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Out-of-plane transverse resistivity in high-T_c superconductors as a signature of flow of rigid vortex lines

Zhidong Hao
Texas Center for Superconductivity, University of Houston, Houston, Texas 77204

Chia-Ren Hu
Department of Physics, Texas A&M University, College Station, Texas 77843

C.-S. Ting
Texas Center for Superconductivity, University of Houston, Houston, Texas 77204

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When the transport current is applied parallel to the CuO_2 layers, say, along the a axis, of a high-T_c superconductor, and the magnetic field **B** is in a direction which makes a polar angle θ with the c axis and an azimuthal angle ϕ with the ac plane, for the case of rigid flux lines, in addition to the usual longitudinal resistivity $\rho_{\parallel}$, there should also exist an out-of-plane transverse resistivity $\rho_{\perp}$, which is of the same order of magnitude as $\rho_{\parallel}$ and satisfies the relation $|\rho_{\perp}/\rho_{\parallel}|=\tan\theta\cos\phi$ in the high anisotropy limit and for θ being not very close to $\pi/2$. For less rigid flux lines, reduction in $|\rho_{\perp}/\rho_{\parallel}|$ from this prediction should be observed, and for a set of decoupled pancake vortices, $\rho_{\perp}$ should vanish entirely.

Because of the layered crystal structures, magnetic vortices in the high-temperature superconductors (HTSCs) may be considered as stacks of two-dimensional pancake vortices coupled via magnetic and Josephson forces.[1] The rigidity of the vortices in these layered materials, or the strength of the interlayer coupling, has been the subject of several recent experimental studies.[2–6] In the flux transformer experiments of Busch *et al.*[2] and of Safar *et al.*,[3,4] a magnetic field was applied along the c axis, a transport current was injected along one surface ($\|ab$) of the sample, and voltages along both sides were measured simultaneously. In the case of Bi2212, it was found that the voltage signal on the side of current contacts was much greater then that on the other side, indicating that vortices were sheared under the influence of a highly nonuniform Lorentz force. (Due to the smallness of the c axis conductivity in comparison with the ab plane conductivity, current flew mainly in the layers close to the surface of current contacts.)[2,3] In the case of Y-Ba-Cu-O (YBCO), in spite of the nonuniform current distribution, voltage signals on both sides were essentially identical, indicating that vortices were moving as rigid lines.[4] In the experiment of decorating vortices on both sides of a Bi2212 single crystal by Yao *et al.*,[5] vortices (in the low-field regime) were found to be linelike objects. Lee *et al.*[6] measured flux noise generated by films and crystals of Bi2212 and YBCO (in zero applied magnetic field) at opposing surfaces; their results indicates that the thermally activated vortices in both Bi2212 and YBCO move as rigid lines (at specific temperatures).

In this paper we suggest an experiment to study the rigidity of the vortices in HTSCs in the flux-flow state. When the current is applied along the ab plane and magnetic field **B** is tilted away from the c axis, we show that if the vortices move as rigid lines, an out-of-plane transverse resistivity should be observed, which is of the same order of magnitude as the in-plane longitudinal resistivity, and there exists a quantitative relation between them, which, for **B** not too close to the ab plane, is a function only of the orientation of **B** relative to the applied current and the c axis. For less rigid flux lines, deviations from this relation (i.e., smaller transverse resistivity) should be observed, and if the vortices move as decoupled pancake vortices, the transverse resistivity should vanish entirely. Thus, by measuring the relation between the transverse and longitudinal resistivities, one can infer the rigidity of the vortices as a function of temperature, the magnitude of the transport current, and the magnitude and orientation of the applied field. Results obtained from such an experiment can complement those from other experiments (for example, Refs. 2–6).

