ADDRESSING PRACTICAL ISSUES IN DESIGNING WEATHER INSURANCE CONTRACTS FOR RISK MANAGEMENT APPLICATIONS IN DEVELOPING COUNTRIES

A Dissertation

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ABSTRACT

In this dissertation we address practical issues in designing weather insurance contracts for risk management in developing countries in three different scenarios. First, we develop an innovative contract design strategy based on agronomic considerations that can be implemented in situations where only short and/or aggregate data series are available. We attempt to mitigate both the aggregate nature of yield data and the need for data-demanding analysis by looking at areas sharing the same growing conditions and using agronomic requirements to specify contract parameters. We find that the proposed contracts do not achieve the same degree of risk reduction as the contracts that can be constructed using no data limitations, but they do provide meaningful risk protection and typically at lower premiums. The implication is that the proposed methodology can be used to design weather derivatives for developing countries, where paucity of data often renders the conventional design approaches unworkable.

The second essay aims to derive a general-form optimal payoff of an index contract that takes into account potentially nonlinear dependence between the index underlying the contract and the loss that is insured. We find that the quasi-linear contract payoff structure may not be the optimal choice if the dependence between the index and the yield/revenue is nonlinear. The implication is that the proposed methodology can help to improve risk-reducing capabilities of weather derivatives particularly in situations where the effect of weather on yield is complex and not obvious.

The third essay analyzes the use of weather derivatives in managing water supply risk arising in making water allocation decisions. The specific application is developed for the Alto Rio Lerma Irrigation District (ARLID) in the state of Guanajuato in Mexico. We argue that incorporation of weather derivatives in water allocation decisions can improve overall well-being of producers and allow shift water allocations from the wet to the dry season with the assumption that the wet season farmers can cope with the risk of water shortages by using weather derivatives. We find that use of weather derivatives does lead to better water allocation policies that allow the representative farmer to reach higher levels of utility. The implication is that introduction of weather derivatives can help to improve water management decisions in developing countries where agriculture heavily depends on irrigation and can be severely affected by extreme weather events.

DEDICATION

To my wife, Rosanna Huayamave, for giving me her beautiful smile during the darkness and her peace during the light.

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1. INTRODUCTION

Adverse selection and moral hazard has been cited as the main reasons of failure of private crop insurance markets. As a result, insurers may not be able to provide any type of crop insurance in developing countries where the problem is compounded by the fact that the insurance markets may be incomplete or missing due to poor contract enforcement mechanisms and government inability to support crop insurance programs.

In the last years, there has been extensive research on the advantages of weather derivatives relative to traditional crop insurance (Skees and Barnett 1999); however, demand for these weather instruments has been lower than expected. While partly attributable to lack of familiarity with the products, the problem can be also traced back to the issues of contract design (Skees and Barnett 2006; Skees 2008; Miranda and Farrin 2012). This dissertation deals with practical issues arising in designing weather derivatives for risk management in developing countries in three essays.

The first essay attempts to deal with the yield data limitations by introducing a semi-naïve contract structure based on agronomic considerations and identification of homogeneous production regions. The approach is much less data-intensive than the design methods previously used in the literature and can be implemented in situations where only short and/or aggregate data series are available. In particular, we look at production areas sharing the same soil conditions rather than just the ones encompassed by arbitrary administrative boundaries. We use a simple index (one weather variable) and determine the parameters of the contract by using agronomical considerations. In order to evaluate the effectiveness of this simple semi-naïve approach to index construction, we use Arkansas soybeans as a test case. The extensive historical yield data available for this crop and region allows us to compare the performance of both the conventional contract designs and the ones developed in the essay. Given the similarities between growing conditions in Arkansas and South America, it is expected that the results would transfer to the case of soybean production in that region as well.

The research objective of the second essay is to derive an optimal form of payoff of an index contract that takes into account potentially nonlinear dependence between the index underlying the contract and the loss that is insured using the contract. Most of the existing papers on weather derivatives use a "standard" piecewise-linear contract to define the payoffs structure of the analyzed contracts. However, these contract payoff structures may not be the optimal choice if the dependence between the index and the yield/revenue is nonlinear. This framework is illustrated using weather insurance contracts for Arkansas soybean as a case study. The results are then compared with those obtained in the first essay.

In the third essay, we look at potential improvements in water allocation strategies that could be achieved by using weather derivatives. In many Latin American countries, the changes in temperature and shifts in precipitation patterns could affect the water supply, thus making the water allocation a major problem for agriculture.

A particularly interesting situation arises when there are two growing seasons, each characterized by different rainfall patterns but both dependent on irrigation. Weather derivatives can then incentivize adoption of allocation patterns that shift water allocations to the dry season from the wet season with the assumption that the wet season farmers can cope with the risk of water shortages by using weather derivatives. These financial instruments might also induce an inter-temporal reallocation of water in irrigation districts, increasing the efficiency of water use in the long term. The essay applies the analytical model to the Alto Rio Lerma Irrigation District in the state of Guanajuato in Mexico.

2. PRACTICAL APPROACHES TO DESIGNING WEATHER DERIVATIVES UNDER YIELD DATA LIMITATIONS

In the last 10 years, methodological advances in designing weather-based insurance instruments have increased expectations for their performance, mainly in rural areas of developing countries (World Bank 2005; Hazell et al. 2010). Pilot programs have been developed for Mexico, India, Malawi, China, Nicaragua, India, Morocco, among others. For the exception of Mexico and India, demand for these weather instruments have been lower than expected (Hess 2003; Barnett and Mahul 2007; Giné and Yang 2009; Giné et al. 2010)[1].

Factors like the lack of appropriate formal insurance markets, the absence of institutional framework to support trading between international and local institutions, potential basis risk, no consensus between farmers and insurers on which weather variables affect yields, and the lack of agreement over a common pricing model are the most typical explanations for that behavior given in the literature (Dischel 2002; Richards et al. 2004). In this context, basis risk emerges as the prevalent contract design problem, which affects the reliability of protection that index insurance contract may offer to small famers (Miranda 1991; Doherty and Richter 2002; Cummins et al. 2004; Barnett and Mahul 2007).

[1] Depending on the context, weather-based insurance instruments can be treated either as insurance contracts or as options written on realization of the index. However, there is no difference between these two frameworks from the standpoint of contract design and risk-reducing efficiency. For the rest of this essay the weather-based risk management instruments will be referred to as weather derivatives or index insurance contracts interchangeably.

Basis risk arises when a policyholder receives an indemnity payment that does not match the actual loss (Varangis et al. 2003). The aggregate nature of yield data, complex relationship between weather measurements and actual loss, and spatial variability of weather conditions are the most commonly cited sources of basis risk (Manfredo and Richards 2005). This risk can be reduced through product design (Skees 2008), with several approaches available. The existing literature primarily concentrates on finding the most accurate relationship between weather and losses. Other approaches consider limiting index insurance to low-frequency, high-impact events such as hurricanes or extreme droughts. It is thought that, under such extreme conditions, farmers' losses may be better correlated to the underlying weather variable.

Basis risk can be magnified even more in developing countries where data are often limited and unreliable. In such situations, insurance companies use aggregate data to develop insurance products whose payments are contingent upon indices presumably correlated with individual loss (Goodwin and Mahul 2004). The shortness of data series also contributes to the basis risk, since the available data is insufficient for establishing weather-loss relationship using the conventional econometric methods.

The design methodology presented in this essay attempts to circumvent the limitations of available yield data and reduce the basis risk inherent in the contracts. First, the contracts are designed for homogeneous production areas with the expectation that yield variability in the area is comparable to that on a single farm. In this case, the available aggregate yield data can be considered as more accurately representing the distribution of yields of individual farms in the area. In particular, we look at production areas sharing the same soil conditions rather than just the ones encompassed by arbitrary administrative boundaries. Furthermore, instead of constructing the indexes based on econometric models, we use a simple index (one weather variable) and determine the parameters of the contract based on agronomic considerations.

The research objective is not to develop a new or better way of constructing index insurance contracts, but rather to evaluate the effectiveness of a simple semi-naïve approach to index construction that can be implemented in situations where only short and/or aggregate data series are available. While the potential of this approach can be mostly appreciated in developing countries, we use Arkansas soybeans as a test case. On the one hand, long and reliable data series are available for this crop and location, which allow us to validate this approach. On the other hand, there are similarities between soybean productions in Arkansas and Latin America, which would allow us to transfer the results to that region.

The rest of the chapter is organized as follows. Literature on index insurance contracts is reviewed first, with particular attention to various design procedures. We then briefly explain the growth process of soybean plants, identify environmental conditions required for optimal growth, and determine soil types best suited for soybean production. The second subsection presents the methodology used to design the proposed weather derivative contracts and evaluate their effectiveness as a risk reduction tool. The third subsection describes characteristics of soybean production in Arkansas, data collection process, and identification of homogeneous production zones. The fourth subsection presents and discusses the results. The final subsection concludes and discusses directions for future research.

2.1 Literature Review

2.1.1 Risk Management in Developing Countries

Unfavorable weather conditions are one of the main risk factors affecting agricultural production and agri-business (Dercon 2002). These factors have a significant impact on farmers' decisions related to production and investment, on their ability to service debts, and on their standards of living. Traditionally, farmers have utilized nonmarket institutions[2] such as family, local, or community lending institutions as informal risk transfer mechanisms in rural areas (Ellis 2000). Informal loans, diversification of income sources, and crop diversification have also been mechanisms used by rural household to smooth consumption (Morduch 1995; Fafchamps and Pender 1997; Zimmerman and Carter 2003). However, when an extreme weather event occurs, these nonmarket institutions fail as risk management tools because of their limited capacity to spread correlated risks affecting farmers in the same area at the same time (Skees et al. 1999).

The extreme weather events such as drought, floods and windstorms strangle rural household economy which owns few assets. Due to high risk exposure, rural household become more risk averse and adopt low risk investment strategies associated with low return, which is not enough to allow rural households to escape of the poverty trap (Carter and Barrett 2006).

[2] Besley (1995) used this term as a catch-all for many different arrangement.

Insurance companies are often reluctant to conduct business in rural areas due to poor contract enforcement mechanisms. Because of the asymmetric information problems, insurance companies have to invest in monitoring mechanisms, require tradable collaterals, and impose high deductibles and co-payments (Hess et al. 2002). Since losses are spatially correlated across farmers, an extreme event could increase the number of defaults among farmers which in turn would represent additional liquidity problems (Skees and Barnett 2006; Skees et al. 2007). All these factors increase premiums and thus reduce demand for crop insurance.

Countries respond to weather-related risks by taking action both before and after the extreme weather events. As ex-ante strategy, governments have supported a variety of crop insurance programs. All of these have relied on government subsidies and yielded mixed results (Goodwin and Smith 1995). As ex-post strategy, governments often redirect resources used usually in activities such as education or health to cover damage caused by natural disaster. Because of these programs, farmers would not internalize the costs of weather risks and would be more dependable on public relief (Skees et al. 1999). In general, both governments and rural household of developing countries have not been effective in managing risk transfer neither ex-ante nor ex-post of a shock (Hazell 1992; Barnett et al. 2008).

2.1.2 Reinsurance and Securitization

Weather insurance was first conceived for the purposes of reinsurance of systemic risks. Extreme weather risks represent an enormous financial problem for the insurance companies because thousands of claims have to be paid within a relatively short period. In order to deal with these losses, insurance companies traditionally looked to unload weather risk using reinsurance as an ex-ante funding source. It allowed the insurer to raise its capital in the aftermath of the natural disasters by hedging its risk exposure with reinsurance companies which, in turn, diversified their portfolios by taking risks in other regions.

In spite of the advantage of reinsurance, insurance companies could not always transfer their risk exposure to reinsurers. Froot (2007) gave several explanations for this result, such as lack of reinsurance supply, market power of reinsurers, the price of the reinsurance contracts, and asymmetric information between insurer and reinsurer[3].

Securitization could be a natural way to introduce market efficiency and to provide an affordable insurance for weather-related risks. Securitization pools certain types of assets and repackage those into interest-bearing securities (Simmons 2003). In case of weather risks, these were tied to a specific weather event and were divisible so as to allow an investor to buy any amount of risk exposure.

The literature on catastrophe securities provides a number of arguments in favor of this approach (Lewis and Davis 1998). In particular, insurance companies could get sufficient capital to cover their exposure to catastrophe risk from the financial market. Since weather risks are not related to the performance of capital markets, insurers could get cheaper source of funds.

[3] Also see Skees (2000)

2.1.3 Weather Insurance

Weather insurance contracts were proposed in the literature as a natural extension of weather-linked reinsurance arrangements to primary insurance markets, in particular to deal with risks of agricultural production (Miranda 1991; Miranda and Vedenov 2001; Hess et al. 2002).

These insurance contracts are typically modeled as options whose payoffs are linked to realizations of specific weather variables such as number of heating or cooling degree days, rainfall level, etc. Most of the literature structures these contracts as a put (call) option with payoff triggered by a specific weather variable falling below (rising above) a pre-specified level. These payoffs can also be triggered by realizations of an index, which correlates losses and weather. The latter can be constructed based on an econometric relationship between weather and yield (Martin et al. 2001; Turvey 2001; Vedenov and Barnett 2004).

The key advantages of these insurance contracts over the traditional insurance products usually mentioned in the literature are as follows (Miranda and Vedenov 2001; Barnett and Mahul 2007):

- The contract structure is simpler than that of traditional insurance.
- The weather variables or indices are measured objectively and transparently and do not depend on actions of either farmers or insurers. Neither farmers nor insurers have better information about the future realization of the index or the dependence relationship between losses and the index.
- Farmers do not need to be classified according to their risk exposure

- Operating costs are low due to lack of asymmetric information or moral hazard.

The major disadvantage of index insurance is the so-called basis risk. It arises when farmers' losses are poorly correlated to the index used in designing a weather-based contract. In this case, a farmer could receive an indemnity payment that does not match the actual loss (Varangis et al. 2003).

There are three potential sources of basis risk. First, specific relationship between weather and yields is rather complex and not fully understood. Most of the approaches to designing indexes are based on econometric estimation of weather-yield relationship function which is then used as an index (Martin et al. 2001; Turvey 2001; Vedenov and Barnett 2004). This method, however, requires adequate data series, which may be a problem in developing countries.

The second potential source of basis risk is the yield data used for such estimations is typically aggregated over a larger production area (e.g. a county), which could also affect the performance of the insurance contracts. Ideally, farmers would prefer contracts written on a weather index designed for their individual farms, but farm-level data are rarely available especially in developing countries. Contracts based on weather measured at a specific farm maybe less attractive to outside investors and would affect the possibility of risk transfer to capital markets. These insurance contracts are often designed based on more easily available area yields. The downside of this approach is that the variability of aggregate yields is typically lower than those of individual farmers.

Third, the spatial variability of weather conditions affects the reliability of weather contracts. Given that the underlying weather variables are measured at specific locations, the impact of this measure can be diluted as one moves away from the weather station (Manfredo and Richards 2005).

All these problems can hurt the performance of the weather insurance product; however, the basis risk can be reduced through product design (Skees 2008).

2.1.4 Application of Weather Derivatives to Agricultural Risks

Turvey (2001) examined if weather derivatives could provide a hedge against production risk in Ontario. The relationship between crop productivity and weather events was estimated assuming a 2-input production function (rain and degree-day heat). The quadratic and Cobb-Douglas production functions were considered, although the goodness of fit was low. Given that no other specifications were explored and the limited evidence provided for these function, contract parameters based on this estimations could be inconsistent. The author's approach relies on daily data, which could be a challenge to obtain in developing countries. In addition, the author also assumed that farmers know what weather events to be insured against because he allowed farmers to choose contract parameters.

His concluded that farmers could reduce risk exposure to weather events purchasing these instruments. His models however, did not consider specifics of crop growth which could affect the performance of the weather derivatives. The author pointed out the need to minimize basis risk.

Martin et al. (2001) considered European put options on precipitation. These contracts start paying when the index falls below a specified strike. Once the index falls below a limit, the payoff "maxes out" at the maximum indemnity level. When the index falls between the strike and the limit, the contract pays a proportion of the maximum indemnity. This type of contract is completely designed once the values of strike, limit and maximum indemnity are specified. The authors used cumulative daily precipitation for September and October in Stoneville, Greenville and Cleveland counties in Mississippi as the index. Farmers were allowed to choose the parameters of the contract according to their risk management needs. Using extended time series of weather data, the authors estimated expected loss cost from the simulated historical loss costs. They used a gamma distribution to model cumulative precipitation. Their results encourage the use of weather derivatives within the US agriculture.

Vedenov and Barnett (2004) evaluated the efficiency of weather derivatives constructed as put or call options. Similar to Martin et al., these contracts start paying when the index ε falls below/exceeds a specified strike ε^*. Once the index falls below/exceeds a limit $\mu\varepsilon^*$, the insured receives the maximum indemnity z. When the index falls between the strike and the limit, the contract pays a proportion of the maximum indemnity. A formal payoff schedule for the put option can be written as

$$
(2.1) \qquad I(\varepsilon|z,\varepsilon^*,\mu) = z \times \begin{cases} 0 & if\ \varepsilon > \varepsilon^*, \\ \dfrac{\varepsilon^* - \varepsilon}{\varepsilon^*(1-\mu)} & if\ \ \mu\varepsilon^* < \varepsilon \le \varepsilon^*, \\ 1 & if\ \varepsilon \le \mu\varepsilon^*, \end{cases}
$$

where the parameter μ varies between 0 and 1, with the limiting case of 0 corresponding to the conventional proportional payoff with deductible, and 1 corresponding to a "lump-sum" payment once the contract is triggered regardless of the severity of the shortfall. The payoff for the call option can be written in a similar way with obvious changes.

The authors used district level yield data for corn, soybeans, and cotton in major respective production areas in the U.S. in order to construct weather indexes. They found that constructed weather derivatives may provide risk reduction for the considered crop/district combinations. Though they estimated models with complex combinations of weather variables, they obtained relatively low goodness of fit (36% at most). They used an ad hoc selection of weather variables (e.g. monthly average temperatures and cumulative monthly rainfalls) The weather derivatives were designed for relatively large geographic areas in order to avoid problems such as weather data availability and allow for risk transfer to capital markets, but this came at a cost of added basis risk.

Since the late 1990s, a number of studies considered implementation of index insurance for agriculture in developing countries (Hazell 1992; Miranda and Vedenov 2001; World Bank 2005). The majority of these papers relied on availability of extended series of weather and crop yield data, which is often not the case in developing countries. For instance, Skees et al. (1999) examined the performance of rainfall insurance in

Nicaragua, which is affected by insufficient or excess rainfall. In order to ensure the sustainability of such an insurance scheme, they recommended the development of extensive crop yield data sets to design and price the insurance.

Linear dependency between crop yield and the index has been usually assumed in literature (Skees et al. 2001; Turvey 2001; Hess 2003; Deng et al. 2008). These approaches may reduce the effectiveness of index insurance contracts, since they only rely on the strong monotonic dependence between crop yield and the index rather than a linear dependence.

Except for Vedenov and Barnett (2004), the authors typically assume that farmers have a complete knowledge about what type of weather instrument satisfy their requirements. Finally, there is no information on specifics of plant growth incorporated in the design of weather contracts.

2.2 Modelling Approach

We assume that we are presented with a short yield data series (20 years or less) averaged over an area such as a county or a comparable administrative unit. We propose to mitigate the problems with data by designing weather insurance contracts in the following way.

First, based on agronomic criteria such as soil pH, soil texture and drainage, we identify the composite types of soils best suited for the production of crop under investigation. Then, we use the developed classification to detect homogeneous production areas among the administrative units for which we have data. Specifically, we try to identify counties with a single or a predominant soil type. We expect that the variability of yield aggregated over these areas is comparable to that of any given farm within the area. We perform the analysis looking at areas sharing the same soil conditions rather than just the ones encompassed by arbitrary administrative boundaries.

Second, we select indexes based on growing requirements for the crop during each phenological stage and for the entire growth season.

Third, we determine the parameters of the contract by using agronomical considerations, viz. the minimum and maximum requirements of the weather variable chosen as an index.

In order to illustrate the proposed methodology, we apply it to the design of weather derivatives to Arkansas soybeans.[4] In particular, we use rainfall as an index and construct cumulative daily rainfall variables for each stage of soybean growth and the entire crop season. We consider both the excess and lack of water to be equally detrimental. Therefore, the designed contract is set to trigger when the rainfall falls outside of the optimal range suggest by agronomist. We call the proposed contract the agronomic contracts.

In order to evaluate the effectiveness of the agronomic contract, we compare it with the contracts designed according to the methodology proposed by Vedenov and Barnett (2004).

[4] As pointed earlier, while this case does not represent the situation with data we are trying to address, availability of long data series allows us to evaluate the performance of the proposed contracts against the benchmarks established in the literature.

2.2.1 Overview of Soybeans Production

The most comprehensive reference on soybean production is published by The Center for Agricultural Bioscience International (Singh 2010). The latter describes factors affecting soybean growing —geography, climate, land and soil. Other comprehensive sources include Heatherly and Hodges (1999) and Arkansas Soybean Handbook which describe phenological growth stages of soybeans and how various aspects such as soil texture affect soybean production. A comprehensive manual on crop water needs is written by Brouwer and Heibloem (1986). The remainder of this section presents summary of relevant information contained in these sources.

2.2.1.1 Phenology of the Soybean Plant

Soybeans are classified as a short-day plant, because the duration of darkness regulates the flowering. The cumulative water requirement from planting to harvest is between 450 and 700 mm, with daily consumption that varies as the crop develops. Soybean production requires good drainage as standing water can increase the incidence of diseases. Well-planned drainage provides better soil aeration, higher soil temperatures, better impact of herbicides, better soil structure and higher yields. Poor drainage increases the chance of plant diseases and insects/pests because herbicides cannot reach the soybean root zone.

Soybean plant develops over three distinct sequential stages (vegetative, reproductive and mature) that influence each of the three yield components —the number of pods per plant, the number of seeds per pod and seed size.

The vegetative stage begins when seed are exposure to moisture and soil temperature of 55 to 60°F. Usually in the first 15 days after planting, the cotyledon emerges above the soil surface and nodes appear on the main stem with fully developed leaves during the next 36 days. The water demand is around 15-26% of the total water requirement and needs to be applied at uniform rates during the 51-day period. Short-term excess water during the early vegetative stages can cause yield reductions depending on soil texture.

The reproductive stage begins when a soybean plants blossom and lasts until pods and leaves are developed. The process takes about 64 days to complete, during which seeds reach their full size. The water need at this stage is around 55-64% of the total water requirement. Water stress reduces number of pods per plant and affects grain-filling.

The maturity stage begins when pods on the main steam have reached their mature color and lasts 18 days. During this time, dry weather is required to reduce the moisture content in soybeans. The water need on this stage is approximately 10-20% of the total water requirement. At this stage, drain decisions may be critical. Early drainage speeds up the harvest but can negatively impact the grain filling, which in turns, affect the soybean weight.

The growth of soybean plants can also be negatively affected by critical temperatures (below 55°F and above 95°F) across different growth stages. Table B-1 summarizes the water requirement and the duration of each stage.

2.2.1.2 Soil Characteristics

Soybean productivity also depends greatly on soil characteristics. Soil texture determines the amount of water available to plants, how well-aerated the soils are, and the rate at which water moves to roots. Based on the proportion of various particle sizes present in soils, their texture is classified as fine, medium, or coarse. Fine-textured soils are dominated by tiny clay particles, while coarse-textured are characterized by larger-size sand particles. Medium-textured soils contain various proportions of clay, sand and silt. Coarse-textured soils such as loamy sand and silt loam are better suited for soybean growing than clay and medium-textured soils because the former maintains well-balanced nutrient levels, reduces erosion, moves air and water to the roots and supports rapid growth.

Soybean plants also require a right balance of acidity for high quality yield and healthy crops. The soil pH is a measure of acidity or alkalinity in soils and affects the ability of soil organisms to survive, which in turn transform organic matter into plant nutrients. Soybean can grow in soils with a wide range of pH, from 5.8 (slightly acid) to 7 (neutral). Soils with pH above 7 are considered marginal and not suitable for soybean production. High pH levels are detrimental for soybeans, because nutrients such as phosphorus, calcium and nitrogen are unavailable in such soils. Yields are also negatively affected when soil becomes too acidic (low pH), although this can be mitigated by adding liming materials before fertilizing.

2.2.1.3 Proposed Soil Classification

Based on the information above, we attempt to identify the composite types of soils best suited for soybean production.

We designate the optimally suited soil as Soil Type I. The latter is characterized by coarse texture, pH level 5.8 to 7 and good drainage. As mentioned above, coarse-textured soils are more fertile than fine-textured because the former has a relatively high amount of water available to plants. Also, coarse-textured soils do not require much attention because they can maintain a steady pH level so that farmers would not need to reduce soil acidity. The pH level between 5.8 and 7 guarantees the availability of soil nutrients to plants and avoid damage to rotational crops. These characteristics together with good drainage enable soybean cultivation.

Fine texture soils regardless of pH level and soil drainage are designated Soil Type III because it is the least suitable type for soybean production. Fine-textured soils have small amount of water available to plants because of their small-sized particles. Large economic losses can occur when soybeans are cultivated on fine-textures soils because of the Phytophthora diseases.

All other combinations of soil characteristics are grouped as Soil Type II. The latter combine coarse and medium texture and pH level less than 7 regardless of soil drainage. The properties of all constructed soil types are summarized in figure A-1.

2.2.2 Contract Design and Analysis

We follow the general contract design approach of Martin et al. (2001) and Vedenov and Barnett (2004), with necessary modifications to accommodate out modeling strategy discussed above. In particular, cumulative rainfall over a specific phenological stage is used as an index. The contract is set to trigger only when the rainfall falls outside of the optimal range since both the excess and lack of water can be equally detrimental to plan growth. In other words, the contract has two triggers suggested by agronomic criteria. Formally, the contract indemnity (expressed in units of yield) can be presented as:

$$(2.2) \quad I(x|x_{min}, x_{max}, \beta, \rho, \alpha) = \beta \times \rho \times \begin{cases} \alpha x_{min} & if\ x \leq (1-\alpha)x_{min} \\ x_{min} - x & if\ (1-\alpha)x_{min} < x \leq x_{min} \\ 0 & if\ x_{min} < x < x_{max} \\ x - x_{max} & if\ \ x_{max} \leq x < x_{max} + (1-\alpha)x_{min} \\ \alpha x_{min} & if\ x \geq x_{max} + (1-\alpha)x_{min} \end{cases}$$

where x is the cumulative rainfall level over a specific period, the trigger points x_{min} and x_{max} correspond to the minimum and maximum water requirement during the period based on agronomic recommendations, ρ is a conversion factor between the units of rainfall and units of yield[5]. The parameter $\alpha \leq 1$ is used to cap the contract payoff for

[5] We define this as the ratio of average crop yield and average cumulative rainfall during the entire crop season.

tractability purposes.[6] Since index contracts are subject to basis risk, the buyer is also allowed to increase or decrease the amount of insurance protection by the scale factor β, similar to the Group Risk Plan (GRP) offered in the U.S. (Deng et al. 2007). The scale factor adjusts the indemnities so that those could better track actual losses. The indemnity schedule (2.2) is illustrated in figures A-2 and A-3.

Figure A-2 shows payoffs of two contracts with the scale factor of 100 percent and the cap factors of 80 and 150 percent. Figure A-3, shows two contracts with the cap factor of 100 percent and scale factors of 75 and 100 percent.

2.2.2.1 Measuring Risk Reduction

The effectiveness of the designed contracts in reducing risk is measured within the expected utility framework. The analysis is performed from the standpoint of an economic agent who is not necessarily directly involved with the production, but is involved with the economic activity directly affected by the agricultural production risks (i.e. a "risk aggregator"). For example, county cooperatives that give loans to farmers are directly affected by the farmers' losses because the latter affect the probability of payoffs. The rationale behind this approach is that the index insurance contract protect better against systemic risks rather than idiosyncratic risks of individual producers. A portfolio of risks aggregated over a properly defined area would diversify away such

[6] For the tractability of this contract, its payoff must be at least constrained on the excess side where it can be potentially infinite. Since the payoff is naturally limited on the deficiency side (at $x = 0$), this provides for a convenient overall cap which corresponds to $\alpha = 1$. Values of $\alpha < 1$ are considered to allow for a possibility of a cap set at a fraction of the maximum payoff on the deficiency side. The latter is similar to the maximum payoff constraint parameter utilized in Vedenov and Barnett (2004).

idiosyncratic risks, but would still be exposed to the area-wide risks typically associated with the extreme weather events (e.g. droughts, floods, hurricanes, etc.).

We assume that there is one such "risk aggregator" in a given county/region, that aggregator's choice of insurance is driven by expected utility maximization motives, and that its preferences over risky alternatives can be represented by a utility function $u(\cdot)$[7] defined over the total revenues expressed in units of yield.[8]

If no insurance is available, the aggregator utility is simply

$$EU_{without} = Eu(y) \tag{2.3}$$

where y is crop yield, and the expectation is taken over its distribution. If we assume that a random variable x (weather index) can communicate information about y (crop yield), and an insurance contracts on x is available, then the utility becomes

$$EU_{with} = Eu(y + I(x) - P) \tag{2.4}$$

where the indemnity function $I(x)$ is as in equation (2.2), P is the contract premium[9], and the expectation is taken over the joint distribution of the index x and yield y.

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[7] This aggregator derives utility from the county-level yield, but the nature of that utility depends on the nature of the aggregator. For purpose of this analysis, we do not specify the latter.
[8] This assumption can be relaxed in a trivial way if prices are fixed and nonrandom. Stochastic prices can be accommodated within the same framework, although practical application would require additional historical price data, which may or may not be available.
[9] Both the indemnity and premium are expressed in terms of yield.

Without loss of generality, we assume that the premium is actuarially-fair and is equal to the expected payoff of the contract[10], i.e.

$$P = EI(x \mid \alpha, \pi, x_{min}, x_{max}) \quad (2.5)$$

where all parameters are the same as in equation (2.2) and the expectation is taken over the distribution of the index x.

Under these conditions the agent would decide to buy the insurance contract if the expected utility of revenue with the contract is greater than the expected utility without the contract. For illustrative purpose the expected utility can be conveniently represented by the certainty equivalent levels (Schnitkey et al. 2003), namely:

$$CE = u^{-1}[EU(\cdot)] \quad (2.6)$$

The risk reduction due to the weather derivative can be then computed as $\Delta CE = CE_{with} - CE_{without}$. The contract has a value to the aggregator and reduces its risk exposure if $\Delta CE > 0$.

2.2.2.2 Estimation of Distributions

In order to compute premiums and certainty equivalent revenues in equations (2.5) and (2.6), the joint distribution of yield and rainfall $h(y, x)$ and the marginal distribution of rainfall $h_x(x)$ are required.

[10] Loaded premiums can also be considered within the same framework.

A typical approach here is to assume a parametric functional form and then estimate the unknown parameter(s) based on historical data. Given the shortness of data series and the lack of valuable prior information about the underlying data generation process of yield and the index, parametric estimations can be unreliable.

The alternative is to use nonparametric methods, which impose fewer assumptions and rely on data to determine the shape of the distributions. In particular, we use kernel-density method (Wand and Jones 1994) to estimate the marginal distributions of rainfall and yield, viz.

$$h_{\xi}(\xi) = \frac{1}{T\delta_{\xi}} \sum_{j=1}^{T} K\left(\frac{\xi - \xi_j}{\delta_{\xi}}\right) \tag{2.7}$$

where ξ is the random variable of interest (either the index x or the yield y), $K(\cdot)$ is a kernel function, δ_{ξ} is the degree of smoothness, and $\{\xi_j\}_{j=1}^{T}$ are the observations (historical realizations) of interest.

Following Charpentier et al. (2007), we estimate non-parametric copula density

$$c(u, v) = \frac{1}{T\delta_y \delta_x} \sum_{i=1}^{T} K\left(\frac{u - H_{y,T}(y_i)}{\delta_y}, \frac{v - H_{x,T}(x_i)}{\delta_x}\right) \tag{2.8}$$

where $K(\cdot)$ is a bivariate kernel function, δ_y and δ_x are the degrees of smoothness (Li and Racine 2011), and $H_{\xi,T}$ is the empirical distribution functions defined as

$$H_{\xi,T}(\xi) = \frac{1}{T+1}\sum_{i=1}^{T} \mathbb{I}(\xi_i \leq \xi) \tag{2.9}$$

where the indicator function $\mathbb{I}(A)$ takes the value of one if the condition A is satisfied and zero otherwise. The marginal distributions (2.7) and the copula estimator (2.8) are combined to construct the joint probability distribution using the Sklar's theorem (Sklar 1959). The latter postulates that any joint distribution can be decomposed into its marginal distributions and a copula function which captures the dependence structure between variables, namely,

$$h(y,x) = c\left(H_y, H_x\right)h_y(y)h_x(x) \tag{2.10}$$

2.2.2.3 Efficiency Analysis

In order to evaluate the effectiveness of the agronomic contract, we compare its risk-reducing capability with the benchmark contract designed according to the methodology in Vedenov and Barnett (2004). In addition, we analyze variations of the agronomic contract constructed both for the entire season and for each stage. We also consider agronomic contracts written on the excess of water, the lack of water or both. All variations of the agronomic contracts and the Vedenov and Barnett (2004)' contracts are constructed based on the same data set.

We express the risk reduction in terms of the certainty-equivalent payouts of these contracts. The goal is not to obtain the best contracts (by construction there are not) but rather to see how close the performance of the optimal contracts can be approached by the agronomic contracts in the situations when one cannot rely on extended data series or when data aggregation may create potential problems.

2.3 Application: Arkansas Soybean Production

Soybean production in Arkansas has characteristics similar to those found in South America, especially in Argentina, Brazil, Bolivia and Paraguay. Many producers plant soybean on marginal lands without irrigation, especially in the lower Mississippi River valley. The increasing value of water together with the reduced amount of inputs required make farmers increase the amount of acreage devoted to non-irrigated soybeans. Under these conditions, the effects of drought or flood could increase the variability in soybean production.

Arkansas, located about 35°N of the equator, has temperatures influenced by the Mississippi River and the Ozark and Ouachita mountains. Argentina and Brazil, the biggest soybean producers in South America, have temperatures more stable than Arkansas. Argentina soybeans are grown in temperate regions (35°S of the equator) with rainfall during the growing season. Brazil, with soybeans regions closer to the equator, has a wetter climate and higher rainfall than Arkansas.

