Proceedings of the 5th International Conference on Nonlinear Dynamics
ND-KhPI2016
September 27-30, 2016, Kharkov, Ukraine

R-Functions and WA-Systems of Functions in Modern Information Technologies

Victor F. Kravchenko[1,2,3], Oleg V. Kravchenko[1,2,3], Yaroslav Yu. Konovalov[3], Vladislav I. Pustovoit[2], and Dmitry V. Churikov[1,2,4]

Abstract
The review report consists of five parts. It describes the main physical applications of atomic, WA-systems and R–functions.

Keywords
atomic functions, R-functions, WA-systems of function, wavelets, DSP

[1]Kotel'nikov Institute of Radio Engineering and Electronics, Russian Academy of Sciences
[2]Scientific and Technological Center of Unique Instrumentation, Russian Academy of Sciences
[3]Bauman Moscow State Technical University
[4]Moscow Institute of Physics and Technology (State University)

* **Corresponding author:** kvf-ok@mail.ru, olekravchenko@gmail.com, mpio_nice@mail.ru

Introduction

More than 50 years theory of R- and atomic functions (AF) and also WA-systems of functions are developed in wide range of physical applications [1-22]. Modern development of the theory of AF in combination with R-function (V.L. Rvachev functions) is produced in physics and technology for a wide range of frequencies from microwaves to optics.

Basic applications of atomic, WA-systems and R-functions are as follows:

1. Generalized Kotelnikov series based on AF $\mathrm{h}_a(x)$ and $\mathrm{fup}_N(x)$.
2. Generalized *N*-dimensional Whittaker-Kotelnikov-Shannon (WKS) theorem based on AF.
3. Levitan and the Strang-Fix polynomials on the basis of the AF.
4. Kravchenko-Rvachev (KR) WA-systems of functions and their application for the detection of alternating short-term and ultra-wideband (UWB) physical processes.
5. UWB signals spectral processing based on AF.
6. Analytic Kravchenko-Kotelnikov (KK) wavelets in digital signal processing (DSP).
7. Kravchenko-Pustovoit (KP) temporal weight functions in DSP devices on surface acoustic waves.
8. KK weight functions in digital signals spectroscopy.
9. Space-time distribution of Cohen class with AF in a nonlinear DSP.
10. AF and atomic-fractal functions in antenna theory.
11. Study of the field pulse of an electric dipole behavior.
12. Digital signal processing in the synthetic aperture radar.
13. Orthogonal wavelet bases in digital signal and image processing.
14. Atomic function in the theory of probability and stochastic processes.
15. Synthesis of two-dimensional digital filter with complex shape geometry.
16. Multidimensional filtering.
17. Nonparametric signal estimation.
18. Constructing of Kravchenko-Kotelnikov -Gauss (KKG) and Kravchenko-Kotelnikov -Levitan-Gauss (KKLG) weight functions.

19. New class of wavelets on the basis of $\mathrm{h}_a(x)$.
20. New class of analytic KR wavelets in problems of UWB signals and processes.
21. Digital processing and spectral estimation of UWB signals using AF and wavelets.
22. KR weight functions in problems of radar images construction and antenna aperture synthesizing.
23. Atomic function in the theory of probability and stochastic processes.
24. Atomic functions in problems of physical electronics.
25. Application of the theory of R-functions and WA-function systems to solving boundary value problems of mathematical physics.

1. Generalization of Sampling Theorem on Basis of Atomic Functions

Generalization of WKS sampling theorem on basis of atomic functions, application of atomic functions in probability theory and stochastic processes, interpolation of stationary probability process by atomic functions, new class of probability weight functions in the digital signal and image processing are considered.

It is known that in communications technology in the transmission of various signals usually deal with functions of time the range of which is limited (in the spectrum does not contain frequencies above a certain boundary). Such functions have unique properties which was first established in 1933 by V.A. Kotelnikov. They expressed it in the theorem [19] which plays a fundamental role in the theory of communication and informatics as well as in various other physical applications. For interpolation signals with finite spectrum can also be used Fourier transform (FT) AF [1-7].This is due to the fact that the zeros of these transformations are arranged in a regular manner. In addition, the spectra of AF tend to zero at infinity much faster function sinc(x) that allows you to restrict the relatively small number of members of the interpolation series. We consider a generalization of the KK sampling theorem as a one-dimensional, and for the N-dimensional cases, as well as on the basis of Fourier transforms atomic functions $\mathrm{fup}_n(\omega)$.

