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Inverse Bifurcation Problem as a Tool For Rapid Identification of Progressive Collapse for Thin-Walled Systems

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Abstract
Notwithstanding recent advances in robust design, the problem of vulnerability of structures is still open. On the one hand, this leads to various structure collapses; on the other hand, this prompts researchers to develop models and methods to identify a state of progressive collapse and estimate lifetime and residual functionality of perturbed structure. An inverse bifurcation problem implies that one identifies a pre-bifurcation state of a perturbed thin-walled system. The topological precursor (a tool to solve an inverse bifurcation problem) used is based on typical sequences of deformed states extracted from clustered post-critical solutions of non-linear boundary problem of thin-walled systems theory. It implies that complete bifurcation structure of the non-linear boundary problem (including primary, secondary and tertiary bifurcation paths) are constructed The proposed approach was employed to identify a pre-bifurcation state of a cylindrical shell under uniform pressure (close to the critical) subjected to a pulse impact.

Keywords
Direct and inverse bifurcation problems, non-linear boundary problems, non-linear partial differential equations, von Karman equation

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Introduction

For anti-missile defence systems as well as for air and space ones, accident prevention calls for to study system's vulnerability that is its inability to function properly under progressive collapse in emergency situation. The main focus here is to estimate residual lifetime and identify a post-critical shape for a time less than that of progressive collapse. This rules out extensive and prolonged calculation with the employment of CAD\CAE\CAM systems altogether and poses the problem to assess lifetime rapidly using automated monitoring system.

Recent advances [4, 9] in the field (see also reviews [2, 10]) usually deal with methods that investigate system's reliability in the design stage and do not take into account real state of thin-walled structure in emergency situation. This one can be rather complex [12] due to essential non-linearity of thin-walled systems. A number of papers indicate conclusively that finite elements method, conventional for the design stage, is computationally prohibitive for rapid assessment and simultaneously propose various simplified approaches to assess vulnerability (say, the method of ideal structural elements [15]). The paper [10] analyzes progressive collapse for systems made up of beams with box-like sections in order to reveal its starting mechanisms. It is of fundamental importance (especially for thin-walled systems) to trace a route from the initial stage of the progressive collapse to various variants shell losses its stability; by way of illustration, one may consider a paper [14], which discusses buckling shapes of axially-compressed thin-walled systems of square section.

To summarize briefly, most papers focuses on progressive collapse and rapid assessment of stress-strain state in the design stage for beams and thin plates.

The present paper proposes an approach to assess (in real time) thin-walled shells vulnerability under emergency impact; the method involves identification for multi-dimensional time series associated with dynamics of loaded thin-walled system subjected to a pulse impact. The necessary precondition for employing the approach in question successfully is information about bifurcation paths of the respective static non-linear boundary problem.

1. Problem statement

The paper considers thin-walled cylindrical shell (defined on the domain $\Omega = \{0 \le x_1 \le L, 0 \le x_2 \le 2\pi R\}$) under stationary external pressure λ subjected to localized pulse impact $Af(t,\tau)$ at $t = t_0$. Hereinafter, R, L, h are radius, length, and thickness of shell, respectively; A is a pulse amplitude; τ determines pulse effective time. The external load λ is related to the estimate of critical load obtained in the frameworks of linear shell theory for stationary uniform external load.

To formulate the problem, we assume the following:

1. Emergency situation is a single one.

2. The time required to complete prognostic calculations t_p should be significantly less than that of calculations using CAD\CAE\CAM system simulation t_s, $t_p << t_s$, and less than that of progressive collapse process t^*, $t_p < t^*$.

3. Vulnerability criterion involves observation of post-critical shape with preset value of maximum displacement.

It is possible to tackle the problem considered along two different lines of attack.

1. Recorded data are used to solve inverse problem of mechanics of deformed solid body and thereby identify perturbing impact. In turn, it makes possible to solve direct problem and determine actual stress-strain state.

2. Alternatively, typical sequences of deformed shape are utilized to identify a pre-bifurcation state; the typical sequences are results of clustering of deformed shape sequences observed on bifurcation paths of static non-linear boundary problem.