Arkansas soybean growers apply various production systems under different tillage regimes which may or may not include irrigation. Under non-irrigated system, farmers can stabilize yield from year to year when they combine high-yielding varieties with different maturity date (Ashlock, Mayhew, et al. 2000).

2.3.1 Soil Structure

In order to apply the classification suggested in figure A-1, we combined the Arkansas maps of landforms, soil texture, pH level and soil drainage provided by Soil Survey Staff, et al. The resulted map was combined with the soybean area map provided by USDA/NASS Cropland Data Layer and USDA/SSURGO.

We identified counties that are composed mostly of a single soil type or combination of at most two types. Six counties selected for analysis are listed in table B-2 along with the distribution of soil types in each.

Independence County mostly has Soil Type I (63.1 percent), while Jackson County is predominantly Soil Type II (78.6 percent). Crittenden mostly has soil type III (78.6 percent). Phillips and Saint Francis Counties have a mixed soil composition — a combination of soil types II and III. Finally, Pulaski County combines all identified soil types. Mixed soil type counties are included in the analysis in order to verify the conjecture that agronomic contracts are more effective when constructed for areas with similar (homogeneous) growing conditions. Figure A-4 shows the location of selected counties.

2.3.2 Weather Data

Daily precipitation and temperature data recorded at the weather stations nearest to the selected counties were collected from the NOAA/NCDC. Locations of the weather stations are shown in figure A-4. The data were used to construct cumulative rainfall variables by stage and for the entire growing season. Since Arkansas farmers use different soybean varieties with different growing periods, it was impossible to determine specific planting dates and stage durations for each individual producer or the county as a whole. Instead, we used average planting dates and average stage durations reported in Ashlock and Purcell (2000) (see table B-1).

Average temperatures by stage and for the entire season were generated in a similar fashion. Tables B-3 and B-4 show descriptive statistics of both cumulative rainfall and average temperature by stage, assuming June 15 as planting date (Ashlock, Klerk, et al. 2000). Data show that daily temperatures in selected counties do not fall outside of the agronomic requirement, for that reason temperature was excluded from our analysis. However, availability of rainfall is critical and it is the main constraint during the crop season.

2.3.3 Yield Data

Historical county-level yield data for non-irrigated soybean production in the selected counties for 1972-2012 were collected from USDA/NASS. The descriptive statistics of the yields are listed in table B-5. These selected counties accounted for 63.6 percent of Arkansas non-irrigated soybean production in 2012.

KPPS test, Dickey-Fuller and Phillip-Perron tests were performed to detect whether yields have stochastic trends. All these tests agree that yield series are trend stationary.[11]

New disease-resistant and high-yielding soybean varieties have been introduced in Arkansas over time. These improvements make soybean yields incomparable across years. To address this problem, yields series were detrended following Vedenov et al.

[11] Unit root results are available upon request.

(2006). In particular, a piecewise log-linear trend equation was fitted for each yield series. The general form of the estimated equation is

$$\ln(y_t) = a_0 + a_1 t + b_1(t - t_1)d_1 + b_2(t - t_2)d_2 + \cdots + b_k(t - t_k)d_k + u \tag{2.11}$$

where $\ln(y_t)$ is the natural logarithm of yield in year t; t_i, for $i = 1, \ldots, k$, represent the years at which the slope of the trend line changes, d_i are dummy variables which are equal to 1 for all observations such that $t_i \leq t$, and 0 otherwise and u is the error term.

We used a nonlinear least square procedure to estimate model (2.11). This method allowed us to find the points t_i that result in the best fitting model. Table B-6 shows the best fit models and their statistics for selected counties.

Based on these estimations, the detrended yields were then calculated as:

$$y_t^{det} = y_t \frac{y_{2012}^{tr}}{y_t^{tr}} \tag{2.12}$$

where y_t^{tr} is the trend predicted for year t.

2.3.4 Simulation Parameters

Epanechnikov kernel was used to estimate both the marginal CDFs and PDFs for yield and rainfall. The same kernel was also used to estimate the copula function. The *rule of thumb* was used to estimate the bandwidth (Li and Racine 2011). CRRA power utility function

$$u(w, \gamma) = \frac{w^{1-\gamma}}{1-\gamma} \tag{2.13}$$

was used to reflect the preferences, with the risk aversion parameter γ ranging from 1 to 3 (as per Myers (1989) and Wang et al. (1998)).

The expected utilities in equations (2.3) and (2.4) were integrated numerically using Simpson's quadrature method (Miranda and Fackler 2002) on a 501×501 grid over the ranges of yield and rainfall distributions. The trigger points x_{min} and x_{max} for each stage were set according to table B-1. The range of cap factor α in equation (2.2) was set from 70 to 100 percent and the scale factor β from 85 to 150 percent, both in 5% increments. The conversion factor ρ was set equal to the ratio of average yield and average water needed in the entire season. The risk reduction of *agronomic contract* specified in equation (2.2) was evaluated for situations when the risk aggregator is allowed to purchase contracts on excess of rain, lack of rain or both.

The effectiveness of the agronomic contracts was compared with that of the contract designed according to Vedenov and Barnett (2004). The weather index ε was constructed using parametric regressions between soybean yield and cumulative precipitation for each growth stage. Table B-7 shows the "best fit" models and their statistics.

Indices estimated by these models were then used to calculate payoffs of the standard contracts in equation (2.1). The optimal values of the strike ε^*, the limit μ and the maximum indemnity z were determined so as to maximize the expected utility with the contract. The risk reduction was calculated following equation (2.6).

2.4 Results

The risk-reducing effectiveness of agronomic contracts varies both across counties (or soil types) and stages. Figures A-5 through A-10 show the estimated joint distributions of soybean yield and rainfall for each growth stages for selected counties. The vertical lines indicate the minimum and maximum water requirements in each stage. The horizontal line represents the average soybean yield. Numbers in bold represent the joint probability of drawing a rain-yield from the respective ranges of rain and yield.

Unclear dependence between crop yield and rainfall were found for the vegetative stage regardless of soil types (except for Phillips and Pulaski Counties) suggesting a substantial amount of basis risk (see figures A-5 – A-10). This pattern continues in the maturity stage as well. However, there appears to be a more pronounced dependence between yields and rainfall in the reproductive stage. In particular, the probability of obtaining low yields when the rainfall is outside of the optimal range is higher at this stage than at any other. Finally, the joint distributions of yields and rainfall for the entire season exhibit a stronger dependence structure than the individual stages.

Tables B-8 and B-9 summarize parameters and risk reduction effectiveness of the best agronomic contracts for each combination of soil type, contract type (excess, lack, or both), and growth stage.

The cap and scale factor parameters turned out to be the same for all combinations and equal to the lower bounds of their respective range. This result confirms the presence of the basis risk indicated by the distribution plots, with the risk aggregator attempting to reduce the former by selecting lower coverage levels and scaling down the payments.

Nevertheless, for most counties, agronomic contracts can provide at least some degree of risk reduction. The best results as seen in table B-8 are obtained with the contracts written on lack of water during the reproductive stage for the counties with soil types I and II (Independence and Jackson counties). This seems to confirm the conjecture that the weather insurance contracts perform better when written for areas with similar growing conditions. An interesting result is that the risk reduction for soil type II is higher than that for soil type I. A possible explanation is that soil type II is less suitable for soybean growth, and therefore plants are more sensitive to variations in weather. At the same time, the agronomic contracts seem to be making no difference for the soil type III (Crittenden county), which could be due to the poor overall growing conditions provided by this soil type.

Results for counties with a mixture of soil types are less consistent, but it appears as if contracts written on lack of water during the reproductive stage are performing reasonably well for the mixture of soil type II and III (Phillips and St. Francis). The contracts written on the entire season’s rainfall appear to be rather ineffective for the homogeneous growing areas, but do provide some risk reduction in the counties with the heterogeneous soils (Phillips and St. Francis counties).

Efficiency analysis was also carried out for the “optimal” contracts in the sense of V&B. As expected, the “optimal” contracts would offer risk reduction for all soil types (see table B-10), with the levels of reduction generally higher than those offered by the agronomic contracts. However, these higher levels of risk reduction come at the price of much higher premium rates (up to 88%). Furthermore, no connection between the degree of risk reduction and soil quality is reflected in these results. The contract for Jackson County achieved the highest risk reduction (2.71%) although with the premium rate above 60%.

2.5 Conclusions

A simple semi-naïve approach to designing weather insurance contracts is proposed in this study. The potential of this approach lies primarily in its low yield data requirements, which is a typical situation in developing countries. Lack of long, reliable farm-level yield data series is a major hindrance in applying the conventional methods of designing weather contracts outlined in the literature. Weather data, on the other hand, are usually more readily available and more objectively measured. In such circumstances, the proposed methodology allows one to design practical instruments that still provide some degree of risk protection. The two key points of the presented methodology are (a) the use agronomic information in order to set contract parameters, and (b) construction of contracts for homogeneous production areas.

Soybean production in Arkansas is used as the case study in order to test the validity of our approach. In order to mitigate the aggregate nature of the available yield data, we attempt to look at areas sharing the same growing conditions rather than simply located within the same administrative boundaries. To that end, we design classification of soils to reflect their suitability for soybean production, which then allows us to identify counties that are composed mostly of a single soil type or are combinations of soil types. We also attempt to avoid the need for data-demanding analysis of weather-yield relationship by using agronomic requirements in order to specify parameters of weather insurance contracts.

The risk reduction of thus constructed weather index contracts, called *agronomic contracts* in this study, are evaluated and compared with the performance of the "optimal" contracts suggested in the literature. While the agronomic contracts do not achieve the same degree of risk reduction as the "optimal" contracts, they do provide meaningful risk protection and typically at lower premiums. As expected, the agronomic contracts perform better in homogeneous production areas. Furthermore, the best risk reduction is achieved when the contracts are written on rainfall during a specific stage of plan growth rather than the entire season. Finally, the agronomic contracts seem to provide the highest risk reduction on second-best soils, which could be explained by higher sensitivity of production on such soils to weather.

Future research should investigate the inclusion of additional weather variables to measure their influences on the risk reduction in each stage. Further research could address the potential for reducing basis risk by defining homogenous production areas at higher definition. Also, future research could use spatial smoothing on weather measurements. Soil-crop simulation model could help to define specific trigger points for each soil type.

3. OPTIMAL CONTRACT STRUCTURE FOR WEATHER DERIVATIVES

Many of the existing papers on weather derivatives use a “standard” piecewise-linear structure to define the payoffs of the analyzed contracts (Turvey 2001; Vedenov and Barnett 2004; Collier et al. 2009; Giné et al. 2010). It is commonly claimed that these contracts provide sufficient flexibility to construct different instruments. The intuition behind this claim goes back to the seminal work of Arrow (1974) who showed that the optimal payoff on an insurance contract is proportional to the loss. However, Arrow’s result is derived under the condition that the insurance contract is written on the actual loss.

In case of index contracts (e.g. weather derivatives), the payoff of the contract depends on realization of a random variable that is related to the loss but not exactly equal to it. Therefore, the piecewise-linear contract payoff structure may not be the optimal choice if the dependence between the index and the yield/revenue is nonlinear. The research objective of this essay is to derive an optimal payoff of an index contract that takes into account potentially nonlinear dependence between the index underlying the contract and the loss that is insured using the contract.

This framework is illustrated by constructing index insurance contracts for the case of Arkansas soybean considered in Essay 1. The results are then compared with those presented in Section 2.4, which utilizes the standard (quasi-linear) contracts with the goal of measuring the improvements in risk reduction that can be obtained by using the optimal contract payoff structure.

The rest of the essay is organized as follows. The first subsection reviews literature on contract design. The second subsection presents the theoretical model of the optimal index contract and the numerical solution procedure. The third subsection briefly describes the specific application and data collection. The fourth subsection presents and discusses the results. The final subsection concludes and discusses directions for future research.

3.1 Literature Review

Literature on index insurance contracts and their applications for agricultural production was already reviewed in Sections 2.1.1 and 2.1.2 (Miranda 1991; Martin et al. 2001; Miranda and Vedenov 2001; Turvey 2001; Hess et al. 2002; Vedenov and Barnett 2004). The common feature of these papers is that they model index insurance contracts as options whose payoffs are linked to realizations of specific index variables such as area yield, number of heating or cooling degree days, rainfall level, etc.

The theory of the optimal contract design goes back to the seminal works of Arrow (1974). Arrow derived a Pareto optimal insurance policy within the complete market framework and showed that it necessarily has to include elements of coinsurance and deductibility. Raviv (1979) expanded on Arrow's results by identifying the form of a Pareto optimal insurance contract under fairly general assumptions. He also found that the optimal contract structure has a deductible and coinsurance of losses above the deductible.

The main problem in applying the results of Arrow and Raviv to the case of index contracts stems from the fact that, unlike the conventional insurance, the payoff of the index contract depends on realizations of a random variable that is not the same as the loss. Therefore, the deductible-with-coinsurance structure may no longer be optimal.

Mahul (1999) attempted to design an optimal insurance contract for the case where the indemnity depends on an aggregate yield of a specific area. However, in order to obtain a closed-form expression for the optimal contract and make conclusions similar to those of Arrow, he had to assume a linear relationship between the farm yield and area yield, with all of dependence between the two captured by the linear correlation. Under these restrictive assumptions, Mahul found was that the optimal insurance contract structure conforms to that of Arrow, with the payoff depending on the ratio of the covariance of the farmer's yield and area yield to the variance of the area yield.

Mahul and Wright (2003) investigated the design of optimal contract when farmers face joint yield and price risk under an incomplete market framework. However, they also specify farmers' price and yield as linear functions of single price and yield index in order to derive the optimal contract structure. Under these conditions, they do obtain the Arrow's results but only under the assumption of linear dependence between the loss and the index.

3.2 The Model

Consider an economic agent who receives a risky income y from an economic activity[12]. We assume that agent's decisions are driven by the expected utility

[12] The derivations in this subsection follow the general approach of Raviv (1979) but extends it to the case of an index contract.

maximization motives and her preferences can be represented by a conventional utility function $u(y)$ with $u'(y) > 0$ and $u''(y) < 0$ for all y.

The agent has an opportunity to purchase an insurance contract with a payoff contingent on an observable random variable x, which, in turn, can potentially communicate information about y. For the purposes of this essay, we refer to x as an index[13].

The insurance contract is described by a pair $[I(x), P]$, where $I(x)$ is the indemnity payment made by the insurer and P is the premium, which is the price paid by the insured. This contract should satisfy certain feasibility conditions, viz. (1) the indemnity function $I(\cdot)$ is nonnegative for all realization of x, and (2) for any indemnity schedule $I(\cdot)$, a premium P can be determined. If this type of insurance contract is available, the economic agent could purchase it to cope with, for instance, weather-related risks.

Specifically, the economic agent would purchase the insurance contract $[I(x), P]$ if her expected utility with the contract is greater than the expected utility without it,

$$Eu(y + I(x) - P) \geq Eu(y) \tag{3.1}$$

where the expectation is taken over the joint distribution of the index x and income y on the left hand side, and over the probability distribution of y on the right hand side.

[13] In the crop insurance context, this can be, for example, a weather variable, an aggregate yield, or any other variable that had a dependence relationship with the yields or revenues.

We assume that the insurer is a risk-neutral agent with an initial wealth W_0, who is willing to offer the contract under a competitive environment. Let $v(\cdot)$ be his utility function defined over his final wealth such that $v'(\cdot) \geq 0$ and $v''(\cdot) = 0$. Without loss of generality, we can assume that

$$v(z) = az + b \tag{3.2}$$

Provision of insurance is costly due to administrative cost and other expenses. In general, the insurance cost $c(\cdot)$ defined over the indemnity schedules I is increasing ($c'(\cdot) \geq 0$) and convex ($c''(\cdot) \geq 0$) for all indemnity schedule I. In practice, the insurance cost is usually set to be directly proportional to the indemnity, i.e. set equal to the indemnity multiplied by a so called *load factor*. We adopt this approach for the purposes of this essay[14] and define the cost function $c(\cdot)$ as

$$c[I] = \theta I \tag{3.3}$$

where $\theta > 0$ is a load factor and $c(0) \geq 0$. If the insurance company offers the contract $[I(x), P]$ and the realization of x represents a loss to the agent, the final wealth of the insurer is $W_0 + P - I(x) - c(I)$. In this case, a necessary condition for the insurance company to offer such a contract is

[14] More general forms of the cost function can be incorporated in the presented framework in a straightforward fashion. We use a simpler form primarily for the reason of tractability.

$$Ev\big(W_0 + P - I(x) - c(I)\big) \geq Ev(W_0) \tag{3.4}$$

Proposition 1. If insurer exhibits risk-neutral preferences (equation 3.2) and the insurance cost follows the structure of equation (3.3), then under (3.4) the constraint on the insurance contract $[I(x), P]$ is

$$P \geq (1+\theta)E\big(I(x)\big) \tag{3.5}$$

(after Raviv (1979)).

Proof: Given the linearity of the utility function (3.2), the condition (3.4) becomes

$$a\Big(W_0 + P - E\big(I(x)\big) - (1+\theta)E\big(I(x)\big)\Big) + b \ \geq aW_0 + b \tag{3.6}$$

Solving for P results in (3.5). ■

The optimal insurance contract is formed by the pair $[I^*(x), P^*]$ that maximizes the expected utility of the economic agent while at the same time not decreasing the expected utility of the insurer. Formally,

$$I^* = \underset{I}{\operatorname{argmax}} \int_{supp\ x} dx \int_{supp\ y} u(y + I(x) - P)h(x,y)dy \tag{3.7}$$

$$\text{s.t. (3.5) and } I(x) \geq 0$$

where $h(x,y)$ is the joint probability density function of the index x and income y. This is a calculus of variations problem with the optimal solution $I(\cdot)$ being a function rather and just a single point (Chiang 1999).

3.2.1 Optimality Conditions

Raviv (1979) showed that the constraint in (3.5) is binding at the optimum. Therefore the problem in (3.7) can be solved rewritten as:

$$I^*(x) = \underset{I(x)}{\operatorname{argmax}} \int_{suppx} dx \int_{supp\, y} u(y + I(x) - P)h(x,y)dy \tag{3.8}$$

Subject to:

$$P = (1+\theta)\int I(x)h_x(x)dx \quad \text{and} \quad I(x) \geq 0$$

where $h_x(x)$ is the probability density function of the index x. The corresponding Hamiltonian can be written as:

$$H = \int u(y + I(x) - P)h(x,y)dy - \lambda\left[(1+\theta)\int I(x)h_x(x)dx - P\right]$$

Since the latter does not depend on $I'(x) \equiv \frac{\partial I(x)}{\partial x}$, the optimal solution is determined by the Euler-Lagrange conditions.

$$\frac{\partial H}{\partial I} = \int_{supp\ x} u'(y + I(x) - P)h(x,y)dy - \lambda(1+\theta)h_x(x) = 0 \tag{3.9}$$

$$\frac{\partial H}{\partial \lambda} = (1+\theta)\int I(x)h_x(x)dx - P = 0$$

The conditions expressed in equation (3.9) define a set of index values X over which the contract pays an indemnity. Arrow (1974) and Raviv (1979) demonstrated that this set is a continuous interval $X = [0, \bar{x}]$ in the case of a traditional insurance contract where the payoff is determined by the realization of a loss. However, in the case of an index contract this result does not necessarily hold true. In fact, any meaningful statements regarding this interval can be only made by making specific (if rather limited) assumptions about the utility function. For example, the result of Mahul (1999) follows from equation (3.9) under the assumption of quadratic utility.

Integrating the first condition (3.9) with respect to x and taking into account that $\int h_x(x) = 1$, gives

$$\lambda = \frac{1}{1+\theta}\iint u'(y + I(x) - P)h(x,y)dxdy \tag{3.10}$$

Together, the conditions (3.9)-(3.10) indirectly define the optimal contract indemnity $I^*(x)$.

While these equations do not have closed-form solutions, they can be solved numerically for any given choice of the utility function u and the joint distribution function $h(x, y)$.

3.2.2 Algorithm for Solving the Euler-Lagrange Equations

The first order conditions expressed in equations (3.9)-(3.10) can be solved iteratively using the following algorithm. For convenience, denote

$$A(x) = \int u'(y + I(x) - P)h(x, y)dy \tag{3.11}$$

Then the optimality conditions (3.9)-(3.10) can be rewritten as

$$A(x) - (1 + \theta)\lambda h_x(x) = F(x) = 0 \text{ at all } x \tag{3.12}$$

and

$$\lambda = \frac{1}{1 + \theta}\int A(x)dx \tag{3.13}$$

From the numerical standpoint, equation (3.12) represents a set of nonlinear equations problems that need to be satisfied at all values of the index x, while equation (3.13) is a cumulative condition on the whole indemnity function. Newton methods and function iteration method are commonly used to find solutions to nonlinear equations

(Miranda and Fackler, 2002). Newton methods are generally considered faster, however they require calculation of derivatives and are best suited for continuously differentiable function. In our case, the indemnity function is expected to be equal to zero on one or several intervals, which would result in discontinuity of first derivatives. Therefore, we opt for the function iteration method in order to ensure convergence.

In order to use function iteration, the root-finding problem in equation (3.12) is recast as a fixed-point problem

$$I(x) = I(x) - Scale \cdot F(x) \tag{3.14}$$

where $Scale$ is scaling factor used to regulate the speed of convergence and ensure stability of the iterative process. The solution algorithm then can be written as follows.

Step 0: Select an initial guess for the indemnity function $I_0(x)$.

Step k: Use $I_{k-1}(x)$ to

- Compute the premium $P_k = (1+\theta)\int I_{(k-1)}(x)h_x(x)dx$
- Update $A_k(x) = \int u'(y + I_{k-1}(x) - P_k)h(x,y)dy$
- Compute $\lambda_k = (1+\theta)^{-1}\int A_k(x)dx$
- Calculate $F_k(x) = A_k(x) - (1+\theta)\lambda_k h_x(x)$
- Stop if $\|F_k(x)\| \le \varepsilon$, where ε is a chosen tolerance level. Otherwise, update $I_k(x)$ as: $I_k(x) \leftarrow I_{k-1}(x) - Scale \cdot F_k(x)$.
- At all points x where the updated value of $I_k(x)$ turns out to be negative, set $I_k(x) = 0$.

- Repeat the above as needed

Generally speaking, the first order conditions equations (3.9)-(3.10) have to be satisfied at all x, i.e. at an infinite number of points. For practical purposes, a grid of 1001 nodes in the domain of x is used to provide a reasonably accurate approximation of the optimal contract payoff function.

3.3 Application Procedure and Data

In order to illustrate the presented method for constructing the optimal index insurance contract, we use the case of Arkansas soybeans introduced in Section 2.2. In particular, we design rainfall insurance contracts for representative farmers for a subset of counties and phenological growth stages discussed in Section 2.2.1. Table B-11 shows the counties and the growth stages selected.

The general approach is to use the cumulative rainfall during a specific period as an index x, and the per-acre yield as a measure of the income y in equation (3.7). Without loss of generality, the indemnity $I(x)$ and premium P of the insurance contracts are assumed to be denominated in units of yield. The distribution $h(x, y)$ then reflects the joint distribution of rainfall and yields for the area and growth stage considered.

As in Section 2, we assume that the representative farmer's preference is described by the CRRA power utility function

$$u(z, \gamma) = \frac{z^{1-\gamma}}{1-\gamma} \tag{3.15}$$

where γ is the risk aversion parameter ranging from 1 to 3 (as per Myers (1989) and Wang et al. (1998)). For purposes of this Essay the load factor θ is set equal to 0, i.e. the premiums are assumed actuarially-fair.

3.3.1 Estimation of Distributions

We use the kernel-density method to estimate the marginal distribution of index and yield, viz (Wand and Jones 1994).

$$h_\xi(\xi) = \frac{1}{T\delta_\xi}\sum_{j=1}^{T} K\left(\frac{\xi - \xi_j}{\delta_\xi}\right) \tag{3.16}$$

where ξ is the random variable of interest (either the index x or the yield y), $K(\cdot)$ is a kernel function, δ_ξ is the degree of smoothness, and $\{\xi_j\}_{j=1}^{T}$ are the observations (historical realizations) of interest.

We estimate the joint probability distribution using the copula method (Sklar 1959), namely

$$h(y, x) = c(H_y, H_x)h_y(y)h_x(x) \tag{3.17}$$

where $h_x(x)$ and $h_y(y)$ are the marginal densities from equation (3.16) and $c(H_y, H_x)$ is the copula density function evaluated at corresponding CDFs. Following Charpentier et al. (2007), we estimate the non-parametric copula density

$$c(u,v)=\frac{1}{T\delta_y\delta_x}\sum_{i=1}^{T}K\left(\frac{u-H_{y,T}(y_i)}{\delta_y},\frac{v-H_{x,T}(x_i)}{\delta_x}\right) \tag{3.18}$$

where $K(\cdot)$ is a bivariate kernel function, δ_y and δ_x are the degrees of smoothness and $H_{\xi,T}$ is the empirical distribution functions defined as

$$H_{\xi,T}(\xi)=\frac{1}{T+1}\sum_{i=1}^{T}\mathbb{1}(\xi_i\leq\xi) \tag{3.19}$$

where the indicator function $\mathbb{1}(A)$ takes the value of one if the condition A is satisfied and zero otherwise. The overall joint distribution is estimated on a 1001×1001 grid over the ranges of yield and index distributions, respectively.

Epanechnikov kernel was used to estimate the marginal PDFs for yield and index. The same kernel was also used to estimate the copula function. The *rule of thumb* was used to estimate the degree of smoothness, or bandwidth (Li and Racine 2011).

3.3.2 Initial Guess for Indemnities Function

The numerical algorithm used to solve the optimal control problem in (3.7) requires an initial guess of the indemnity function. For the purposes of this essay, we used the "standard" contracts designed based on methodology in Vedenov and Barnett (2004) (see Section 2.1.3). Recall that the "standard" contract starts paying when an index ε falls below a specified strike ε^*. Once the index falls below a limit, the insured receives the maximum indemnity z. When the index falls between the strike and the limit, the contract pays a proportion of the maximum indemnity. A formal payoff schedule can be written as

$$I(\varepsilon|z,\varepsilon^*,\mu) \;=\; z \times \begin{cases} 0 & if\ \varepsilon > \varepsilon^*, \\ \dfrac{\varepsilon^* - \varepsilon}{\varepsilon^*(1-\mu)} & if\ \ \mu\varepsilon^* < \varepsilon \le \varepsilon^*, \\ 1 & if\ \varepsilon \le \mu\varepsilon^*, \end{cases} \tag{3.20}$$

where the parameter μ varies between 0 and 1, with the limiting case of 0 corresponding to the conventional proportional payoff with deductible, and 1 corresponding to a “lump-sum” payment once the contract is triggered regardless of the severity of the shortfall[15].

The index ε is constructed by regressing crop yields on a set of relevant weather variables (e.g. rainfall).

3.3.3 Weather and Yield Data

Table B-12 shows descriptive statistics of cumulative rainfall for the stages and counties selected, and table B-13 displays the descriptive statistics of the soybean yields[16].

3.4 Results

Figures A-11 through A-13 show probability distribution of yield and rainfall for the selected counties. The marginal distributions of soybean rainfall and yield are shown

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[15] The described contract protects against a shortfall of the index. With trivial modifications, the contract can be also constructed to protect against an excess of the index.
[16] For more details about data description refers to section 2.

in panels (a) and (b), respectively. The contour plots of the joint probability distributions of these variables are shown in panels (c). The estimated joint distributions reflect different degrees of dependence from a fairly pronounced in Pulaski to almost nonexistent in Phillips and Saint Francis.

The scatter plots of data and the index fitted following the approach of Vedenov and Barnett (2004) are shown in panels (d). In particular, the weather index was constructed using the parametric regressions between the soybean yield and the cumulative precipitation received during the corresponding phenological stages (table B-11). Insignificant variables were dropped from all models, and models with the highest adjusted R^2 were used to derive the weather indices for each county. The selected models and their corresponding statistics are presented in table B-14.

The goodness of fit of selected models ranges from 7% (Phillips and Saint Francis) to 20% (Pulaski). Linear models turned out to be the best in describing the relationship between the yields and cumulative rainfall. The parameters of the standard contracts constructed for the risk aversion parameter equal to 2 are reported in table B-15. [17] The corresponding indemnity functions were then used as initial guesses for the optimal contracts.

Shown in figures A-14 through A-16 are indemnities of the standard and optimal contracts for the selected counties and growth stages for the risk aversion parameter equal to 2. The optimal contracts have more irregular payoff structure, which

[17] The parameters calculated using risk aversion parameter γ of 1.5 and 3 turned out to be fairly similar and did not lead to qualitative changes in the results.

presumably better reflect the dependence between the yields and the rainfall. For each county, optimal contracts start paying indemnities earlier than do the standard contracts. In other words, the “strikes” of the optimal contracts are higher than those of standard contracts. Results suggest that the indemnities of optimal contracts are higher than those of standard contracts for any precipitation level.

The risk-reducing effectiveness of the standard and optimal contracts are compared in table B-16, which also reports premiums and liabilities for each contract under different risk aversion parameters. Results suggest the variation of certainty equivalent (ΔCER) is positive for both contracts, meaning that both reduce the risk exposure to lack-of-water- event. By construction, optimal contracts provide higher risk reduction than the standard contracts.

Results show that optimal contracts would offer higher levels of risk reduction than those offered by the standard contracts across counties. For instance, percentage change of CER between both contracts varies 0.35 to 0.80 percent when γ equal to 2 (see table B-16). It seems that counties (Pulaski) with higher degree of dependence between crop yield and rainfall (see figures A-11 through A-13) reach higher risk reduction than otherwise.

Results suggest that the optimal contracts also provide higher risk protection than that provided by the standard contracts when the risk aversion parameter increases. In fact, due to this higher protection, the indemnities of optimal contracts are higher than those of standard ones. As results, the former exhibits higher premiums and maximum indemnities. This pattern is hold across counties.

3.5 Conclusions

This essay proposes a procedure for deriving the optimal payoffs of index insurance contracts. We formulate the expected utility maximization problem with insurance à la Raviv (1979), where the indemnity function is conditional on realization of an index related but not equal to the actual loss. The solution to the problem is given by the Euler-Lagrange equations, which does not have a closed-form solution but can be solved numerically.

Soybean production in Arkansas is used as case study in order to test the validity of our approach. We apply the methodology for selected counties and phenological growth stages of soybean plant. We use the "standard" contracts in the sense of Vedenov and Barnett (2004) as a benchmark.

As expected, we find that the risk reduction provided by the optimal contracts is better than that of the standard contracts. The degree of improvement varies from county to county and seems to be affected by the dependence structure between the index and yield. We find that the optimal contract remains attractive to the insured under different risk aversion parameter. Although the optimal payoff structures can be derived for different risk aversion parameter, results cannot be generalized across counties.

Future research should concentrate on analyzing the behavior of the optimal contract payoffs under as general conditions as possible. Furthermore, the effect of different loading factors and preference specifications can be considered.

4. USING WEATHER DERIVATIVES IN WATER ALLOCATION DECISIONS: A CASE OF GUANAJUATO, MEXICO[18]

Extreme weather events such as droughts could affect the water supply, thus making the water allocation a major problem for agriculture. Although water markets can increase the efficiency of water use, the implementation of a price system for water is not always feasible because of its institutional, social and political connotations. In contrast, because of network externalities, water is typically managed as a natural monopoly and is underpriced by regulatory authorities.

In this essay, we look at potential improvements in water allocation strategies that could be achieved by using weather derivatives. A particularly interesting situation arises when there are two growing seasons, each characterized by different rainfall patterns but both dependent on irrigation. Weather derivatives can then incentivize adoption of allocation patterns that shift water allocations to the dry season from the wet season with the assumption that the wet season farmers can cope with the risk of water shortages by using weather derivatives. In other words, these financial instruments could not only smooth farmers' income, but might also induce an inter-temporal reallocation of water in irrigation districts, increasing the efficiency of water use in the long term.

[18] This section was part of the project "Effectiveness of Weather Derivatives as a Cross-Hedging Instrument against Climate Change: The Cases of Reservoir Water Allocation Management in Guanajuato, Mexico and Lambayeque, Peru" co-authored with Miriam Juarez-Torres and supported by the Inter-American Development Bank and the Latin American and Caribbean Environmental Economics Program.

The analytical model is applied to the Alto Rio Lerma Irrigation District (ARLID) in the state of Guanajuato in Mexico. Precipitation variability is of particular concern in this area. In the early 1970s, the average return period of extreme events was 12 years, in the early 2000s it was estimated to be 5 years (Groisman et al. 2005). Heavy rainfall events have increased during the rainy season, followed by more severe droughts in the dry season

By 2030 the population growth in Central Mexico is expected to place additional pressure on the hydrological regions of Lerma-Santiago-Pacifico and Valle de Mexico (Comision Nacional del Agua (National Water Commission) 2010). In fact, this system could collapse if precipitation levels in the main basins declines by 7 and 12% as the Mexican Institute of Water Technology expects and river flows diminish (Martínez Austria 2007). This situation may affect the way the Mexican water authority allocate the irrigation water for agricultural districts[19].

This section attempts to provide a modeling support for this task and incorporates an analysis of current legal and institutional framework, water tariff system, and irrigation infrastructure management based on water rights.

The rest of the section is organized as follows. The next subsection reviews the literature on water allocation and applications of weather derivatives in this context. Then, we describe the organization of irrigation districts in Mexico, current allocation policies, and agricultural activities of the region under consideration. The third subsection presents the general modeling approach proposed in this study. The fourth

[19] Ibid.

subsection describes data and model parameterization. Subsection 4.5 discusses the results, and assesses the effect of weather derivatives on water allocation patters. The last subsection concludes and discusses directions for future research.

4.1 Literature Review

Developing countries tend to be more vulnerable to extreme weather events such as intense rainfall or droughts because they are more dependent on agriculture (Barnett et al. 2008). Losses due to extreme weather as a proportion of GDP in these countries have been historically higher than in developed countries (Linnerooth-Bayer et al. 2005). In order to deal with the effects of the disasters, governments often have to redirect resources that would be otherwise used in activities such as education, health, etc. (Barnett et al. 2008).

An approach that emerged in recent years uses index insurance products as a way to alleviate the effect of weather events in developing countries (Barnett and Mahul 2007). Since the early 2000s, numerous cases have demonstrated possible benefits of weather index insurance in transferring weather risk to financial markets (Miranda and Vedenov 2001; World Bank 2005). These instruments can quickly provide financial resources to people at risk of natural disasters because loss assessment is not required.

