OBSERVER-BASED FAULT DIAGNOSIS: APPLICATIONS TO EXOTHERMIC CONTINUOUS STIRRED TANK REACTORS

A Thesis

by

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ABSTRACT

For chemical engineering dynamic systems, there is an increasing demand for better process performance, high product quality, absolute reliability & safety, maximum cost efficiency and less environmental impact. Improved individual process components and advanced automatic control techniques have brought significant benefits to the chemical industry. However, fault-free operation of processes can not be guaranteed. Timely fault diagnosis and proper management can help to avoid or at least minimize the undesirable consequences.

There are many techniques for fault diagnosis, and observer-based methods have been widely studied and have proved to be efficient for fault diagnosis. The basic idea of an observer-based approach is to generate a specific residual signal which carries the information of specific faults, as well as the information of process disturbances, model uncertainties, other faults and measurement noises. For fault diagnosis, the residual should be sensitive to faults and insensitive to other unknown inputs. With this feature, faults can be easily detected and may be isolated and identified.

This thesis applied an observer-based fault diagnosis method to three exothermic CSTR case studies. In order to improve the operational safety of exothermic CSTRs with risks of runaway reactions and explosion, fault diagnostic observers are built for fault detection, isolation and identification. For this purpose, different types of most common faults have been studied in different reaction systems. For each fault, a specific observer and the corresponding residual is built, which works as an indicator of that fault and is robust to other unknown inputs. For designing linear observers, the original nonlinear system is linearized at steady state, and the observer is designed based on the linearized system. However, in the simulations, the observer is tested on the nonlinear system instead of the linearized system. In addition, an efficient & effective general MATLAB program has been developed for fault diagnosis observer design. Extensive simulation studies have been performed to test the fault diagnostic observer on exothermic CSTRs. The results show that the proposed fault diagnosis scheme can be directly implemented and it works well for diagnosing faults in exothermic chemical reactors.

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NOMENCLATURE

C_A	Concentration of A component in the reactor
T	Temperature of the mixture in the reactor
T_w	Temperature of the coolant
F	Feed flow rate
V	Volume of the reactor
C_{Ain}	Inlet feed concentration
T_{in}	Inlet feed temperature
V_W	Volume of the cooling jacket
T_{win}	Inlet coolant temperature
F_W	Inlet coolant flow rate
C_p	Heat capacity of the reacting mixture
C_{pw}	Heat capacity of the coolant
ρ	Density of the reacting mixture
ρ_w	Density of the reacting coolant
U	Overall heat transfer coefficient
E	Activation energy
ΔE	Uncertainty in the activation energy
ΔU	Uncertainty in the overall heat transfer coefficient
f	Fault
d	Disturbance

A	Overall heat transfer area

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1. INTRODUCTION

1.1 Motivation

1.1.1 Why Fault Diagnosis?

For chemical dynamic systems, there is an increasing demand for better process performance, high product quality, absolute reliability & safety, maximum cost efficiency and less environmental impact. There are many ways to achieve these objects. Traditionally, industry struggles to improve individual process components, including plant equipment, controllers, sensors and actuators. Improvement of individual components can somehow lower the risk of faults. However, fault-free processes cannot be guaranteed during operation. Minor equipment damage as well as sensor & actuator malfunction may result in unexpected events. Even if the hardware equipment have no problems during operation, process abnormalities, due to complexity of chemical process, still have risks to cause faults.

When faults occur, the operating points will always go away from desired points. Small deviation may affect product quality, and large deviation may cause unplanned shut down, which increases the operating cost. Larger deviation without timely detection and management will inevitably result in safety issues, and even human casualties and environmental problems.

Timely diagnosis (including detection, isolation and identification) of faults and proper management (including timely maintenance, necessary switch off some processes and reconfiguration of controllers) can help to avoid or at least minimize the undesirable consequences. Thus fault diagnosis is very important for chemical processes.

1.1.2 Why Observer Based Methods?

For fault diagnosis, there are two schemes: hardware redundancy based method and analytical redundancy based method. Hardware redundancy is a traditional method, using the identical (redundant) hardware components parallel to process components. If an output of a process component differs from the output of its identical component, the fault can be detected. The obvious advantages of this method are its direct fault isolation and high reliability. But the disadvantages are also apparent: this scheme can only be applied on some components with outputs or sensors. Also, some expensive components with hardware redundancies or some equipment with limited space require higher technology and cost.

The other scheme is the analytical redundancy, replacing the hardware redundancy by a mathematical model. Many digital computers can simulate mathematical models and estimate the outputs. The difference between measured variables and estimated output can help for fault diagnosis. Currently, there are three main analytical redundancy based methods: observer-based method, parity-space based method and parameter-identification based method. Observer-based method is easy to be used for online implementation and for quick detection. Also, observer based method is more flexible, because parity-space and parameter identification based method are special forms of observer based method.

1.1.3 Why for Linear Systems?

Linear system is relatively simple and easier to study. Many systems can be replaced by reasonable linear models, which have the characteristics of processes. If the operating region is not too wide and the linearization error is not too large, linearized model is a good option.

1.2 Objectives

Just like the sensors, if we can build soft sensors which not only indicate the existence of faults, but also the location and size of the faults, fault diagnosis is completely realized. Previously, state observers have been used to estimate the state variables. Comparing the difference, called residual, between estimated states and measurements, we can somehow notice the faults and disturbances in the system. With disturbances and faults, the residuals are always nonzero. But we are more interested in the faults instead of disturbances. If we can build residuals which are only affected by faults, the residuals will work as indicators of faults. When faults occur, the residuals are nonzero. Without faults, the residuals stay at zero. Thus the main objective of this thesis is to design special observers which get the disturbances decoupled from the residuals. The residuals work as soft sensors of faults. Then apply this method to chemical processes. Even though this method has been studied for two decades and has wide applications in electrical and aerospace engineering, it has had limited application in chemical engineering. The objective of this thesis is to present unknown input diagnostic observer technique and apply this method to chemical processes.

If effective fault diagnosis can be achieved, the fault indicators can be connected to the alarms system. It will be very convenient and efficient for chemical engineers to diagnose faults at an early stage in the chemical plant. Also, the signals of fault soft sensors can be transferred to the Distributed Control Systems (DCS) or Programmable Logic Controller (PLC). With the information of faults, DCS or PLC will correct the measurements and controllers' output. In this way, chemical plant safety can be improved.

1.3 Thesis Outline and Contributions

The following overview briefly describes its major contributions and presents an outline of this dissertation.

Chapter 2: Review of fault diagnosis techniques – introduces the basic concepts used in fault diagnosis and presents an overview of fault diagnosis methods. Basic concepts include types of faults, fault detection, fault isolation and fault identification. Different fault diagnosis methods have been reviewed.

Chapter 3: Unknown input diagnosis observer (UIDO) design - reviews Luenberger functional observer, unknown input observer (UIO) and unknown input diagnostic observer. Detailed derivation of UIDO is presented in this chapter. The design procedure is summarized and a MATLAB program is also developed.

Chapter 4: Application to exothermic CSTRs – presents the application of UIDO method to three representative exothermic CSTR systems. The first one is a CSTR consisting of two possible faults in two different sensors. The second CSTR has one possible fault in the heat exchanger and the other possible fault in the reactor temperature sensor. The third CSTR considers two possible faults in an analytical sensor and heat exchanger, and one disturbance in reaction activation energy. These three cases consider different reaction systems.

Chapter 5: Conclusions and future directions - presents a summary of this thesis, discusses the conclusions of the applications and gives several possible future directions.

2. REVIEW OF FAULT DIAGNOSIS TECHNIQUES

2.1 Basic Concepts

2.1.1 Types of Faults

Location-based Categories

Sensor Faults

Sensor faults: In the broadest definition, a sensor is an object whose purpose is to detect events or changes in its environment, and then provide a corresponding output. In Process Control, information is gathered automatically from various sensors or other devices in the plant. This information is used to control different types of equipment and thereby to control operation of the plant. These sensors detect temperatures, pressures, fluid flow rates and levels, etc. [2].

Therefore, a sensor fault may degrade performance of decision-making systems, including feedback control system, safety control system, quality control system, state estimation system, optimization system[3]. For instance, in a CSTR system, the temperature and pressure sensors are used to measure the temperature and pressure of the reactor, and transfer the measured signal to the feedback control system. In closed loop control system, the outputs from sensors are used as the input of the controllers to maintain the operating points within a desired range. The presence of sensor faults would affect the decision making of the feedback controller. Any fault in sensors may cause the operating points deviating from the set points. If the deviation is out of desired range, several issues may occur, including product quality and safety, etc.

There are five major sensor categories of collecting various types of variables in industrial [4].

1. Physical parameter sensors: temperature, pressure, density, weight, etc.;

2. Spatial parameter sensors: state, position, level, depth, interface, etc.;

3. Sensors for detecting abnormal phenomena: flame, smoke, ATEX-rated atmosphere, hazardous gaseous/liquid/solid substances, video-monitoring, etc.;

4. Kinematic parameter sensors: flow rate, velocity, acceleration, vibration, rotation, mechanical stress, etc.;

5. Physicochemical parameter sensors: pH, rH, conductivity, resistivity, radioactivity, intensity, voltage, metal content, etc.

The first three categories are involved in over 90% of the accidents. Thus it is important to detect sensor faults.

Common sensor faults/failures include: (a) bias; (b) drift; (c) performance degradation (or loss of accuracy); (d) sensor freezing; and (e) calibration error [3]. Figure 2.1 depicts common types of sensor faults.

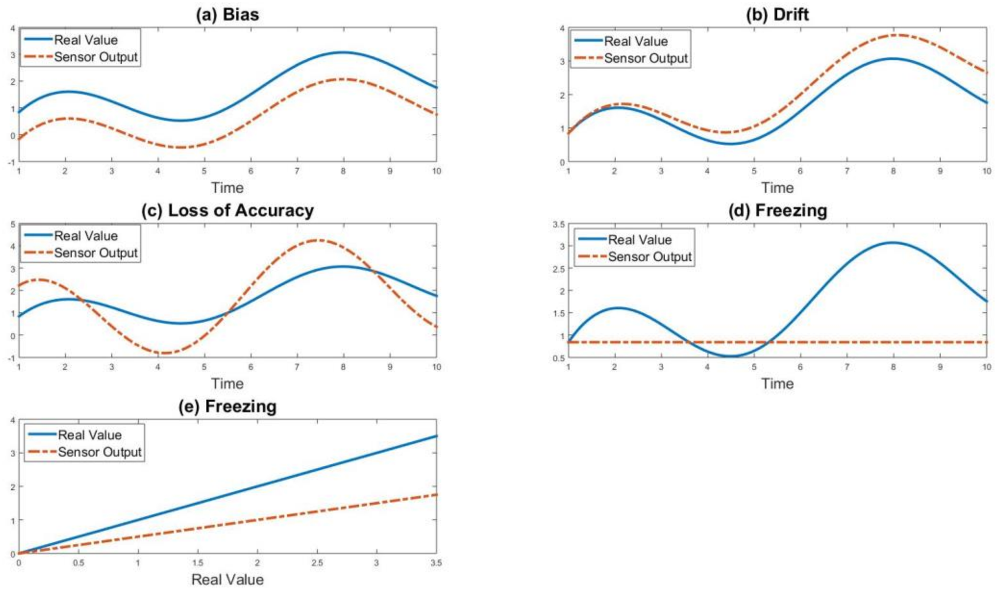

Fig. 2.1 Common types of sensor faults

The mathematical representation of the sensor faults [5]:

$$y = \begin{cases} x & \text{No failure} \\ x + b, \quad \dot{b} \equiv 0 & \text{Bias} \\ x + b(t), \quad |b(t)| = ct, \;\; 0 < c \ll 1 & \text{Drift} \\ x + b(t), \quad b(t) \in \left[-\bar{b}, \bar{b}\right], \;\; \dot{b} \in \mathcal{L}^{\infty} & \text{Loss of Accuracy} \\ b, \quad \dot{b} \equiv 0 & \text{Freezing} \\ k(t)x, \quad 0 < k \leq 1 & \text{Calibration Error} \end{cases}$$

Actuator Faults

An actuator is a type of motor that is responsible for moving or controlling a mechanism or system. It is operated by a source of energy, typically electric current, hydraulic fluid pressure, or pneumatic pressure, and converts that energy into motion [6]. For process control system, actuators are necessary to transform output of controllers (or control signal) into motion to control processes. A fault in actuator may cause loss of control. Actuator faults include, for example, stuck-up of control valves and faults in pumps, etc. Several common faults in servomotors include Lock-in-Place (LIP), Float, Hard-over Failure (HOF) and Loss of Effectiveness (LOE). In the case of LIP case, the actuator “freezes” at a particular condition and will not respond to subsequent commands. In the case of HOF, the actuator moves to the lower or upper position limit independent of subsequent commands. When float failure occurs, the actuator output stick to zero and will not respond to the commands. Loss of effectiveness is characterized by lowering the actuator’s gain respecting to its nominal value [3, 7]. Figure 2.2 depicts common types of actuator faults.

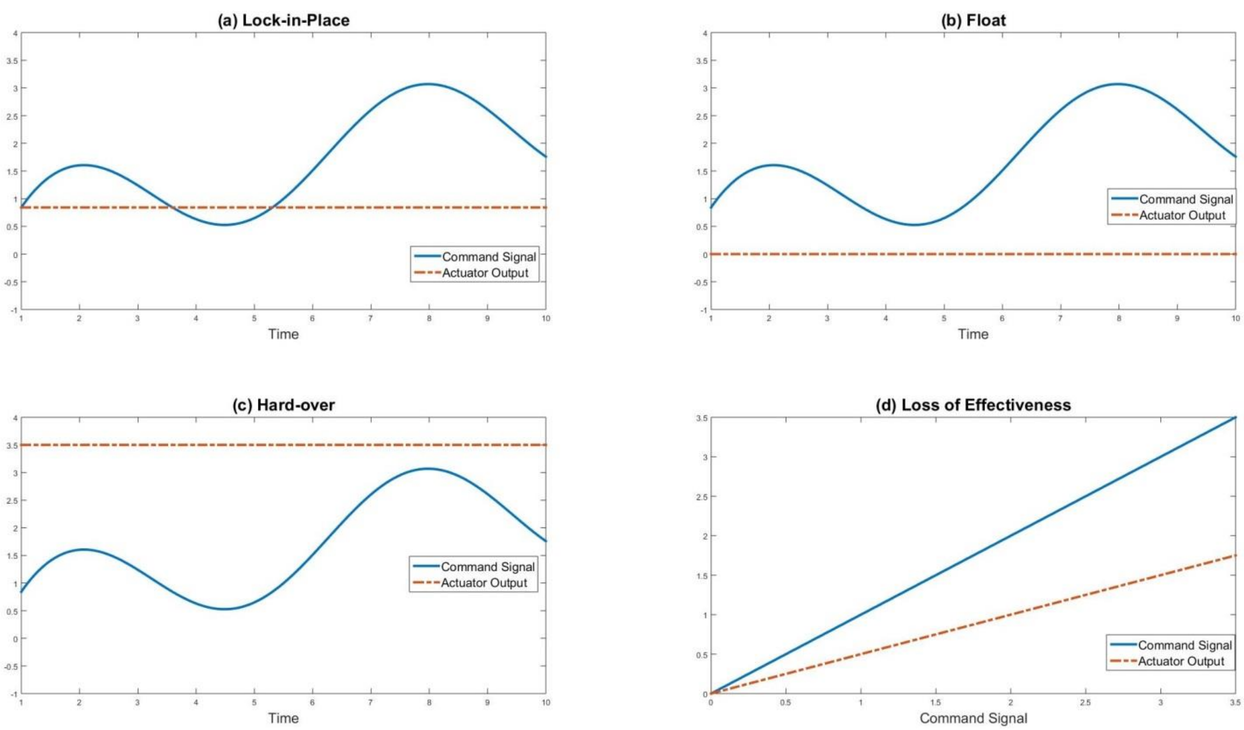

Fig. 2.2 Common types of actuator faults

Different types of actuator faults can be mathematically represented by:

$$u_a = \begin{cases} u_a(t), & & \text{No Failure} \\ b, & \dot{b} \equiv 0 & \text{Lock} - \text{in} - \text{Place} \\ 0, & & \text{Float} \\ b_{max} \text{ (or } b_{min}), & \dot{b}_{max} \equiv 0 \left(\text{or } \dot{b}_{min} \equiv 0\right) & \text{Hard} - \text{over} \\ k(t)u_c, & 0 < k \leq 1 & \text{Loss of Effectiveness} \end{cases}$$

Component Faults

These faults occur in the equipment of plant. When component faults occur, the physical parameters of that component change, and then the plant dynamics may change. The main reason for equipment faults is wear and tear. Sometimes the leakage in the tank or pipeline, or the fouling in the heat exchanger may cause serious consequences and immeasurable loss. Thus it is important to detect equipment faults during process [1].

Heat exchangers and heat exchanger networks are frequently used for cooling and recovering heat for safety concerns and energy requirements. For a fiercely exothermic-reaction reactor, the heat exchanger is necessary to cool down the temperature in case of runaway reactions. Also in the oil refining plant, the amount of energy used is enormous. Thus it is important to know the performance of heat exchangers. The well-known problem of heat exchangers is the fouling, which affects heat transfer, and the temperature of products, and then safety issues and energy waste occur. Thus production engineers need monitoring methods to answer practical questions including: What’s the actual performance of the particular heat exchanger at particular time? Which moment is ideal or necessary to shut down the process and maintain the heat exchanger?

Since the fouling mainly affects the heat transfer coefficient, and fouling process is quite slow comparing to the dynamics of chemical process, it can be mathematically represented by:

$$U(t) = {}_0 - b(t), \qquad b > 0, \qquad \dot{b} \rightarrow 0^+$$

where U is the overall heat transfer coefficient at given time t, U0 is the initial overall heat transfer coefficient, b(t) exists because of fouling.

The Way of Affecting System Dynamics

Due to the way of affecting the system dynamics, faults can be classified into two categories: additive fault and multiplicative fault. It has to be pointed out that, while in many cases a particular fault can be classified as additive or multiplicative according to its nature, sometimes it may also be arbitrary. As we will see, the additive fault is much easier to detect than multiplicative fault, therefore it is better to consider a fault as an additive fault whenever possible [8].

Additive Fault

In general, additive fault is assumed to be the deviation from the normal behavior, but independent of system configuration. Normally, the values of additive faults are zero. When an additive fault occur, it will cause changes of system variables [8]. For a process control system, an offset in a sensor or an actuator can be considered as a constant, and a drift in a sensor as a ramp. It is a typical additive fault. For a disturbance, it is also an extra unknown input. It is reasonable to consider that there is no physical difference between a disturbance and an additive fault.

Multiplicative Fault

Multiplicative fault will change as some plant parameters change [8]. Typically, model errors or model uncertainties are considered as multiplicative faults. Model uncertainties are the gap between the real system and the model. In practice, we could not get the exact model of a real system. But if the model uncertainty is very small and will not cause relatively large model configuration, the model is always to be considered as the perfect model without model uncertainties.

2.1.2 Fault Diagnosis

- **Fault Detection:** Detection of the occurrence of faults in the functional units of the process, which lead to undesired or intolerable behavior of the whole system.
- **Fault Isolation:** Localization (classification) of different faults.
- **Fault Identification:** Determination of the type, magnitude and cause of the fault.

2.2 Classification of Fault Detection Schemes

2.2.1 Hardware Redundancy Based FD

Hardware (or physical/parallel) redundancy is a traditional method for fault diagnosis. This approach uses multiple sensors to measure a particular variable. Also, a voting scheme is used to decide if and when a fault has occurred and its likely location amongst redundant system [1]. But the applicability is limited because of the extra cost and additional space required. Additionally, its application is limited on sensor faults.

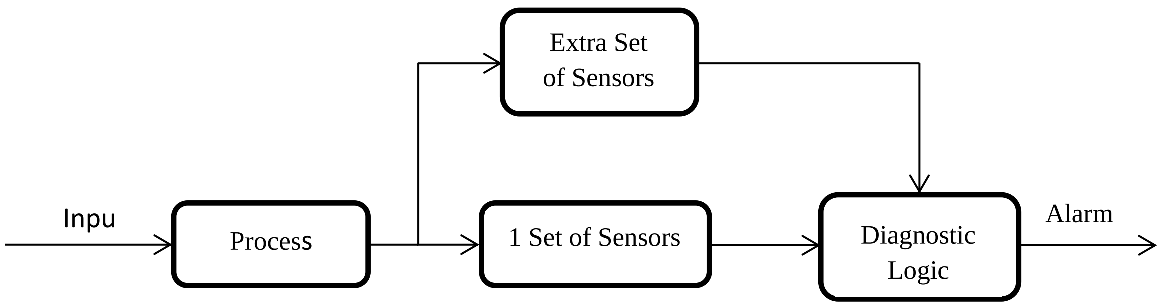

Fig. 2.3 Schematic description of the hardware redundancy scheme

2.2.2 Plausibility Test

Plausibility test assumes a fault which results in the loss of plausibility. This technique evaluates outputs of process and compares with their rough behavior under normal operation. Examples include the sign and size of the measurements. But this method is less efficient in detecting faults and difficult for complex system [1, 9].

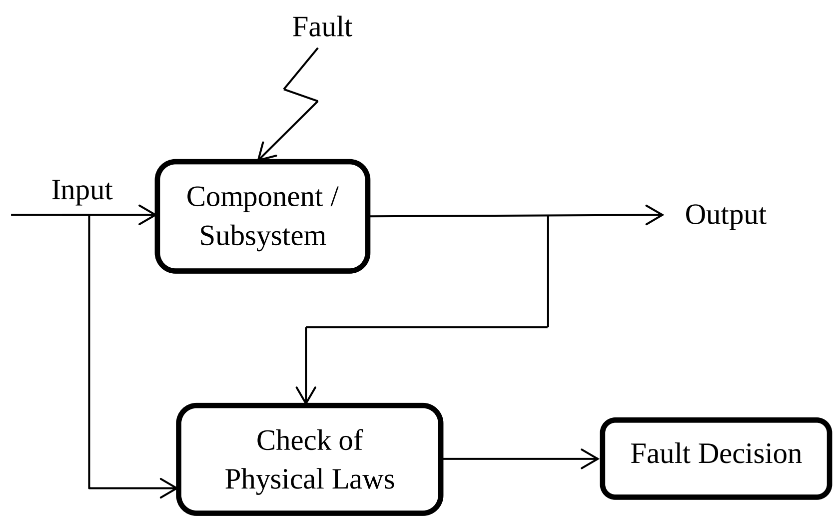

Fig. 2.4 Schematic description of plausibility scheme

2.2.3 Signal-based FD

This approach is based on the properties of measured signals. Typical properties include magnitude and trend checking from the derivative, mean and variance, spectral power densities, correlation coefficient, etc., of the measured signals. Among these treatments of signal, absolute value and derivative (trend) of measured signal are the two most simply and widely used methods for fault detection. In signal-based fault detection, suitable upper and lower bound are set based on the knowledge of the system and required performance of the process. If the absolute value or derivative, etc., cross the limit, it means a fault occurs.

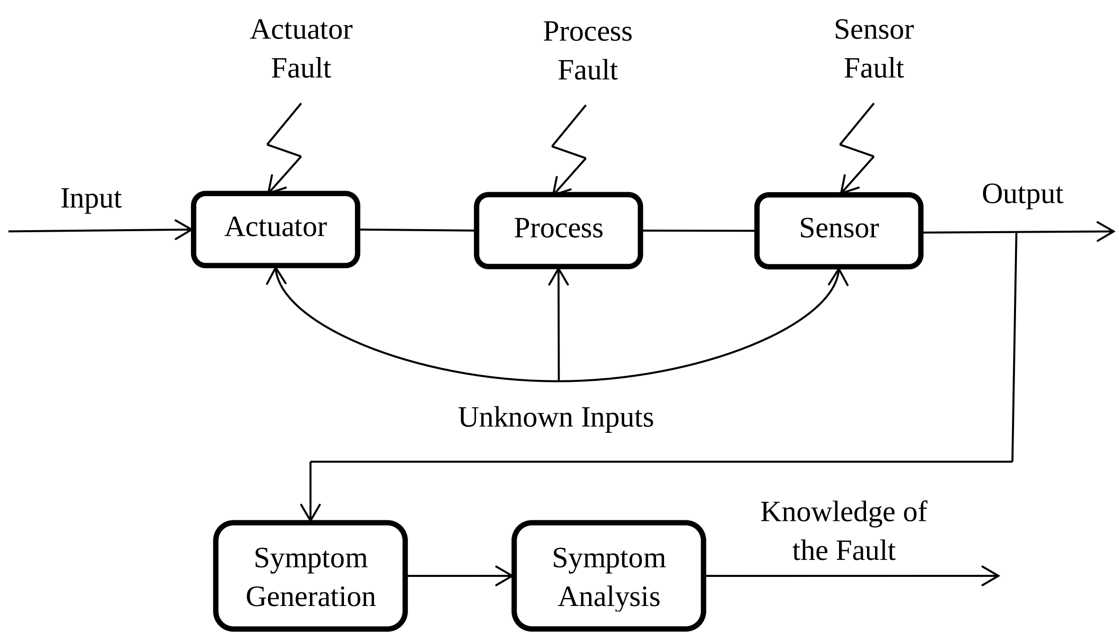

Fig. 2.5 Schematic description of signal processing based scheme

y(t): measured signal

Y(t): processed signal (absolute value, derivative, etc.)

$$Y_{min} \leq Y(t) \leq Y_{max} \Longrightarrow fault - free$$

$$Y(t) < Y_{min} \; or \; Y(t) > Y_{max} \;\; \Rightarrow faulty$$

This method is simple and easily implemented. But the disadvantages are obvious: it cannot detect a small fault when the signal is still within the desired range. Also, the efficiency is limited when the operating range is wide because of the possible large noise, disturbances and variation of input signals.

2.2.4 Model-based FD

The intuitive idea of model-based fault detection is from hardware redundancy. Model-based fault detection (or analytical redundancy) uses redundant analytical relationship between various measured variables rather than single variable. That is to say, model-based fault detection replaces the hardware redundancy by a mathematical model. This approach is achieved by comparing measured variables with their estimations from the mathematical model [1]. The differences between measured variables and estimations are called residuals. For a fault-free system, the residuals are zero. If a fault occurs, the corresponding residuals are not zero. Thus a residual, similar to the difference amongst hardware redundancy system, is a fault indicator of monitored process.

The major advantage of this method is that no extra hardware components, but a control computer with related software, are required for fault detection. And fast development of computer technique makes this approach feasible and practicable [10].

