PERSON-BASED ADAPTIVE PRIORITY SIGNAL CONTROL WITH CONNECTED-VEHICLE TECHNOLOGY

A Thesis

by

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ABSTRACT

This thesis proposes a TSP (transit signal priority) strategy of person-based adaptive priority signal control with connected-vehicle information (PAPSCCI). By minimizing the total person delay at an isolated intersection, PAPSCCI can assign signal priorities to transit vehicles due to their high occupancies, while minimize the negative impact to the auto traffic. With the accurate vehicle information provided by connected-vehicle technology, PAPSCCI can estimate person delay for each passenger directly and form a MILP (mixed-integer linear program) for the optimization.

Performances of PAPSCCI were evaluated through simulations. Results show decreases of both vehicle delay and person delay of all vehicle types when there are up to three bus routes running through the intersection.

How different penetration rates of the connected-vehicle technology affect the performance of the PAPSCCI were tested. Necessary revisions were made to the PAPSCCI model considering different penetration rates. Results show that the effectiveness of PAPSCCI worsens with the lowering of penetration rate. The delay improvements, however, were still promising when the penetration rate is above 40%.

PAPSCCI model were also developed and tested with communication range of 2000 m, 1000 m, 500 m and 250 m. Expect that the 1000 m case has the best delay improvements after PAPSCCI optimization, the effectiveness of the model worsens when the communication range getting smaller. Even when the communication range is down to 250 m, PAPSCCI can still reduce the delay for all vehicle types.

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CHAPTER I

INTRODUCTION

Traffic congestion is a serious problem that are bothering modern cities almost every day. Improving the mobility of the transportation system is the key in addressing this problem, therefore an efficient multimodal transportation system is required. Within such transportation system, transit vehicles, as designed to be capable of transporting a large number of passengers through areas at one time, should be preferentially treated, especially at signalized intersections, where a large portion of their delay occurs.

Transit signal priority (TSP) is such an operational strategy that favors transit vehicles at signalized intersections. By assigning signal priorities to transit vehicles, such as buses, TSP strategies can significantly reduce transit delay at intersections, thereby improving their operation efficiencies and schedule reliabilities. The study on the TSP strategies has been on for decades and the success of TSP strategies in reducing transit vehicles' intersection delay and the improvement of their operation efficiencies has been demonstrated by many researches. Several cities in the U.S. and Europe have now been implemented with TSP strategies.

Most of the conventional TSP strategies are triggered by receiving the signal priority requests from the coming transit vehicles, and then assign signal priorities following pre-defined logics such as green extension or red truncation. To decide which rule to follow is based on transit vehicle's arrival time which is acquired from some prediction models using the detection data. However, the conventional detection systems, such as loop detectors, cannot provide enough accuracy in predicting transit vehicles' arrival time, and may affect the efficiency of the TSP strategies. Besides, there are two major problems with the traditional ways of assigning signal priorities. First, they often leads to delay increases with auto traffic. Second, they cannot perform well when two or more transit vehicles are traveling on conflicting routes. Therefore, a more sophisticated TSP algorithm should be developed. By optimizing the traffic signals, the new algorithm should be able to perform TSP while minimizing the negative impacts caused to auto traffic, as well as coordinating among conflicting bus routes. A more advanced detection technology is needed to support such algorithm.

Connected-vehicle technology is such a new emerged and advanced technology. By equipping vehicles with diagnostic sensors, connected-vehicle technology can make wireless communications among vehicles and between vehicles and nearby infrastructures possible. By collecting and providing vehicle data individually, connected-vehicle technology has no doubt brought and will continue to bring more possibilities to the development of TSP strategies. AASHTO Connected Vehicle Infrastructure Deployment Analysis stated that to apply connected-vehicle technology to TSP strategies is important and can greatly enhance mobility *(1)*.

In this thesis, a TSP strategy of person-based adaptive signal priority control with connected-vehicle information (PAPSSCI) is proposed. By minimizing the total person delay at an intersection, the PAPSCCI model can assign signal priorities to transit vehicles based on their high occupancies. By using vehicle speed and location information available from the connected vehicle technology, the PAPSCCI model is able to avoid estimating vehicle delay using second order queueing model but to directly formulate delays in a MILP (mixed-integer linear program). Restrictions on fixed cycle length and fixed phase sequence are freed in the algorithm, providing more flexibility in assigning signal priorities. Development on the basic PAPSCCI model are made to expand the adaptability of the model to different sizes of communication range at the intersection. Traffic simulations will be used to evaluate the performance of the PAPSCCI model under different scenarios, including auto only, single bus route operation and multiple bus routes operations. Performances of the PAPSCCI model with different penetration rates of the connected vehicle technology will also be tested, since the technology is still new to the market and it is now impossible to have full penetration rate in reality.

The organization of the remainder of the thesis is as follows: Chapter II is a literature review on the research background and the need for this research; the description on the methodology and the simulation testbed are presented in Chapter III and Chapter IV; followed by the simulation results and analysis in Chapter V; Chapter VI summarizes and concludes this thesis with further discussions.

CHAPTER II

LITERATURE REVIEW

This chapter reviews related studies on different transit signal priority strategies and their implementing environments, in order to have general understandings on the background of this research area, as well as the need to conduct this thesis. Two main aspects are focused to depict the background thoroughly on this research: transit signal priority and data collecting systems.

2.1 Transit Signal Priority

Transit, as being capable of transporting a large number of passengers at one time, is considered to be a key to relieve traffic congestion in densely populated cities. It is believed that, with proper treatment, transit has the potential to achieve high operation efficiencies. Hence, TSP strategies are designed to offer preferential treatments to transit vehicles at signalized intersections. Passive, active and adaptive signal control are the three main categories of existing TSP strategies *(2)*.

2.1.1 Passive TSP Strategies

Passive TSP strategies are inexpensive since they don’t require vehicle detections. Their preferential treatments for transit vehicles are pre-set and are realized through changes in the signal timings, such as extending the green time of those phases with transit vehicles, or reducing the cycle length to increase the turnover of all the phases. Once implemented, the passive TSP strategies will not change according to the real traffic conditions. Therefore, only when the bus operation is predictable and frequent, and when the non-transit traffic flow is comparatively low, will passive TSP strategies perform well *(2,3)*.

2.1.2 Active TSP Strategies

Contrary to passive TSP strategies, which are developed off-line, active TSP strategies can react to real-time traffic through vehicle detectors placed at signalized intersections. The signal timings are adjusted based on some pre-defined rules when the TSP requests from approaching transit vehicles are received. The need for detection and communication systems increases the cost of active TSP strategies, but improves their effectiveness as well.

The most commonly adopted rules are green extension of the current transit phase and red truncation of the non-transit phase. The former one applies when the signal is green for the approaching transit vehicles. And the latter one is used only when red signals of the transit serving phase are on. Thereby, active TSP strategies often adopt these two rules together to achieve better performances *(2,4)*. Phase insertion and phase rotation are two other commonly used logic rules in active TSP strategies as well. Known by name, they decrease the transit delays either through inserting a new phase or rotating the transit phase to serve the approaching transit vehicles *(3)*. Based on these basic logic rules, many researches have been done to develop active TSP strategies. Janos et al. designed a rule-based TSP strategy for a corridor with high frequency of TSP requests. The model adopted early green, green extension and early red together, and was able to quickly respond to and recover from signal priority conductions *(5)*. Lee et al. proposed a rule-based TSP method using online transit travel time prediction model to select the most appropriate TSP plan from six commonly used priority plans, including green extension only, green extension and red truncation, green truncation only, green truncation and red truncation, queue clearance, and queue clearance and red truncation *(6)*. Apart from those, many researches and field tests have been conducted to demonstrate the efficiency of active TSP strategies *(7,8)*.

2.1.3 Adaptive TSP Strategies

Studies show that attempting to assign signal priorities to transit vehicles inevitably causes negative impacts on the operation of non-transit traffic. With the development of TSP strategies gets further, people find this problem hard to ignore. Adaptive TSP strategies are developed due to this concern. They try to find the balance points where transit vehicles get their signal priorities and the negative impacts on other vehicles are minimized. To make a good trade-off between transit and non-transit vehicles, it is important for adaptive TSP strategies to have an effective signal control algorithm. A number of mathematical models have been built in previous researches. Ma, et al. built an optimization model to minimize the bus delay by generating optimal servicing sequence for multiple bus lines. A rolling time horizon approach is adopted to implement the model in real-time situation *(9)*. Li, et al. optimized green splits to minimize a weighted delay of both buses and other vehicles. Single bus requests were assumed in this model *(10)*. Instead of using vehicle delay as the basis for optimization, Christofa, et al. minimized person delay by multiplying the estimated occupancies of each type of vehicles and their vehicle delays in the objective function. This person-based algorithm has good performance in the simulation test, by effectively decreasing the delay of bus passengers while only increasing the delay of auto passengers by a little amount *(11)*. He, et al. proposed a platoon-based formulation to optimize the arterial signal timings based on the clustered signal requests with different priority levels. Vehicles, including buses, are treated as different types of platoons through a platoon recognition algorithm *(12)*. Zeng, et al. proposed a stochastic mixed-integer model to minimize the deviation of the phase split times from the optimal background split times in the objective function *(13)*. He, et al. used a heuristic method to solve simultaneous multiple transit signal priority requests at an isolated intersection *(14)*. Zhou, et al. optimizes the signal control in order to perform TSP using a parallel genetic algorithm (PGA) *(15)*.

However, if considering auto traffic, a majority of previous studies on adaptive TSP strategies treated auto traffic other than transit vehicles as a group. To estimate vehicle delays, deterministic queueing model is the best available method without individual vehicle information.

2.2 Data Collecting Systems

Apart from a good formulated signal control algorithm, an informative and accurate detection system is also essential to adaptive TSP strategies. From loop detectors, to mobile sensors, detection technologies are kept emerging during the past decades and continuously brought new ideas to accurately estimating the queue length, as well as other performance measurements of signalized intersections.

2.2.1 Transit Vehicle Detection Systems

Transit vehicle detection is important to TSP strategies. Many concerns are involved in selecting an appropriate detection system, including the accuracy requirements of the TSP algorithm, the stability of the detection system, as well as the implementation and maintenance costs of the system. Based on the detecting range, most of the existing transit vehicle detection systems can be divided into three categories: point detectors, area detectors and zone detectors *(2)*.

Point detectors, such as loop detectors, radar detectors and video detectors, are commonly used in conventional TSP strategies. Point detectors can quickly report the existence of a transit vehicle at a certain point, but cannot provide more information on the transit vehicle's behavior between different detection points, such as vehicle's speed change. Due to this reason, point detectors have less accuracy in predicting transit vehicles' arrival time to the intersection.

Area detectors, by name, can monitor transit vehicle movements through an area. They can provide continuous vehicle data so that have higher accuracy in predicting transit vehicles arrival time than point detectors. Therefore, area detectors, such as the AVL technology (Automatic Vehicle Location) based on GPS (Global Positioning System), have become more and more popular with current developed TSP strategies.