High variability of precipitation and temperature in irrigation districts could exponentially increase competition for water among users (Valera and Institute 1992; Pereira et al. 2002). There is evidence that water authorities already have problems managing water supply, given the inflexibility of irrigation infrastructure and the instances of high and diversified demand (Brown and Carriquiry 2007)

In basins where irrigation districts are typically located, drought is a phenomenon that builds slowly over time based on shortages of runoffs from daily rainfall. Weather derivatives can transfer the additional costs associated with the provision of water during contingency situations to the financial markets.

There have been a few previous attempts to introduce the weather derivatives to the problem of water allocation. Zeuli and Skees (2005) designed a rainfall index contract for correcting the inefficiencies produced by water management systems in a drought situation. According to the authors, this instrument might reduce the uncertainty in supply and demand associated with the extremely conservative estimations of available water, which create inefficiencies in the allocation. The paper demonstrates that the index insurance creates an incentive for the authority to more accurately estimate the availability of water supply and demand sides. The authors did not investigate the effect of weather derivatives on water demand.

Leiva and Skees (2008) designed a financial instrument based on river flows accumulations as a market-based alternative to administrate water supply risk. Using the Rio Mayo irrigation system in northwestern Mexico as case study, the authors developed a stochastic model that incorporated water release rules and plating response functions. This model was used to evaluate the effectiveness of the proposed insurance scheme. The authors concluded that the insurance is a viable option from both the demand and supply sides, and it could be cost effective for mitigating water supply risk.

Block et al. (2008) observed that although river inflows are a direct measure of the available water in single-reservoir systems, it could be a poor option for hydrological systems with multiple reservoir systems and significant diversion of upstream flows. Systems of the latter type are predominant in Mexico and other Latin America countries, where irrigation districts are located within hydrological basins.

Brown and Carriquiry (2007) proposed an index insurance based on reservoir inflows to cover the financial needs of water supplier in situations where droughts are persistent[20]. They suggested that inflows have advantages over storage levels, because inflows to reservoir represent integration over space and time of the rainfall in a basin, while reservoir storage levels can be manipulated by the water authority.

Although the previous studies provide a useful insight on application of index insurance to water management and allocation problems, their value is somewhat limited in developing adaptation strategies for climate change. In particular, they do not incorporate the water allocation decision into a dynamic programming framework, where central planner makes intertemporal decisions.

The contribution of this essay is to incorporate new dimensions and challenges to the problem initially formulated by Leiva and Skees (2008). The district studies in this chapter currently allocates water collected during the rainy season across the wet and dry season farmers, but a rainfall shortfall could lead to insufficient allocations in both seasons to a particular detriment of dry season farmers. Addition of weather derivatives can improve the efficiency of water allocation by redirecting more water from wet to dry

[20] The authors based their study on the Angat reservoir in Manila, Philippines.

season and by managing wet farmers' increased risk exposure using weather derivatives[21].

4.2 Irrigation Districts in Mexico

CONAGUA[22], a technical and consultative agency, manages water resources in Mexico and carries out its functions through 13 river basin organizations. These organizations are defined according to the limits of the country's catchments[23] (Comision Nacional del Agua (National Water Commission) 2008)

By the second week in September of each year, CONAGUA determines water supply based on calculated basin's runoff generated from previous November to August and forecasted rainfall for September and October (Comision Nacional del Agua (National Water Commission) 2006). Once the annual volume of restitution run-off is calculated, CONAGUA provides water needed to carry our planting activities to irrigation districts. CONAGUA also determines fees based on the allocated volume of water and receives part of these fees as a recuperation payment (Kloezen and Garcés-Restrepo 1997).

The Lerma-Chapala river basin system, one of the river basin organizations, covers an area of 54,451 km^2 (around 3% of Mexico's territory) and crosses five states: Queretaro, Guanajuato, Michoacán, Mexico and Jalisco. It also serves nine irrigation districts and it is a source of water for 11 million people (Mestre 2001). The total runoff

[21] While Skees and Leiva (2005) include two productive cycles completely dependent on irrigation water, this paper more broadly considers two seasons in the case of Mexico: a Fall-Winter season totally dependent on irrigation and a Spring-Summer season depending mainly on rain, with minimum irrigation requirements.
[22] CONAGUA stands for Comisión Nacional del Agua (National Water Commission).
[23] The area drained by a river or body of water.

of this basin is 4,740 million cubic meters, of which on average 43% is available to the irrigation districts (Kloezen et al. 1997).

4.2.1 The Alto Rio Lerma Irrigation District (ARLID)

Among the nine irrigation districts, ARLID is the largest and accounts for approximately 44% of total water stored. The district receives an average rainfall of 670 mm during the wet season (from May to October) and 80 mm during the dry season (from November to April). Farmers within ARLID, which has an average temperature between 18 and 20°C and favorable soils, competitively produces a wide range of crops including grains, perennials and vegetables for export (Comision Nacional del Agua (National Water Commission) 2010).

Four dams provide surface water to ARLID. Tepuxtepec with 538 million cubic meters (Mm^3), Solis (1,217 Mm^3) and lake Yuriria (188 Mm^3) are interconnected by the Lerma river. The Purisima dam (196 Mm^3) is fed independently from the other three dams. All these dams provide a combined storage capacity of 2,138 Mm^3 serving 77,697 hectares.

For the sake of the operation and management, ARLID is organized into 11 modules, each managed by an individual water user association (WUA)[24] and its operations are based on a water rights system which awards property rights and assigns

[24] The 11 modules are: Acambaro, Salvatierra, Jaral del Progreso, Valle de Santiago, Cortazar, Salamanca, La Purisima, Irapuato, Abasolo, Corralejo, Huanimaro, and Pastor Ortiz.

detailed roles and responsibilities to modules.[25] Each module is entitled to a proportional share of water available for the irrigation district. These modules are also in charge of carrying out the final water allocation to its users and collecting fees from them.

A limited liability company (LLC) operates, manages, conserves, and maintains the irrigation network of ARLID — which includes primary canals, secondary canals, and drainage — and coordinates and monitors modules. The LLC plans delivery of water resources to the modules on a weekly basis and checks ditch tender reports at each module. Due to a growing water shortage and low average efficiency in transmission (65 percent), ARLID provides irrigation water for only 70 percent of the registered physical surface, where the property rights to water are concentrated.

ARLID's irrigation cycle starts in early November (when the hydrological cycle of the basin begins) and encompasses two production cycles (seasons). The dry season (mid-October to April) has been the priority of ARLID because it depends only on irrigation water. The wet season (May to mid-October) depends on both rainfall and irrigation for satisfying crops' water requirements.

4.2.2 The Allocation of Water for Irrigation

The volume of water that every module receives is based on an irrigation plan, which is the result of negotiations among the CONAGUA, the LLC and the modules (Kloezen et al. 1997). The water demand is projected based on the intended planting

[25] The water-rights system requires the concessionaire to pay for the volume of extracted water. The payment is set in relation to shortages in every region of the country and with different rates for every use. Industry and services pay more than urban users, while water for agriculture and farm-related activities is free. Thus, the fees that water users pay are related to the cost of operation fee for the irrigation district infrastructure and for the use of the main infrastructure (dams, channels, etc.) that CONAGUA operates.

estimates that LLC submits to CONAGUA. The latter makes up the difference between water demand and supply in this period, which is estimated based on the amount of water available at the end of the previous period.

This section will focus on the module Valle de Santiago (Valle), located in the Municipality of Valle de Santiago — the center of ARLID. The module is the third largest in terms of irrigated area and was selected because it is the most efficient (the transmission rate is 92 percent). Valle de Santiago has a well-organized ownership structure, which is extremely useful for the functioning of insurance schemes.

Valle has two main crop activities mainly irrigated by gravity and each one is carried out during different seasons. The farmers grow barley during the dry season and cultivate sorghum in the wet one.

4.3 General Modeling Approach

For the sake of the analysis, it is assumed that the same representative farmer cultivates both crops within the Valle module. This assumption has powerful implications because the irrigation districts are water rights systems that provide water users in the module with the allocation determined by their non-transferable water rights, established by law and linked to a particular piece of land property. While somewhat circumventing the issue of property right, this assumption simplifies the problem to an intertemporal reallocation of the same volume of water, which allows us to concentrate on improvements in the efficiency of water use.[26]

[26] Since water rights are non-transferable, the introduction of insurance does not automatically affect those. However, this assumption can be relaxed for a deeper analysis of the issue.

For the purposes of this study, the representative farmer is composed of the aggregate of all farmers located in the module Valle. This approach conceptualizes all producers located in the same region as a single farmer who makes production decisions and has water rights.

The analysis is carried out in two stages. The first stage considers a baseline scenario, which solves the dynamic water allocation model under uncertainty originally developed by Miranda and Fackler (2002). Based on stochastic prediction of the rainfall, the model characterizes authorities' optimal water allocation between the two crop seasons for a single representative farmer.

The second stage introduces a weather insurance contract that can compensate the farmer for precipitation shortfall (the contract payoff is tied to the amount of rainfall received during wet season). The optimal allocation strategy is now calculated by the authorities under the assumption that weather derivatives can partially substitute for decreased water allocation.

Farmer's welfare is evaluated in both scenarios by simulating multiple water consumption paths over the planning horizon and averaging the utilities over each path.

4.3.1 Baseline Water Allocation Model

Consider a water authority who acts as a central planner and allocates reservoir water among modules. The central planner makes decisions based on how much water is available, how many hectares will be planted, and how much water is needed per hectare. It is assumed that the central planner knows the structure of representative farmers' profit functions to the extent necessary to determine the amount of water needed for the farmer[27] to maximize his profits.

We assume that a representative farmer cultivates barley and sorghum. Barley is cultivated in the dry season and depends only on water allocated by the central planner. Sorghum, which is raised during the wet season, has two sources of water — rainfall and water allocated from the reservoir. Furthermore, we assume that each crop is only grown in its respective season, i.e. the farmer uses the same land for different crops in different seasons.

Let s_t be the amount of water available for a given module at the beginning of the year t. This water is held in an upstream reservoir and the water authority decides to release the amount w_t^{dry} of water during the dry season and w_t^{wet} during the wet season so that $0 \leq w_t^{dry} + w_t^{wet} \leq s_t$. During the rainy season, the reservoir levels are replenished by random inflows ε such as rainfall. Water available to the module at the beginning of period $t+1$ is then represented by a controlled Markov process [28]

$$s_{t+1} = s_t - w_t^{dry} - w_t^{wet} + \alpha\, \varepsilon_{t+1} \tag{4.1}$$

where α is a proportion of rainfall water attributable to this module. Thus, the distribution of the next period's state (the transition probability matrix), conditional on

[27] Henceforth, the terms "farmer" and "representative farmer" will be used interchangeably.
[28] Markov process is a random process in which the probability of any outcome in a given period depends only on the events in the previous period (no long-term memory). A controlled Markov process is a Markov process in which the outcome is also affected by a deterministic decision made each period.

all currently available information, depends on the current state (amount available to the module), the water allocated to the farmer, and the rainfall at the reservoir, namely,

(4.2) $$Pr\left(s_{t+1} = s' \mid s_t = s, w_t^{dry} = w^{dry}, w_t^{wet} = w^{wet}, \alpha\varepsilon_t\right) = Pr(\varepsilon_t = \varepsilon)$$

where the right hand side refers to a probability of a particular rainfall level that would result in s' in the next period, given that the level of reservoir in the current period is s and that the authority has allocated w^{dry} and w^{wet}.

We assume that the representative farmer is risk-averse and his preferences can be represented by a utility function $u(\cdot)$ defined over the profits π, which in turn depend on the total water received for crop production.

In particular,

(4.3) $$\pi_t^{dry} = P_{dry} y_t^{dry} - P_w w_t^{dry}$$

(4.4) $$\pi_t^{wet} = P_{wet} y_t^{wet} - P_w w_t^{wet}$$

where $P_{(\cdot)}$ represents the output prices for the wet and dry-season crops[29], and the price of irrigation water is denoted by P_w. This price represents the cost of water allocation that must be paid by the farmers and is assumed to be constant over time.[30]

We assume that the farmer uses a divisible technology to produce $y_t^{(\cdot)}$, which is characterized by a quadratic production function. For the purposes of this analysis, we assume that the crop production functions $y_t^{(\cdot)}$ depend explicitly only on the total amount of water received either from irrigation or from rainfall. Thus, we consider both sources of water as perfect substitutes. Specifically, the production functions are described by

$$y_t^{dry} = a_0 + a_1 w_t^{dry} + a_2\left(w_t^{dry}\right)^2 \tag{4.5}$$

$$y_t^{wet} = b_0 + b_1(w_t^{wet} + x_t) + b_2(w_t^{wet} + x_t)^2 \tag{4.6}$$

[29] Each farmer is assumed to be small enough so that input and output prices are not affected by farmer's decisions.

[30] If it were not constant, it would be a decision variable for the planner. In that case the profit maximization problem for each farmer would give us water demand as a function of water price. Thus, the dynamic optimal allocation would depend on the optimal path of the water price established by the planner. Analysis of such a model can be carried out within the presented framework, but is outside of the scope of this Essay.

where x_t is the rainfall level observed in the module. Because the dynamic model evaluates the marginal effects of water on crop yields, we assume that the farmer has already decided on the amount of other inputs[31].

The objective of the central planner is then to seek the optimal water allocation strategy $\left(\widetilde{w}_t^{dry}, \widetilde{w}_t^{wet}\right)$ that prescribes the actions $w_t^{dry} = \widetilde{w}_t^{dry}(s_t)$ and $w_t^{wet} = \widetilde{w}_t^{wet}(s_t)$ that should be taken in each state at each point in time so as to maximize the total expected discounted value of farmer's utility over an infinite lifetime. Namely, we solve

$$\text{(4.7)} \qquad \max_{w_t^{dry};\, w_t^{wet}} \quad E_0 \sum_{t=0}^{\infty} \delta^t \left[u\left(\pi(w_t^{dry})\right) + u(\pi(w_t^{wet}))\right]$$

subject to the transition equation (4.1), where δ is the discount factor. This is a discrete time, discrete state Markov decision problem and can be analyzed using dynamic programming methods based on Bellman's Principle of Optimality (Miranda and Fackler 2002).

In particular, this principle implies that the problem in equation (4.7) can be rewritten as a condition that the value function $V(s)$, which specifies the maximum

[31] The constant terms in equations (4.5) and (4.6) may represent conventional inputs (e.g., labor and capital) evaluated at "optimal" levels. However, the input usage depends on the weather because of pest intensity. Thus, it could be the case that input costs increase over time, especially if extreme weather events occur. In this study, we are not considering how this input issue affects the farmer's profit and welfare.

attainable sum of current and expected future rewards given that the current reservoir level is s, should satisfy the Bellman equation

$$(4.8) \qquad V(s) = \max_{w^{dry};\, w^{wet}} \left\{ u\big(\pi(w_t^{dry})\big) + Eu(\pi(w_t^{wet})) + \delta EV(s') \right\}$$

where $s' = s - w^{dry} - w^{wet} + \alpha\varepsilon'$ as per (4.1). Note that equation (4.8) includes the expected value of the current-period utility of wet season profits, because the latter depends on random rainfall during the season.

4.3.2 Incorporating Weather Derivatives

Literature on weather derivatives utilizes a variety of indexes such as cumulative rainfall over a defined period, heating degree days, or cooling degree days (Brockett et al. 2009). Most contracts are structured as put (call) options which pay an indemnity if a specific weather variable falls below (rises above) a pre-specified level (Martin et al. 2001; Turvey 2001).

For purposes of this study, we use a contract structure based on V&B. Namely, we define the contract as a put option without imposing any limits on contract payoff (the "fully proportional" contract of V&B). This contract pays nothing as long as the index is in the acceptable range and pays a proportional indemnity whenever the index falls below a specified strike.

The cumulative rainfall in the module is used as the index, and the strike is set equal to the expected rainfall, $E(x)$, in the module multiplied by the coverage level θ, viz.

(4.9) $$strike = \theta E(x)$$

The profit at the strike

(4.10) $$\bar{\pi} = P_{wet}\,[\,b_0 + b_1 strike + b_2 strike^2\,]$$

is then used to set the level of protection provided, so that for a given amount w^{wet} of water allocated, the contract pays an indemnity according to the schedule

(4.11) $$I(x \mid \theta, w^{wet}) = \begin{cases} \bar{\pi} - \pi(w^{wet}) & if\ \ x \leq strike \\ 0 & if\ \ x > strike \end{cases}$$

Figure A-17 illustrates the indemnity function in equation (4.11). Without loss of generality, we assume that the premium is actuarially-fair and is set equal to the expected payoff of the contract[32]

(4.12) $$P(\theta, w^{wet}) = E[I(x \mid \theta, w^{wet})].$$

[32] Loaded premiums can also be considered within the same framework.

Figure A-18 shows the total profits of the wet-season farmer with and without the weather derivative, with the vertical distance between both functions representing the indemnity payment.

The insurance payment modifies the profit in the wet season, but otherwise the water allocation model in equation (4.7) remains the same. The optimal water allocation policies $\widetilde{w}_t^{dry}(s)$ and $\widetilde{w}_t^{wet}(s)$ in the presence of insurance can be again calculated by solving the Bellman equation (4.8) with the appropriate modifications.

4.3.3 Numerical Solution of Bellman Equation

Generally, the Bellman equation (4.8) does not have a closed form solution, but can be solved numerically. In order to do so, we assume that the reservoir level s takes only discrete value in the state space S, which enumerates all the states attainable by the system. Similarly, the action space W enumerates all possible (discrete) actions or water allocations $w^{dry}$and w^{wet} that may be taken by the water authority. Both spaces are assumed to be finite and the water allocation decisions should satisfy the conditions

$$w^{dry} \leq s \tag{4.13}$$

$$w^{wet} \leq s - w^{dry} \tag{4.14}$$

The probability distributions of rainfall $x \in X$ in the module and the total annual precipitation $\varepsilon \in \mathrm{H}$ at the reservoir are also discretized over the respective domains.

Assuming the infinite horizon, the Bellman equation (4.8) can be then expressed as a functional fixed-point equation

$$(4.15)\quad V(s) = \max_{w^{dry};\, w^{wet}} \left\{ u(\pi(w_t^{dry})) + \sum_{x \in X} p_X(x) u(\pi(w_t^{wet})) + \delta \sum_{\varepsilon \in \mathrm{H}} P(s'|s, w, \varepsilon) V(s') \right\}$$

where the expectations in equation (4.8) are replaced by their discrete form. If the discount factor δ is less than one and the utility functions for both wet and dry seasons are bounded, then by the Contraction Mapping Theorem, this Bellman equation possesses a unique solution.

The policy iteration algorithm (Miranda and Fackler 2002) can be used to solve equation (4.15) numerically. To implement this iteration process, we recast the Bellman fixed-point equation as a root finding problem

$$(4.16)\quad \begin{aligned} V(s) \; - \max_{w^{dry};\, w^{wet}} &\left\{ u(\pi(w_t^{dry})) + \sum_{x \in X} p_X(x) u(\pi(w_t^{wet})) \right. \\ &\left. + \delta \sum_{\varepsilon \in E} P(s'|s, w, \varepsilon) V(s') \right\} = 0. \end{aligned}$$

By the Envelope Theorem, the derivative of the left-hand side with respect to V is $I - \delta P(w)$, which leads to the iteration rule

$$V \leftarrow [I - \delta P(w)]^{-1}\left[u(\pi(w_t^{dry})) + \sum_{x \in X} p_X(x) u(\pi(w_t^{wet}))\right] \quad (4.17)$$

The solution steps would then involve making a guess as to the initial value of V, solving the optimization problem in equation (4.15) to find the optimal w^{dry} and w^{wet} for the current value of V, and updating V as per equation (4.17). The solution is found when the convergence is achieved. Since the total number of states is finite, the total number of admissible policies is also finite. The policy iteration algorithm will terminate after finitely many iterations with an exact optimal solution.

4.3.4 Dynamic Simulation Analysis

The optimal policy functions w^{dry} and w^{wet} provide rules as to how the water authority should allocate water given the reservoir level. The dynamics of the model can be then studied using the dynamic path and the steady-state analysis.

The dynamic path analysis evaluates the expected path followed by both the reservoir level and the optimal irrigation policy over time starting from an initial value of the reservoir level. The expectation is taken by averaging a large number of paths generated by the Monte Carlo method based on the optimal probability transition matrix, the optimal policy, and the vector of initial reservoir levels.

The steady-state distribution is obtained as the limit of the transition probability matrices $Q_t = Pr(\varepsilon_t = \varepsilon)$ as $t \to \infty$.

4.4 Data and Problem Parameterization

Before calculating the optimal policies, we need to specify the functional forms of the utility, parameterize the profit and productions functions (4.3)-(4.6), and discretize the distributions of the rainfall at the module and at the reservoir.

4.4.1 Utility Function

We assume that the representative farmer's preference is described by the Constant Relative Risk Aversion (CRRA) utility function

$$u(z,\gamma) = \frac{z^{1-\gamma}}{1-\gamma} \tag{4.18}$$

where γ is the risk aversion parameter that reflects producers' willingness to forgo a certain amount of risk-premium in exchange for elimination of uncertainty. Financial and economics literature suggest the use of CRRA to represent the agent's preferences (Boulier et al. 2001; Cairns et al. 2006). Brandt et al. (2009) pointed out that CRRA possesses desirable properties such as double differentiability and continuity that increases the efficiency of numerical optimization algorithms, while incorporating preferences toward higher-order moments in a simpler way.

4.4.2 Yield Data

Historical module-level yield series were collected from the SIAP (2013)[33]. They represent data on sorghum and barley crops in module Valle from 1985 to 2011.

[33] SIAP stands for Sistema de Información Agropecuaria y Pesquera (Information System for Agricultural and Fisheries).

Historical water allocations for both crops were provided by Comision Nacional del Agua (National Water Commission) (2010). Table B-17 displays the range of available data, and descriptive statistics for both crop yields and irrigation water. The average barley and sorghum yields during the analyzed period were 5.06 and 6.66 tons/ha, respectively. Historically, barley farmers have received more water than sorghum farmers (5.77 versus 2.92 TCM/ha)[34].

Several unit root tests such as KPPS test, Dickey-Fuller and Phillip-Perron were performed to detect whether yields and historical water allocations have stochastic trends. All these tests agree that yield series are trend stationary, while all other series are stationary.[35]

New barley and sorghum varieties have introduced in Valle de Santiago over time. These improvements make crop yields incomparable across years. To address this problem, yields were detrended following Vedenov et al. (2006). In particular, a piecewise log-linear trend equation was fitted for each yield series. The general form of the estimated equation is

$$\text{(4.19)} \quad \ln(y_t) = a_0 + a_1 t + b_1(t - t_1)d_1 + b_2(t - t_2)d_2 + \cdots + b_k(t - t_k)d_k + u$$

[34] TCM stands for Thousands of Cubit Meter. 1 TCM is equivalent to 100 mm per hectare.

[35] Unit root results are available upon request.

where $\ln(y_t)$ is the natural logarithm of yield in year t; t_i, for $i = 1, \ldots, k$, represent the years at which the slope of the equation changes, d_i are dummy variables which are equal to 1 for all observations such that $t_i \leq t$, and 0 otherwise and u is the error term.

We used a nonlinear least square procedure to estimate model equation (4.19). This method allowed us to find the points t_i that yield the best fitting model. Table B-18 shows the best fit models and their statistics.

Results suggest that both barley and sorghum yields exhibit two trend breaks over the time period analyzed. Based on these estimations, the detrended yields were then calculated as:

$$y_t^{det} = y_t \frac{y_{2011}^{tr}}{y_t^{tr}} \tag{4.20}$$

where y_t^{tr} is the predicted trend in year t. Figure A-19 and A-20 display historical and detrended yields for barley and sorghum, respectively. Barley yields exhibit an overall upward trend, while sorghum seems to have a period of downward trend between 1994 and 2000.

4.4.3 Rainfall Data

We collected daily precipitation data from the National Meteorological System in Mexico (2011) for two weather stations corresponding to module Valle (municipio of

Valle de Santiago) [36] and dam Solis (municipio of Acambaro)[37]. Table B-19 displays the descriptive statistics for monthly precipitation data at both stations. Unit root tests suggest that all series are stationary.[38] Data show that precipitation at dam Solis has a higher mean and lower variability than that at Valle.

The policy iteration algorithm requires the computation of expectations in a numerically practical way. The expected value of wet-season utility (see equations 4.15 and 4.17) requires estimating the probability distribution $p_x(x)$ of rainfall in Valle. Given that sorghum is grown throughout the wet season (planted mainly in April and harvested in June), we calculated the cumulative rainfall over this period. The continuous distribution was replaced by a discrete approximant using the gamma quadrature. In particular, we fit gamma distribution to the observations of the cumulative rainfall for module Valle and then used the estimated parameters to generate quadrature nodes and weights. Figure A-21 shows the estimated probability distribution and the histogram of the data.

In a similar way, the discretized distribution of total annual precipitation $\boldsymbol{\varepsilon}$ at dam Solis was obtained by fitting a gamma distribution to the cumulative precipitation observed from November in year t through October $t+1$. The estimated parameters were then used to determine the quadrature nodes and weights. The latter were used in

[36] Municipio is an administrative division in Mexico similar to county in USA.
[37] Rainfall data were collected from weather station 11079 for Valle de Santiago and from 11076 for Dam Solis.
[38] Unit root results are available upon request.

constructing the transition probability matrix $P(s'|s, w, \varepsilon)$ in equations (4.15) and (4.17). Figure A-22 shows the estimated probability distribution and the histogram of the data.

4.4.4 Parameterization

Table B-20 summarizes all parameter values used in the analysis. The collected data were used to parameterize the functional forms in equations (4.3)-(4.6). Parameters of the production function were estimated using the OLS procedure. For barley production function, we ran the regression between barley yields versus water allocated during the dry season (see equation 4.5). Results suggest that all parameters were statistically significant at 2% level, except for the constant term.

For sorghum production function (see equation 4.6), we ran the regression between sorghum yield and water allocated during the wet season plus precipitation received from April to June. Results suggest that all parameters are statistically significant at 8% level, except for the constant term.

The prices of crop yields were set equal to their levels in 2010, the last data available, and expressed in pesos per tons. We allow $s_t \in [0, \bar{S}]$, where $\bar{S}$ is the maximum possible allocation level set to 10 TCM/ha. The proportion of rainfall water attributable to module Valle α is set equal to 0.11, which is the water right assigned by ARLID to module Valle. The risk aversion parameter ranges from 1 to 3 (as per Myers (1989) and Wang et al. (1998)). We set the coverage level θ equal to 100% and the expected rainfall in module Valle was calculated using the corresponding probability distribution. The profit at strike level was calculated using equation (4.10). Finally, all prices are expressed relative to price of barley.

4.5 Results and Sensitivity Analysis

4.5.1 Water Allocation Strategies

Figure A-23 shows the optimal water allocation policy without insurance for both crops: barley (dry season) and sorghum (wet season). The results suggest that regardless of water available, central planner allocates more water to the dry-season farmers than to the wet-season farmers. Furthermore, when there is not enough water to satisfy farmers' needs in both seasons, it is optimal not to allocate water to wet season farmers. As expected, the more water is available, the more water is allocated in both seasons. These policy functions prescribe the optimal action to be taken at a given reservoir level.

Figure A-24 displays the expected paths of the reservoir level and the optimal water allocation levels simulated over a 50 year horizon from the initial reservoir level equal to zero. Results suggest that in the long-run the reservoir level averages 1.41 TCM/ha and this level is reached in 10 years. Given that level, the optimal water allocation policy in the long-run is around 0.6 and 0.3 TCM/ha for the dry and wet season. The steady-state distribution also suggests that in the long-run the most common state (i.e. the most frequent reservoir level) would be around 1.4 TCM/ha (figure A-25).

Figure A-26 shows the optimal irrigation policy with insurance for both crops: barley (dry season) and sorghum (wet season). Here, the central planner still allocates more water to dry season farmers than to wet season farmers regardless of the level of reservoir. The dry-season farmers receive more water with the insurance than without it. The situation is completely different for the wet-season farmers. Not only the central planner forgoes the needs of the wet-season farmers when there is not enough water, he now does not starts allocating water to them until the reservoir reaches higher levels than in the scenario without the insurance.

In the long-run, the inclusion of the weather index insurance has also an impact on the expected path followed by the reservoir water and the optimal irrigation policy (figures A-27 and A-28). While the steady-state reservoir level remains relatively unchanged (1.42 TCM/ha vs. 1.41 TCM/ha without the insurance), the steady-state allocations are redistributed from wet- to dry-season farmers. The former receive only 0.05 TCM/ha vs. 0.20 TCM/ha without insurance, while the latter receive 0.76 TCM/ha vs. 0.65 TCM/ha without insurance.

Given a rather small change in the steady-state level of the reservoir from one scenario to the other, it seems that the availability of weather derivatives induces a substitution effect between the two seasons in the sense that the central planner now allocates an additional amount of water to dry season farmers — the amount that previously was a part of wet season farmers' allocation.

Note that despite redistribution of water allocation, the overall welfare of the farmers increase. Figure A-29 shows the value function with and without the insurance. This value function represents the maximum sum of current and expected future rewards attained by both wet- and dry- season farmers, given a specific reservoir level. Results suggest that regardless of water available, both wet-and-dry season farmers reach higher rewards when the central planner provide the weather index insurance to wet season farmers. As expected, the more water is available, the higher the reward is.

The availability of weather index insurance results in a better water allocation policy. The exchange of water between both seasons allows the representative farmer to reach higher levels of utility. In order to verify the robustness of this result, we perform sensitivity analysis by varying some of the model parameters.

4.5.2 Sensitivity to Coverage Level

Figure A-30 shows the optimal water allocation for dry and wet season farmers with different coverage levels. Regardless of the coverage levels, the substitution effect is still present albeit at a diminishing scale. Intuitively, coverage levels below 100% do not fully protect wet-season farmers against rainfall shortfall, and therefore the central planner reallocates smaller proportion of their water share to the dry-season farmers. The overall welfare of both farmers is still higher with insurance than without it even at the coverage levels below 100% (figure A-31). As expected, the higher the coverage level is, the higher the benefit of the insurance.

4.5.3 Sensitivity to Prices

Next, we compare the optimal water allocation policies with and without insurance for different ratios P_{wet}/P_{dry} between the prices of sorghum (wet season) and barley (dry season). The baseline case corresponds to the ratio of 1.15 (table B-20), which we varied up and down by 25%. Figures A-32 and A-33 show the resulting optimal water allocation policies and figure A-34 the value functions.

Results suggest that higher relative prices of wet-season crop result in a reallocation effect either with or without the weather derivative. The central planner allocates less water to wet-season farmers than to dry- season farmers when relative prices of wet-season crop increases (see figure A-32 and A-33), as losses of the dry-season crop have higher impact on the overall welfare. In fact, the relative price of wet-season crop increases the overall welfare, represented by the value function, increases (see figure A-34).

As expected, when the dry-season crop is relatively more profitable, the opposite holds true.

4.5.4 Effect of the Relative Risk Aversion Parameter

Next, we compare the optimal water allocation policies with and without insurance for different the risk aversion parameter γ. Figure A-35 through A-37 show the resulting optimal water allocation policies and corresponding value functions for values of risk aversion parameter ranging from 1.5 to 2.5

Results suggest that when farmers are more risk averse, the central planner tends to allocate less water to dry-season crop than to wet-season crop. In this case, the central planner would keep the wet-season farmer's share, since they prefer certain allocation from the reservoir to the uncertainty of the rainfall even with the insurance (see figures A-35 and A-36). The opposite hold true when the farmers become less risk averse. Figure A-37 shows the overall welfare under different risk aversion parameter.

4.5.5 Sensitivity to Distribution of Rainfall

Finally, we consider the effect of changes in precipitation pattern on the water allocation policies. Namely, we vary the shape parameters α of the gamma distributions used to model the rainfall in module Valle and dam Solis. Figure 38 displays variations in the corresponding distributions when the shape parameter decreases by 50% relative to the maximum likelihood estimator (table B-20). The scale parameter β is kept constant at its estimated. Generally speaking, lower values of the shape parameter shifts the rainfall distributions to the left thus increasing the probability of low precipitation levels.

Figures A-39 and A-40 shows the optimal irrigation policy for both seasons assuming that the shape parameter of the rainfall distribution in module Valle decreases by 50%. This corresponds to the situation when the farmers receive less rainfall, even though the amount of water at reservoir remains the same. Without insurance, the central planner increases water allocation to the wet-season farmers, since they no longer can rely on rainfall to meet crop water needs. When the insurance is available, central planner increases the amount of water allocated to dry season farmers, but does not change the amount assigned to wet-season farmers. In this case, the weather insurance effectively compensates the wet-season farmers for the loss of rainfall.

Similarly, figure A-41 shows the optimal irrigation policy for both seasons assuming that the shape parameter of the rainfall probability distribution at dam Solis decreases by 50%. This corresponds to the situation of normal precipitation, but lower overall amount of water available for irrigation. As expected, when the precipitation level at dam Solis is low, the central planner allocates less water in both seasons relative to the baseline scenario. Availability of the proposed insurance scheme, makes it optimal to allocate no water at all to wet season farmers while increasing the overall welfare.

4.6 Conclusions

This essay introduces a weather-based insurance contract as a tool for managing water supply risk in the Alto Rio Lerma Irrigation District (ARLID) in the state of Guanajuato in Mexico. The analysis is performed from the standpoint of a central planner allocating water for irrigation between the representative farmers operating during dry and wet season.

We look at the potential improvement in water allocation strategies that can be achieved using such an insurance tool. The results suggest that these financial instruments can incentivize the adoption of new allocation patterns which considers more generous allocations for dry-season farmers while providing reduced allocations for wet-season farmers. The latter are assume to be able to cope with the risk of rainfall shortages by using the derivatives. At the same time, the overall welfare of the farmers increases.

As expected, higher coverage levels result in higher amounts of water reallocated from the wet- to dry-season farmers, although higher degree of risk aversion reduces the attractiveness of the insurance. An interesting result is that the weather derivatives can help to mitigate the effect of lower water availability (e.g. due to less frequent rainfall at the reservoir) as long as the distribution of the rainfall at the farmer location remains the same.

While the analysis is based on a somewhat stylized model, it can be expanded in a straightforward way to incorporate more realistic features. For instance, crop productivity may depend on other factors (e.g. labor, capital) rather than total water received only. The insurance scheme can also be designed for both lack and excess rainfall.