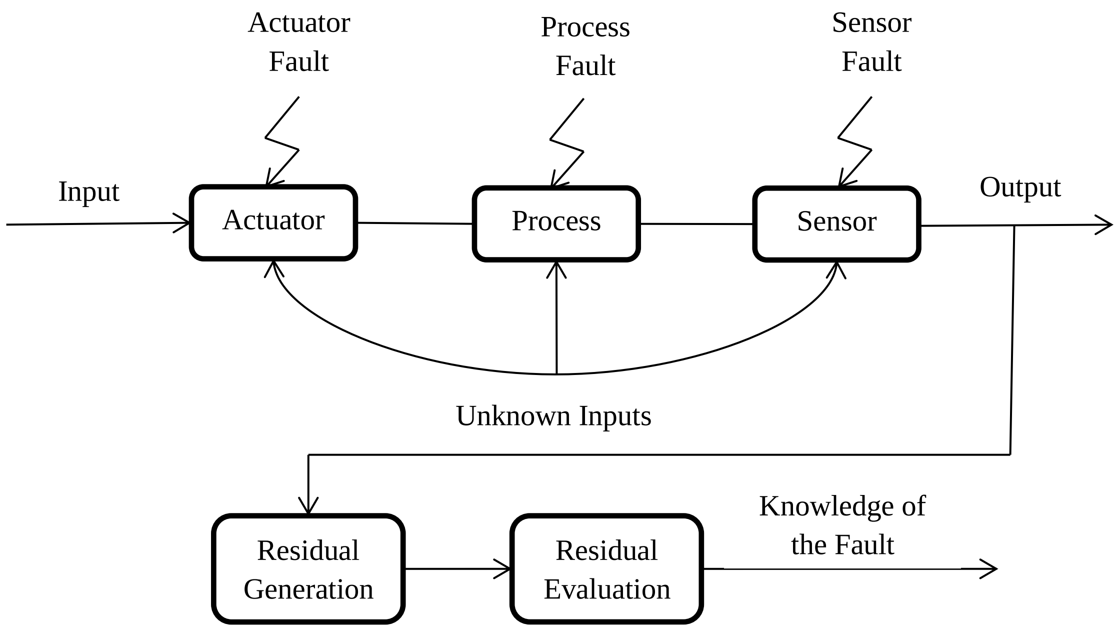

Fig. 2.6 Schematic description of model-based fault diagnosis scheme

Knowledge-based FD

For large-scale systems, detailed quantitative mathematical model may not be available or may be costly and time-consuming to obtain. In this situation, it is better to choose knowledge-based methods which are based on qualitative models for process monitoring. These methods include expert systems, artificial neural networks, fuzzy logic, etc. [11].

Observer-based FD

The first observer-based FDI system was proposed by Beard and Jones in the early 1970s [12], which marked a historical milestone in the development of the model-based fault diagnosis [1]. And then Luenberger observers were first applied for fault detection [13] and isolation [14-16]. In the late 1980s, the position of observer-based approach for FDI was established [17].

Observer-based method is one of the most applied model-based techniques for FDI. By comparing the process output with estimation, the residual is obtained.

Unknown Input Observer (UIO)

The idea of robust fault detection is to perfectly decouple the estimated state from unknown input (disturbances) based on the disturbances distribution. If the disturbances cannot affect the estimated states, the residual should be independent of the disturbances. It is reported [18] that Frank [19] firstly used robust observer-based fault detection schemes for instrument failure detection. And then robust unknown input observers were intensively studied [20-26].

The main advantage of unknown input observer is the decoupling of disturbances. But the drawback is also obvious: model uncertainties would affect the performance of UIO. If the model uncertainties can be modeled as the additive term as external disturbances, the UIO can be used to decouple the effect of model uncertainties. However, it will inevitably increase the number of disturbances. If the number of unknown inputs including disturbances and faults is larger than the number of independent measurements, it is almost impossible to isolate the faults. For chemical processes, only a few state variables can usually be measured. Thus it is difficult to deal with model uncertainties.

2.2.5 A Comparison of Different Fault Detection Methods

It has to be noted that comparing different fault detection methods is not easy. Choices are affected by several factors including information of the process, availability of the process mathematical model, complexity of the mathematical model, system nonlinearity, safety requirement, etc. If the analytical model is available and easy to be implemented, analytical model-based approach is preferred, since it is faster for fault detection and easier for on-line implementation. If the mathematical model is difficult to obtain, or if it is quite complex to implement, knowledge-based approach or signal-based method is preferred.

2.3 Fault Diagnosis Approach for Chemical Process

Chemical industry is one of the most important economic forces in the world [27]. Modern chemical plant is large-scale and highly complex [28] and operates under closed loop control for product quality and production efficiency. However, unexpected consequences, including major production loss, human injury and environmental impact, may occur when faults make the operating points deviating from the designed range. Only petrochemical industries may lose 20 billion dollars every year [29]. Therefore, there is considerable research in fault diagnosis [29-31], and various approaches for process monitoring and fault diagnosis have been studied and developed. These methods may be classified into three categories: process model-based method, multivariate statistical process monitoring and knowledge-based approaches [32].

One widely used approach in chemical industry is multivariate statistical process monitoring [33-37]. For a plant with normal operation, process data can be collected to build an empirical correlation model by multivariate latent variable methods [38], including principal component analysis (PCA) and partial least squares (PLS), which have successful application in process industries. This empirical model with low dimension is able to capture the key information from normal operating data. The comparison between the current data and the empirical model is used to detect abnormal behavior by statistical tests. The contribution plots method [39] is then used for simple fault isolation. Past fault data is also required to improve fault diagnosis by isolating complex faults [40]. This method is able to handle the relatively complex system with a large number of measurements or without first principle analytical model. However, the fault-isolation design relies heavily on the past data with faults. These data are always unavailable or are difficult and expensive to obtain [32].

Considering the drawbacks of multivariate statistical process monitoring method, an alternative approach for fault diagnosis is analytical model-based method. Process model-based method was first proposed and has also been received significant attention [1, 10, 41-46]. The limited information from measurements can not determine the presence of faults in chemical processes. In this method, the dynamics of systems and relationship between various variables (process model) can offer more information including the faults of the system. The extracted information from the process model and input/output data is called analytical redundancy. For fault diagnosis, a residual is always generated as a fault indicator of a particular fault through analytical redundancy. Fault is then detected and isolated by checking the value of a residual. If the relationship between a fault and the corresponding residual can be found, that fault can be identified.

Process model-based approach has been studied extensively over several decades for linear systems [1, 47, 48] and nonlinear systems [49-52]. But in the process, there are still some disturbances and model uncertainties. Rajaraman and Mannan et al. used parameter estimation method to estimate parameter with uncertainties [53]. But the nonlinear model is repeatedly linearized step by step, which takes a lot of computation time. Kazantzis and Kravaris proposed a systematic observer design framework for estimating unmeasured sate variables and applied this observer to a batch reactor and a CSTR [54]. But it is very difficult to use a similar method to design an observer for fault diagnosis. Therefore, an simple and efficient residual only sensitive to a particular fault but robust to other unknown inputs is desired. Unknown input observer is developed for decoupling the effects of other unknown inputs [47]. However, this method requires accurate mathematical model, which is very difficult to achieve or time-consuming to obtain or even unavailable for some complex chemical processes. Thus the application of model-based method is limited [55].

Another fault diagnosis method is the knowledge-based approach. Among knowledge-based approaches, wavelet neural network has been successfully applied in a wide range of applications [56]. Zhou and Mannan et al. applied feed forward neural networks to a batch reactor and a distillation column. However, the drawbacks of this method is long training time and it is difficult to extract symbolic knowledge from trained network [57].

It is difficult to determine which method above is the best because every method has its own advantages and disadvantages. But when the analytical process model is available or easy to obtain, model-based method is always the first option.

2.4 Unknown Input Observers

In modeling, there are various types of uncertainties including model mismatches, parameter changes and unknown external excitation, which can be conveniently represented as unknown inputs or disturbances [58].

The observer was first proposed and developed by Luenberger [59-61]. After the early development, observers with unknown inputs have been developed as the so-called unknown input observer (UIO) or disturbance-decoupled observers [62-72]. The first unknown input observer was proposed in 1973 [72] (the earliest document this thesis can find) by the response of a suitably selected dynamic system. Another approach was proposed in 1975 by designing a reduced-order observer without any knowledge of unknown inputs [62]. The existence conditions for the reduced-order observer was proposed by Kudva et al [64]. Bhattacharyya proposed a unknown input observer by geometric approach [63]. Geometric approach is one of the fields in the control theory, but the application of this approach requires a deep understanding of mathematics. Miller et al. proposed a reduced-order Luenberger observer for a linear time-invariant system by simplest matrix generalized inverse [66]. Fairman et al. designed disturbance decoupled observer via singular value decomposition [67]. Hou et al. derived an equivalent system, which is free of unknown inputs, for designing the disturbance decoupled observer [58]. Yang et al. used straightforward matrix calculations to design a full-order and a reduced-order observer [71].

Because of the special feature of disturbance decoupling, all of the methods mentioned above helped to develop unknown input fault diagnostic observers.

2.5 Unknown Input Diagnostic Observer

The objective of unknown input diagnostic observer (UIDO) is to make the residual decoupled from unknown inputs (disturbances). But the original idea is to make the state estimation error decoupled from disturbances, and thus get residual independent disturbances. Watanabe et al. first proposed this approach for sensor fault detection and isolation by decoupling uncertainty. After that, Frank et al. generalized this method for fault detection and isolation [20, 73, 74]. Chen et al. applied this method to a realistic chemical process system example for robust FDI [75]. For robust FDI, many researchers made contributions to this area [26, 76-78]. Frank et al. used canonical form transformation method to simplify the computation [73]. Ding et al. generalized all of the methods above and used a very simple and general numerical method to design UIDO [1]. This design method has more freedom and the computation is quite efficient. This thesis mainly refers this method.

3. UNKNOWN INPUT DIAGNOSTIC OBSERVER (UIDO) DESIGN

Luenberger first proposed the concept of observer to estimate system states in 1964 [59], and then first proposed the functional observer for various purposes in 1971 [61]. After that, observers have been studied extensively. The Unknown Input Observer (UIO) has received considerable attention in the literature [47, 62-66, 68-71, 73, 75, 78, 79]. A brief introduction on the UIO has already been given in chapter 2. The study of UIO helps to design a special observer for fault diagnosis [1, 17, 18, 20, 22, 26, 29, 75-81]. This kind of observer is called Unknown Input Diagnostic Observer (UIDO) [1]. The introduction can also be found in chapter 2.

In the following subsections, Luenberger functional observer is first introduced. Then unknown input observer. Finally, UIDO design procedure and derivation is reviewed. The derivation mainly based on Ding's book [1]. The only exception is the last subsection where instead of using parity space approach of Ding's book [1], a simple alternative method is used to derive the inequality conditions and obtain the same results.

3.1 Luenberger Functional Observer

Considering continuous-time linear time-invariant (LTI) system as following:

$$\dot{x} = Ax + Bu \tag{3.1}$$

$$y = Cx + Du \tag{3.2}$$

where $x \epsilon \mathcal{R}^n$ is a vector of state variables, $u \epsilon \mathcal{R}^{k_u}$ is a vector of input, and $y \epsilon \mathcal{R}^m$ is a vector of output variables.

As for system (3.1)-(3.2), the Luenberger type Functional Observer [61] is built for various purposes including feedback control and state estimation [59].

$$\dot{z} = Gz + Hu + Ly \tag{3.3}$$

where $z \epsilon \mathcal{R}^s$ is a vector of observer state variables. G matrix is a Hurwitz matrix. The observer state z represents an estimation of Tx, where T is a transformation matrix. By modifying the matrix T, functional observer (3.3) can be identity state observer, reduced order observer, or other kinds of observers for various purposes [61].

3.2 Unknown Input Observer

Considering continuous-time linear time-invariant (LTI) system with unknown inputs as follows:

$$\dot{x} = Ax + Bu + E_d d \tag{3.4}$$

$$y = Cx + Du + F_d d \tag{3.5}$$

where $x \epsilon \mathcal{R}^n$ is a vector of state variables, $u \epsilon \mathcal{R}^{k_u}$ is a vector of input, and $y \epsilon \mathcal{R}^m$ is a vector of output variables. $d \epsilon \mathcal{R}^{k_d}$ is a vector of unknown input (uncertainty) vector.

As for systems (3.4)-(3.5), unknown input observer is designed as following:

$$\dot{z} = Gz + Hu + Ly \tag{3.6}$$

where $z \epsilon \mathcal{R}^s$ is a vector of observer state variables. G matrix is a Hurwitz matrix. Similar to Luenberger functional observer, there is still a transformation matrix T between system state x and observer state z. The form of (3.6) is the same as Luenberger functional observer, but UIO is able to get the disturbances decoupled and still able to achieve various objectives by modifying the T matrix. Many researchers have been focused on this area to design a particular observer for a specific purpose, including state estimation and feedback control purpose. Luenberger functional observer is a very general form, not only for the nominal system, but also for the system with disturbances or faults. The designing method is not introduced here. Interested readers can find introduction in chapter 2.

3.3 Unknown Input Diagnostic Observer

The objective of UIDO is to detect, isolate and identify the faults in the system. The method used is to generate a special residual which is sensitive to a specific fault but insensitive to other faults or disturbances, instead of trying to estimate the entire state vector. Through this way, the design freedom is significantly increased. The order of the UIDO could be equal to the order of the system, or it could be lower order or higher order.

3.3.1 Problem Formulation

The continuous-time linear time-invariant (LTI) system is given by

$$\dot{x} = Ax + Bu + E_d d + E_f f \tag{3.7}$$

$$y = Cx + Du + F_d d + F_f f \tag{3.8}$$

where $x \epsilon \mathcal{R}^n$ is a vector of state variables, $u \epsilon \mathcal{R}^{k_u}$ is a vector of input, and $y \epsilon \mathcal{R}^m$ is a vector of output variables. $d \epsilon \mathcal{R}^{k_d}$ is a vector of unknown input (uncertainty) vector, and $f \epsilon \mathcal{R}^{k_f}$ is a vector of fault. Matrices A, B, E_d, E_f, C, D, F_d and F_f are appropriately dimensioned real constant matrices. We define

$$E_f = \begin{bmatrix} E_{f_a} & E_{f_p} & E_{f_s} \end{bmatrix} \tag{3.9}$$

$$F_f = \begin{bmatrix} F_{f_a} & F_{f_p} & F_{f_s} \end{bmatrix} \tag{3.10}$$

$$f = \begin{bmatrix} f_a \\ f_p \\ f_s \end{bmatrix} \tag{3.11}$$

where

$$f = \begin{cases} f_a\text{: actuator fault } (F_{f_a} = D, E_{f_a} = B) \\ f_p\text{: process fault } (F_{f_p} = F_p, E_{f_p} = E_p) \\ f_s\text{: sensor fault } (F_{f_S} = I, E_{f_S} = 0) \end{cases} \tag{3.12}$$

Apply (3.9)-(3.12), we get

$$E_f = [B \quad E_p \quad 0] \tag{3.13}$$

$$F_f = [D \quad F_p \quad I] \tag{3.14}$$

Based on the Luenberger type observer, The Unknown Input Diagnostic Observer (UIDO) is formulated for given system (3.7)-(3.8) as following.

$$\dot{z} = Gz + Hu + Ly \tag{3.15}$$

And the residual is

$$r = vy - wz - qu \tag{3.16}$$

where $z \epsilon \mathcal{R}^s$ is a vector of observer state variables, and the residual is given by $r \epsilon \mathcal{R}^1$. The G matrix is a Hurwitz stable. There is a state transformation matrix: T. The relationship between the state variables and the observer variables is that when that is no fault in the system and the sensors,

$$\lim_{t \to \infty} z(t) \Big|_{f=0} = Tx(t) \tag{3.17}$$

The matrices T, G, H, L, v, w, q are to be selected. Some conditions are required to determine these matrices. The objective of the residual is to decouple the effect of the disturbance and to be sensitive to faults. In order to make the residual as the indicator of faults, the residual must be zero when there are no faults in the system or sensors. When there are faults in the system or sensors, the residual must not be zero.

Let's define

$$e \equiv Tx - z \tag{3.18}$$

From (3.18), the error dynamics is represent by

$$\dot{e} = (TAx - Gz - LCx) + (TB - H - LD)u + (TE_f - LF_f)f + (TE_d - LF_d) \tag{3.19}$$

If the following conditions hold,

$$TA = GT + LC \tag{3.20}$$

$$H = TB - LD \tag{3.21}$$

$$TE_d - LF_d = 0 \tag{3.22}$$

$$TE_f - LF_f \neq 0 \tag{3.23}$$

the error dynamics can be simplified to

$$\dot{e} = Ge + (TE_f - LF_f)f \tag{3.24}$$

At steady state,

$$\dot{e} = Ge + (TE_f - LF_f)f = 0 \tag{3.25}$$

From (3.25) we can get

$$e = -G^{-1}(TE_f - LF_f)f \tag{3.26}$$

The residual r becomes

$$r = vy - wz - qu$$

$$= v(Cx + Du + F_d d + F_f f) - w(Tx - e) - qu$$

$$= we + (vC - wT)x + (vD - q)u + vF_f f + vF_d d \tag{3.27}$$

$$= -wG^{-1}(TE_f - LF_f)f + (vC - wT)x + (vD - q)u + vF_f f + vF_d d$$

$$= [-wG^{-1}(TE_f - LF_f) + vF_f]f + (vC - wT)x + (vD - q)u + vF_d d$$

If the following additional conditions hold

$$vC - wT = 0 \tag{3.28}$$

$$q = vD \tag{3.29}$$

$$VF_d = 0 \tag{3.30}$$

$$-wG^{-1}(TE_f - LF_f) + vF_f \neq 0 \tag{3.31}$$

then

$$r = [-wG^{-1}(TE_f - LF_f) + vF_f]f \tag{3.32}$$

If $-wG^{-1}(TE_f - LF_f) + vF_f$ is invertable,

$$f = [-wG^{-1}(TE_f - LF_f) + vF_f]^{-1}r \tag{3.33}$$

Conditions (3.20)-(3.23) and (3.28)-(3.31) are called Luenberger conditions.

Summary of design condition as follows:

$$TA = GT + LC$$

$$H = TB - LD$$

$$TE_d - LF_d = 0$$

$$TE_f - LF_f \neq 0$$

$$vC - wT = 0$$

$$q = vD$$

$$VF_d = 0$$

$$-wG^{-1}(TE_f - LF_f) + vF_f \neq 0$$

For convenience, these eight conditions are called Luenberger conditions in this thesis.

3.3.2 Observer Design

The observer design problem is to solve the Luenberger conditions. Because the canonical form is always an extremely convenient starting point for certain design problems [82], the pair (C, A) and the pair (w, G) is given in the canonical form [1].

Derivation Based on the $(\bar{C}, \bar{A})$ Canonical Form

Canonical form for the system (3.7)-(3.8) is constructed by transforming the state vector to a new coordinate system where the system equations have a particular form [83].

$$\bar{x} = Px \leftrightarrow x = P^{-1}\bar{x} \tag{3.34}$$

Following Luenberger's work [82], Korovin and Fomichev [83] found a way to select P matrix. From the observability, a composed matrix is selected as following.

$$v = \begin{bmatrix} C_1 \\ C_1 A \\ \vdots \\ C_1 A^{\sigma_1 - 1} \\ - - - \\ C_2 \\ \vdots \\ C_2 A^{\sigma_2 - 1} \\ - - - \\ \vdots \\ C_m A^{\sigma_m - 1} \end{bmatrix} \in R^{n\times n},\ C = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_m \end{bmatrix},\ \sigma_i\text{: } observability\ index \tag{3.35}$$

Then inverse the v matrix

$$v^{-1} = [g_1 \quad g_2 \quad \cdots \quad g_m \quad \cdots \quad g_n] \tag{3.36}$$

Let

$$P^{-1} = [g_1 \quad Ag_1 \quad \cdots \quad A^{\sigma_1}g_1 \quad g_2 \quad \cdots \quad A^{\sigma_2}g_2 \quad \cdots \quad A^{\sigma_m}g_m] \tag{3.37}$$

System (3.7)-(3.8) is then represented by observer canonical form

$$\dot{\bar{x}} = PAP^{-1}\bar{x} + PBu + PE_d d + PE_f f \tag{3.38}$$

$$y = CP^{-1}\bar{x} + Du + F_d d + F_f f \tag{3.39}$$

Let

$$\bar{A} = PAP^{-1}, \bar{B} = PB, \overline{E_d} = PE_d, \bar{E_f} = PE_f, \bar{C} = CP^{-1} \tag{3.40}$$

System (3.38)-(3.39) becomes

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u + \bar{E}_d d + \bar{E}_f f \tag{3.41}$$

$$y = \bar{C}\bar{x} + Du + F_d d + F_f f \tag{3.42}$$

where

$$\bar{A} = \begin{bmatrix} A_{1,1} & \cdots & A_{1,m} \\ \vdots & \ddots & \vdots \\ A_{m,1} & \cdots & A_{m,m} \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} c_{1,1} & 0 & 0 & \cdots & 0 \\ c_{2,1} & c_{2,2} & 0 & \cdots & 0 \\ c_{3,1} & c_{3,2} & c_{3,3} & \cdots & 0 \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ c_{m,1} & c_{m,2} & c_{m,3} & \cdots & c_{m,m} \end{bmatrix} \tag{3.43}$$

$$A_{i,i} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & * \\ 1 & 0 & 0 & \cdots & 0 & * \\ 0 & 1 & 0 & \cdots & 0 & * \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & \ddots & 0 & 1 & 0 & * \\ 0 & \cdots & 0 & 0 & 1 & * \end{bmatrix} \in R^{\sigma_i \times \sigma_i}, A_{i,j} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & * \\ 0 & 0 & 0 & \cdots & 0 & * \\ 0 & 0 & 0 & \cdots & 0 & * \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & \ddots & 0 & 0 & 0 & * \\ 0 & \cdots & 0 & 0 & 0 & * \end{bmatrix} \in R^{\sigma_i \times \sigma_j} \tag{3.44}$$

$$c_{i,i} = (0 \quad 0 \quad \cdots \quad 0 \quad 1) \in \mathcal{R}^{1\times\sigma_i}, c_{j,i} = (0 \quad 0 \quad \cdots \quad 0 \quad *) \in \mathcal{R}^{1\times\sigma_i} \tag{3.45}$$

Let's split $\bar{A}$ matrix into two parts [1], because there is one property of A_0 matrix we can use later.

$$\bar{A} \equiv A_0 + L_0\bar{C} \tag{3.46}$$

where

$$A_0 = \mathrm{diag}(A_{01}, \cdots, A_{0m}) \tag{3.47}$$

$$A_{0i} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix} \in R^{\sigma_i \times \sigma_i}, i = 1, \cdots, m \tag{3.48}$$

$L_0\bar{C}$ is the rest of $\bar{A}$ matrix.

Let's take matrix G, w in the observer canonical form [1].

$$G = [G_0 \quad g] \tag{3.49}$$

where

`

$$G_0 = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} \in R^{s \times (s-1)} \tag{3.50}$$

$$g = \begin{bmatrix} g_1 \\ \vdots \\ g_s \end{bmatrix} \in R^s \tag{3.51}$$

$$w = [0 \quad \cdots \quad 0 \quad 1] \tag{3.52}$$

(w,G) is set in the canonical observer form. Because every observer pair can be similarity transformed into canonical form, this observer canonical form does not lose generality.

Derivation from Luenberger Condition TA=GT+LC [1]

From (3.20) and (3.40),

$$TP^{-1}\bar{A} = GTP^{-1} + L\bar{C} \tag{3.53}$$

Let

$$\overline{\mathrm{T}} = \mathrm{TP}^{-1}, \text{ and } \mathrm{L} \equiv \overline{\mathrm{L}} + \overline{\mathrm{T}}\mathrm{L}_0 \tag{3.54}$$

From (3.46),(3.49) ,(3.53) ,(3.54)

$$\overline{\mathrm{T}}\mathrm{A}_0 = [\mathrm{G}_0 \quad \mathrm{g}]\overline{\mathrm{T}} + \overline{\mathrm{L}}\overline{\mathrm{C}} \tag{3.55}$$

Let

$$\overline{\mathrm{L}} \equiv \begin{bmatrix} \mathrm{l}_1 \\ \vdots \\ \mathrm{l}_s \end{bmatrix}, \text{ and } \overline{\mathrm{T}} = \begin{bmatrix} \mathrm{t}_1 \\ \vdots \\ \mathrm{t}_s \end{bmatrix} \tag{3.56}$$

Then

$$\begin{bmatrix} \mathrm{t}_1 \\ \vdots \\ \mathrm{t}_s \end{bmatrix} \mathrm{A}_0 - \mathrm{g}\mathrm{t}_s = \mathrm{G}_0 \begin{bmatrix} \mathrm{t}_1 \\ \vdots \\ \mathrm{t}_s \end{bmatrix} + \begin{bmatrix} \mathrm{l}_1 \\ \vdots \\ \mathrm{l}_s \end{bmatrix} \overline{\mathrm{C}} \tag{3.57}$$

Expand (3.57) from the last row, we can get

$$\mathrm{t}_{s-1} = \mathrm{t}_s\mathrm{A}_0 - (\mathrm{l}_s\overline{\mathrm{C}} + \mathrm{g}_s\mathrm{t}_s)$$

From the last second row

$$\mathrm{t}_{s-2} = \mathrm{t}_s\mathrm{A}_0^2 - (\mathrm{l}_s\overline{\mathrm{C}} + \mathrm{g}_s\mathrm{t}_s)\mathrm{A}_0 - (\mathrm{l}_{s-1}\overline{\mathrm{C}} + \mathrm{g}_{s-1}\mathrm{t}_s)$$

From the last third row

$$\mathrm{t}_{s-3} = \mathrm{t}_s\mathrm{A}_0^3 - (\mathrm{l}_s\overline{\mathrm{C}} + \mathrm{g}_s\mathrm{t}_s)\mathrm{A}_0^2 - (\mathrm{l}_{s-1}\overline{\mathrm{C}} + \mathrm{g}_{s-1}\mathrm{t}_s)\mathrm{A}_0 - (\mathrm{l}_{s-2}\overline{\mathrm{C}} + \mathrm{g}_{s-2}\mathrm{t}_s)$$

$$\vdots$$

$$\mathrm{t}_2 = \mathrm{t}_s\mathrm{A}_0^{s-2} - (\mathrm{l}_s\overline{\mathrm{C}} + \mathrm{g}_s\mathrm{t}_s)\mathrm{A}_0^{s-3} - (\mathrm{l}_{s-1}\overline{\mathrm{C}} + \mathrm{g}_{s-1}\mathrm{t}_s)\mathrm{A}_0^{s-4} - \cdots - (\mathrm{l}_3\overline{\mathrm{C}} + \mathrm{g}_3\mathrm{t}_s)$$

$$t_1 = t_s A_0^{s-1} - (l_s \bar{C} + g_s t_s) A_0^{s-2} - (l_{s-1} \bar{C} + g_{s-1} t_s) A_0^{s-3} - \cdots$$
$$- (l_2 \bar{C} + g_2 t_s)$$

From the first row

$$t_1 A_0 - g_1 t_s = l_1 \bar{C}$$

Derivation from Luenberger Condition vC-wT=0 [1]