The first variant implies that the time required to perform prognosis is dictated by the time required to solve repeatedly numerous direct problems of mechanics of deformed solid body that violates the condition $t_p < t^*$. The second one (based upon preliminary clustering of available data) ensures that this condition is satisfied, but leads to less accurate approximation of the solution.

2. Vulnerability model for thin-walled system

We define vulnerability model as a set of mappings $\Im = \{F : D \to Y\}$, where D is a time series that represent deformed shapes over a period $0 < t < t^*$ and Y is the final state of the system to be ascertained.

A series $D_s = \{u_0^{(s)}, u_1^{(s)}, ..., u_{t_s}^{(s)}\}$, $s = \overline{1,S}$, describes behaviour of thin-walled system after a pulse impact. It comprises states $u_t^{(s)} = \{u_{kt}^{(s)}\}$ (observed at equally spaced moments of time $t = \overline{0,t_s}$), which are described by a vector of displacements at fixed set of points $k = \overline{1,K}$ on the shell surface.

Such model suggests that system is monitored using a set of displacement sensors and then the observations are normalized to their maximum value. The final state Y may be close to the initial one (it means that the system functions properly) or may differ from it significantly.

By way of illustration, Fig. 1a displays typical behaviour of time series; $\lambda = 0.92, A = 50.0\ N, \tau = 0.002\ sec., t_0 = 0.0025\ sec.$, $f(t,\tau)$ is a step function. The plot represents normal displacement w/h at the point of the central cross-section ($x_1 = L/2$) opposite to that of pulse application. Fig. 1b shows the section of the series for $0 \le t \le 1.5 * 10^{-2}$, Fig. 1c depicts deformed shapes corresponding to the oscillations. It is quite obvious that the process is chaotic (cf. (4)), and the shell therewith oscillates elastically in the vicinity of the initial state; oscillations amplitude amounts to a single thickness of the shell.

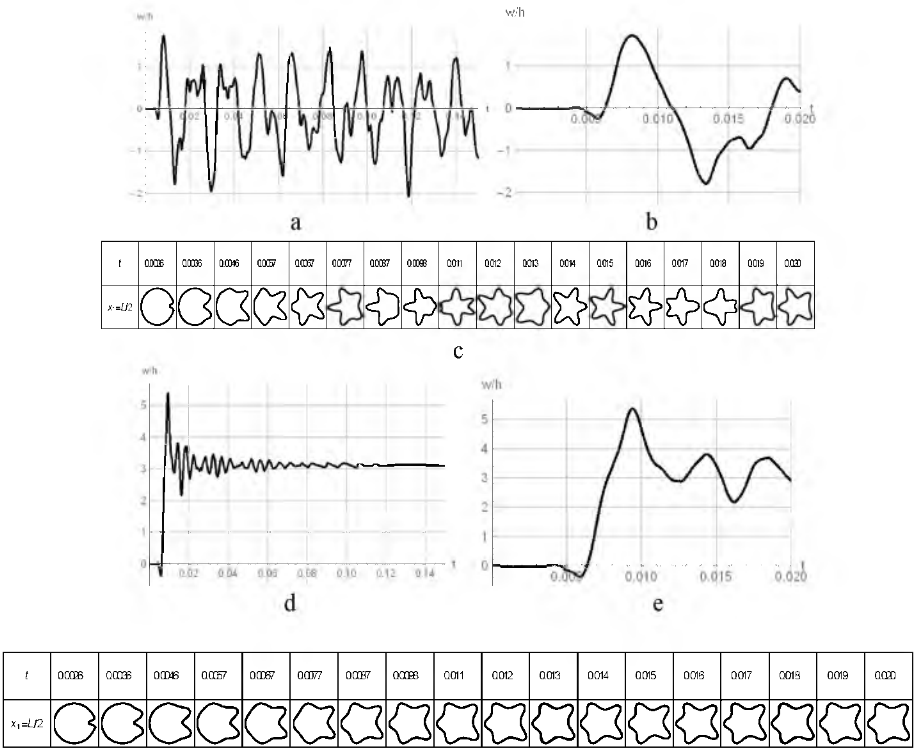


Figure 1. Typical variants of perturbed shell dynamics along with the respective central cross-sections corresponding to chaotic behaviour (a, b, c) and to loss of stability (d, e, f)