This is an exploratory study about how a new insurance scheme could improve water allocation policies. We assume that farmers have water rights, but it is the central planner that manages them in order to maximize the total welfare. In reality, the proposed insurance scheme would require modifications of the legal arrangements underlying these rights.

Nevertheless, the adoption of water allocation policies supported by weather derivatives can be an effective strategy of water allocation that could be utilized in irrigation districts. Modules of irrigation districts are potential targets for deployment of these instruments. Governments could support the operations of this scheme as an integral strategy against emergencies and disasters. Also they could assist modules in developing the institutional features needed for weather insurance such as legal and regulatory framework, data collection and management, training of insurance suppliers and consumer education. Once weather insurance is working, it is likely to be an effective tool in improving water management in irrigation districts.

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Figure A-1. Suggested classification of soil types for soybean production

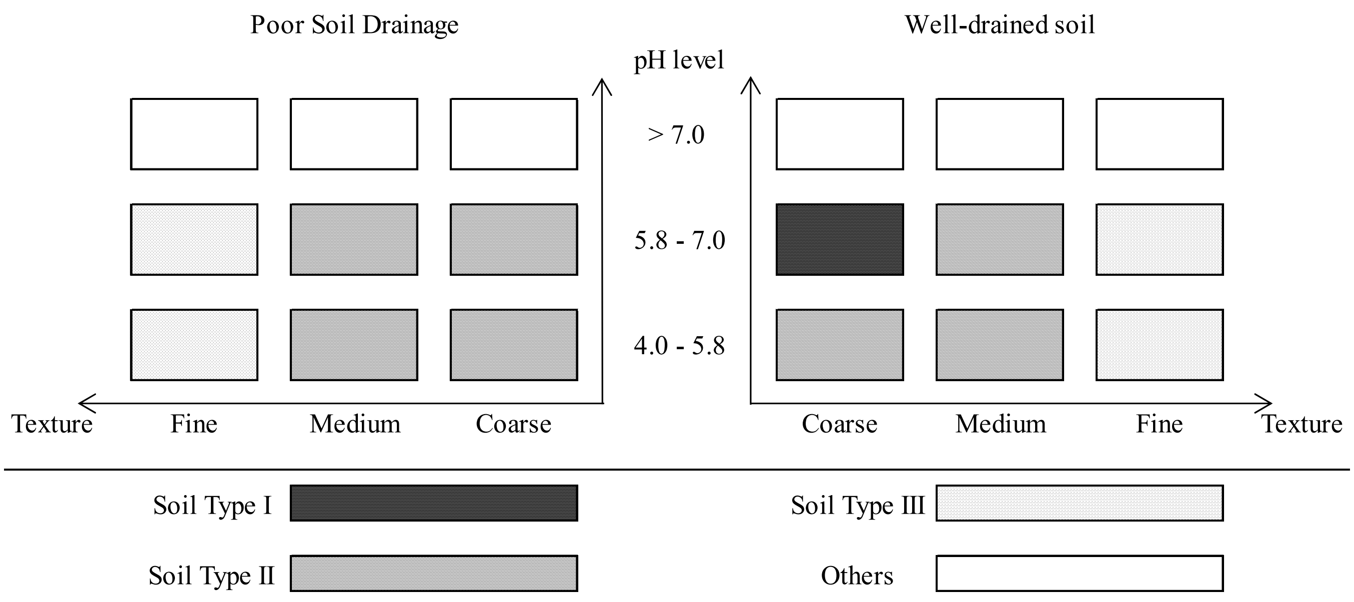


Figure A-2. Payoff structure of the agronomic contract with different cap factor α

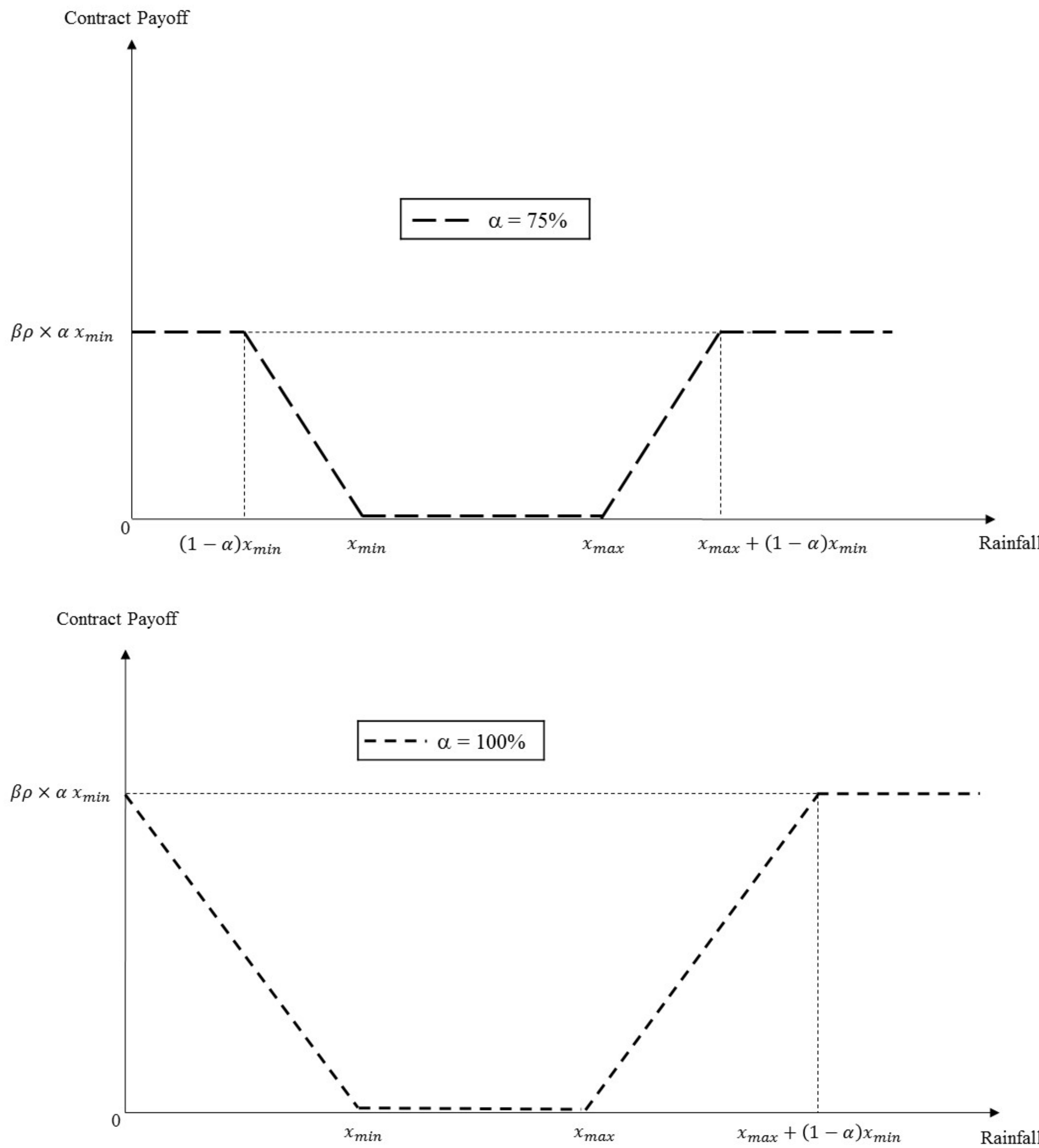


Figure A-3. Payoff structure of the agronomic contract with different scale factor β

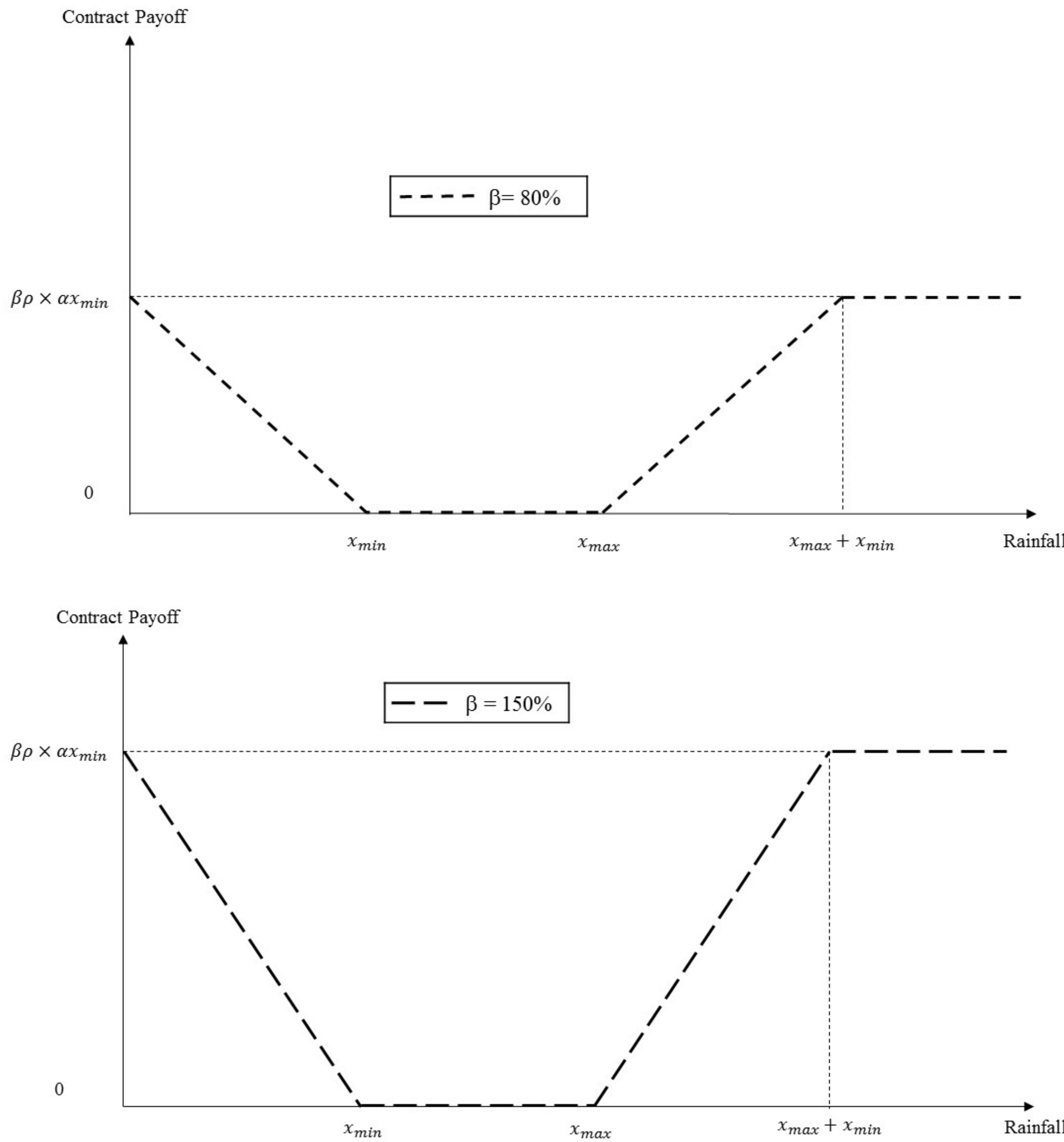


Figure A-4. Locations of counties selected for analysis and corresponding weather stations in Arkansas state

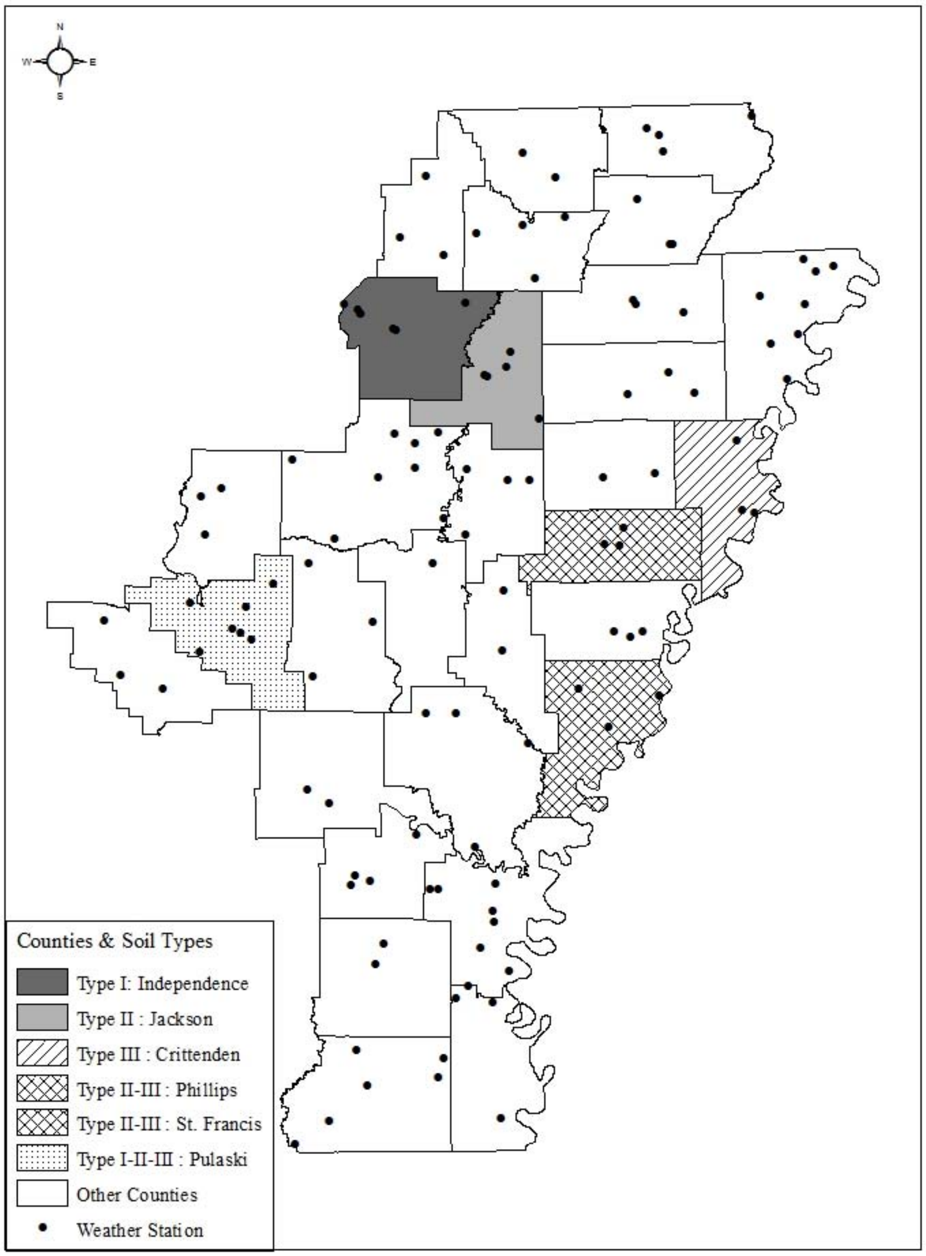


Figure A-5. Joint probability distributions of rainfall and soybean yields, Independence County

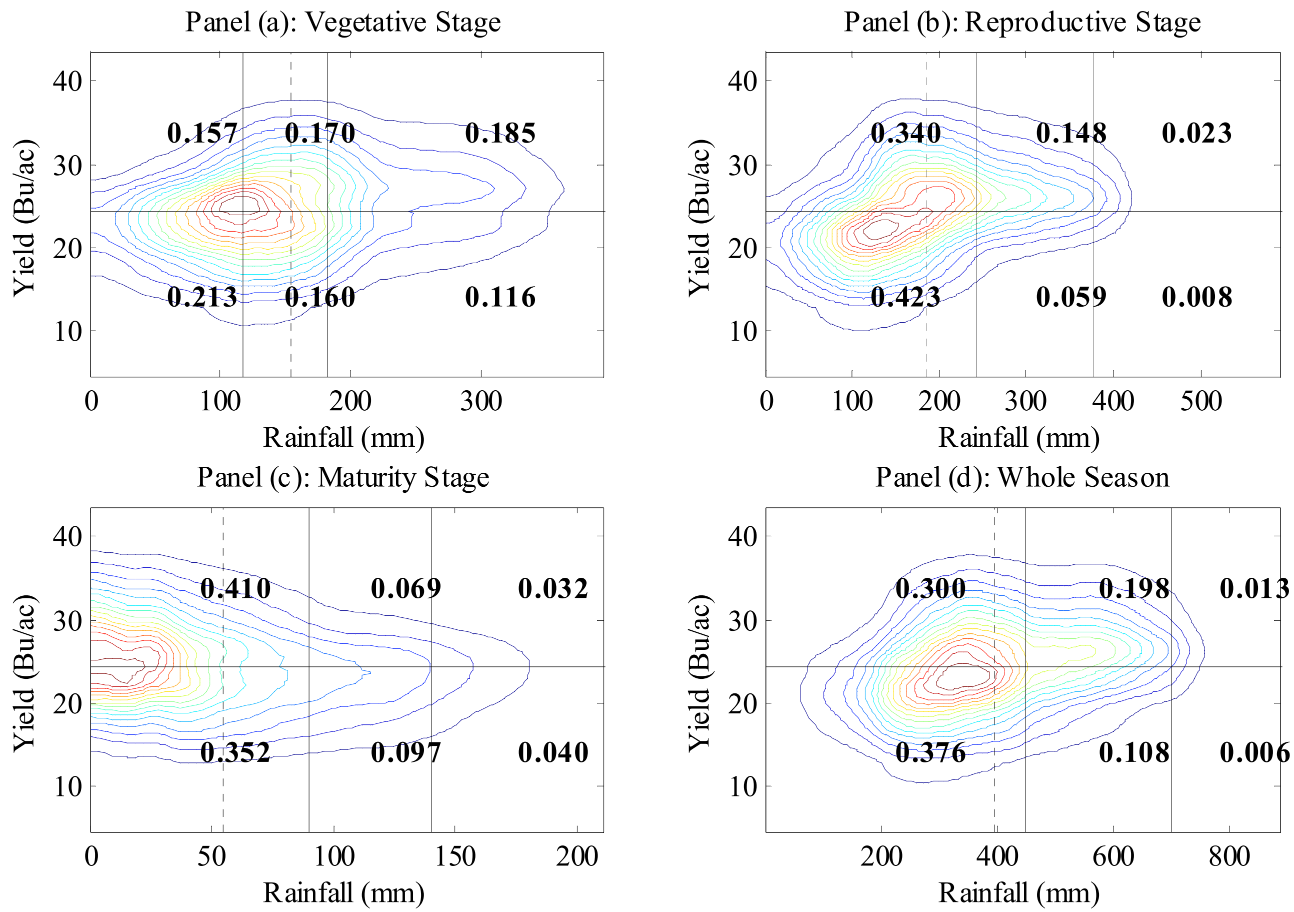


Figure A-6. Joint probability distributions of rainfall and soybean yields, Jackson County

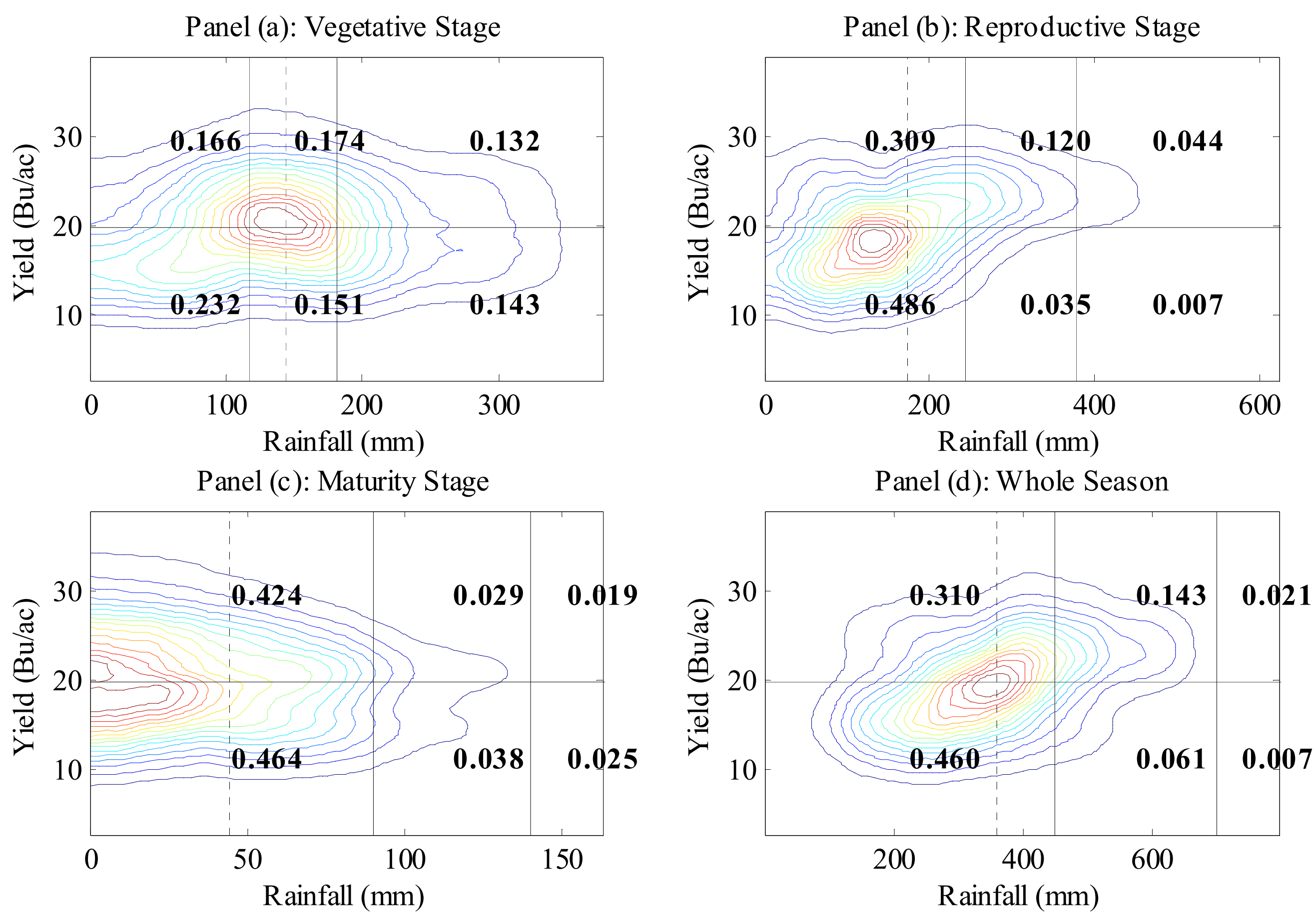


Figure A-7. Joint probability distributions of rainfall and soybean yield, Crittenden County

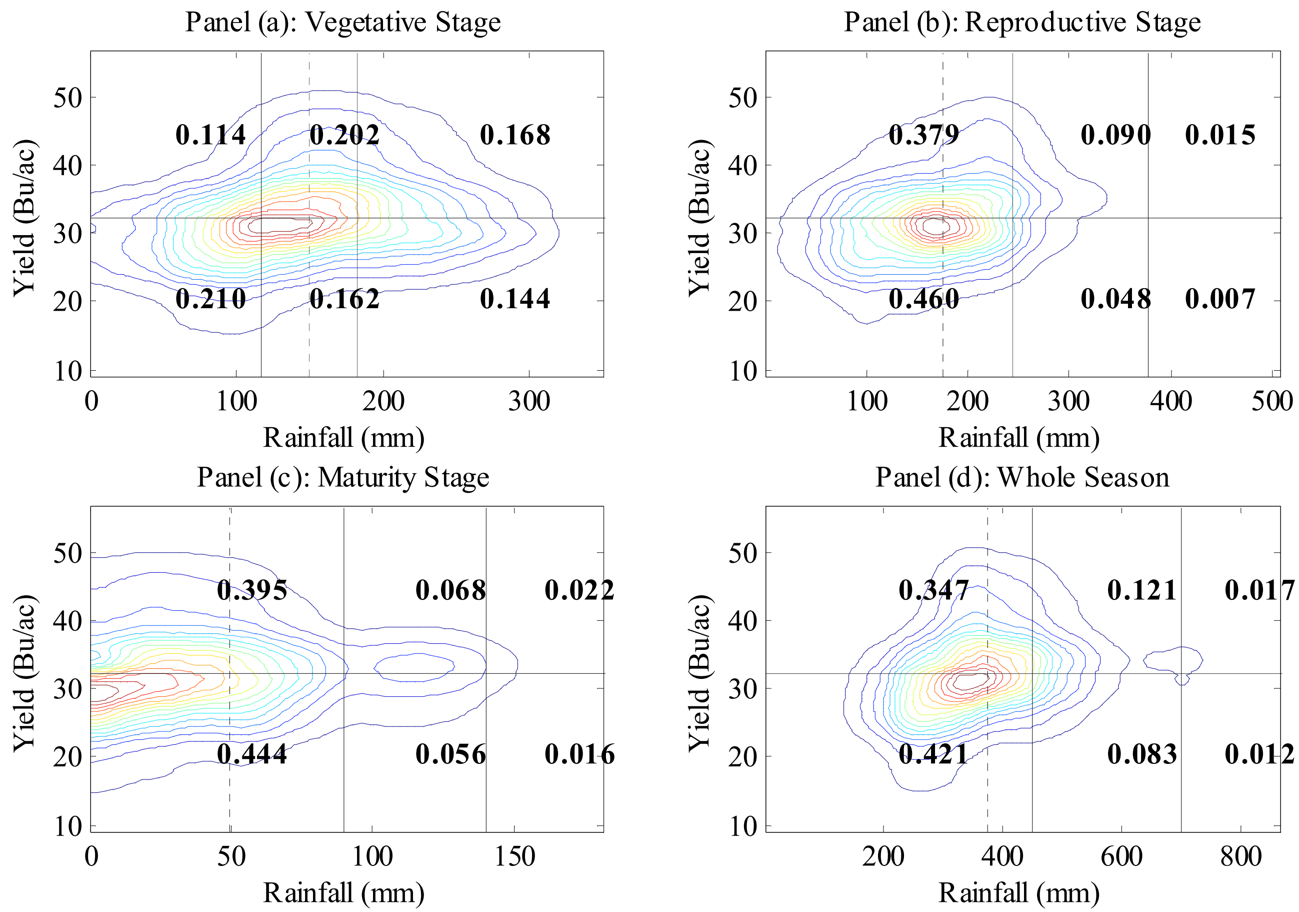


Figure A-8. Joint probability distributions of rainfall and soybean yield, Phillips County

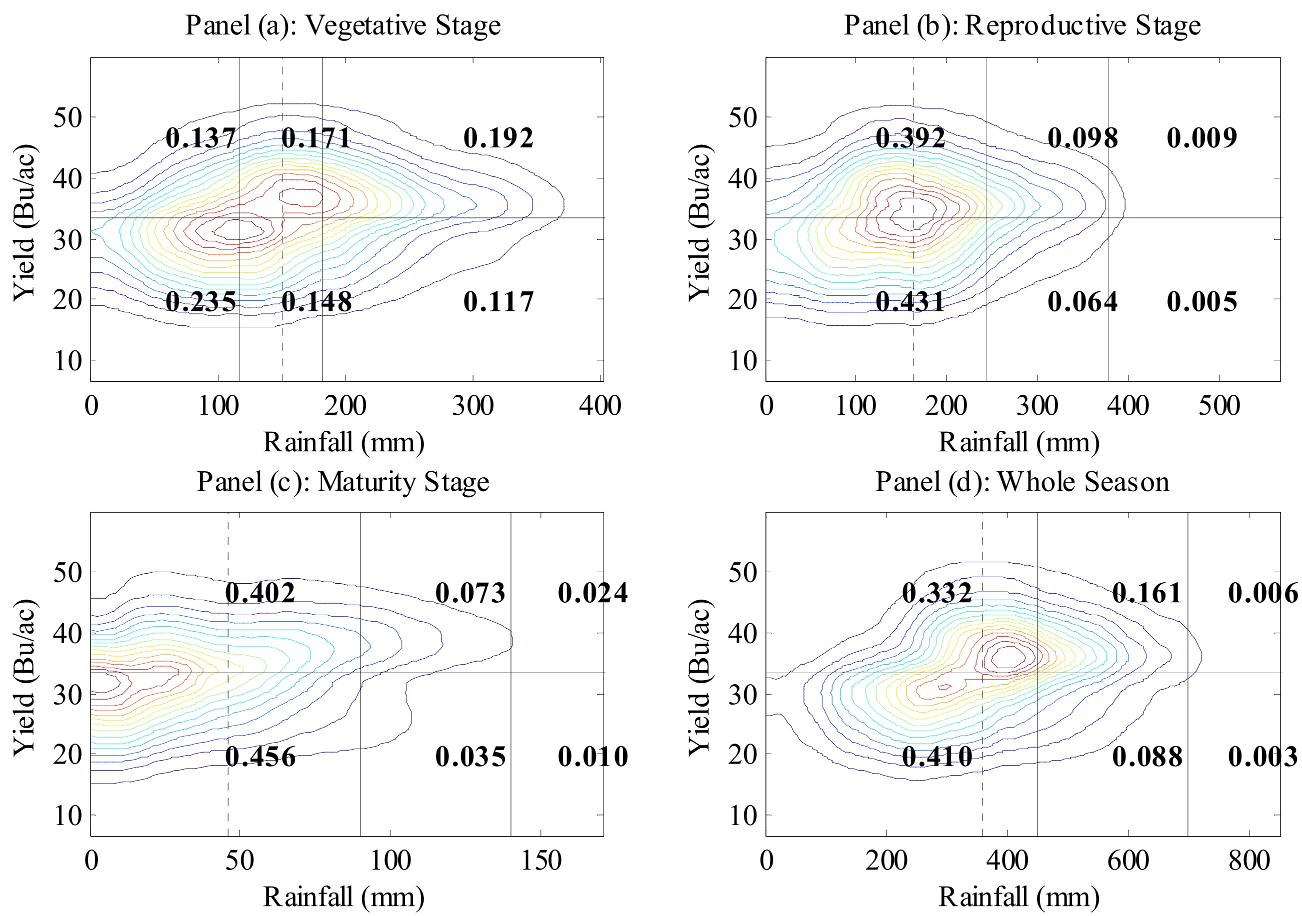


Figure A-9. Joint probability distributions of rainfall and soybean yield, Saint Francis County

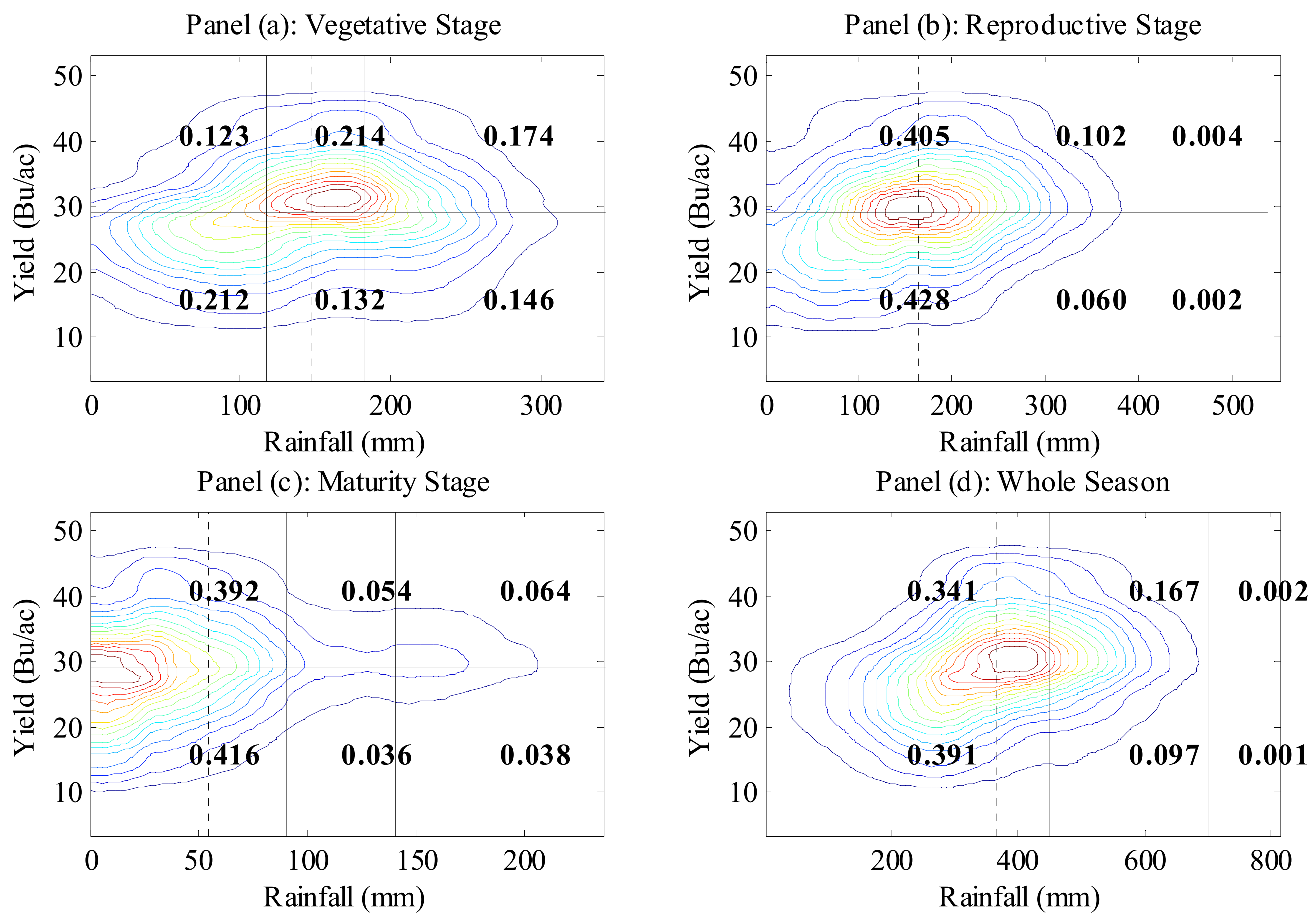


Figure A-10. Joint probability distributions of rainfall and soybean yield, Pulaski County

