

Density matrix expansion for the isospin- and momentum-dependent MDI interactionJun Xu¹ and Che Ming Ko²¹*Cyclotron Institute, Texas A&M University, College Station, Texas 77843-3366, USA*²*Cyclotron Institute and Department of Physics and Astronomy, Texas A&M University, College Station, Texas 77843-3366, USA*

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By assuming that the isospin- and momentum-dependent MDI interaction has a form similar to the Gogny-like effective two-body interaction with a Yukawa finite-range term and the momentum dependence originates only from the finite-range exchange interaction, we determine its parameters by comparing the predicted potential energy density functional in uniform nuclear matter with what has been usually given and used extensively in transport models for studying isospin effects in intermediate-energy heavy-ion collisions as well as in investigating the properties of hot asymmetric nuclear matter and neutron star matter. We then use the density matrix expansion to derive from the resulting finite-range exchange interaction an effective Skyrme-like zero-range interaction with density-dependent parameters. As an application, we study the transition density and pressure at the inner edge of neutron star crusts using the stability conditions derived from the linearized Vlasov equation for the neutron star matter.

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I. INTRODUCTION

One of the phenomenological nucleon-nucleon interactions that have been extensively used in transport models to study heavy-ion collisions is the isospin- and momentum-dependent MDI interaction [1]. By using this interaction in the isospin-dependent Boltzmann-Uhling-Uhlenbeck (IBUU) transport model, an extensive amount of work has been carried out to study various isospin-sensitive observables in intermediate-energy heavy-ion collisions (for recent reviews, see Refs. [2,3]). From comparisons of the isospin diffusion data from the National Superconducting Cyclotron Laboratory (NSCL) at Michigan State University (MSU) for $^{124}\text{Sn} + ^{112}\text{Sn}$ reactions at $E = 50$ MeV/nucleon [4] with results from the IBUU model, a relatively stringent constraint on the density dependence of the nuclear symmetry energy at subsaturation densities has been obtained [5,6]. The resulting symmetry energy has further been used to impose constraints on both the parameters in the Skyrme effective interactions and the neutron skin thickness of heavy nuclei [7]. The MDI interaction with the constrained isospin dependence has also been used to study the properties of hot asymmetric nuclear matter [8] as well as those of neutron stars [9–14], including the transition density that separates their liquid core from their inner crust [15]. New constraints on the masses and radii of neutron stars were then obtained from comparing the resulting crustal fraction of the moment of inertia of neutron stars with that of the Vela pulsar extracted from its glitches [15].

Although the momentum-dependent nuclear mean-field potential or the energy density functional of uniform nuclear matter was used in these applications of the MDI interaction, the explicit form of the MDI interaction has never been given in the literatures. It was, however, implicitly mentioned [1,16] that this momentum-dependent nucleon mean-field potential was constructed according to a Gogny-like interaction [17] consisting of a zero-range Skyrme-like interaction [18] and a finite-range Yukawa interaction as in the momentum-dependent Yukawa interaction (MDYI) [16]. Similar to the

Gogny interaction, the momentum dependence in the nucleon mean-field potential from the MDI interaction is then from the contribution of the finite-range Yukawa interaction to the exchange energy density of uniform nuclear matter. The effect of the finite-range part of the MDI interaction in nonuniform nuclear matter was in previous applications either neglected or inconsistently included by using the average density-gradient terms from the phenomenological Skyrme interactions. From a comparison of the potential energy density functional obtained from above assumed interaction with that used in previous applications of the MDI interaction, we are able to find unique relations among the parameters of this interaction and those used in the potential energy density functional for uniform nuclear matter.

As a first step to facilitate the consistent application of the finite-range MDI interaction, it is of interest to derive an effective zero-range interaction with density-dependent parameters using the density matrix expansion first introduced in Ref. [19]. As shown in Ref. [20] for other finite-range interactions, corresponding zero-range interactions derived from the density matrix expansion reproduce reasonably well in the self-consistent Hartree-Fock approach the binding energies and radii of nuclei obtained with the original interactions, particularly if the density matrix expansion is used only for the exchange energy from the finite-range interaction [19,20]. In the present paper, we carry out such a study as an attempt to include consistently the effect due to the finite-range part of the MDI interaction. As an application, we use the resulting interaction to study the transition density and pressure in neutron stars and compare the results with those from previous studies using the MDI interaction but with inconsistent density-gradient coefficients [15,21].