The probe vehicle data have been used in a number of researches have used in transit vehicle arrival time prediction, as well as queue length estimation. Dailey, et al. proposed an algorithm using the time and location data from the AVL system to predict transit vehicles' future arrival time *(16)*. Mishalani, et al. presented an evaluation on the benefits of reliable bus arrival time information system to the waiting passengers using AVL information *(17)*. Bie, et al. developed a travel time prediction model to forecast transit vehicle's arrival time at next bus stop based on GPS data *(18)*. Ferman, et al. developed an analytical model to evaluate the feasibility of a real-time traffic information system based on probe vehicle data *(19)*. Comert, et al. proposed an estimation algorithm of the queue lengths at an isolated intersection using the location information from probe vehicles in the queue *(20)*.

Zone detectors are less accurate in reporting the transit vehicles location, but can sense the existence of transit vehicles within the detection zone. However, if properly provided with additional vehicle location information, zone detectors can achieve better performance in TSP strategies.

Other types of detectors including: driver activated detection system and exit detection systems. The former one is not an ideal way to detect target vehicles since it increases the workload on drivers. The latter one has been used in many TSP systems. By detecting the time when buses leave the intersection, it can help the system to decide when to terminate granting the signal priorities, thus increase the efficiency of the traffic operations *(2)*.

2.2.2 Connected-vehicle Technology

"Connected-vehicle" is a new emerged technology that provides enriched data for new types of TSP models to be developed. Equipped with wireless communications onboard units (OBU), vehicles can transmit messages to other vehicles and nearby infrastructures over the air. Messages, such as vehicle speed, location, acceleration/deceleration rate, queue length, occupancy, and stopped time may be collected and transmitted *(21)*. With these information, connected-vehicle technology has made many applications so far. Badillo, et al. proposed a queue length estimation algorithm using data from conventional vehicle detectors and data provided by the connected vehicle technology. The proposed algorithm improved the prediction accuracy to within the length of a single vehicle *(22)*. Venkatanarayana, et al. developed a signal control strategy for oversaturated traffic conditions using connected vehicle data in the IntelliDrvie environment. In the algorithm, the queue length was estimated according to the position of the last vehicle equipped with connected vehicle devices *(23)*. Christofa, et al. adopted connected vehicle technology in the queue spillback detections and achieved promising results from test simulations *(24)*.

As a new technology, time is needed for connected-vehicle technology to reach a certain level of market penetration rate. Several studies indicated that better performance of connected-vehicle technology in transportation applications comes with higher penetration rate *(12,25)*. Goodall concluded in his researches that, though varies from model to model. the minimum required penetration rate for the connected-vehicle technology to demonstrate any benefit is typically near 20% to 30% *(26)*.

Being able to gather individual vehicle information and trajectory data continuously and accurately, connected vehicle technology has no open a door for the development of TSP strategies. However, researches adopting this new technology into developing TSP strategies are not often seen since it is still new to the market. A majority of previous studies on adaptive TSP strategies treated auto traffic other than transit vehicles as a group. To estimate vehicle delays, deterministic queueing model is commonly assumed for auto delay calculation, as it is proved the best available method without individual vehicle information. The algorithms are often formulated non-linearly when estimating auto delay on a rate based method. This non-linearity increases the complexity in solving the optimization. However, under the connected vehicle paradigm, individual vehicle delays can be calculated, and overall vehicle delays can be aggregated linearly. Such a signal optimization algorithm named PAPSCCI (person-based adaptive priority signal control with connected-vehicle information) will be proposed in this thesis. This model treats auto delay individually as with bus delay. Calculating each vehicles' delay using its own travelling data can significantly improve the estimation accuracy, and helps the model to make quicker reactions to the traffic variations at the intersection. A MILP (mixed-integer linear program) is adopted to reduce the complexity in solving the optimization algorithm.

CHAPTER III

METHODOLOGY

A person-based adaptive priority signal control with connected-vehicle information (PAPSCCI) is introduced in this chapter. PAPSCCI model is designed for isolated intersections, minimizing the total person delay of all directions based on adaptive signal control method. Christofa et al. *(11)* used the person-based idea to develop a mathematical model and proved that the person-based models had advantages over traditional vehicle-based TSP methods. Person-based TSP models, on one hand, ensure vehicles with high occupancy, such as buses, to have signal priorities over the general traffic; on the other hand, they minimize the potential negative impacts on auto delays by including them in the objective function. Moreover, person-based TSP models are more flexible in assigning signal priorities especially when conflicting bus lines are competing for signal priorities at approximately the same time.

Connected-vehicle information is another important characteristic of the PAPSCCI model. With the help of connected-vehicle technology, which makes continuous and accurate vehicle data possible, PAPSCCI model can treat every vehicle individually. This treatment not only improves the delay estimation, especially for auto traffic, but also leads to a MILP (mixed-integer linear program), making it easier to generate optimal solutions.

This chapter will begin with introducing the basic PAPSCCI model. This basic model assumes perfect information are available, including 100% market penetration rate of the connected-vehicle technology and large enough data collecting area. Based on this basic PAPSCCI model, concerns for the model with lower penetration rates will be considered. How to revise the model under different penetration rates of the connected-vehicle technology will be presented. Last but not least, to adapt the model to situations where the data collecting area is limited, further developments are made to the basic PAPSCCI model and will be introduced at the end of this chapter.

3.1 Basic PAPSCCI Model

3.1.1 Objective Function

This basic PAPSCCI model is built assuming perfect vehicle and intersection information is available. It minimizes the total person delay, which is the summation of each vehicle's delay multiplying by its occupancy. The objective function of PAPSCCI model is shown in Equation [1]. Vehicle occupancies and other information, such as vehicle location and speed, are collected through connected-vehicle technology. For each optimization, all the vehicles are present at or will arrive to the intersection during the planning horizon are considered in this objective function.

$$Min\,Z = \sum_{j=1}^{J}\sum_{i=1}^{I_j^Q} o_{i,j} d_{i,j}^Q + \sum_{j=1}^{J}\sum_{i=I_j^Q+1}^{I_j} o_{i,j} d_{i,j} \qquad [1]$$

where

J is the total number of phases adopted in this intersection. Normally, it equals to 8.

I_j^Q is the number of vehicles that are waiting in the queue of phase *j* at the intersection when optimization begins.

I_j is the total number of vehicles that are either presenting at or will arrive to the intersection before the end of the planning horizon for phase *j*.

$o_{i,j}$ is the occupancy of the *i*th vehicle of phase *j*.

$d_{i,j}^Q$, $d_{i,j}$ are both the delay of the *i*th vehicle of phase *j*. The only difference is that $d_{i,j}^Q$ applies to vehicles that are waiting in the queues when optimization begins, while $d_{i,j}^Q$ refers to vehicles that will approach the intersection during the planning horizon.

3.1.2 Timing Structure

The planning horizon is set as two cycles in PAPSCCI model. That is to say, when cycle *k* is under optimization, the delays of those vehicles approaching the intersection during cycle *k+1* are also considered. In this way, the possible influences of each optimization on the following cycles can be taken under control. Therefore, the PAPSCCI model can ensure each optimization would not leave accumulated negative impacts that might deteriorate the intersection's traffic operation in the future.

Though the duration of a planning horizon is two cycles, optimization in the PAPSCCI model is conducted at the beginning of every cycle. Before every optimization process begins, the system is assumed to be able to "know" the total number of vehicles of each phase that would be included in this optimization (I_j). Each vehicle's arrival time to the stop bar with no delays ($T_{i,j}^{r}$) is predicted in the system by a simple calculation using vehicle speed ($v_{i,j}$) and location information from the connected-vehicle technology. Vehicle's acceleration and deceleration processes are ignored. All the vehicles included in one optimization process are given a unique index *i*, according to their arrival sequences or queueing positions (for vehicles that are queueing at the intersection before optimization) to the stop bar.

PAPSCCI model uses a standard ring-barrier signal timing structure, which can be mathematically described as Equations [2] to [8]. This precedence relationship was proposed by Head et al. *(27)*.

$$t_{1,1} = t_{5,1} = 0 \qquad [2]$$

$$t_{2,k} = t_{1,k} + v_{1,k};\ t_{3,k} = t_{2,k} + v_{2,k};\ t_{4,k} = t_{3,k} + v_{3,k} \qquad [3]$$

$$t_{6,k} = t_{5,k} + v_{5,k};\ t_{7,k} = t_{6,k} + v_{6,k};\ t_{8,k} = t_{7,k} + v_{7,k} \qquad [4]$$

$$t_{1,k+1} = t_{4,k} + v_{4,k};\ t_{5,k+1} = t_{8,k} + v_{8,k} \qquad [5]$$

$$t_{1,k} = t_{5,k};\ t_{3,k} = t_{7,k} \qquad [6]$$

$$t_{1,\mathrm{k}+1} - t_{1,k} = t_{5,k+1} - t_{5,k} = C \qquad [7]$$

$$v_{j,k} = g_{j,k} + Y + R \qquad [8]$$

where

$t_{j,k}$ is the start time of green of phase *j* in cycle *k*.

$v_{j,k}$ is the phase split of phase *j* in cycle *k*.

$g_{j,k}$ is the green time of phase *j* in cycle *k*.

C is background cycle length.

Y is yellow time.

R is all-red time.

However, different from Christofa's person-based TSP model, which used fixed cycle length and fixed signal phase sequence in the formulation *(11)*, PAPSCCI model does not strictly require a fixed cycle length, but a fixed planning horizon instead. Besides, PAPSCCI model also allows phase sequence exchange between every two signal phases that are on the same side of each barrier and ring in the signal timing diagram. For example, phase *1* can be leading or lagging behind phase *2*, but cannot change its position in the ring diagram with other phases. To achieve this mathematically, eight phases are divided into four pairs according to their positions to the rings and barriers. Within each signal phase pair, for example, phase *1* and phase *2*, a virtual phase (phase *1'*) is inserted following phase *2*, as shown in Figure 1. A set of binary variables ($r_{1,k}$ and $r_{1',k}$) are introduced here to decide whether the time duration of phase *1* or phase *1'* should be zero. In this way, whether phase *1* should be leading or lagging behind phase *2* can be decided. The complete signal timing structure used in PAPSCCI model are mathematically expressed through Equation [9] to [19].

$$t_{1,1} = t_{5,1} = 0 \qquad [9]$$

$$t_{2,k} = t_{1,k} + v_{1,k};\ t_{1',k} = t_{2,k} + v_{2,k};\ t_{3,k} = t_{1',k} + v_{1',k};\ t_{4,k} = t_{3,k} + v_{3,k} \qquad [10]$$

$$t_{6,k} = t_{5,k} + v_{5,k};\ t_{5',k} = t_{6,k} + v_{6,k};\ t_{7,k} = t_{5',k} + v_{5',k};\ t_{8,k} = t_{7,k} + v_{7,k} \qquad [11]$$

$$t_{1,k+1} = t_{3',k} + v_{3',k};\ t_{5,k+1} = t_{7',k} + v_{7',k} \qquad [12]$$

$$t_{1,k} = t_{5,k};\ t_{3,k} = t_{7,k} \qquad [13]$$

$$t_{7',k+1} + v_{7',k+1} - t_{1,k} = 2C \qquad [14]$$

$$v_{j,k} = g_{j,k} + Y + R \quad \forall j \in \{2,4,6,8\} \qquad [15]$$

$$v_{j',k} + v_{j,k} = g_{j,k} + Y + R \quad \forall j \in \{1,3,5,7\} \qquad [16]$$

$$v_{j,k} \le r_{j,k} M;\ v_{j',k} \le r_{j',k} M \quad \forall j \in \{1,3,5,7\} \qquad [17]$$

$$v_{j,k}, v_{j',k} \geq 0 \qquad [18]$$

$$r_{j,k} + r_{j',k} = 1 \quad \forall j \in \{1,3,5,7\} \qquad [19]$$

where

$t_{j,k}, t_{j',k}$ are the start time of green of phase *j* or virtual phase *j'* in cycle *k*.