Fig. 1d displays another typical variant for shell after pulse action; these results correspond to nearly the same set of values ($\lambda = 0.92$, $A = 100.0\ N$, $\tau = 0.002$ sec., $t_0 = 0.0025$ sec.), but, after some oscillations, the system transits to post-critical state (the beginning is about $t \approx 1.5 * 10^{-2}$); Fig. 1e shows the initial section of the series; Fig. 1f depicts the respective deformed shapes sequence. The characteristic displacement for this case comes to three thicknesses. If one compares the series and the respective shape sequences, one can infer that these sequences are substantially dissimilar, although separate shapes from different sequences may be quite the same. This fact is used to construct an algorithm to solve inverse bifurcation problem for theory of thin-walled systems.

3. Method to construct typical deformed shape sequences

First of all, a wide-ranging finite-element simulation was carried out using COSMOSM 2.6 to construct time series corresponding to various values of external pressure λ, of a pulse amplitude A, and of its effective time τ. The time series under consideration appeared to be complex and diverse to such an extent that it was impossible to apply to them conventional time series forecasting algorithms (6), which imply a single prediction model and identification of its parameters. This made necessary to employ predictive clustering algorithms [5]. In the frameworks of this paradigm, the problem to be solved is classified as a pattern discovery problem that is the one to extract typical sequences from a set of time series and then identify given dynamics using the sequences. Recent review [1] provides ones with possible formulations of the problem. Unfortunately, owing to the aforesaid series diversity, it is a quite tricky thing to apply such algorithms to them straightforward. It leads to immense number of clusters of doubtful ability. To overcome the problem, we, first of all, ascertained that the shape sequence corresponding to bifurcation paths of static problem (traced with the employment of the iterative decomposition method) [12] correlated well with those of dynamics finite element simulation.

It allowed us to restrict ourselves to clustering of shape sequences for bifurcation paths of static non-linear boundary problem.

To put it mathematically, one considers sections of “similar” K-dimensional time series $u_0^{(s)}, u_1^{(s)}, \ldots, u_{t_s}^{(s)}$, $u_t^{(s)} = \{u_{kt}^{(s)}\}, t = \overline{1, t_s}, k = \overline{1, K}$, $s = \overline{1, S}$, where S is the total number of series, t_s is the number of observations for s-th series, and K is the number of points at which the displacements are recorded. The problem is to assign given sequence of K observations $\tilde{u}_1, \ldots, \tilde{u}_P$ to a certain cluster corresponding to a typical dynamics pattern or to ascertain that it is impossible. In the latter case, the observed dynamics is considered not to lead to loss of stability.

Interestingly, sequences comprised of successive observations have proven less efficient than those comprised of non-successive ones according to a predefined pattern. Here, a pattern is a fixed sequence of distances between positions of observations that are to be neighbours in a vector to be comprised.

Each algorithm belonging to this class consists of two subalgorithms. The first one analyzes time series in order to cluster shape sequences comprised in compliance with a pattern from a set of predefined patterns [7]. The second subalgorithm implies dynamics identification using obtained typical sequences. The clustering subalgorithm is applied separately to different samples constructed in compliance with all possible patterns; modified Wishart [11] one is used to cluster the samples.

The centres of the clusters (that is typical sequences) constructed for all patterns are used to identify dynamics. Namely, starting from the first observation, the subalgorithm, for current position, constructs vectors from previous observation in accordance with each possible pattern in such a way that the last position of a pattern coincides with the current position. Then it calculates the Euclidean distances between the sequences constructed from observations and the centres of clusters and seeks for the minimum distance. If this distance appears to be less than the threshold value ρ, then dynamics is considered to have been identified and led to the respective bifurcation path. Otherwise, the dynamics is considered non-identifiable and that that occurs in the vicinity of pre-critical equilibrium path.

