MODERN PID CONTROLLER PARAMETER DESIGN

FOR DC MOTOR

Thesis

by

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Submitted to the Office of Graduate and Professional Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

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December 2015

Major Subject: Electrical Engineering

ABSTRACT

DC motor is widely applied in industry practices. The speed control is a key procedure of the design, where PID controller is still the main stream. Stability and transient response are two significant design requirements for the DC motor control system. To decrease overshoot and rising time, a lot of tuning methods were invented, which were used in DC motor design. However, most of rules are based on empirical observation and experiment experience.

The purpose of this thesis is to theoretically design PID parameter sets which can guarantee both system stability and performance. The thesis starts from theoretical signature PID stabilizing sets design method and finds all PID parameters for DC motor control system. Then the thesis proposes a modified characteristic ratio assignment criterion to optimize previous stabilizing sets. Utilizing the parameter from new sets, better transient response performance can be reached. To demonstrate the method, a PID controller and PD controller design are integrated into the DC motor velocity control and DC motor position control problems separately.

Next, the DC motor system is discretized. The digital PI and PID stabilizing sets are designed by using root counting algorithm.

To sum up, the thesis gives a completed theoretical procedure to design PID controller parameters for DC motor system. The optimized parameter sets can not only satisfy stability criterion, but also achieve better transient performance.

DEDICATION

To my mother,

my father,

and

my grandparents

ACKNOWLEDGEMENTS

I want to first thanks for my advisor Professor Shankar Bhattacharyya for his support and kind guidance. Two years working with him not only inspired me how to do research, but also inspired me how to think about life. This experience will be my fortune forever.

Thanks also to Professor Hsieh for his valuable guidance and continuous support. I really appreciate his help to guide me how to find a good research topic.

Thanks also to Professor Huang for your great instruction in power system and nonlinear control classes. I learned a lot from him.

I also would like to extend my thanks to my committee members Professor Cui and Professor Zou for your advice and comments.

I am also grateful to my friends in Texas A&M University. Thanks for support and help. Discussions with them give me too much courage and power.

Finally, at most, I want to thank my father and mother on the other side of Pacific Ocean. Thanks for endless support, inspiration and love.

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1. INTRODUCTION

1.1 Problem Formulation

DC motor is popularly used in industry applications. The speed control is a key procedure of the design, where PID controller is still the main stream. Stability and transient response are two significant design requirements for the DC motor control system. To achieve better transient response, Ziegler-Nichols tuning formula was firstly introduced in 1942[1]. To decrease overshoot and rising time, a lot of tuning methods were invented, which were used in DC motor design.

However, most rules are developed according to empirical observation and experiment experience. In this thesis, a theoretical PID stabilizing sets design method will be integrated into the DC motor control design, which the system stability will be guaranteed. Then the thesis proposes a modified characteristic ratio assignment criterion to optimize previous stabilizing sets. Utilizing the parameter from new sets, better transient response performance can be reached. To demonstrate the method, a PID controller and PD controller design are integrated into the DC motor velocity control and DC motor position control problems separately. In addition, the digital PI and PID stabilizing sets are designed by using root counting algorithm. Taking w transform for discrete time system, the optimized digital PI sets are also calculated by modified characteristic ratio assignment method.

1.2 Research Objective

The research objectives are:

1. For a continuous time DC motor system, use the signature method to design PI and PID parameter sets for the controller.
2. Find PI and PD parameter sets that achieve desired transient response via modified characteristic ratio assignment for DC motor speed control and position control system.
3. For a discrete time DC motor model, find the digital PI and PID controller stabilizing sets.
4. Apply the w transform to the system, calculate the optimized parameter sets for digital PI controller that could improve transient performance.

2. SURVEY OF PREVIOUS WORK

Starting from classical PID control design method, Neenu Thomas[2] used Genetic Algorithm to improve the speed of transient response. However, GA method is complex and fussy. El-Gammal[3] tried to minimize the maximum overshoot, minimize the settling time and minimize the rise time by multi objective particle swarm optimization.

Similarly, in 2010, Nedim Tutkun used Gravitational Search Algorithm for tune the PID controller to achieve less rise time, less settling time and less maximum overshoot[4].

While the previous method got better results, they all needed DC model to work. In 2011, the iterative feedback tuning method was used to DC motor controller[5], in which no model is needed and uncertainty will not influence.

One year later, Rohit G Kanojiya and PM Meshram [6] separately implemented PI- Partial Swarm Optimization controller, PID-Ziegler Nichols controller and PID-Modified Ziegler Nichols controller to minimize the rise time, the settling time and minimize the maximum overshoot. R. Bindu[7] presented a flexible and fast tuning Genetic Algorithm to solve the position control of DC motor. At the same year, Azadi Controller was used to stabilize the DC motor controller system[8].

In 2014, an optimized PID parameter for DC motor position control was developed through multi objective genetic algorithm [9].

From the literature above, we could realize that a lot of tuning method are provided to design DC motor controller. They make it lucrative and scientifically important to investigate the DC motor controller design. However, by checking their system after design, their stability is guaranteed.

In my thesis, the modern design of PID controller[10] is implemented to solve the speed control of DC motor, which the system stability is guaranteed in advance. Additionally, through the modified characteristic ratio assignment, the transient response is improved and the PID sets are optimized.

Moreover, we also give the solution of the digital PID controller system design which guaranteed system stability and achieve desired transient response performance.

3. PID CONTROLLER STABILIZING SETS DESIGN FOR CONTINUOUS TIME DC MOTOR SYSTEMS

3.1 Continuous Time DC Motor Mathematic Model

The DC motor we introduced is separately excited[11] , which changes the motor velocity through control the armature voltage. The equivalent circuit is shown in [11].

Where R_a is resistance of armature; L_a is inductance of armature; i_a is current of armature; e_a is the input voltage; e_b is the back electromotive force(EMF); i_f is field current; T_M is defined as motor torque; J is the rotating inertia. Let ω be the angular velocity. K_b is the constant of EMF. B is defined as the constant of friction.

According to the physical law,

$$\text{Back EMF } e_b(\mathrm{t}) = K_b \frac{d_\theta}{d_t} = K_b \omega(t) \tag{3.1}$$

$$\text{Motor torque } T_m(t) = J \frac{d^2\theta(t)}{dt} + \mathrm{B}\frac{d\theta}{dt} = K_T\, i_a(t) \tag{3.2}$$

According to the KCL law,

$$\text{Voltage } e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_b(t) \tag{3.3}$$

Take Laplace transform to the above three equations,

$$E_a(s) = (R_a + L_a s)\, I_a(s) + E_b(s) \tag{3.4}$$

$$T_m(s) = B\omega(s) + Js\omega(s) = K_T I_a(s) \tag{3.5}$$

$$E_b(s) = K_b \omega(s) \tag{3.6}$$

To sum up, we could calculate the transfer function of speed by input voltage as follows,

$$G(s) = \frac{\omega(s)}{E_a(s)} = \frac{K_T}{(L_a s+R_a)(Js+B)+K_b K_T} \tag{3.7}$$

3.2 PID Stabilizing Sets Design Signature Method

In the following figure 3.1, it is a general feedback control system design diagram[12]

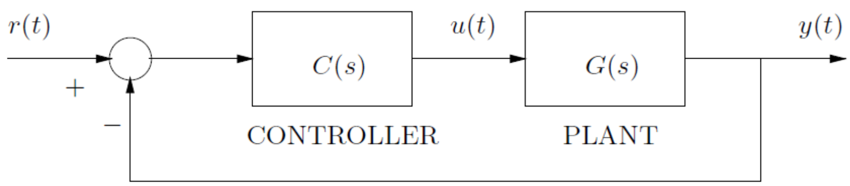

Figure 3.1 Continuous Time Feedback Control System

In the above figure, r(t) is defined as the reference input. Let y(t) denote the output signal. Let G(s) be the plant that needs to be controlled. Let C(s) be the controller.

$$C(s) = k_p + \frac{k_i}{s} + k_d s \tag{3.8}$$

In which k_p, k_i and k_d are the proportional, integral and derivative gains.

Assume $G(s) = \frac{N(s)}{D(s)}$, hence the characteristic polynomial of the continuous time feedback control system is

$$\delta(s, k_p, k_i, k_d) = sD(s) + (k_i + k_d s^2)N(s) + k_p sN(s) \tag{3.9}$$

Design stabilizing sets (k_p, k_i, k_d) is to determine those values that can make characteristic polynomial satisfy Hurwitz condition. In other words, all roots should be in the left half plane.

The new and high efficient method to determine stabilizing set is developed by Dr. Shankar Bhattacharyya[12, 13], which uses root counting and signature formulas.

Signature Formulas:

Assume p(s) is a polynomial as follows

$$\mathrm{p(s)} \coloneqq p_0 + p_2 s^2 + \cdots + s(p_1 + p_3 s^2 + \cdots) \tag{3.10}$$

where coefficients are real and there is no zeros on the jω axis.

$$p_{even}(s^2) = p_0 + p_2 s^2 + \cdots \tag{3.11}$$

$$P_{odd}(s^2) = p_1 + p_3 s^2 + \cdots \tag{3.12}$$

Therefore

$$\mathrm{p(j\omega)} = p_r(\omega) + j p_i(\omega) \tag{3.13}$$

Definition 1:

We define a signum function here. If a real number y is positive, signum function of y is sgn[y] $= 1$. If y is 0, signum function sgn[y] $= 0$. If real number y is negative, signum function of y is sgn[y] $= -1$.

Definition 2:

Assume p is a polynomial and define l as the amount of roots at left half plane. Also define r as the amount of roots at the right half plane[12].

$\Delta_{\omega_1}^{\omega_2} \angle$ p(jω) is the change of angle as ω runs from ω_1 to ω_2.

Lemma:

$$\Delta_0^{\infty}\angle \mathrm{p}(\mathrm{j}\omega) = \frac{\pi}{2}(\mathrm{l} - \mathrm{r}) \tag{3.14}$$

Theorem:

The $\mathrm{p}(\mathrm{j}\omega)$ can be written as

$$\mathrm{p}(\mathrm{j}\omega) = p_r(\omega) + jp_i(\omega) \tag{3.15}$$

and $\omega_0, \omega_1, \omega_3, \dots, \omega_{l-1}$ indicate the real nonnegative roots of $p_i(\omega)$.