$v_{j,k}, v_{j',k}$ are the phase split of phase *j* or virtual phase *j'* in cycle *k*.

$r_{j,k}, r_{j',k}$ are binary variables. When $r_{j,k} = 0$, it makes $v_{j,k} = 0$, meaning in cycle *k* phase *j* is lagging behind phase *j+1*. (Only when j is odd do $t_{j',k}, v_{j',k}, r_{j',k}$ make sense.)

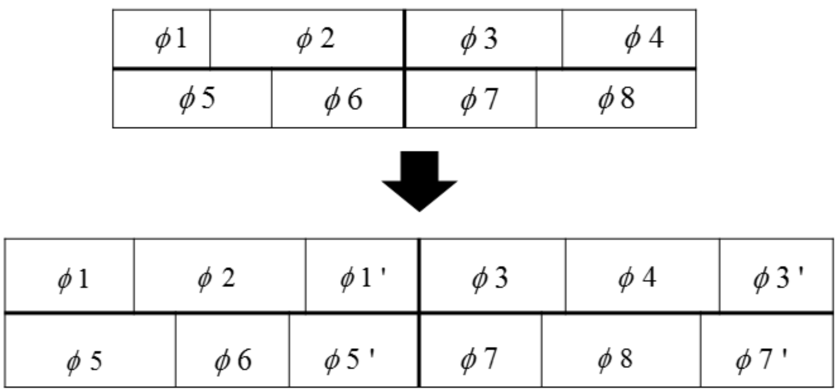


Figure 1 Signal Timing Structure

The optimization is realized by changing the start time ($t_{j,k}$) and the green duration ($g_{j,k}$) of each phase within the planning horizon. The duration of each phase's green time is limited by a lower bound, depending on the v/c ratios.

$$g_{j,k} \geq g_{\min,j} \qquad [20]$$

3.1.3 Vehicle Delay

There are two delay categories in the objective function – delay of vehicles that are queueing at the intersection by the time the optimization begins, and of those that will approach within the following planning horizon. Delay of the first category is easy to calculate. Take the *i*th vehicle in the queue as an example. The number of vehicles queueing ahead of it is $\left\lfloor \frac{i-1}{N_j} \right\rfloor$ (N_j is the number of lanes of phase *j*). These vehicles will clear the intersection at the saturation flow rate (s_j) when the green begins ($t_{j,k}$). Therefore, delay of the *i*th vehicle can be calculated using Equation [21].

$$d_{i,j}^{Q} \geq \frac{\left\lfloor (i-1)/N_j \right\rfloor}{s_j} + t_{j,k} - t_{i,j}^{s} \quad \forall i \in [1, I_j^{Q}] \qquad [21]$$

However, for vehicles that are not currently present but will approach the intersection during the following planning horizon, the delay estimation gets a bit more complicated. Based on each vehicle's arrival time to the stop bar ($T_{i,j}^{r}$), approaching vehicles can be divided into three groups (as shown in Figure 2): those arrive before the end of green time in cycle *k* (arrival A); those arrive after the end of green in cycle *k* but before the end of green in cycle *k+1* (arrival B); and those arrive after the end of green in cycle *k+1* (arrival C). Two binary variables, $y_{i,j}^{k}$ and $y_{i,j}^{k+1}$, are introduced to help distinguish vehicles from these three arrival types. The relationships between the binary variables and the vehicle's arrival time to the stop bar are expressed as Equations [22] to [25].

$$T_{i,j}^{r} > t_{j,k} + g_{j,k} - y_{i,j}^{k}M \qquad [22]$$
$$T_{i,j}^{r} \le t_{j,k} + g_{j,k} + (1 - y_{i,j}^{k})M \qquad [23]$$
$$T_{i,j}^{r} > t_{j,k+1} + g_{j,k+1} - y_{i,j}^{k+1}M \qquad [24]$$
$$T_{i,j}^{r} \le t_{j,k+1} + g_{j,k+1} + (1 - y_{i,j}^{k+1})M \qquad [25]$$

where

$T_{i,j}^{r}$ represents the time of the i^{th} vehicle of phase j to arrive to the stop bar of the intersection with no delay.

$y_{i,j}^{k}$ and $y_{i,j}^{k+1}$ are binary variables. When $y_{i,j}^{k}$ equals to 1, it means the i^{th} vehicle of phase *j* will theoretically arrive to the stop bar before the end of green in cycle *k*. When it equals to 0, it means the opposite. The meaning of $y_{i,j}^{k+1}$ is similar, but is for cycle *k+1*. For any vehicle, if it arrives before the end of green in cycle *k*, it must arrive before the end of green in cycle *k+1*. Therefore, $y_{i,j}^{k}$ is never greater than $y_{i,j}^{k+1}$.

M is the big number constraint.

From the equations above, the value of $(y_{i,j}^{k}, y_{i,j}^{k+1})$ for each vehicle may have three possible combinations. (1,1) refers to vehicle arrival type A, (0,1) points to arrival B, and (0,0) shows the vehicles belong to arrival C. Since delay estimation varies among different arrival types, the value of $(y_{i,j}^{k}, y_{i,j}^{k+1})$ helps to find the right delay calculations for each vehicle, as shown in Equations [26] to [31].

$$d_{i,j} \ge d_{i,j}^{A} - (1 - y_{i,j}^{k})M - (1 - y_{i,j}^{k+1})M \qquad [26]$$
$$d_{i,j} \le d_{i,j}^{A} + (1 - y_{i,j}^{k})M + (1 - y_{i,j}^{k+1})M \qquad [27]$$
$$d_{i,j} \ge d_{i,j}^{B} - y_{i,j}^{k}M - (1 - y_{i,j}^{k+1})M \qquad [28]$$
$$d_{i,j} \le d_{i,j}^{B} + y_{i,j}^{k}M + (1 - y_{i,j}^{k+1})M \qquad [29]$$

$$d_{i,j} \geq d_{i,j}^{C} - y_{i,j}^{k} M - y_{i,j}^{k+1} M \quad [30]$$

$$d_{i,j} \leq d_{i,j}^{C} + y_{i,j}^{k} M + y_{i,j}^{k+1} M \quad [31]$$

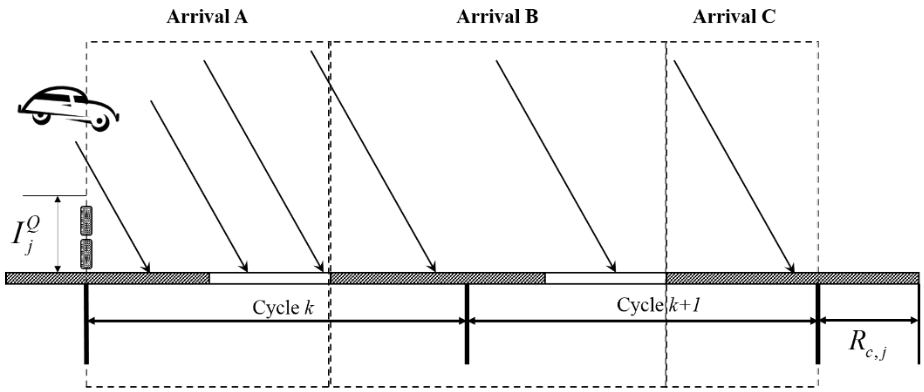


Figure 2 Three Arrival Types for Vehicles Arriving the Intersection

Two other important binary variables, $\theta_{i,j}^{k}$ and $\theta_{i,j}^{k+1}$, are introduced in the formulation to describe the situation when vehicles are leaving the intersection. A vehicle can clear the intersection before the end of green of phase *j* in cycle *k* when its $\theta_{i,j}^{k}$ equals to 1, otherwise, it cannot clear the intersection in cycle *k* ($\theta_{i,j}^{k}$ equals 0).

$$\theta_{i,j}^{k} \leq y_{i,j}^{k} \quad [32]$$

$$\theta_{i,j}^{k} + \theta_{i,j}^{k+1} \leq y_{i,j}^{k+1} \quad [33]$$

Equations [32] and [33] ensure three basic rules. First, vehicles arrive after the end of green in cycle *k* cannot clear the intersection during cycle *k*; second, vehicles arrive after the end of green in cycle *k+1* cannot clear the intersection in neither cycle *k* nor *k+1*; third, vehicles cannot leave the intersection in cycle *k* and cycle *k+1* both, but there is a chance that vehicles do not clear the intersection either in cycle *k* or cycle *k+1*.

3.1.3.1 Formulations for Arrival A

Delay occurs when the vehicle's arrival time is earlier than the start of the green, or when some of those vehicles arrived prior to it have queued up at the intersection. Hence, for the i^{th} vehicle of phase j, the queue length (in vehicles) upon its arrival ($q_{i,j}^{A}$) and the time it joins the queue ($t_{i,j}^{r,A}$) are the two important variables in its delay estimation. **(**The superior characteristic *A* in every variable in this part is only used to indicate arrival type A. So do the superior characteristics B and C in the following parts.**)**

Vehicle's time to join the queue at the intersection ($t_{i,j}^{r,A}$) is related to the vehicle's travel speed (v_j), its predicted time to arrive to the stop bar ($T_{i,j}^{r}$), and the queues upon its arrival ($q_{i,j}^{A}$). Since the queues are calculated in number of vehicles, while the unit of speed is m/s, an average length of vehicles (L_s) is assumed to convert the units. For each vehicle, the time to join the queue is calculated as in Equation[34].

$$t_{i,j}^{r,A} = T_{i,j}^{r} - \frac{q_{i,j}^{A} L_s}{v_j} \qquad [34]$$

For the i^{th} vehicle of phase j, there are clearly $(i-1)$ vehicles arrived prior to it, since the index i is numbered according to the vehicle's arrival sequence to the stop bar. To estimate its delay, it is important to find out how many of these vehicles may have left the intersection by the time the i^{th} vehicle approaches. Figure 3 shows the three typical situations of calculating the queues upon each vehicles arrival ($q_{i,j}^{A}$).