1.1. The Analysis of the Physical Meaning of the Generalized KK Series

As is well known [1,6,19] WKS theorem can transmit continuous signals with limited range using pulse methods. Such a transmission system comprises the following main components:

- limiting a band pass filter with a cutoff frequency Ω,
- sampler with pitch x_Δ,
- communication link,
- linear output filter, restoring the signal from the discrete samples sequence.

For accurate reproduction of the transmitted signals the filter should make an infinite delay time. In the formation of the signal values at a point between samples involved the entire set of WKS or KK kernels. Consequently in order to restore the value of the signal at a point between samples. It is necessary to take samples of the signal at infinite time interval. The further away from the observation point count, as well as decreasing faster than interpolation kernel, the filter makes a smaller contribution to the formation of the signal in it. Therefore it is always possible to determine the number of samples in the neighborhood of the observation point at which the signal is restored to the required accuracy.

The application of AF [19] to the probability theory and the problems of radio physics and its applications as well as questions relate to the probabilistic sense of the “mother” AF $\mathrm{up}(x)$ and $\mathrm{h}_a(x)$ family of functions. KR distributions are introduced. The basic numerical characteristics of KR distribution (moments, asymmetry, excess, entropy) are considered. Next construction of N-dimensional probability density for independent random variables is discussed. Substantiates the use of the cumulant analysis in the study of the distribution KR. Abstract narrowband random processes on the basis of the KR distribution. The application of AF to spectral analysis of random processes, namely atomic distribution KR and its moments, asymmetry, excess and the random variable entropy having KR probability density function, N-dimensional probability density, statistical independence and mixed expressions for probability methods of digital simulation of random variables with KR distribution.

1.2. The Interpolation of Stationary Random Processes Atomic Functions

Problems of sampling and interpolation of continuous signals are important in the theory of random processes [19].When selecting step sampling error estimation problems arise replace discrete continuous processes and optimization interpolating filter by the criterion of minimum mean square error (RMSE) of interpolation characteristics. In [19] discussed the possibilities for the use of interpolation methods of random processes on the basis of atomic functions (AF) and their Fourier transforms (PF). The aim is to study the methods of interpolation of stationary random processes (MSP) based on the AF as well as their comparison with the difficult to implement in practice the best method for the criterion of the minimum MSE interpolation. The first stage deals with the general relations for the correlation function of the energy spectrum and the dispersion of arbitrary SSP interpolation error and are determined by the parameters of the optimal reconstruction filter by the criterion of minimum MSE interpolation. In a second stage various interpolation schemes are analyzed for specific random process (step linear and sinc- interpolation) versus optimal. Special attention is given to studying the SSP interpolation method based on the theory of AF [1-6]. The analysis of general expressions for the dispersion of stationary random processes of interpolation errors over samples is given. Interpolation of stationary random processes atomic functions, n-th class probability weighting functions in the digital signal and image processing is considered.

2. Kravchenko WA-Systems of Functions

This part is devoted to the application of Kravchenko WA-systems of functions to various physical applications. Through to local properties both in time and frequency areas, orthogonally, zero moments and the multiresolution analysis (MRA) the computing algorithms on the basis of wavelets have essential advantages over the Fourier transform. Therefore great scientific and practical interest represents construction of new classes of orthogonal and also analytical WA-systems of Kravchenko functions on the basis of the atomic functions. Efficiency of this approach is shown on the concrete physical examples relating to digital signal processing (DSP). Thus, the proposed design is reasonable and wavelets Kravchenko has all the properties of MRA. To move it to other families of AF present scheme of constructing a universal algorithm. We considered in detail and algorithm is constructing orthogonal Kravchenko WA-function systems on the basis of atomic functions. Proposed orthogonal Kravchenko WA-system functions which have fractal properties. Orthogonal WA-system functions Kravchenko built on the basis of the AF have a smooth Fourier transform. This allows for a better time localization of wavelets in comparison with Kotelnikov-Shannon. Their construction by means of conjugate mirror filters. It is shown that using the properties of AF is possible to simplify the output scaling function $\hat{m}_0(\omega)$. It is known that wavelet basis functions have ample opportunities in digital data processing, and the problems of physical modeling. However the most widely received wavelet -analysis in digital signal processing (compression, encryption, filtering, contour analysis, recognition, detection, etc.). Consider the use of orthogonal Kravchenko wavelets for filtering and compression of one-dimensional and multidimensional signals. The analytical WA-system functions Kravchenko and offered quality functional selection of wavelet basis for the analysis of signals. To improve the quality of the physical signal analysis should choose the most appropriate system of wavelets. The wavelet function must have characteristics consistent with the analyzed signal. Quality functional write as