Let the coordinate axes x_1, x_2, and x_3 be parallel to the a, b, and c axes, respectively, and consider the configuration in which the applied current $\mathbf{J}^{\text{ext}}$ is parallel to x_1, and the magnetic field **B** is in the direction specified by the polar and azimuthal angles θ and ϕ.[7] The macroscopic electric field $\mathbf{E}=(E_1,E_2,E_3)$ induced by the flux motion can be determined by measuring the voltages along the three axes. The longitudinal, in-plane transverse, and out-of-plane transverse resistivities are defined, respectively, by

$$\rho_{\parallel}=\frac{E_1}{J^{\text{ext}}},\quad \rho_{\perp}^{(2)}=\frac{E_2}{J^{\text{ext}}},\quad \rho_{\perp}^{(3)}=\frac{E_3}{J^{\text{ext}}}. \tag{1}$$

In the following we first deduce some general, model-independent expressions for $\rho_{\parallel}$, $\rho_{\perp}^{(2)}$, and $\rho_{\perp}^{(3)}$ for the case of rigid flux lines, and then compare the results with the predictions based on the assumption of decoupled pancake vortices.

The macroscopic electric field, induced by a uniform motion of vortices with velocity **v**, obeys[8] $\mathbf{E}=-(\mathbf{v}\times\mathbf{B})/c$, which shows that in the mixed state **E** is always perpendicular to **B**; i.e.,

$$\mathbf{E}\cdot\mathbf{B}=0. \tag{2}$$

In a recent work[9] we have pointed out that, when this equation is combined with the linear response relation

$$E_i=\rho_{ij}J_j^T, \tag{3}$$

where ρ_{ij} is the flux-flow resistivity tensor and $\mathbf{J}^T$ is the dissipative transport current density, it gives

$$\rho_{11}J_1^TB_1+\rho_{22}J_2^TB_2+\rho_{33}J_3^TB_3=0, \tag{4}$$

where we have neglected the Hall elements of the tensor ρ_{ij}, since they are usually smaller than the smallest of the diagonal elements by $O(10^{-3})$.

As we have emphasized in Ref. 9, Eq. (4) implies a constraint on the relative orientation of $\mathbf{J}^T$ and $\mathbf{B}$. Thus in many experimental configurations, $\mathbf{J}^T$ actually cannot be identified with $\mathbf{J}^{\text{ext}}$, which, being an externally applied quantity, can be arbitrary. (Here both $\mathbf{J}^T$ and $\mathbf{J}^{\text{ext}}$ are assumed to be uniform.) Instead, in such cases, we have to identify $\mathbf{J}^{\text{ext}}$ as a sum of $\mathbf{J}^T$ and a nondissipative supercurrent density $\mathbf{J}^S$ along $\mathbf{B}$,[9] i.e.,

$$\mathbf{J}^{\text{ext}}=\mathbf{J}^T+\mathbf{J}^S. \tag{5}$$

The current $\mathbf{J}^S\|\mathbf{B}$ corresponds to a uniform translation of the whole "superfluid" along $\mathbf{B}$ and is not included in Eq. (3).

In Ref. 9 (where the problem of Lorentz force independence of the longitudinal resistivity $\rho_\|$ is discussed), for the purpose of presenting the main conceptual idea using the simplest mathematical expressions, we have restricted $\mathbf{J}^T$ to be in the x_1x_3 plane. Then for an extremely anisotropic system we showed there that $\mathbf{J}^{\text{ext}}$ is very nearly along x_1, but it is not exactly along this direction. A similar but slightly more complicated calculation can be carried out, in which $\mathbf{J}^{\text{ext}}$ is strictly in the x_1 direction. We then find that all the components of $\mathbf{J}^T$ and $\mathbf{J}^S$ are nonzero in general, as given below:

$$J_1^T=J^{\text{ext}}-J_1^S=J^{\text{ext}}(\rho_{22}\sin^2\theta\,\sin^2\phi+\rho_{33}\cos^2\theta)/\tilde{\rho}, \tag{6}$$

$$J_2^T=-J_2^S=-J^{\text{ext}}\rho_{11}\sin^2\theta\,\cos\phi\,\sin\phi/\tilde{\rho}, \tag{7}$$

$$J_3^T=-J_3^S=J^{\text{ext}}\rho_{11}\sin\theta\,\cos\theta\,\cos\phi/\tilde{\rho}, \tag{8}$$

where

$$\tilde{\rho}=\rho_{11}\sin^2\theta\,\cos^2\phi+\rho_{22}\sin^2\theta\,\sin^2\phi+\rho_{33}\cos^2\theta. \tag{9}$$

This calculation uses only Eqs. (3)–(5) and the fact that $\mathbf{J}^S\|\mathbf{B}$. It is therefore completely general and model independent.