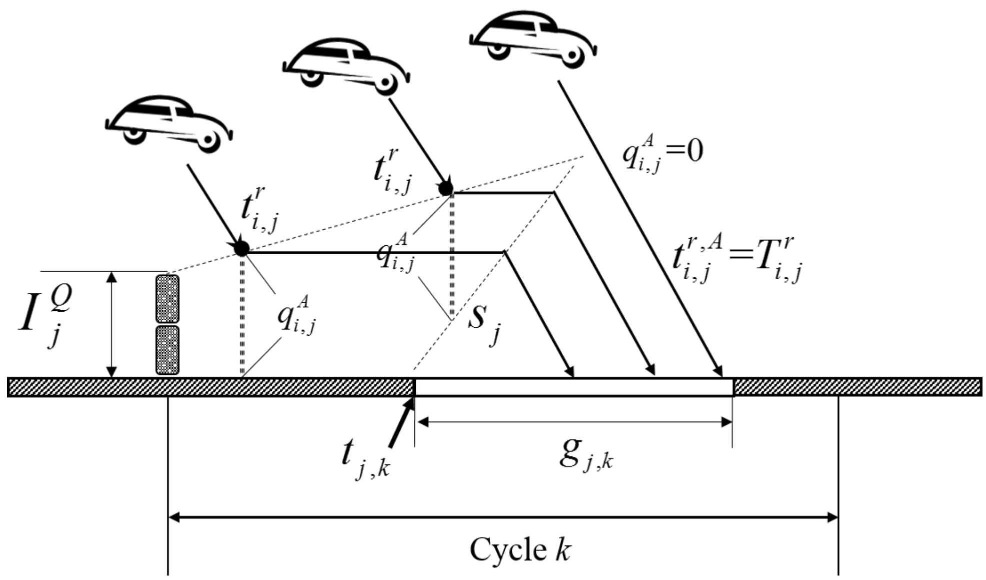


Figure 3 Three Types of Queues upon Approaching

In Figure 3, the vehicle on the left arrives before the green of cycle k starts. Therefore, the $(i-1)$ vehicles arrived prior all remains waiting at the intersection when it approaches. However, the green has been on for $(t^r_{i,j}-t_{j,k})$ when the middle vehicle approaches the intersection. Some of the vehicles arrived prior to it have left and they are leaving at the saturation flow rate (s_j). Thereby, the queues upon its arrival reduces to $\left\lfloor \frac{i-1}{N_j} \right\rfloor - s_j(t^r_{i,j}-t_{j,k})$ (N_j is the number of lanes of phase j). Vehicles arrive late enough may have $\left\lfloor \frac{i-1}{N_j} \right\rfloor \le s_j(t^r_{i,j}-t_{j,k})$, meaning all vehicles on its lane that arrives prior to it have cleared the intersection, and there would be no queues ahead of it by the time it arrives. This situation is represented by the vehicle on the right in Figure 3.

A binary variable ($\sigma_{i,j}^{A}$) is used in Equations [35] and [36] to decide whether a vehicle arrives before or after the start of green. When $\sigma_{i,j}^{A}$ equals to 1, it means the vehicle approaches (joins the queue at the intersection) before its green starts, and when $\sigma_{i,j}^{A}$ equals to 0, it means approaching after the start of green.

$$t_{i,j}^{r,A} \leq t_{j,k} + (1-\sigma_{i,j}^{A})M \qquad [35]$$

$$t_{i,j}^{r,A} > t_{j,k} - \sigma_{i,j}^{A}M \qquad [36]$$

Therefore, the queues upon approaching the intersection ($q_{i,j}^{A}$) for each vehicle of arrival A can be estimated in Equations [37] through [39], which correspond to the three types of queues illustrated in Figure 3.

$$q_{i,j}^{A} \geq \left\lfloor \frac{i-1}{N_j} \right\rfloor - (1-\sigma_{i,j}^{A})M \qquad [37]$$

$$q_{i,j}^{A} \geq \left\lfloor \frac{i-1}{N_j} \right\rfloor - s_j(t_{i,j}^{r,A} - t_{j,k}) - \sigma_{i,j}^{A}M \qquad [38]$$

$$q_{i,j}^{A} \geq 0 \qquad [39]$$

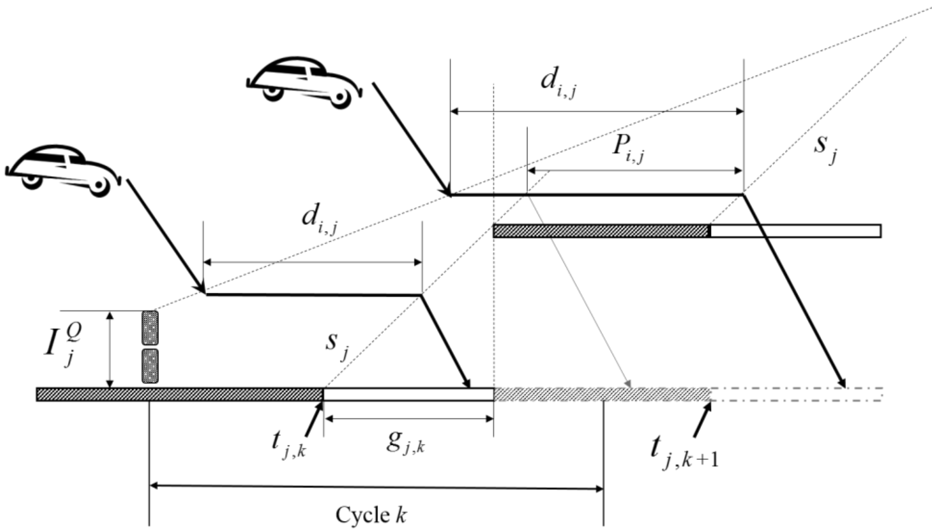


Figure 4 Two Types of Vehicle Delay

After approaching the intersection, the vehicle either clears the intersection in the current cycle k ($\theta_{i,j}^k = 1$) or waits for the green in next cycle ($\theta_{i,j}^k = 0$). Only if the queues upon a vehicle's arrival can be dissipated during the rest of green time in cycle k would the vehicle be able to clear the intersection during the current cycle. This condition can be described mathematically in Equations [40] to [43].

$$\frac{q_{i,j}^A}{s_j} \le g_{j,k} + (1-\sigma_{i,j}^A)M + (1-\theta_{i,j}^k)M \qquad [40]$$

$$\frac{q_{i,j}^A}{s_j} > g_{j,k} - (1-\sigma_{i,j}^A)M - \theta_{i,j}^k M \qquad [41]$$

$$\frac{q_{i,j}^A}{s_j} + t_{i,j}^{r,A} \le t_{j,k} + g_{j,k} + \sigma_{i,j}^A M + (1-\theta_{i,j}^k)M \qquad [42]$$

$$\frac{q_{i,j}^A}{s_j} + t_{i,j}^{r,A} > t_{j,k} + g_{j,k} - \sigma_{i,j}^A M - \theta_{i,j}^k M \qquad [43]$$

A vehicle clears the intersection only if all other vehicles from the queues upon its arrival have left. This waiting time is its delay. However, those cannot clear the intersection in cycle *k* has to wait longer for the next green. This extra waiting time ($P_{i,j}^{k}$) equals to the red duration between the two adjacent green lights of its phase, shown in Figure 4. Delays of all the vehicles belonging to arrival type A are calculated as from Equations [44] to [48].

$$d_{i,j}^{A} \geq \frac{q_{i,j}^{A}}{s_j} + t_{j,k} - t_{i,j}^{r,A} - (1-\sigma_{i,j}^{A})M - (1-\theta_{i,j}^{k})M \quad [44]$$

$$d_{i,j}^{A} \geq \frac{q_{i,j}^{A}}{s_j} + t_{j,k} - t_{i,j}^{r,A} + P_{i,j}^{k} - (1-\sigma_{i,j}^{A})M - \theta_{i,j}^{k}M \quad [45]$$

$$d_{i,j}^{A} \geq \frac{q_{i,j}^{A}}{s_j} - \sigma_{i,j}^{A}M - (1-\theta_{i,j}^{k})M \quad [46]$$

$$d_{i,j}^{A} \geq \frac{q_{i,j}^{A}}{s_j} + P_{i,j}^{k} - \sigma_{i,j}^{A}M - \theta_{i,j}^{k}M \quad [47]$$

$$P_{i,j}^{k} = t_{j,k+1} - (t_{j,k} + g_{j,k}) \quad [48]$$

3.1.3.2 Formulations for Arrival B

Delay estimation of vehicles from arrival B are similar to those in arrival A. Cycle number is the only factor to change in most of the formulations. However, it should be noticed that the number of vehicles that have cleared the intersection during the past cycle *k* (V_j^k) should be taken into consideration when using the index *i* to calculate the queues upon each vehicle's approaching in arrival B. Changes are made in Equations [49] to [52].

$$V_j^k = \sum_{i=I_j^Q+1}^{I_j} \theta_{i,j}^k \quad [49]$$

$$q_{i,j}^B \geq \left\lfloor \frac{i-1-V_j^k}{N_j} \right\rfloor - (1-\sigma_{i,j}^B)M \quad [50]$$

$$q_{i,j}^B \geq \left\lfloor \frac{i-1-V_j^k}{N_j} \right\rfloor - s_j(t_{i,j}^{r,B} - t_{j,k+1}) - \sigma_{i,j}^B M \quad [51]$$

$$q_{i,j}^B \geq 0 \quad [52]$$

Similar to $P_{i,j}^k$ in arrival A, vehicles that cannot clear the intersection in cycle *k+1* ($\theta_{i,j}^{k+1} = 0$) have a longer delay with an extra waiting time ($P_{i,j}^{k+1}$). This extra delay time should equal to the red duration between two adjacent green durations. However, the start time of green in cycle *k+2* is out of the planning horizon. To make the delay calculation of each phase equally weighted, a red time compensation is assumed ($R_{c,j}$), as illustrated on the right of the time bar in Figure 2. Calculation is shown in Equation [53].

$$P_{i,j}^{k+1} = t_{8,k+1} + v_{8,k+1} + R_{c,j} - (t_{j,k+1} + g_{j,k+1}) \quad [53]$$

3.1.3.3 Formulations for Arrival C

Delay calculations of vehicles belonging to arrival type C are comparatively simple. Vehicles all approach on red, and will not clear the intersection within the planning horizon. To calculate the queues upon each vehicle's arrival in arrival C, the number of vehicles that have cleared the intersection during cycle *k+1* (V_j^{k+1}) should be further subtracted. Calculations are as follows:

$$V_j^{k+1} = \sum_{i=I_j^Q+1}^{I_j} \theta_{i,j}^{k+1} \quad [54]$$

$$d_{i,j}^C \geq \frac{q_{i,j}^C}{s_j} + t_{8,k+1} + v_{8,k+1} + R_{c,j} - t_{i,j}^{r,C} \quad [55]$$

$$q_{i,j}^C \geq \left\lfloor \frac{(i-1) - V_j^k - V_j^{k+1}}{N_j} \right\rfloor \quad [56]$$

There are a few more constraints on the green durations in cycle *k* and *k+1*. Equation [57] ensures those vehicles that have been queueing at the intersection before the optimization begins get to clear the intersection during cycle *k*. Similarly, equation [59] ensures the vehicles that have arrived before the end of green in cycle *k* but cannot clear the intersection can pass during the next green in cycle *k+1*.These constraints are to make sure that the intersection won't become too much over-saturated due to prioritizing buses, even for one or two cycles.

$$g_{j,k} \geq \frac{\left\lfloor I_j^Q / N_j \right\rfloor}{s_j} \quad [57]$$

$$A_j = \sum_{i=I_j^Q+1}^{I_j} y_{i,j}^k \quad [58]$$

$$g_{j,k+1} \geq \frac{\left\lfloor \left(I_j^Q + A_j - V_j^k\right) / N_j \right\rfloor}{s_j} \quad [59]$$

where

A_j is the total number of vehicles belonging to arrival type A.

3.2 Penetration Rate Revision

So far, PAPSCCI model is built on the ideal assumption that all the vehicles are equipped with connected-vehicle devices. However, this is not realistic in the foreseeable future. Therefore, the impacts of the market penetration on the performance of the PAPSCCI model need to be evaluated.