This paper is organized as follows. In Sec. II, we review the isospin- and momentum-dependent MDI interaction and determine its underlying nucleon-nucleon (NN) interaction by fitting the energy density functional used in previous applications of the MDI interaction. In Sec. III, the density matrix expansion is then used to derive from the finite-range

exchange interaction a zero-range effective interaction with density-dependent parameters. The application of the resulting interaction to study the transition density and pressure at the inner edge of neutron star crusts is given in Sec. IV, using the stability conditions that are derived from the linearized Vlasov equation for the neutron star matter. Finally, the summary is given in Sec. V. Details on the determination of the underlying nucleon-nucleon interaction potential in the MDI interaction and the application of the density matrix expansion to the exchange energy of nuclear matter are given in Appendixes A and B, respectively.

II. THE MDI INTERACTION

In applications of the momentum-dependent MDI interaction for studying isospin effects in intermediate-energy heavy-ion collisions as well as the properties of hot asymmetric nuclear matter and neutron star matter, one usually uses the following potential energy density:

$$H(\rho, \delta) = \frac{A_1}{2\rho_0}\rho^2 + \frac{A_2}{2\rho_0}\rho^2\delta^2 + \frac{B}{\sigma+1}\frac{\rho^{\sigma+1}}{\rho_0^\sigma}(1-x\delta^2) + \frac{1}{\rho_0}\sum_{\tau,\tau'}C_{\tau,\tau'}\iint d^3pd^3p'\frac{f_\tau(\vec{r},\vec{p})f_{\tau'}(\vec{r},\vec{p}')}{1+(\vec{p}-\vec{p}')^2/\Lambda^2} \quad (1)$$

for infinite nuclear matter of density ρ and isospin asymmetry $\delta = (\rho_n - \rho_p)/\rho$, with ρ_n and ρ_p being, respectively, the neutron and proton densities. In the above, τ (τ') is the nucleon isospin; $f_\tau(\vec{r}, \vec{p})$ is the nucleon phase-space distribution function at position \vec{r} ; and $\rho_0 = 0.16 \text{ fm}^{-3}$ is the saturation density of normal nuclear matter. Values of the parameters $A_1 = (A_l + A_u)/2$, $A_2 = (A_l - A_u)/2$, B , σ , Λ , $C_l = C_{\tau,\tau}$, and $C_u = C_{\tau,-\tau}$ can be found in Refs. [1,5]. For symmetric nuclear matter, this interaction gives a binding energy of -16 MeV per nucleon and an incompressibility K_0 of 212 MeV at saturation density.

The parameter x in Eq. (1) is used to model the density dependence of the symmetry energy. Based on analyses of the isospin diffusion in intermediate-energy heavy-ion collisions and the neutron skin thickness of heavy nuclei [4–6], the slope parameter $L = 3\rho_0[\partial E_{\text{sym}}(\rho)/\partial\rho]_{\rho=\rho_0}$ of the symmetry energy has been constrained to $L = 86 \pm 25 \text{ MeV}$, which corresponds to $-1 < x < 0$. More recent analyses of experimental data on isospin diffusion, double n/p ratio, and neutron skin thickness favor, however, a softer symmetry energy of $L = 58 \pm 18 \text{ MeV}$ [22–24].

The momentum dependence in the MDI interaction can have a number of different origins. In addition to the finite-range exchange term in the nucleon-nucleon interaction, the intrinsic momentum dependence in the nucleon-nucleon interaction and the effects of short-range nucleon-nucleon correlations [25] can also contribute to its momentum dependence. For simplicity, we assume in the present study that all of the momentum dependence in the MDI interaction comes from the finite-range exchange term. In this case, the explicit form for the MDI interaction can be obtained from the energy density given in Eq. (1) by assuming that the interaction potential between two nucleons located at \vec{r}_1 and \vec{r}_2 has a form similar

to the Gogny interaction [1,17] but with its Gaussian form in the finite-range term replaced by a Yukawa form, that is,

$$v(\vec{r}_1, \vec{r}_2) = \frac{1}{6}t_3(1+x_3P_\sigma)\rho^\alpha\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)\delta(\vec{r}_1-\vec{r}_2) + (W+BP_\sigma-HP_\tau-MP_\sigma P_\tau)\frac{e^{-\mu|\vec{r}_1-\vec{r}_2|}}{|\vec{r}_1-\vec{r}_2|}. \quad (2)$$

