velocity in pre-collision state in the following formula:

$$\omega_{Pi,c1.arrival} = \frac{\omega_{0i}}{r_i}\sqrt{\frac{\dot{x}_{0i}^2}{\omega_{0i}^2} + \left(x_{0i}^2 - x_{i,c1}^2 + 2l_{0i}\sqrt{b_{0i}^2 + x_{i,c1}^2} - 2l_{0i}\sqrt{b_{0i}^2 + x_{0i}^2}\right)},\quad i = 1,2 \tag{7}$$

Then by means of expressions (3) and (7) we obtain outgoing angular velocities of the both balls in the state post-collision, which are starting for next interval between first and second collision. In Figure 5.b* at phase trajectory branches representative points of first collision are presented for both rolling balls and also jumps of angular velocities and corresponding jumps of ball's energy.

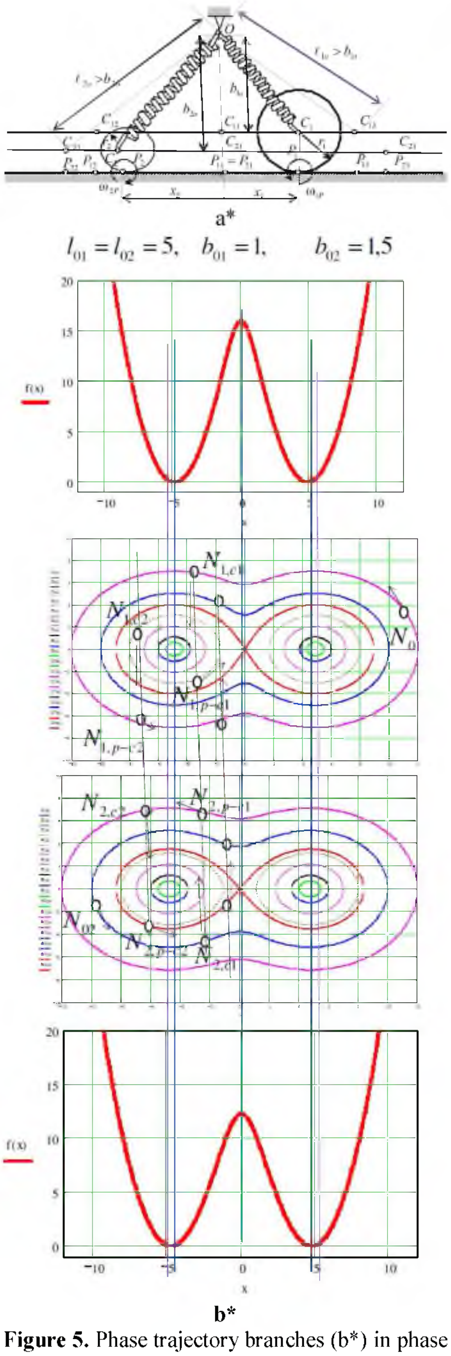


Figure 5. Phase trajectory branches (b*) in phase portraits, potential energy curves and total mechanical energy branches of two rolling disks for motion in interval between initial condition configuration and configurations of pre-first-collision and post-first-collision between two rolling disks (a*) with vibro-impact dynamics along a line.

Conclusions

We present necessary methods and models of collision of two rolling balls with new and original expressions of outgoing post-collision angular velocities. Also, by kinematical analysis we present traces of rolling balls pre- and post-collision kinetic state in relation to collision tangent plane between rolling balls through common contact point. By considered with vibro-impact dynamics in a model of real system, a methodology for investigation is founded by combination description of dynamics in phase planes and results of extended theory of collision of two rolling balls or disks in the system. From phase portraits for both rolling balls in conservative system it is possible to conclude that total mechanical energy for the case of ideal elastic collision is constant, but in collision appear jump energy between balls.

Acknowledgments

Parts of this research were supported by the Ministry of Sciences and Technology of Republic of Serbia through Mathematical Institute SANU Belgrade Grant ON174001 "Dynamics of hybrid systems with complex structures.", Mechanics of materials and Faculty of Mechanical Engineering University of Niš.

References

[1] Coriolis G. *Théorie mathématique des effets du jeu de billard; suivi des deux celebres memoires publiés en 1832 et 1835 dans le Journal de l'École Polytechnique: Sur le principe des forces vives dans les mouvements relatifs des machines & Sur les équations du mouvement relatif des systèmes de corps* (Or. publ. by Carilian-Goeury, 1835 ed.). Éditions Jacques Gabay; 1990.