In PAPSCCI model, delay estimation is closely related to each vehicle's unique index *i*, which is numbered according to their approaching sequence. However, when the penetration rate gets lower, more vehicles without connected-vehicle devices cannot be recognized and indexed by the system when approaching the intersection. This causes two problems with the model. First, if not all vehicles are indexed by the system, it would be inaccurate to use $(i-1)$ as the number of vehicles that have arrived prior to vehicle i when calculating the queue position of the i^{th} vehicle. This would affect the accuracy in delay estimation, and further affect the efficiency of the PAPSCCI model. Second, with less autos recognized by the system, the person delay of buses gain higher weights in the objective function. In order to remain the delay estimation of each vehicle's delay as accurate as possible, and to properly weight the auto traffic in the objective function, necessary revisions are needed to the related formulations.

To address the first problem, a gross estimation on the portion of vehicles that are not "seen" by the connected-vehicle system are made, by dividing the penetration rate from the total number of vehicles actually "seen" and indexed by the system. Original constraints [21], [37], [38], [50], [51], [56], [57] and [59] are revised into Equations [60] to [67]. To account for the second problem, the objective function is also revised by dividing the person delay of autos by the penetration rate.

$$d_{i,j}^{Q} \geq \left\lfloor \frac{i-1}{N_j} \right\rfloor \Big/ \left(s_j \times PenRate\right) + t_{j,k} + t_{stop,i,j} \quad \forall i \in [1, I_j^Q] \qquad [60]$$

$$q_{i,j}^{A} \geq \left\lfloor \frac{i-1}{N_j} \right\rfloor \Big/ PenRate - (1-\sigma_{i,j}^{A})M \qquad [61]$$

$$q_{i,j}^{A} \geq \left\lfloor \frac{i-1}{N_j} \right\rfloor \Big/ PenRate - s_j(t_{i,j}^{r,A} - t_{j,k}) - \sigma_{i,j}^{A}M \qquad [62]$$

$$q_{i,j}^{B} \geq \left\lfloor \frac{i-1-V_j^k}{N_j} \right\rfloor \Big/ PenRate - (1-\sigma_{i,j}^{B})M \qquad [63]$$

$$q_{i,j}^{B} \geq \left\lfloor \frac{i-1-V_j^k}{N_j} \right\rfloor \Big/ PenRate - s_j(t_{i,j}^{r,B} - t_{j,k+1}) - \sigma_{i,j}^{B}M \qquad [64]$$

$$q_{i,j}^{C} \geq \left\lfloor \frac{(i-1)-V_j^k - V_j^{k+1}}{N_j} \right\rfloor \Big/ PenRate \qquad [65]$$

$$g_{j,k} \geq \left\lfloor \frac{I_j^Q}{N_j} \right\rfloor \Big/ \left(s_j \times PenRate\right) \qquad [66]$$

$$g_{j,k+1} \geq \left\lfloor \frac{I_j^Q + A_j - V_j^k}{N_j} \right\rfloor \Big/ \left(s_j \times PenRate\right) \qquad [67]$$

3.3 PAPSCCI Model with Limited Communication Range

The communication range of the connected-vehicle technology at an intersection decides how much information can be collected. Therefore, the sizes of the communication range can affect the performance of PAPSCCI model. However, in reality, the communication range at an intersection is usually very limited. Considering this, the system is unable to detect and collect information from all the vehicles included in a whole planning horizon at the beginning of each optimization process. To address this problem, the real-time vehicle information can be combined with some predicted vehicle information to complete an optimization.

During each optimization, due to the limit of the communication range, only part of the vehicles involved in a complete planning horizon can be detected by the system. The information of those vehicles that cannot be collected ahead of the optimization will be substituted by some predicted vehicle information. The predicted vehicle information is assumed based on an average vehicle arrival rate of each phase, using historical traffic data, such as the volume of each phase. Vehicles outside the communication range of the intersection are assumed to approach following this estimated arrival rate. The predicted vehicles all travels at a constant speed, and they are all assumed to be auto traffic with a constant vehicle occupancy.

CHAPTER IV

REAL-TIME EVALUATION

Traffic simulation is used to evaluate the performance of PAPSCCI model. This chapter gives a brief introduction on this traffic simulation environment, from the evaluation platform, the setting up of the simulation testbed, to the design of different simulation scenarios.

4.1 Evaluation Platform

A real-time traffic simulation is built to evaluate the performance of PAPSCCI model. The simulation platform is adopted from Zeng's previous research *(13)*. It consists of three main modules: optimization, signal control and simulation module. At the beginning of every cycle, the signal control module gathers information of all the vehicles that are present at or will approach the intersection during the following planning horizon. These pieces of information include vehicle speed, vehicle location, occupancy and vehicle type, all of which can be obtained through connected-vehicle technology in real-time. All information is extracted from simulation module and is supplied to the optimization module, where PAPSCCI model is coded. Optimization is conducted with IBM CPLEX and follows the PAPSCCI model. The optimized signal timing data are then sent back to simulation module to continue the traffic simulation. Simulation is conducted with PTV VISSIM, and the simulation platform is coded using Microsoft Visual Studio C++ compiler.

4.2 Simulation Testbed

The test intersection is designed as a typical four-leg intersection shown in Figure 5. Auto traffic consists of OBUs (autos with connected vehicle with on-board units) and regular cars (autos without connected vehicle units). Both types of vehicles travel at 60 kph. The occupancy in simulation follows a uniform distribution from 1 to 4 passengers. There are three bus lines designed for different test scenarios. Bus route 202 travels eastbound and requests for phase *2*. It travels with a headway of 300 seconds and occupancy randomly selected from 20 to 50 passengers. Route 303 requests phase *3* at this intersection. Its headway is 360 seconds with passengers randomly from 10 to 40 on each bus. Route 401 enters the intersection from phase *4*. The bus headway for the route is 400 seconds, and each bus carries passengers uniformly distributed from 20 to 40. All buses travel around 60 kph (40 mph).

Since the basic PAPSCCI model needs to collect data from all the vehicles that are presenting at or will approach the intersection during a planning horizon (i.e., two cycles or 120 s), the approach length of the eight movements are set to be a slightly over 2000 meters in the simulation testbed to ensure perfect information are available. For testing the performance of PAPSCCI model with different communication ranges, the approach length can be decreased as long as it is slightly larger than the communication range.

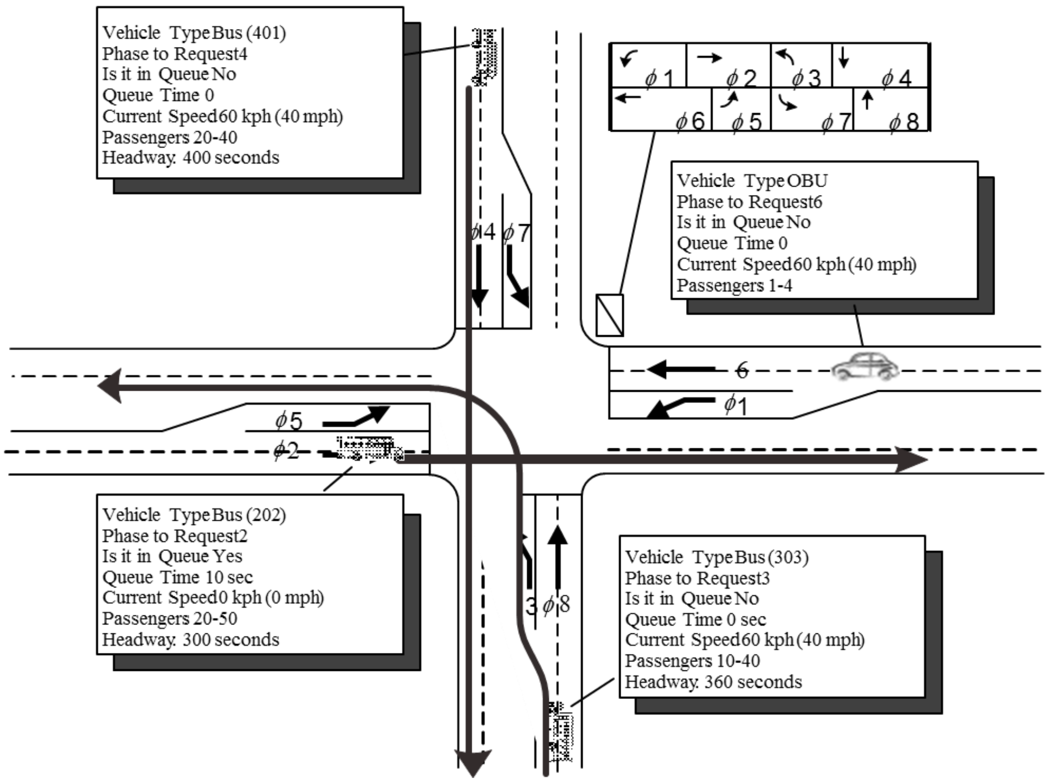


Figure 5 Intersection Used in the Simulation Study (adopted from *(13)*)

The default of the phase sequence on this test intersection is shown in Figure 5. Other important parameters about this test intersection, including the number of lanes of each phase, traffic volumes, and background signal timing plans, are listed in Table 1. The phase splits and cycle length (60 seconds) are optimized in SYNCHRO based on pre-set traffic volumes, and this baseline timing serves as a background setup for the simulation and evaluation on PAPSCCI model.

Table 1 Background Setup for the Test Intersection

Phase	$\phi1$	$\phi2$	$\phi3$	$\phi4$	$\phi5$	$\phi6$	$\phi7$	$\phi8$
# of lanes	1	2	1	2	1	2	1	2
Volume	112	616	90	381	78	784	101	280
Optimal splits (s)	11	24	11	14	10	25	11	14

4.3 Simulation Scenarios

Different simulation scenarios are designed to comprehensively evaluate the performance of PAPSCCI model. For the evaluation of the basic PAPSCCI model, three scenarios are designed as: auto only, single bus route and multiple bus routes. Comparing with the optimal signal timings obtained from SYNCHRO, the auto only scenario will offer a preliminary evaluation on how basic PAPCSSI model performs in a situation when no TSP needed. Single bus line scenario will test the ability of the basic PAPSCCI model to perform TSP with different bus line characteristics. Combining with multiple bus line scenarios, how the effectiveness of the basic PAPSCCI model would be affected by the increasing number of bus lines would be revealed. Though the PAPSCCI model allows changeable phase sequences, only the transit phase was considered as changeable in simulation in order to save time for optimization. For example, if bus route 202 (requesting phase *2*) is running, phase *1* may allow to leading or lagging behind phase *2*, while other phases remain as default.

In the penetration rate test, only the single bus route operation scenario will be performed. Simulation will be conducted under different penetration rates, ranging from 100% to 20%.

Last but not least, PAPSCCI model will be tested with different communication ranges at the intersection (2000 m, 1000 m, 500 m and 250 m). Only the typical single bus line operation scenario (bus route 202) will be performed. The focus of this test is to explore the relationship between the performance of PAPSCCI model and the different sizes of the communication range.

CHAPTER V

RESULTS ANALYSIS

This chapter presents the simulation results with different test scenarios of the basic PAPSCCI model, the penetration rate tests and different communication ranges tests. Five runs are made with different random seeds for each simulation. Results from these five runs will be averaged to get the final results of each simulation. Analysis will be made based on these simulation results, and lead to a comprehensive evaluation of the PAPSCCI model in this chapter.