Using Eqs. (6)–(8) and the fact that $E_i\simeq\rho_{ii}J_i^T$, we can easily calculate the three quantities defined in Eq. (1). The results are

$$\rho_\|=\rho_{11}(\rho_{22}\sin^2\theta\,\sin^2\phi+\rho_{33}\cos^2\theta)/\tilde{\rho} \tag{10}$$

$$\rho_\perp^{(2)}=-\rho_{11}\rho_{22}\sin^2\theta\,\cos\phi\,\sin\phi/\tilde{\rho} \tag{11}$$

$$\rho_\perp^{(3)}=-\rho_{11}\rho_{33}\sin\theta\,\cos\theta\,\cos\phi/\tilde{\rho}. \tag{12}$$

If the supercurrent $\mathbf{J}^S$ along $\mathbf{B}$ is large enough, it can induce a helical instability in the flux-line lattice,[10] which in turn invalidates our assumption of rigid (straight) flux lines. Thus, Eqs. (10)–(12) are valid only under the assumption that no such instability occurs. For our case of $\mathbf{J}^{\text{ext}}\|x_1$ and assuming the extreme-anisotropy condition, $\rho_{11}\simeq\rho_{22}\ll\rho_{33}$, as can be seen in Eqs. (6)–(8), J^S is indeed always very small, except in the limit of $\theta\to\pi/2$, with the ratio J^S/J^{ext} being only $O(\rho_{11}/\rho_{33})$. [In Ref. 9, below its Eq. (10), the magnitudes of (ρ_{11}/ρ_{33}) for various HTSCs have been given, and are indeed all very small.] Thus except for $\theta\to\pi/2$ we do not have to worry about helical instabilities (unless the applied current is extremely large), and the above results are valid. In this case we also have $J_2^T/J_1^T\sim O(\rho_{11}/\rho_{33})$ and $J_3^T/J_1^T\sim O(\rho_{11}/\rho_{33})$. But the tiny current J_3^T in the x_3 direction can induce an electric field E_3 of the same order of magnitude as E_1, because $\rho_{11}J_1^T\sim\rho_{33}J_3^T$.

Restricting here to the case that θ is not very close to $\pi/2$, and $\rho_{11}\simeq\rho_{22}\ll\rho_{33}$, Eqs. (10)–(12) reduce to

$$\rho_\|=\rho_{11}[1+O(\rho_{11}/\rho_{33})], \tag{13}$$

$$\rho_\perp^{(2)}=\rho_{11}O(\rho_{11}/\rho_{33}), \tag{14}$$

$$\rho_\perp^{(3)}=-\rho_{11}\tan\theta\,\cos\phi[1+O(\rho_{11}/\rho_{33})]. \tag{15}$$

Clearly, for the in-plane transverse resistivity, the ratio $\rho_\perp^{(2)}/\rho_\|\sim O(\rho_{11}/\rho_{33})$ is negligibly small, but for the out-of-plane transverse resistivity, we have