From Shankar Bhattachayya's results in[12], we have

When n is even,

$$\delta(\mathrm{p}) = \mathrm{sgn}[p_i(0^+)](sgn[p_r(0)] + 2\sum_{j=1}^{l-1}(-1)^j sgn\left[p_r\left(\omega_j\right)\right] + (-1)^l sgn[p_r(\infty)]) \tag{3.16}$$

When n is odd,

$$\delta(\mathrm{p}) = \mathrm{sgn}[p_i(0^+)](sgn[p_r(0)]+2\sum_{j=1}^{l-1}(-1)^j sgn\left[p_r\left(\omega_j\right)\right]) \tag{3.17}$$

3.3 PID Parameter k_p, k_s, k_d Design for the DC Motor Control

$$\mathrm{G(s)} = \frac{\omega(s)}{E_a(s)} = \frac{K_T}{(L_a s+R_a)(Js+B)+K_b K_T} \tag{3.18}$$

One set of DC motor parameters [11] are shown in following table 3.1.

Table 3.1 DC MOTOR PARAMETERS

Parameters	values
$R_a(\Omega)$	2
$L_a(H)$	0.5
J(Kgm^2)	0.02
B(Nms)	0.2
$K_T(Nm/A)$	0.015
$K_B(Vs/rad)$	0.01

Therefore,

$$\mathrm{G(s)} = \frac{0.015}{0.01s^2+0.14s+0.40015} \tag{3.19}$$

We have,

$$\mathrm{N(s)} = 0.015 \tag{3.20}$$

$$\mathrm{D(s)} = 0.01s^2 + 0.14s + 0.40015 \tag{3.21}$$

So degree of N(s) is 2, and order of D(s) is 0.

$$\sigma(\mathrm{s}, k_p, k_i, k_d) = 0.01s^3 + 0.14s^2 + 0.40015s + (k_i + k_d s^2) * 0.015 + 0.015k_p s \tag{3.22}$$

$$\mathrm{v(s)} = \sigma(\mathrm{s}) = 0.014s^2 + (k_i + k_d s^2) * 0.015 + 0.01s^3 + 0.40015s + 0.015k_p s \tag{3.23}$$

$$v(j\omega) = -0.14\omega^2 + (k_i - k_d\omega^2) * 0.015 + j\omega\left(-0.01\omega^2 + 0.40015 + 0.015k_p\right) \tag{3.24}$$

The closed loop system is stable when

$$\sigma(\nu) = n + 1 = 3 \tag{3.25}$$

$$p(\omega) = -0.14\omega^2 + (k_i - k_d\omega^2) * 0.015 \tag{3.26}$$

$$q(\omega) = \omega\left(-0.01\omega^2 + 0.40015 + 0.015k_p\right) \tag{3.27}$$

There should be at least one positive zero of $q(\omega)$

$$k_p = \frac{0.01\omega^2 - 0.40015}{0.015} \tag{3.28}$$

$\omega \geq 0$, so $k_p \geq -26.6767$

Fix $k_p = 1$ and calculate real, positive frequencies of $\nu_{odd}\left(-\omega^2, k_p^*\right) = 0$. It will have $\omega_1 = 6.44321$. In addition, $j = \text{sgn}[\nu_{odd}\left(0^+, k_p^*\right)] = 1$.

Then we have the signature balance equation,

$$j * (i_0 - 2i_1) = 3 \tag{3.29}$$

Hence $i_0 = 1, i_1 = -1$.

The stabilizing sets of k_i, k_d can be got by linear inequalities

$$\nu_{even}(-\omega_t^2, k_i, k_d)\ i_t > 0 \tag{3.30}$$

$$\begin{cases} k_i > 0 \\ 0.015k_i - 0.6227k_d < 5.8121 \end{cases}$$

According to the linear inequalities, the stabilizing parameter sets can be plotted as following figure 3.2.

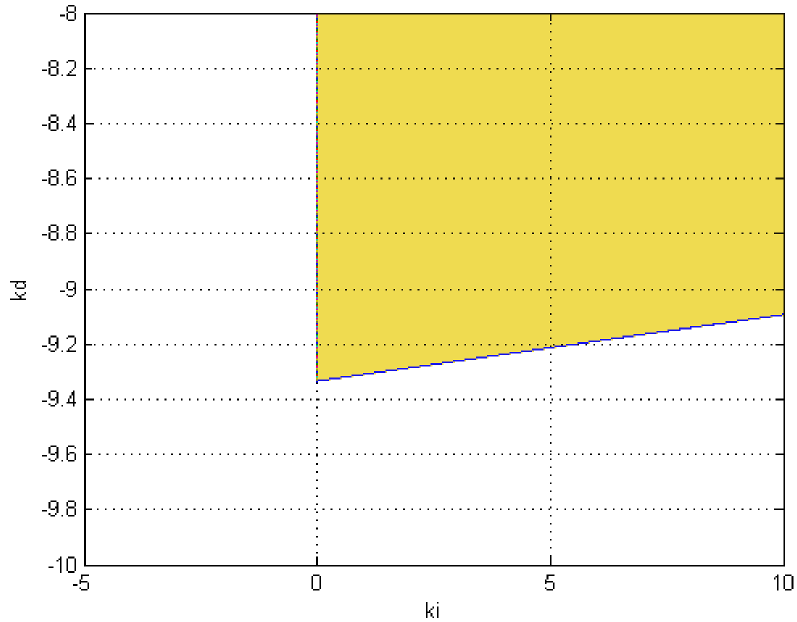

Figure 3.2 The Stabilizing Parameter Sets (k_i, k_d) Values with Fixed k_p

If we sweep k_p, and repeat the procedure of getting the linear equalities, we will have the following stabilizing sets figure 3.3:

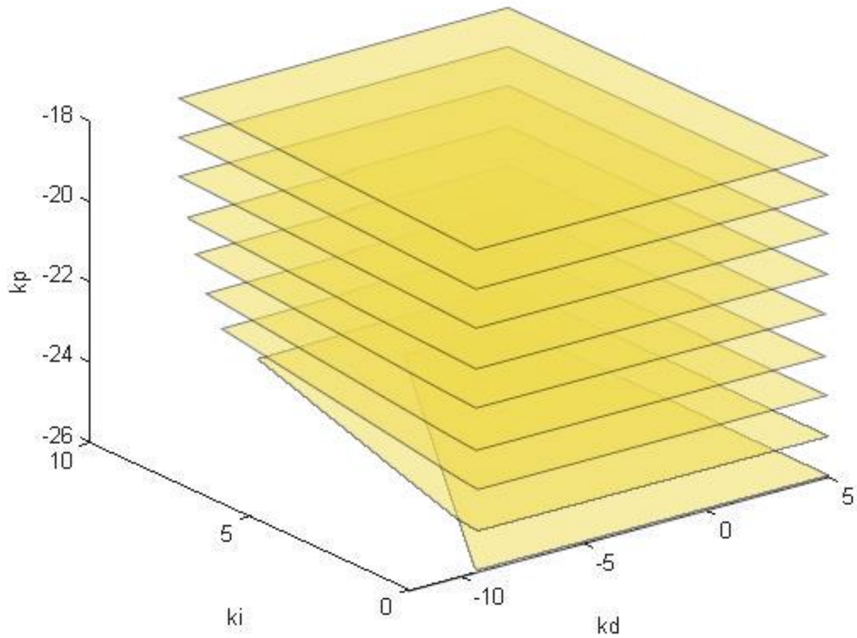

Figure 3.3 The Stabilizing Parameter Sets (k_p, k_i, k_d)

We could pick up a set of parameter to design the controller.

If we choose $k_p = 1, k_i = 20, k_d = 1$, $C(s) = \frac{s+20+s^2}{s}$. The diagram can be shown in figure 3.4.

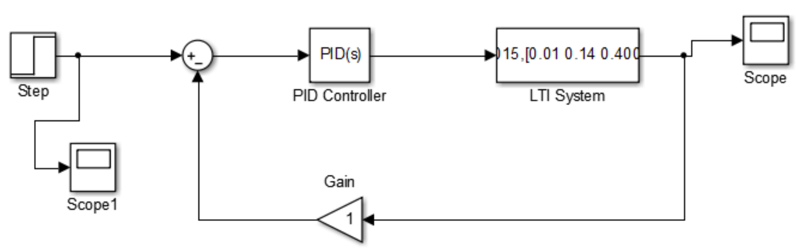

Figure 3.4 Continuous Motor Control System Simulink Diagram

Step response with the controller is as figure 3.5:

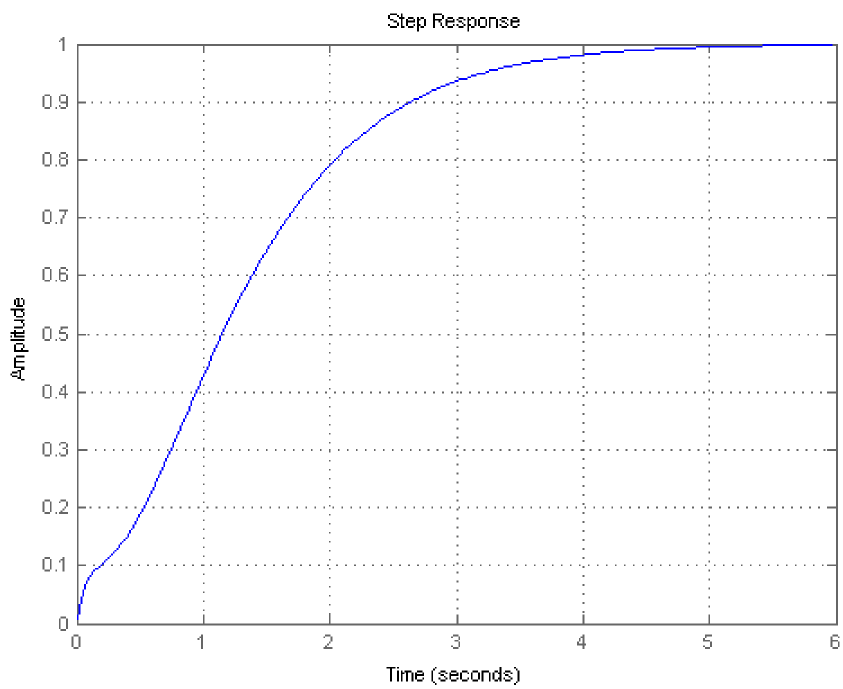

Figure 3.5 Step Response of Continuous System 1

If we choose $k_p = 1, k_i = 100, k_d = 1,$

$$C(s) = \frac{s + 100 + s^2}{s}$$

Step response with above controller is as figure 3.6:

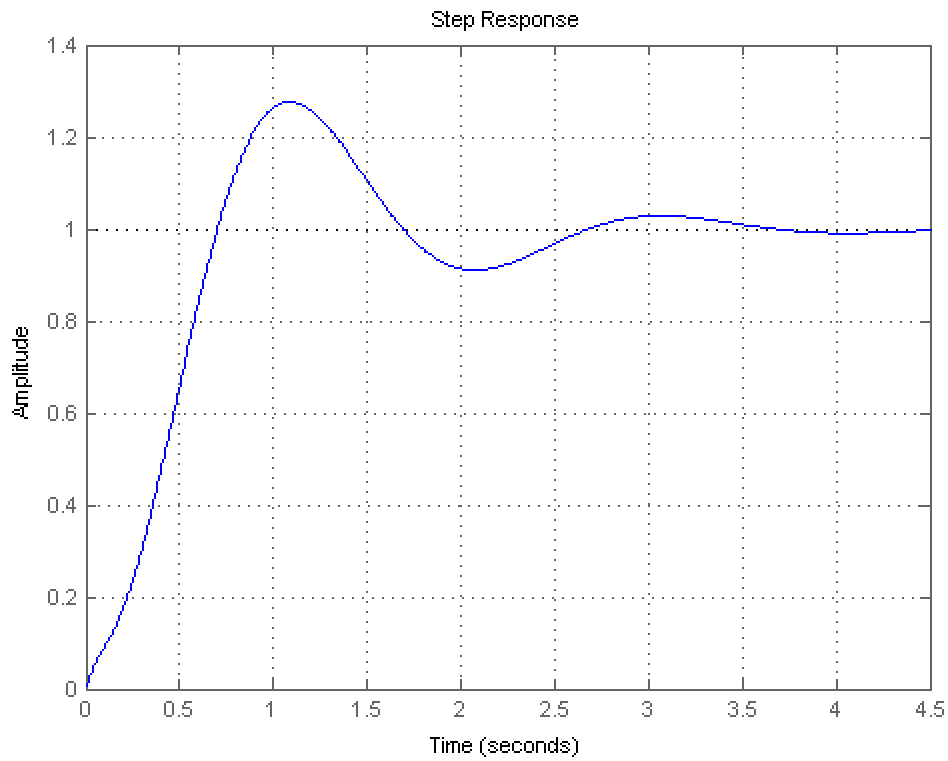

Figure 3.6 Step Response of Continuous System 2

From the above figure, we could see the system is stable and could track step signal. However, the rising time is a little bit slow of system 1 and overshoot is too high for system 2, which means transient response needs improvement. In next chapter, we would propose the method to get optimized stabilizing sets which will improve transient response.

4. OPTIMIZED PID CONTROLLER PARAMETER DESIGN FOR DC MOTOR

4.1 Introduction to Characteristic Ratio Assignment

Transient response is very significant while designing the practical system. For the second order system, we knew the exact relationship between transient response and system coefficient. However, for the higher order system, the relationship is complex and hard to describe.

Naslin[14, 15] first introduced the characteristic ratio and tried to give some methods to answer this question.

Consider the following characteristic polynomial:

$$\sigma(s) = a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \cdots + a_1 s + a_0 \tag{4.1}$$

in which the coefficients $a_i > 0$.

The characteristic ratio are:

$$\alpha_1 = \frac{a_1^2}{a_0 a_2},\quad \alpha_2 = \frac{a_2^2}{a_1 a_3},\quad \cdots \quad ,\ \alpha_{n-1} = \frac{a_{n-1}^2}{a_{n-2} a_n} \tag{4.2}$$

The time constant is defined as

$$\tau = \frac{a_1}{a_0} \tag{4.3}$$

There are some results[16] [17] showing the transient response related with the characteristic ratio and time constant. There are several theorems for all pole transfer function system.

Theorem:

Consider two all pole system of degree n:

$$G_1(s) = \frac{c_0}{P_1(s)} = \frac{c_0}{c_n s^n + c_{n-1}s^{n-1} + \cdots + c_1 s + c_0}, \qquad for\ all\ c_i > 0 \tag{4.4}$$

$$G_2(s) = \frac{d_0}{P_2(s)} = \frac{d_0}{d_n s^n + d_{n-1}s^{n-1} + \cdots + d_1 s + d_0}, \qquad for\ all\ d_i > 0 \tag{4.5}$$

Then, we have

$$\tau_1 = \frac{c_1}{c_0} \quad and \quad \tau_2 = \frac{d_1}{d_0}$$

To an arbitrary input, let zero state response of $G_i(s)$ be $y_i(t)$.

Consequently, we have

$$y_1(t) = y_2\left(\frac{\tau_1}{\tau_2}t\right) \tag{4.6}$$

When both $P_1(s)$ and $P_2(s)$ have the same characteristic ratios:

$$\frac{c_i^2}{c_{i-1}c_{i+1}} = \frac{d_i^2}{d_{i-1}d_{i+1}} = \alpha_i, for\ i = 1,2,\dots,n-1$$

Proof.

Sufficiency:

Assume that $\frac{c_i^2}{c_{i-1}c_{i+1}} = \frac{d_i^2}{d_{i-1}d_{i+1}} = \alpha_i$ holds. Then for an arbitrary input r(t), we have

$$c_0 r(t) = c_n \frac{d^n y_1(t)}{dt^n} + c_{n-1}\frac{d^{n-1}y_1(t)}{dt^{n-1}} + \cdots + c_1 \frac{dy_1(t)}{dt} + c_0 y_1(t) \tag{4.7}$$

Rewrite the coefficients c_i by its characteristic ratios $\alpha_i s$,

$$\mathrm{r(t)} = \frac{\tau_1^n}{\alpha_{n-1}\alpha_{n-2}^2 \cdots \alpha_1^{n-1}} \frac{d^n y_1(t)}{dt^n} + \frac{\tau_1^{n-1}}{\alpha_{n-2}\alpha_{n-3}^2 \cdots \alpha_1^{n-2}} \frac{d^{n-1} y_1(t)}{dt^{n-1}} + \cdots + \frac{\tau_1^2}{\alpha_1} \frac{d^2 y_1(t)}{dt^2} + \tau_1 \frac{dy_1(t)}{dt} + y_1(t) \tag{4.8}$$

Similarly

$$\mathrm{r(t)} = \frac{\tau_2^n}{\alpha_{n-1}\alpha_{n-2}^2 \cdots \alpha_1^{n-1}} \frac{d^n y_2(t)}{dt^n} + \frac{\tau_2^{n-1}}{\alpha_{n-2}\alpha_{n-3}^2 \cdots \alpha_1^{n-2}} \frac{d^{n-1} y_2(t)}{dt^{n-1}} + \cdots + \frac{\tau_2^2}{\alpha_1} \frac{d^2 y_2(t)}{dt^2} + \tau_2 \frac{dy_2(t)}{dt} + y_2(t) \tag{4.9}$$

For $y_2(t)$, $r$eplace

$$\mathrm{t} \leftarrow \frac{\tau_1}{\tau_2} t \coloneqq \varepsilon$$

So $y_2(t)$ becomes $y_2\left(\frac{\tau_1}{\tau_2} t\right)$.

$$\frac{dy_2\left(\frac{\tau_1}{\tau_2}t\right)}{dt} = \frac{dy_2(\varepsilon)}{d\left(\frac{\tau_1}{\tau_2}\varepsilon\right)} = \frac{\tau_1}{\tau_2}\frac{dy_2(\varepsilon)}{d\varepsilon} \tag{4.10}$$

$$\frac{d^2 y_2\left(\frac{\tau_1}{\tau_2}t\right)}{dt^2} = \left(\frac{\tau_1}{\tau_2}\right)^2 \frac{d^2 y_2(\varepsilon)}{d\varepsilon^2} \tag{4.11}$$

$$\frac{d^n y_2\left(\frac{\tau_1}{\tau_2}t\right)}{dt^n} = \left(\frac{\tau_1}{\tau_2}\right)^n \frac{d^n y_2(\varepsilon)}{d\varepsilon^n} \tag{4.12}$$

Hence, the time domain r(t) can be written as

$$\mathrm{r}(\varepsilon) = \frac{\tau_1^n}{\alpha_{n-1}\alpha_{n-2}^2 \cdots \alpha_1^{n-1}} \frac{d^n y_2(\varepsilon)}{dt^n} + \frac{\tau_1^{n-1}}{\alpha_{n-2}\alpha_{n-3}^2 \cdots \alpha_1^{n-2}} \frac{d^{n-1} y_2(\varepsilon)}{dt^{n-1}} + \cdots + \frac{\tau_1^2}{\alpha_1} \frac{d^2 y_2(\varepsilon)}{dt^2} + \tau_1 \frac{dy_2(t)}{dt} + y_2(\varepsilon) \tag{4.13}$$

It is obvious that the solution of (4.13) is the same as time domain of (4.8) .

Therefore

$$y_2(\varepsilon) = y_2\left(\frac{\tau_1}{\tau_2}t\right) = y_1(t)$$

Necessity is similarly to be proved.

Theorem:

For all pole system:

$$\mathrm{G(s)} = \frac{a_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_0} \tag{4.14}$$

$$a_1 = a_0\tau$$

$$a_i = \frac{a_0\tau^i}{\alpha_{i-1}\alpha_{i-2}^2\alpha_{i-3}^3 \cdots \alpha_2^{i-2}\alpha_1^{i-1}} \qquad for\ i = 2, \dots, n$$

For all pole system whose frequency response magnitude is monotonically decreasing[17]:

$$|G(j\omega)|^2 = \frac{a_0^2}{\delta(j\omega)\delta(-j\omega)} = \frac{1}{\bar{Q}^2(\omega)} \tag{4.15}$$

Define:

$$\Delta_i^j \coloneqq \begin{cases} \Pi_{k=i,i<j}^{j}\alpha_k, & if\ i < j \\ \alpha_i, & if\ i = j \end{cases}$$

$$\bar{Q}^2(\omega) = 1 + \eta_1\tau^2\omega^2 + \frac{\eta_2\tau^4}{(\Delta_1^1)^2} + \cdots \frac{\eta_n\tau^{2n}}{(\Delta_1^1\Delta_1^2\Delta_1^3 \cdots \Delta_1^{n-1})^2}\omega^{2n}$$

Where $\eta_k \coloneqq 1 - \frac{2}{\alpha_k} + \frac{2}{\alpha_k\Delta_{k-1}^{k+1}} - \frac{2}{\alpha_k\Delta_{k-1}^{k+1}\Delta_{k-2}^{k+2}} + \cdots + (-1)^k\frac{2}{\alpha_k\Pi_{j=1}^{k-1}\Delta_{k-j}^{k+j}}$

We consider the approximated curve of $|G(j\omega)|^2$ $vs.$ frequency [17], which is shown as Pseudo-asymptotic Diagram of $|G(j\omega)|^2$.