5.1 Basic PAPSCCI Model Test

Basic PAPSCCI model is built assuming 100% penetration rate of the connected-vehicle technology and large enough communicate range for perfect vehicle and intersection information. The model will be evaluated in different test scenarios, including auto only scenario, simulation with single bus route operation and simulation with multiple bus routes. Following are the simulation results of the basic PAPSCCI model in different test scenarios.

5.1.1 Basic PAPSCCI Model with Only Auto Traffic

In this test scenario, simulations of “SYNCHRO optimization” and “PAPSCCI optimization” are both conducted with no bus operations. “SYNCHRO optimization” refers to the background settings, whose signal timings are obtained from SYNCHRO based on the pre-set volume for each phase. Vehicle delays and person delays are listed in Table 2, as well as the delay changes in terms of percentage between these two optimization types.

Table 2 Auto Only Scenario for Basic PAPSCCI Model

Delay Type	SYNCHRO Optimization	PAPSCCI Optimization	Delay Changes
Vehicle Delay (s)	21.74	19.45	-10.57%
Person Delay (s)	21.64	19.20	-11.27%

From the data, it is clear that signal timings after the basic PAPSCCI optimization generates less vehicle delay and person delay than without PAPSCCI. Unlike the fixed signal timings from SYNCHRO, in PAPSCCI model the green duration of each phase can change according to the traffics on the road. The cycle length is also allowed to change, which renders even more flexibility to timing adjustability. It should be noted, however, the planning horizon (120 seconds) is fixed.

5.1.2 Basic PAPSCCI Model with Single Bus Line Operation

In this test scenario, simulations of "SYNCHRO optimization" and "PAPSCCI optimization" are both conducted with only one bus line running through the test intersection. Bus route 202 requests phase *2*, which has an hourly volume of 616 vehicles and background green split of 24 seconds. Contrary to bus route 202, bus route

301 is on phase *3* with an hourly volume of only 90 vehicles, and the background green time is 11 seconds. Both bus routes are chosen in this test. Table 3 lists the changes of both vehicle delay and person delay of each vehicle type from SYNCHRO optimization to basic PAPSCCI optimization with either bus route 202 or 301 running through the intersection.

Table 3 Basic PAPSCCI Model with Single Bus Line Operation

Delay Type	Vehicle Type	SYNCHRO	PAPSCCI	Delay Change
Bus 202				
Vehicle delay (s)	Auto	21.78	20.31	-6.79%
	Bus	22.54	16.02	-28.93%
	Total	21.78	20.31	-6.79%
Person delay (s)	Auto	21.74	20.03	-7.88%
	Bus	22.54	15.78	-29.97%
	Total	21.78	19.75	-9.35%
Bus 301				
Vehicle delay (s)	Auto	21.88	20.22	-7.59%
	Bus	41.26	30.68	-25.64%
	Total	21.96	20.30	-7.56%
Person delay (s)	Auto	21.78	20.04	-7.98%
	Bus	41.26	30.14	-26.96%
	Total	22.54	20.38	-9.58%

In general, the basic PAPSCCI model can effectively decrease both vehicle delays and person delays of buses. Around 29% and more improvements were observed for the bus route 202 and 25% and more for bus route 301. Moreover, the delays of auto traffic experienced a slight decrease after PAPSCCI optimization, even though bus vehicles gain much more weights in competing for signal priorities.

Comparing the simulation results between only bus route 202 or 301 running through the test intersection, bus route 202 has slight more delay improvements than that of bus route 301. This result is quite as expected, since the additional signals assigned to bus route 301 may be limited by the much less traffic demand (inevitably much less passengers) that phase *3* has.

However, the delay improvements of the total traffic operations after PAPSCCI optimization are quite similar, with around 7% and 9% delay decreases for vehicle delay and person delay respectively, demonstrating PAPSCCI a comprehensive model that can balance delays among all types of vehicles while operating TSP needs.

To further test how vehicle occupancy can affect the performance of the PAPSCCI model, a test was designed as follows. The scenario with single bus line operation (bus route 202 was chosen) was run twice. In the first time (normal occupancy setting), all auto vehicles were assigned occupancy uniformly distributed from 1 to 4. However, in the second time (biased occupancy setting), vehicles requesting phase *2* (the same phase that bus route 202 is requesting) were assigned occupancies randomly from 1 to 2, while vehicles on other routes may either have 3 or 4 passengers on board. Table 4 shows the simulation results and the comparison between these two situations.

Table 4 Delay Changes after PAPSCCI with Different Occupancy Settings

	Vehicle Delay			Person Delay		
	Auto	Bus	Total	Auto	Bus	Total
Normal Occupancy	-6.79%	-28.93%	-6.79%	-7.88%	-29.97%	-9.35%
Biased Occupancy	-7.99%	-20.85%	-7.90%	-6.99%	-19.64%	-7.71%

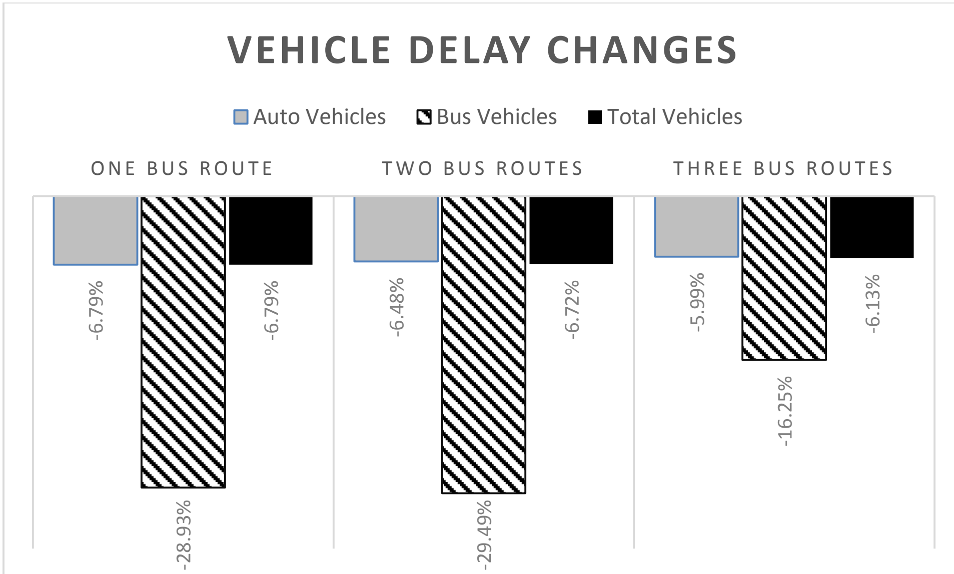
Though with similar numbers of vehicles on the road, the delay improvements of the transit vehicles are significantly reduced when the vehicles on the non-transit phases have much higher occupancies than those on the transit phase. This result reveals the "person-based" characteristics of PAPSCCI model. Since PAPSCCI optimizes the signal timings based on person delay, the prioritized green time that was meant to be assigned to transit vehicles has been shared with other phases in the biased occupancy setting. This also explains why the improvement of vehicle delay of auto traffic with the biased occupancy setting is better than that with the normal occupancy setting. As to person delay, since in the biased occupancy setting, the total number of passengers at the intersection is greatly increased, the improvements of person delay of all types of vehicles are inevitably affected and are less than that of the normal occupancy settings.

5.1.3 Basic PAPSCCI Model with Multiple Bus Lines Operations

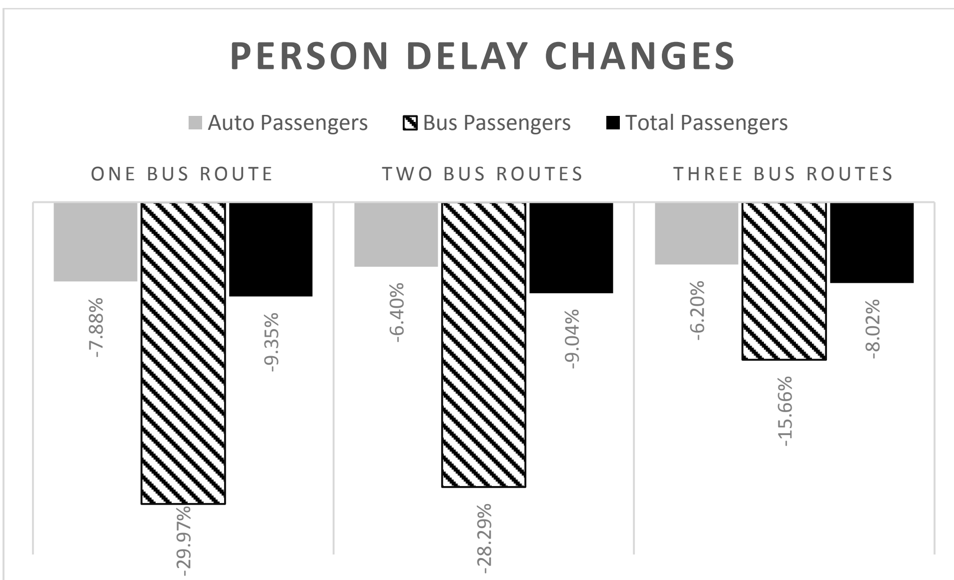
In this part, the basic PAPSCCI model is tested with up to 3 bus lines running through the intersection. Multiple bus lines in conflicting movements make it more restrictive in terms of timing adjustments. This is partly because the optimization program has to balance not only between buses and autos, but also among buses. This scenario is designed to further test the performance of PAPSCCI model in assigning signal priorities.

Column charts displayed in Figure 6 show the delay changes after PAPSCCI optimization for multiple bus lines scenarios. In each chart, from left to right, the columns show the delay changes for different vehicle types with one bus line (bus route 202), two bus lines (bus routes 202 and 301) and three bus lines (bus routes 202, 301 and 402) running through the intersection respectively.

Similar to the single bus line scenario, bus and auto delays all get different degrees of reduction after PAPSCCI optimizations. In all three cases, buses experienced approximately 22%, 22% and 10% more delay improvements than the auto vehicles respectively. Though with one exception, the general trend is that, with more bus routes in operations, the decreases of bus delays, as well as those of auto delays, are getting smaller. This is an interesting trend and somewhat counter-intuitive. Because one would expect the more bus lines getting priority, the higher person savings would result. However, priorities are relative. When two buses with lots of passengers on board come in conflicting movements, not all passengers are getting faster passages, and some of them may even have to wait longer than without TSP operations. This is even more obvious when conflicting bus routes are more than 2. This interesting result shed lights on a critical property of the general TSP operations. That is a TSP operation is most effective when there is only one priority request in the intersection.