From (3.28), (3.40) and (3.54),

$$v\bar{C} - w\bar{T} = 0 \tag{3.58}$$

From (3.52), (3.56) and (3.58)

$$t_s = v\bar{C} \tag{3.59}$$

Substitute (3.59) into the expansion of (3.57)

$$t_{s-1} = v\bar{C}A_0 - (l_s + g_s v)\bar{C}$$

$$t_{s-2} = v\bar{C}A_0^2 - (l_s + g_s v)\bar{C}A_0 - (l_{s-1} + g_{s-1} v)\bar{C}$$

$$\vdots$$

$$t_2 = v\bar{C}A_0^{s-2} - (l_s + g_s v)\bar{C}A_0^{s-3} - \cdots - (l_4 + g_4 v)\bar{C}A_0$$
$$- (l_3 + g_3 v)\bar{C}$$

$$t_1 = v\bar{C}A_0^{s-1} - (l_s + g_s v)\bar{C}A_0^{s-2} - \cdots - (l_3 + g_3 v)\bar{C}A_0$$
$$- (l_2 + g_2 v)\bar{C}$$

$$v\bar{C}A_0^s - (l_s + g_s v)\bar{C}A_0^{s-1} - \cdots - (l_2 + g_2 v)\bar{C}A_0 - (l_1 + g_1 v)\bar{C} = 0 \tag{3.60}$$

Lemma 3.1:

$\mathbf{p_0 C + p_1 C A_0 + \cdots + p_s C A_0^s = 0}$**, s ≥ 0,**

holds if and only if $\mathbf{p_i C A_0^i = 0}$**.**

Further $\mathbf{p_i = 0}$ **for** $\mathbf{0 \le i \le \sigma_{min-1}}$**.**

And $\mathbf{p_i C A_0^i = 0}$ **for** $\mathbf{\sigma_{min} \le i \le \sigma_{max-1}}$

From lemma 3.1 and (3.60)

$$v\bar{C}A_0^s = 0$$

$$(l_s + g_s v)\bar{C}A_0^{s-1} = 0$$

$$\vdots$$

$$(l_{\sigma_{min+1}} + g_{\sigma_{min}+1} v)\bar{C}A_0^{\sigma_{min}} = 0$$

$$l_{\sigma_{min}} + g_{\sigma_{min}} v = 0$$

$$\vdots$$

$$l_1 + g_1 v = 0$$

Then the expansion of (3.57) become

$$t_s = v\bar{C}$$

$$t_{s-1} = v\bar{C}A_0 - (l_s + g_s v)\bar{C}$$

$$\vdots$$

$$t_{\sigma_{min}} = v\bar{C}A_0^{s-\sigma_{min}} - (l_s + g_s v)\bar{C}A_0^{s-\sigma_{min}-1} - \cdots - (l_{\sigma_{min}+1} + g_{\sigma_{min}+1} v)\bar{C}$$

$$t_{\sigma_{min}-1} = v\bar{C}A_0^{s-\sigma_{min}+1} - (l_s + g_s v)\bar{C}A_0^{s-\sigma_{min}} - \cdots - (l_{\sigma_{min}+1} + g_{\sigma_{min}+1} v)\bar{C}A_0$$

$$\vdots$$

$$t_2 = v\bar{C}A_0^{s-2} - (l_s + g_s v)\bar{C}A_0^{s-3} - \cdots - (l_{\sigma_{min}+1} + g_{\sigma_{min}+1} v)\bar{C}A_0^{\sigma_{min}-2}$$

$$t_1 = v\bar{C}A_0^{s-1} - (l_s + g_s v)\bar{C}A_0^{s-2} - \cdots - (l_{\sigma_{min}+1} + g_{\sigma_{min}+1} v)\bar{C}A_0^{\sigma_{min}-1}$$

Let define:

$$\bar{v}_i \equiv -(l_{i+1} + g_{i+1} v), i = \sigma_{min}, \sigma_{min} + 1, \cdots, s-1 \qquad (3.61)$$

And

$$\bar{v}_s \equiv v, \text{ and } \bar{v} \equiv [\bar{v}_{\sigma_{min}} \quad \bar{v}_{\sigma_{min}+1} \quad \cdots \quad \bar{v}_s] \tag{3.62}$$

Then we can get

$$\bar{v}\begin{bmatrix} \bar{C}A_0^{\sigma_{min}} \\ \bar{C}A_0^{\sigma_{min}+1} \\ \vdots \\ \bar{C}A_0^{s} \end{bmatrix} = 0 \tag{3.63}$$

Thus

$$t_s = \bar{v}_s\bar{C}$$

$$t_{s-1} = \bar{v}_s\bar{C}A_0 + \bar{v}_{s-1}\bar{C}$$

$$\vdots$$

$$t_{\sigma_{min}} = \bar{v}_s\bar{C}A_0^{s-\sigma_{min}} + \bar{v}_{s-1}\bar{C}A_0^{s-\sigma_{min}-1} + \cdots + \bar{v}_{\sigma_{min}}\bar{C}$$

$$t_{\sigma_{min}-1} = \bar{v}_s\bar{C}A_0^{s-\sigma_{min}+1} + \bar{v}_{s-1}\bar{C}A_0^{s-\sigma_{min}} + \cdots + \bar{v}_{\sigma_{min}}\bar{C}A_0$$

$$\vdots$$

$$t_2 = \bar{v}_s\bar{C}A_0^{s-2} + \bar{v}_{s-1}\bar{C}A_0^{s-3} + \cdots + \bar{v}_{\sigma_{min}}\bar{C}A_0^{\sigma_{min}-2}$$

$$t_1 = \bar{v}_s\bar{C}A_0^{s-1} + \bar{v}_{s-1}\bar{C}A_0^{s-2} + \cdots + \bar{v}_{\sigma_{min}}\bar{C}A_0^{\sigma_{min}-1}$$

Thus

$$\bar{T} = \begin{bmatrix} 0 & \cdots & 0 & \bar{v}_{\sigma_{min}} & \cdots & \bar{v}_s \\ 0 & \cdots & \bar{v}_{\sigma_{min}} & \cdots & \bar{v}_s & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & \bar{v}_{\sigma_{min}} & \cdots & \bar{v}_s & \cdots & 0 \\ \bar{v}_{\sigma_{min}} & \cdots & \bar{v}_s & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \bar{v}_{s-1} & \bar{v}_s & 0 & 0 & \cdots & 0 \\ \bar{v}_s & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}\begin{bmatrix} \bar{C} \\ \bar{C}A_0 \\ \vdots \\ \bar{C}A_0^{s-\sigma_{min}-1} \\ \bar{C}A_0^{s-\sigma_{min}} \\ \vdots \\ \bar{C}A_0^{s-2} \\ \bar{C}A_0^{s-1} \end{bmatrix} \tag{3.64}$$

During the calculation, it is more convenient to use matrix A rather than matrix A_0. Thus it is necessary to convert matrix A_0 to matrix A.

$$\begin{bmatrix} \bar{C} \\ \bar{C}A_0 \\ \vdots \\ \bar{C}A_0^i \end{bmatrix} = \begin{bmatrix} I & 0 & \cdots & 0 \\ -\bar{C}L_0 & I & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ -\bar{C}A_0^{i-1}L_0 & \cdots & -\bar{C}L_0 & I \end{bmatrix} \begin{bmatrix} \bar{C} \\ \bar{C}(A_0 + L_0\bar{C}) \\ \vdots \\ \bar{C}(A_0 + L_0\bar{C})^i \end{bmatrix} \tag{3.65}$$

The formula of (3.65) is obvious, thus there is no need to prove.

According to (3.46) and (3.65), we can get:

$$\begin{bmatrix} \bar{C} \\ \bar{C}A_0 \\ \vdots \\ \bar{C}A_0^{s-1} \end{bmatrix} = \begin{bmatrix} I & 0 & \cdots & 0 \\ -\bar{C}L_0 & I & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ -\bar{C}A_0^{s-2}L_0 & \cdots & -\bar{C}L_0 & I \end{bmatrix} \begin{bmatrix} \bar{C} \\ \overline{CA} \\ \vdots \\ \overline{CA}^{s-1} \end{bmatrix} \tag{3.66}$$

According to (3.66)

$$\begin{bmatrix} \bar{C}A_0^{\sigma_{min}} \\ \bar{C}A_0^{\sigma_{min}+1} \\ \vdots \\ \bar{C}A_0^{s} \end{bmatrix} = \begin{bmatrix} -\bar{C}A_0^{\sigma_{min}-1}L_0 & \cdots & -\bar{C}L_0 & I & 0 & \cdots & 0 \\ -\bar{C}A_0^{\sigma_{min}}L_0 & \cdots & -\bar{C}A_0L_0 & -\bar{C}L_0 & I & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ -\bar{C}A_0^{s-1}L_0 & \cdots & -\bar{C}A_0^{s-\sigma_{min}}L_0 & -\bar{C}A_0^{s-\sigma_{min}-1}L_0 & \cdots & -\bar{C}L_0 & I \end{bmatrix} \begin{bmatrix} \bar{C} \\ \overline{CA} \\ \vdots \\ \overline{CA}^{s} \end{bmatrix} \tag{3.67}$$

Let's define:

$$H_1 = \begin{bmatrix} -\bar{C}A_0^{\sigma_{min}-1}L_0 & \cdots & -\bar{C}L_0 & I & 0 & \cdots & 0 \\ -\bar{C}A_0^{\sigma_{min}}L_0 & \cdots & -\bar{C}A_0L_0 & -\bar{C}L_0 & I & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ -\bar{C}A_0^{s-1}L_0 & \cdots & -\bar{C}A_0^{s-\sigma_{min}}L_0 & -\bar{C}A_0^{s-\sigma_{min}-1}L_0 & \cdots & -\bar{C}L_0 & I \end{bmatrix} \tag{3.68}$$

Then (3.67) becomes

$$\begin{bmatrix} \bar{C}A_0^{\sigma_{min}} \\ \bar{C}A_0^{\sigma_{min}+1} \\ \vdots \\ \bar{C}A_0^{s} \end{bmatrix} = H_1 \begin{bmatrix} \bar{C} \\ \overline{CA} \\ \vdots \\ \overline{CA}^{s} \end{bmatrix} \tag{3.69}$$

According to (3.63) and (3.69), we can get:

$$\bar{v}\begin{bmatrix}\bar{C}A_0^{\sigma_{min}}\\ \bar{C}A_0^{\sigma_{min}+1}\\ \vdots\\ \bar{C}A_0^{s}\end{bmatrix}=\bar{v}H_1\begin{bmatrix}\bar{C}\\ \bar{C}\bar{A}\\ \vdots\\ \bar{C}\bar{A}^{s}\end{bmatrix}=0 \tag{3.70}$$

Let's define:

$$v_s\equiv\bar{v}H_1 \tag{3.71}$$

where:

$$v_s\equiv[v_{s,0}\quad v_{s,1}\quad\cdots\quad v_{s,s}] \tag{3.72}$$

According to (3.70)-(3.71), we can get:

$$v_s\begin{bmatrix}\bar{C}\\ \bar{C}\bar{A}\\ \vdots\\ \bar{C}\bar{A}^{s}\end{bmatrix}=0 \tag{3.73}$$

According to (3.64), (3.66), (3.68), (3.71) and (3.72), we can get:

$$\bar{\bar{T}}=\begin{bmatrix}v_{s,1}&v_{s,2}&\cdots&v_{s,s-1}&v_{s,s}\\ v_{s,2}&\cdots&\cdots&v_{s,s}&0\\ \vdots&\cdots&\cdots&\vdots&\vdots\\ v_{s,s}&0&\cdots&\cdots&0\end{bmatrix}\begin{bmatrix}\bar{C}\\ \bar{C}\bar{A}\\ \vdots\\ \bar{C}\bar{A}^{s-1}\end{bmatrix} \tag{3.74}$$

According to Lemma 3.1 and (3.61), we can get:

$$\begin{cases}l_i+g_iv=-\bar{v}_{i-1}, & i=\sigma_{min}+1,\sigma_{min}+2,\cdots,s\\ l_i+g_iv=0, & i=1,2,3,\cdots,\sigma_{min}-1,\sigma_{min}\end{cases} \tag{3.75}$$

From (3.75)

$$\bar{L}=\begin{bmatrix}l_1\\ l_2\\ \vdots\\ l_1\\ l_1\\ \vdots\\ l_{s-1}\\ l_s\end{bmatrix}=\begin{bmatrix}-g_1v\\ -g_2v\\ \vdots\\ -g_{\sigma_{min}}v\\ -\bar{v}_{\sigma_{min}}-g_{\sigma_{min}+1}v\\ \vdots\\ -\bar{v}_{s-2}-g_{s-1}v\\ -\bar{v}_{s-1}-g_sv\end{bmatrix} \tag{3.76}$$

From (3.54), (3.76)

$$L = \begin{bmatrix} -g_1 v \\ -g_2 v \\ \vdots \\ -g_{\sigma_{min}} v \\ -\bar{v}_{\sigma_{min}} - g_{\sigma_{min}+1} v \\ \vdots \\ -\bar{v}_{s-2} - g_{s-1} v \\ -\bar{v}_{s-1} - g_s v \end{bmatrix} + \begin{bmatrix} 0 & \cdots & 0 & \bar{v}_{\sigma_{min}} & \cdots & \bar{v}_s \\ 0 & \cdots & \bar{v}_{\sigma_{min}} & \cdots & \bar{v}_s & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & \bar{v}_{\sigma_{min}} & \cdots & \bar{v}_s & \cdots & 0 \\ \bar{v}_{\sigma_{min}} & \cdots & \bar{v}_s & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \bar{v}_{s-1} & \bar{v}_s & 0 & 0 & \cdots & 0 \\ \bar{v}_s & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \bar{C} \\ \bar{C}A_0 \\ \vdots \\ \bar{C}A_0^{s-\sigma_{min}-1} \\ \bar{C}A_0^{s-\sigma_{min}} \\ \vdots \\ \bar{C}A_0^{s-2} \\ \bar{C}A_0^{s-1} \end{bmatrix} L_0 \tag{3.77}$$

According to and (3.62), (3.68), (3.71), (3.72)

$$v_s = [\bar{v}_{\sigma_{min}} \quad \bar{v}_{\sigma_{min}+1} \quad \cdots \quad \bar{v}_s] \begin{bmatrix} -\bar{C}A_0^{\sigma_{min}-1} L_0 & \cdots & -\bar{C}L_0 & I & 0 & \cdots & 0 \\ -\bar{C}A_0^{\sigma_{min}} L_0 & \cdots & -\bar{C}A_0 L_0 & -\bar{C}L_0 & I & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ -\bar{C}A_0^{s-1} L_0 & \cdots & -\bar{C}A_0^{s-\sigma_{min}} L_0 & -\bar{C}A_0^{s-\sigma_{min}-1} L_0 & \cdots & -\bar{C}L_0 & I \end{bmatrix} \tag{3.78}$$

According to (3.77), (3.78)

$$L = -\begin{bmatrix} v_{s,0} \\ v_{s,1} \\ \vdots \\ v_{s,s-1} \end{bmatrix} - gv \tag{3.79}$$

Derivation Based on Original System

The results above are based on the $(\bar{C}, \bar{A})$ observer canonical form. Now it's time to remove the canonical form.

Let

$$\bar{A} = PAP^{-1}, \bar{B} = PB, \overline{E_d} = PE_d, \bar{E}_f = PE_f, \bar{C} = CP^{-1} \tag{3.80}$$

According to (3.73) and (3.80)

$$v_s \begin{bmatrix} C \\ CA \\ \vdots \\ CA^s \end{bmatrix} = 0 \tag{3.81}$$

According to (3.74) and (3.80)

$$T = \begin{bmatrix} v_{s,1} & v_{s,2} & \cdots & v_{s,s-1} & v_{s,s} \\ v_{s,2} & \cdots & \cdots & v_{s,s} & 0 \\ \vdots & \cdots & \cdots & \vdots & \vdots \\ v_{s,s} & 0 & \cdots & \cdots & 0 \end{bmatrix} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{s-1} \end{bmatrix} \tag{3.82}$$

Derivation from Luenberger Condition: TEd - LFd =0 and VFd=0 [1]

According to (3.79) and (3.82),

$$\begin{bmatrix} v_{s,1} & v_{s,2} & \cdots & v_{s,s-1} & v_{s,s} \\ v_{s,2} & \cdots & \cdots & v_{s,s} & 0 \\ \vdots & \cdots & \cdots & \vdots & \vdots \\ v_{s,s} & 0 & \cdots & \cdots & 0 \end{bmatrix} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{s-1} \end{bmatrix} E_d + \begin{bmatrix} v_{s,0} \\ v_{s,1} \\ \vdots \\ v_{s,s-1} \end{bmatrix} F_d + gvF_d = 0 \tag{3.83}$$

According to (3.30) and (3.83)

$$\begin{bmatrix} v_{s,1} & v_{s,2} & \cdots & v_{s,s-1} & v_{s,s} \\ v_{s,2} & \cdots & \cdots & v_{s,s} & 0 \\ \vdots & \cdots & \cdots & \vdots & \vdots \\ v_{s,s} & 0 & \cdots & \cdots & 0 \end{bmatrix} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{s-1} \end{bmatrix} E_d + \begin{bmatrix} v_{s,0} \\ v_{s,1} \\ \vdots \\ v_{s,s-1} \end{bmatrix} F_d = 0 \tag{3.84}$$

Let’s expand (3.84) from the last row to the first row:

$$v_{s,s}CE_d + v_{s,s-1}F_d = 0$$

$$v_{s,s}CAE_d + v_{s,s-1}CE_d + v_{s,s-2}F_d = 0$$

$$v_{s,s}CA^2E_d + v_{s,s-1}CAE_d + v_{s,s-2}CE_d + v_{s,s-3}F_d = 0$$

$$\vdots$$

$$v_{s,s}CA^{s-1}E_d + v_{s,s-1}CA^{s-2}E_d + \cdots + v_{s,2}CAE_d + v_{s,1}CE_d + v_{s,0}F_d = 0$$

From the expanded formula of (3.84) and (3.30), we can construct a new form as following:

$$[v_{s,0} \quad v_{s,1} \quad \cdots \quad v_{s,s}]\begin{bmatrix} F_d & 0 & \cdots & 0 \\ CE_d & F_d & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}E_d & \cdots & CE_d & F_d \end{bmatrix} = 0 \tag{3.85}$$

Thus

$$v_s\begin{bmatrix} F_d & 0 & \cdots & 0 \\ CE_d & F_d & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}E_d & \cdots & CE_d & F_d \end{bmatrix} = 0 \tag{3.86}$$

Derivation from Luenberger Condition: $\mathbf{TE_f - LF_f \neq 0}$ and $\mathbf{-wG^{-1}(TE_f - LF_f) + vF_f \neq 0}$

The parity space approach can solve these two Luenberger conditions [1]. But this thesis uses a different but more direct and simple method to derive from these two conditions.

In order to simplify the problem, let's make

$$vF_f = 0 \tag{3.87}$$

According to (3.23), (3.79) and (3.82)

$$\begin{bmatrix} v_{s,1} & v_{s,2} & \cdots & v_{s,s-1} & v_{s,s} \\ v_{s,2} & \cdots & \cdots & v_{s,s} & 0 \\ \vdots & \cdots & \cdots & \vdots & \vdots \\ v_{s,s} & 0 & \cdots & \cdots & 0 \end{bmatrix}\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{s-1} \end{bmatrix}E_f + \begin{bmatrix} v_{s,0} \\ v_{s,1} \\ \vdots \\ v_{s,s-1} \end{bmatrix}F_f \neq 0 \tag{3.88}$$

Let's expand (3.88)

$$\begin{bmatrix} v_{s,s}CA^{s-1}E_f + v_{s,s-1}CA^{s-2}E_f + \cdots + v_{s,2}CAE_f + v_{s,1}CE_f + v_{s,0}F_f \\ \vdots \\ v_{s,s}CA^2E_f + v_{s,s-1}CAE_f + v_{s,s-2}CE_f + v_{s,s-3}F_f \\ v_{s,s}CAE_f + v_{s,s-1}CE_f + v_{s,s-2}F_f \\ v_{s,s}CE_f + v_{s,s-1}F_f \end{bmatrix} \neq 0$$

From the expansion of (3.88), we can construct a new form as following:

$$v_s \begin{bmatrix} F_f & 0 & \cdots & 0 \\ CE_f & F_f & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}E_f & \cdots & CE_f & F_f \end{bmatrix} \neq 0 \tag{3.89}$$

3.3.3 Summary of UIDO Design for Fault Diagnosis

For the unknown input observer,

$$\dot{z} = Gz + Hu + Ly$$

$$r = vy - wz - qu$$

The design procedure has the following steps [1].

Step 1: Solve $\mathbf{v_s}[\mathbf{H_{f,s}} \quad \mathbf{H_{o,s}} \quad \mathbf{H_{d,s}}] = [* \quad \mathbf{0} \quad \mathbf{0}]$ for vs;

where

$$v_s = [v_{s,0} \quad v_{s,1} \quad \cdots \quad v_{s,s-1} \quad v_{s,s}]$$

$$H_{f,s} = \begin{bmatrix} F_f & 0 & \cdots & 0 \\ CE_f & F_f & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}E_f & \cdots & CE_f & F_f \end{bmatrix}$$

$$H_{o,s} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^s \end{bmatrix}$$

$$H_{d,s} = \begin{bmatrix} F_d & 0 & \cdots & 0 \\ CE_d & F_d & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}E_d & \cdots & CE_d & F_d \end{bmatrix}$$

It is solvable if and only if

$$\text{rank}[H_{f,s} \quad H_{o,s} \quad H_{d,s}] > \text{rank}[H_{o,s} \quad H_{d,s}]$$

Step 2: Given vs, calculate v and T;

$$\mathrm{v} = \mathrm{v}_{s,s}$$

$$\mathrm{T} = \begin{bmatrix} \mathrm{v}_{s,1} & \mathrm{v}_{s,2} & \cdots & \mathrm{v}_{s,s-1} & \mathrm{v}_{s,s} \\ \mathrm{v}_{s,2} & \cdots & \cdots & \mathrm{v}_{s,s} & 0 \\ \vdots & \cdots & \cdots & \vdots & \vdots \\ \mathrm{v}_{s,s} & 0 & \cdots & \cdots & 0 \end{bmatrix} \begin{bmatrix} \mathrm{C} \\ \mathrm{CA} \\ \vdots \\ \mathrm{CA}^{s-1} \end{bmatrix}$$

Step 3: Select G and w;

$$\mathrm{G} = \begin{bmatrix} 0 & 0 & \cdots & 0 & g_1 \\ 1 & 0 & \cdots & 0 & g_2 \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 & g_{s-1} \\ 0 & \cdots & 0 & 1 & g_s \end{bmatrix} \in \mathrm{R}^{s\times s}$$

where $g = \begin{bmatrix} g_1 \\ \vdots \\ g_s \end{bmatrix} \in \mathrm{R}^s$ is chosen to make G Hurwitz.

$$\mathrm{w} = [0 \quad \cdots \quad 0 \quad 1]$$

Step 4: Calculate L, H, q;

$$\mathrm{L} = -\begin{bmatrix} \mathrm{v}_{s,0} \\ \mathrm{v}_{s,1} \\ \vdots \\ \mathrm{v}_{s,s-1} \end{bmatrix} - g\mathrm{v}_{s,s}$$

$$\mathrm{H} = \mathrm{TB} - \mathrm{LD}$$

$$\mathrm{q} = \mathrm{vD}$$

3.3.4 MATLAB Program for UIDO Design

The purpose of the MATLAB program is to design an unknown input diagnostic observer (3.15)-(3.16) for linear system (3.7)-(3.8).

The input of this MATLAB program is the observer order s, and linear system matrices A, B, E_d, E_f, C, D, F_d and F_f. Based on the linear system, the unknown input diagnostic observer can be designed automatically. And the observer eigenvalues are designed to be faster than the linear system but slower than the ten times fast of the linear system. The purpose of the upper limit is to be tolerant to noises. The output of this MATLAB program is the UIDO matrices T, G, H, L, v, w and q.

The MATLAB program is as follows:

```
function [Tout,Gout,Hout,Lout,vout,wout,qout] = ObserDesign( s,A,B,Ed,Ef,C,D,Fd,Ff )
%% The objective of this function is to design UIDO
%% Detailed explanation goes here
%% Input
% s: Observer order
% A,B,Ed,Ef,C,D,Fd,Ff are the system matrices
% dx/dt = A*x + B*u + Ed*d + Ef*f
%     y = C*x + D*u + Fd*d + Ff*f

%% Output
% Tout,Gout,Hout,Lout,vout,wout,qout are the observer matrices
% dz/dt = G*z + H*u + L*y
%     r = v*y - w*z - q*u
 % The eigenvalues of Observer are designed to be faster than the system but slower than 10
times fast % of the system.

%% Calculation begins
n=size(A,1);
m=size(C,1);
ku=size(B,2);
```

```
kf=size(Ef,2);
kd=size(Ed,2);

Hos=zeros((s+1)*m,n);
Hfs=zeros((s+1)*m,(s+1)*kf);
Hds=zeros((s+1)*m,(s+1)*kd);

%get Hos
for i=1:s+1
   row_start=(i-1)*m+1;
   row_end=i*m;
   Hos(row_start:row_end,1:n)=C*A^(i-1);
end

%get Hfs and Hds
for i=1:s+1 %i represent row i
   for j=1:s+1 %j represent column j
      row_start=(i-1)*m+1;
      row_end=i*m;
      column_start_d=(j-1)*kd+1;
      column_end_d=j*kd;
      column_start_f=(j-1)*kf+1;
      column_end_f=j*kf;
      if i==j
```

```
            Hfs(row_start:row_end,column_start_f:column_end_f)=Ff;

            Hds(row_start:row_end,column_start_d:column_end_d)=Fd;

        elseif i-j>=1

            Hfs(row_start:row_end,column_start_f:column_end_f)=vpa(C*A^(i-j-1)*Ef,100);

            Hds(row_start:row_end,column_start_d:column_end_d)=vpa(C*A^(i-j-1)*Ed,100);

        end

    end

end

H_od_s=[Hos Hds];

H_fod_s=[Hfs Hos Hds];


eigOfSys=max(real(eig(A))); % find the system speed


%% step 1: Check rank and original system stability, and calculate vs

if rank(H_fod_s)>rank(H_od_s) && eigOfSys<0

    vs=vpa(null(H_od_s')',100);

    row_vs=size(vs,1);

    numi=1;

    vs_correct=vs(numi,:);

    while rank(vs_correct*Hfs)==0 && numi<row_vs

        numi=numi+1;

        vs_correct=vs(numi,:);

    end
```

```
%% Step 2: Given vs, calculate v and T

Vs_matrix=zeros(s+1,m);

for i=1:s+1

    Vs_matrix(i,1:m)=vs_correct( ((i-1)*m+1) : i*m );

end

v=Vs_matrix(s+1,:);


% get T_x_z

Vs_T=zeros(s,m*s);

for i=1:s

    for j=1:s

        if i+j<=s+1

            Vs_T(i,(j-1)*m+1:j*m)=Vs_matrix(i+j,:); %5.71

        end

    end

end

T_x_z=Vs_T*Hos(1:s*m,:);


%% Step 3: get G and w

G0=[zeros(1,s-1);eye(s-1)];

eigens=zeros(s,1);

temp=(10-1)*eigOfSys/(s+1);

for i=1:s

    eigens(i)=eigOfSys+i*temp;
```

```
end
g0=poly(eigens)';
g=zeros(s,1);
for i=1:s
    g(i)=-g0(s+2-i);
end
G=[G0,g];
eig(G);

% get w
w=zeros(1,s);
w(1,s)=1;

%% Step 4: Calculate L, H, q
% get L
L=-Vs_matrix(1:s,:)-g*v;
% get H q
q=v*D;
H=T_x_z*B-L*D;

%% Prepare to return designed matrix
Tout=T_x_z;
Gout=G;
Hout=H;
```

```
    Lout=L;
    vout=v;
    wout=w;
    qout=q;
else
    Tout='No Solution';
    Gout='No Solution';
    Hout='No Solution';
    Lout='No Solution';
    vout='No Solution';
    wout='No Solution';
    qout='No Solution';
end
end
```

3.3.5 UIDO Design for Fault Diagnosis for Linear Systems

Consider a linear system with multiple faults and disturbances:

$$\dot{x} = Ax + Bu + \begin{bmatrix} E_{d,1} & E_{d,2} & \cdots & E_{d,k_d} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_{k_d} \end{bmatrix} + \begin{bmatrix} E_{f,1} & E_{f,2} & \cdots & E_{f,k_f} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_{k_f} \end{bmatrix} \quad (3.90)$$

$$y = Cx + Du + \begin{bmatrix} F_{d,1} & F_{d,2} & \cdots & F_{d,k_d} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_{k_d} \end{bmatrix} + \begin{bmatrix} F_{f,1} & F_{f,2} & \cdots & F_{f,k_f} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_{k_f} \end{bmatrix} \quad (3.91)$$

There are k_d unknown disturbances and k_f unknown faults in this system. For fault diagnosis, k_f UIDO need to be designed.