Let us focus on some critical frequencies:

$$\omega_i := \sqrt{\frac{\eta_i}{\eta_{i+1}}\frac{\Delta_1^i}{\tau}}, \qquad \mathrm{i} = 0{,}1, \dots, \mathrm{n}-1 \tag{4.16}$$

And define:

$$l_i := log\omega_i - log\omega_{i-1} \tag{4.17}$$

Therefore we have

$$l_0 = -\left(log\tau + \frac{1}{2}log\eta_1\right) \tag{4.18}$$

$$l_i = log\eta_i - \frac{1}{2}(log\eta_{i-1} + log\eta_{i+1}) + \log\alpha_i, \tag{4.19}$$

In which i=1,2,…..,n-1.

It is evident that $\eta_i \approx \eta_{i+1}\ for\ i = 1{,}2, \dots n-1$ when α_i is larger than 2. And we also could see $l_i \approx log\alpha_i$ and $\omega_i < \omega_{i+1}$.

For the time domain response of strictly proper system, frequency magnitude of low frequency dominantly influence the system. We could find that l_0, l_1, l_2, l_3 play the key role corresponding to response. Hence, α_1,$\alpha_2, \alpha_3\ and\ \tau$ have much more influence to the transient response

4.2 Modified Characteristic Ratio Assignment Method Used in Optimized Parameter Sets Design for DC Motor

We have got the stabilizing sets for DC motor design. However, the good performance of transient response could not be guaranteed if we arbitrarily choose the parameters. It will cause severe extreme problems for practical design.

In order to optimize the stabilizing sets to guarantee the performance of transient response. We will propose a modified characteristic ratio assignment method to solve this problem.

From the previous chapter, we have got the closed loop system transfer function for DC motor design.

The transfer function is:

$$\mathrm{G(s)} = \frac{0.015 * k_d s^2 + 0.015 k_p s + 0.015 k_i}{0.01 * s^3 + (0.015 k_d + 0.14) s^2 + \left(0.015 k_p + 0.40015\right) s + 0.015 * k_i} \tag{4.20}$$

We have used signature method and found all stabilizing sets.

For this model, it is evident that the transfer function is not all pole system. It means that we could not apply the theorem directly.

Revisit the theorem, we could find that the numerator is a constant of all pole system. Therefore, if we could add some specific criterions to high order coefficients of numerator, we could apply the characteristic ratio assignment method.

In this function, if the second and first order coefficients is very small compared to the constant term, it would not influence the overall performance in low frequency magnitude. For the high frequency situation, the denominator is third order and it would dominant the system response. So we could add criterion to this transfer function and use a modified characteristic ratio assignment method.

For the above system:

Characteristic ratio is:

$$\alpha_1 = \frac{\left(0.40015+0.015*k_p\right)^2}{k_i*0.015*(0.14+0.015*k_d)} \tag{4.21}$$

$$\alpha_2 = \frac{(0.14+0.015*k_d)^2}{0.01*(0.40015+0.015*k_p)} \tag{4.22}$$

Time constant is:

$$\tau = \frac{0.40015+0.015*k_p}{0.015*k_i} \tag{4.23}$$

Fix kp=1,

and we let the stabilizing sets satisfy time response criterion:

$$\begin{cases} \alpha_1 > 2 \\ \alpha_2 > 2 \\ 0.45 < \tau < 1 \end{cases} \quad \& \quad \begin{cases} \frac{0.015k_i}{0.015k_d} > 20 \\ \frac{0.015k_i}{0.015k_p} > 20 \end{cases} \tag{4.24}$$

After adding this criterion, we would have optimized parameter set as figure 4.1:

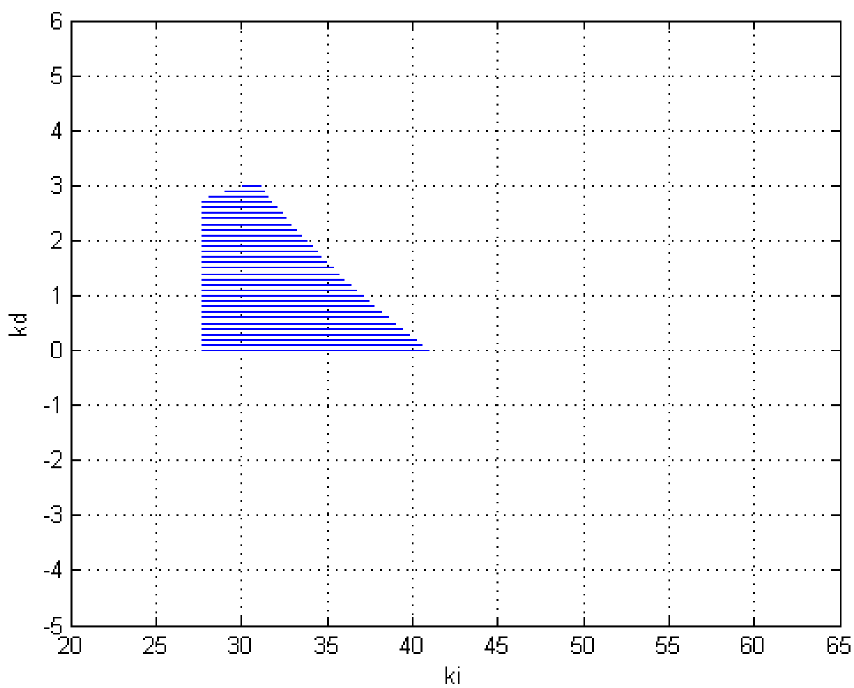

Figure 4.1 Optimized Parameter Sets for PID Controller with Fixed k_p

This parameter sets are much smaller than stabilizing sets.

We could choose 3 groups of parameter to show the step response (system 3 is desired system) . The parameters are in Table 4.1.

Table 4.1 Three System PID Parameters

system	k_p	k_i	k_d
#1	1	100	1
#2	1	20	1
#3	1	30	3

In Table 4.2, the system's characteristic ratio and time constant are given.

Table 4.2 Three System Characteristic Ratio and Time Constant Comparison

systems	τ	α_1	α_2
#1	0.27	0.62	5.78
#2	1.38	3.71	5.78
#3	0.92	2.07	8.24

The step responses are as following figure 4.2:

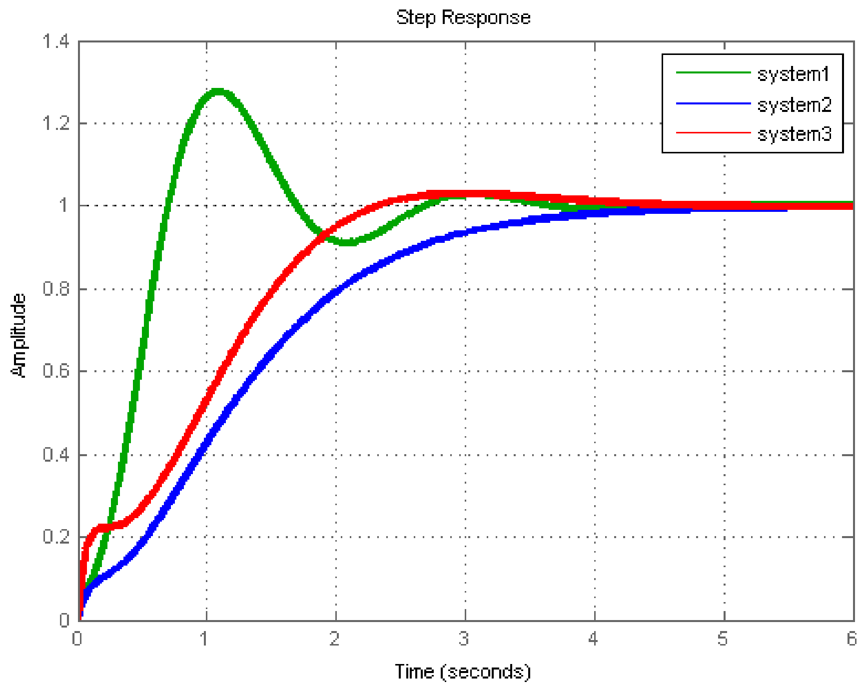

Figure 4.2 Comparison of Three System Step Response

The overshoot of system 3 is much less than system 1. And the rising time of system 3 is better than system 2.

If we sweep k_p, we could get following optimized stabilizing sets as figure 4.3:

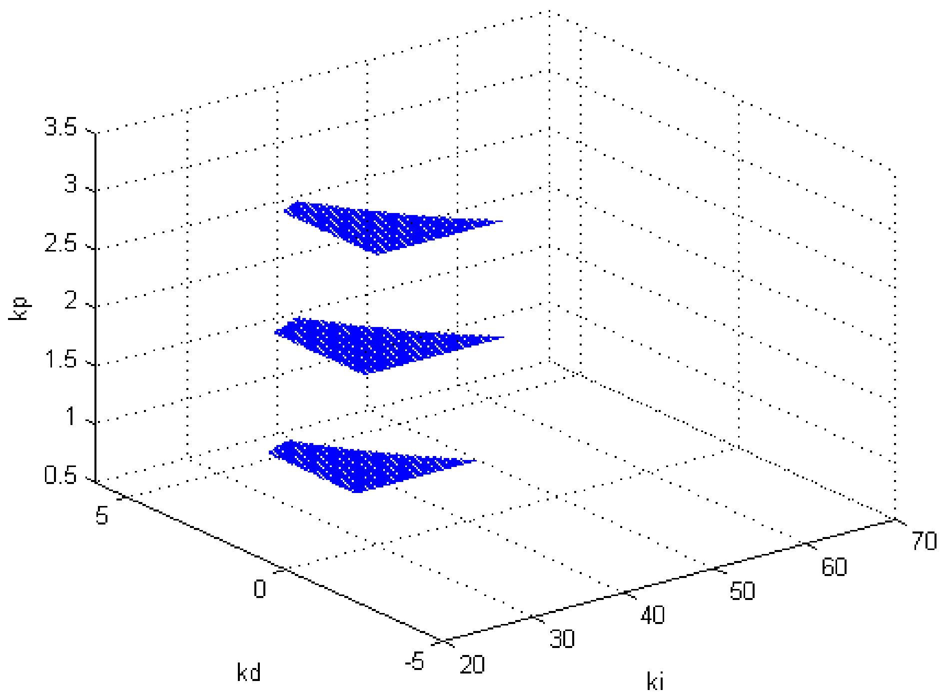

Figure 4.3 Optimized Parameter Sets (k_p, k_i, k_d) for PID controller

4.3 Modified Characteristic Ratio Assignment Method Used in Optimized Parameter Sets Design for DC Motor Position Control

4.3.1 DC Motor Position Control Model[2]

DC motor position control transfer function is:

$$\frac{\theta(s)}{V(s)} = \frac{K_b}{JL_a s^3 + (R_a J + BL_a)s^2 + (K_b^2 + R_a B)s} \qquad (4.25)$$

From the results in [2], there is one set of parameters in Table 4.3.