$$J(\psi, y) = \sum_{k=0}^{4} \left| \frac{\gamma_k^{\psi} - \gamma_k^{y}}{\gamma_k^{y}} \right|^2,$$

where $\psi(x)$ is wavelet function, $y(x)$ is signal, γ_k^{ψ} and γ_k^{y} are physical characteristics, $\gamma_0 = \mu$. The optimal choice of the base corresponds to the minimum value of the functional. For best results of the wavelet function and the signal must be close to each other broadband indicators ($\mu^{\psi} \approx \mu^{y}$). Analytic Kravchenko wavelets can be applied to the analysis of time series of different physical nature [6,20]. Examples of such problems include the study of celestial bodies. Here, we consider the construction and application of Kravchenko-Kaiser functions to the problems of weight averaging of difference frequency. Application of R-functions (V.L. Rvachev functions) [1-4,20] is based on the technique of description of complex domains and new relations to construct two-dimensional analytic wavelets (2DWA) KR.

3. New Construction of Weight Functions on Basis of Atomic Functions

This part is devoted to the utilization of families of atomic functions to the construction of weight functions on the basis of the convolutions. This approach can significantly reduce the level of side lobes of weight functions in problems of nonparametric estimation of probability density and also in systems of phase synchronization with the samples.

It is known [1-6, 21] that the weight functions built on AF have been widely used in digital signal processing (DSP), image analysis, synthesis of antennas, and also in numerical solution of boundary value problems of mathematical physics, imaging, astronomy, statistical data processing methods, etc. During the numerical solution of such problems [1-6, 21] the attention is paid on the following physical characteristics: the maximum side lobe level, the coherent gain, the maximum conversion loss, the width of the window at 3 and 6 dB. It is required to have a good side lobe level, high coherent gain and low conversion loss. The proposed approach makes it possible to reach a compromise between these requirements. It should be noted that the convolution of AF are also AF themselves [21] and it can be found as the solution of the corresponding functional differential equation.

Phase synchronization systems (SPS) [21] are widely used in problems of generation of high-precision oscillation (frequency synthesizer), optimum reception and coherent processing of discrete and continuous signals, the organization of parallel calculations in multiprocessing (cluster) systems. The exact mathematical model of the SPS samples with a difference equation. Most methods of research with samples of the SPS built on differential equations is not based on rigorous methods for digital mapping theory [21] and sometimes lost some of the physical effects observed in SPS. It should be a detailed study of the dynamics of the SPS works with samples on the basis of methods of the theory of discrete mappings. An important feature inherent in the SPS with samples is the presence of a large number of synchronous modes zones fractional frequency ratio. It use for the synthesis task frequency is limited by the fact that within the physical zone synchronism fractional multiplicities range of the oscillations generated by parasitic components contaminated due to the fact that the input signal samples are placed unevenly on the time axis. When approaching the boundaries of natural areas synchronism spectrum signals generated is destroyed. One method of combating "pollution spectrum" is to use different forms of input signals. At present widely used for applications of ultra-wideband radar digital signal and image processing] received theory of AF [1-6]. However the literature is almost no discussion of signal generation based on AF. The work pays special attention with regard to the problem of obtaining the oscillation spectrum in a pure SPS with samples. It is shown that there is a continuous SPS only one physical area of the synchronous mode band limited synchronism [21]. A study of dynamic processes a standard method for studying discrete mappings diagrams Lamerey-Koenigs. The study of dynamic processes with SPS samples reveals a large number of physical synchronous zone modes - zones dividing and frequency multiplication. It is shown that the usage of such signals is expanding the physical area of synchronous mode fractional multiplicities at low frequency regulation factor.