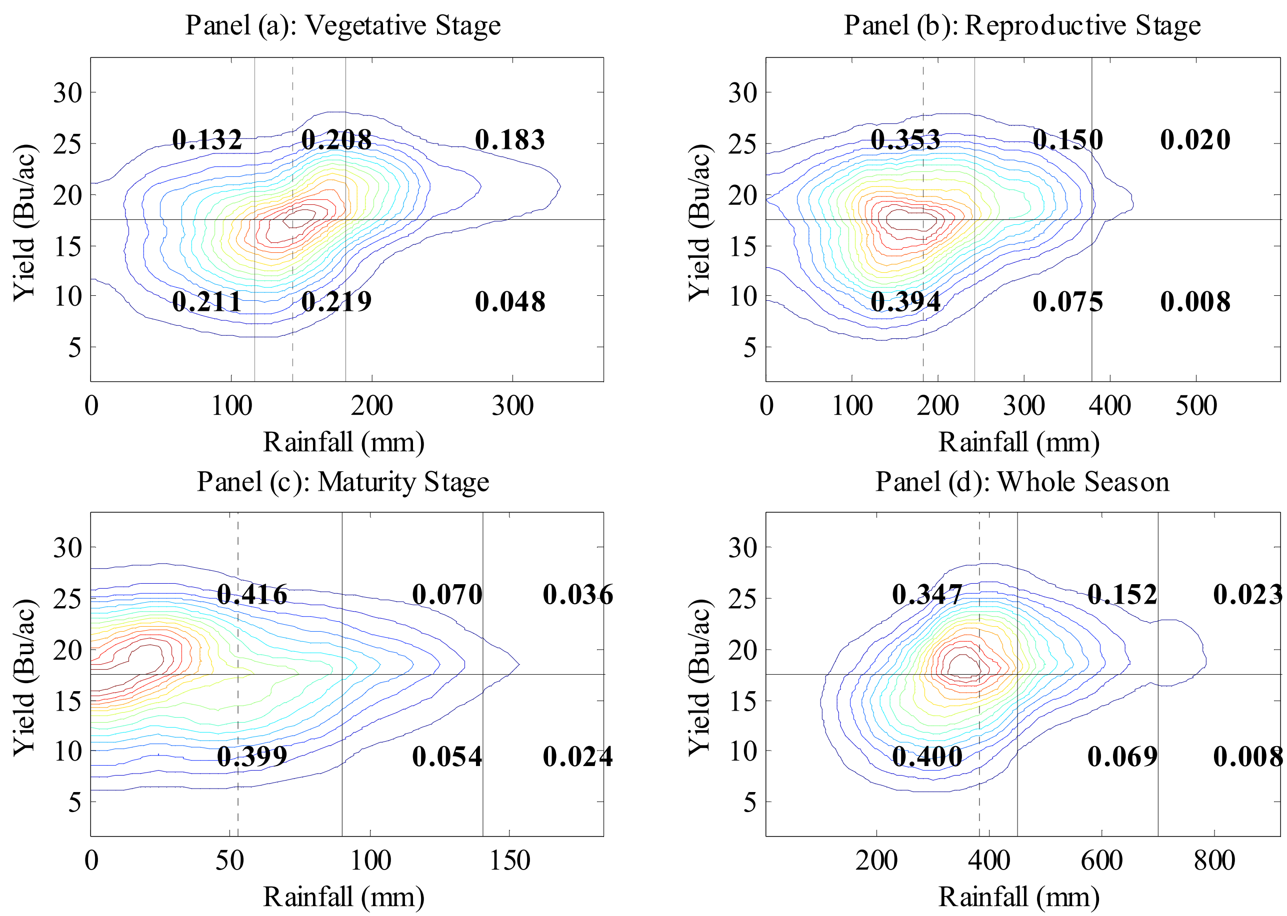


Figure A-11. Joint and marginal probability distributions for rainfall and soybean yields, Phillips county, maturity stage

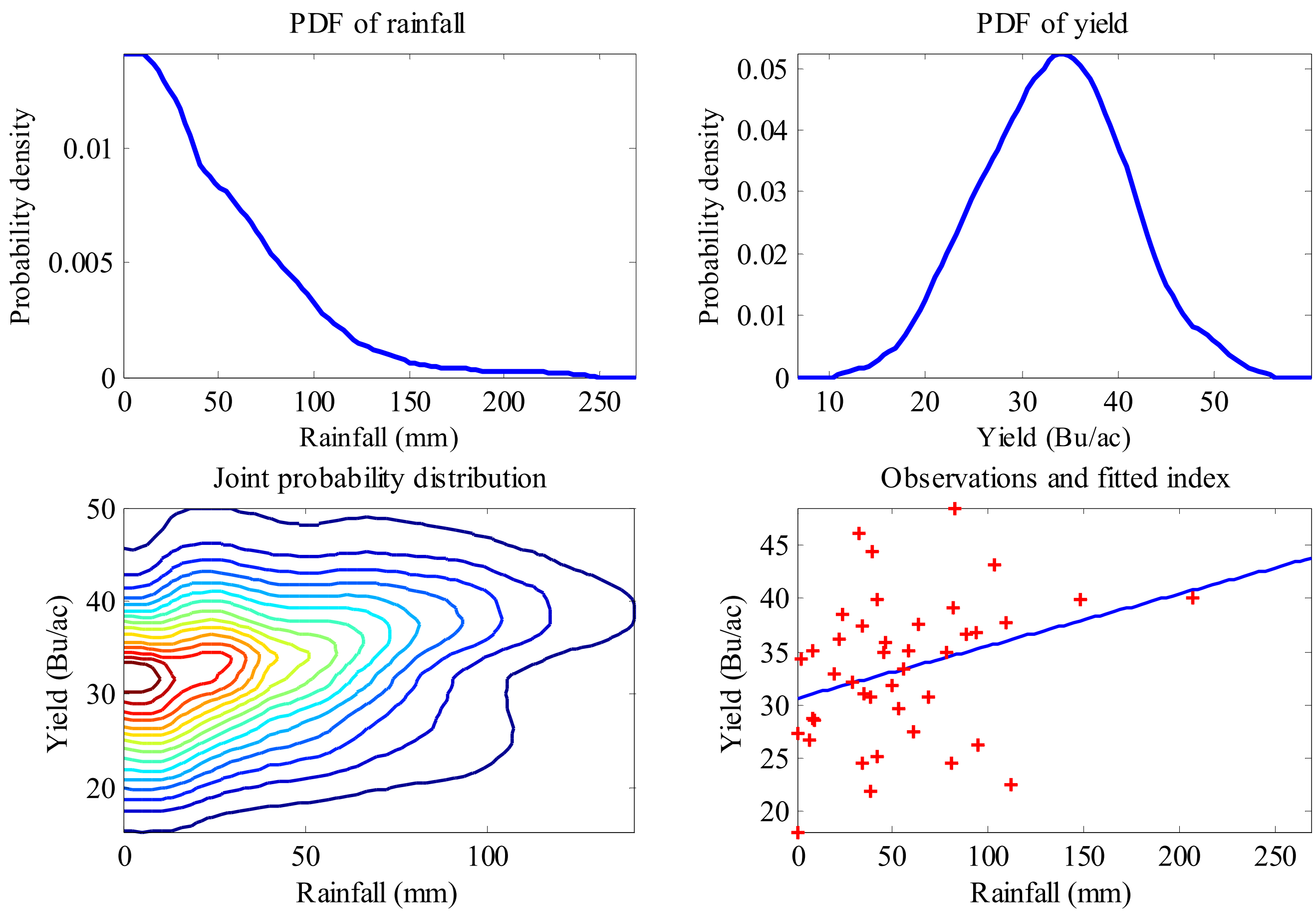


Figure A-12. Joint and marginal probability distributions for rainfall and soybean yields, Saint Francis county, maturity stage

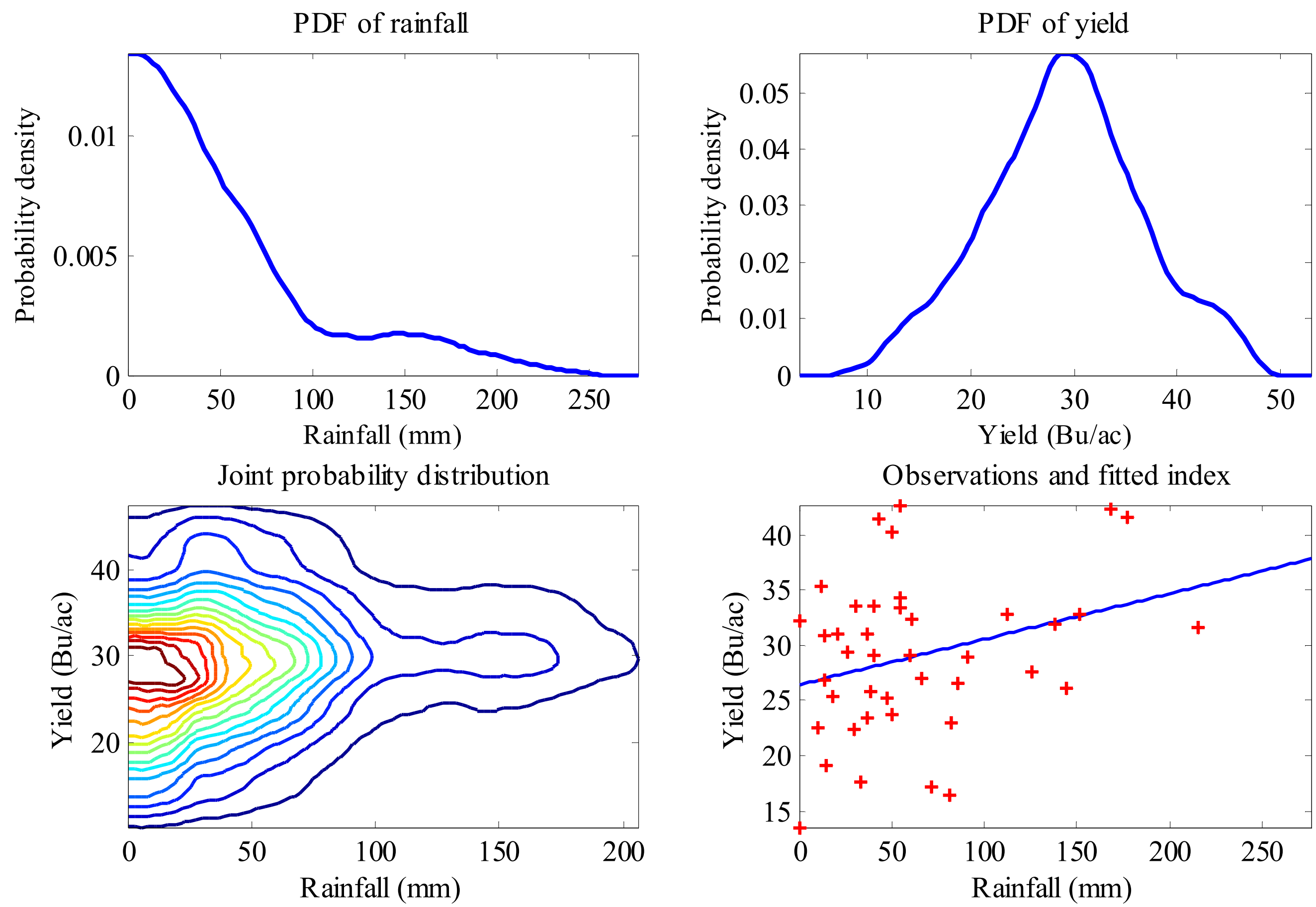


Figure A-13. Joint and marginal probability distributions for rainfall and soybean yields, Pulaski county, vegetative stage

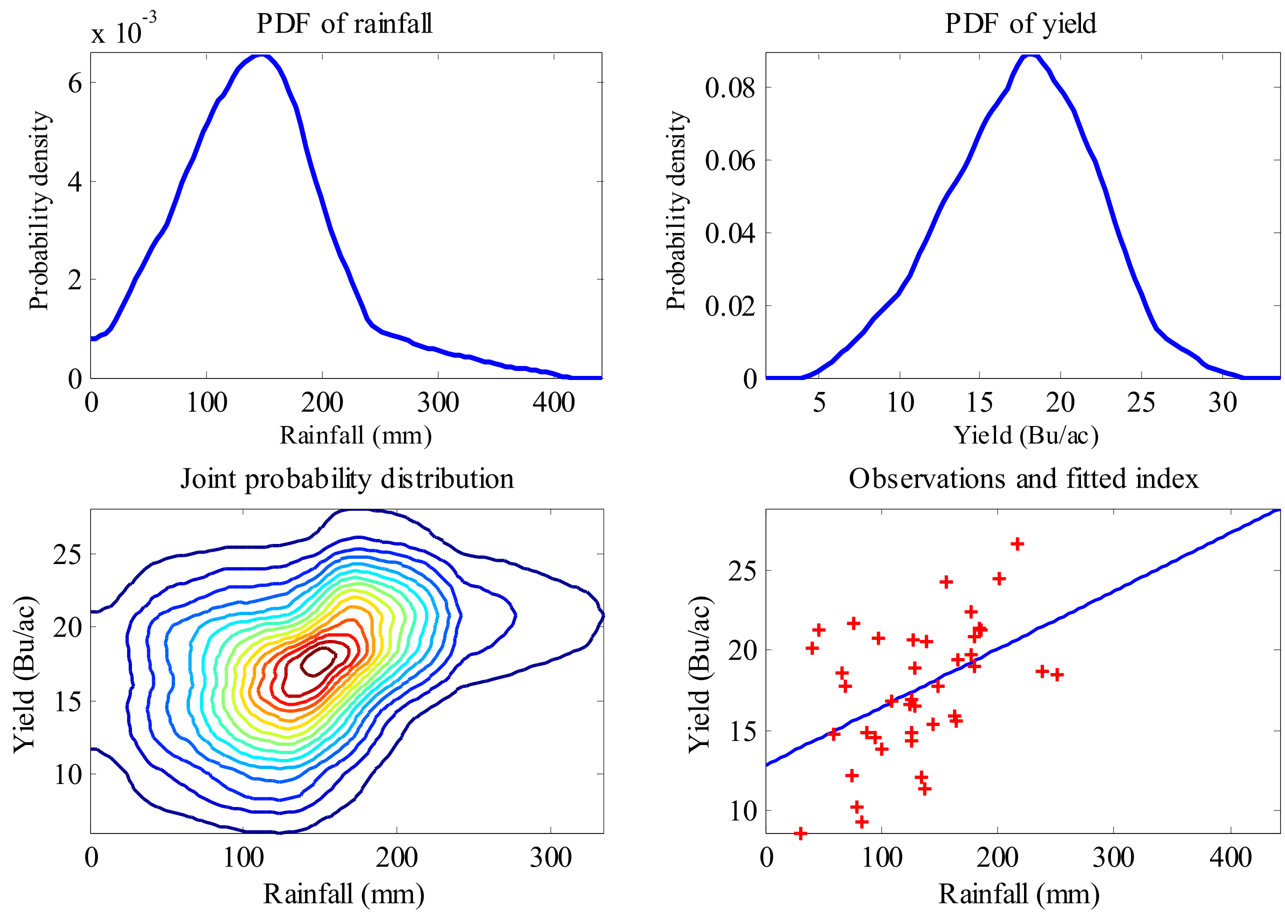


Figure A-14. Payoff structure of the standard and optimal contract, Phillips county

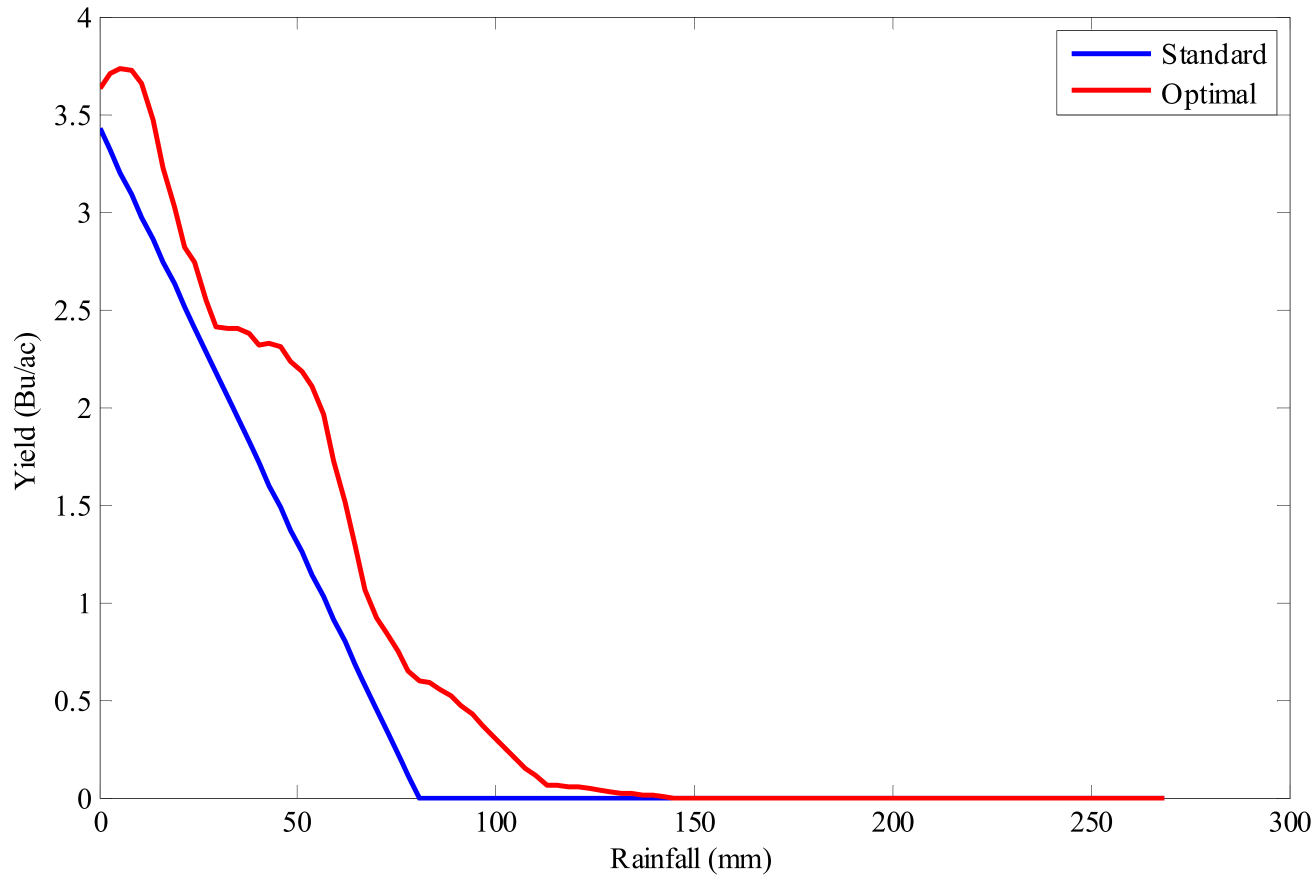


Figure A-15. Payoff structure of the standard and optimal contract, Saint Francis county

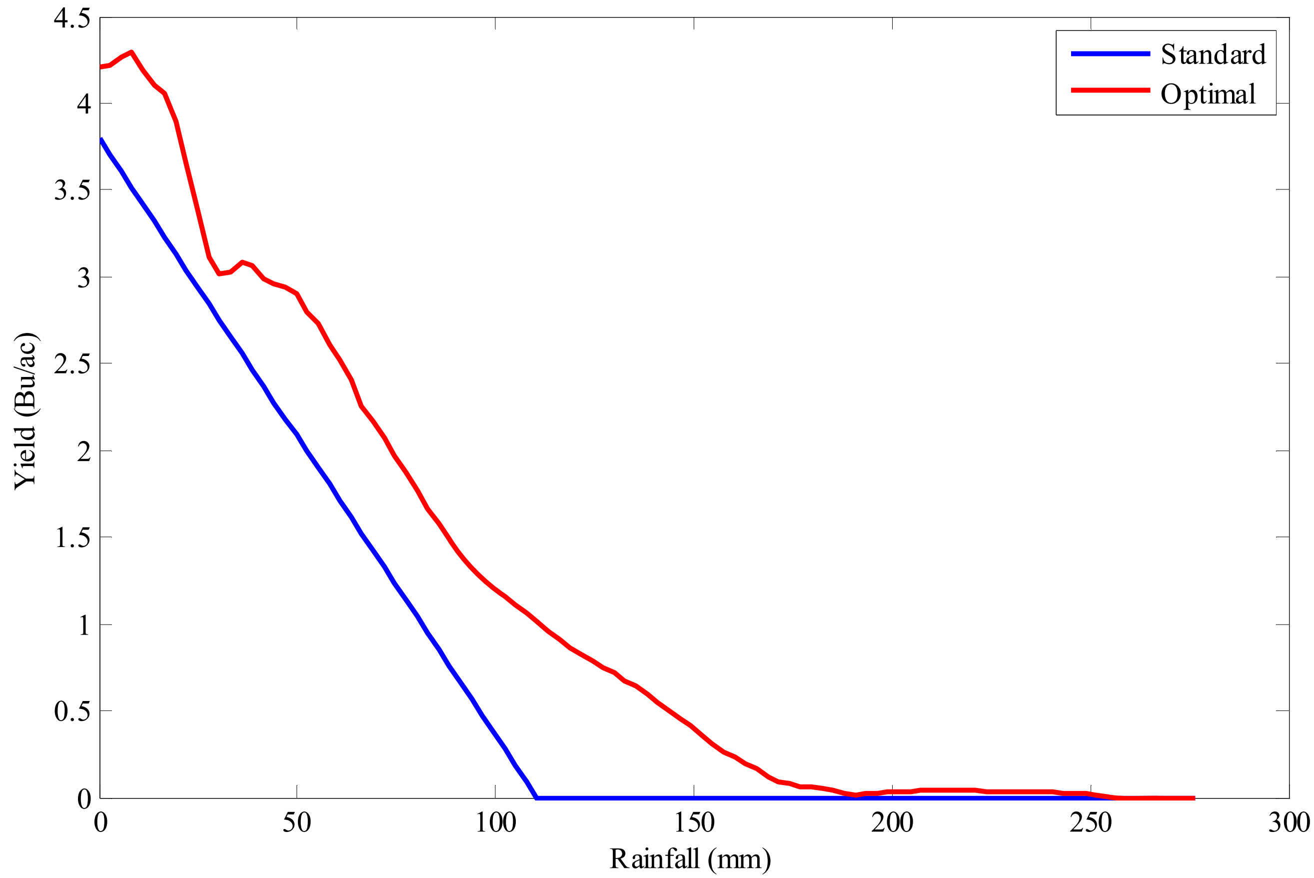


Figure A-16. Payoff structure of the standard and optimal contract, Pulaski county

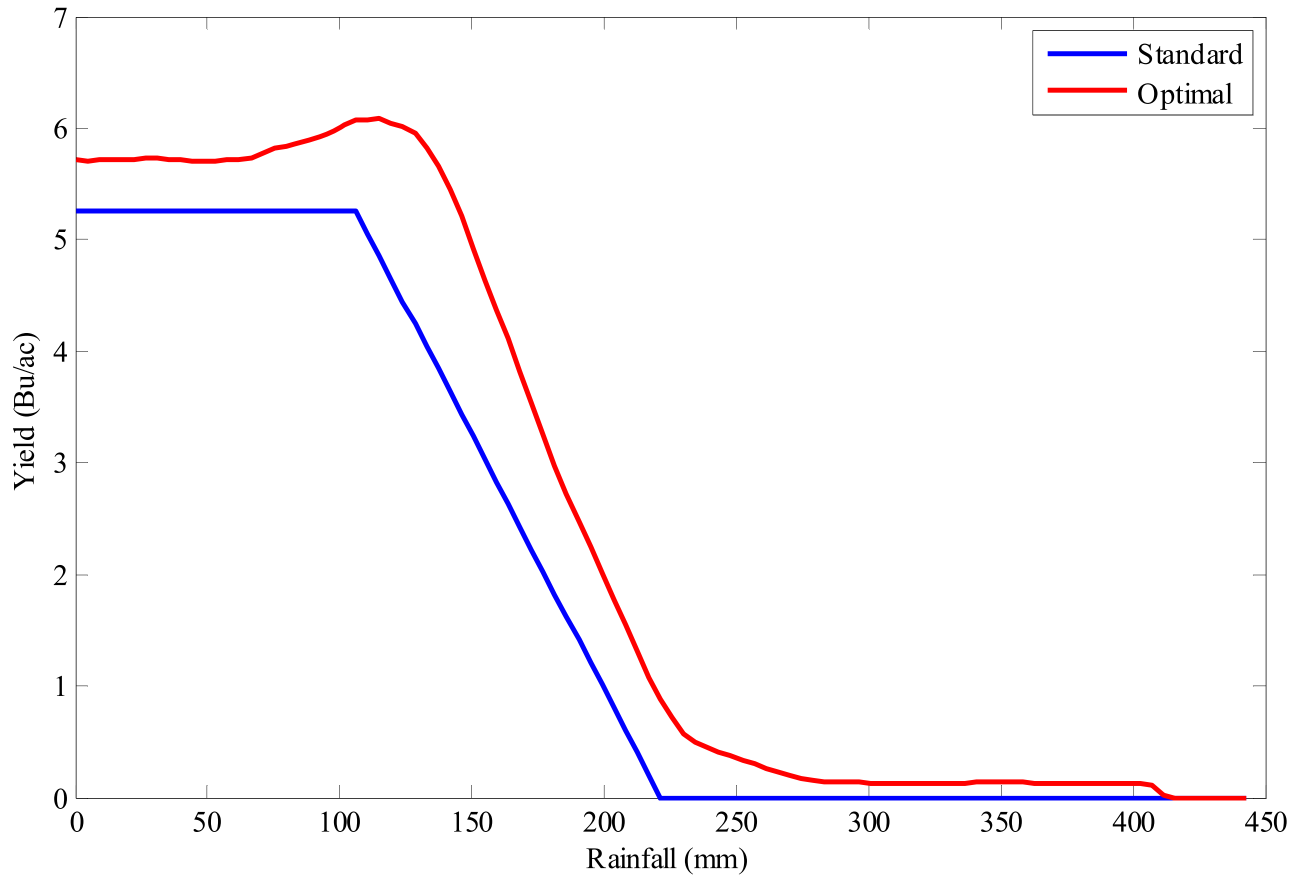


Figure A-17. The profit functions π of the wet-season farmers with and without weather derivatives

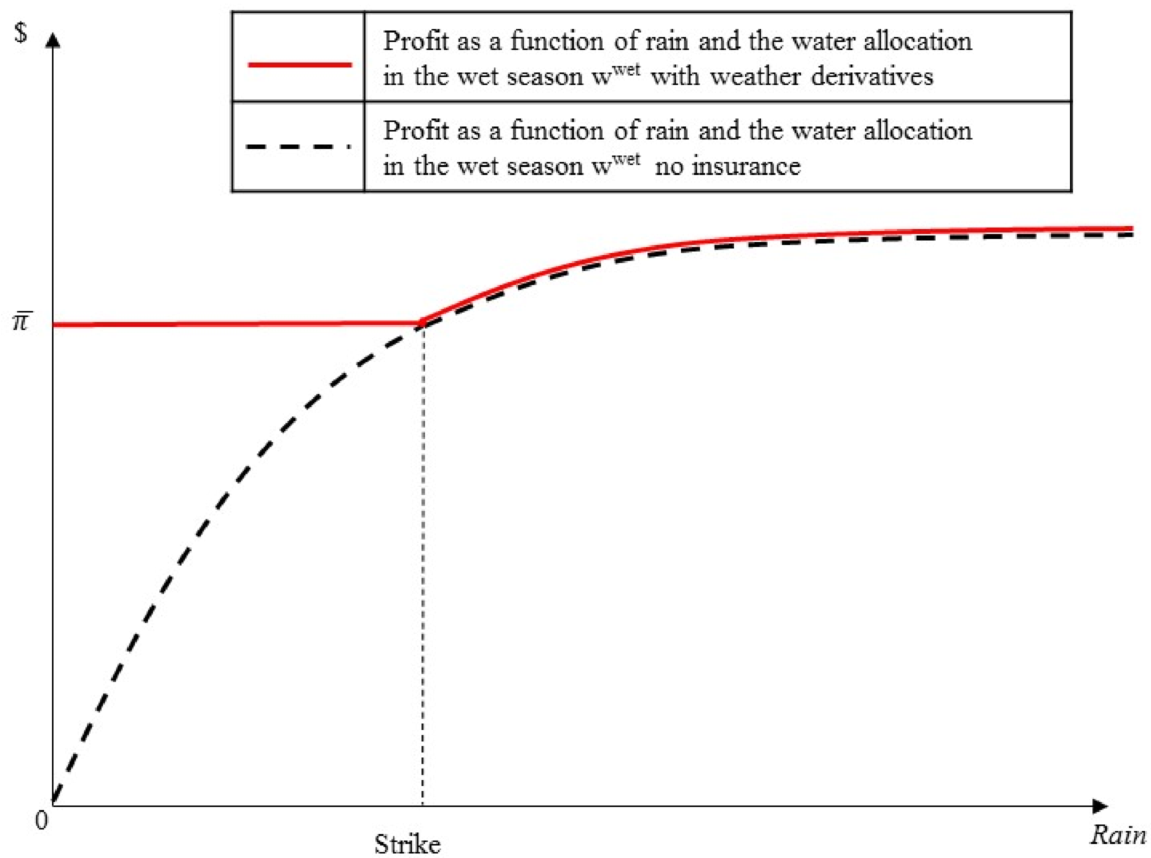


Figure A-18. Payoff structure of the weather derivatives contract

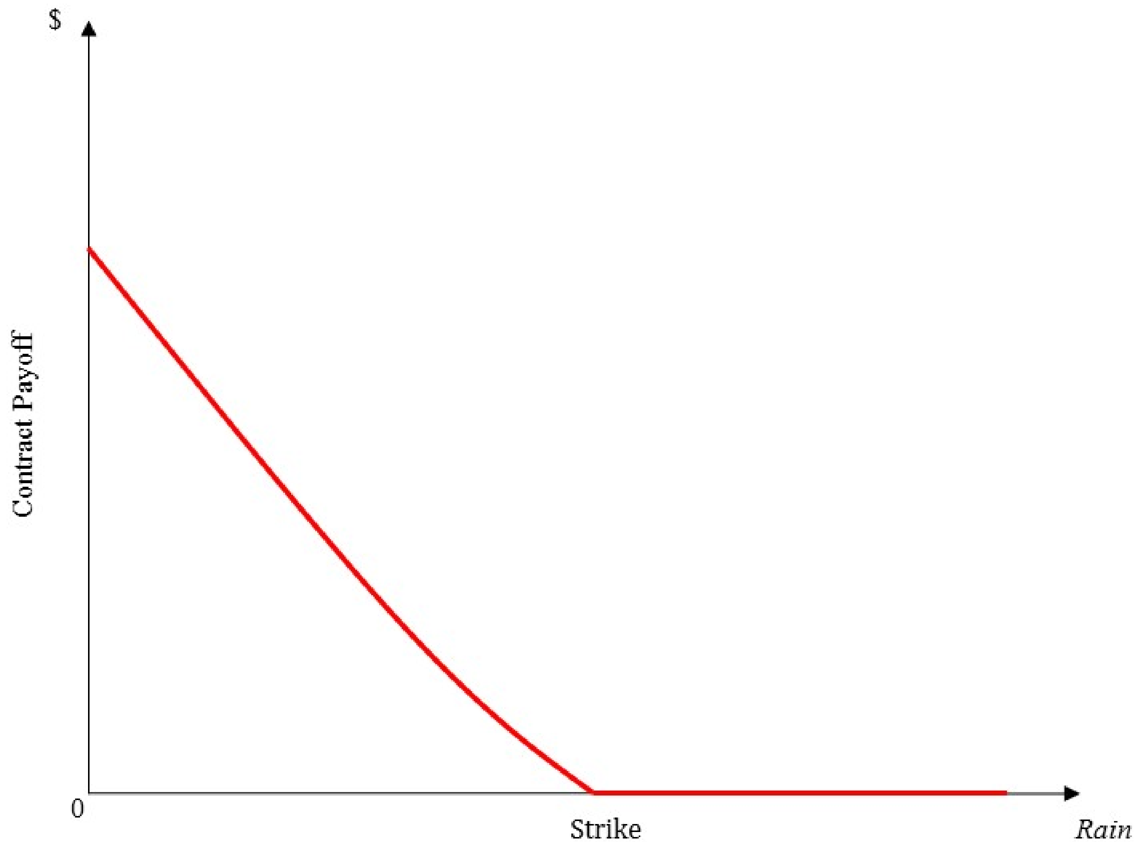


Figure A-19. Barley (dry season) yields in module Valle de Santiago, Mexico, 1989-2010