$$\rho_\perp^{(3)}/\rho_\|=E_3/E_1=-\tan\theta\,\cos\phi. \tag{16}$$

We now compare the above results with the predictions based on the assumption of a system of decoupled pancake vortices. In the latter case, only the field component parallel to the c (x_3) axis is responsible for forming the vortices, so that $\rho_\|(\mathbf{B})=\rho_\|(B\cos\theta)$ (for θ being not too close to $\pi/2$) and is clearly Lorentz force independent (i.e., for a given B, it depends only on the angle θ between $\mathbf{B}$ and the c axis, but is independent of the angle between $\mathbf{B}$ and $\mathbf{J}^{\text{ext}}$).[11] The Lorentz-force independence (LFI) of $\rho_\|$ has been observed experimentally (including also the case of $\theta=\pi/2$) (see, for example, Refs. 12–16) and was first explained in terms of the formation of pancake vortices.[11] However, in Eq. (13), $\rho_\|\simeq\rho_{11}$ is also Lorentz-force independent, because ρ_{11}, a linear transport coefficient, cannot depend on the driving current $\mathbf{J}^{\text{ext}}$, but can only depend on the equilibrium properties of the sample. This explanation of the LFI of $\rho_\|$ in terms of rigid flux lines was given in Ref. 9. It also has a $\rho_\|(\mathbf{B})\simeq\rho_{11}[B/H_{c2}(\theta,\phi)]\simeq\rho_\|(B\cos\theta)$ field dependence (for the latter point, see the discussion and the references cited in Ref. 9). Thus, from the LFI, and/or the $\mathbf{B}$ dependence of $\rho_\|$, one actually cannot tell whether the vortices in HTSCs are decoupled pancake vortices or rigid flux lines, as the predictions for the behavior of $\rho_\|$ based on the two assumptions are practically the same.

As to the transverse resistivities, we have $\rho_\perp^{(2)}=\rho_\perp^{(3)}=0$ for a system of decoupled pancake vortices, since the motion of the pancake vortices in the x_2 direction [under the driving force $(\hat{\mathbf{x}}_1J^{\text{ext}})\times(\hat{\mathbf{x}}_3B\cos\theta)$] can only induce an electric field in the x_1 direction (neglecting Hall effect).

For the cases of intermediate interlayer coupling strengths, it is necessary to consider the contributions from the Josephson vortices which fit in the interlayer regions. In the presence of $\mathbf{J}^{\rm ext}=J^{\rm ext}\hat{x}_1$, the driving force acting on the Josephson vortices is always in the x_3 direction. Because of the substantial energy barrier against direct hoping of the Josephson vortices across superconducting layers, it is more likely that the hoping is mediated by the creation of pancake and antipancake vortex pairs, as described in Ref. 15. Motion of the so-created pancake and antipancake vortices in opposite directions along a Josephson vortex can contribute to both E_1 and E_2, but not to E_3, and the hopping of the Josephson vortices in the x_3 direction cannot contribute to E_3 either. Although this qualitative argument suggests that both E_1 and E_2 can likely exist in the intermediate interlayer-coupling regime, it is likely true that E_2 will be negligibly small in comparison with E_1 for extremely anisotropic systems, since it is already found here to be true for both extreme limits of strongly coupled layers and completely decoupled layers.

Since the driving (Lorentz) force acting on the Josephson vortices is always zero in any direction parallel to the layers, they can only be dragged to move along the layers by the pancake vortices in the superconducting layers. This motion can then induce an electric field E_3 in the x_3 direction, but slippage may occur between the motion of the pancake vortices and that of the Josephson vortices to make E_3 smaller than that given by Eq. (16). We expect such slippage to occur with larger frequency for weaker interlayer coupling, and therefore E_3 should be a monotonic function of the interlayer coupling strength; i.e., it is larger for stronger interlayer coupling, having maximum in the limit of rigid flux lines and minimum $\rho_\perp^{(3)}=0$ in the limit of decoupled pancake vortices.

Experimentalists can of course measure E_2 to see whether it is indeed always small in comparison with E_1 in extremely anisotropic superconductors, but the main point of this paper is that the most significant difference between the predictions based on the two limiting assumptions (of rigid vortex lines vs decoupled pancake vortices) is in the out-of-plane transverse resistivity $\rho_\perp^{(3)}$: it is finite and satisfies Eq. (16) for rigid flux lines, but is zero for decoupled pancake vortices. For intermediate cases, $\rho_\perp^{(3)}$ may be nonzero, but should lie between zero and that given by Eq. (16). It is also interesting to note that $\rho_\perp^{(3)}$ is strongly ϕ dependent [see Eq. (16)], whereas $\rho_\parallel$ is ϕ independent (in the extremely anisotropic limit, and neglecting any in-plane anisotropy). Thus for experimentalists to find out whether the electric transport properties of extremely anisotropic, high-T_c or other superconductors are truly "Lorentz-force independent" (more precisely, ϕ independent), they should measure the ϕ dependence of E_3 and $\rho_\perp^{(3)}$, and not that of E_1 and $\rho_\parallel$.

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