Table 4.3 DC Motor Model Parameters [2]

Parameters	Values
$R_a(\Omega)$	2.45
$L_a(H)$	3.5×10^{-2}
$K_b(V \cdot rad/s)$	1.2
$\mathrm{J}(\mathrm{kg}m^2/rad)$	2.2×10^{-2}
$\mathrm{B}(N \cdot m \cdot rad/s)$	5×10^{-4}

Therefore we have:

$$\frac{\theta(s)}{V(s)} = \frac{1.2}{0.00077s^3 + 0.0539s^2 + 1.441s} \tag{4.26}$$

4.3.2 DC Motor Position PD Control Stabilizing Sets Design

Define PD controller as:

$$\mathrm{C(s)} = k_p + k_d s \tag{4.27}$$

$$\mathrm{N(s)} = 1.2 \tag{4.28}$$

$$\mathrm{D(s)} = 0.00077s^3 + 0.0539s^2 + 1.441s \tag{4.29}$$

So n=3, m=0.

Therefore the characteristic polynomial of the overall system is

$$\mathrm{v(s)} = \delta(\mathrm{s}) = 0.00077s^3 + 0.0539s^2 + 1.441s + 1.2k_d s + 1.2k_p \tag{4.30}$$

Next

$$\delta(\mathrm{j}\omega) = -0.0539\omega^2 + 1.2k_p + \mathrm{j}\omega(-0.00077\omega^2 + 1.441 + 1.2k_d) \tag{4.31}$$

It needs to satisfy following conditions to make the system stable:

$$\sigma(\nu) = n = 3 \tag{4.32}$$

$$p(\omega) = -0.0539\omega^2 + 1.2k_p \tag{4.33}$$

$$q(\omega) = \omega(-0.00077\omega^2 + 1.441 + 1.2k_d) \tag{4.34}$$

$q(\omega)$ needs at least 1 positive zero.

$$k_d = \frac{0.00077\omega^2 - 1.441}{1.2} \tag{4.35}$$

$\omega \geq 0$, therefore $k_d \geq -1.2$

Fix $k_d = 1$ and calculate real positive frequencies of $\nu_{odd}(-\omega^2, k_p^*) = 0$

$$\omega_1 = 58.56 \tag{4.36}$$

$$j = \mathrm{sgn}[\nu_{odd}(0^+, k_p^*)] = 1 \tag{4.37}$$

Write signature balance equation,

$$j * (i_0 - 2i_1) = 3 \tag{4.38}$$

Therefore $i_0 = 1, i_1 = -1$.

The Stabilizing sets can be given by linear inequalities:

$$\begin{cases} k_p > 0 \\ -184.87 + 1.2k_p < 0 \end{cases} \tag{4.39}$$

Hence,

$$0 < k_p < 154$$

Sweep k_d, we could get following stabilizing sets as figure 4.4.

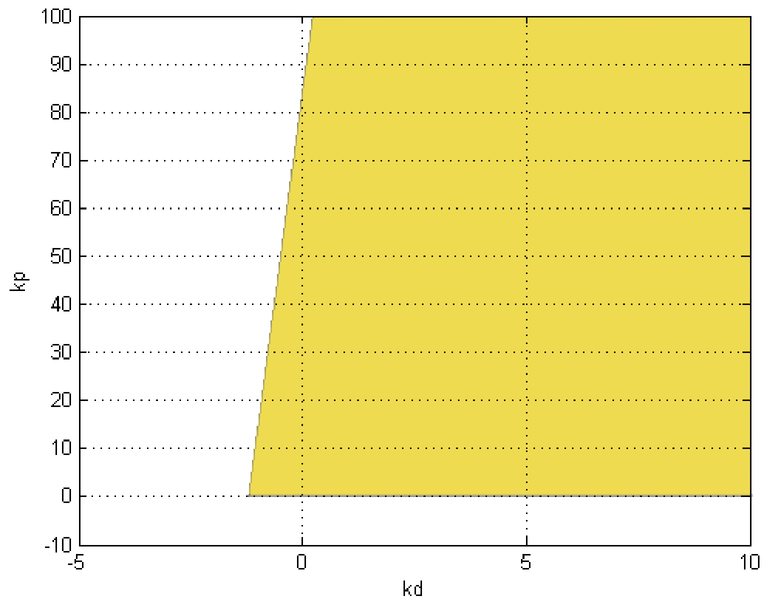

Figure 4.4 Stabilizing Sets (k_p, k_d) for PD Controller

4.3.2 Optimized Sets of PD Parameters

The overall transfer function of above system is

$$\mathrm{G(s)} = \frac{1.2k_d s + 1.2k_p}{0.00077s^3 + 0.0539s^2 + 1.441s + 1.2k_d s + 1.2k_p} \quad (4.40)$$

Time Constant:

$$\tau = \frac{1.441 + 1.2k_d}{1.2k_p} \quad (4.41)$$

Characteristic Ratio:

$$\alpha_1 = \frac{(1.441 + 1.2k_d)^2}{0.0539 \times 1.2k_p} \quad (4.42)$$

$$\alpha_2 = \frac{0.0539^2}{0.00077 \times (1.441 + 1.2k_d)} \quad (4.43)$$

Use our modified characteristic ratio assignment method and we have the following criterions:

$$\begin{cases} 0.1 < \frac{1.441+1.2k_d}{1.2k_p} < 0.6 \\ \frac{(1.441+1.2k_d)^2}{0.0539\times 1.2k_p} > 2 \\ \frac{0.0539^2}{0.00077\times(1.441+1.2k_d)} > 2 \end{cases} \quad \& \quad \frac{k_p}{k_d} > 10 \tag{4.44}$$

Fix $k_d = 1$ and we will have

$$3.6 < k_p < 22$$

We could use specific values to demonstrate our method.

For first system, we choose $k_d = 1,\ k_p = 1$, that is in stabilizing sets but not in optimized sets. For second system, we choose $k_d = 0.1,\ k_p = 10$, that is in optimized sets. The rising time of second is much better than the first one. The step response is as following figure 4.5.

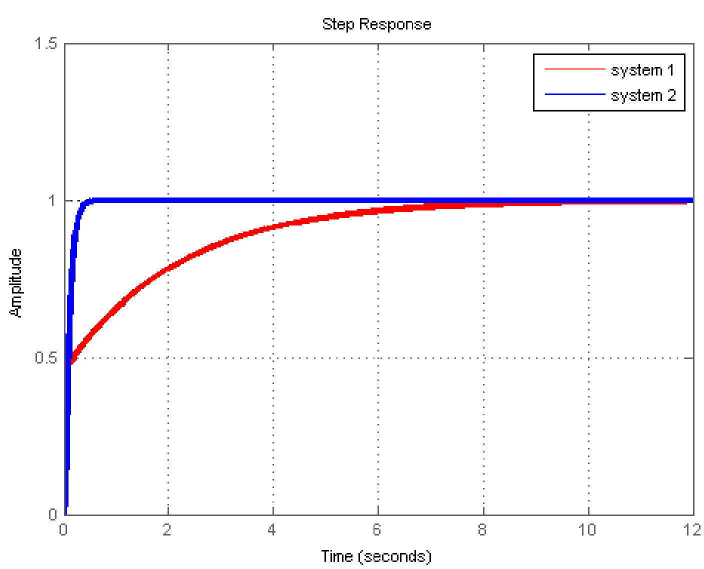

Figure 4.5 Comparison of System 1 and System 2

Sweep kd, all the optimized sets are as following figure 4.6:

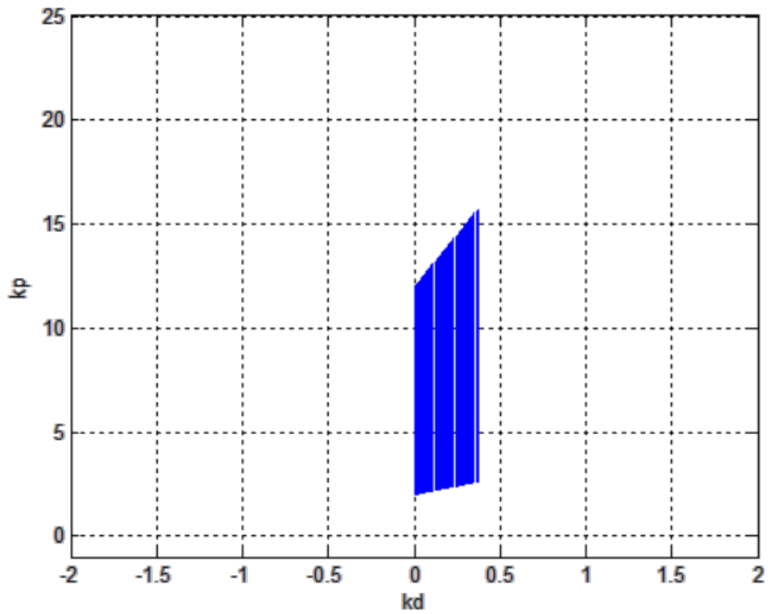

Figure 4.6 Optimized Sets (k_p, k_d) for PD Controller

The optimized sets are very small compared with stabilizing sets.

5. DIGITAL PID CONTROLLER DESIGN FOR DC MOTOR

5.1 Introduction to Tchebyshev Representation

Through implementing the Tchebyshev representation for a z domain system and implement the root counting method[12], we could design PID stabilizing parameters for DC motor.

Consider z domain polynomial $\mathrm{P(z)} = a_n z^n + \cdots + a_0$ with real coefficients. We could draw the image of P(z) in 2-D plane, that is evaluated on the circle C_ρ.

$$\{\mathrm{P(z)}: \mathrm{z} = \rho e^{j\theta}, 0 \le \theta \le 2\pi\} \tag{5.1}$$

Because coefficients a_i of real $\mathrm{P}(\rho e^{j\theta})$ and $\mathrm{P}(\rho e^{-j\theta})$ are in conjugate complex form, it can be described using the circle as follows:

$$\{\mathrm{P(z)}: z = \rho e^{j\theta}, 0 \le \theta \le \pi\} \tag{5.2}$$

As

$$z^k|_{z=\rho e^{j\theta}} = \rho^k(cosk\theta + jsink\theta),$$

So

$$\mathrm{P}(\rho e^{j\theta}) = (a_n\rho^n cosn\theta + \cdots a_1 cos\theta + a_0) + j(a_n\rho^n sinn\theta + \cdots a_1\rho sin\theta)$$

$$= \bar{R}(\rho,\theta) + j\bar{I}(\rho,\theta) \tag{5.3}$$

By using Tchebyshev polynomials, we could rewrite coskθ, and $\frac{sink\theta}{sin\theta}$ by cosθ. Let $\mathrm{u} = -\mathrm{cos}\theta$. When θ changes from 0 to π, u will change from -1 to +1,

$$e^{j\theta} = cos\theta + jsin\theta = -u + j\sqrt{1-u^2}$$

In addition, we have

$\cos k\theta =: c_k(u)$ and $\frac{sink\theta}{sin\theta} =: s_k(u)$

In which $c_k(u)$ and $s_k(u)$ are polynomials with real coefficients and is written by u[12]. They are known as Tchebyshev Polynomials.