Design of i^{th} UIDO: detect, isolate and identify f_i

Step 1 Rearrangement of the System

In this step, only f_i is considered as the fault, and the other faults and disturbances are all considered as disturbances

$$\dot{x} = Ax + Bu + \begin{bmatrix} E_{f,1} & \cdots & E_{f,i-1} & E_{f,i+1} & \cdots & E_{f,k_f} & E_{d,1} & \cdots & E_{d,k_d} \end{bmatrix} \begin{bmatrix} f_1 \\ \vdots \\ f_{i-1} \\ f_{i+1} \\ \vdots \\ f_{k_f} \\ d_1 \\ \vdots \\ d_{k_d} \end{bmatrix} + E_{f,i} f_i \quad (3.92)$$

$$y = Cx + Du + \begin{bmatrix} F_{f,1} & \cdots & F_{f,i-1} & F_{f,i+1} & \cdots & F_{f,k_f} & F_{d,1} & \cdots & F_{d,k_d} \end{bmatrix} \begin{bmatrix} f_1 \\ \vdots \\ f_{i-1} \\ f_{i+1} \\ \vdots \\ f_{k_f} \\ d_1 \\ \vdots \\ d_{k_d} \end{bmatrix} + F_{f,i} f_i \quad (3.93)$$

Step 2 Design the UIDO Based on System (3.92)-(3.93)

According to section 3.3.3, the ith UIDO can be designed as follows.

$$\dot{z}^i = G^i z^i + H^i u^i + L^i y^i \tag{3.94}$$

$$r^i = v^i y^i - w^i z^i - q^i u^i \tag{3.95}$$

Equations (3.94)-(3.95) can be designed automatically using the MATLAB program in section 3.3.4

4. APPLICATION TO EXOTHERMIC CSTRS

This chapter presents the results obtained by the application of the algorithms in Chapter 3 to exothermic Continuous Stirred-tank Reactors (CSTRs). The chemical reaction rate is quite sensitive to the system temperature. For an exothermic CSTR system, runaway reaction or thermal explosion may occur if the heat generation rate exceeds the heat removal rate. This energy accumulation may result from malfunction of cooling system or temperature sensors and it could seriously affect the closed-loop temperature-control system. Therefore, it is very important to detect faults in sensors and faults in cooling jacket in the CSTR system.

As for the model of CSTR systems, it will be assumed that the exothermic CSTR is adequately modeled by three differential equations:

- A component mass balance for the reactant.
- An energy balance for the reactor.
- An energy balance for the cooling jacket.

Therefore the model includes three state variables: component concentration (C_A), reactor temperature (T) and cooling jacket temperature (T_w). Three representative case studies will be presented here:

- Case 1: two possible faults in the reactor and cooling jacket temperature sensors.
- Case 2: one possible fault in the reactor temperature sensor, and another one in heat exchanger.

- Case 3: location of possible faults is the same as case 2, but a model uncertainty of the reaction activation energy is introduced in the model.

Each of the case studies will involve a different reaction system.

4.1 Case 1: Two Temperature Sensor Faults

For some highly exothermic CSTRs with explosive reactants or products typically in munitions factories, the reactor and the cooling system have to be well maintained. And the fouling in the cooling system is relatively easy to be regularly maintained. The possible risks may come from the presence of faults in the temperature sensors during the operation process. Therefore, this case only considers one possible fault in the temperature sensor of reactor and the other possible fault in the temperature sensor of cooling jacket.

4.1.1 Introduction

RDX, known as Research Department Formula X, is an explosive nitroamine which is widely used in military and industrial applications. It was developed as an explosive which was several times as powerful as TNT, and RDX was widely used during World War II. RDX is stable in storage and is considered one of the most powerful and brisant of the military high explosives [84]. During World War II, the US produced about 15,000 long tons (15,000 t) per month and Germany about 7,000 long tons (7,100 t) per month [85]. RDX has several advantages for advanced propulsion, including better performances (large amount of gas, high energy, high specific impulse for rockets and high impetus for guns), safety (difficult accidental ignition and low sensitivity in open air), and environment friendliness (smokeless, nontoxic, no corrosive combustion products) [86].

There are several methods of RDX manufacture including: (1) the reaction of nitric acid with hexamine [87], (2) reaction of the mixture of hexamine, ammonium nitrate and nitric acid [88], (3) preparation from formaldehyde, sulphamic acid and nitric acid [88], (4) preparation from paraformaldehyde, ammonium nitrate and acetic anhydride [88], (5) preparation from hexamine dinitrate, ammonium dinitrate and acetic anhydride [89]. The most widely used method in munitions factory is the reaction of hexamine with excess concentrated nitric acid. But this method is very dangerous in its reaction process [90].

4.1.2 Reactive System

The chemical reaction of hexamine and nitric acid to manufacture RDX is expressed by the following equation including main reaction and side reaction [90].

$$C_6H_{12}N_4 + 13HNO_3 \rightarrow \frac{1}{2}C_3H_6N_6O_6 + \frac{9}{2}CH_2(ONO_2) + \frac{5}{2}NH_4NO_3 + \frac{3}{2}H_2O \tag{4.1}$$

(Hexamine) (RDX)

A B C D E F

The reaction rate is:

$$r_A = k_0 C_B \exp(-\frac{E_a}{RT}) C_A^{1.28} \tag{4.2}$$

where C_B is the major component in the mixture, and is assumed to be constant during the process.

4.1.3 Modeling

Model of CSTR

The mass balance in the reactor is:

$$\dot{C_A} = \frac{F}{V}(C_{Ain} - C_A) - r_A \tag{4.3}$$

The heat balance in the reactor is:

$$\dot{T} = \frac{F}{V}(T_{in} - T) + \frac{(-\Delta H_R)}{\rho C_p} r_A - \frac{UA(T - T_w)}{V\rho C_p} \tag{4.4}$$

The heat balance in the cooling jacket is:

$$\dot{T}_w = \frac{F_w}{V_w}(T_{win} - T_w) + \frac{UA(T - T_w)}{V_w \rho_w C_{pw}} \tag{4.5}$$

Table 4.1 gives the process parameters. These parameters are mainly taken from [91], except that instead of assuming constant cooling temperature, the cooling jacket dynamics is included in the model.

Modeling of Faults

Two faults will be considered:

- An additive fault in reactor temperature sensor
- An additive fault in cooling jacket temperature sensor

No disturbances and other faults are considered in this case.

The system output is:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} T + f_1 \\ T_w + f_2 \end{bmatrix} \tag{4.6}$$

Table 4.1: Process parameters of RDX manufacturing in CSTR

Parameter	Description	Value	Unit
V	Volume of the reactor	630	L
k_0	Frequency factor of Arrehenius form	2.06×10^4	$(L/mole)^{1.28}\cdot s^{-1}$
E_a	Activation energy	47149	J/mol
ΔH	Enthalpy of reaction (exothermic)	87319.5	J/mol
ρ	Density of the reacting mixture	1317.5	g/L
C_p	Heat capacity of the reacting mixture	1.989	J/(g·K)
F_{in}	Feed flow rate	0.79	L/s
C_{Ain}	Inlet feed concentration of component A	0.9851	mol/L
C_{Bin}	Inlet feed concentration of component B	20.9087	mol/L
T_{in}	Inlet feed temperature	298.15	K
V_w	Volume of the cooling jacket	60	L
ρ_w	Density of the reacting coolant	1000	g/L
C_{pw}	Heat capacity of the coolant	4.2	J/(g·K)
U	Overall heat transfer coefficient	1400	$w/m^2\cdot K$
$Å$	Overall heat transfer area	7	m^2
T_{win}	Inlet coolant temperature	293.15	K
F_{win}	Inlet coolant flow rate	2	L/s
R	Gas constant	8.3144621	J/(K·mol)

C_A, T, T_w are state variables. At steady state:

$C_{A,s}$=0.3615 mol/L

T_s=301.2448K

T_w=297.5088 K

4.1.4 Observer Design

There are two possible faults in the system, thus two observers are required. Each observer is to estimate one fault, and the effect of the other fault is decoupled on the observer and residual.

Observer 1: Estimate the Reactor Temperature Sensor Fault

The objective of this observer is to estimate the possible fault in the reactor temperature sensor. In order to decouple the effect of the other possible fault in the cooling jacket temperature sensor, cooling jacket fault is considered to be disturbance.

The model of CSTR is linearized at steady state (fault and disturbance are zero). The linearized system is as follows (3.7)-(3.8):

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{E_d d} + \mathbf{E_f f}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du} + \mathbf{F_d d} + \mathbf{F_f f}$$

where:

$$x = \begin{bmatrix} C_A - C_{A,s} \\ T - T_s \\ T_w - T_{w,s} \end{bmatrix} \qquad u = \left[F_{w_{in}} - F_{w_{in,s}}\right]$$

$$\mathrm{A} = \begin{bmatrix} -0.0040 & 0 & 0 \\ 0.0923 & -0.0056 & 0.0059 \\ 0 & 0.0389 & -0.0722 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ -0.0726 \end{bmatrix}$$

$$\mathrm{C} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathrm{E_d} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mathrm{E_f} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$F_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad F_f = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Based on the linearized system, the unknown input diagnostic observer is as follows (3.15)-(3.16):

$$\dot{\mathbf{z}} = \mathbf{Gz} + \mathbf{Hu} + \mathbf{Ly}$$

$$\mathbf{r} = \mathbf{vy} - \mathbf{wz} - \mathbf{qu}$$

Based on the observer, the estimated fault is as follows (3.33):

$$\mathbf{f} = [-\mathbf{w}\mathbf{G}^{-1}(\mathbf{T}\mathbf{E_f} - \mathbf{L}\mathbf{F_f}) + \mathbf{v}\mathbf{F_f}]^{-1}\mathbf{r}$$

According to section (3.3.3), set s=3, then:

$$H_{f,s} = \begin{bmatrix} \mathrm{F_f} & 0 & \cdots & 0 \\ C\mathrm{E_f} & \mathrm{F_f} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}\mathrm{E_f} & \cdots & C\mathrm{E_f} & \mathrm{F_f} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$H_{o,s} = \begin{bmatrix} C \\ C\mathrm{A} \\ \vdots \\ C\mathrm{A}^s \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.0923 & -0.0056 & 0.0059 \\ 0 & 0.0389 & -0.0722 \\ -0.0009 & 0.0003 & -0.0005 \\ 0.0036 & -0.0030 & 0.0054 \\ 0 & 0 & 0 \\ -0.0003 & 0.0002 & -0.0004 \end{bmatrix}$$

$$H_{d,s} = \begin{bmatrix} \mathrm{F_d} & 0 & \cdots & 0 \\ C\mathrm{E_d} & \mathrm{F_d} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}\mathrm{E_d} & \cdots & C\mathrm{E_d} & \mathrm{F_d} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$rank[H_{f,s} \quad H_{o,s} \quad H_{d,s}] = 8 > rank[H_{o,s} \quad H_{d,s}] = 7$$

Thus it is solvable of $\mathrm{v_s}[\mathrm{H_{f,s}} \quad \mathrm{H_{o,s}} \quad \mathrm{H_{d,s}}] = [* \quad 0 \quad 0]$ for $\mathrm{v_s}$

Step 1 Solve $\boldsymbol{v_s}[\boldsymbol{H_{f,s}} \quad \boldsymbol{H_{o,s}} \quad \boldsymbol{H_{d,s}}] = [* \quad \mathbf{0} \quad \mathbf{0}]$ for $\boldsymbol{v_s}$;

After calculation,

$$\mathrm{v_s} = [\mathrm{v_{s,0}} \quad \mathrm{v_{s,1}} \quad \mathrm{v_{s,2}} \quad \mathrm{v_{s,3}}]$$

Step 2 According to vs, get v and T;

$$v = v_{s,s} = [0.9967 \quad 0]$$

$$T = \begin{bmatrix} v_{s,1} & v_{s,2} & \cdots & v_{s,s-1} & v_{s,s} \\ v_{s,2} & \cdots & \cdots & v_{s,s} & 0 \\ \vdots & \cdots & \cdots & \vdots & \vdots \\ v_{s,s} & 0 & \cdots & \cdots & 0 \end{bmatrix} \begin{bmatrix} C \\ C\mathrm{A} \\ \vdots \\ C\mathrm{A}^{s-1} \end{bmatrix} = \begin{bmatrix} 0.0066 & 0.0003 & 0.0000 \\ 0.0920 & 0.0760 & 0.0059 \\ 0 & 0.9967 & -0.0000 \end{bmatrix}$$

Step 3 Choose g to make G stable, and w is also determined.

$$\mathrm{G} = [\mathrm{G}_0 \quad \mathrm{g}] = \begin{bmatrix} 0 & 0 & -0.0000 \\ 1 & 0 & -0.0008 \\ 0 & 1 & -0.0519 \end{bmatrix}$$

$$\mathrm{w} = [0 \quad 0 \quad 1]$$

Step 4 Get L, H, q;

$$L = -\begin{bmatrix} v_{s,0} \\ v_{s,1} \\ \vdots \\ v_{s,s-1} \end{bmatrix} - g v_{s,s} = \begin{bmatrix} 0.0000 & 0 \\ 0.0004 & 0 \\ -0.0298 & 0 \end{bmatrix}$$

$$H = TB - LD = 10^{-3} \times \begin{bmatrix} -0.0017 \\ -0.4298 \\ 0 \end{bmatrix}$$

$$q = vD = 0$$

Observer 2: Estimate Cooling Jacket Temperature Sensor Fault

The objective of this observer is to estimate the possible fault in the cooling jacket temperature sensor. In order to decouple the effect of the other possible fault in the reactor temperature sensor, reactor temperature sensor fault is considered to be disturbance.

The model of CSTR is linearized at steady state (fault and disturbance are zero). The linearized system is as follows (3.7)-(3.8):

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{E_d d} + \mathbf{E_f f}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du} + \mathbf{F_d d} + \mathbf{F_f f}$$

where:

$$x = \begin{bmatrix} C_A - C_{A,s} \\ T - T_s \\ T_w - T_{w,s} \end{bmatrix} \qquad u = \left[F_{w_{in}} - F_{w_{in,s}}\right]$$

$$\mathrm{A} = \begin{bmatrix} -0.0040 & 0 & 0 \\ 0.0923 & -0.0056 & 0.0059 \\ 0 & 0.0389 & -0.0722 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ -0.0726 \end{bmatrix}$$

$$\mathrm{C} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathrm{E_d} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mathrm{E_f} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$F_d = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad F_f = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Based on the linearized system, unknown input diagnostic observer is as follows (3.15)-(3.16):

$$\mathbf{\dot{z} = Gz + Hu + Ly}$$

$$\mathbf{r = vy - wz - qu}$$

Based on the observer, the estimated fault is as follows (3.33):

$$\mathbf{f = [-wG^{-1}(TE_f - LF_f) + vF_f]^{-1}r}$$

According to (3.3.3), set s=3, then:

$$T = \begin{bmatrix} 0.0036 & 0.0002 & 0 \\ 0 & 0.0388 & 0.0096 \\ 0 & 0 & 0.9967 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0 & -0.0000 \\ 1 & 0 & -0.0008 \\ 0 & 1 & -0.0519 \end{bmatrix}$$

$$H = \begin{bmatrix} 0 \\ -0.0007 \\ -0.0724 \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & 0.0000 \\ 0 & 0.0004 \\ 0 & -0.0298 \end{bmatrix}$$

$$v = [0 \quad 0.9967]$$

$$w = [0 \quad 0 \quad 1]$$

$$q=0$$

4.1.5 Simulation

The system model (4.3)-(4.6) and observers are simulated by MATLAB. The initial state of the system is the steady state without fault. White noises with normal distribution have been added to the temperature sensor. Two step faults in the temperature sensors occur at different times. The initial state of the observers is zero. The eigenvalues of observers are set negative making sure the observers are stable.

Simulation Conditions for the Reactor

Initial Conditions

$$\begin{bmatrix} C_{A0} \\ T_0 \\ T_{w0} \end{bmatrix} = \begin{bmatrix} 0.3615 \\ 301.2448 \\ 297.5088 \end{bmatrix}$$

Noise

Normally distributed random noises with zero mean and standard deviation 0.1 were simulated with the MATLAB function "randn" and were added to the simulated values of T and T_w.

Faults

$$\mathrm{f}_1 = 20 \text{ K (it occurs at t=300s)}$$

$$\mathrm{f}_2 = -10\text{K (it occurs at t=100s)}$$

Time

Initial Time: $t_0 = 0\ s$

Final Time: $t_f = 5000\ s$

Simulation Conditions for the Observers

Initial Conditions

$$\text{Observer 1: } \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{Observer 2: } \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Eigenvalues

Based on the eigenvalues of linearized systems, observer eigenvalues are selected faster than the linearized system but slower than ten times the speed of the linearized system.

Linearized System 1: eig = { -0.0031 + 0.0019i, -0.0031 - 0.0019i, -0.0755};

Linearized System 2: eig = { -0.0031 + 0.0019i, -0.0031 - 0.0019i, -0.0755};

Observer 1: eig = {-0.0102, -0.0173, -0.0244};

Observer 2: eig = {-0.0102, -0.0173, -0.0244};

4.1.6 Simulation Results and Discussion

To validate the results, nonlinear model of the CSTR system (4.3)-(4.6), the corresponding linearized system (3.7)-(3.8) and the observers (3.15)-(3.16) have been programmed and simulated by MATLAB. The simulation results are described in the following subsections. In order to better evaluate the performance of the unknown input diagnostic observer, states of system and observers along with residual signal and estimated faults are plotted.

System Results for the Reactor

The CSTR system starts at steady state. Fig. 4.1 shows that at time t=100s and t=300s, there are temperature step changes. But we can not determine if the step signals from the measurements are caused by system state changes or sensor faults. Therefore, it is necessary to build an indicator signal (residual) for fault detection.

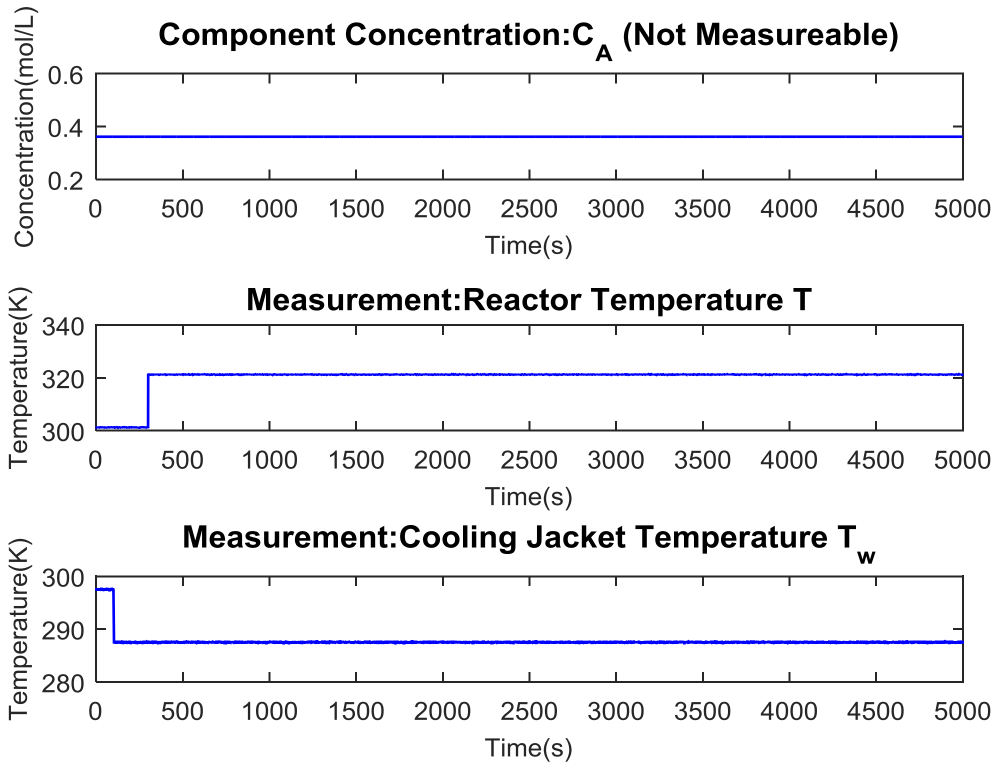

Fig. 4.1 CSTR state variables over time

Simulation Results for the Observers

Observer 1

Fig. 4.2 shows that observer 1 only responds at t = 300s (when reactor temperature sensor fault occurs), and has no response at t = 100s (when cooling jacket temperature sensor fault occurs).

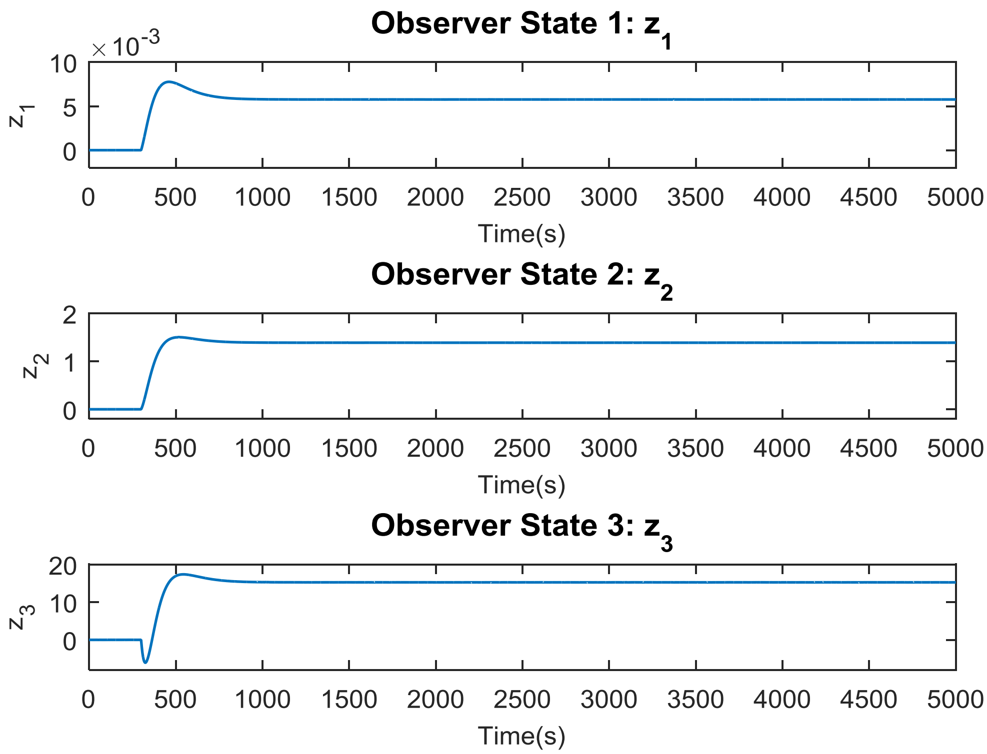

Fig. 4.2 Observer 1 state variables over time

Observer 2

Fig. 4.3 shows that observer 2 responds at t = 100s (when cooling jacket temperature sensor fault occurs).

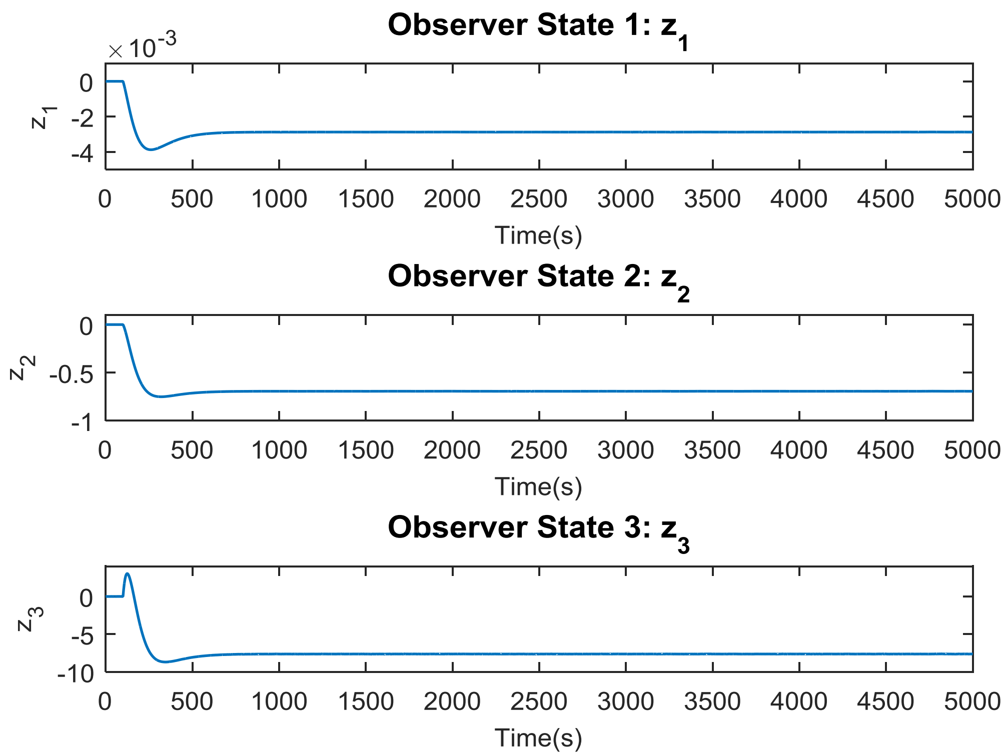

Fig. 4.3 Observer 2 state variables over time

Fault Diagnosis

Even though the observer states can represent the presence of a specific fault, observer states are also affected by the system state. We can not determine the presence of faults by observer states. It is necessary to check the residual signal, which is only sensitive to a specific fault at steady state.