In the above equation, the first term is the density-dependent zero-range interaction, which can be considered as an effective three-body interaction, whereas the second term is the density-independent finite-range interaction. In terms of this nucleon-nucleon interaction, the total potential energy of a nuclear system can be calculated from

$$E = \frac{1}{2}\sum_{i,j}\langle ij|v(1-P_rP_\sigma P_\tau)|ij\rangle, \quad (3)$$

where

$$|i\rangle = |i_r i_\sigma i_\tau\rangle \quad (4)$$

is the quantum state of nucleon i with the spatial state i_r , the spin state i_σ , and the isospin state i_τ , and P_r , P_σ , and P_τ are, respectively, the space, spin, and isospin exchange operators.

As shown in Appendix A, where the detailed derivation of Eq. (1) from Eq. (2) is given, the first and second terms in Eq. (1) come from the direct contribution [the first term of Eq. (3)] of the finite-range term in the NN interaction. The third term in Eq. (1) is simply from the contribution of the zero-range term. Although different density-dependent zero-range terms can lead to different symmetry energies and/or potentials [26], we use in the present study the one in the standard Skyrme interaction [27]. The momentum-dependent terms in Eq. (1) come from the exchange contribution [the second term of Eq. (3)] of the finite-range interaction. Comparison of the resulting energy density functional with Eq. (1) leads to the following unique relations among the eight parameters in the NN interaction and those in the energy density functional of Eq. (1):

$$t_3 = \frac{16B}{(\sigma+1)\rho_0^\sigma}, \quad (5)$$

$$x_3 = \frac{3x-1}{2}, \quad (6)$$

$$\alpha = \sigma - 1, \quad (7)$$

$$\mu = \Lambda, \quad (8)$$

$$W = \frac{\Lambda^2}{3\pi\rho_0}(A_1 - A_2 + C_l - C_u), \quad (9)$$

$$B = \frac{\Lambda^2}{6\pi\rho_0}(-A_1 + A_2 - 4C_l + 4C_u), \quad (10)$$

$$H = \frac{\Lambda^2}{3\pi\rho_0}(-2A_2 - C_u), \quad (11)$$

$$M = \frac{\Lambda^2}{3\pi\rho_0}(A_2 + 2C_u). \quad (12)$$

It is seen that the symmetry energy parameter x is related to the coefficient of the spin exchange term in the density-dependent zero-range interaction, whereas the width Λ in the

momentum dependence is related to the range of the finite-range interaction.

In the original MDI interaction [1], the symmetry energy parameter x can only have the value of 0 or 1. To get a larger range of density dependence for the symmetry energy while fixing the value of $E_{\text{sym}}(\rho_0) = 30.5$ MeV, the parameters A_l and A_u have been expressed as [5]

$$A_l = -120.57 + x \frac{2B}{\sigma + 1}, \quad A_u = -95.98 - x \frac{2B}{\sigma + 1}, \quad (13)$$

which results in the x dependence of A_2 . Therefore, with this constraint the x dependence also appears in the finite-range direct interaction.

III. THE DENSITY MATRIX EXPANSION

Although the contribution of a finite-range interaction to the direct energy density of nuclear matter can be treated exactly, it is numerically challenging to evaluate its contribution to the exchange energy density. The latter can be, however, approximated by that from a Skyrme-like zero-range interaction using the density-matrix expansion of Ref. [19]. As shown in Appendix B, the resulting exchange energy density from the finite-range interaction at position \vec{r} in a nuclear system can be expressed in terms of densities ρ_n and ρ_p as well as kinetic energy densities τ_n and τ_p as

$$\begin{aligned} H_{\text{SL}}^E(\vec{r}) = & A[\rho_n(\vec{r}), \rho_p(\vec{r})] \\ & + B[\rho_n(\vec{r}), \rho_p(\vec{r})]\tau_n(\vec{r}) + B[\rho_p(\vec{r}), \rho_n(\vec{r})]\tau_p(\vec{r}) \\ & + C[\rho_n(\vec{r}), \rho_p(\vec{r})]|\nabla\rho_n(\vec{r})|^2 \\ & + C[\rho_p(\vec{r}), \rho_n(\vec{r})]|\nabla\rho_p(\vec{r})|^2 \\ & + D[\rho_n(\vec{r}), \rho_p(\vec{r})]\nabla\rho_n(\vec{r}) \cdot \nabla\rho_p(\vec{r}), \end{aligned} \quad (14)$$