They have:

$$s_k(u) = -\frac{c_k'(u)}{k} \text{ k=1,2,...} \tag{5.4}$$

If in the form

$$\left(\rho e^{j\theta}\right)^k = \rho^k cosk\theta + j\rho^k sink\theta \tag{5.5}$$

We could define generalized Tchebyshev polynomials:

$$c_k(u,\rho) = \rho^k c_k(u), s_k(u,\rho) = \rho^k s_k(u), k = 0,1,2 \ldots \tag{5.6}$$

Therefore, we have

$$\mathrm{P}\left(\rho e^{j\theta}\right) = R(u,\rho) + j\sqrt{1-u^2}T(u,\rho) =: P_c(u,\rho) \tag{5.7}$$

In which

$$\mathrm{R}(\mathrm{u},\rho) = a_n c_n(u,\rho) + a_{n-1}c_{n-1}(u,\rho) + \cdots + a_1 c_1(u,\rho) + a_0 \tag{5.8}$$

$$\mathrm{T}(\mathrm{u},\rho) = a_n s_n(u,\rho) + a_{n-1}s_{n-1}(u,\rho) + \cdots + a_1 s_1(u,\rho) \tag{5.9}$$

5.2 Discrete Time System Root Clustering Method

Consider the discrete time system with single input and single output. In the following figure 5.1, G(z) is the discrete time plant transfer function and C(z) is the controller.

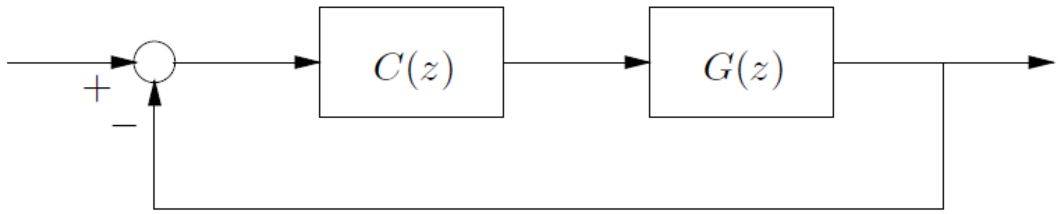

Figure 5.1 Discrete Time System Diagram [12]

The transfer functions are

$$\mathrm{G(z)} = \frac{N(z)}{D(z)}, \qquad \mathrm{C(z)} = \frac{N_C(z)}{D_C(z)}$$

Then it leads to characteristic polynomial

$$\Pi(\mathrm{z}) \coloneqq D_C(z)D(z) + N_C(z)N(z) \tag{5.10}$$

The system is stable only when characteristic roots have magnitude less than unity, which is well known and called Schur Stability of $\Pi(\mathrm{z})$.

5.2.1 Interlacing Conditions for root clustering

Define P(z) as real polynomial and its degree is n,

$$\mathrm{P}\left(\rho e^{j\theta}\right) = \bar{R}(\theta,\rho) + j\bar{I}(\theta,\rho)$$

$$= \mathrm{R(u,\rho)} + \mathrm{j}\sqrt{1-u^2}\,T(u,\rho) \tag{5.11}$$

In which $\mathrm{R(u,\rho)}$ is real polynomial and its order is n, while $T(u,\rho)$ is polynomial of degree n-1.

For Schur stability, $\rho = 1$.

So we have following theorem 5.1:

All zeros of P(z) are within C_ρ if and only if

(a) There are n real distinct zeros r_i inside (-1,+1) for $R(u,\rho)$.

(b) There are n-1 real distinct zeros t_j in (-1,+1) for $T(u,\rho)$.

(c) The zeros interlace between r_i and t_j.

5.2.2 Root Counting formulas

Lemma:

For P(z), if there are j roots within circle C_ρ and no roots on the circle.

We have

$$\Delta_0^{\pi}[\emptyset_P] = \pi j \tag{5.12}$$

Theorem 5.2

Define P(z) as a real polynomial which does not have roots on the circle C_ρ. And assume $T(u,\rho)$ contains p roots at u=-1.

Next we have the total amount of roots i of P(z) within circle[12]:

$$\mathrm{i} = \frac{1}{2} sgn\left[T^{(p)}(-1,\rho)\right](sgn[R(-1,\rho)] + 2\sum_{j=1}^{k}(-1)^j sgn\left[R\left(t_j,\rho\right)\right] + (-1)^{k+1} sgn[R(+1,\rho)]) \tag{5.13}$$

Theorem 5.3

Assume $\mathrm{H(z)} = \frac{P_1(z)}{P_2(z)}$, in which $P_1(z)$ and $P_2(z)$ are real polynomials. There are i_1 zeros for $P_1(z)$ and i_2 zeros for $P_2(z)$ inside circle C_ρ without zero on it.

Then

$$i_1 - i_2 = \frac{1}{2} sgn[T^{(p)}(-1,\rho)](sgn[R(-1,\rho)] + 2\sum_{j=1}^{k}(-1)^j sgn\left[R\left(t_j,\rho\right)\right] +$$

$$(-1)^{k+1} sgn[R(+1,\rho)]) \qquad (5.14)$$

5.3 Stabilizing Sets Design for PI Controller

Define plant $\mathrm{G(z)} = \frac{N(z)}{D(z)}$, in which N(z) , D(z) are polynomials which have real number coefficients[18]. D(z)'s degree is n and N(z)'s degree is less or equal to n.

Implement a discrete time PI controller as follows:

$$\mathrm{C(z)} = \frac{K_0 + K_1 z}{z-1} \qquad (5.15)$$

The characteristic polynomial of overall system is

$$\sigma(\mathrm{z}) = (z-1)D(z) + (K_0 + K_1 z)N(z) \qquad (5.16)$$

The overall system is stable only when the σ(z) is Schur stable.

Multiplying $\mathrm{N}(z^{-1})$ to the characteristic polynomial, we get

$$\sigma(\mathrm{z})\mathrm{N}(z^{-1}) = \left(-u - 1 + j\sqrt{1-u^2}\right)\left(P_1(u) + j\sqrt{1-u^2}P_2(u)\right)$$

$$+\mathrm{j}K_1\sqrt{1-u^2}P_3(u) - K_1 u P_3(u) + K_0 P_3(u) \qquad (5.17)$$

Hence,

$$\sigma(\mathrm{z})\mathrm{N}(z^{-1})|_{z=e^{j\theta},u=-cos\theta} = \frac{\sigma(z)N_r(z)}{z^l}|_{z=e^{j\theta},u=-cos\theta}$$

$$= \mathrm{R}(\mathrm{u}, K_0, K_1) + \sqrt{1-u^2}\,\mathrm{T}(\mathrm{u}, K_1)$$

$$(5.21)$$

In which N_r(z) is defined as the reverse polynomial of N(z)

$$\mathrm{R}(\mathrm{u}, K_0, K_1) = -(u+1)P_1(u) - (1-u^2)P_2(u) - (K_1 u - K_0)P_3(u) \qquad (5.22)$$

$$\mathrm{T}(\mathrm{u}, K_1) = P_1(u) - (u+1)P_2(u) + K_1 P_3(u) \tag{5.23}$$

For the fixed value of K_1, we need to calculate real distinct roots t_i of $\mathrm{T}(\mathrm{u}, K_1)$ in $\mathrm{u} \in (-1, +1)$:

$$t_i < t_{i+1}$$

Define i_δ is the number of zeros of $\sigma(\mathrm{z})$ and i_{N_r} is the amount of zeros of $N_r(\mathrm{z})$ inside the circle with radius 1[18], then

$$\begin{aligned} i_\sigma + i_{N_r} - l = {} & \frac{1}{2} Sgn[T^{(p)}(-1)](Sgn[R(-1, K_0, K_1)] \\ & + 2\sum_{j=1}^{k} (-1)^j Sgn\left[R\left(t_j, K_0, K_1\right)\right] + (-1)^{k+1} Sgn[R(+1, K_0, K_1)]) \end{aligned} \tag{5.24}$$

To guarantee the overall system closed loop stability, it is required $i_\sigma = n$.

Applying this with above methods, we can get the signum function of real part. Next we have the linear inequalities in K_0. Sweep the K_1 and we will have the whole sets.

5.4 Digital PI Stabilizing Sets Design for DC Motor System

DC motor discrete time model:

$$P(z) = \frac{0.004802z + 0.003013}{z^2 - 1.038z + 0.2466} \tag{5.25}$$

Applying the PI controller

$$C(z) = \frac{k_0 + k_1 z}{z - 1} \tag{5.26}$$

For the discrete time plant,

$$D(z) = z^2 - 1.038z + 0.2466 \tag{5.27}$$

$$N(z) = 0.004802z + 0.003013 \tag{5.28}$$

$$D\left(e^{j\theta}\right) = e^{2j\theta} - 1.038e^{j\theta} + 0.2466$$

$$= 2u^2 + 1.038u - 0.7534 + j\sqrt{1-u^2}(-2u - 1.038) \tag{5.29}$$

Then we have

$$R_D(u) = 2u^2 + 1.038u - 0.7534 \tag{5.30}$$

$$T_D(u) = -2u - 1.038 \tag{5.31}$$

$$N\left(e^{j\theta}\right) = 0.004802e^{j\theta} + 0003013$$

$$= 0.004802cos\theta + j0.004802sin\theta + 0.003013$$

$$= -0.004802u + 0.003013 + j0.004802\sqrt{1-u^2} \tag{5.32}$$

$$R_N(u) = -0.004802u + 0.003013 \tag{5.33}$$

$$T_N(u) = 0.004802 \tag{5.34}$$

$$P_1(u) = 0.006026u^2 - 0.0029u - 0.0073 \tag{5.35}$$

$$P_2(u) = -0.006026u + 0.0005 \tag{5.36}$$

$$P_3(u) = -0.000014468u + 0.000032059 \tag{5.37}$$

Then we could get

$$\mathrm{R}(\mathrm{u}, k_0, k_1) = -0.012052u^3 - 0.0026u^2 + 0.0162u + 0.0068$$

$$-(k_1 u - k_0)(-0.000014468u + 0.000032059) \tag{5.38}$$

$$\mathrm{T}(\mathrm{u}, k_1) = 0.012052u^2 + 0.0026u - 0.0078 + k_1(-0.000014468u +$$

$$0.000032059) \tag{5.39}$$

Therefore, in order to guarantee the system stability, the amount of roots of σ(z) inside circle with radius 1 should be 3.

$$i_\sigma = 3$$

The amount of roots of the Nr(z): $i_{N_r} = 0$

The order of N(z) is $\mathrm{l} = 1$

Therefore,

$$i_1 - i_2 = \left(i_\sigma + i_{N_r}\right) - l = 2$$

Hence, the $\mathrm{T}(\mathrm{u}, k_1)$ needs to have two real roots to make the system stable.