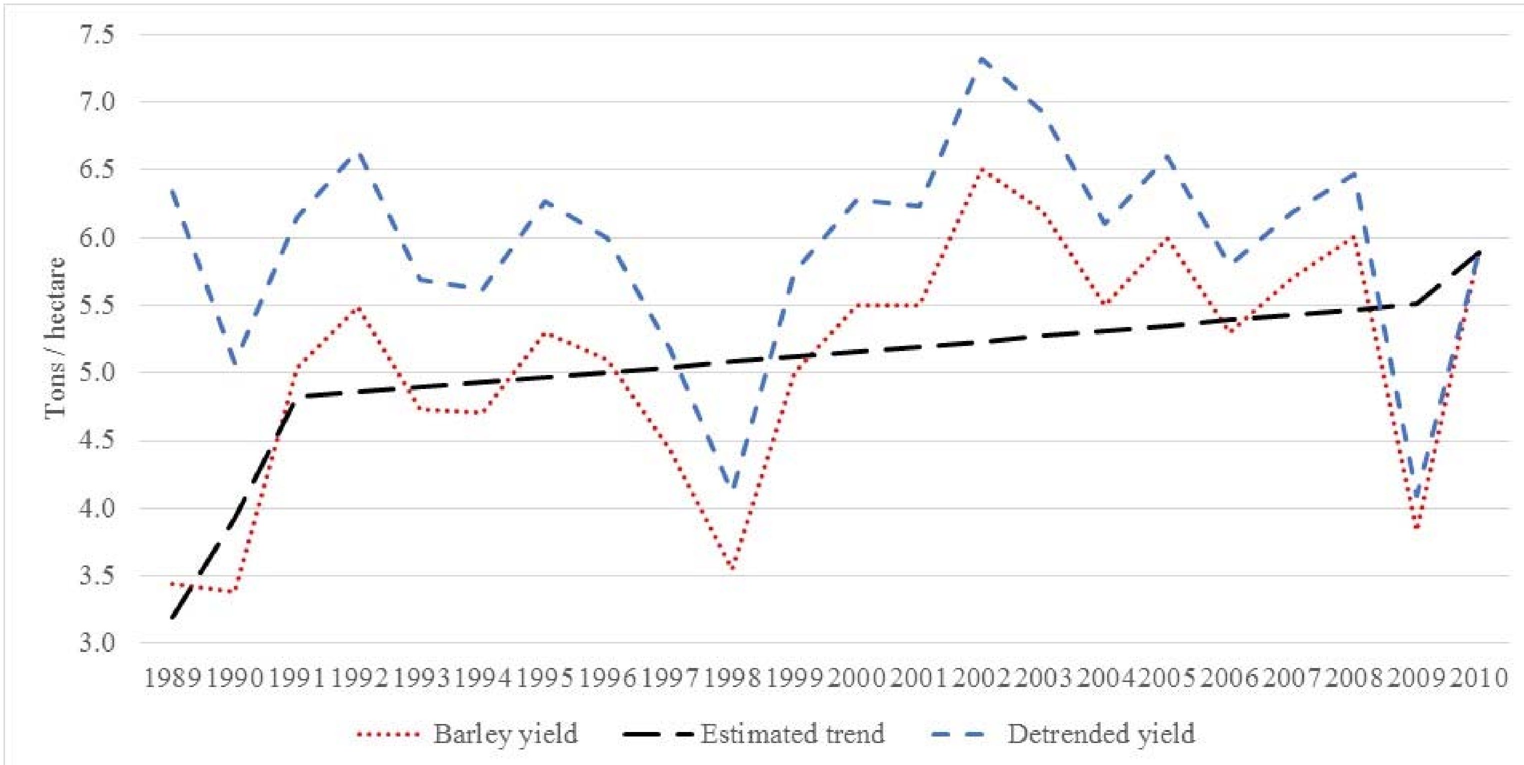


Figure A-20. Sorghum (wet season) yields in module Valle de Santiago, Mexico, 1989-2005

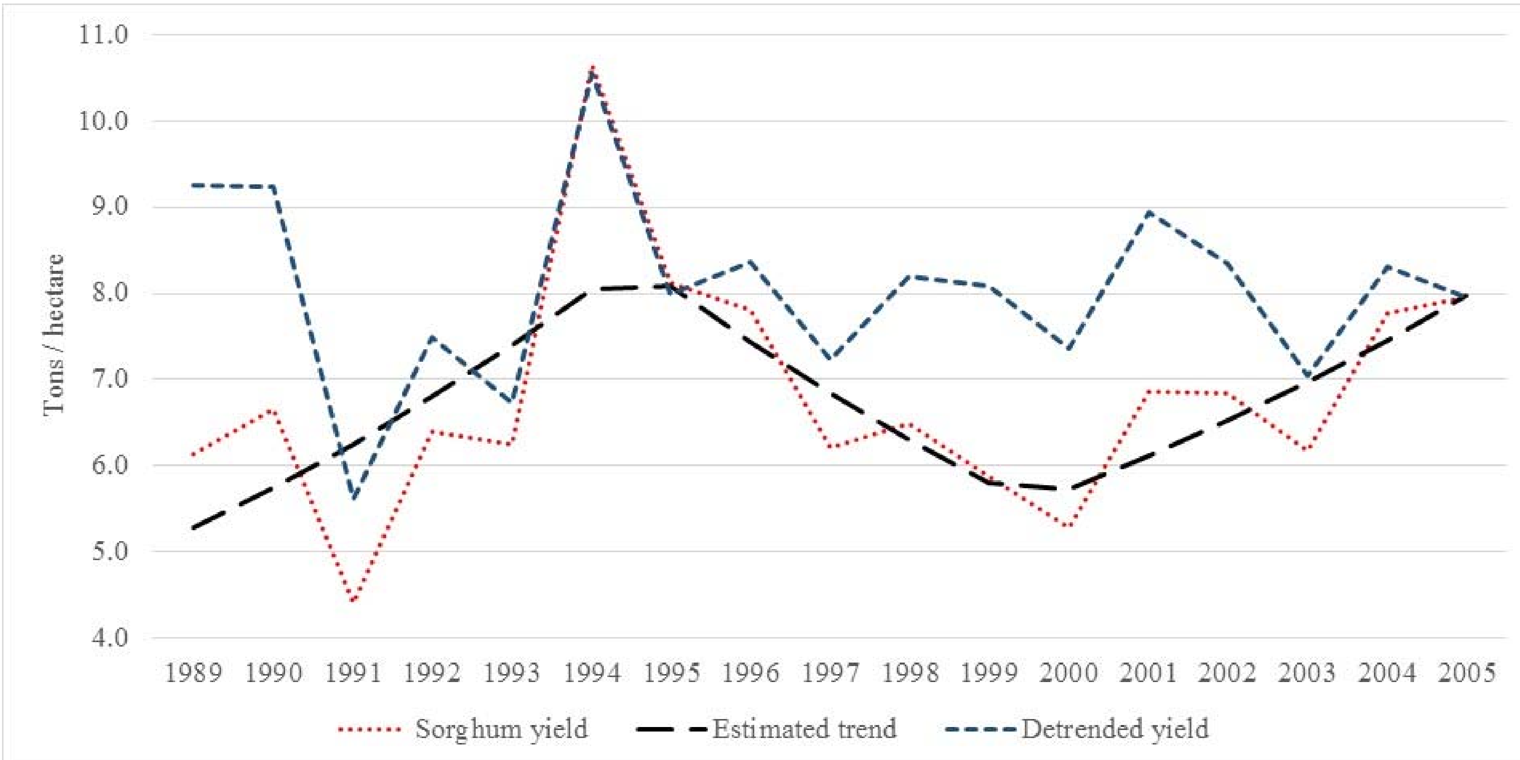


Figure A-21. Relative frequency distribution of cumulative rainfall (from April to June) in the municipality of Valle de Santiago, Mexico, 1942-2010

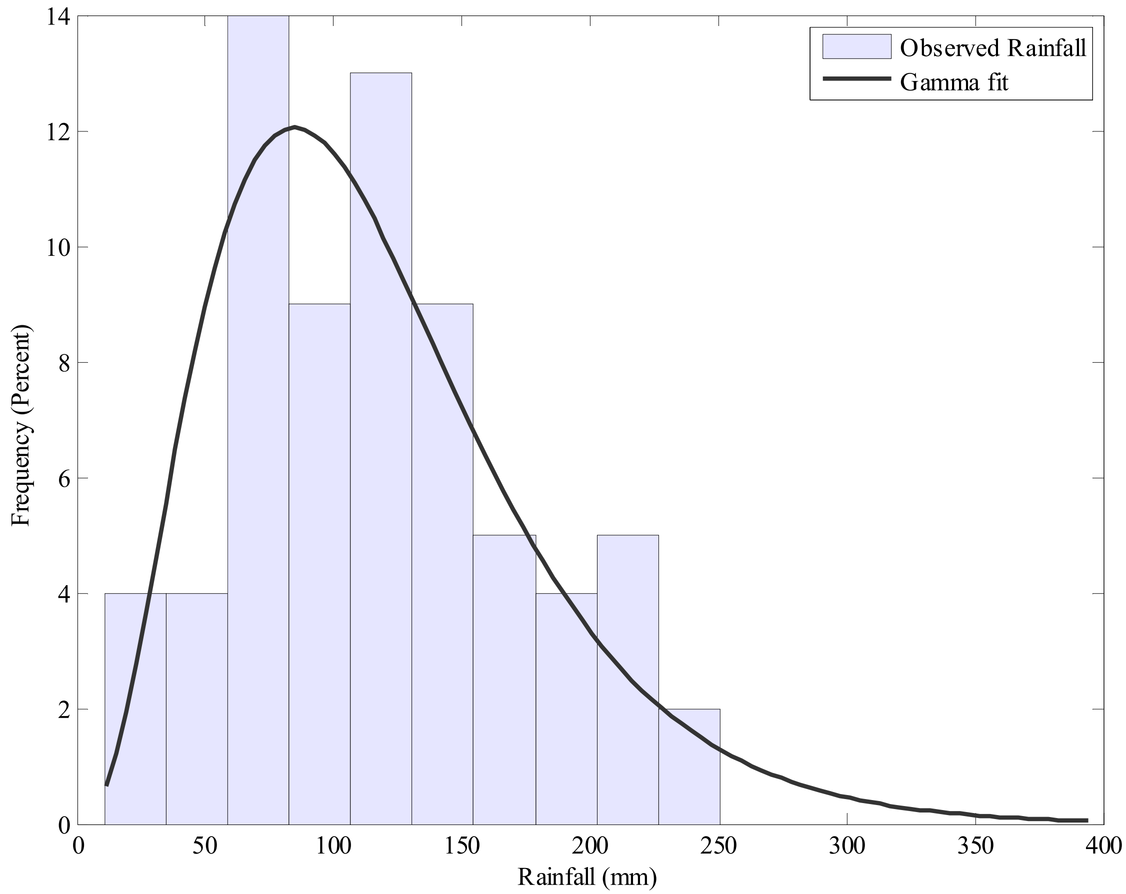


Figure A-22. Relative frequency distribution of cumulative rainfall (from November to October, next year) at the dam Solis, Mexico, 1961-2011

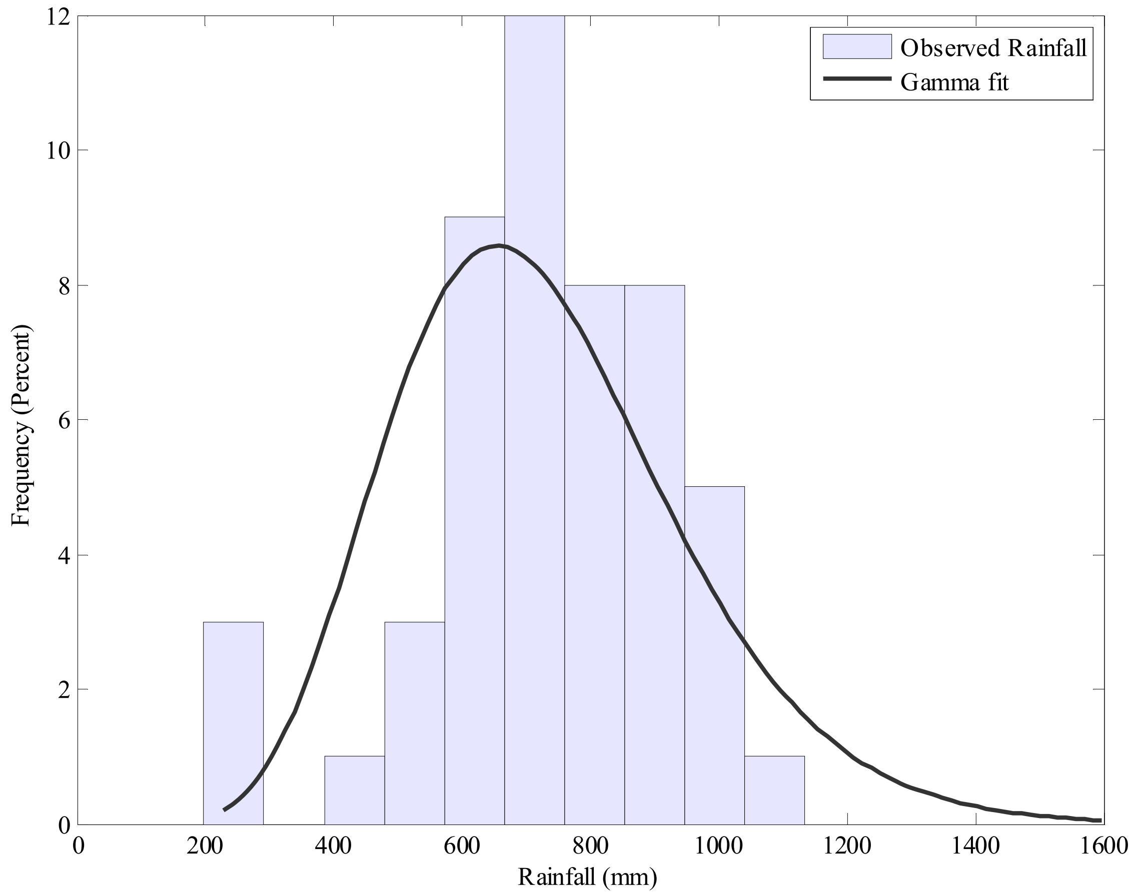


Figure A-23. Optimal water allocation policy without insurance

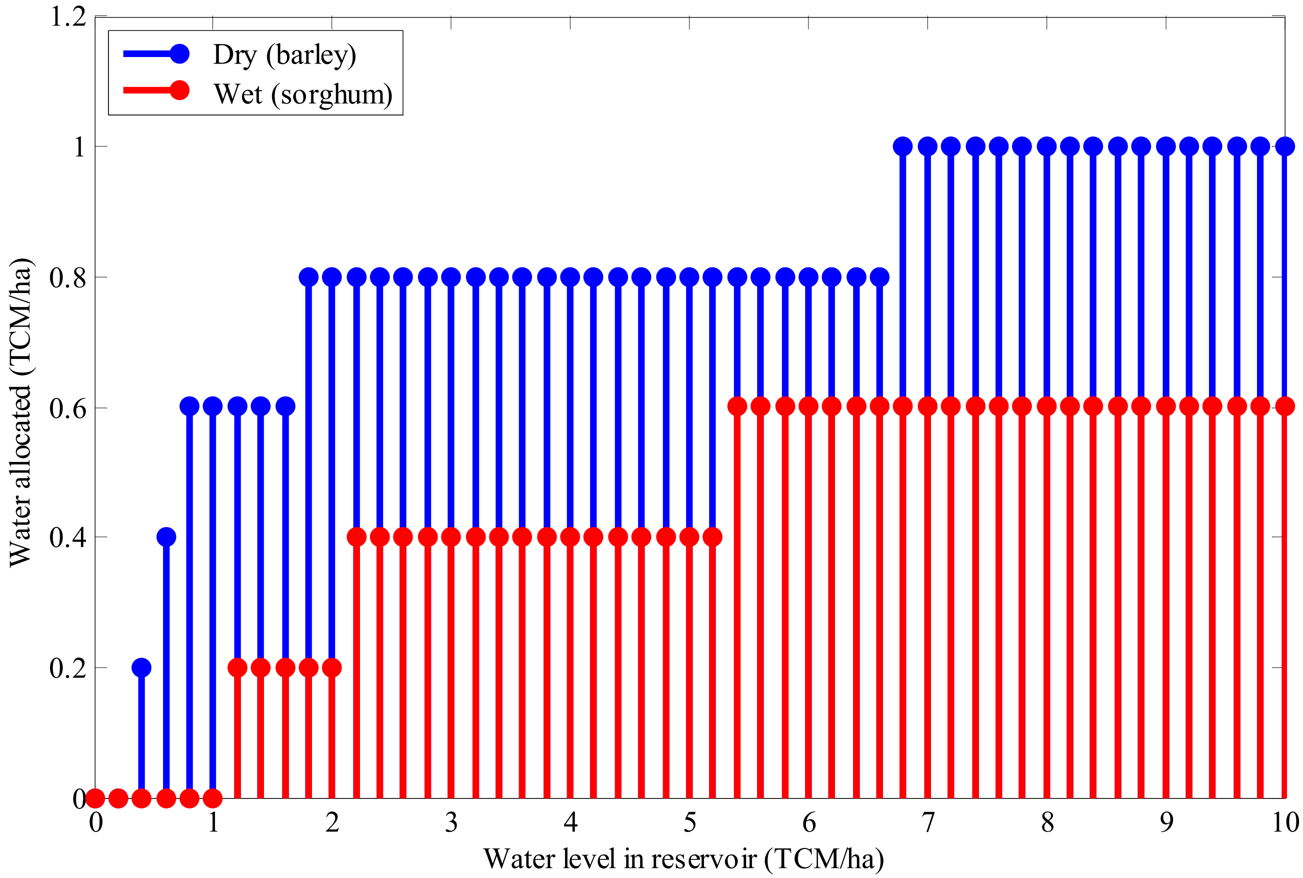


Figure A-24. Optimal state path without insurance

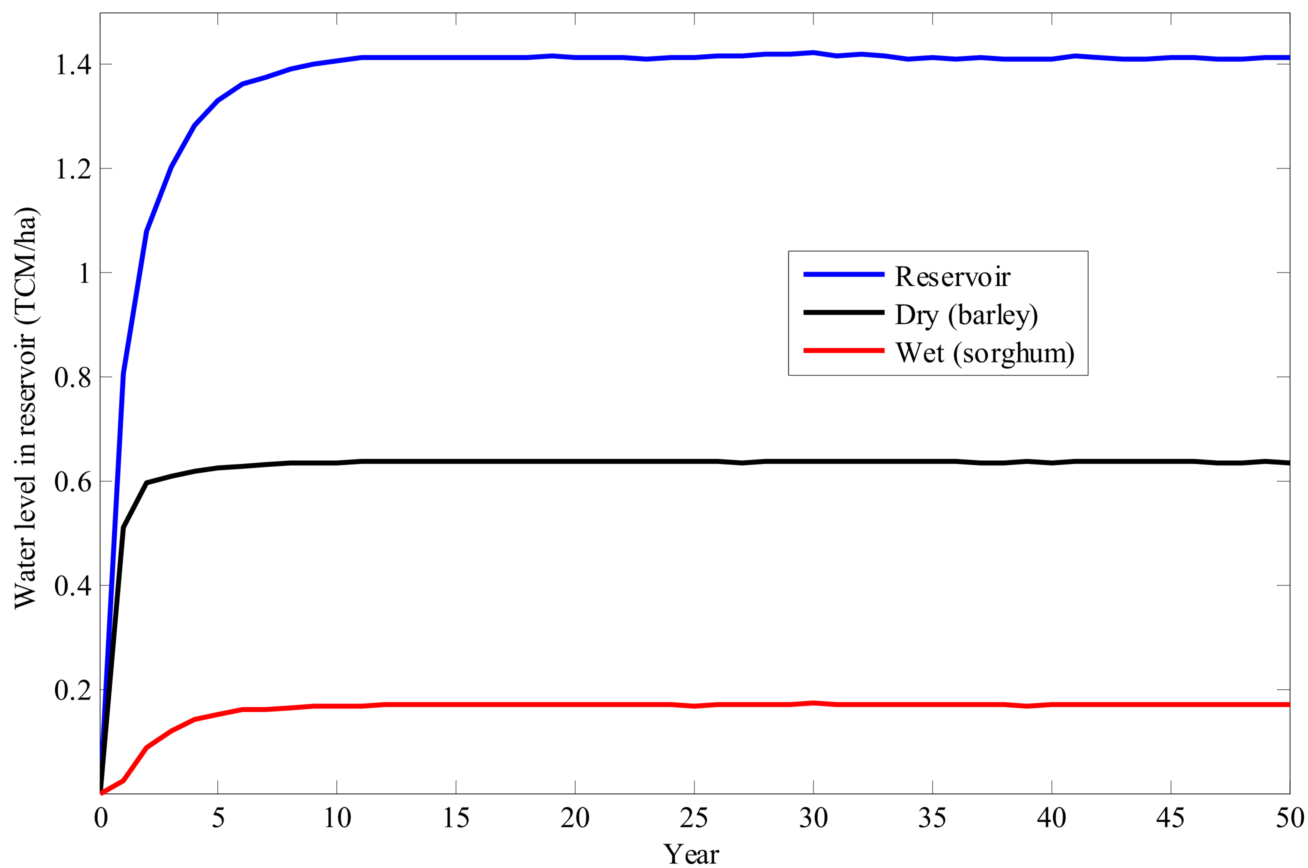


Figure A-25. Steady state distribution without insurance

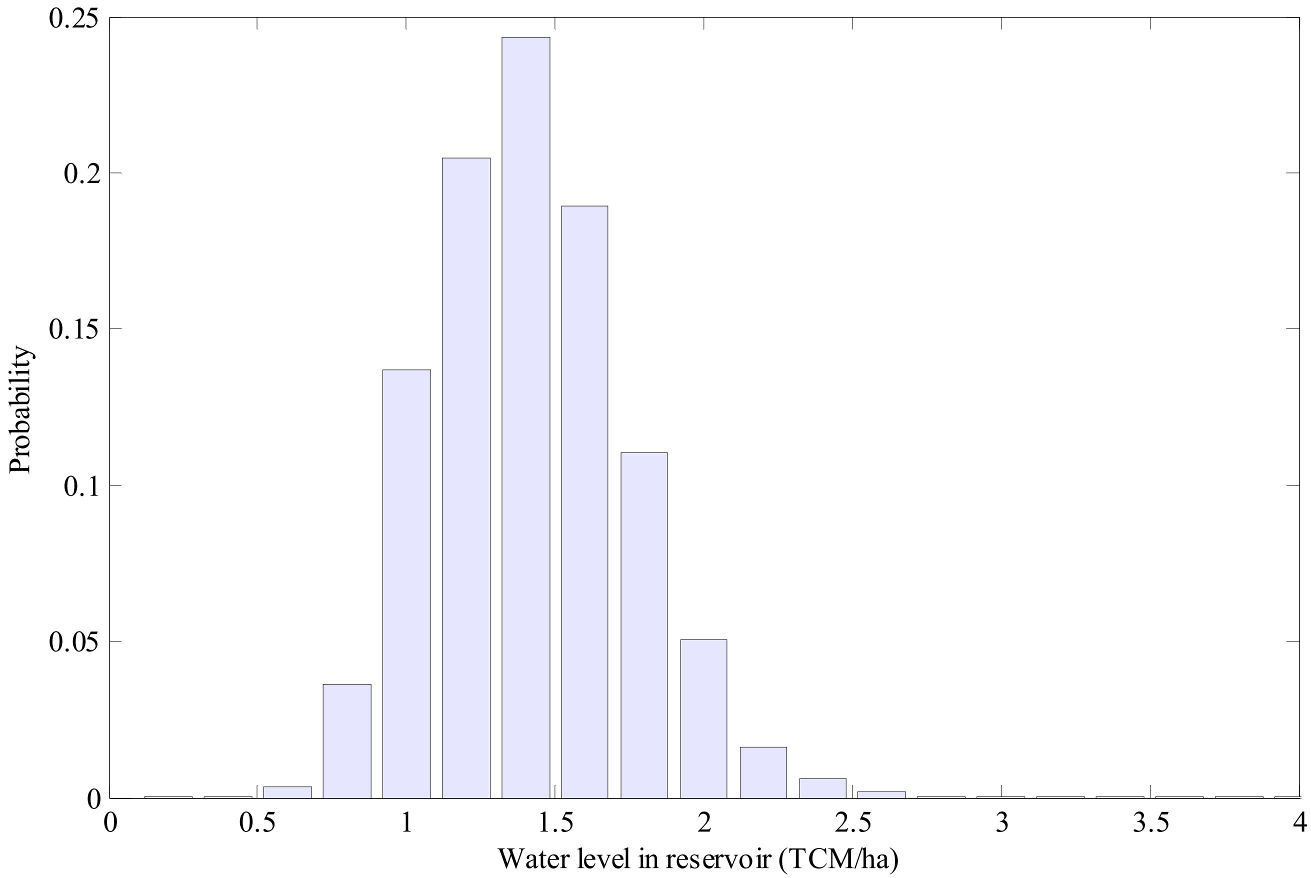


Figure A-26. Optimal water allocation policy with insurance

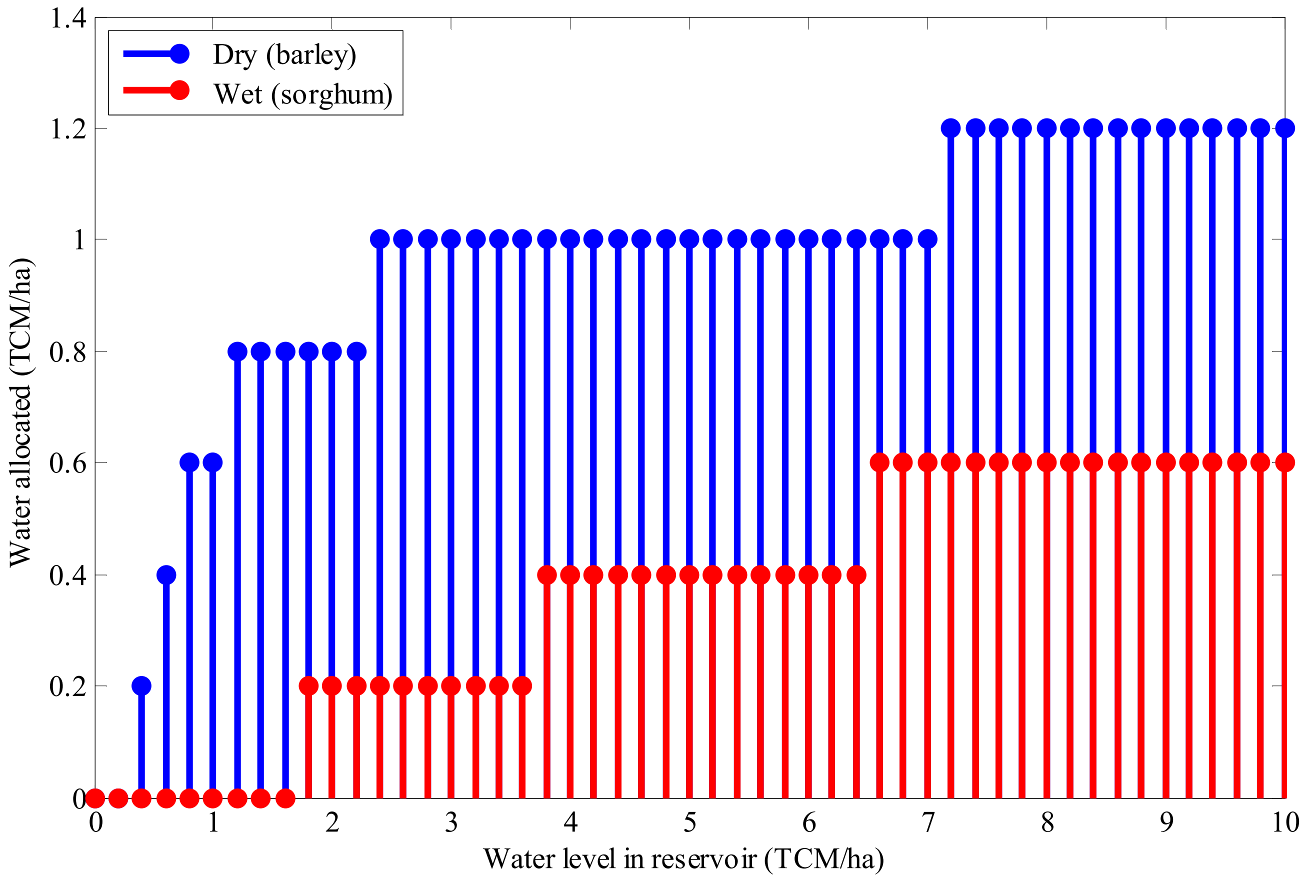


Figure A-27. Optimal state path with insurance

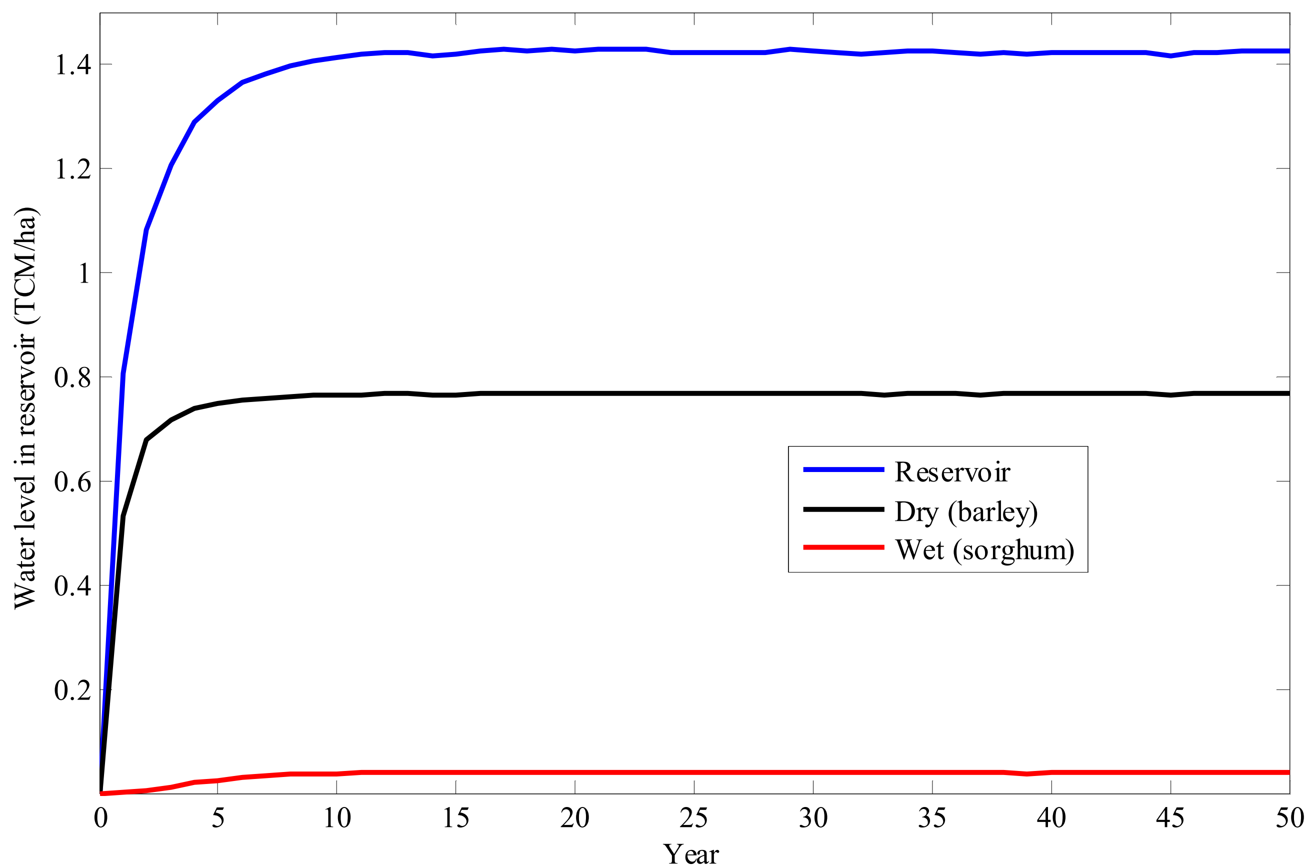


Figure A-28. Steady state distribution of water level in the reservoir with insurance