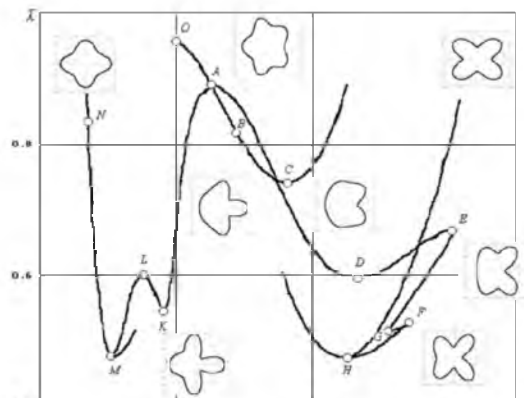


Figure 2. Typical structure of bifurcation paths of non-linear boundary problem for cylindrical shell under uniform external pressure

4. Typical variants of behaviour

It should be emphasized that typical shape sequences associated with bifurcation paths are of interest as they represent possible scenarios of loss of stability and thereby are the basis for vulnerability analysis. Fig. 2 exhibits typical structure of bifurcation paths [12]; central-cross-sections of deformed shapes are placed near the sections of bifurcation path to which the shapes correspond.

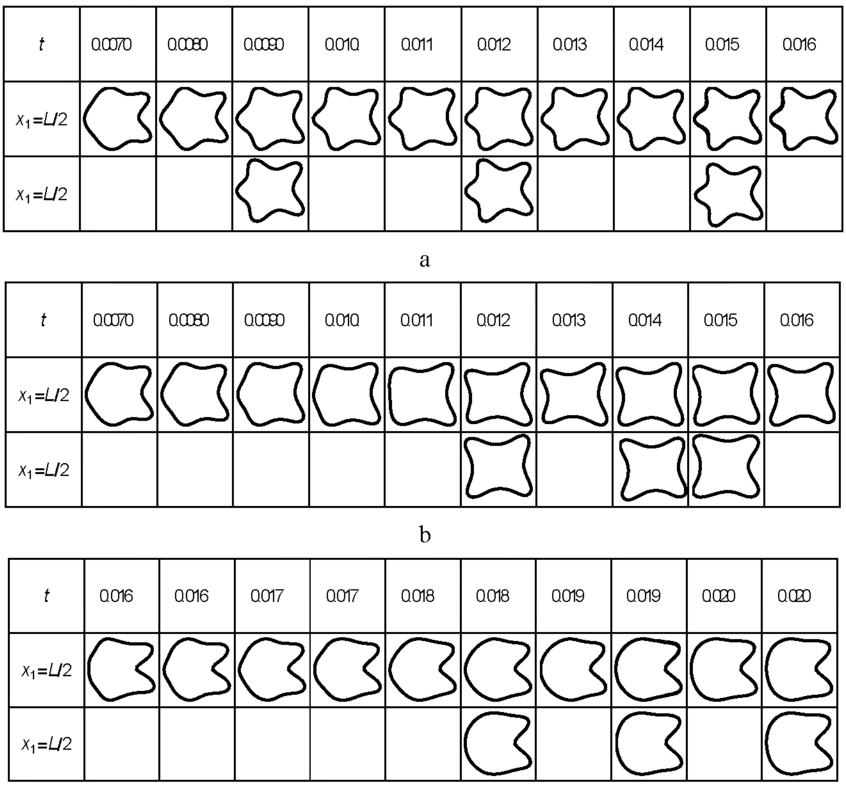
The clustering allows revealing that the number of clusters associated with a single bifurcation path is limited, and as far as the number of bifurcation paths situated lower than value of parameter load corresponding to the first bifurcation point at the pre-critical equilibrium path is rather small, the total number of clusters are limited too.

Typical sequences associated with dynamics of perturbed shell can be separated (in accordance with their final states) into the following groups:

1. Return of the system to the initial state.
2. Deformed shape similar to that of statics loss of stability.
3. Deformed shape similar to that of dynamics loss of stability.
4. Local deformed shape.

Each variant correlates with certain intervals of both static and pulse loadings. Each final shape is associated with large displacements and the time necessary to develop such displacements amounts to $t \sim 10^{-1}$, while the onset of loss of stability correlates with $t \sim 10^{-2}$. These interval, $10^{-2} < t < 10^{-1}$, determines time available to make decision.