(a) Vehicle Delay



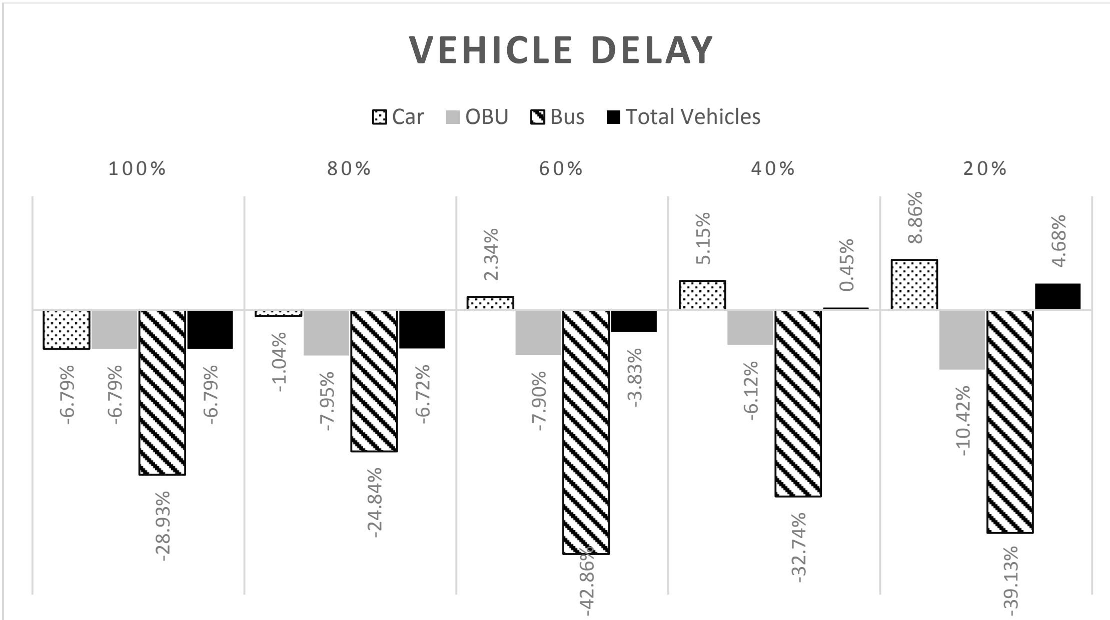
(b) Person Delay

Figure 6 Vehicle Delay and Person Delay Changes with Multiple Bus Lines

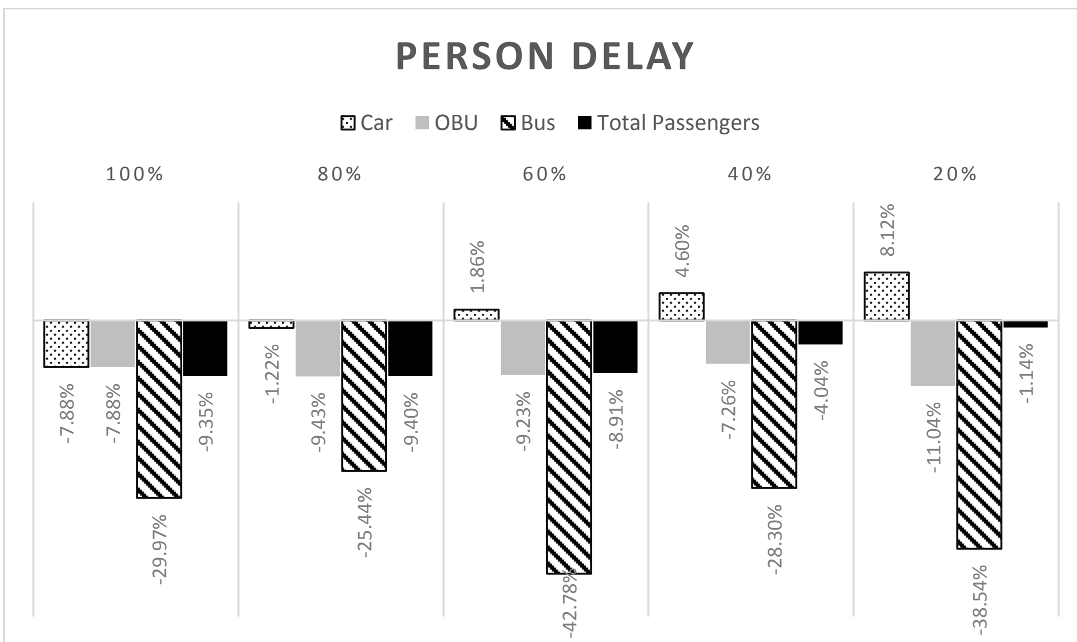
5.2 Penetration Rate Test

The basic PAPSCCI model is built on the ideal assumption that all the vehicles are equipped with connected-vehicle devices. However, this is not realistic in a foreseeable future. Therefore, an evaluation about the impacts of market penetration on the performance of the PAPSCCI model is needed. The PAPSCCI model performances were tested with penetration rate of 100%, 80%, 60%, 40% and 20% for the single bus line operation scenario (bus route 202). The communication range used here is 2000 m.

The column charts in Figure 7 shows the percent delay changes after PAPSCCI optimization. The improvement of both vehicle delay and person delay for the total traffic at the intersection worsens with the decrease of the penetration rate. However, the decrease of the delay changes is gentle when the penetration rate is above 40%. Since the PAPSCCI model calculates the vehicle delay individually, plus the revisions that were made to the formulations mentioned in Chapter III, the change of penetration rates affects much less on the variables of the model, especially compared to those models using estimated arrival rates in auto delay calculations.



(a) Vehicle Delay Changes



(b) Person Delay Changes

Figure 7 Percent Delay Changes after PAPSCCI with Different Penetration Rates

Still, the revisions are based on gross estimations on the vehicles without connected-vehicle devices. It works well when only a small portion of the vehicle information is not available. But the error may easily get magnified when penetration rate is very low. Besides, less vehicle information included in the model inevitably leads to less basis for the optimization decisions. These are explanations why the performance of the PAPSCCI model worsens quickly when penetration rate very low.

Among all the vehicle types, cars (vehicles without connected-vehicle onboard units) are the most affected by the lowering of penetration rate, since the PAPSCCI model cannot make estimations on their delays. When the penetration rate drops below around 60%, cars are starting to experience delay increases after the PAPSCCI optimization. However, the performances on the total traffic in terms of both vehicle delay and person delay are still promising, only when the penetration rate is lowered to around 20%, would the delay of the total traffic to increase after PAPSCCI optimization. Buses still experience much greater delay decreases after PAPSCCI optimization than any other types of vehicles. And their delay improvements after PAPSCCI optimization seem to be less affected by the change of penetration rate. The fluctuation trend for bus delays with lowering the penetration rate may be caused due to different occupancies in different simulation runs.

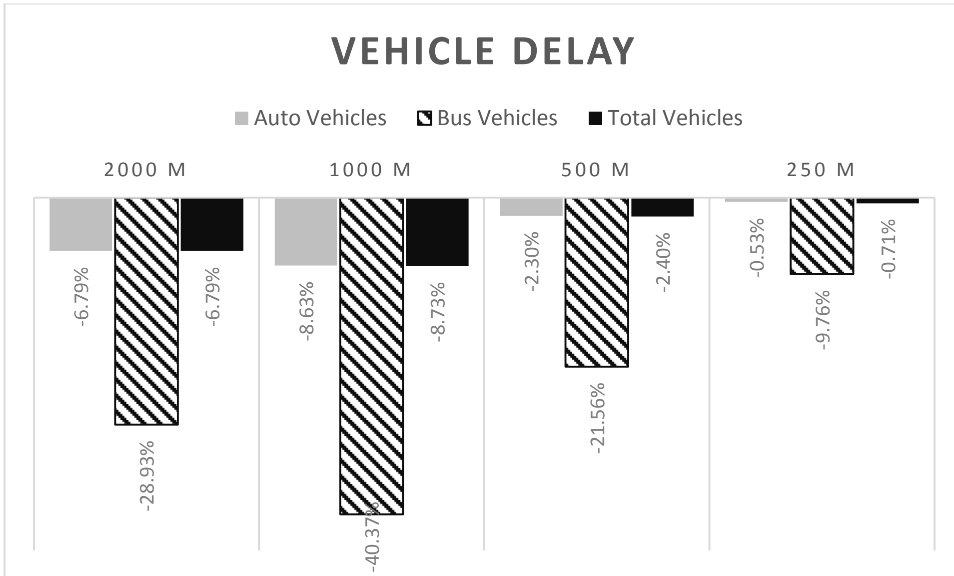
5.3 PAPSCCI Model with Limited Communication Ranges

The size of the communication range directly affects how much vehicle information the system could get before each optimization. In the basic PAPSCCI model, the communication range is set as slightly over 2000 m (assuming vehicles travel at a speed of 60 kph) to ensure information of all vehicles involving in one planning horizon (120 s) are available to the system. However, in this part, the PAPSCCI model will be adjusted and implemented with different communication ranges at the intersection, respectively 2000 m, 1000 m, 500 m and 250 m.

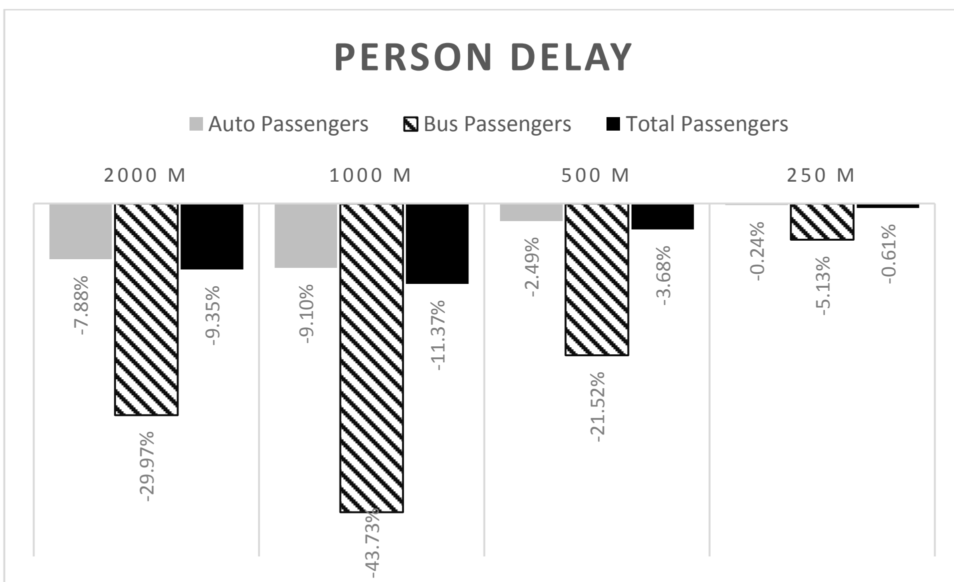
For the 2000 m communication range, the information of all the vehicles throughout each planning horizon are available, and the optimization process of the PAPSCCI model is the same as the basic model that has been tested before.

For the 1000 m, 500 m and 250 m communication range, only information of vehicles approaching the intersection during the first 60 seconds (30 seconds for the 500 m communication range, and 15 seconds for 250 m) of the planning horizon are available, and information of vehicles coming during the rest of the planning horizon are assumed based on a constant arrival rate. More details are illustrated in Chapter III.

The simulation scenario used here is the single bus scenario (bus route 202).



(a) Vehicle Delay



(b) Person Delay

Figure 8 Delay Changes with Different Communication Ranges

Seen from the column charts in Figure 8, the improvements of both vehicle delay and person delay of the total traffic at the intersection worsens as the communication range decreases, except for the 1000 m communication range case. Theoretically shorter communication range means less available real-time vehicle information, thus leads to the decrease of the accuracy in estimating vehicle delay, and affect the effectiveness of the PAPSCCI optimization. However, in the delay estimation, some assumptions were made to simplify the vehicle operations, such as constant travel speed for every vehicle. These assumptions inevitably raise some inaccuracies in estimating the vehicle operations from the real situation. Besides, these inaccuracies may be magnified for vehicles that are far from the intersection when optimization begins. Replacing the information of these faraway vehicles with some predictions from the historical data may somehow compensate for this kind of inaccuracies. That's why the 1000 m communication range case has a better performance than the 2000 m range. However, delay estimated from the connected-vehicle technology is still better than merely using historical data, otherwise, the delay improvements for all vehicle types wouldn't worsen when the communication range is only 500 and 250 meters.