Let $\mathrm{T}(\mathrm{u}, k_1) = 0$, we have $k_1 = \frac{0.012052u^2+0.0026u-0.0078}{0.000014468u-0.000032059}$. So the range of k_1 is $k_1 \in [-389.5174, 243.8580]$.

To give an specific controller, we fix $k_1 = 200$.

The roots of $\mathrm{T}(\mathrm{u}, k_1)$ inside unit circle are -0.327426 and 0.351787.

$$\mathrm{Sgn}[\mathrm{T}(-1)] = +1$$

And since

$$i_1 - i_2 = 2$$

Therefore,

$$\frac{1}{2}Sgn[T(-1)](Sgn[R(-1)] - 2Sgn[R(-0.3274)] + 2Sgn[R(0.351787)] - Sgn[R(+1)]) = 2 \tag{5.40}$$

We could set

$$Sgn[R(-1)] = 1$$

$$Sgn[R(-0.3274)] = -1$$

$$Sgn[R(0.351787)] = 1$$

$$Sgn[R(+1)] = 1$$

Therefore we have following linear inequalities:

$$\begin{cases} 4.65k_0 + 935.7 > 0 \\ 3.68k_0 + 404.96 < 0 \\ 2.70k_0 + 975.5 > 0 \\ 1.76k_0 + 482.98 > 0 \end{cases} \tag{5.41}$$

These inequalities describes the stability region in k_0 space for fixed $k_1 = 200$. By repeating the procedure and change k_1, we could get stability sets shown in figure 5.2.

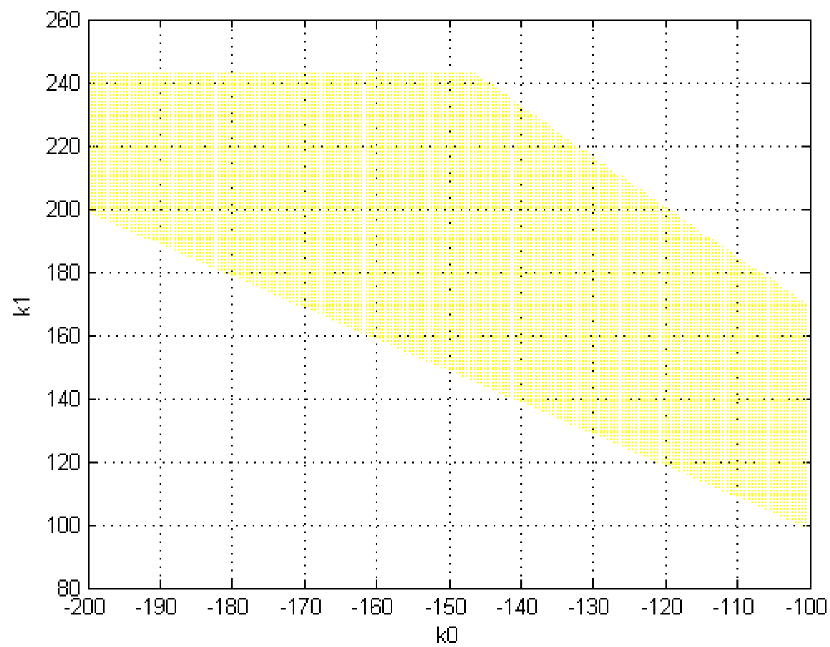

Figure 5.2 Discrete Time PI Controller Stabilizing Sets (k1,k0)

If we set $k_0 = -150$, we could get the PI controller as

$$C(z) = \frac{-150+200z}{z-1} \tag{5.42}$$

The system is shown in following figure 5.3:

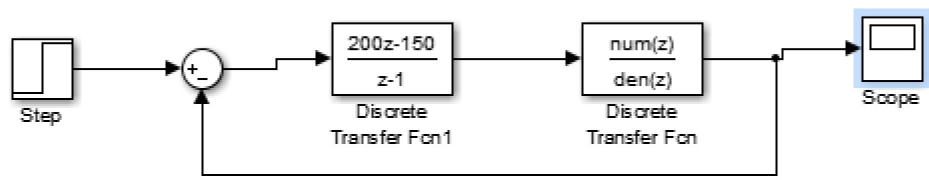

Figure 5.3 Discrete Time Motor Control with PI Controller Simulink

The step response is as following figure 5.4:

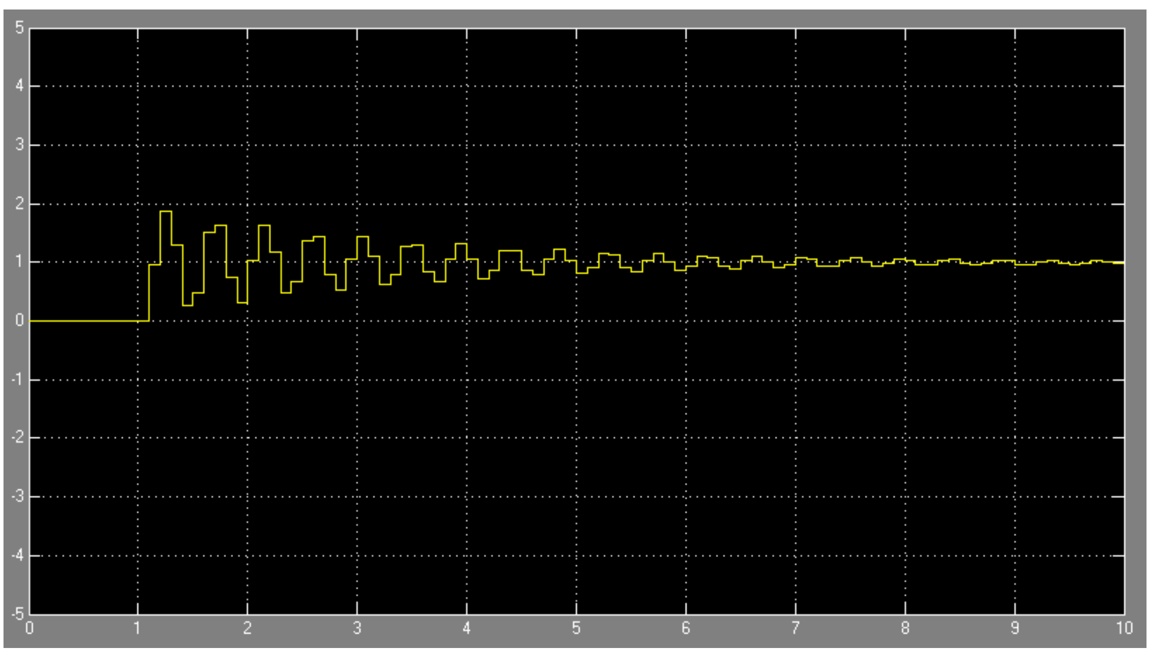

Figure 5.4 Discrete Time Motor Control with PI Controller Step Response

5.5 Digital PI Optimized Parameter Design for DC Motor

Y.C. Kim [19] showed that w-domain can preserve z-domain in transient response design. The w transform is given by: z $\coloneqq \frac{2+T_s\omega}{2-T_s\omega}$.

We will use this transformation to solve our problem with $T_s = 0.1$.

Apply w transform to our DC motor model:

$$\mathrm{P}(\omega) = \frac{-0.000018\omega^2 - 0.0012052\omega + 0.0313}{0.022846\omega^2 + 0.30136\omega + 0.8352} \tag{5.43}$$

Apply w transform to PI controller:

$$\mathrm{C}(\omega) = \frac{0.1\omega(k_1 - k_0) + 2(k_0 + k_1)}{0.2\omega} \tag{5.44}$$

Therefore, the closed loop system transfer function will be:

$$G(\omega) = \frac{\begin{array}{c}-0.0000018\omega^3(k_1 - k_0) - \omega^2(0.00015652k_1 - 0.00008452k_0) \\ -\omega(0.0055403k_0 - 0.0007196k_1) + 0.0626(k_0 + k_1)\end{array}}{\begin{array}{c}\omega^3(0.0045692 - 0.0000018k_1 + 0.0000018k_0) + \omega^2(0.060272 - 0.00015652k_1 \\ +0.00008452k_0) + \omega(0.16704 - 0.0055404k_0 + 0.0007196k_1) + 0.8352 + 0.0626(k_0 + k_1)\end{array}} \tag{5.45}$$

Define $a_0 = 0.8352 + 0.0626(k_0 + k_1)$

$a_1 = (0.16704 - 0.0055404k_0 + 0.0007196k_1)$

$a_2 = 0.060272 - 0.00015652k_1 + 0.00008452k_0$

$a_3 = 0.0045692 - 0.0000018k_1 + 0.0000018k_0$

$b_0 = 0.0626(k_0 + k_1)$

$b_1 = 0.0055403k_0 - 0.0007196k_1$

$b_2 = 0.00015652k_1 - 0.00008452k_0$

$b_3 = 0.0000018(k_1 - k_0)$

Apply modified characteristic ratio assignment criterion:

$$\begin{cases} \alpha_1 = \frac{a_1^2}{a_0 a_2} > 2 \\ \alpha_2 = \frac{a_2^2}{a_1 a_3} > 2 \\ 0.1 < \tau < 0.5 \end{cases} \quad \&\& \quad \begin{cases} \frac{b_0}{b_1} > 10 \\ \frac{b_0}{b_2} > 10 \\ \frac{b_0}{b_3} > 10 \end{cases} \tag{5.46}$$

Based on the stabilizing sets and above criterion, the optimized sets can be got as following figure 5.5:

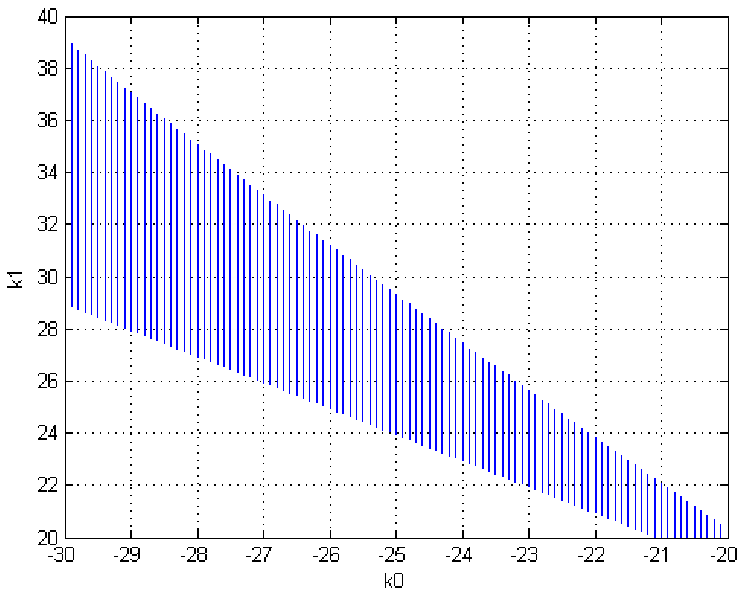

Figure 5.5 Optimized (k1,k0) sets of Digital PI Controller

If we select k0=-30, k1=39, then the step response is as figure 5.6:

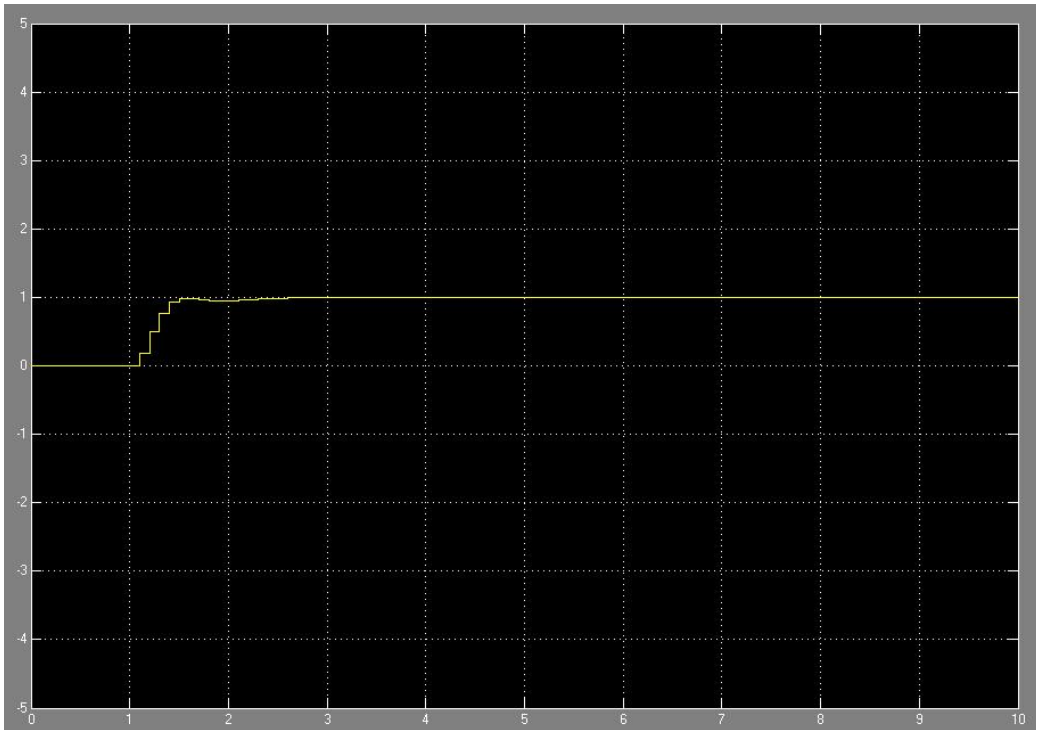

Figure 5.6 The Step Response of Optimized Digital PI Controller

It is evident that the overshoot is much less than the previous result.

5.6 Digital PID Controller Stabilizing Sets Design for DC Motor

Discrete time PID controller is:

$$C(z) = \frac{K_2 z^2 + K_1 z + K_0}{z(z-1)} \tag{5.47}$$

$$P(z) = \frac{0.004802z + 0.003013}{z^2 - 1.038z + 0.2466} \tag{5.48}$$

The characteristic polynomial is

$$\sigma(z) = z(z-1)D(z) + (K_2 z^2 + K_1 z + K_0)N(z) \tag{5.49}$$

Applying the Tchebyshev representations, then

$z^{-1}\delta(z)N(z^{-1})$

$$= -(u+1)P_1(u) - (1-u^2)P_2(u) - [(K_0+K_2)u - K_1]P_3(u)$$

$$+ j\sqrt{1-u^2}[-(u+1)P_2(u) + P_1(u) + (K_2-K_0)P_3(u)]$$

$$= R(u, K_0, K_1, K_2) + j\sqrt{1-u^2}T(u, K_0, K_2) \tag{5.50}$$

Let

$$K_3 \coloneqq K_2 - K_0$$

Then we have as follows:

$$R_D(u) = 2u^2 + 1.038u - 0.7534u \tag{5.51}$$

$$T_D(u) = -2u - 1.038 \tag{5.52}$$

$$R_N(u) = -0.004802u + 0.003013 \tag{5.53}$$

$$T_N(u) = 0.004802 \tag{5.54}$$

$$P_1(u) = 0.006026u^2 - 0.0029u - 0.0073 \tag{5.55}$$

$$P_2(u) = -0.006026u + 0.0005 \tag{5.56}$$

$$P_3(u) = 0.000014468u + 0.000032059 \tag{5.57}$$

Therefore

$$R(u, K_1, K_2, K_3) = -0.012052u^3 - 0.002626u^2 + 0.016226u + 0.0068 - [(2K_2 - K_3)u - K_1](0.000014468u + 0.000032059) \tag{5.58}$$

$$\mathrm{T}(\mathrm{u}, K_3) = 0.012052u^2 + 0.002626u - 0.0078 + K_3(0.000014468u + 0.000032059) \tag{5.59}$$

Let T=0

Then $K_3 = -\frac{0.012052u^2 + 0.002626u - 0.0078}{0.000014468u + 0.000032059}$

As $-1 \le u \le 1$

$$-92 \le K_3 \le 271$$

The amount of roots of $\sigma(z)$ inside the circle with radius 1 should be 4.

$$i_\sigma = 4$$

$$i_{N_r} = 0$$

$$l = 1$$

$$i_1 - i_2 = \left(i_\sigma + i_{N_r}\right) - (l+1) = 2$$

Fix $K_3 = 0$,

For $T(u, K_3)$, the real roots in the range (-1,1) are -0.920760 and 0.703120

$$Sgn[T(-1)] = 1$$

$$\frac{1}{2} Sgn[T(-1)](Sgn[R(-1)] - 2Sgn[R(-0.920760)] + 2Sgn[R(0.703120)] - Sgn[R(1)])=2 \tag{5.60}$$

$$Sgn[R(-1)] = 1$$

$$Sgn[R(-0.920760)] = -1$$

$$Sgn[R(0.703120)] = 1$$

$$Sgn[R(1)] = 1$$

Then the following linear inequalities can be got

$$-(-2K_2 - K_1) * 0.000017591 > 0$$

$$-0.00001494 - (-1.84K_2 - K_1) * 0.00001874 > 0$$

$$0.012721 - (2K_2 - K_1) * 0.00004223 > 0$$

$$0.008348 - (2K_2 - K_1) * 0.000046527 > 0$$

We could choose $K_3 = 0$, that is $K_2 - K_0 = 0$

Let us set $K_1 = K_2 = K_0 = 1$

Therefore

$$C(z) = \frac{z^2 + z + 1}{z(z-1)}$$

The system diagram is shown as following figure 5.7:

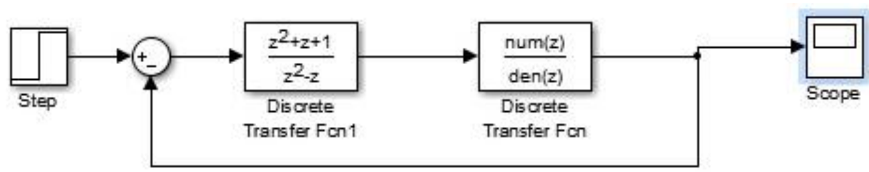

Figure 5.7 Discrete Time Motor Control with PID Controller Simulink

The step response of this system is as figure 5.8:

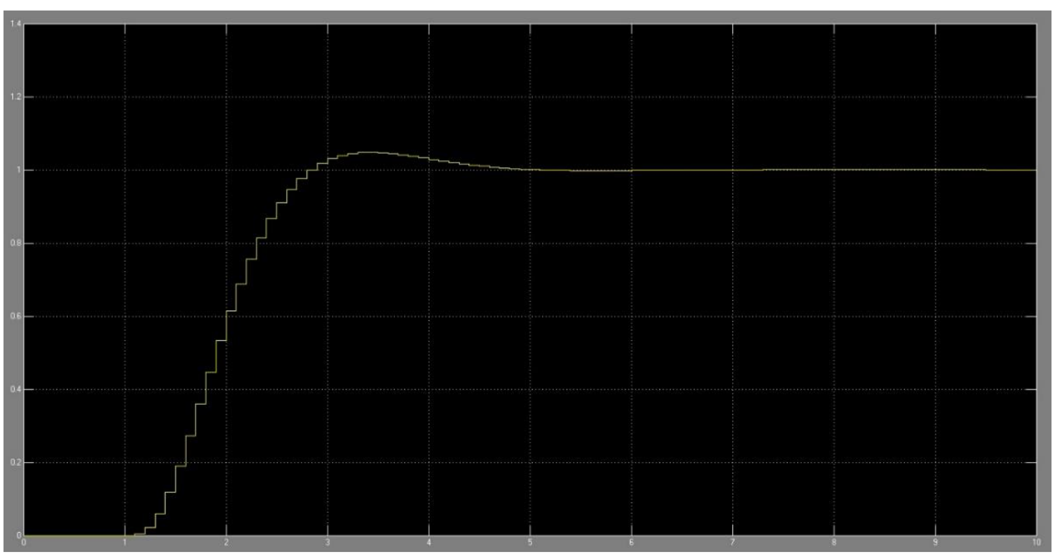

Figure 5.8 Discrete Time Motor Control with PID Controller Step Response

From above figure, the system is stable by applying the PID controller designed by stabilizing sets.

6. CONCLUSION

This thesis provides a complete theoretical procedure to design PID controller parameters for DC motor system.

For the continuous time DC motor system, signature method gives us a tool to find all stabilizing sets. But the transient response performance are not considered. To decrease overshoot and rising time, the thesis proposed the modified characteristic ratio assignment method. Using the proposed criterion, the new parameter sets can guarantee better transient response performance.

Using above method, the PID controller and PD controller design are integrated into the DC motor velocity control and DC motor position control problems separately. The selected parameter successfully design the DC motor control system and achieve a good step response.

For the discrete system, digital PI and PID stabilizing sets are designed by using root counting algorithm. In addition, by using w transform and modified characteristic ratio assignment method, the optimized digital PI parameter sets are also got.

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