Residual works as an indicator of a specific fault. Fig. 4.4 shows that at t = 300s (and Fig. 4.6 at t = 100s), there is a big spike. This change is caused by the reactor temperature sensor fault. Sensor output signals are the input of observers. Step change in the sensor signal may cause instant large deviations of observer sates and residual at transient period can not be used to identify the size of fault. After a while, the residual comes back to steady state.

From Fig. 4.4 and Fig. 4.6, the new steady states of residuals are nonzero, which successfully indicates the presence of faults. That is to say, residual signals are enough for fault detection and isolation. But just from the residual signal, it is still difficult to evaluate the size of fault. From (3.33), we can estimate the value of fault based on the residual signal. Fig. 4.5 and Fig. 4.7 show that the estimated faults are around the real faults within an error band caused by sensor noises.

Observer 1

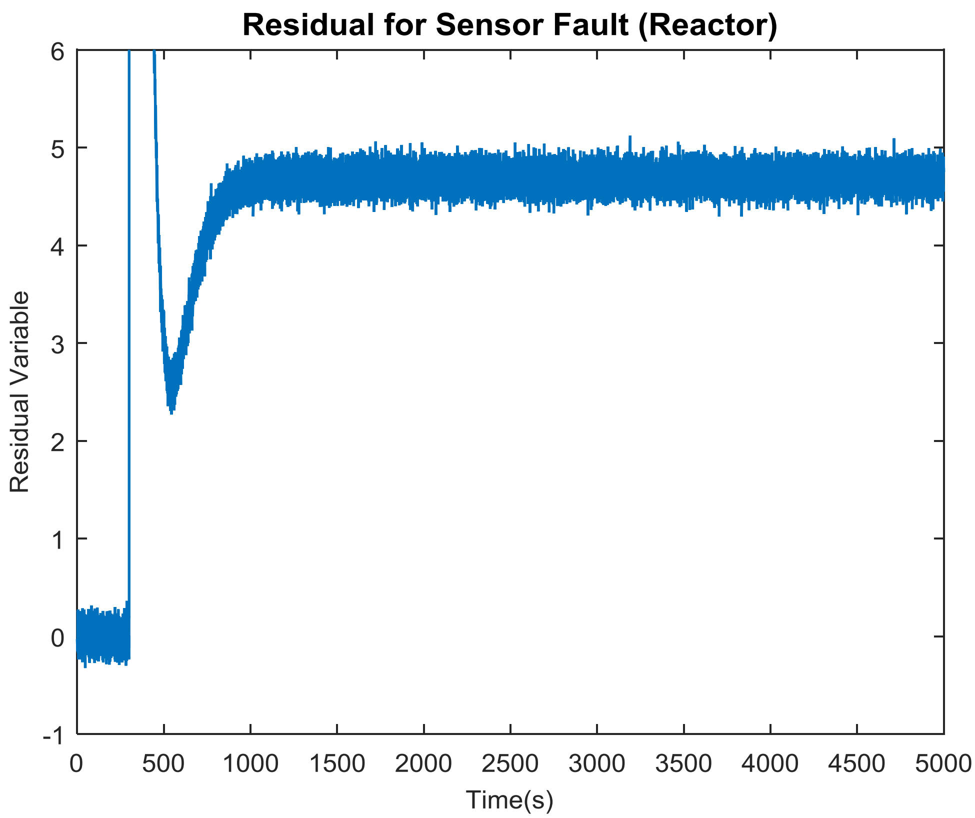

Fig. 4.4 Residual for reactor temperature sensor fault over time

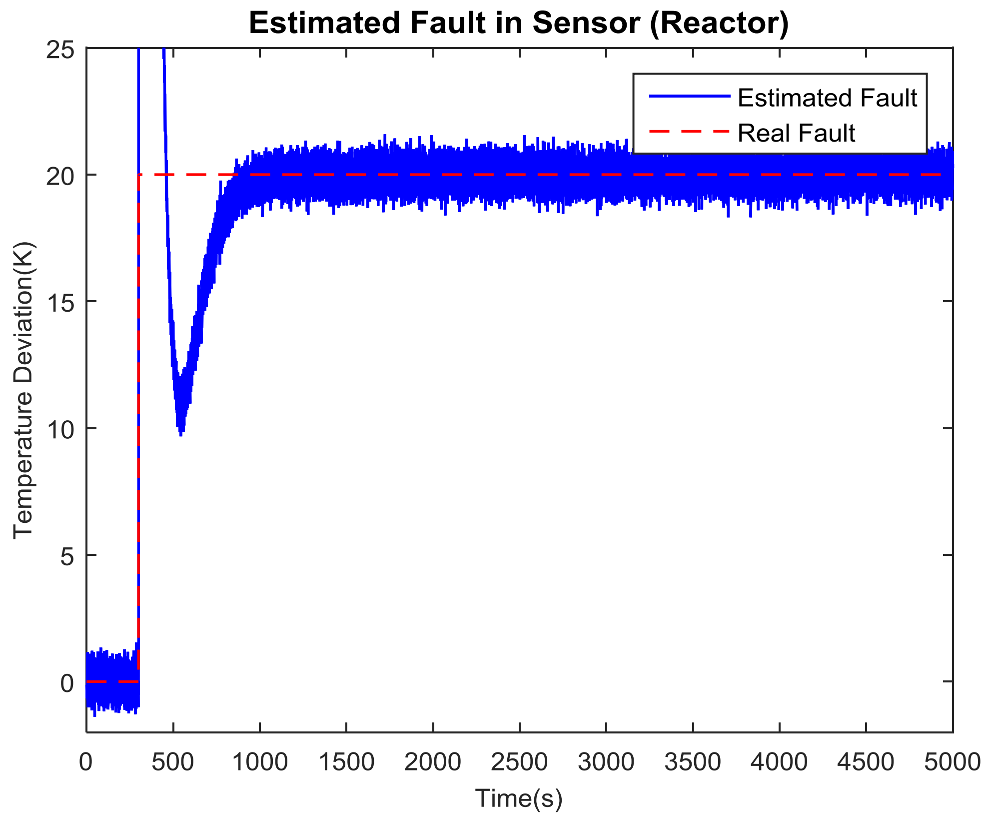

Fig. 4.5 Estimated fault compared with real fault in reactor temperature sensor

Observer 2

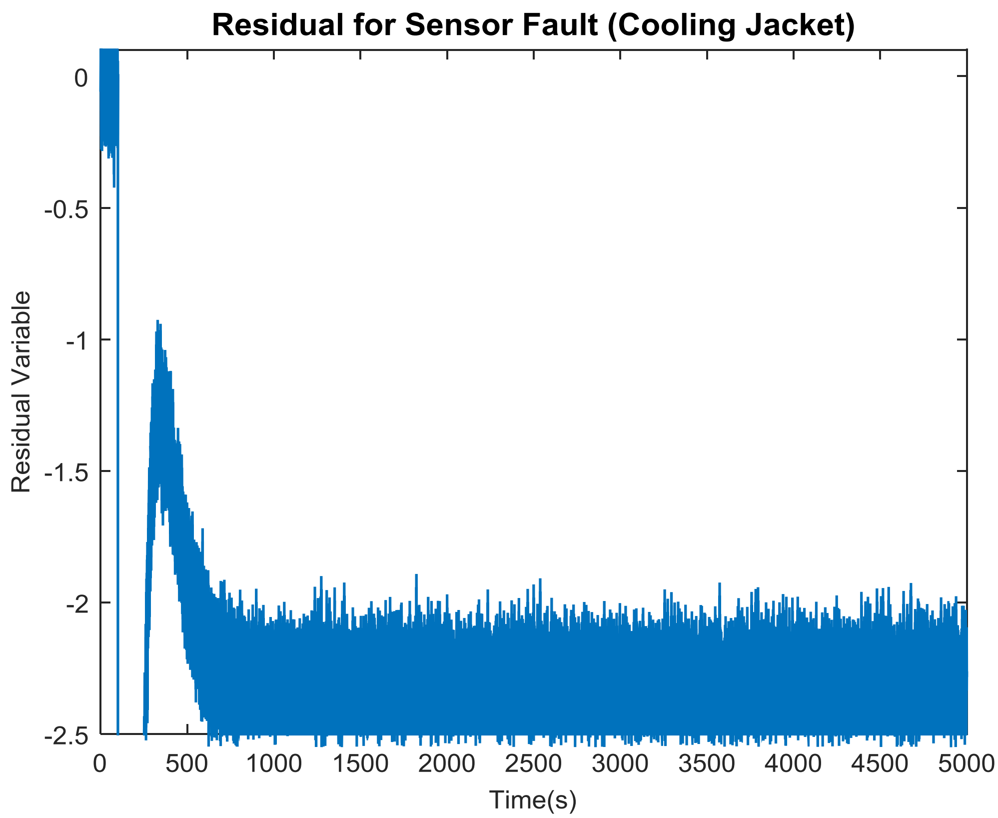

Fig. 4.6 Residual for cooling jacket temperature sensor fault over time

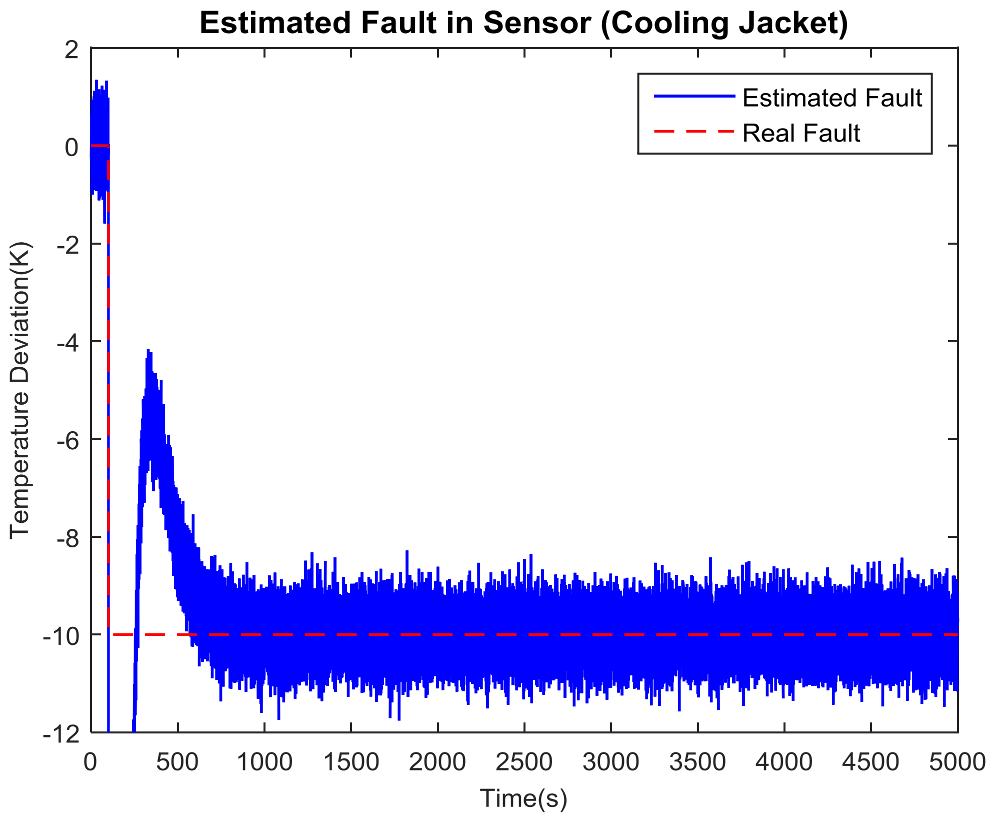

Fig. 4.7 Estimated fault compared with real fault in cooling jacket temperature sensor

So far, fault detection, isolation and identification have been achieved at the same time. By observing the simulation results, it is easy to immediately notice that the unknown input diagnostic observer works well on this exothermic reactor.

4.2 Case 2: One Sensor Fault and One Component Fault

In this case, two possible faults are concerned: one possible fault is in the reactor temperature sensor and the other one is in the heat exchanger caused by fouling. The other parts of CSTR system are well maintained.

4.2.1 Introduction

Acetic anhydride is an organic compound widely used in the production of cellulose acetate, explosives, aspirin, aceticacid, and in others organic synthesis. Its handling can be dangerous. It is an irritant and highly flammable liquid and, in gaseous phase, it can release toxic vapors. Moreover, the acetic anhydride vapor/air mixtures, at temperatures above 322K, maybe come explosive. The acetic anhydride hydrolysis is another reaction with high thermal sensitivity [92, 93].

4.2.2 Reactive System

The hydrolysis of acetic anhydride, is an exothermic reaction in liquid phase catalyzed by sulfuric acid. It can be written as follows:

$$\underset{A}{(CH_3CO)_2O_{(l)}} + \underset{B}{H_2O_{(l)}} \xrightarrow{H_2SO_4 \text{ Catalyst}} \underset{C}{2CH_3COOH_{(l)}} \tag{4.7}$$

The reaction rate is:

$$r_A = k_0 C_s e^{\left(\frac{-E}{RT}\right)} C_A \tag{4.8}$$

where C_s is the sulfuric acid concentration.

4.2.3 Modeling

Model of CSTR

The mass balance in the reactor is:

$$\dot{C_A} = \frac{F}{V}(C_{Ain} - C_A) - r_A \tag{4.9}$$

The heat balance in the reactor is:

$$\dot{\mathrm{T}} = \frac{\mathrm{F}}{\mathrm{V}}(\mathrm{T_{in}} - \mathrm{T}) + \frac{(-\Delta \mathrm{H_R})}{\rho \mathrm{C_p}} \mathrm{r_A} - \left(\frac{(\mathrm{U} - \mathrm{f_2})\mathrm{A}(\mathrm{T} - \mathrm{T_w})}{\mathrm{V}\rho \mathrm{C_p}}\right) \quad (4.10)$$

The heat balance in the cooling jacket is:

$$\dot{\mathrm{T_w}} = \frac{\mathrm{F_w}}{\mathrm{V_w}}(\mathrm{T_{win}} - \mathrm{T_w}) + \left(\frac{(\mathrm{U} - \mathrm{f_2})\mathrm{A}(\mathrm{T} - \mathrm{T_w})}{\mathrm{V_w}\rho_\mathrm{w} \mathrm{C_{pw}}}\right) \quad (4.11)$$

Table 4.2 gives the values of parameters. The kinetic parameters are from [92, 93].

Modeling of Faults

Two faults will be considered:

- An component fault in cooling jacket because of fouling
- An additive fault in reactor temperature sensor

No disturbances and other faults are considered in this case.

The system output is:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \mathrm{T} + \mathrm{f_1} \\ \mathrm{T_j} \end{bmatrix} \quad (4.12)$$

Table 4.2: Process parameters of acetic anhydride hydrolysis in CSTR

Parameter	Description	Value	Unit
V	Volume of the reactor	100	L
k0	Frequency factor of Arrehenius form	1.85×10^{13}	L/(mole·s)
Ea	Activation energy	93446	J/mol
ΔH	Enthalpy of reaction (exothermic)	58520	J/mol
ρ	Density of the reacting mixture	1050	g/L
Cp	Heat capacity of the reacting mixture	3.533	J/(g·K)
Fin	Feed flow rate	1.5	L/s
CAin	Inlet feed concentration of component A	5	mol/L
Cs	Sulfuric acid concentration in CSTR	2	mol/L
Tin	Inlet feed temperature	323.15	K
Vw	Volume of the cooling jacket	30	L
ρw	Density of the reacting coolant	1000	g/L
Cpw	Heat capacity of the coolant	4.2	J/(g·K)
U	Overall heat transfer coefficient	400	$w/m^2\cdot K$
Å	Overall heat transfer area	10	m^2
Twin	Inlet coolant temperature	293.15	K
Fwin	Inlet coolant flow rate	2	L/s
R	Gas constant	8.3144621	J/(K·mol)

C_A, T, T_w are state variables. At steady state:

$C_{A,s}$=0.0439 mol/L

T_s=365.9042 K

$T_{w,s}$=316.6191 K

4.2.4 Observer Design

There are two possible faults in the system, thus two observers are required. Each observer is to estimate one fault, and the effect of the other fault is decoupled on the observer and residual.

Observer 1: Estimate the Reactor Temperature Sensor Fault

The objective of this observer is to estimate the possible fault in the reactor temperature sensor. In order to decouple the effect of the other possible fault in the heat exchange coefficient caused by fouling in the heat exchanger, heat exchanger fault is considered to be disturbance.

The model of CSTR is linearized at steady state (f1, d are zero). The linearized system is as follows (3.7)-(3.8):

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{E_d d} + \mathbf{E_f f}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du} + \mathbf{F_d d} + \mathbf{F_f f}$$

where:

$$x = \begin{bmatrix} C_A - C_{A,s} \\ T - T_s \\ T_w - T_{w,s} \end{bmatrix} \qquad u = \left[F_{w_{in}} - F_{w_{in,s}}\right]$$

$$\mathrm{A} = \begin{bmatrix} -1.7077 & -0.0062 & 0 \\ 26.7025 & 0.0727 & 0.0108 \\ 0 & 0.0317 & -0.0984 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ -0.7823 \end{bmatrix}$$

$$\mathrm{C} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathrm{E_d} = \begin{bmatrix} 0 \\ 0.0013 \\ -0.0039 \end{bmatrix} \qquad \mathrm{E_f} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$F_d = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad F_f = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Based on the linearized system, the unknown input diagnostic observer is as follows (3.15)-(3.16):

$$\dot{\mathbf{z}} = \mathbf{Gz} + \mathbf{Hu} + \mathbf{Ly}$$

$$\mathbf{r} = \mathbf{vy} - \mathbf{wz} - \mathbf{qu}$$

Based on the observer, the estimated fault is as follows (3.33):

$$\mathbf{f} = [-\mathbf{wG}^{-1}(\mathbf{TE_f} - \mathbf{LF_f}) + \mathbf{vF_f}]^{-1}\mathbf{r}$$

According to section (3.3.3), set s=2, then:

$$H_{f,s} = \begin{bmatrix} \mathrm{F_f} & 0 & \cdots & 0 \\ C\mathrm{E_f} & \mathrm{F_f} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}\mathrm{E_f} & \cdots & C\mathrm{E_f} & \mathrm{F_f} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$H_{o,s} = \begin{bmatrix} C \\ C\mathrm{A} \\ \vdots \\ C\mathrm{A}^s \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 26.7025 & 0.0727 & 0.0108 \\ 0 & 0.0317 & -0.0984 \\ -43.6595 & -0.1610 & -0.0003 \\ 0.8477 & -0.0008 & 0.0100 \end{bmatrix}$$

$$H_{d,s} = \begin{bmatrix} \mathrm{F_d} & 0 & \cdots & 0 \\ C\mathrm{E_d} & \mathrm{F_d} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}\mathrm{E_d} & \cdots & C\mathrm{E_d} & \mathrm{F_d} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.0013 & 0 & 0 \\ -0.0039 & 0 & 0 \\ 0.0001 & 0.0013 & 0 \\ 0.0004 & -0.0039 & 0 \end{bmatrix}$$

$$rank[H_{f,s} \quad H_{o,s} \quad H_{d,s}] = 6 \; > rank[H_{o,s} \quad H_{d,s}] = 5$$

Thus it is solvable of $\mathrm{v_s}[\mathrm{H_{f,s}} \quad \mathrm{H_{o,s}} \quad \mathrm{H_{d,s}}] = [* \quad 0 \quad 0]$ for $\mathrm{v_s}$

Step 1: Solve $\boldsymbol{v_s}[\boldsymbol{H_{f,s}} \quad \boldsymbol{H_{o,s}} \quad \boldsymbol{H_{d,s}}] = [* \quad \mathbf{0} \quad \mathbf{0}]$ for $\boldsymbol{v_s}$;

After calculation,

$$\mathrm{v_s} = [\mathrm{v_{s,0}} \quad \mathrm{v_{s,1}}] = [-0.01189 \quad -0.01905 \quad -0.8003 \quad -0.297 \quad -0.4927 \quad -0.1674]$$

Step 2: According to vs, get v and T;

$$\mathrm{v} = \mathrm{v}_{\mathrm{s,s}} = [-0.4927 \quad -0.1674]$$

$$T = \begin{bmatrix} v_{s,1} & v_{s,2} & \cdots & v_{s,s-1} & v_{s,s} \\ v_{s,2} & \cdots & \cdots & v_{s,s} & 0 \\ \vdots & \cdots & \cdots & \vdots & \vdots \\ v_{s,s} & 0 & \cdots & \cdots & 0 \end{bmatrix} \begin{bmatrix} C \\ C\mathrm{A} \\ \vdots \\ C\mathrm{A}^{s-1} \end{bmatrix} = \begin{bmatrix} -13.1572 & -0.8414 & -0.2858 \\ 0 & -0.4927 & -0.1674 \end{bmatrix}$$

Step 3: Choose g to make G stable, and w is also determined.

$$\mathrm{G} = [\mathrm{G}_0 \quad \mathrm{g}] = \begin{bmatrix} 0 & -0.0132 \\ 1 & -0.2388 \end{bmatrix}$$

$$\mathrm{w} = [0 \quad 1]$$

Step 4: Get L, H, q;

$$L = -\begin{bmatrix} v_{s,0} \\ v_{s,1} \\ \vdots \\ v_{s,s-1} \end{bmatrix} - g v_{s,s} = \begin{bmatrix} 0.0054 & 0.0168 \\ 0.6826 & 0.2570 \end{bmatrix}$$

$$H = TB - LD = \begin{bmatrix} 0.2236 \\ 0.1309 \end{bmatrix}$$

$$q = vD = 0$$

Observer 2: Estimate heat exchange coefficient

The objective of this observer is to estimate the possible fault in the heat exchanger. In order to decouple the effect of the other possible fault in the reactor temperature sensor, reactor temperature sensor fault is considered to be disturbance.

The model of CSTR is linearized at steady state (f2, d are zero). The linearized system is as follows (3.7)-(3.8):

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{E}_{\mathbf{d}}\mathbf{d} + \mathbf{E}_{\mathbf{f}}\mathbf{f}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du} + \mathbf{F_d d} + \mathbf{F_f f}$$

where:

$$x = \begin{bmatrix} C_A - C_{A,s} \\ T - T_s \\ T_w - T_{w,s} \end{bmatrix} \qquad u = \begin{bmatrix} F_{w_{in}} - F_{w_{in,s}} \end{bmatrix}$$

$$\mathrm{A} = \begin{bmatrix} -1.7077 & -0.0062 & 0 \\ 26.7025 & 0.0727 & 0.0108 \\ 0 & 0.0317 & -0.0984 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ -0.7823 \end{bmatrix}$$

$$\mathrm{C} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathrm{E_d} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mathrm{E_f} = \begin{bmatrix} 0 \\ 0.0013 \\ -0.0039 \end{bmatrix}$$

$$F_d = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad F_f = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Based on the linearized system, the unknown input diagnostic observer is as follows (3.15)-(3.16):

$$\dot{\mathbf{z}} = \mathbf{Gz} + \mathbf{Hu} + \mathbf{Ly}$$

$$\mathbf{r} = \mathbf{vy} - \mathbf{wz} - \mathbf{qu}$$

Based on the observer, the estimated fault is as follows (3.33):

$$\mathbf{f} = [-\mathbf{wG^{-1}}(\mathbf{TE_f} - \mathbf{LF_f}) + \mathbf{vF_f}]^{-1}\mathbf{r}$$

According to section (3.3.3), set s=3, then:

$$T = \begin{bmatrix} 0.4214 & 0.0270 & 0.0212 \\ 0 & 0.0158 & 0.8128 \\ 0 & 0 & 0.4971 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0 & -0.0014 \\ 1 & 0 & -0.0404 \\ 0 & 1 & -0.3582 \end{bmatrix}$$

$$H = \begin{bmatrix} -0.0165 \\ -0.6359 \\ -0.3889 \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & -0.0011 \\ 0 & -0.0809 \\ 0 & -0.6837 \end{bmatrix}$$

$$v = [0 \quad 0.4971]$$

$$w = [0 \quad 0 \quad 1]$$

$$q=0$$

4.2.5 Simulation

The system model (4.9)-(4.12) and observers are simulated by MATLAB. The initial state of the system is the steady state without fault. White noises with normal distribution have been added to temperature sensors. One step fault in the temperature sensor and one ramp fault in heat exchanger occur at different time. The initial state of the observers is zero. The eigenvalues of observers are set negative making sure the observers are stable.

Simulation Conditions for the Reactor

Initial Conditions

$$\begin{bmatrix} C_{A0} \\ T_0 \\ T_{w0} \end{bmatrix} = \begin{bmatrix} 0.0439 \\ 365.9042 \\ 316.6191 \end{bmatrix}$$

Noise

Normally distributed random noises with zero mean and standard deviation 0.1 were simulated with the MATLAB function "randn" and were added to the simulated values of T and T_w.

Faults

$f_1 = 20$ K (it occurs at t=300s)

$f_2 = 0 \sim 0.1 \times U$ (it occurs at t=100s and increases continuously. f2 is linear to time. At t=5000s, f2=0.1U)

Time

$$\text{Initial Time: } t_0 = 0\ s$$

$$\text{Final Time: } t_f = 5000\ s$$

Simulation Conditions for the Observers

Initial Conditions

$$\text{Observer 1: } \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\text{Observer 2: } \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Eigenvalues

Based on the eigenvalues of linearized systems, observer eigenvalues are selected faster than the linearized system but slower than ten times the speed of the linearized system.

Linearized System 1: eig = { -0.0217, -0.1032, -1.6086};

Linearized System 2: eig = { -0.0217, -0.1032, -1.6086};

Observer 1: eig = {-0.0868, -0.1520};

Observer 2: eig = {-0.0706, -0.1194, -0.1683};

4.2.6 Simulation Results and Discussions

To validate the results, nonlinear model of the CSTR system (4.9)-(4.12), the corresponding linearized system (3.7)-(3.8) and the observers (3.15)-(3.16) have been programmed and simulated by MATLAB. The simulation results are described in the following subsections. In order to better show out the superiority of the unknown input diagnostic observer, states of system and observers along with residual signal and estimated faults are plotted.

System Results for the Reactor

The CSTR system starts at steady state. Fig. 4.8 shows that at time t = 300s and t = 100s, there is a step change in reactor temperature and a ramp change in cooling jacket, respectively. But we still can not determine that the signal changes from the measurements are caused by system state changes or unknown inputs. Therefore, it is necessary to build an indicator signal (residual) for fault detection.