4. Atomic and R- Functions in Boundary-Value Problems of Mathematical Physics

This section is devoted to the application of atomic and R-functions to problems of locus description, contour and structural image analysis, and electromagnetic analysis of waveguides of complex cross-section waveguides [4-7, 22]. Interest in this issue due to the fact that it covered a wide range of areas [22] the development of which is of paramount importance for the modern scientific and technological progress, such as the thermal physics and laser physics, a gyroscope, the electrodynamics of superconducting structures, the theory of analysis and synthesis of antennas, remote sensing, space technology, communication, radar and navigation, the scattering of waves on a complex shape bodies, the digital processing of multidimensional signals, pattern recognition, mathematical programming, boundary-value problems of mathematical physics, neural networks. These problems concern a lot of attention is paid to analytical description of complex objects. Here, a decisive role is played by the algebra of logic methods, R-functions and atomic functions (AF) [1-22]. Methods for constructing the function $\omega(x,y)$ describing locus (see. [4, 22]) of complex shape, based on the theory of R-functions [1,4].To construct the equation of a boundary of the region must be the first step to write a logical formula for it (predicate equation). Let in $\mathbf{R}^2$ given area S with piecewise smooth boundary ∂S. It is necessary to construct a function $\omega(x,y)$ is positive in S and is negative in outside and equal to zero on ∂S. The resulting equation $\omega(x,y)=0$ will implicitly define the locus of points representing the boundary of the region. Denote the characteristic function corresponding to the area S, across $\chi_i = [\omega_i(x,y) \geq 0]$ and having a system $\chi_i = [\omega_i(x,y) \geq 0]$ characteristic functions and Boolean expressions $Y = F(x_1 \ldots x_m)$ we can build a predicate

$$\chi = F(\chi_1,...\chi_m) = F[(\omega_1(x,y) \geq 0],...[\omega_m(x,y) \geq 0)]$$

to define the area of *S*, built from the auxiliary areas $S_1 \ldots S_m$ with well-known equation of the boundary on logical rules defined by the Boolean function *F*. Designating the following logical operations on the sets: "$\cap$" is intersection", "$\cup$" is association, "$\neg$" is additionally. Then we can write

$$S = F([S_1,...S_m],[\cap,\cup,\neg]).$$

R-functions method [4,22] allows of set-theoretic description of the field *S* to obtain a closed form equation of its boundaries $\omega(x,y)=0$. It is known that one of the properties of the image is closed and the continuity of their lines. This circuit is completely defines the form and contains the required information for the recognition of the object [22]. When considering the wide class of boundary value problems of different physical nature there is a need for solving differential equations in partial derivatives, in which the investigated area has a complex configuration. In such cases, usually used numerical methods: grid (method of finite differences, finite elements, boundary elements), variational and projection (Ritz method, Bubnov-Galerkin-Petrov, collocation, Trefftz, least squares method the method of fictitious domains, R -functions). It is necessary to highlight the method of R-functions [22] which has geometric flexibility and versatility with respect to the selected method of minimizing a functional. Such an approach requires significant computational cost. This is due to the use of structural formulas, which are based constructed using R-operations function area. Such features may have a complex structure and to calculate integrals of them are irregularly shaped area must use the quadrature formula with high order accuracy. Wavelet -bases circumvent the above drawbacks due to its unique properties and develop adaptive design scheme without the integration operation. One of the key trends observed in the development of devices operating in the Microwave and EHF bands, is to try to accurately calculate and numerically optimize the necessary physical characteristics, reducing the experimental verification. In this regard the role of electrodynamics calculation methods is increasing. It is known that in the design of microwave devices as the base structure element is preferred composite sectional waveguides. This is due to several advantages when compared with other transmission lines. However an effective selective excitation waves, especially higher in BCC presents considerable difficulties. To solve this problem it is necessary to know precisely the critical wave numbers and impedances excited BCC waves, as well as the distribution of the electromagnetic fields in its complex cross section. This question is the subject of many works which focuses on the analysis of the P and H-waveguides including dielectric filling [22].

5. Methods of Optimal Processing of Space-Time Signals in Active, Passive and Combined Active-Passive Radio Systems

Using the concept of the search for the optimal signal parameter estimates and related to them parameters of electromagnetic and regression models of scattering and radio-thermal radiation of electromagnetic waves by various objects and natural medias on the basis of modern theory of statistical decisions and estimates of probability distribution parameters, a number of fundamental results developing the theory and methods of optimal time-space signal processing are obtained for the first time [4,6,8,16]. Corresponding directions of their practical application in active, passive, and combined (complex) active-passive radar and remote-sensing radio systems are considered. Special attention is given to application of Kravchenko weight functions in problems of radio engineering systems synthesis.

References

[1] Kravchenko V. F. *Lectures on the Theory of Atomic Functions and Their Some Applications*. Radiotekhnika, Moscow, 2003 [in Russian].

[2] Kravchenko V.F. and Basarab M.A. *Boolean algebra and methods of approximation in the boundary value problems of electromagnetics*, Ed. by V.F. Kravchenko. Moscow, Fizmatlit, 2004 [in Russian].

[3] Zelkin E.G., Kravchenko V.F., and Gusevskii V.I. *Approximation design methods in the antenna theory*. Sains–Press, Moscow, 2005 [in Russian].