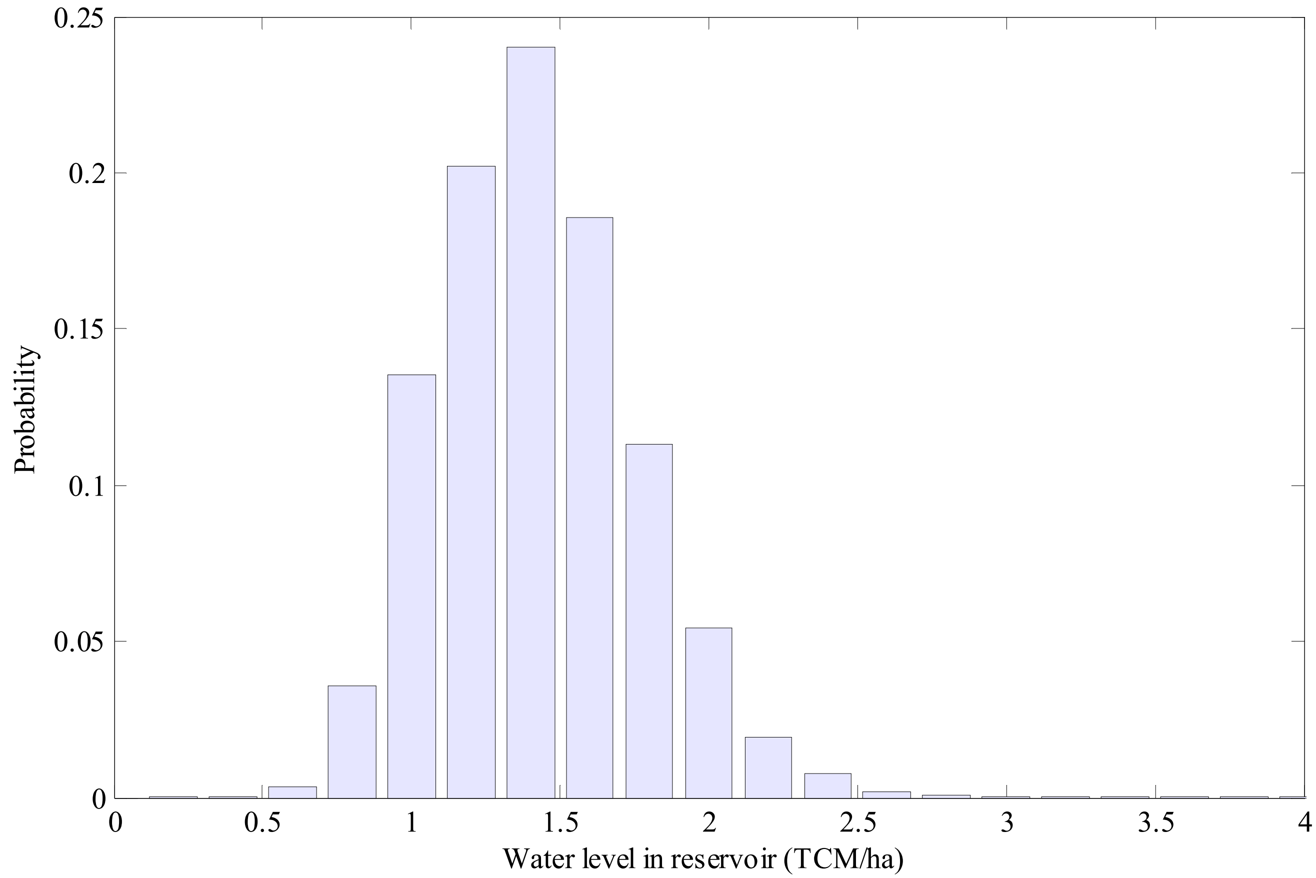


Figure A-29. Optimal value function

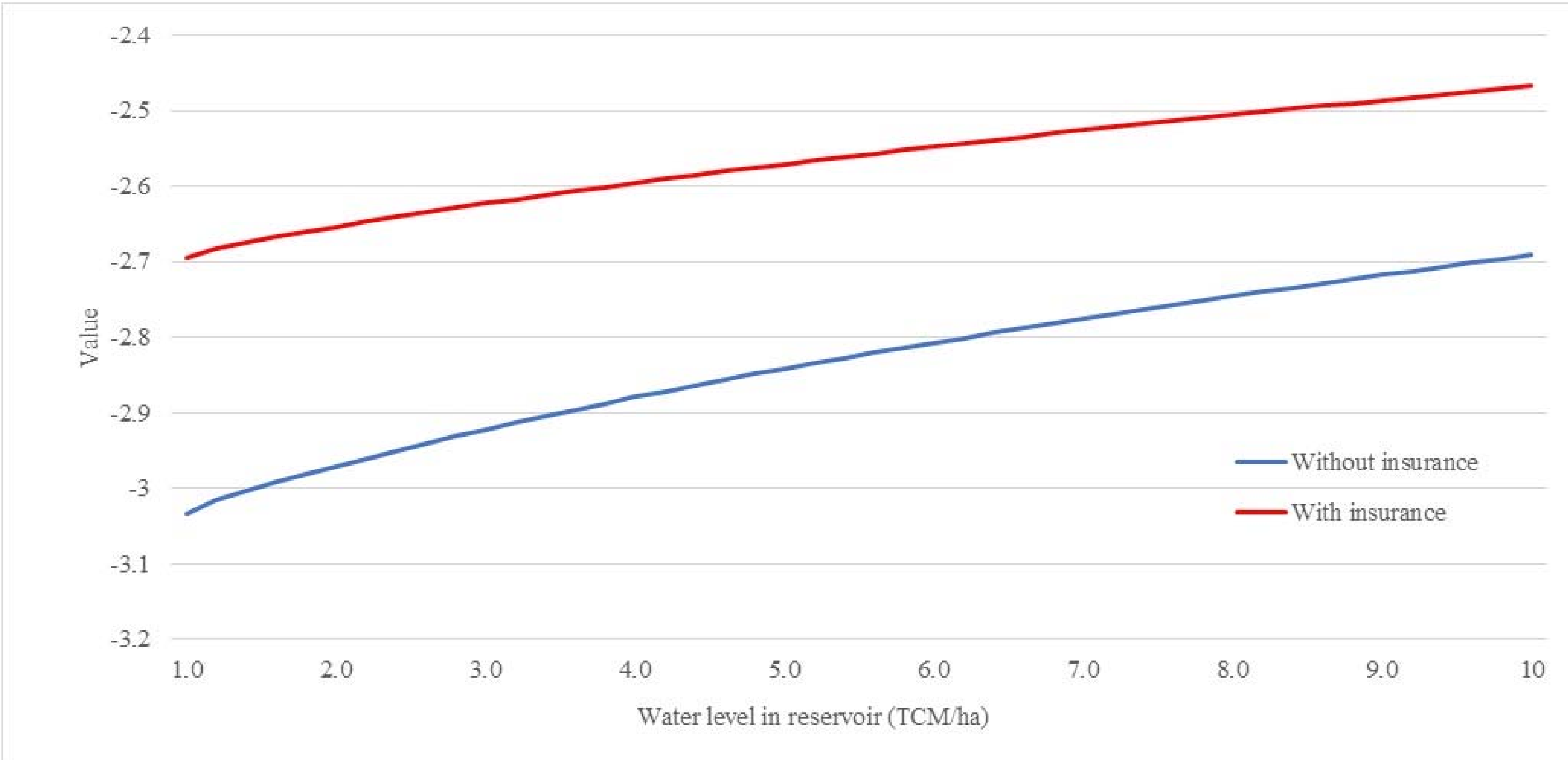
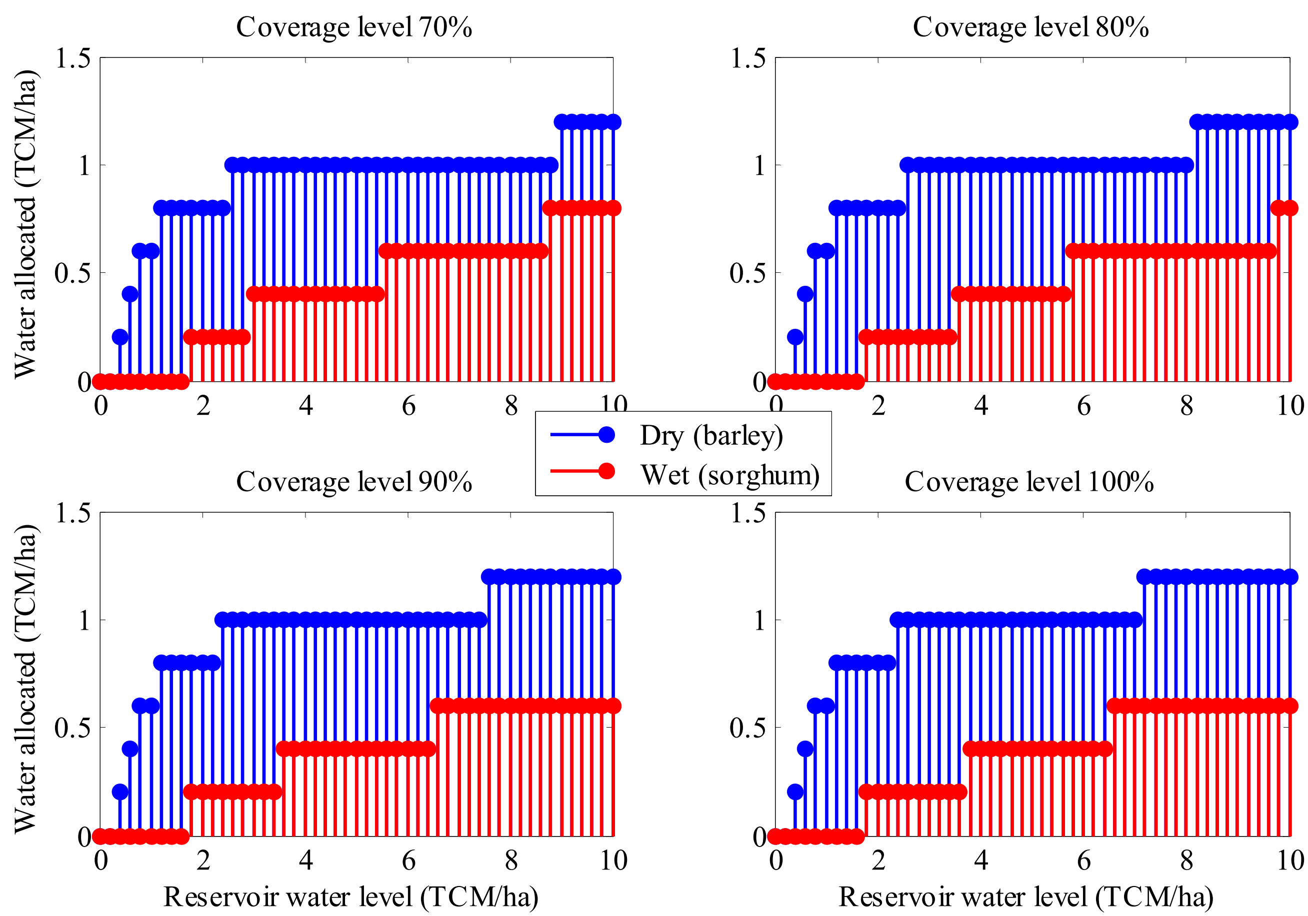


Figure A-30. Optimal water allocation policy with insurance for different coverage levels



Note: coverage level is measured as percent of expected rainfall in the module

Figure A-31. Optimal value function for different coverage levels

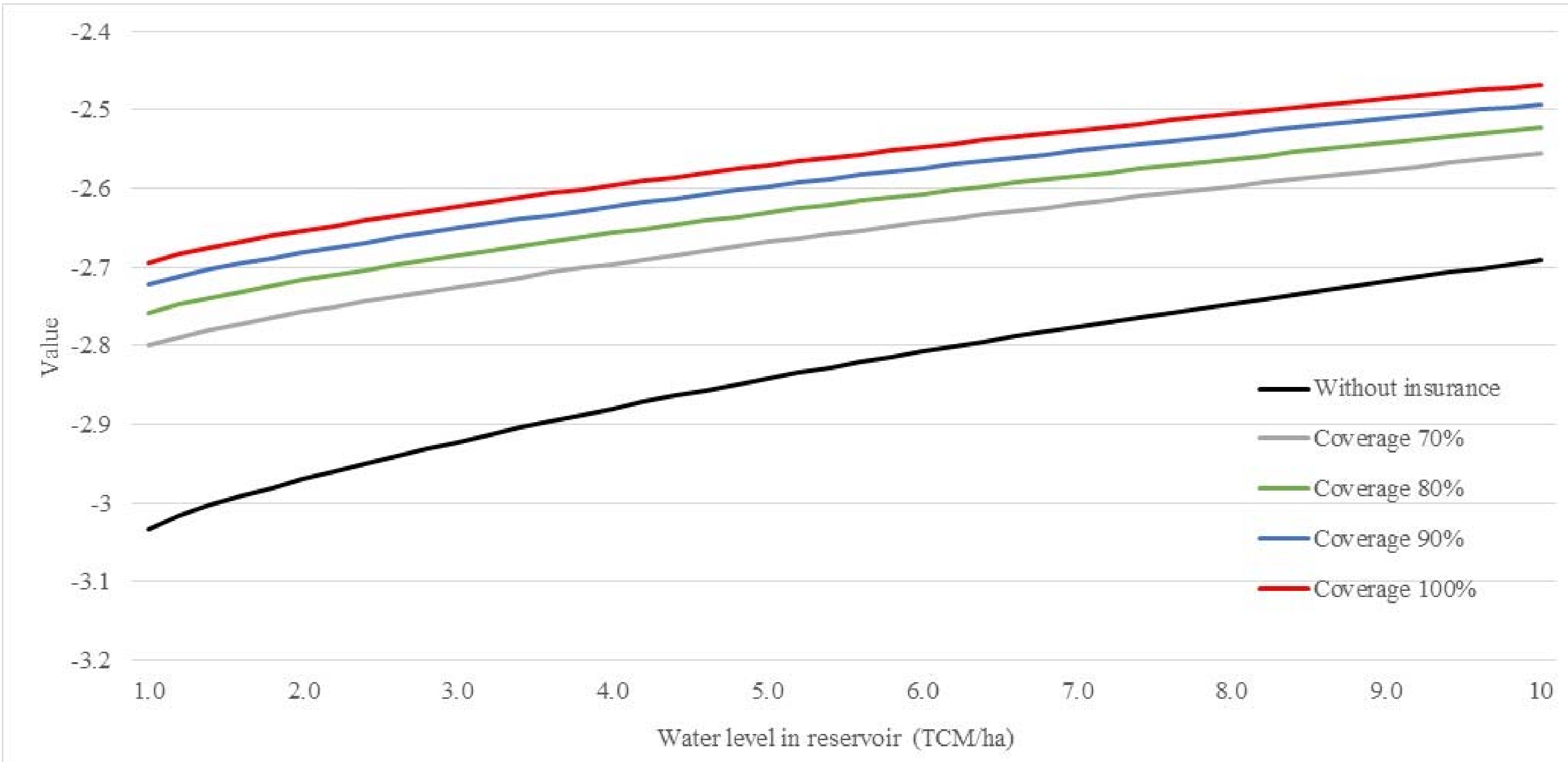


Figure A-32. Water allocation decisions under different ratios of wet- and dry-season crop prices without weather derivatives

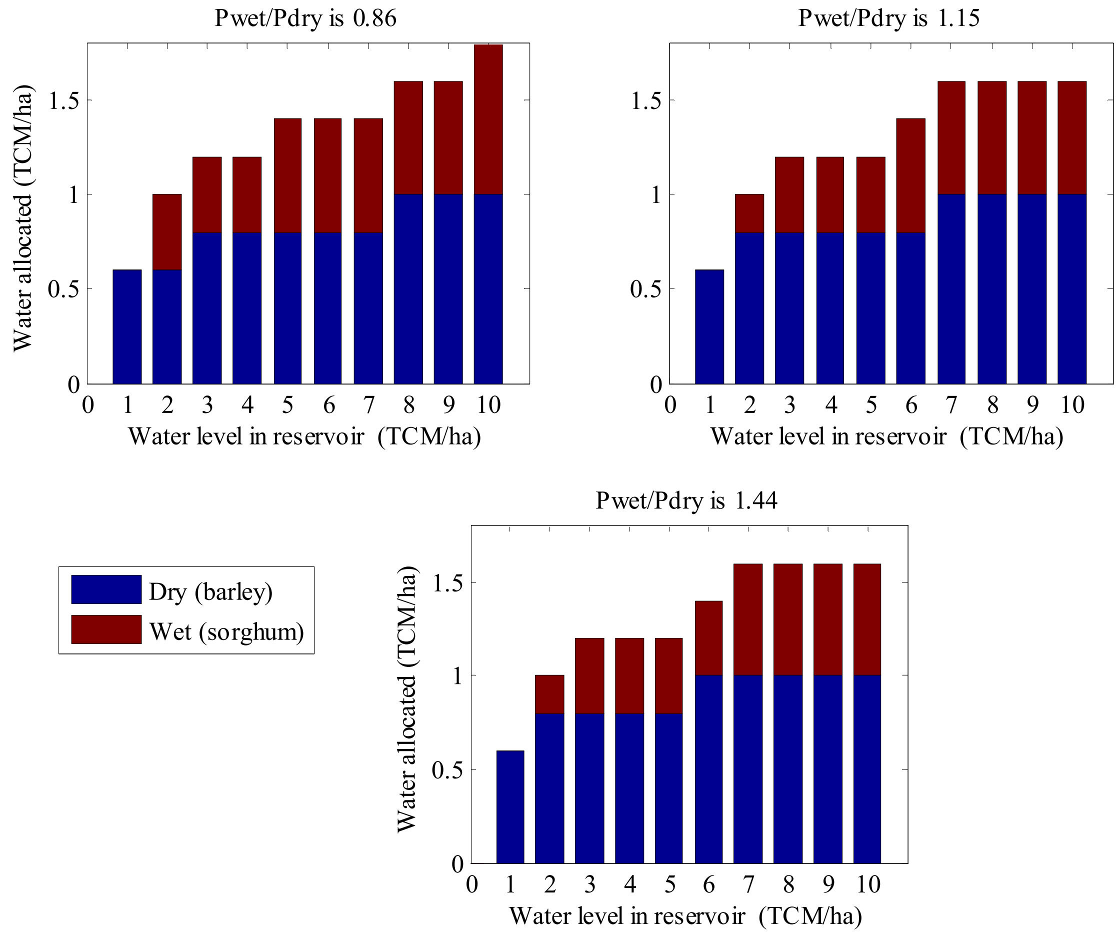


Figure A-33. Water allocation decisions under different ratios of wet- and dry-season crop prices with weather derivatives

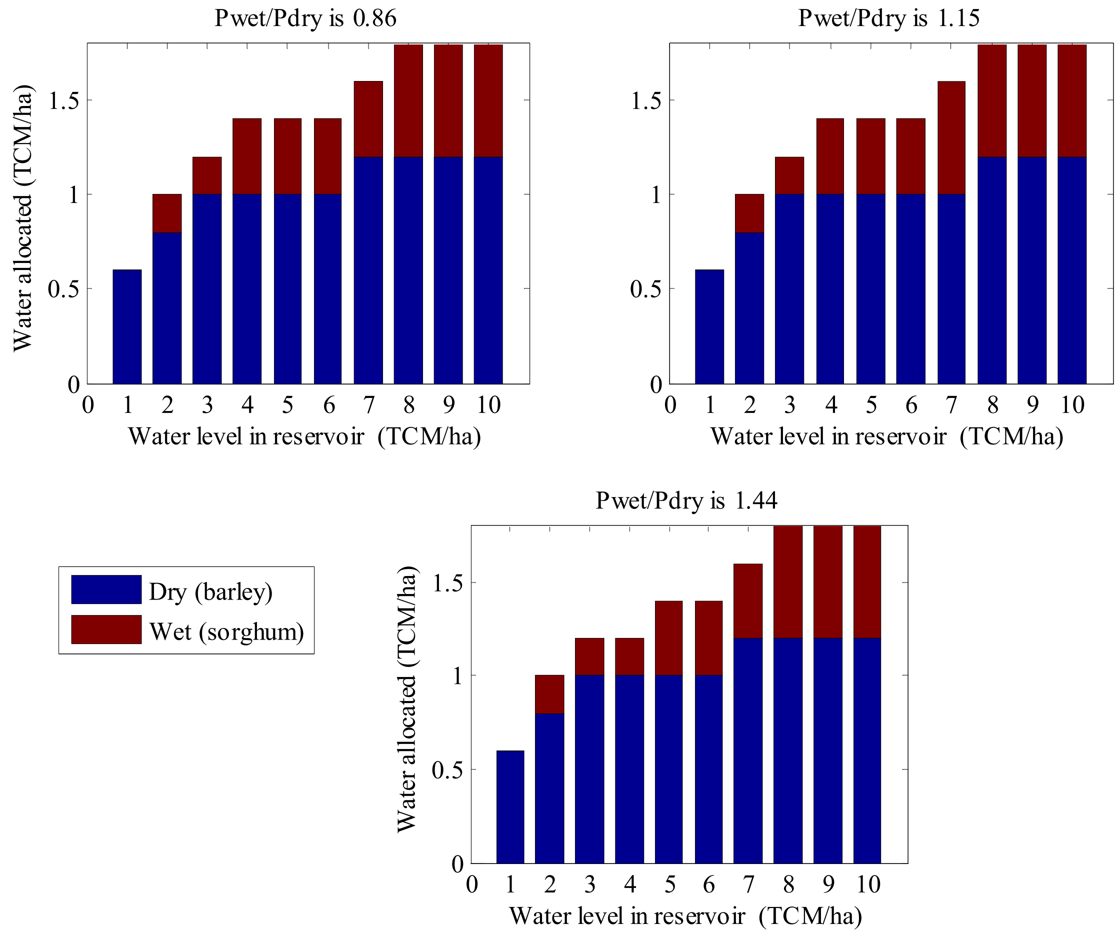


Figure A-34. Value functions for different ratios of wet- and dry-season crop prices

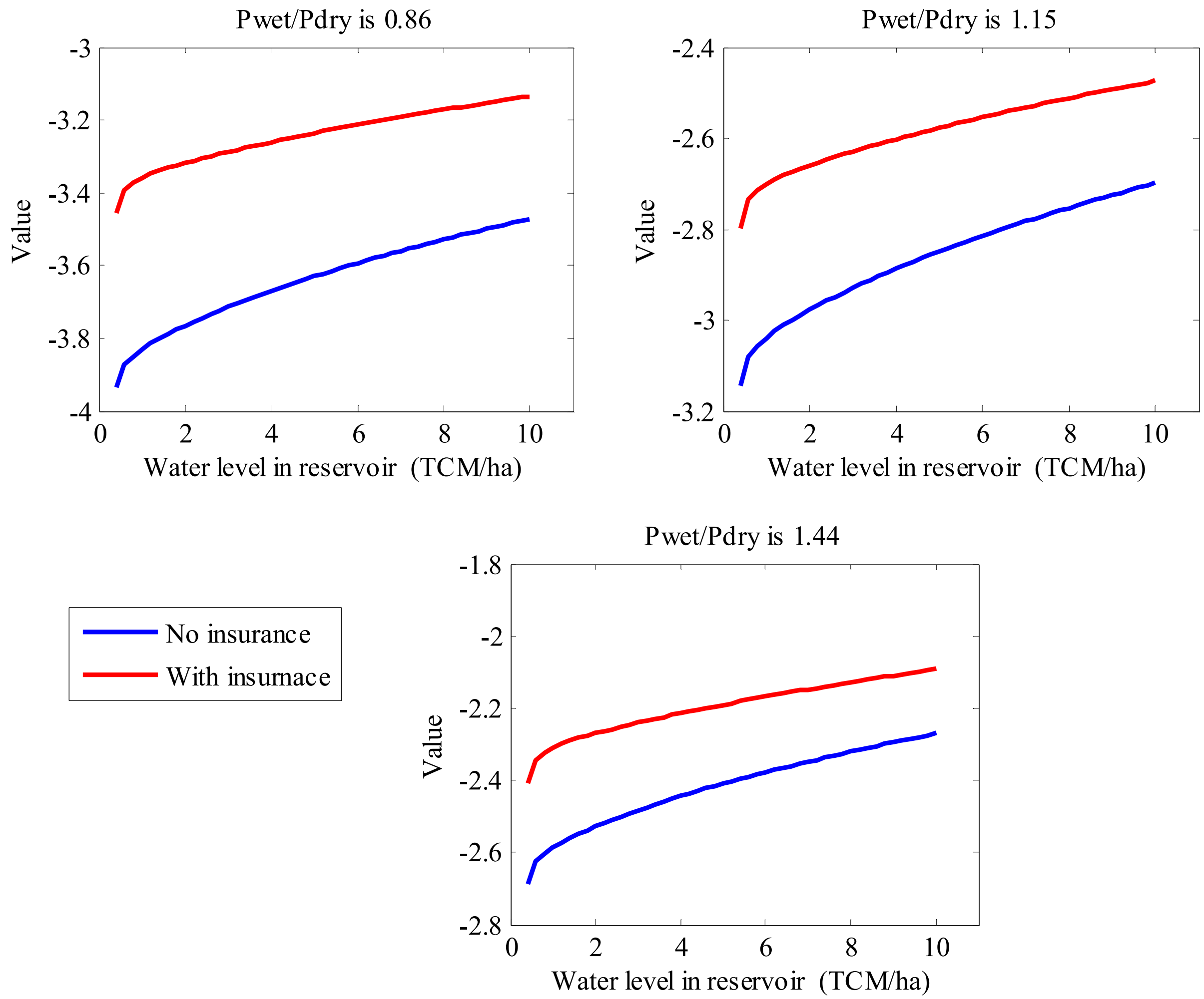


Figure A-35. Water allocation decisions under different relative risk aversion parameter without weather derivatives

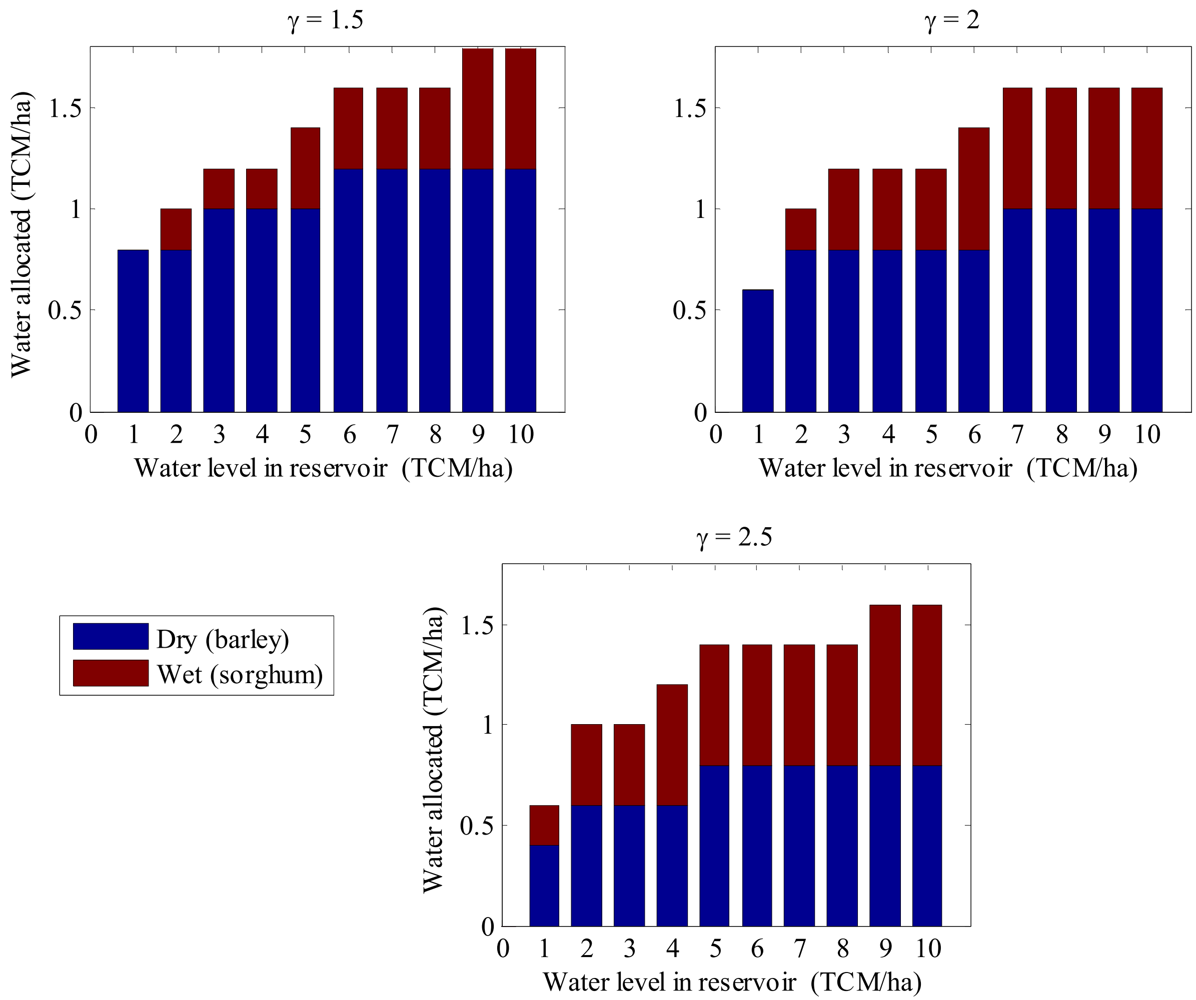


Figure A-36. Water allocation decisions under different relative risk aversion parameter with weather derivatives

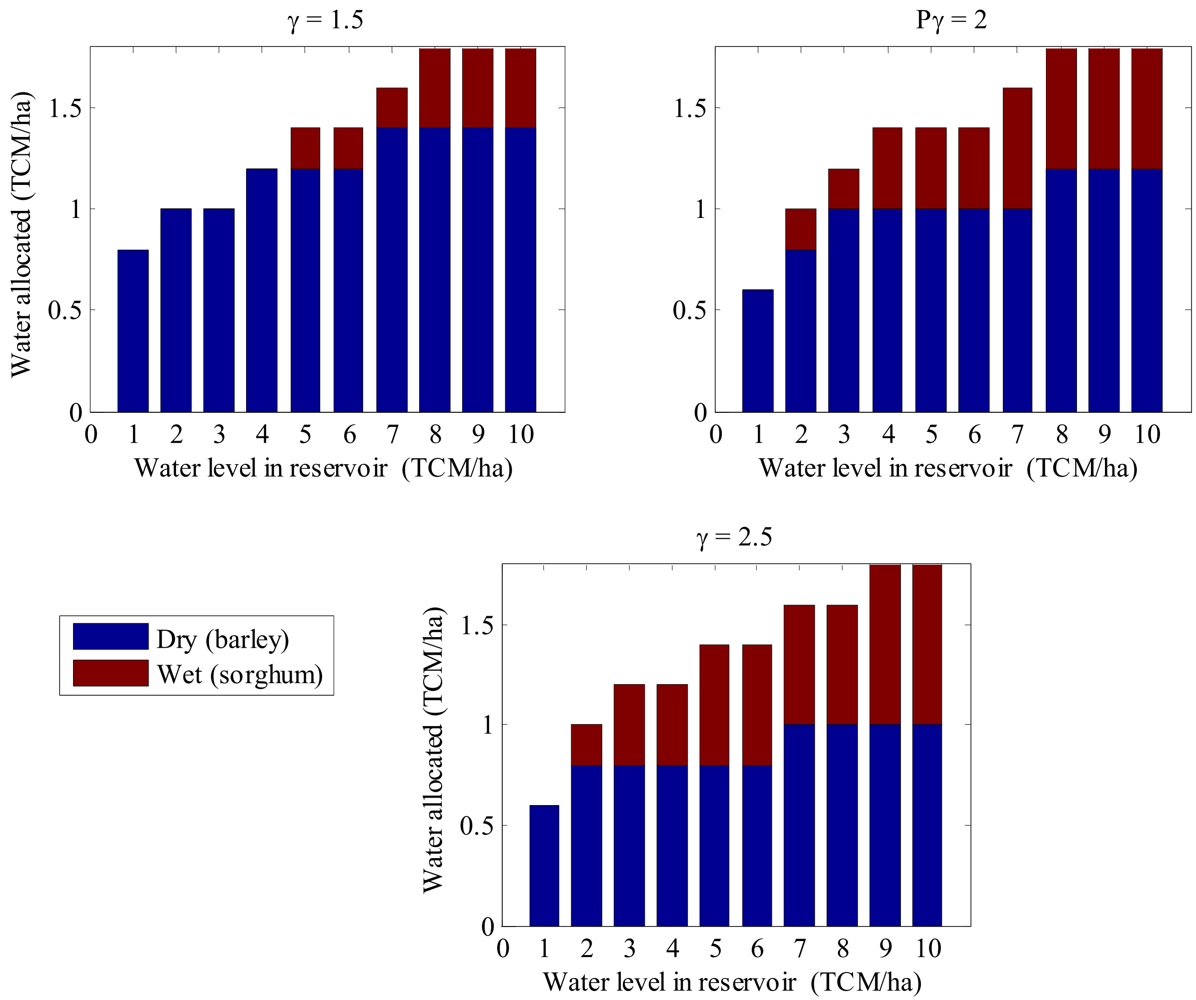


Figure A-37. Value functions for different relative risk aversion parameter

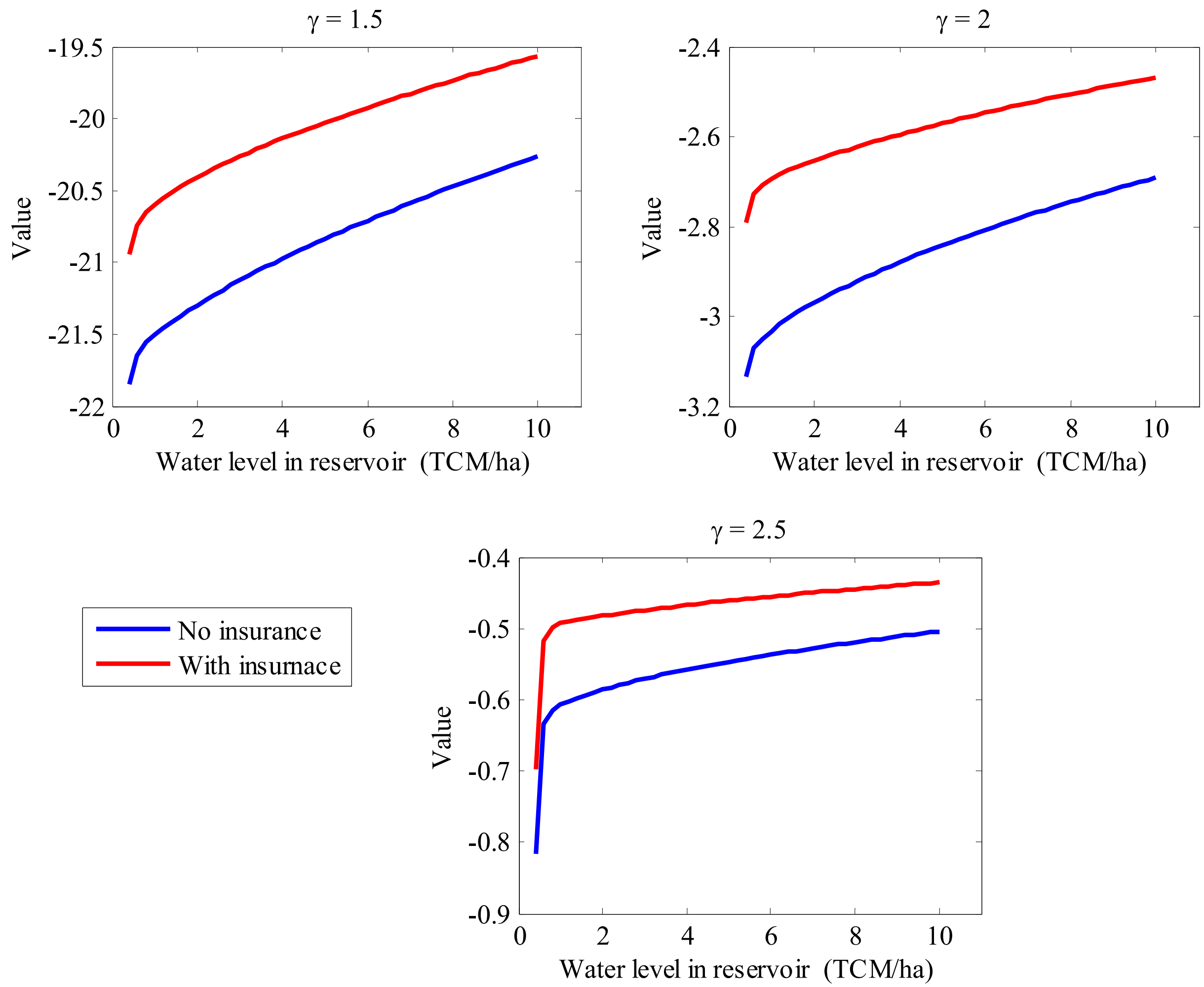


Figure A-38. Comparison of gamma probability distributions with different shape parameters α