where

$$\begin{aligned} A(\rho_n, \rho_p) = & V_{\text{NM}}(\rho_n, \rho_p) - \frac{3}{5}(3\pi^2)^{2/3}[\rho_n^{5/3}B(\rho_n, \rho_p) \\ & + \rho_p^{5/3}B(\rho_p, \rho_n)], \end{aligned} \quad (15)$$

$$B(\rho_n, \rho_p) = -2[\rho_n V^L(\rho_n) + \rho_p V^U(\rho_p, \rho_n)], \quad (16)$$

$$C(\rho_n, \rho_p) = -\frac{\partial F(\rho_n, \rho_p)}{\partial \rho_n}, \quad (17)$$

$$D(\rho_n, \rho_p) = -\frac{\partial F(\rho_n, \rho_p)}{\partial \rho_p} - \frac{\partial F(\rho_p, \rho_n)}{\partial \rho_n}. \quad (18)$$

In above equations, $V_{\text{NM}}(\rho_n, \rho_p)$ is the potential energy density of infinite nuclear matter from the finite-range exchange interaction [i.e. the momentum-dependent terms in Eq. (1)] and is given by

$$\begin{aligned} V_{\text{NM}}(\rho_n, \rho_p) = & \rho_n^2 V_{\text{NM}}^L(\rho_n) + \rho_p^2 V_{\text{NM}}^L(\rho_p) \\ & + 2\rho_n \rho_p V_{\text{NM}}^U(\rho_n, \rho_p), \end{aligned} \quad (19)$$

with

$$\begin{aligned} V_{\text{NM}}^L(\rho_\tau) = & \frac{1}{2} \left(M + \frac{H}{2} - B - \frac{W}{2} \right) \\ & \times \int d^3s \rho_{\text{SL}}^2(k_\tau s) \frac{e^{-\mu s}}{s}, \end{aligned} \quad (20)$$

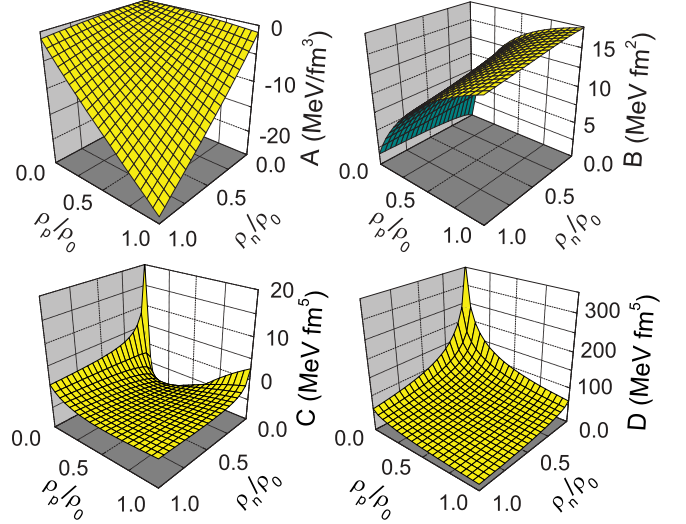


FIG. 1. (Color online) Dependence of $A(\rho_n, \rho_p)$, $B(\rho_n, \rho_p)$, $C(\rho_n, \rho_p)$, and $D(\rho_n, \rho_p)$ on ρ_n and ρ_p at subsaturation densities.

$$\begin{aligned} V_{\text{NM}}^U(\rho_\tau, \rho_{\tau'}) = & \frac{1}{2} \left(M + \frac{H}{2} \right) \\ & \times \int d^3s \rho_{\text{SL}}(k_\tau s) \rho_{\text{SL}}(k_{\tau'} s) \frac{e^{-\mu s}}{s}. \end{aligned} \quad (21)$$