[4] Kravchenko V.F. and Rvachev V.L. Boolean Algebra, *Atomic Functions and Wavelets in Physical Applications*. Moscow, Fizmatlit, 2006 [in Russian].

[5] Kravchenko V.F. *Electrodynamics of superconducting structures*. Moscow, Fizmatlit, 2006 [in Russian].

[6] *Digital signal and image processing in radio physical applications*, Ed. by V. F. Kravchenko. Moscow, Fizmatlit, 2007 [in Russian].

[7] Basarab M.A., Kravchenko V. F., Matveev V. A. *Methods for modeling and digital signal processing in a gyroscope*. Moscow, Fizmatlit, 2007 [in Russian].

[8] Volosyuk V.K. and Kravchenko V.F. *Statistical theory of radio technical systems of remote sensing and radar*, Ed. by V.F. Kravchenko. Fizmatlit, Moscow, 2008 [in Russian].

[9] Kravchenko V.F., Perez-Meana H.M., and Ponomaryov V.I. *Adaptive digital processing of multidimentional signals with applications*. Moscow, Fizmatlit. 2009.

[10] Kravchenko V. F., Labun'ko O. S., Lerer A. M., and Sinyavskii G.P. *Computing Methods in Modern Radio Physics*. Moscow, Fizmatlit, 2009 [in Russian].

[11] Lazorenko O. V., Chernogor L. F. *Ultra wideband Signals and Processes*. V.N. Karazin Kharkov: Nats. Univ., Kharkov, 2009 [in Russian].

[12] Totsky A.V., Zelensky A.A., Kravchenko, V.F. *Bispectral methods of signal processing. Applications in radar, Telecommunications and Digital Image Restoration*. Berlin: Walter de Gruyter, 2015.

[13] Kravchenko V. F., Lutsenko V. I., Lutsenko I. V. *Scattering of radio waves by sea and detection of objects on the its background*. Moscow, Fizmatlit, 2015.

[14] Kravchenko V.F., Kravchenko O.V., Pustovoit V.I., and Churikov D.V. Atomic functions in modern problems of radio physics. Review. *Physical Bases of Instrumentation*. 2011, p. 3-48.

[15] Kravchenko, V. F., Application of the theory of atomic, WA-systems and R-functions in information technology. *Proceedings of the International scientific seminar "Actual problems of mathematical physics". Moscow. MSU M. V. Lomonosov*. November 28-29, 2014, p. 28-33.

[16] Volosyuk V. K., Gulyaev Yu. V., Kravchenko V. F., Kutuza, B. G., Pavlikov V. V., Pustovoit V. I. Modern methods for optimal spatio-temporal signal processing in active, passive, and combined active-passive radio-engineering systems. Review. *Journal of Communications Technology and Electronics*. 2014 , Vol. 59. Iss. 2, p. 97-118

[17] Kravchenko V. F., Kravchenko O. V., Churikov, D. V. Atomic functions in problems of filtering and digital signal processing. *Physical Bases of Instrumentation*. 2015, Vol. 4. No. 2 (15), p. 5-53.

[18] Kravchenko V.F., Kravchenko O.V., and Konovalov Ya.Yu. Prouhet-Thue-Morse sequence and atomic functions in applications of physics and techniques. *Journal of Measurement Science and Instrumentation*. 2015, Vol. 6. No. 2, p. 128-141.

[19] Kravchenko V.F., Kravchenko O.V., Pustovoit V.I., and Churikov D.V. Atomic, WA-systems, and R-functions applied in modern radio physics problems: Part I. *Journal of Communications Technology and Electronics*, 2014, Vol. 59, No. 10, P. 981–1009.

[20] Kravchenko V.F., Kravchenko O.V., Pustovoit V.I., Churikov D.V., and Yurin A.V. Atomic and WA-systems and R-functions applied in modern radio physics problems: Part II. *Journal of Communications Technology and Electronics*. 2015, Vol. 60. Iss. 2, p. 103-141.

[21] Kravchenko V.F., Kravchenko O.V., Pustovoit V.I., Konovalov Ya.Yu., and Churikov D.V. Atomic and WA-systems and R-functions applied in modern radio physics problems: Part III. *Journal of Communications Technology and Electronics*. 2015, Vol. 60. Iss. 7, p. 707-736.

[22] Kravchenko V.F., Kravchenko O. V., Pustovoit V. I., Churikov D. V., and Yurin A. V. Atomic, WA-systems, and R-functions applied in modern radio physics problems: Part IV. *Journal of Communications Technology and Electronics*. 2015, Vol. 60. Iss. 11, p. 1153-1190.