System State Variable

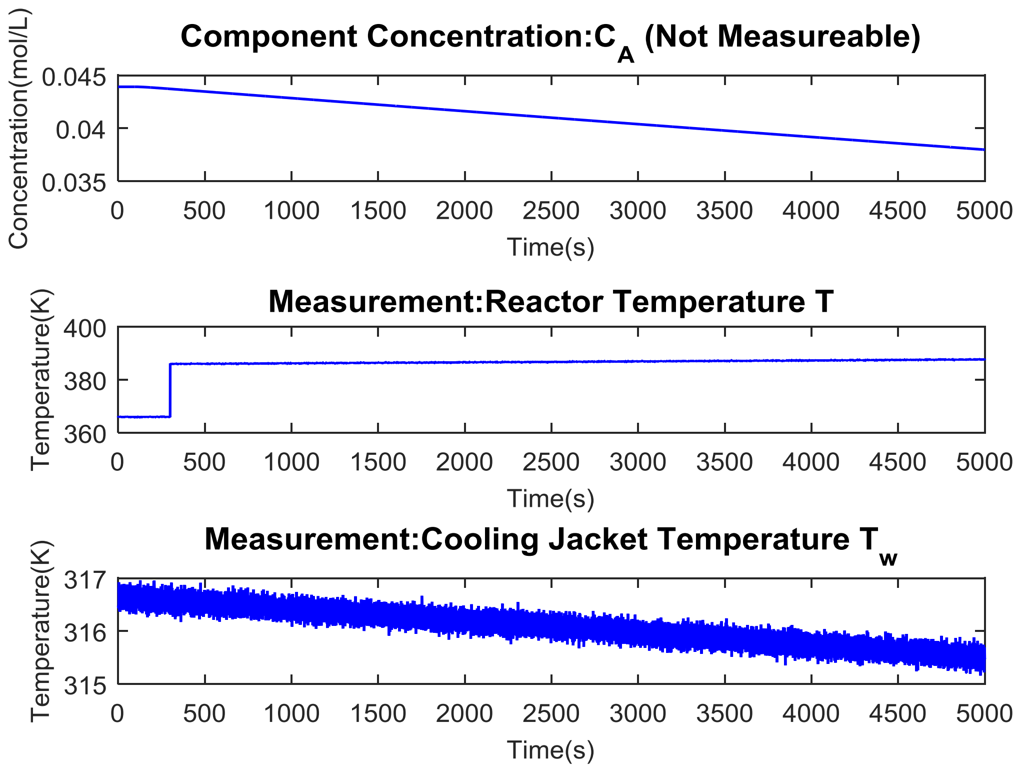

Fig. 4.8 CSTR state variables over time

Simulation Results for the Observers

Observer 1

Fig. 4.9 shows that observer 1 has step change at t=300s (when reactor temperature sensor fault occurs) and has little ramp change since t=100s (when heat exchanger fouling begins).

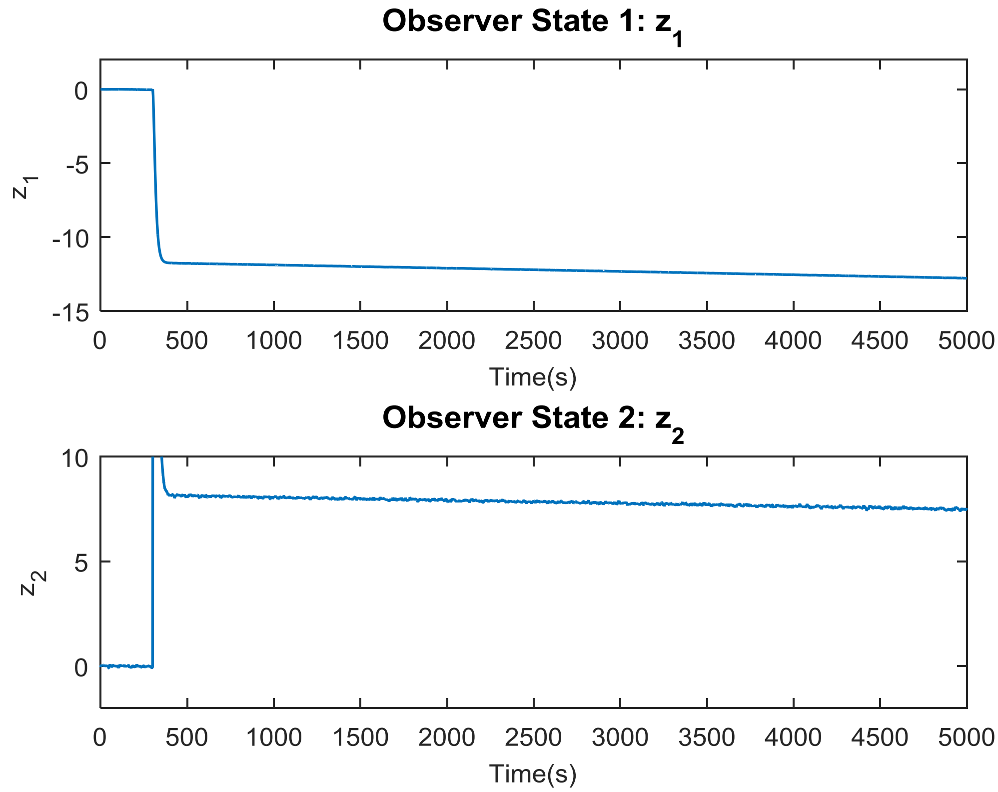

Fig. 4.9 Observer 1 state variables over time

Observer 2

Fig. 4.10 shows that observer 2 responses at t = 100s (when heat exchanger fouling begins).

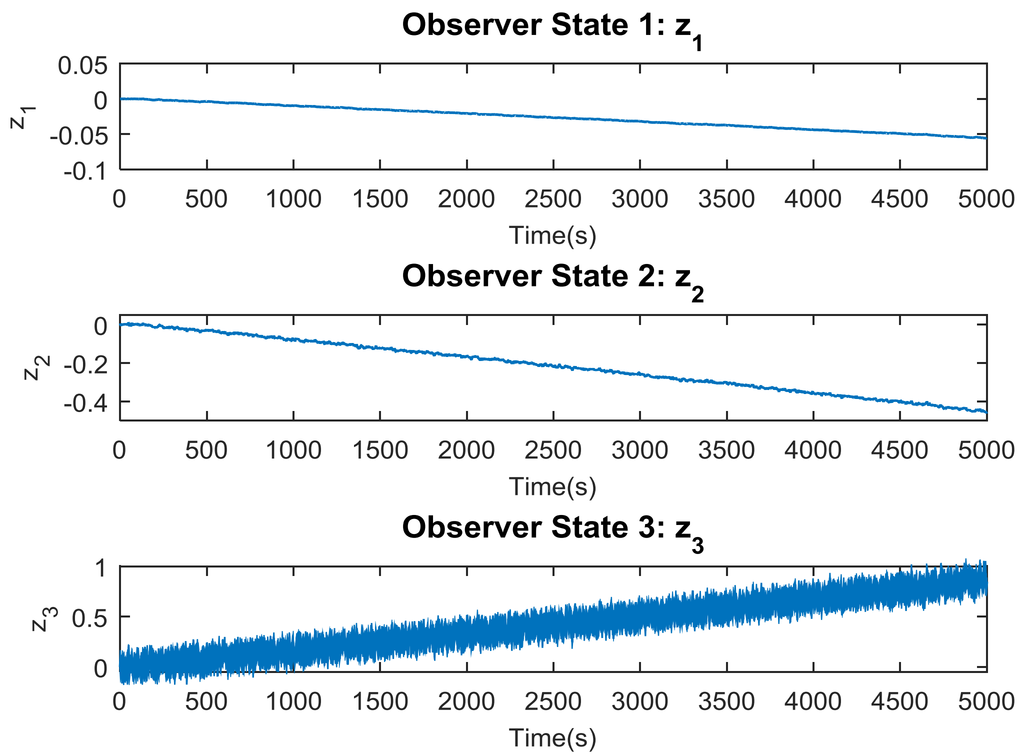

Fig. 4.10 Observer 2 state variables over time

Fault Diagnosis

Fig. 4.9 shows that the observer state may be affected by both system state and faults. We can not determine the presence of faults by observer states. It is necessary to check the residual signal, which is only sensitive to a specific fault at steady state.

Residual works as an indicator of a specific fault. Fig. 4.11 shows that at t = 300s, there is a big spike. This change is caused by the reactor temperature sensor fault. Sensor output signals are the input of observers. Step change in the sensor signal may cause instant large deviations of observer sates and residual at transient period can not be used to identify the size of fault. After a while, the residual comes back to steady state.

From Fig. 4.11 and Fig. 4.13, the new states of residuals are nonzero, which successfully indicate the presence of faults. That is to say, residual signals are enough for fault detection and isolation. But just from the residual signal, it is still difficult to evaluate the size of fault. From (3.33), we can estimate the value of fault based on the residual signal. Fig. 4.12 and Fig. 4.14 show that the estimated faults are around the real faults within an error band caused by sensor noises.

Observer 1

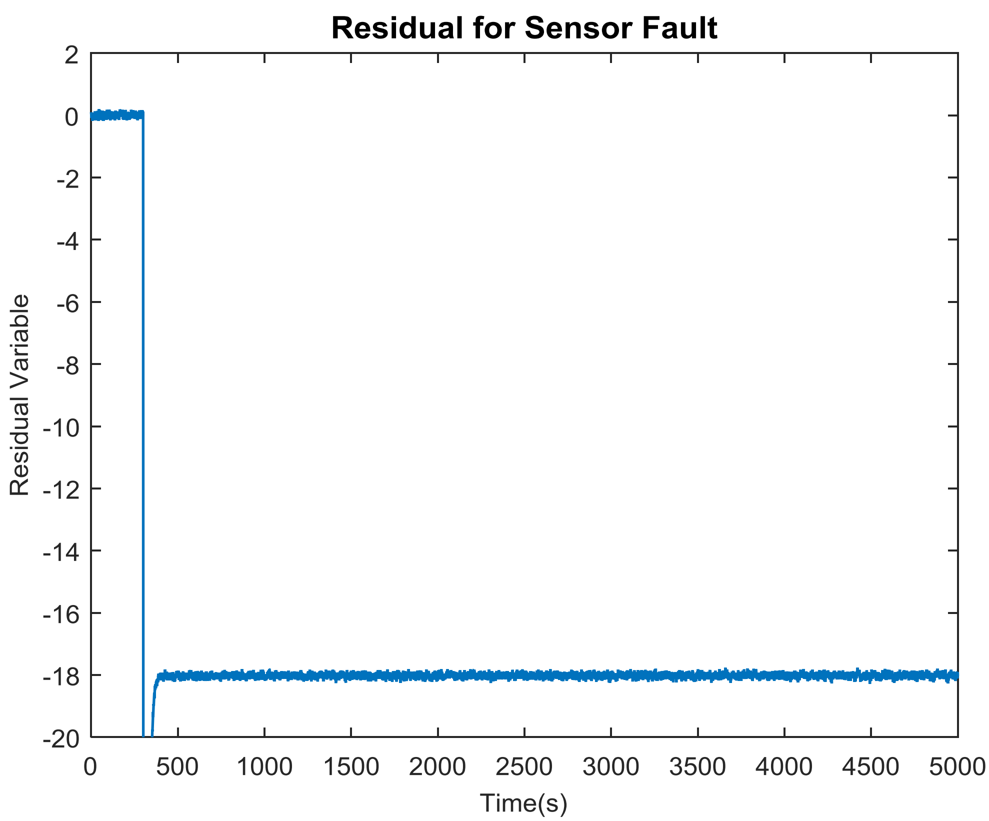

Fig. 4.11 Residual for reactor temperature sensor fault over time

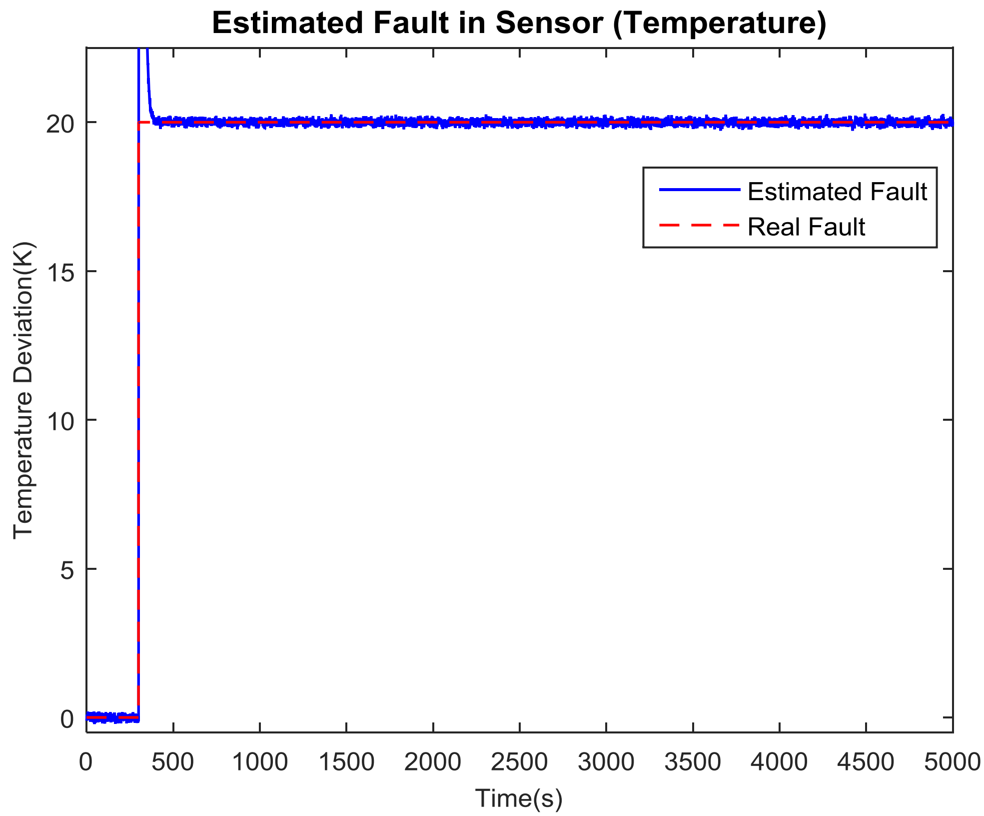

Fig. 4.12 Estimated fault compared with real fault in reactor temperature sensor

Observer 2

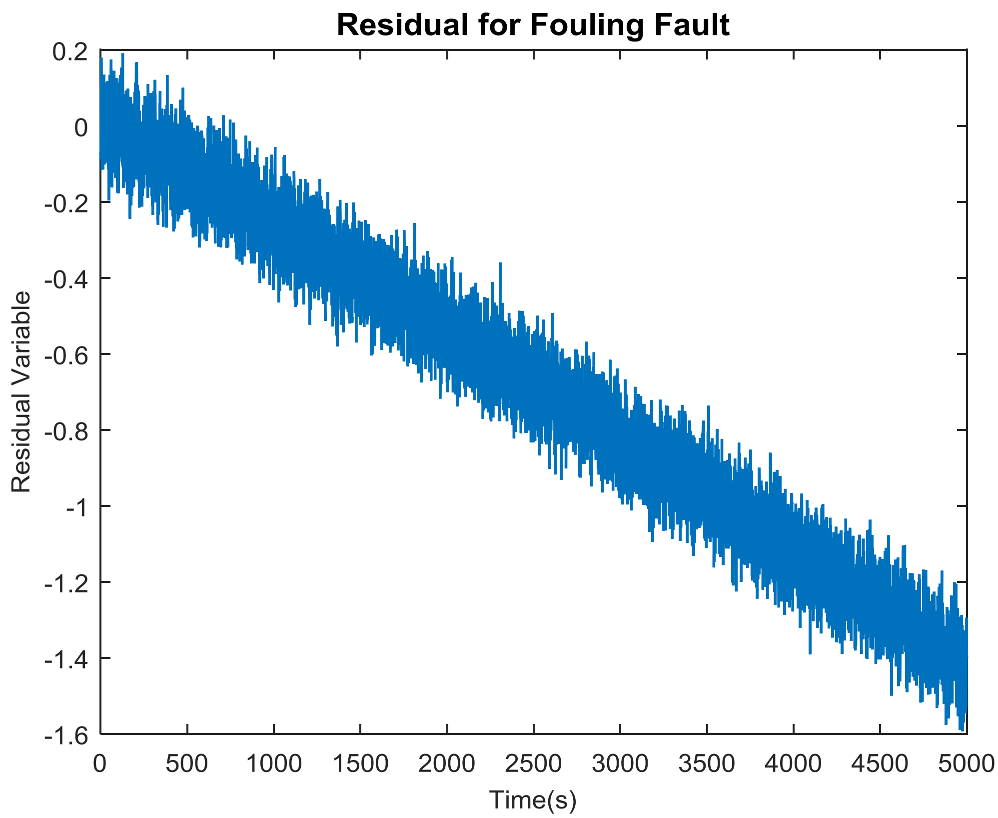

Fig. 4.13 Residual for heat exchanger fouling fault over time

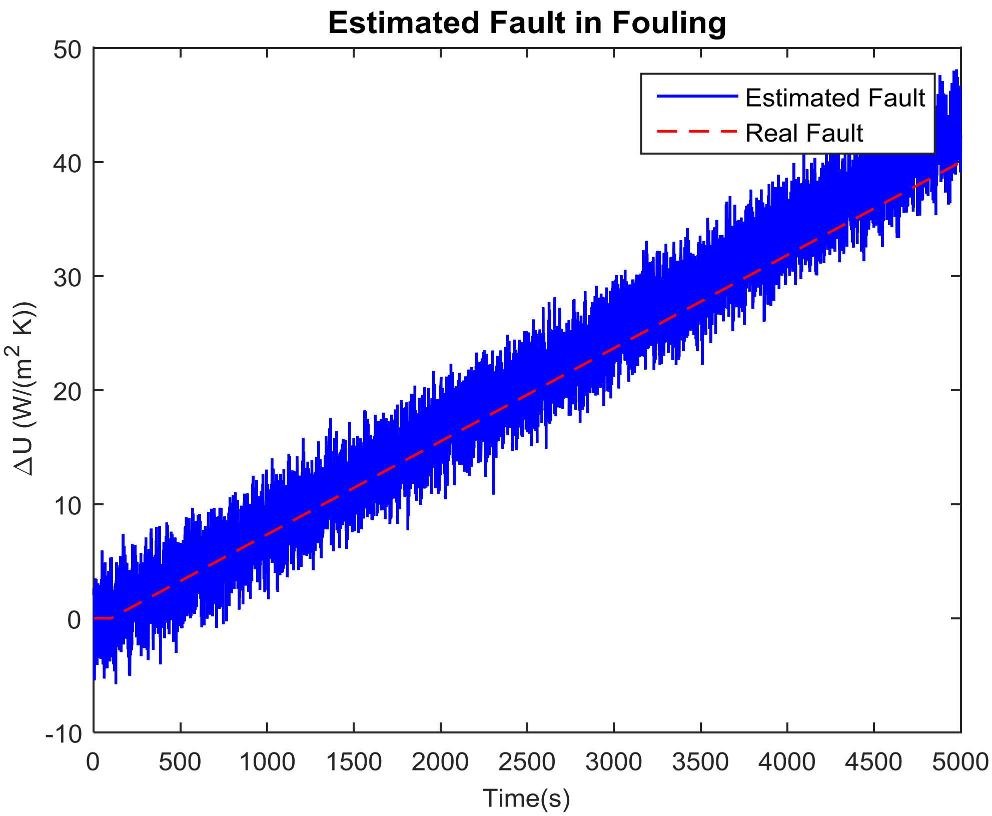

Fig. 4.14 Estimated fault compared with real fault in heat exchanger

So far, fault detection, isolation and identification have been achieved at the same time. By observing the simulation results, it is easy to immediately notice that unknown input diagnostic observer works well for fault diagnosis.

4.3 Case 3: Dealing with Model Uncertainties

In this case, an analytical instrument is used to measure the component concentration. The analytical sensor is more likely to have fault. In this case, another possible fault is in the heat exchanger. Also, model uncertainty of reaction activation energy is present in the model. This case is different from the previous two cases: first, a model uncertainty exists in the reaction activation energy; second, the fault in the heat exchanger may be as high as 50% of the nominal value. Thus model mismatch increases in this case. In order to investigate the effect of model uncertainty on fault diagnosis, two alternative pairs of observers have been examined. The first pair was designed on the basis of the linearized system, as in the previous case study. The second pair considered (i) model uncertainty as an additive disturbance to the reaction rate and (ii) the overall heat exchange rate as fault.

4.3.1 Introduction

A non-isothermal continuous stirred tank reactor (CSTR) is considered with coolant jacket dynamics, where the following exothermic irreversible reaction between sodium thiosulfate and hydrogen peroxide is taking place [53, 54].

4.3.2 Reactive System

$$\underset{A}{2Na_2S_2O_3} + \underset{B}{4H_2O_2} \rightarrow \underset{C}{Na_2S_3O_6} + \underset{D}{Na_2S_2O_4} + \underset{E}{4H_2O} \tag{4.13}$$

The reaction rate is:

$$r_A = 2k_0 e^{\left(\frac{-E+\boldsymbol{d_1}}{RT}\right)} {C_A}^2 \tag{4.14}$$

where d_1 is the uncertainty of the reaction activation energy, and E is the nominal activation energy.

4.3.3 Modeling

Model of CSTR

The mass balance in the reactor is:

$$\dot{C_A} = \frac{F}{V}(C_{Ain} - C_A) - r_A \tag{4.15}$$

The heat balance in the reactor is:

$$\dot{T} = \frac{F}{V}(T_{in} - T) + r_A \frac{(-\Delta H_R)}{\rho C_p} - \left(\frac{(U - \mathbf{f_2})A(T - T_w)}{V\rho C_p}\right) \tag{4.16}$$

The heat balance in the cooling jacket is:

$$\dot{T_w} = \frac{F_w}{V_w}(T_{win} - T_w) + \left(\frac{(U - \mathbf{f_2})A(T - T_w)}{V_w \rho_w C_{pw}}\right) \tag{4.17}$$

The parameters are given in Table 4.3. The parameters are mainly from [54].

Modeling of Faults

Two faults will be considered:

- An additive fault $\mathbf{f_1}$ in the analytical sensor
- An fault f_2 in heat exchanger because of fouling

Modeling of Disturbances

One disturbance will be considered:

- The reaction activation energy has an uncertainty $\boldsymbol{d_1}$

The system output is:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} C_A + \mathbf{f_1} \\ \mathrm{T} \\ \mathrm{T_w} \end{bmatrix} \tag{4.18}$$

The concentration of component A is assumed to be measured by an analytical sensor.

Table 4.3: Process parameters of reaction sodium thiosulfate and hydrogen peroxide in CSTR

Parameter	Description	Value	Unit
V	Volume of the reactor	100	L
k_0	Frequency factor of Arrehenius form	6.85×10^{11}	L/(s·mol)
E_a	Activation energy	76534.704	J/mol
ΔH	Enthalpy of reaction (exothermic)	596.619×10^{3}	J/mol
ρ	Density of the reacting mixture	1000	g/L
C_p	Heat capacity of the reacting mixture	4.2	J/(g·K)
F_{in}	Feed flow rate	1	L/s
C_{Ain}	Inlet feed concentration of component A	1	mol/L
C_{Bin}	Inlet feed concentration of component B	2	mol/L
T_{in}	Inlet feed temperature	278.15	K
V_w	Volume of the cooling jacket	30	L
ρ_w	Density of the reacting coolant	1000	g/L
C_{pw}	Heat capacity of the coolant	4.2	J/(g·K)
U	Overall heat transfer rate	500	w/(m^2·K)
$Å$	Overall heat transfer area	10	m^2
T_{win}	Inlet coolant temperature	278.15	K
F_{win}	Inlet coolant flow rate	10	L/s
R	Gas constant	8.3144621	J/(K·mol)

$\mathrm{C_A}$, T, $\mathrm{T_w}$ are state variables. At steady state:

$\mathrm{C_{A,s}}$=0.0555 mol/L

$\mathrm{T_s}$=343.1617 K

$\mathrm{T_{w,s}}$=285.0661 K

4.3.4 Observer Design

There are two possible faults in the system, thus two observers are required. Each observer is to estimate one fault, and the effect of the other fault and disturbance are decoupled on the residual. But in order to deal with the model uncertainties, an alternative pair of observers was also considered and compared to the first pair of observers.

Observer 1: Estimate Analytical Sensor Fault $\mathbf{f_1}$, and Consider $\mathbf{f_2}$ as Disturbance $\mathbf{d_2}$

The objective of this observer is to estimate the possible fault in the analytical sensor. In order to decouple the effect of the other possible fault in the heat exchange coefficient caused by fouling in the heat exchanger, heat exchanger fault is considered to be disturbance.

The model of CSTR is linearized at steady state (fault and disturbance are zero). The linearized system is as follows (3.7)-(3.8):

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{E_d d} + \mathbf{E_f f}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du} + \mathbf{F_d d} + \mathbf{F_f f}$$

where:

$$x = \begin{bmatrix} C_A - C_{A,s} \\ T - T_s \\ T_w - T_{w,s} \end{bmatrix} \qquad u = \left[F_{w_{in}} - F_{w_{in,s}}\right]$$

where:

$$f = f_1 \qquad\qquad d = \begin{bmatrix} f_2 \\ d_1 \end{bmatrix}$$

$$\mathrm{A} = \begin{bmatrix} -0.3506 & -0.0007 & 0 \\ 48.3797 & 0.0830 & 0.0119 \\ 0 & 0.0397 & -0.3730 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ -0.2305 \end{bmatrix}$$

$$\mathrm{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathrm{E_d} = \begin{bmatrix} 0 & 0.0000 \\ 0.0014 & -0.0005 \\ -0.0046 & 0 \end{bmatrix} \qquad \mathrm{E_f} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$F_d = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad F_f = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Based on the linearized system, the unknown input diagnostic observer is as follows (3.15)-(3.16):

$$\dot{\mathbf{z}} = \mathbf{Gz} + \mathbf{Hu} + \mathbf{Ly}$$

$$\mathbf{r} = \mathbf{vy} - \mathbf{wz} - \mathbf{qu}$$

Based on the observer, the estimated fault is as follows (3.33):

$$\mathbf{f} = [-\mathbf{wG^{-1}}(\mathbf{TE_f} - \mathbf{LF_f}) + \mathbf{vF_f}]^{-1}\mathbf{r}$$

According to section (3.3.3), set s=1, then:

$$v = v_{s,s} = [-0.9999 \quad -0.0070 \quad -0.0021]$$

$$T = \begin{bmatrix} v_{s,1} & v_{s,2} & \cdots & v_{s,s-1} & v_{s,s} \\ v_{s,2} & \cdots & \cdots & v_{s,s} & 0 \\ \vdots & \cdots & \cdots & \vdots & \vdots \\ v_{s,s} & 0 & \cdots & \cdots & 0 \end{bmatrix} \begin{bmatrix} C \\ C\mathrm{A} \\ \vdots \\ C\mathrm{A}^{s-1} \end{bmatrix}$$

$$= [-0.9999 \quad -0.0070 \quad -0.0021]$$

$$\mathrm{G} = [\mathrm{G}_0 \quad \mathrm{g}] = -0.1406$$

$$\mathrm{w} = 1$$

$$L = -\begin{bmatrix} v_{s,0} \\ v_{s,1} \\ \vdots \\ v_{s,s-1} \end{bmatrix} - g v_{s,s} = [-0.1306 \quad -0.0009 \quad 0.0004]$$

$$H = TB - LD = 4.8684 \times 10^{-4}$$

$$q = vD = 0$$

Observer 2: Estimate Heat Exchanger Fault f_2, and Consider f_1 as Disturbance d_2

The objective of this observer is to estimate the possible fault in the heat exchanger. In order to decouple the effect of the other possible fault in the reactor temperature sensor, reactor temperature sensor fault is considered to be disturbance.