Other terms are defined as

$$\begin{aligned} V^L(\rho_\tau) = & \frac{1}{2} \left(M + \frac{H}{2} - B - \frac{W}{2} \right) \\ & \times \int d^3s s^2 \rho_{\text{SL}}(k_\tau s) g(k_\tau s) \frac{e^{-\mu s}}{s}, \end{aligned} \quad (22)$$

$$\begin{aligned} V^U(\rho_\tau, \rho_{\tau'}) = & \frac{1}{2} \left(M + \frac{H}{2} \right) \\ & \times \int d^3s s^2 \rho_{\text{SL}}(k_\tau s) g(k_\tau s) \frac{e^{-\mu s}}{s}, \end{aligned} \quad (23)$$

where

$$\rho_{\text{SL}}(k_\tau s) = \frac{3}{k_\tau s} j_1(k_\tau s), \quad (24)$$

$$g(k_\tau s) = \frac{35}{2(k_\tau s)^3} j_3(k_\tau s), \quad (25)$$

with j_1 and j_3 being, respectively, the first- and third-order spherical Bessel functions and $k_\tau = (3\pi^2 \rho_\tau)^{1/3}$ the Fermi momentum. For the function $F(\rho_n, \rho_p)$, it is defined as

$$F(\rho_n, \rho_p) = \frac{1}{2} V^L(\rho_n) \rho_n + \frac{1}{2} V^U(\rho_p, \rho_n) \rho_p. \quad (26)$$

In Fig. 1, we show the dependence of $A(\rho_n, \rho_p)$, $B(\rho_n, \rho_p)$, $C(\rho_n, \rho_p)$, and $D(\rho_n, \rho_p)$ on ρ_n and ρ_p at subsaturation densities. It is seen that $A(\rho_n, \rho_p)$ and $D(\rho_n, \rho_p)$ are symmetric in ρ_n and ρ_p , while $B(\rho_n, \rho_p)$ and $C(\rho_n, \rho_p)$ are not. Furthermore, the density dependences of $B(\rho_n, \rho_p)$, $C(\rho_n, \rho_p)$, and $D(\rho_n, \rho_p)$ are strong at low densities but weak near the saturation density. As these functions are from the exchange contribution of the finite-range interaction, they are independent of x .

IV. APPLICATION: TRANSITION DENSITY AND PRESSURE IN NEUTRON STARS

The explicit form of the finite-range NN interaction and the Skyrme-like zero-range interaction with density-dependent parameters derived in previous sections make it convenient to use the MDI interaction for a wider range of studies. In this section, we discuss its application in studying the transition density and pressure at the inner edge of neutron star crusts based on the stability conditions that are derived from the linearized Vlasov equation for the neutron star matter.

A. The single-particle Hamiltonian

The single-particle Hamiltonian for a nucleon can be obtained from minimizing the total energy of a nuclear system with respect to its wave function. For a neutron, it is given by

$$h_n = -\nabla \cdot \left(\frac{\hbar^2}{2m_n^*} \nabla \right) + U_n(\rho_n, \rho_p) \quad (27)$$

with the neutron effective mass

$$\frac{\hbar^2}{2m_n^*} = \frac{\hbar^2}{2m} + B(\rho_n, \rho_p) \quad (28)$$

and the potential

$$U_n(\rho_n, \rho_p) = U_n^\rho(\rho_n, \rho_p) + U_n^D(\rho_n, \rho_p) + U_n^E(\rho_n, \rho_p), \quad (29)$$

where

$$U_n^\rho(\rho_n, \rho_p) = \frac{\partial H_0(\rho_n, \rho_p)}{\partial \rho_n} \quad (30)$$

is the contribution from the zero-range interaction with

$$H_0(\rho_n, \rho_p) = \frac{1}{24} t_3 \rho^\alpha [(2 + x_3) \rho^2 - (2x_3 + 1)(\rho_n^2 + \rho_p^2)] \quad (31)$$

being the corresponding energy density, and

$$U_n^D(\rho_n, \rho_p) = \left(W + \frac{B}{2} - H - \frac{M}{2} \right) \int d^3 r' \rho_n(\vec{r}') \frac{e^{-\mu|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \\ + \left(W + \frac{B}{2} \right) \int d^3 r' \rho_p(\vec{r}') \frac{e^{-\mu|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \quad (32)$$