Fig. 3 portrays a number of shape sequences (preceding to loss of stability) obtained with the employment of dynamics finite element calculation and the respective typical sequences obtained with the employment of clustering of post-critical shapes.



c

Figure 3. Shapes of finite element simulation before loss of stability (the middle row) and shape patterns (the lowest row)

Particularly, the typical sequence presented in Fig. 3a (pattern consists of three points) corresponds to the beginning of loss of stability with regular shapes with five dents (the path $O-A-B-C$ in Fig. 2), Fig. 3b corresponds to loss of stability with regular shapes with four dents (the branch with lower limit point H), and, finally, Fig. 3b corresponds to loss of stability with localized shape with a single dent (the section $A-D$ of the branch $A-D-E-G-F-H$).

Conclusions

1. It is possible to utilize typical sequences of deformed shape (extracted from clustered post-critical solutions of non-linear boundary problem) to identify and assess pre-bifurcation states.
2. A necessary precondition to generate samples for this clustering is to trace all bifurcation paths of non-linear boundary problem for thin-walled systems theory that is in essence a forward problem of bifurcation analysis.
3. A possible application of the proposed method of pre-bifurcation state identification is to assess progressive collapse of monitored real-world thin-walled system under accidental exposure.

References

[1] Aghabozorgi S., Shirkhorshidi A.S. and Wah T.Y. Time-series clustering – A decade review. *Information Systems*, 2015, Vol. 53, p. 16–38.

[2] Allen J.K., Seepersad C., Choi H.J., Mistree F. Robust design for multiscale and multidisciplinary designs. *J. Mech. Design*, 2006, Vol. 128(4), p. 832–46.

[3] Awrejcewicz J. *Chaos in Structural Mechanics*. N.-Y.: Springer; 2008.

[4] Baker J.W., Schubert M., Faber M.H. On the assessment of robustness. *Structural Safety*, 2008, Vol. 30(3), p. 253–267.

[5] Blockeel H., de Raedt L., Ramon J. Top-down induction of clustering trees. In: *Proceedings of 15th International Conference on Machine Learning*, 1998, Vol. 15, p. 55–63.

[6] Fan J., Yao Q. *Nonlinear time series. Nonparametric and Parametric Methods*. N.-Y.: Springer; 2003.

[7] Gromov V.A., Borisenko E.A. Chaotic time series prediction and clustering methods. *Neural Computing and Applications*, 2015, Vol. 26(8), p. 1827–38.

[8] Gromov V.A., Shulga A.N. Chaotic time series prediction with employment of ant colony optimization. *Expert Systems with Applications*, 2012, Vol. 39(9), p. 8474–78.

[9] Lee M.C.W., Mikulik Z., Kelly D.W., Thomson R.S., Degenhardt R. Robust design – a concept for imperfection insensitive composite structures. *Composite Structures*, 2010, Vol. 92, p. 1469–77.

[10] Kitarovic S., Zanic V. Approximate approach to progressive collapse analysis of the monotonous thin-walled structures in vertical bending. *Marine Structures*, 2014, Vol. 39, p. 255–86.

[11] Lapko A.V., Chentsov S.V. *Non-parametric system of information proceeding*. Novosibirsk: Nauka; 2000. (in Russian)

[12] Obodan N.I., Lebedeyev O.G., Gromov V.A. *Nonlinear behaviour and stability of thin-walled shells*. N.-Y.: Springer; 2013.

[13] Park G.-J., Lee T.-H., Lee K.H., Hwang K.-H. Robust design: an overview. *AIAA J.*, 2006, Vol. 44(1), p. 181–91.

[14] Song J., Zhou Y., Guo F. A relationship between progressive collapse and initial buckling for tubular structures under axial loading. *Int. J. of Mech. Sc.*, 2013, Vol. 75, p. 200–11.

[15] Underwood J.M., Sobey A.J., Blake J.I.R., Shenoi R.A. Ultimate collapse strength assessment of damaged steel-plated structures. *Engineering Structures*, 2012, Vol. 38, p. 1–10.