The model of CSTR is linearized at steady state (f2, d are zero). The linearized system is as follows (3.7)-(3.8):

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{E_d d} + \mathbf{E_f f}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du} + \mathbf{F_d d} + \mathbf{F_f f}$$

where:

$$x = \begin{bmatrix} C_A - C_{A,s} \\ T - T_s \\ T_w - T_{w,s} \end{bmatrix} \qquad u = [F_{w_{in}} - F_{w_{in,s}}]$$

where:

$$f = f_2 \qquad d = \begin{bmatrix} f_1 \\ d_1 \end{bmatrix}$$

$$\mathrm{A} = \begin{bmatrix} -0.3506 & -0.0007 & 0 \\ 48.3797 & 0.0830 & 0.0119 \\ 0 & 0.0397 & -0.3730 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ -0.2305 \end{bmatrix}$$

$$\mathrm{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathrm{E_d} = 10^{-3} \times \begin{bmatrix} 0 & 0.0033 \\ 0 & -0.4703 \\ 0 & 0 \end{bmatrix} \qquad \mathrm{E_f} = \begin{bmatrix} 0 \\ 0.0014 \\ -0.0046 \end{bmatrix}$$

$$F_d = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad F_f = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Based on the linearized system, the unknown input diagnostic observer is as follows (3.15)-(3.16):

$$\dot{\mathbf{z}} = \mathbf{Gz} + \mathbf{Hu} + \mathbf{Ly}$$

$$\mathbf{r} = \mathbf{vy} - \mathbf{wz} - \mathbf{qu}$$

Based on the observer, the estimated fault is as follows (3.33):

$$\mathbf{f} = [-\mathbf{wG}^{-1}(\mathbf{TE_f} - \mathbf{LF_f}) + \mathbf{vF_f}]^{-1}\mathbf{r}$$

According to section (3.3.3), set s=1, then:

$$v = v_{s,s} = [0.0000 \quad 0.0000 \quad 0.9363]$$

$$T = \begin{bmatrix} v_{s,1} & v_{s,2} & \cdots & v_{s,s-1} & v_{s,s} \\ v_{s,2} & \cdots & \cdots & v_{s,s} & 0 \\ \vdots & \cdots & \cdots & \vdots & \vdots \\ v_{s,s} & 0 & \cdots & \cdots & 0 \end{bmatrix} \begin{bmatrix} C \\ C\mathrm{A} \\ \vdots \\ C\mathrm{A}^{s-1} \end{bmatrix}$$

$$= [0.0000 \quad 0.0000 \quad 0.9363]$$

$$\mathrm{G} = [\mathrm{G}_0 \quad \mathrm{g}] = -0.1406$$

$$\mathrm{w} = 1$$

$$L = -\begin{bmatrix} v_{s,0} \\ v_{s,1} \\ \vdots \\ v_{s,s-1} \end{bmatrix} - g v_{s,s} = [0.0000 \quad 0.0372 \quad -0.2176]$$

$$H = TB - LD = -0.2159$$

$$q = vD = 0$$

Observer 1': Estimate Analytical Sensor Fault $\mathbf{f_1}$

Observer 1' considers reaction rate in the form of $\mathrm{r_A} = 2\mathrm{k_0}\mathrm{e}^{\left(-\frac{\mathrm{E}}{\mathrm{RT}}\right)}\mathrm{C_{A_s}}^2 + \mathrm{d_t} = \mathrm{R_s} + \mathrm{d_t}$. Where $\mathrm{d_t}$ is the total disturbance in the reaction rate term. And the entire term $\mathbf{f_2}\mathrm{A}(\mathrm{T} - \mathrm{T_w})$ is also considered as a disturbance $\mathrm{d_2}$. In this way, the system nonlinearity is significantly decreased, and the system model becomes:

$$\dot{\mathrm{C}_\mathrm{A}} = \frac{\mathrm{F}}{\mathrm{V}}(\mathrm{C_{Ain}} - \mathrm{C_A}) - (\mathrm{R_s} + \mathbf{d}_{\boldsymbol{t}})$$

$$\dot{\mathrm{T}} = \frac{\mathrm{F}}{\mathrm{V}}(\mathrm{T_{in}} - \mathrm{T}) + (\mathrm{R_s} + \mathbf{d}_{t})\frac{(-\Delta\mathrm{H_R})}{\rho\mathrm{C_p}} - \left(\frac{\mathrm{UA}(\mathrm{T} - \mathrm{T_w}) + \mathbf{d_2}}{\mathrm{V}\rho\mathrm{C_p}}\right)$$

$$\dot{\mathrm{T}_\mathrm{w}} = \frac{\mathrm{F_w}}{\mathrm{V_w}}\left(\mathrm{T_{jin}} - \mathrm{T_w}\right) + \left(\frac{\mathrm{UA}(\mathrm{T} - \mathrm{T_w}) + \mathbf{d_2}}{\mathrm{V_w}\rho_\mathrm{w}\mathrm{C_{pw}}}\right)$$

The system output is:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} C_A + \mathbf{f_1} \\ \mathrm{T} \\ \mathrm{T_w} \end{bmatrix}$$

The model of CSTR is linearized at steady state (f_2, d are zero). The linearized system is as follows (3.7)-(3.8):

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{E_d d} + \mathbf{E_f f}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du} + \mathbf{F_d d} + \mathbf{F_f f}$$

where:

$$x = \begin{bmatrix} C_A - C_{A,s} \\ T - T_s \\ T_w - T_{w,s} \end{bmatrix} \qquad u = \left[F_{w_{in}} - F_{w_{in,s}}\right]$$

where:

$$f = f_1 \qquad d = \begin{bmatrix} f_2 \\ d_t \end{bmatrix}$$

$$\mathrm{A} = \begin{bmatrix} -0.0100 & 0 & 0 \\ 0 & -0.0219 & 0.0119 \\ 0 & 0.0397 & -0.3730 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ -0.2305 \end{bmatrix}$$

$$\mathrm{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathrm{E_d} = \begin{bmatrix} 0 & -1.0000 \\ 0.0000 & 142.0521 \\ -0.0000 & 0 \end{bmatrix} \qquad \mathrm{E_f} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$F_d = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad F_f = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Based on the linearized system, the unknown input diagnostic observer is as follows (3.15)-(3.16):

$$\dot{\mathbf{z}} = \mathbf{Gz} + \mathbf{Hu} + \mathbf{Ly}$$

$$\mathbf{r} = \mathbf{vy} - \mathbf{wz} - \mathbf{qu}$$

Based on the observer, the estimated fault is as follows (3.33):

$$\mathbf{f} = [-\mathbf{wG^{-1}}(\mathbf{TE_f} - \mathbf{LF_f}) + \mathbf{vF_f}]^{-1}\mathbf{r}$$

According to section (3.3.3), set s=1, then:

$$v = v_{s,s} = [-0.9999 \quad -0.0070 \quad -0.0021]$$

$$T = \begin{bmatrix} v_{s,1} & v_{s,2} & \cdots & v_{s,s-1} & v_{s,s} \\ v_{s,2} & \cdots & \cdots & v_{s,s} & 0 \\ \vdots & \cdots & \cdots & \vdots & \vdots \\ v_{s,s} & 0 & \cdots & \cdots & 0 \end{bmatrix} \begin{bmatrix} C \\ C\mathrm{A} \\ \vdots \\ C\mathrm{A}^{s-1} \end{bmatrix}$$

$$= [-0.9999 \quad -0.0070 \quad -0.0021]$$

$$\mathrm{G} = [\mathrm{G}_0 \quad \mathrm{g}] = -0.0550$$

$$\mathrm{w} = 1$$

$$L = -\begin{bmatrix} v_{s,0} \\ v_{s,1} \\ \vdots \\ v_{s,s-1} \end{bmatrix} - g v_{s,s} = [-0.0450 \quad -0.0003 \quad 0.0006]$$

$$H = TB - LD = 4.8684 \times 10^{-4}$$

$$q = vD = 0$$

Observer 2': Estimate Heat Exchanger Fault f_2 by Estimating $f_2 A(T - T_w)$

Observer 2' considers reaction rate in the form of $\mathrm{r_A} = 2\mathrm{k_0 e}^{\left(-\frac{\mathrm{E}}{\mathrm{RT}}\right)}\mathrm{C_{A_s}}^2 + \mathrm{d_t} = \mathrm{R_s} + \mathrm{d_t}$. Where $\mathrm{d_t}$ is the overall disturbance in the reaction rate term. And sensor fault $\mathrm{f_1}$ is considered as disturbance $\mathrm{d_2}$. This observer is to estimate the entire term $\mathrm{f_3} = \mathrm{f_2 A(T - T_w)}$. Because heat transfer area is known, and T, $\mathrm{T_w}$ can be measured, $\mathrm{f_2}$ is then to be estimated. In this way, the system nonlinearity is significantly decreased, and the system model becomes:

$$\dot{C_A} = \frac{F}{V}(C_{Ain} - C_A) - (R_s + \mathbf{d}_t)$$

$$\dot{T} = \frac{F}{V}(T_{in} - T) + (R_s + \mathbf{d}_t)\frac{(-\Delta H_R)}{\rho C_p} - \left(\frac{UA(T - T_w) + \mathbf{f_3}}{V\rho C_p}\right)$$

$$\dot{T_w} = \frac{F_w}{V_w}\left(T_{jin} - T_w\right) + \left(\frac{UA(T - T_w) + \mathbf{f_3}}{V_w \rho_w C_{pw}}\right)$$

The system output is:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} C_A + \mathbf{f_1} \\ T \\ T_w \end{bmatrix}$$

The model of CSTR is linearized at steady state (f_2, d are zero). The linearized system is as follows (3.7)-(3.8):

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{E_d d} + \mathbf{E_f f}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du} + \mathbf{F_d d} + \mathbf{F_f f}$$

where:

$$x = \begin{bmatrix} C_A - C_{A,s} \\ T - T_s \\ T_w - T_{w,s} \end{bmatrix} \qquad u = \left[F_{w_{in}} - F_{w_{in,s}}\right]$$

$$f = f_3 \qquad d = \begin{bmatrix} f_1 \\ d_t \end{bmatrix}$$

$$A = \begin{bmatrix} -0.0100 & 0 & 0 \\ 0 & -0.0219 & 0.0119 \\ 0 & 0.0397 & -0.3730 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ -0.2305 \end{bmatrix}$$

$$\mathrm{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathrm{E_d} = \begin{bmatrix} 0 & -1.0000 \\ 0 & 142.0521 \\ 0 & 0 \end{bmatrix} \qquad \mathrm{E_f} = 10^{-5} \times \begin{bmatrix} 0 \\ 0.2381 \\ -0.7937 \end{bmatrix}$$

$$F_d = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad F_f = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Based on the linearized system, the unknown input diagnostic observer is as follows (3.15)-(3.16):

$$\dot{\mathbf{z}} = \mathbf{Gz} + \mathbf{Hu} + \mathbf{Ly}$$

$$\mathbf{r} = \mathbf{vy} - \mathbf{wz} - \mathbf{qu}$$

Based on the observer, the estimated fault is as follows (3.33):

$$\mathbf{f} = [-\mathbf{wG^{-1}}(\mathbf{TE_f} - \mathbf{LF_f}) + \mathbf{vF_f}]^{-1}\mathbf{r}$$

According to section (3.3.3), set s=1, then:

$$v = v_{s,s} = [0.0000 \quad 0.0000 \quad 0.9363]$$

$$T = \begin{bmatrix} v_{s,1} & v_{s,2} & \cdots & v_{s,s-1} & v_{s,s} \\ v_{s,2} & \cdots & \cdots & v_{s,s} & 0 \\ \vdots & \cdots & \cdots & \vdots & \vdots \\ v_{s,s} & 0 & \cdots & \cdots & 0 \end{bmatrix} \begin{bmatrix} C \\ C\mathrm{A} \\ \vdots \\ C\mathrm{A}^{s-1} \end{bmatrix} = [0.0000 \quad 0.0000 \quad 0.9363]$$

$$\mathrm{G} = [\mathrm{G_0} \quad \mathrm{g}] = -0.0550$$

$$\mathrm{w} = 1$$

$$L = -\begin{bmatrix} v_{s,0} \\ v_{s,1} \\ \vdots \\ v_{s,s-1} \end{bmatrix} - g v_{s,s} = [0.0000 \quad 0.0372 \quad -0.2978]$$

$$H = TB - LD = -0.2159$$

$$q = vD = 0$$

4.3.5 Simulation

The system model (4.15)-(4.18) and observers are simulated by MATLAB. The initial state of the system is fault free. White noises with normal distribution have been added to the temperature sensor. One step fault in the temperature sensor and one ramp fault in heat exchanger occur at different times. The initial state of the observers is zero. The eigenvalues of observers are set negative making sure the observers are stable.

Simulation Conditions for the Reactor

Initial Conditions

$$\begin{bmatrix} C_{A0} \\ T_0 \\ T_{w0} \end{bmatrix} = \begin{bmatrix} 0.3615 \\ 301.2448 \\ 297.5088 \end{bmatrix}$$

Noise

Normally distributed random noises with zero mean and standard deviation 0.1 (and 0.001) were simulated with the MATLAB function “randn” and were added to the simulated values of T and $\mathrm{T_w}$ (and $\mathrm{C_A}$).

Faults

$\mathrm{f_1} = 20$ K (it occurs at t=300s)

$\mathrm{f_2} = 0 \sim 0.1 \times \mathrm{U}$ (it occurs at t=100s and increases continuously. f2 is linear to time. At t=5000s, f2=0.1U)

Disturbance

Set the reaction activation energy uncertainty as 10% of the nominal value: d1=ΔE= -0.1E

Time

Initial Time: $t_0 = 0\ s$

Final Time: $t_f = 5000\ s$

Observer Part

Initial Conditions

Observer 1: $z = 0$

Observer 2: $z = 0$

Observer 1’: $z = 0$

Observer 2’: $z = 0$

Eigenvalues

Based on the eigenvalues of linearized systems, observer eigenvalues are selected faster than the linearized system but slower than ten times the speed of the linearized system.

Linearized System 1: eig = {-0.0256, -0.2418, -0.3732};

Linearized System 2: eig = {-0.0256, -0.2418, -0.3732};

Linearized System 1’: eig = {-0.0100, -0.0206, -0.3744};

Linearized System 2’: eig= {-0.0100, -0.0206, -0.3744};

Observer 1: eig = -0.1406

Observer 2: eig = -0.1406

Observer 1’:eig = -0.0550

Observer 2’:eig = -0.0550

4.3.6 Simulation Result and Discussion

Similar to previous cases, the system model and observers were simulated. The simulation results are described in the following subsections. In order to better evaluate the performance of the unknown input diagnostic observer, states of system and observers along with residual signals and estimated faults are plotted.

System Results for the Reactor

The CSTR system starts at points near steady state. Fig. 4.15 shows that at time t=300s and t=100s, there is a step change in reactor temperature and a ramp change in cooling jacket, respectively. But we still can not determine that the signal changes from the measurements are caused by system state changes or unknown inputs. Therefore, it is necessary to build an indicator signal (residual) for fault detection.

System State Variable

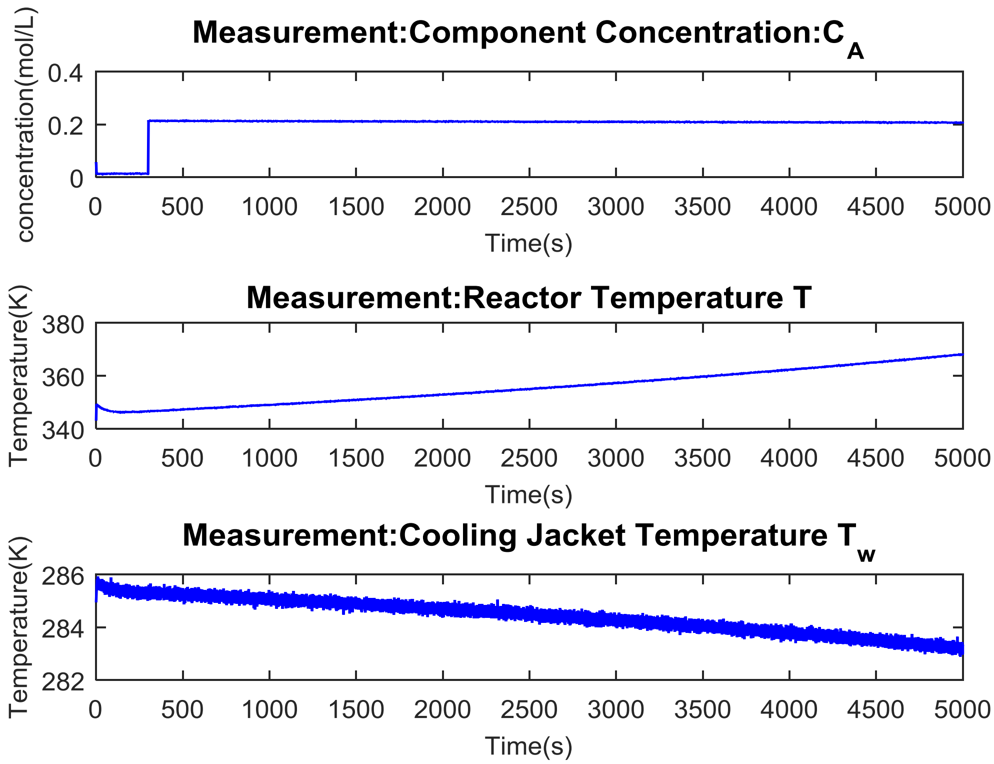

Fig. 4.15 CSTR state variables over time

Simulation Results for the Observers

Observer 1

Fig. 4.16 shows that observer 1 has step change at t=300s (when reactor temperature sensor fault occurs).

Fig. 4.17 shows that observer 2 has obvious ramp change at t=100s.

Fig. 4.18 is the same as Fig. 4.16, and Fig. 4.19 is similar to Fig. 4.17.

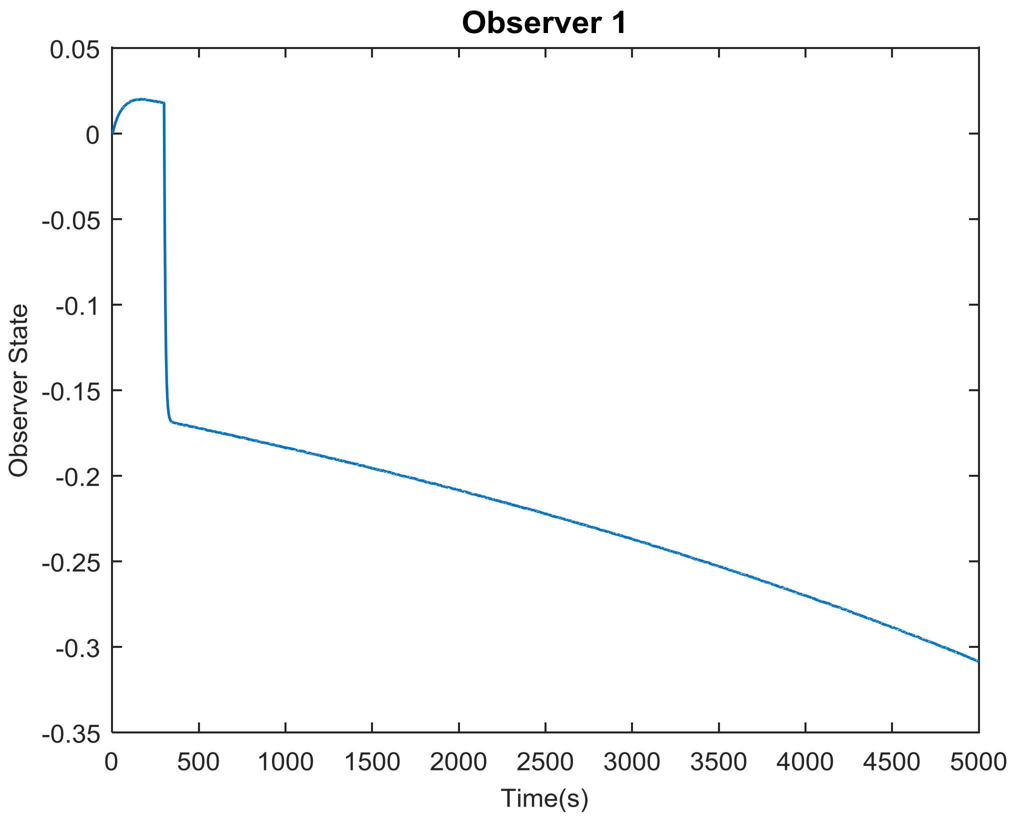

Fig. 4.16 Observer 1 state variables over time

Observer 2

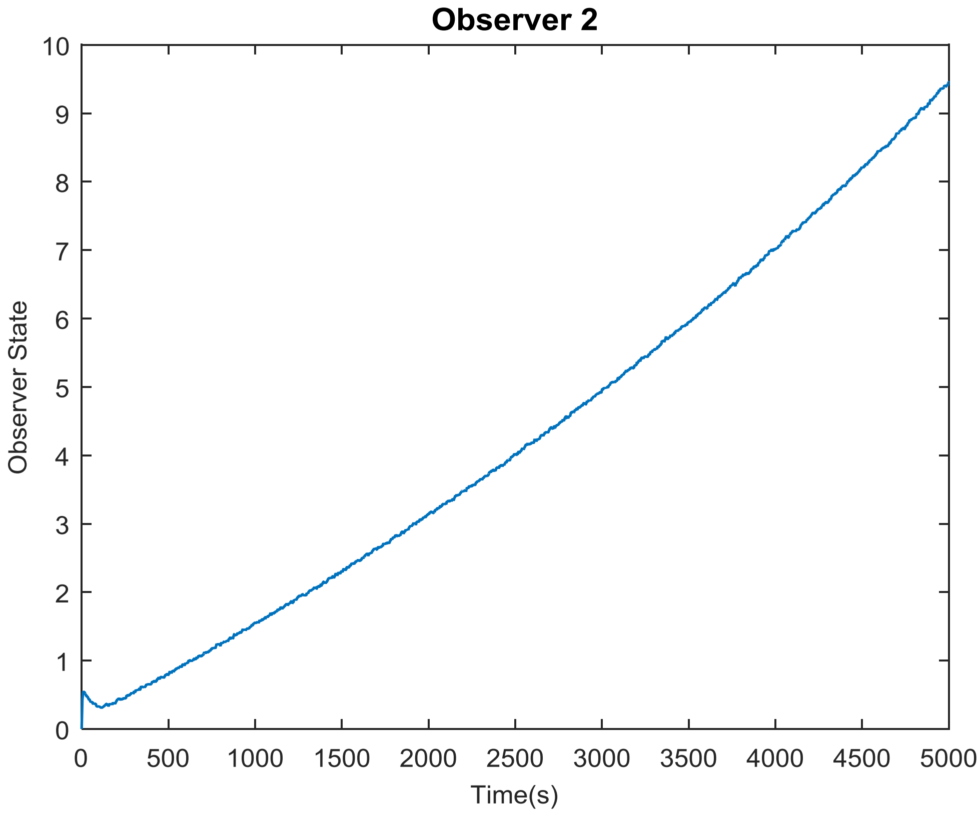

Fig. 4.17 Observer 2 state variables over time

Observer 1'

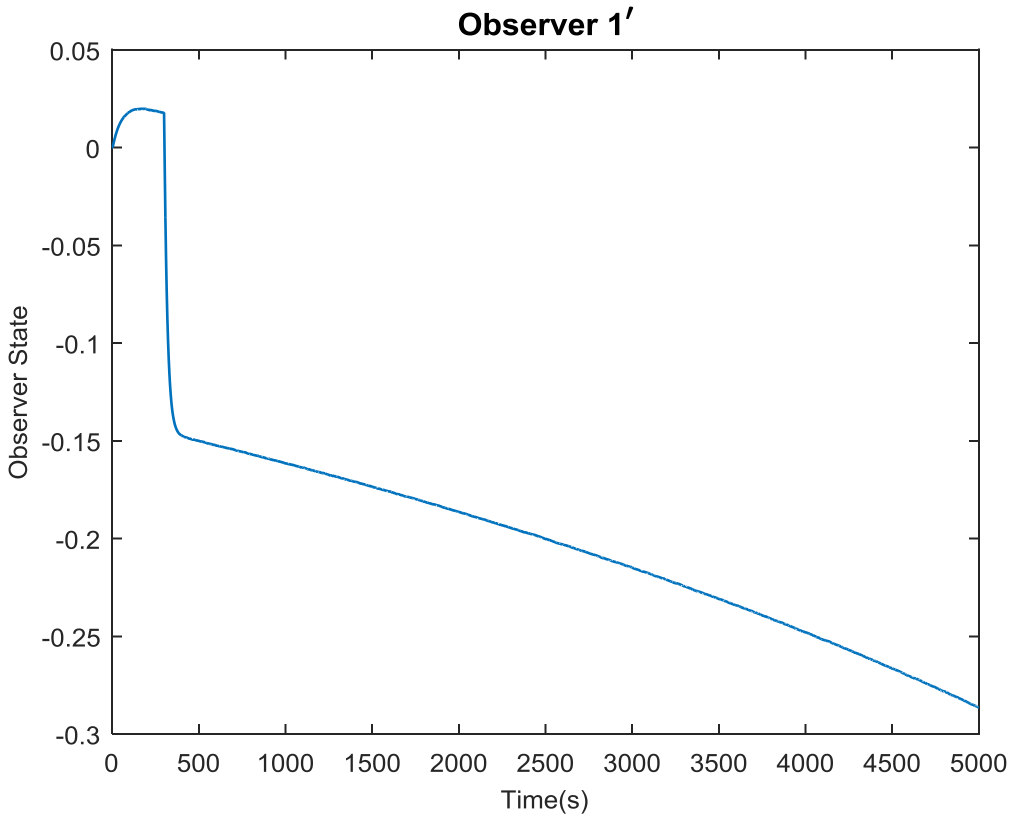

Fig. 4.18 Observer 1' state variables over time

Observer 2’

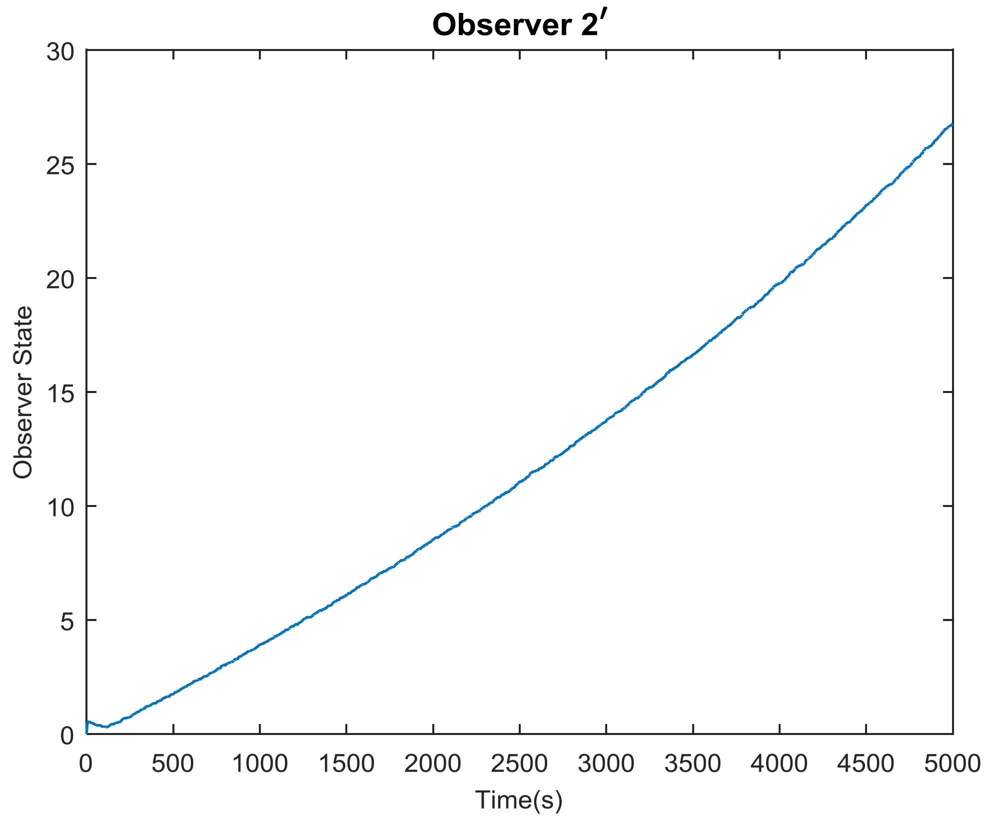

Fig. 4.19 Observer 2’ state variables over time

Fault Diagnosis

Residual works as an indicator of a specific fault. From Fig. 4.20 and Fig. 4.22 (Fig. 4.24 is similar to Fig. 4.20, and Fig. 4.26 is similar to Fig. 4.22), the new states of residuals are nonzero, which successfully indicate the presence of faults. That is to say, residual signals are enough for fault detection and isolation. But just from the residual signal, it is still difficult to evaluate the size of fault. From (3.33), we can estimate the value of fault based on the residual signal. Fig. 4.21 and Fig. 4.25 show that the estimated faults are around the real faults within an error caused by sensor noises. But for Fig. 4.23, there is a big error for the estimation of the fault in heat exchange coefficient because of linearization erorors. However, observer 2' accurately estimates the fault in heat exchange coefficient by decreasing the nonlinearity of the model.

