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Forecasting Bifurcation of Parametrically Excited Systems: Theory & Experiments

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Abstract
A system is parametrically excited when one or some of its coefficients vary with time. Parametric excitation can be observed in various engineered and physical systems. Many systems subject to parametric excitation exhibit critical transitions from one state to another as one or several of the system parameters change. Such critical transitions can either be caused by a change in the topological structure of the unforced system, or by synchronization between a natural mode of the system and the parameter variation. Forecasting bifurcations of parametrically excited systems before they occur is an active area of research both for engineered and natural systems. In particular, anticipating the distance to critical transitions, and predicting the state of the system after such transitions, remains a challenge, especially when there is an explicit time input to the system. In this work, a new model-less method is presented to address these problems based on monitoring transient recoveries from large perturbations in the pre-bifurcation regime. Recoveries are studied in a Poincaré section to address the challenge caused by explicit time input. Numerical simulations and experimental results are provided to demonstrate the proposed method. In numerical simulation, a parametrically excited logistic equation and a parametrically excited Duffing oscillator are used to generate simulation data. These two types of systems show that the method can predict transitions induced by either bifurcation of the unforced system, or by parametric resonance. We further examine the robustness of the method to measurement and process noise by collecting recovery data from an electrical circuit system which exhibits parametric resonance as one of its parameters varies.

Keywords
Parametric excitation, Bifurcation, Forecasting

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A system is parametrically excited when one or some of its coefficients vary with time. Parametric excitation can be observed in various engineered and physical systems such as elastic cables [1], rotating machines [2], electrostatic waves in a plasma with radiation [3] and etc. [4]. Many natural systems are also subject to the variation of parameters due to environmental fluctuations and human activities [5-6].

Many systems subject to parametric excitation exhibit critical transitions from one state to another as one or several of the system parameters change. Such critical transitions can either be caused by a change in the topological structure of the unforced system, or by a synchronization between a natural mode of the system and the parameter variation. Researchers across scientific domains have sought information signaling the imminent critical transition of complex systems. “Tipping points” (i.e., bifurcations) in ecosystems, physiologic regulatory networks, financial markets, are all theoretically foreshadowed by characteristic spatial and temporal patterns [7-9]. Critical slowing down, a dynamical phenomenon in which the system becomes less resilient to perturbations as it approaches the bifurcation [10], is one of the warning signals studied. Past research, however, does not include a quantitative measure of the early warning pattern because of their difficulty in quantitatively detecting the distance to such transitions. That significantly limits their application.

New model-less methods have been developed at UM [11] to anticipate the distance to bifurcation of autonomous systems, and the state of the system after bifurcation is reached [11] by studying the rate of change of amplitude as parameter changes. This work has shown that critical slowing down can be observed at both small and large amplitudes, which makes it possible to predict not only the tipping point, but also the bifurcation diagram when large perturbations to the system exist in the pre-bifurcation regime.

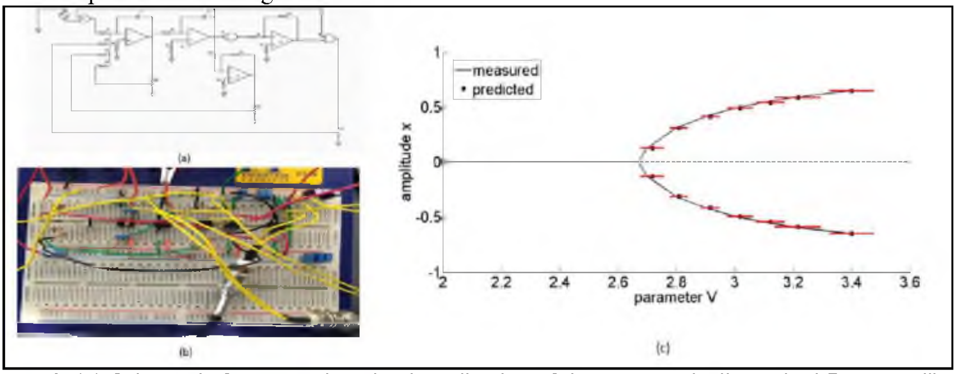

Figure 1. (a) Schematic for an analog circuit realization of the parametrically excited Dung oscillator. (b) The analog circuit as constructed in the laboratory. (c) Prediction results for the constructed Duffing Oscillator. Black solid lines show the measured Hopf bifurcation diagram. Black bullets and red solid lines show the mean prediction results and standard deviation.

In this research we have developed a new, complementary method to predict bifurcation of 1D and 2D, systems with parametric excitation. For the 1D case we focus on systems exhibiting transcritical bifurcations, and in the 2D case we focus on systems exhibiting Hopf bifurcations. In our method, dynamics of the system are studied in a Poincare section, thus eliminating the effects of explicit time inputs. Bifurcation diagrams with Hopf bifurcation are harder to predict because the post bifurcation regime is a limit cycle where the response depends on the phase. To demonstrate the newly-developed forecasting methods, computational and experimental research has been performed. A parametrically excited Duffing oscillator was realized in an analog circuit (Fig. 1b). Its bifurcation diagram was measured experimentally and also predicted using only recoveries in the pre-bifurcation region (Fig. 1 c). A model for the circuit is not needed and not used in the forecasts.

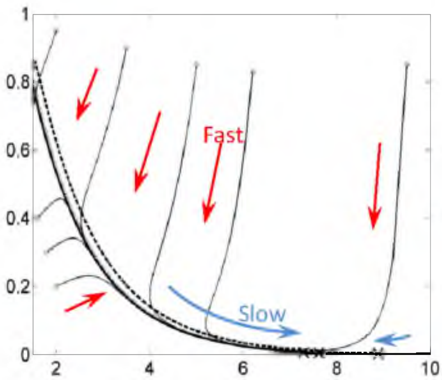

Figure 2. Dynamics in other manifolds decay quicker than in inertial manifold. As parameter changes, inertial manifold changes from solid line to dash line.

The inertial manifold (invariant set in which the dynamics is slowest in time) is used to extract information related to the critical slowing down. When perturbations act on the system, the dynamics in other manifolds decay quickly compared to the dynamics in the inertial manifold. This phenomenon is more pronounced when the system is close to a tipping point. Inertial manifolds have been identified for many systems, including ones we studied computationally and experimentally

(Fig.2) and they exist in much more complex systems also. In our research, we assume that this manifold varies smoothly and slowly with the parameter.

The proposed next steps for this research are to forecast bifurcations of more complex epidemic systems. Methods will be further developed to first identify and measure the dynamics in this manifold and then forecast the shift, creation or disappearance of fixed points, thus predicting possible bifurcation as parameter changes.

The importance of the two methods is twofold: (a) both the distance to bifurcation and the type of bifurcation can be predicted without passing the bifurcation point, which is ideal for early warning and strategy making; (b) our methods are model-less and that shortens the time between data acquisition and decision making while they work for cases where models are hard to build and identify/calibrate.

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