CHAPTER VI

CONCLUSIONS

A real-time signal control optimization model PAPSCCI (person-based adaptive priority signal control with connected-vehicle information) was proposed in this thesis. PAPSCCI optimizes signal timings at an isolated intersection by minimizing the total person delay. This person-based optimization can effectively assign signal priorities to transit vehicles due to their high occupancies, while minimizes the negative impacts that might cause on auto traffic. Prior to conventional TSP strategies, this person-based algorithm is capable of dealing with the cases where multiple bus lines are traveling at conflicting routes. Besides, PAPSCCI model adopts the new emerged connected-vehicle technology into TSP strategy development. By using detailed vehicle information provided by the connected-vehicle technology, the PAPSCCI model can directly computes the person delay for every vehicle running through the intersection, instead of using estimation models for vehicle delays.

The evaluations on the PAPSCCI model were conducted in a traffic simulation environment. Scenarios with different bus lines running through the test intersection were designed to comprehensively test the model performances. Test results showed that the PAPSCCI model can effectively decrease both the vehicle delay and person delay of buses in single and multiple conflicting bus lines scenarios. However, it performs the best when there’s only one bus route running through the intersection. Besides, the PAPSCCI model can still come up with good results by decreasing the intersection delay by around 10% when there is no bus lines running through the intersection, demonstrating PAPSCCI's potential as a general adaptive signal control system. Tests were also performed to evaluate the performance of the PAPSCCI model with different penetration rates. Relative formulations in the PAPSCCI model were revised based on the penetration rate. Results show that even though the delay changes after optimization get smaller with lower penetration rate, the PAPSCCI model can still perform effectively with 40% or higher penetration rates. In addition, PAPSCCI model were also developed to adapt limited communication range at the intersection. Predicted vehicle information based on historical data of the intersection were used to substitute for the "missing" vehicle information due to the decrease in communication range. Tests were run for this developed PAPSCCI model with different communication ranges, and the results turned out to be promising. Even when the communication range is only 500 m, the PAPSCCI can still effectively decrease the transit delay by around 21% and the total delay by around 3%.

Based on the findings from this thesis, recommendations for future studies are listed as follows:

One possible improvement on this model could be to remove the restriction on the fixed length of the planning horizon, in order to provide more flexibilities to making timing adjustments. And this will also allow full adaptability to time-varying traffic conditions.

When the communication range is very limited, a rolling optimization process could be considered in performing PAPSCCI optimization. Instead of performing optimization only at the beginning of each cycle, future studies should focus on how to update vehicle information continuously and practice optimizations during the cycle time as well. This rolling optimization process may offer more accuracy to the model when the communication range and real-time vehicle information is limited.

Last but not least, since calculating the delay for every vehicle in the planning horizon requires large workload, another possible future improvement for this model is to use platoons in vehicle delay calculations instead of considering individual vehicles. This change may effectively speed up the optimization process, especially for intersections with large volumes.

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APPENDIX A

BASIC PAPSCCI MODEL

Objective Function:

$$Min\,Z = \sum_{j=1}^{J}\sum_{i=1}^{I_j^Q} o_{i,j} d_{i,j}^{Q} + \sum_{j=1}^{J}\sum_{i=I_j^Q+1}^{I_j} o_{i,j} d_{i,j}$$

Subject to:

$$t_{1,1} = t_{5,1} = 0$$

$$t_{2,k} = t_{1,k} + v_{1,k};\; t_{1',k} = t_{2,k} + v_{2,k};\; t_{3,k} = t_{1',k} + v_{1',k};\; t_{4,k} = t_{3,k} + v_{3,k}$$

$$t_{6,k} = t_{5,k} + v_{5,k};\; t_{5',k} = t_{6,k} + v_{6,k};\; t_{7,k} = t_{5',k} + v_{5',k};\; t_{8,k} = t_{7,k} + v_{7,k}$$

$$t_{1,k+1} = t_{3',k} + v_{3',k};\; t_{5,k+1} = t_{7',k} + v_{7',k}$$

$$t_{1,k} = t_{5,k};\; t_{3,k} = t_{7,k}$$

$$t_{7',k+1} + v_{7',k+1} - t_{1,k} = 2C$$

$$v_{j,k} = g_{j,k} + Y + R \quad \forall j \in \{2,4,6,8\}$$

$$v_{j',k} + v_{j,k} = g_{j,k} + Y + R \quad \forall j \in \{1,3,5,7\}$$

$$v_{j,k} \le r_{j,k} M;\; v_{j',k} \le r_{j',k} M \quad \forall j \in \{1,3,5,7\}$$

$$v_{j,k}, v_{j',k} \ge 0$$

$$r_{j,k} + r_{j',k} = 1 \quad \forall j \in \{1,3,5,7\}$$

$$g_{j,k} \ge g_{\min,j}$$

$$d_{i,j}^{Q} \ge \frac{\left\lfloor (i-1)/N_j \right\rfloor}{s_j} + t_{j,k} - t_{i,j}^{r} \quad \forall i \in [1, I_j^Q]$$

$$T_{i,j}^{r} > t_{j,k} + g_{j,k} - y_{i,j}^{k} M$$

$$T_{i,j}^{r} \le t_{j,k} + g_{j,k} + (1 - y_{i,j}^{k}) M$$

$$T_{i,j}^{r} > t_{j,k+1} + g_{j,k+1} - y_{i,j}^{k+1} M$$

$$T_{i,j}^{r} \le t_{j,k+1} + g_{j,k+1} + (1 - y_{i,j}^{k+1}) M$$

$$d_{i,j} \geq d_{i,j}^{A} - (1 - y_{i,j}^{k})M - (1 - y_{i,j}^{k+1})M$$
$$d_{i,j} \leq d_{i,j}^{A} + (1 - y_{i,j}^{k})M + (1 - y_{i,j}^{k+1})M$$
$$d_{i,j} \geq d_{i,j}^{B} - y_{i,j}^{k}M - (1 - y_{i,j}^{k+1})M$$
$$d_{i,j} \leq d_{i,j}^{B} + y_{i,j}^{k}M + (1 - y_{i,j}^{k+1})M$$
$$d_{i,j} \geq d_{i,j}^{C} - y_{i,j}^{k}M - y_{i,j}^{k+1}M$$
$$d_{i,j} \leq d_{i,j}^{C} + y_{i,j}^{k}M + y_{i,j}^{k+1}M$$
$$\theta_{i,j}^{k} \leq y_{i,j}^{k}$$
$$\theta_{i,j}^{k} + \theta_{i,j}^{k+1} \leq y_{i,j}^{k+1}$$
$$t_{i,j}^{r,A} = T_{i,j}^{r} - \frac{q_{i,j}^{A} L_{s}}{v_{j}}$$
$$t_{i,j}^{r,A} \leq t_{j,k} + (1 - \sigma_{i,j}^{A})M$$
$$t_{i,j}^{r,A} > t_{j,k} - \sigma_{i,j}^{A}M$$
$$q_{i,j}^{A} \geq \left\lfloor \frac{i-1}{N_{j}} \right\rfloor - (1 - \sigma_{i,j}^{A})M$$
$$q_{i,j}^{A} \geq \left\lfloor \frac{i-1}{N_{j}} \right\rfloor - s_{j}(t_{i,j}^{r,A} - t_{j,k}) - \sigma_{i,j}^{A}M$$
$$q_{i,j}^{A} \geq 0$$
$$\frac{q_{i,j}^{A}}{s_{j}} \leq g_{j,k} + (1 - \sigma_{i,j}^{A})M + (1 - \theta_{i,j}^{k})M$$
$$\frac{q_{i,j}^{A}}{s_{j}} > g_{j,k} - (1 - \sigma_{i,j}^{A})M - \theta_{i,j}^{k}M$$
$$\frac{q_{i,j}^{A}}{s_{j}} + t_{i,j}^{r,A} \leq t_{j,k} + g_{j,k} + \sigma_{i,j}^{A}M + (1 - \theta_{i,j}^{k})M$$
$$\frac{q_{i,j}^{A}}{s_{j}} + t_{i,j}^{r,A} > t_{j,k} + g_{j,k} - \sigma_{i,j}^{A}M - \theta_{i,j}^{k}M$$
$$d_{i,j}^{A} \geq \frac{q_{i,j}^{A}}{s_{j}} + t_{j,k} - t_{i,j}^{r,A} - (1 - \sigma_{i,j}^{A})M - (1 - \theta_{i,j}^{k})M$$
$$d_{i,j}^{A} \geq \frac{q_{i,j}^{A}}{s_{j}} + t_{j,k} - t_{i,j}^{r,A} + P_{i,j}^{k} - (1 - \sigma_{i,j}^{A})M - \theta_{i,j}^{k}M$$

$$d_{i,j}^{A} \geq \frac{q_{i,j}^{A}}{s_j} - \sigma_{i,j}^{A} M - (1-\theta_{i,j}^{k}) M$$

$$d_{i,j}^{A} \geq \frac{q_{i,j}^{A}}{s_j} + P_{i,j}^{k} - \sigma_{i,j}^{A} M - \theta_{i,j}^{k} M$$

$$P_{i,j}^{k} = t_{j,k+1} - (t_{j,k} + g_{j,k})$$

$$V_j^k = \sum_{i=I_j^Q+1}^{I_j} \theta_{i,j}^{k}$$

$$q_{i,j}^{B} \geq \left\lfloor \frac{i-1-V_j^k}{N_j} \right\rfloor - (1-\sigma_{i,j}^{B}) M$$

$$q_{i,j}^{B} \geq \left\lfloor \frac{i-1-V_j^k}{N_j} \right\rfloor - s_j (t_{i,j}^{r,B} - t_{j,k+1}) - \sigma_{i,j}^{B} M$$

$$q_{i,j}^{B} \geq 0$$

$$P_{i,j}^{k+1} = t_{8,k+1} + v_{8,k+1} + R_{c,j} - (t_{j,k+1} + g_{j,k+1})$$

$$V_j^{k+1} = \sum_{i=I_j^Q+1}^{I_j} \theta_{i,j}^{k+1}$$

$$d_{i,j}^{C} \geq \frac{q_{i,j}^{C}}{s_j} + t_{8,k+1} + v_{8,k+1} + R_{c,j} - t_{i,j}^{r,C}$$

$$q_{i,j}^{C} \geq \left\lfloor \frac{(i-1) - V_j^k - V_j^{k+1}}{N_j} \right\rfloor$$

$$g_{j,k} \geq \frac{\left\lfloor I_j^Q / N_j \right\rfloor}{s_j}$$

$$A_j = \sum_{i=I_j^Q+1}^{I_j} y_{i,j}^{k}$$

$$g_{j,k+1} \geq \frac{\left\lfloor \left(I_j^Q + A_j - V_j^k \right) / N_j \right\rfloor}{s_j}$$