Observer 1

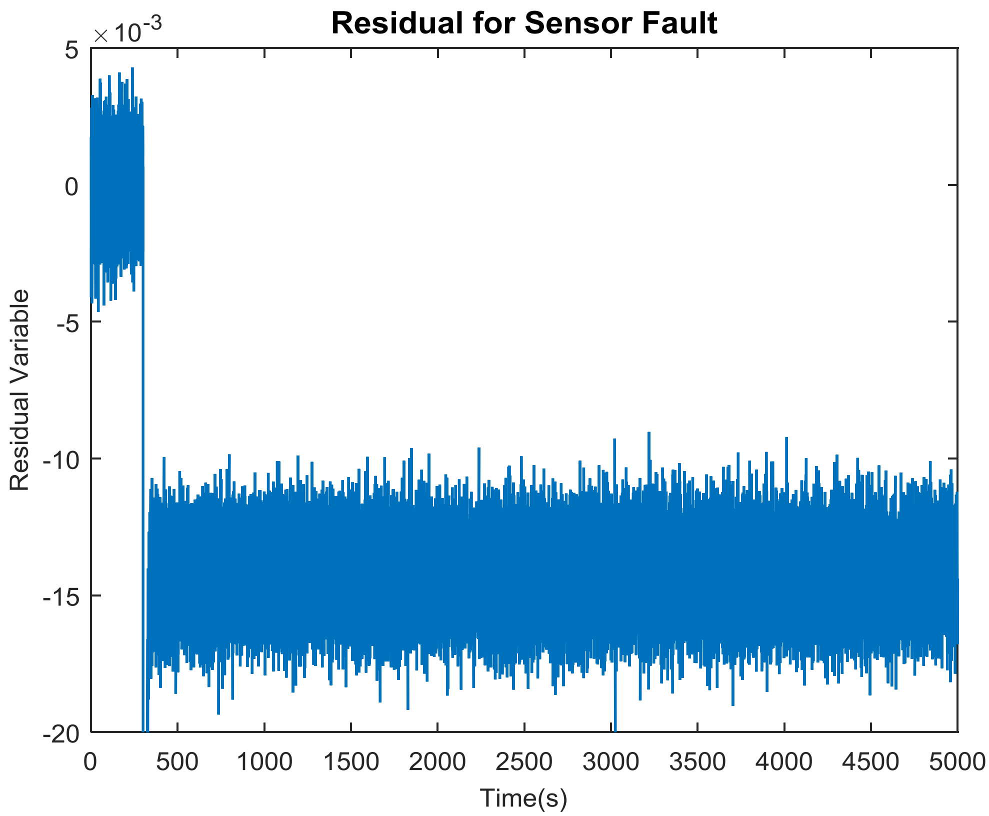

Fig. 4.20 Residual for analytical sensor fault over time

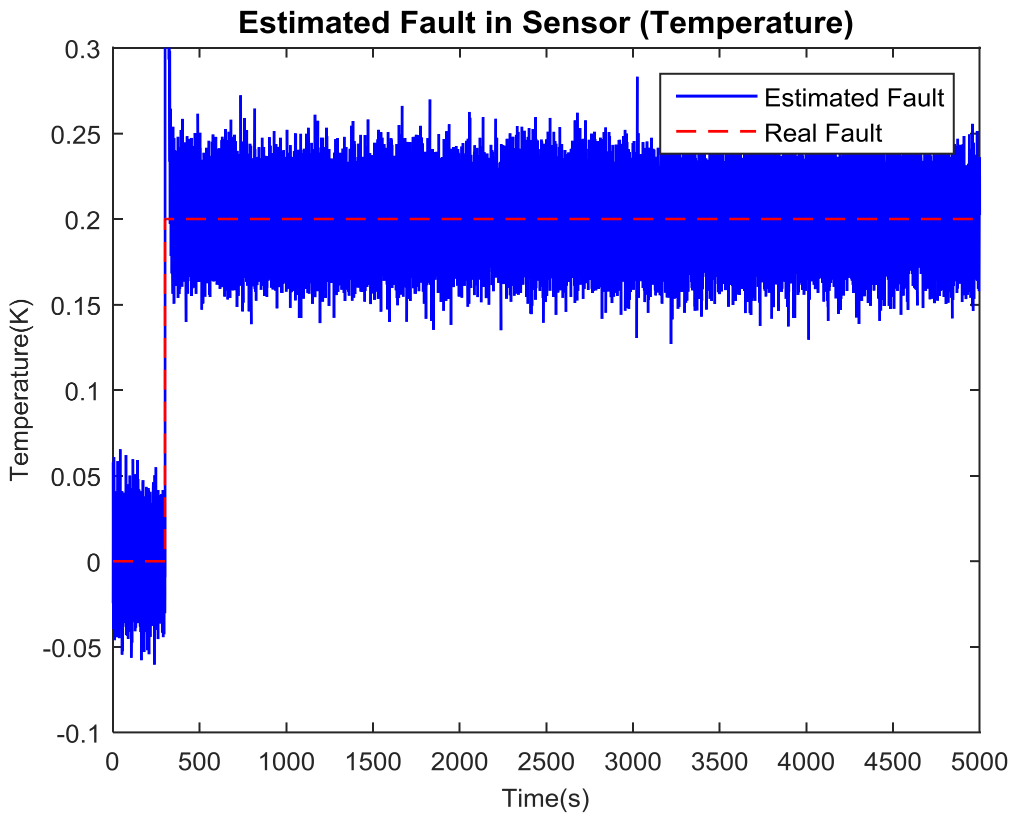

Fig. 4.21 Estimated fault compared with real fault in reactor analytical sensor

Observer 2

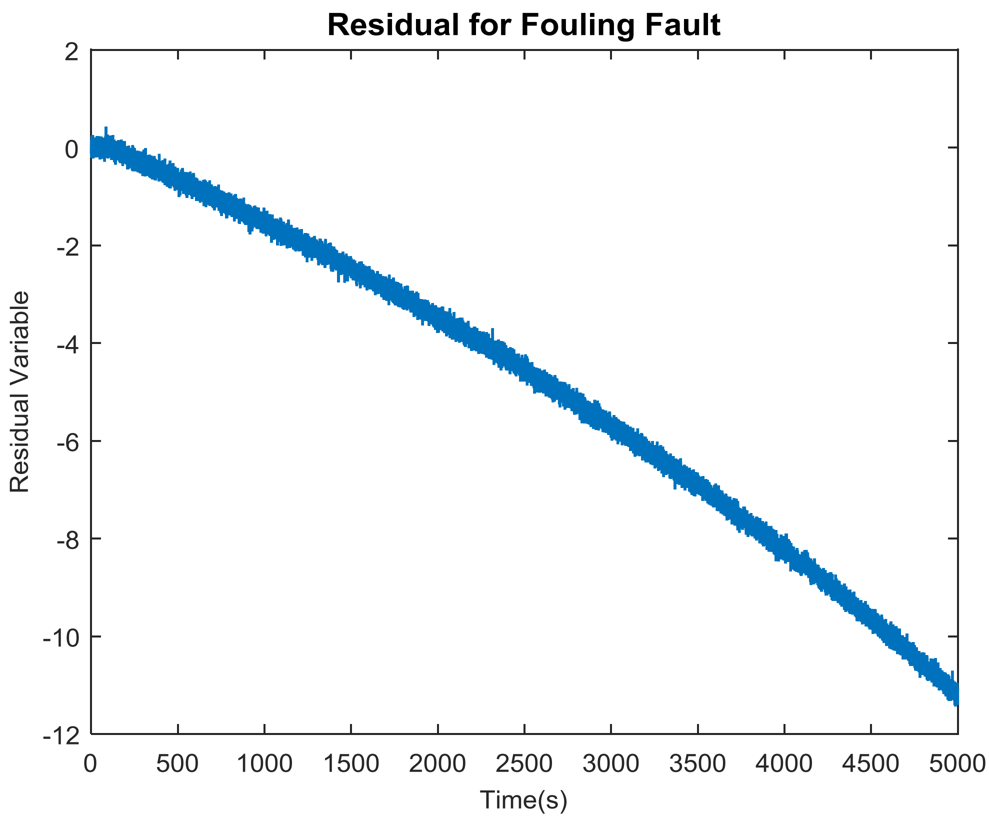

Fig. 4.22 Residual for heat exchanger fouling fault over time

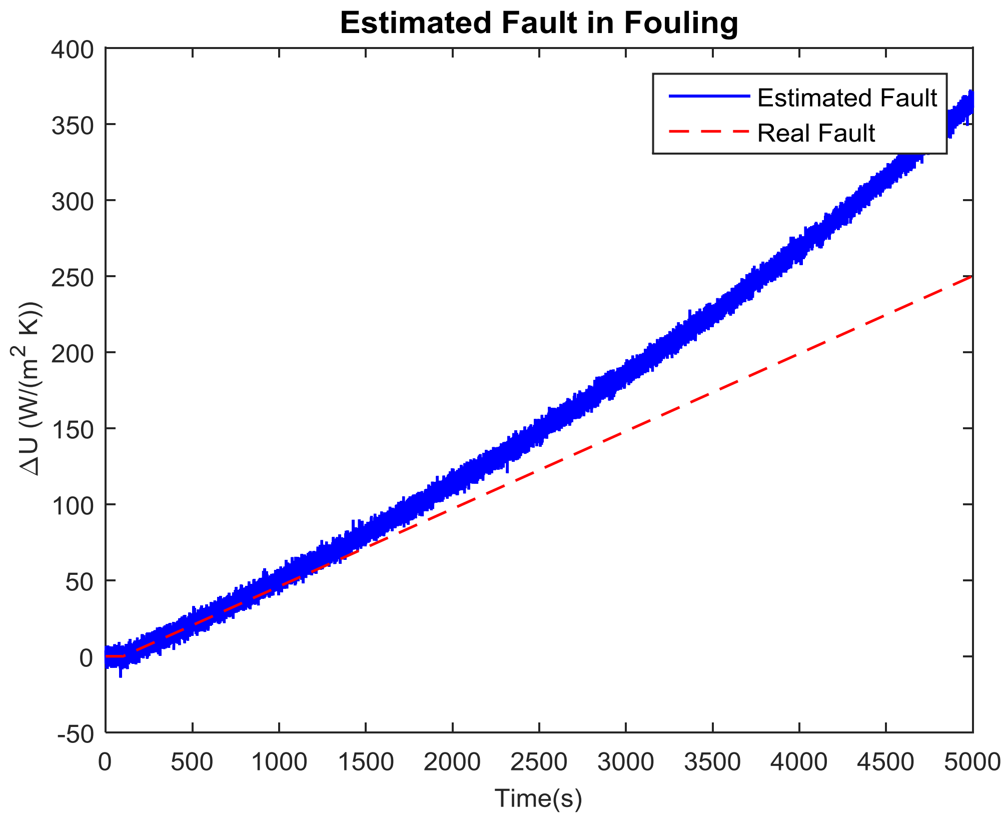

Fig. 4.23 Estimated fault compared with real fault in heat exchanger

Because of linearization errors caused by nonlinearity, the estimated fouling fault in the heat exchanger has large errors. Thus a better observer and residual for fouling fault diagnosis is desired.

Observer 1'

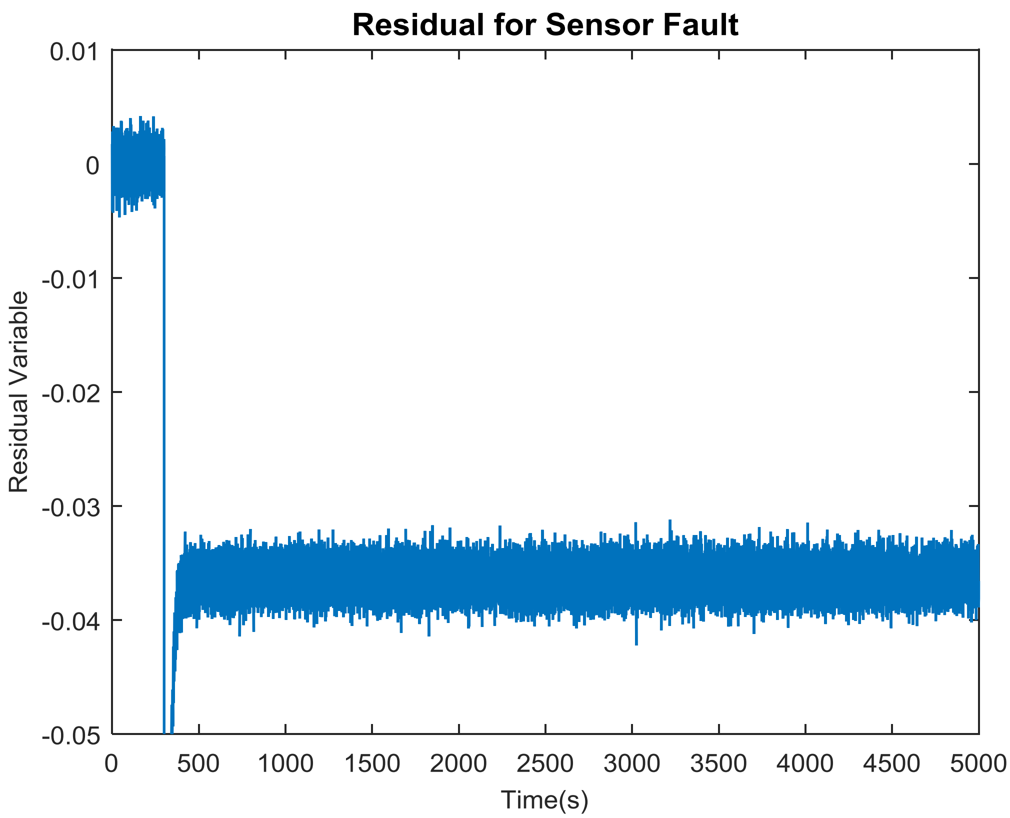

Fig. 4.24 Residual for analytical sensor fault over time

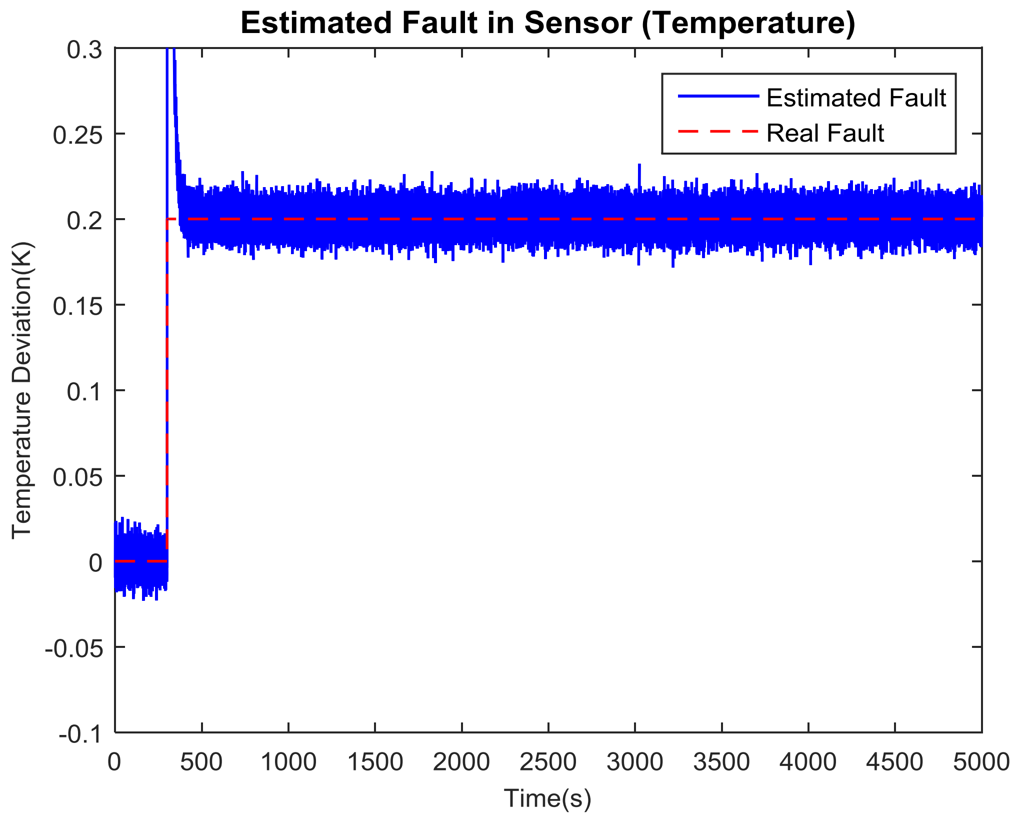

Fig. 4.25 Estimated fault compared with real fault in reactor analytical sensor

Observer 2'

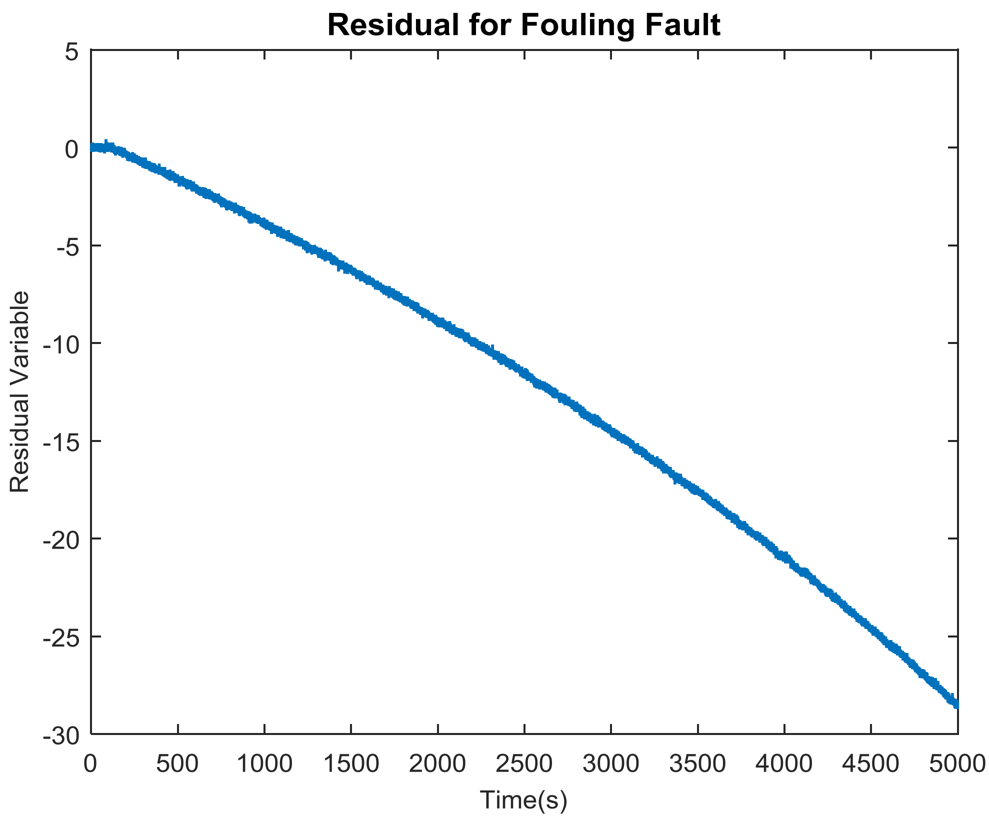

Fig. 4.26 Residual for heat exchanger fouling fault over time

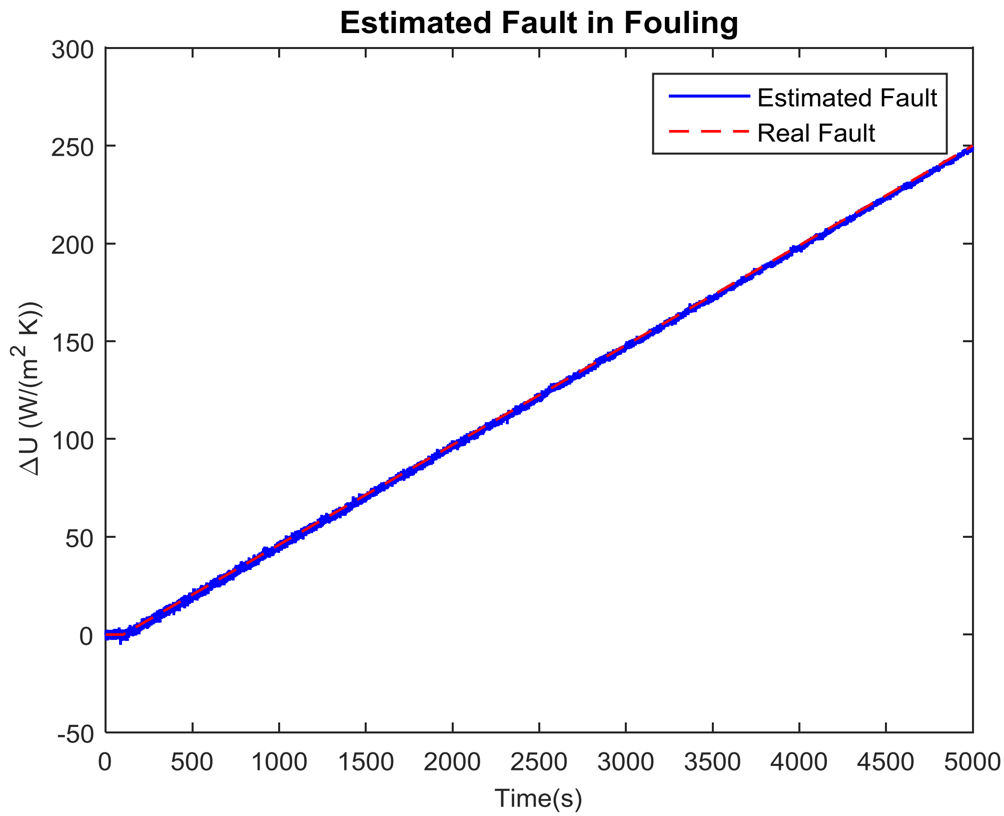

Fig. 4.27 Estimated fault compared with real fault in heat exchanger

From the figures above, we can notice that if a fault or disturbance occurs in the high nonlinearity term in the system model, it is better to consider the whole nonlinear part as fault or disturbance. In this way, the linearization errors can be reduced, and the fault estimation would be more accurate.

5. CONCLUSIONS AND FUTURE DIRECTIONS

5.1 Conclusions

The major focuses of this thesis is on unknown input diagnostic observer for exothermic CSTR systems. The objectives of the thesis have been stated in the first chapter. In order to achieve these objectives, the method of unknown input diagnostic observer has been reviewed. The performance of this approach was illustrated by representative applications to exothermic CSTR cases.

The first objective of the thesis is to find a proper diagnostic technique which can be used in chemical process systems and in particular exothermic CSTRs. To achieve this objective, many papers and books have been studied and three categories of fault diagnosis approaches have been reviewed. These approaches include multivariate statistical process monitoring, analytical model-based method and knowledge-based approach. Considering that a first principle model is often available for CSTRs, observer-based method was selected in this thesis. In order to decouple the effects of disturbances and isolate and identify the faults, the unknown input diagnostic observer (UIDO) was selected for application. The derivation of UIDO was reviewed and a new simple derivation of parts of the Luenberger conditions has been proposed. The UIDO design procedures were presented. A general MATLAB program for UIDO design was developed in this thesis.

The second objective of this thesis is to apply the method selected to exothermic CSTR systems. In order to achieve this goal, the CSTR system is modeled with possible faults and disturbances. Then the CSTR model was linearized at steady state. UIDO was designed based on the linearized model. To better evaluate applicability to real processes, noises were added to the sensor outputs and the observer was tested on the nonlinear system instead of the linearized system. After extensive simulations on the case studies, we conclude that:

- Unknown input diagnostic observe works well on exothermic CSTRs for fault detection, isolation and identification.
- Unknown input diagnostic observe also works on nonlinear system with relatively small model uncertainties.
- MATLAB program for UIDO design works efficiently and effectively.

5.2 Future Directions

Even though the unknown input diagnostic observer works well on exothermic CSTR systems, there is still a room for further improvements. A few possible directions are outlined as follows:

- The linear UIDO works well on the exothermic CSTR systems. But these systems in this thesis are simple and mildly nonlinear. For chemical processes, most systems are highly nonlinear with complex dynamics. Nonlinear unknown input diagnostic observer is one of the directions.
- The design of nonlinear unknown input diagnostic observers requires deep mathematical knowledge even for small scale systems. Even if this objective can be achieved in the future, first principle models of chemical processes are not always available or may be difficult to obtain, in which

case, UIDO would not be applicable. In order to overcome this limitation, multivariate statistical process monitoring methods can be investigated for large and complex systems.

- Multivariate statistical process monitoring methods can be used in complex chemical processes. However, the data obtained from processes provides limited information and fault diagnosis results are not as reliable as the UIDO method. What's worse, statistical-based method requires process data in the presence of faults for faults isolation. This data are always unavailable or difficult and expensive to obtain. An alternative method is neural network fault diagnosis. This method is considered as a middle method between UIDO method and statistical-based method. But neural network method may need long training time.

Currently, no method is absolutely perfect for fault diagnosis. Diverse methods need to be studied under various circumstances.

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