[2] Hedrih (Stevanovic) K.R. Elements of Geometry, Kinematics and Dynamics of Billiards. *MECHATRONICS AND CONTROL, COLLOQUIUM IN HONOUR OF THE 65TH BIRTHDAY OF PROF. NENAD D. PAVLOVIĆ AND PROF. TOMISLAV PETROVIĆ, THE 3rd INTERNATIONAL CONFERE MECHANICAL ENGINEERING IN XXI CENTURY, Proc.* September 17 - 18, 2015, NIŠ, University of Niš, p. 313-318.

[3] Hedrih (Stevanovic) K.R. DYNAMICS OF THE ELLIPTIC BILLIARDS AND ROLLING BALL IMPACTS. *Proc. of the 5th International Congress of Serbian Society of Mechanics.* Arandjelovac, June 15-17, 2015: Published by Serbian Society of Mechanics and Faculty of Technical Sciences Novi Sad, p .98.

[4] Hedrih (Stevanovic) K.R. Rolling heavy disk along rotating circle with constant angular velocity. *Computer Algebra Systems, in Teaching and Research, Chapter 2. Problems of Classical Mechanics* (Eds. A.N. Prokopenya, M. Jakubiak), Siedlce University of Natural Sciences and Humanities, Siedlce, 2015, Vol.V, p. 293-304.

[5] Hedrih (Stevanovic) K.R. Vibro-impact dynamics of the rolling disks along rotate circle in vertical plane. *Dynamical Systems, Control and Stability* (Eds. J. Awrejcewicz, M. Kazmierczak, J. Mrozowski, P. Olejnik), 2015, Vol. 13/3, p. 251-262.

[6] Hedrih (Stevanovic) K.R. Dynamics of impacts and collisions of the rolling balls. *13th Conference on Dynamical Systems. Theory and Applications, DSTA, Abstracts,*(Eds. J. Awrejcewicz, M. Kaźmierczak, P. Olejnik, J. Mrozowski), Lódź, December 7-10, 2015, Lodz University of Technology, 2015, p. 149.

[7] Hedrih (Stevanovic) K.R. Elements of mathematical phenomenology: I. Mathematical and qualitative analogies, *Труды МАИ,* 2015, №84, p. 42 (1-42).

[8] Hedrih (Stevanović) K.R. Elements of mathematical phenomenology: II. Phenomenological approximate mappings. *Труды МАИ,* 2015, №84, p. 29 (1- 29).

[9] Hedrih (Stevanovic) K.R. ELEMENTS OF MATHEMATICAL PHENOMENOLOGY AND PHENOMENOLOGICAL MAPPINGS: UNIVERSAL METHOD ACROS DIFFERENT AREA OF SCIENCES AND TRANSIENT OF KNOWLEDGE FLOWS. Kyiv, February, 20, 2016, XIII Academic Reading in memory of V.I. Striha, Academy of Science of Higher School of Ukraine, 2016.

[10] Hedrih (Stevanovic) K.R. VIBRO-IMPACT DYNAMICS IN SYSTEMS WITH TRIGGER OF COUPED THREE SINGULAR POINTS: COLLISION OF TWO ROLLING BODIES (is accepted). IUTAM ICRAM, Montreal, 2016.

[11] Elements of mathematical phenomenology and phenomenological mapping in non-linear dynamics. *Special Issue of International Journal of Non-Linear Mechanics* (Eds. K.R. (Stevanovic) Hedrih, I. Kosenko, P. Krasilnikov and P.D. Spanos), 2015, Vol.73, p. 1-128.

[12] Kozlov V.V., Treschev D.V. Billiards. *Genetic introduction into dynamics of systems with impacts.* Moscow: Moscow State Univ. Publishing; 1991 (in Russian).

[13] Petrović, *M. Elementi matematičke fenomenologije (Elements of mathematical phenomenology)*, Beograd: Srpska Kraljevska Akademija, 1911.

[14] Petrović M. *Fenomenološko preslikavanje (Phenomenological mapping)*, Beograd: Srpska Kraljevska Akademija; 1933 (In Serbian).

[15] Petrović M. *Mecanismes communs aux phenomenes disparates*. Paris; 1921.

[16] Rašković D. *Mehanika III-Dinamika (Mechanics III-Dynamics.* Beograd: Naučna Knjiga; 1965.