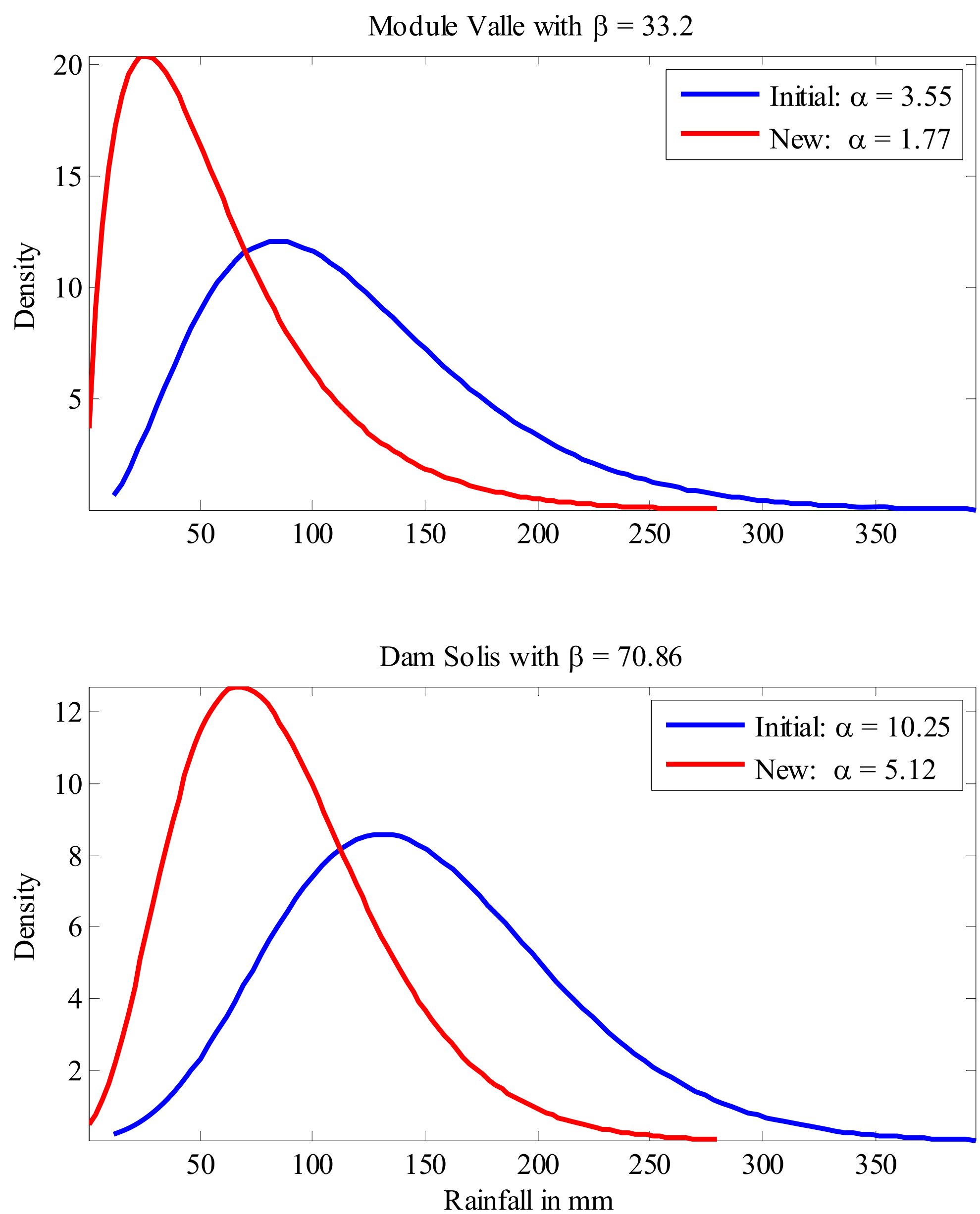


Figure A-39. Water allocation decision without weather derivative when the shape parameter α of the rainfall distribution for Valle and Solis decreases 50%

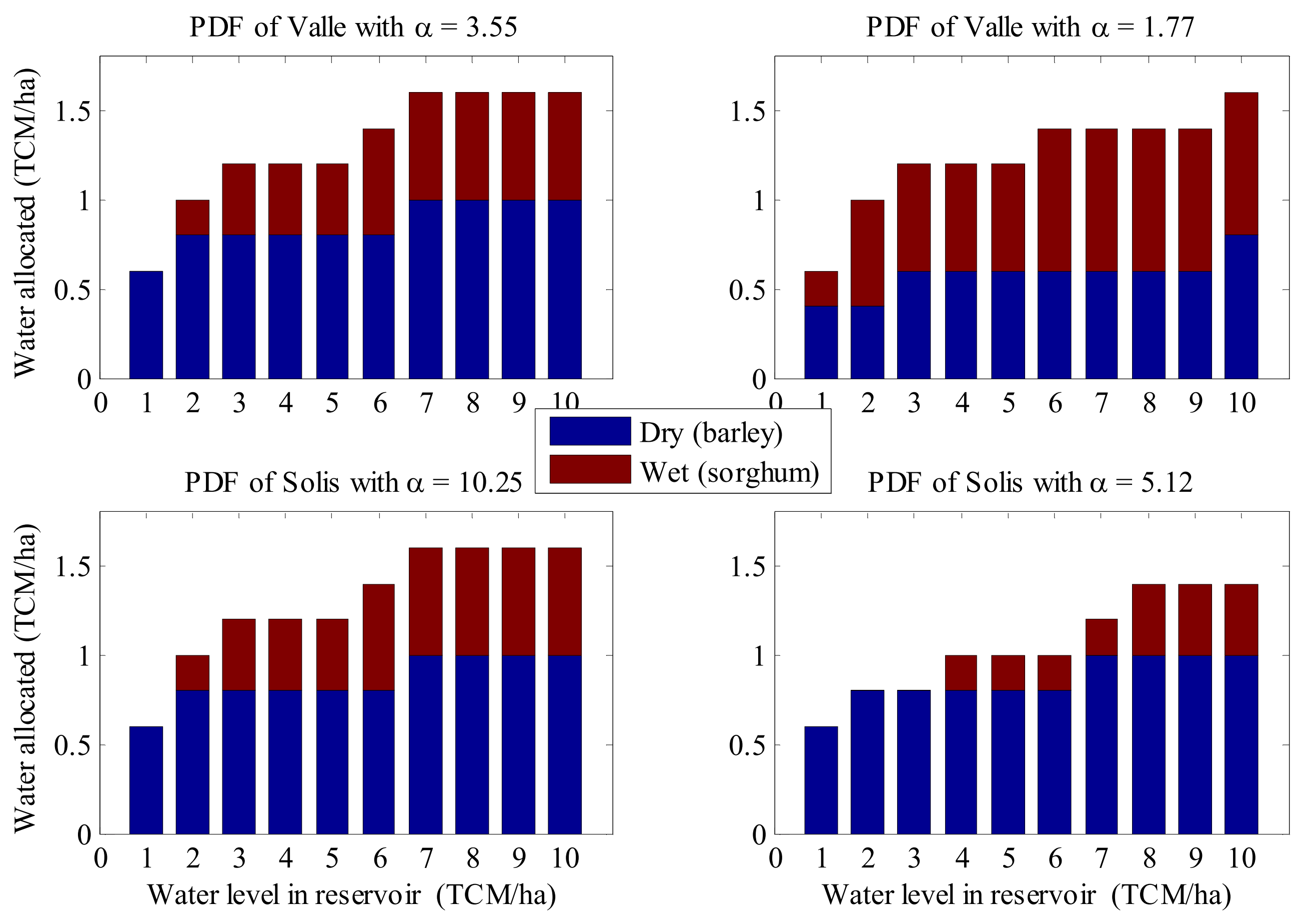


Figure A-40. Water allocation decision with weather derivative when the shape parameter α of the rainfall distribution for Valle and Solis decreases 50%

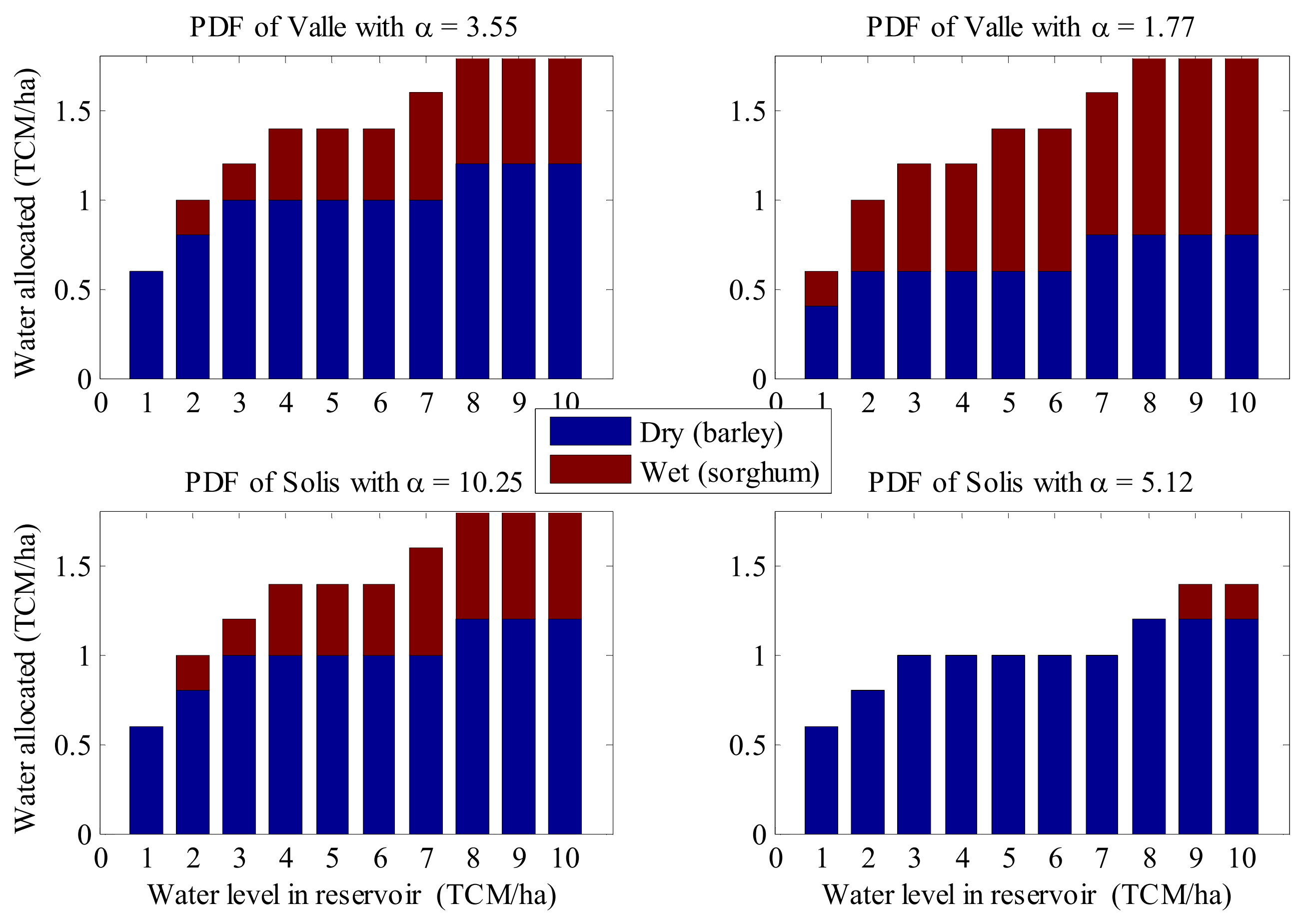
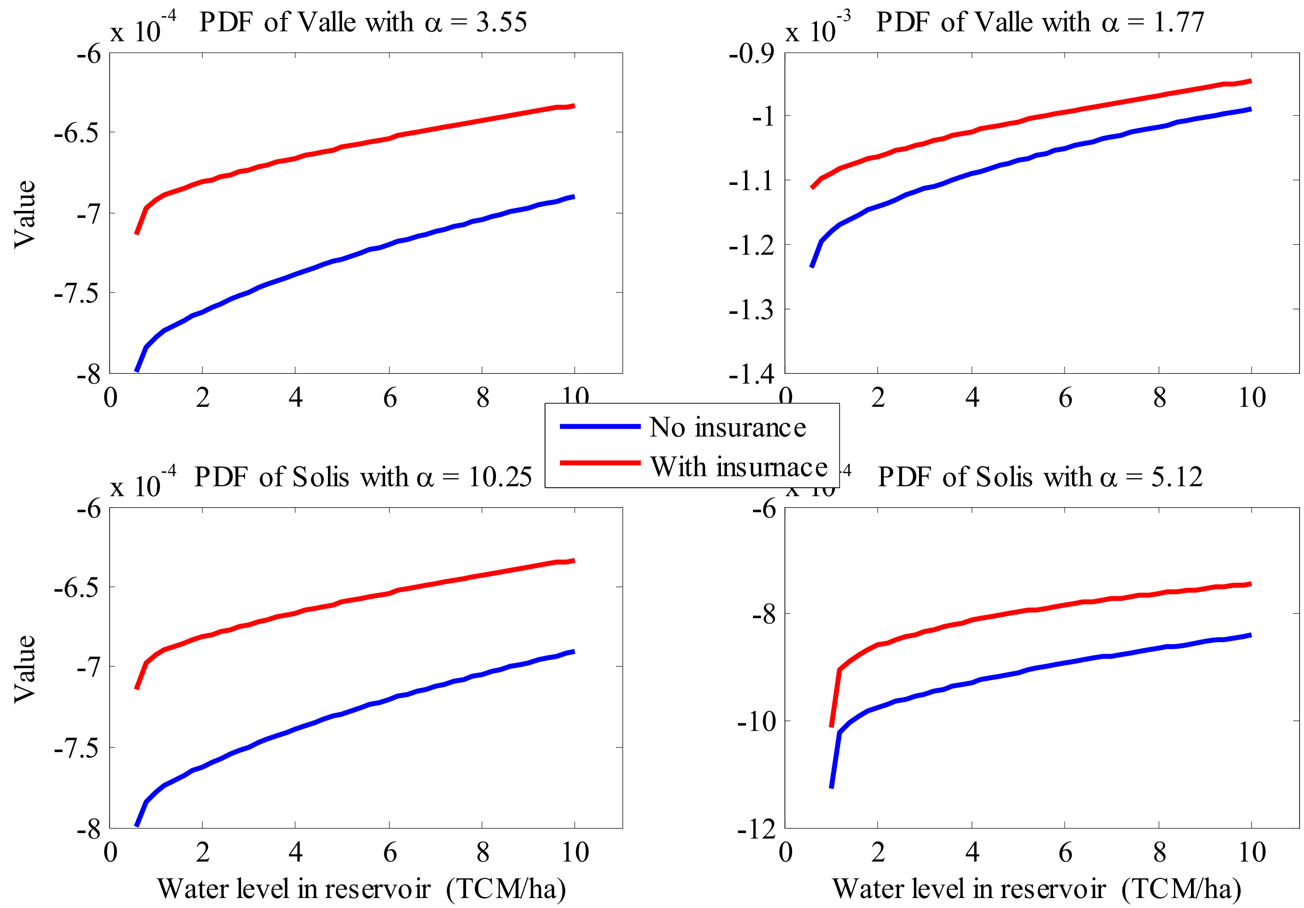


Figure A-41. Value functions when the shape parameter α of the rainfall distribution for Valle and Solis decreases 50%



APPENDIX B: TABLES

Table B-1. Water requirements and duration of each stage of soybean growth

Stages	Water (mm) Minimum	Maximum	Days
Vegetative	119	184	51
Reproductive	291	452	64
Maturity	41	64	18
Entire Season	450	700	133

Table B-2. Distribution of soil type across counties selected for analysis

County	Soil Type I	Soil Type II	Soil Type III	Other	Production Area (acres)
		(%)			
Crittenden	0.4	18.9	78.6	2.1	171,042
Independence	63.1	27.8	9.2	0.0	25,266
Jackson	4.8	78.6	15.2	1.4	134,917
Phillips	0.3	46.8	49.8	3.1	161,460
Pulaski	22.8	30.3	26.9	20.0	25,465
St. Francis	0.3	50.7	49.0	0.0	136,357

Table B-3. Descriptive statistics of precipitation and temperatures for single soil-type counties

	Precipitation (mm)					Temperature (°C)				
Variables\Stages	Mean	Std.Dev	C.V.	Min	Max	Mean	Std.Dev	C.V.	Min	Max
Independence (soil type I) - station code: 30460										
Vegetative	154.3	77.7	0.5	24.8	405.8	26.3	3.9	0.2	19.5	34.1
Reproductive	185.2	85.0	0.5	24.4	412.5	25.4	3.9	0.2	18.1	34.3
Maturity	54.6	49.8	0.9	0.0	236.8	19.0	4.6	0.2	10.1	27.1
Whole season	394.0	135.8	0.3	132.0	672.4	24.9	3.9	0.2	18.5	32.5
Jackson (soil type II) - station code: 35186										
Vegetative	170.1	81.9	0.5	29.4	377.7	23.3	2.3	0.1	19.2	29.7
Reproductive	183.1	82.0	0.5	27.0	413.5	24.9	2.5	0.1	21.2	31.3
Maturity	54.4	50.7	0.9	0.8	230.4	20.2	3.0	0.2	14.9	26.6
Whole season	407.6	134.7	0.3	192.4	872.8	23.7	2.3	0.1	20.0	28.9
Crittenden (soil type III) - station code: 37712										
Vegetative	164.2	79.0	0.5	37.6	375.2	25.6	3.3	0.1	20.7	31.1
Reproductive	174.2	77.7	0.5	19.5	341.0	27.4	3.4	0.1	22.1	34.2
Maturity	53.4	47.0	0.9	0.0	232.7	23.4	4.2	0.2	16.6	31.5
Whole season	391.8	115.0	0.3	122.7	728.9	26.1	3.4	0.1	21.5	32.1

Table B-4. Descriptive statistics of precipitation and temperatures for mixed-soil counties

Variables\Stages	Precipitation (mm) Mean	Std.Dev	C.V.	Min	Max	Temperature (°C) Mean	Std.Dev	C.V.	Min	Max
Phillips (soil types II & III) - station code: 33242										
Vegetative	155.4	80.0	0.5	13.8	415.0	25.2	2.9	0.1	19.2	33.4
Reproductive	162.1	76.8	0.5	13.3	366.6	24.1	3.1	0.1	18.3	31.2
Maturity	48.3	48.5	1.0	0.0	198.5	17.2	4.0	0.2	11.0	26.5
Whole season	365.8	132.5	0.4	78.1	663.4	23.6	3.0	0.1	18.1	30.8
Saint Francis (soil types II & III) - station code: 34528										
Vegetative	160.5	80.0	0.5	36.8	351.8	20.7	0.1	0.01	20.2	21.6
Reproductive	175.6	73.1	0.4	7.4	359.4	22.7	0.3	0.01	20.7	23.7
Maturity	46.7	44.4	1.0	0.0	216.0	18.4	0.7	0.04	15.2	20.7
Whole season	382.8	117.0	0.3	155.8	665.4	21.3	0.2	0.01	20.5	22.5
Pulaski (soil types I, II, and III) - station code: 34010										
Vegetative	145.1	62.6	0.4	21.6	310.7					
Reproductive	188.1	96.8	0.5	19.2	403.8					
Maturity	47.2	43.2	0.9	0.0	185.2					
Whole season	380.4	122.6	0.3	130.3	664.3					

Table B-5. Descriptive statistics of soybean yields for selected counties

County	Soil Type(s)[39]	Mean (Bu/ac)	Std.Dev (Bu/ac)	C.V.	Min	Max
Independence	I	24.4	5.0	0.206	12.5	35.6
Jackson	II	20.0	4.8	0.238	10.7	31.6
Crittenden	III	32.3	6.5	0.201	17.6	48.1
Phillips	II-III	33.3	6.8	0.205	18.0	48.6
Saint Francis	II-III	29.1	7.1	0.244	13.5	42.7
Pulaski	I-II-III	17.5	4.1	0.236	8.6	26.6

[39] See figure A-1 for the definition of soil types.

Table B-6. Estimated yield trend models

	Dependent variable: soybean yield (Bu/ac)					
Coefficients[40]	Independence Soil type I	Jackson Soil type II	Crittenden Soil type III	Phillips Soil types II-III	Saint Francis Soil types II-III	Pulaski Soil types I-II-III
a_0	2.783	2.956	2.447	2.126	2.331	4.714
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)
a_1	0.00963		0.0252	0.0340	0.0252	-0.0470
	(0.507)		(0.104)	(0.038)	(0.198)	(0.142)
b_1	-0.0434	-0.0605	-0.0574	-0.110	-0.0490	0.184
	(0.199)	(0.254)	(0.314)	(0.115)	(0.210)	(0.397)
t_1	1983.0	1987	1986	1987.0	1984	1997.0
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
b_2	0.0982	0.149	0.101	0.167	0.103	-0.507
	(0.380)	(0.254)	(0.196)	(0.068)	(0.442)	(0.094)
t_2	1994.0	1994	1992	1992.5	1994	1999.7
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
b_3	-0.0775	-0.0891	-0.0720	-0.107	-0.0973	0.372
	(0.479)	(0.458)	(0.211)	(0.077)	(0.462)	(0.076)
t_3	1997.2	1998	1998	1998.3	1998	2002.1
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
N. of observ.	41	41	41	41	41	41
Adj. R-square	-0.055	0.020	0.103	0.197	-0.063	0.001

Note: Numbers in parentheses are p-values.

[40] See equation (2.11).

Table B-7. Estimated rainfall-yield models[41]

	Dependent variable: detrended yield (Bu/ac)					
Variables	Independence Soil type I	Jackson Soil type II	Crittenden Soil type III	Phillips Soil types II-III	Saint Francis Soil types II-III	Pulaski Soil types I-II-III
Rainfall vegetative	0.082	0.093	0.142	0.132	0.147	0.034
	(0.017)	(0.003)	(0.022)	(0.027)	(0.011)	(0.003)
Rainfall vegetative square	-0.0002	-0.0003	-0.0004	-0.0003	-0.0004	
	(0.035)	(0.003)	(0.037)	(0.029)	(0.017)	
Rainfall reproductive	0.098	0.058	0.024	0.015	0.031	0.011
	(0.003)	(0.002)	(0.074)	(0.200)	(0.030)	(0.121)
Rainfall reproductive square	-0.0002	-0.00007				
	(0.018)	(0.031)				
Rainfall maturity				0.03	0.032	
				(0.223)	(0.096)	
Constant	6.334	6.593	17.046	18.843	11.192	11.196
	(0.171)	(0.032)	(0.000)	(0.000)	(0.023)	(0.000)
Number of observations	41	41	41	41	41	41
Adj. R-square	0.288	0.379	0.176	0.161	0.257	0.227

Note: Rainfall vegetative/reproductive/maturity is in millimeters and represent the cumulative daily precipitation during the respective stage. Numbers in parentheses are p-values.

[41] The procedure used to estimate these models is based on (Vedenov and Barnett 2004).

Table B-8. Parameter of the best agronomic contracts for single soil-type counties

Stages	Contract	Cap (%)	Scale (%)	Premium (Bu/ac)	Liability (Bu/ac)	Premium Rate (%)	CER with (Bu/ac)	CER w/o (Bu/ac)	ΔCER (%)
Independence (Soil type I)									
Vegetative	lack of water	70	85	0.76	4.43	17.16	22.96	22.97	-0.03
Reproductive	lack of water	70	85	3.91	9.21	42.45	23.10	22.97	0.58
Maturity	excess of water	70	85	0.12	3.41	3.52	22.99	22.97	0.09
Entire season	excess of water	70	85	0.03	17.05	0.18	22.99	22.97	0.09
Jackson (Soil type II)									
Vegetative	lack of water	70	85	0.84	3.89	21.59	18.34	18.28	0.28
Reproductive	lack of water	70	85	4.05	8.07	50.19	18.40	18.28	0.62
Maturity	excess of water	70	85	0.08	2.99	2.68	18.30	18.28	0.11
Entire season	excess of water	70	85	0.15	14.94	1.00	18.23	18.28	-0.31
Crittenden (Soil type III)									
Vegetative	lack of water	70	85	0.95	5.91	16.07	30.78	30.71	0.20
Reproductive	excess of water	70	85	0.10	12.28	0.81	30.71	30.71	-0.01
Maturity	excess of water	70	85	0.10	4.55	2.20	30.72	30.71	0.02
Entire season	excess of water	70	85	0.13	22.74	0.57	30.70	30.71	-0.04

Table B-9. Parameter of the best agronomic contracts for mixed-soil counties

Stages	Contract	Cap (%)	Scale (%)	Premium (Bu/ac)	Liability (Bu/ac)	Premium Rate (%)	CER with (Bu/ac)	CER w/o (Bu/ac)	ΔCER (%)
Phillips (Soil types II & III)									
Vegetative	lack of water	70	85	1.22	6.43	18.97	31.46	31.43	0.13
Reproductive	excess of water	70	85	0.04	13.36	0.30	31.45	31.43	0.09
Maturity	lack of water	70	85	3.21	4.95	64.85	31.50	31.43	0.23
Entire Season	excess of water	70	85	0.02	24.73	0.08	31.46	31.43	0.12
Saint Francis (Soil types II & III)									
Vegetative	lack of water	70	85	1.03	5.58	18.46	26.77	26.69	0.30
Reproductive	excess of water	70	85	0.01	11.59	0.09	26.74	26.69	0.19
Maturity	lack of water	70	85	2.66	4.29	62.00	26.81	26.69	0.45
Entire Season	excess of water	70	85	0.01	21.46	0.05	26.74	26.69	0.19
Pulaski (Soil types I, II, and III)									
Vegetative	lack of water	70	85	0.57	3.42	16.67	16.16	16.09	0.41
Reproductive	excess of water	70	85	0.05	7.10	0.70	16.11	16.09	0.07
Maturity	excess of water	70	85	0.11	2.63	4.18	16.09	16.09	-0.02
Entire Season	excess of water	70	85	0.08	13.15	0.61	16.09	16.09	-0.03

Table B-10. Parameters of the "optimal" contracts based on econometric models[42]

County	Soil type(s)	Strike (Bu/ac)	Limit: Absolute value	Limit: % of strike	Max. liab (Bu/ac)	Premium (Bu/ac)	Premium rate (%)	CER with (Bu/ac)	CER w/o (Bu/ac)	ΔCER %
Independence	I	30.35	21.25	0.70	7.77	4.80	61.80	23.27	22.97	1.28
Jackson	II	24.85	17.40	0.70	7.86	4.88	62.14	18.78	18.28	2.71
Crittenden	III	69.00	27.60	0.40	27.31	24.05	88.04	30.86	30.71	0.48
Phillips	II - III	35.57	30.23	0.85	6.61	2.87	43.36	31.66	31.43	0.74
Saint Francis	II - III	33.21	24.91	0.75	7.33	3.61	49.21	27.10	26.69	1.54
Pulaski	I - II - III	20.77	16.62	0.80	5.56	3.75	67.45	16.44	16.11	2.08

[42] The procedure used to calculate these parameters is based on Vedenov and Barnett (2004)

Table B-11. Counties selected for analysis in Essay 2 with their soil types and phenological growth stages

Counties	Soil types	Stages
Phillips	II-III	Maturity
Saint Francis	II-III	Maturity
Pulaski	I-II-III	Vegetative

Table B-12. Descriptive statistics of precipitation (in millimeters) for selected counties

County	Phenological stages	Mean	Std. Dev.	C.V.	Min	Max
Phillips	Maturity	48.3	48.5	1.0	0.0	198.5
Saint Francis	Maturity	46.7	44.4	1.0	0.0	216.0
Pulaski	Vegetative	145.12	62.63	0.43	21.6	310.7

Table B-13. Descriptive statistics of soybean yields for selected counties

County	Mean (Bu/ac)	Std. Dev. (Bu/ac)	C.V.	Min	Max
Phillips	33.3	6.8	0.205	18.0	48.6
Saint Francis	29.1	7.1	0.244	13.5	42.7
Pulaski	17.5	4.1	0.236	8.6	26.6

Table B-14. Estimated rainfall-yields models

	Dependent variable: detrended yield (Bu/ac)		
Variables	Phillips	Saint Francis	Pulaski
Rainfall	0.049	0.041	0.036
	(0.052)	(0.054)	(0.002)
Constant	30.64	26.44	12.83
	(0.000)	(0.000)	(0.000)
N. of observations	41	41	41
Adj. R-square	0.07	0.07	0.2
Growth stage	Maturity	Maturity	Vegetative

Note: Rainfall is in millimeters and represent the cumulative daily precipitation during the respective growth stage. Numbers in parentheses are p-values.

Table B-15. Parameters of the standard contracts for selected counties

	Phillips	Saint Francis	Pulaski
Growth stage	Maturity	Maturity	Vegetative
Strike (mm)	80.57	110.56	221.27
Limit (mm)	0.00	0.00	106.09

Table B-16. Comparison of the "standard" and optimal contracts for the selected counties

	Phillips			Saint Francis			Pulaski		
	γ = 1.5	γ = 2	γ = 3	γ = 1.5	γ = 2	γ = 3	γ = 1.5	γ = 2	γ = 3
				No contract					
CER (Bu/ac)	31.94	31.43	30.35	27.34	26.69	25.26	16.50	16.09	15.20
Expected utility	-1.12	-0.32	-0.05	-1.21	-0.37	-0.08	-1.56	-0.62	-0.22
				Standard contract					
Max. Liability (Bu/ac)	3.81	3.43	2.93	3.84	3.80	3.42	5.26	5.26	5.26
Premium (Bu/ac)	1.81	1.63	1.02	2.12	2.10	1.89	3.30	3.30	3.30
Premium rate (%)	47.39	47.39	34.59	55.28	55.29	55.28	62.77	62.79	62.77
Expected utility	-1.12	-0.32	-0.05	-1.21	-0.37	-0.08	-1.55	-0.61	-0.20
ΔCER (%)	0.23	0.32	0.40	0.39	0.50	0.81	1.29	1.81	3.06
				Optimal contract					
Max. Liability (Bu/ac)	4.11	4.05	2.98	4.43	4.28	3.91	6.18	5.90	5.78
Premium (Bu/ac)	2.51	2.40	1.48	2.87	2.70	2.43	4.32	4.14	4.19
Premium rate (%)	61.09	59.26	49.50	64.87	62.97	62.27	69.93	70.19	72.52
Expected utility	-1.12	-0.32	-0.05	-1.20	-0.37	-0.08	-1.54	-0.61	-0.20
ΔCER (%)	0.57	0.67	0.80	0.79	0.93	1.31	2.04	2.62	4.08
				Percentage change of CER: Optimal and Standard contracts					
	0.34	0.35	0.39	0.40	0.42	0.49	0.79	0.80	0.99

Note: CER stands for certainty equivalent. ΔCER is the certainty equivalent variation with respect to no-contract case. Premium rate is the ratio between premium and maximum liability.

Table B-17. Descriptive statistics of crop yields and water allocation in module Valle

Variables	Mean	Std. Dev.	CV	Min	Max	Range of data
			Yield (Tons / ha)			
Barley	5.065	0.871	0.172	3.380	6.500	1985-2011
Sorghum	6.662	1.285	0.193	4.408	10.650	1985-2005
			Allocated water (TCM/ha)			
Barley	5.776	0.347	0.060	4.663	6.201	1989-2011
Sorghum	2.920	0.687	0.235	2.081	5.003	1989-2011

CV stands for coefficient of variation TCM/ha.
TCM/ha stands for thousands of cubic meters per hectare.
Barley s the dry-season crop, and sorghum is the wet-season crop

Table B-18. Trend models for barley (dry season) and sorghum (wet season)

Coefficients	Barley yield (Ton./ha)	Sorghum yield (Ton./ha)
a_0	0.375	1.018
	(0.004)	(0.097)
a_1	0.0666	0.0661
	(0.000)	(0.138)
b_1	0.199	0.167
	(0.097)	(0.033)
t_1	1991	1994.5
	(0.000)	(0.000)
b_2	-0.0592	-0.149
	(0.000)	(0.052)
t_2	2009	1999.5
	(0.000)	(0.000)
N. of observ.	22	17
Adj. R-square	0.301	0.209

Note: Numbers in parentheses are p-values.

Table B-19. Descriptive statistics of precipitation at module Valle and dam Solis

	Mean mm	Std. Dev mm	Coef. Variation	Min mm	Max mm
Module Valle (station 11079) - data range 1942-2010					
January	14.50	25.23	1.74	0.00	112.00
February	9.30	19.47	2.09	0.00	138.50
March	5.33	8.46	1.59	0.00	40.00
April	9.51	14.76	1.55	0.00	71.00
May	34.11	30.24	0.89	0.00	115.00
June	113.36	59.86	0.53	0.00	264.00
July	146.44	69.85	0.48	0.00	353.40
August	146.45	61.54	0.42	20.50	371.40
September	113.05	62.92	0.56	9.30	294.50
October	48.49	36.43	0.75	0.00	173.00
November	12.60	15.89	1.26	0.00	58.60
December	7.42	12.85	1.73	0.00	79.20
Dam Solis (station 11076) - data range 1961-2011					
January	12.44	19.21	1.54	0.00	95.00
February	5.62	8.63	1.54	0.00	31.70
March	7.24	11.39	1.57	0.00	61.30
April	9.54	12.82	1.34	0.00	64.60
May	42.76	35.57	0.83	0.00	146.30
June	121.66	52.90	0.43	24.20	246.70
July	166.43	59.85	0.36	44.00	314.80
August	157.82	61.19	0.39	38.40	291.50
September	134.25	69.57	0.52	23.70	413.60
October	51.60	38.33	0.74	0.00	152.50
November	8.61	9.49	1.10	0.00	35.40
December	6.88	10.87	1.58	0.00	51.50

Table B-20. Parameters used in the dynamic model of water allocation

Parameter	Value	Units/Description
$\bar{S}$	10	TCM/ha (maximum reservoir level)
α	0.11	Share of rainwater attributable to Valle
γ	1 to 3	Risk aversion coefficient
P_{dry}	3,900	Pesos/tons (price of barley)
P_{wet}	4,500	Pesos/tons (price of sorghum)
P_w	160	Pesos/TCM
r	5	percent
δ	0.952	It is equal to $1/(1+r)$
a_1	0.4353	Linear coefficient of barley production function
a_2	-0.000377	Quadratic coefficient of barley production function
b_1	0.0983	Linear coefficient of sorghum production function
b_2	-0.000095	Quadratic coefficient of sorghum production function
θ	100%	Coverage level
$E(rain)$	117.74	Expected rainfall in module Valle in mm
α_{Valle}	3.55	Shape parameter of gamma distribution for Valle[43]
β_{Valle}	33.20	Scale parameter of gamma distribution for Valle[44]
α_{Solis}	10.25	Shape parameter of gamma distribution for Solis[45]
β_{Solis}	70.86	Scale parameter of gamma distribution for Solis[46]

[43] Confidence interval: [2.58, 4.88].
[44] Confidence interval: [23.56, 46.78].
[45] Confidence interval: [6.97, 15.07].
[46] Confidence interval: [47.72, 105.22].