$$U_n^E(\rho_n, \rho_p) = \frac{\partial A(\rho_n, \rho_p)}{\partial \rho_n} + \frac{\partial B(\rho_n, \rho_p)}{\partial \rho_n} \tau_n \\ + \frac{\partial B(\rho_p, \rho_n)}{\partial \rho_n} \tau_p - \frac{\partial C(\rho_n, \rho_p)}{\partial \rho_n} (\nabla \rho_n)^2 \\ + \left(\frac{\partial C(\rho_p, \rho_n)}{\partial \rho_n} - \frac{\partial D(\rho_n, \rho_p)}{\partial \rho_p} \right) (\nabla \rho_p)^2 \\ - 2 \frac{\partial C(\rho_n, \rho_p)}{\partial \rho_p} \nabla \rho_n \cdot \nabla \rho_p - 2C(\rho_n, \rho_p) \nabla^2 \rho_n \\ - D(\rho_n, \rho_p) \nabla^2 \rho_p \quad (33)$$

are the direct and exchange contributions from the finite-range interaction, respectively. Expressing the kinetic energy density in the exchange potential in terms of the density via the

extended Thomas-Fermi (ETF) approximation [28,29]

$$\tau_q = a \rho_q^{5/3} + b \frac{(\nabla \rho_q)^2}{\rho_q} + c \nabla^2 \rho_q, \quad q = n, p, \quad (34)$$

where $a = \frac{3}{5}(3\pi^2)^{2/3}$, $b = 1/36$ and $c = 1/3$, the neutron potential can then be written as

$$U_n = U_n^0 + U_n^D + U_n^\nabla, \quad (35)$$

where

$$U_n^0(\rho_n, \rho_p) = U_n^\rho(\rho_n, \rho_p) + \frac{\partial A(\rho_n, \rho_p)}{\partial \rho_n} + a \frac{\partial B(\rho_n, \rho_p)}{\partial \rho_n} \rho_n^{5/3} \\ + a \frac{\partial B(\rho_p, \rho_n)}{\partial \rho_n} \rho_p^{5/3}, \quad (36)$$

$$U_n^\nabla(\rho_n, \rho_p) = G_n^{nn1} (\nabla \rho_n)^2 + G_n^{pp1} (\nabla \rho_p)^2 + G_n^{np} \nabla \rho_n \cdot \nabla \rho_p \\ + G_n^{nn2} \nabla^2 \rho_n + G_n^{pp2} \nabla^2 \rho_p, \quad (37)$$

with

$$G_n^{nn1}(\rho_n, \rho_p) = -\frac{\partial C(\rho_n, \rho_p)}{\partial \rho_n} + \frac{b}{\rho_n} \frac{\partial B(\rho_n, \rho_p)}{\partial \rho_n}, \quad (38)$$

$$G_n^{pp1}(\rho_n, \rho_p) = \frac{\partial C(\rho_p, \rho_n)}{\partial \rho_n} - \frac{\partial D(\rho_n, \rho_p)}{\partial \rho_p} \\ + \frac{b}{\rho_p} \frac{\partial B(\rho_p, \rho_n)}{\partial \rho_n}, \quad (39)$$

$$G_n^{np}(\rho_n, \rho_p) = -2 \frac{\partial C(\rho_n, \rho_p)}{\partial \rho_p}, \quad (40)$$

$$G_n^{nn2}(\rho_n, \rho_p) = -2C(\rho_n, \rho_p) + c \frac{\partial B(\rho_n, \rho_p)}{\partial \rho_n}, \quad (41)$$

$$G_n^{pp2}(\rho_n, \rho_p) = -D(\rho_n, \rho_p) + c \frac{\partial B(\rho_p, \rho_n)}{\partial \rho_n}. \quad (42)$$

Similarly, the proton single-particle Hamiltonian is

$$h_p = h_n(n \leftrightarrow p) + U_p^{CD} + U_p^{CE}, \quad (43)$$

where the first term $h_n(n \leftrightarrow p)$ is due to the nuclear interaction and is obtained from Eq. (27) by interchanging the neutron and proton symbols; the second term U_p^{CD} is the direct Coulomb potential given by

$$U_p^{CD} = e^2 \int \frac{\rho_p(\vec{r}') - \rho_e(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3 r'; \quad (44)$$

and the last term U_p^{CE} is the exchange Coulomb potential which in the first order of the density matrix expansion [19] is given by

$$U_p^{CE} = -e^2 \left(\frac{3}{\pi} \right)^{1/3} \rho_p^{1/3}. \quad (45)$$

B. The Vlasov equation for the neutron star matter

The Vlasov equation has been widely used in studying the collective density fluctuation and spinodal instability in nuclear matter (see Ref. [30] for a recent review). For a β -stable and charge-neutral npe matter, the Vlasov equation can be

written as

$$\frac{\partial f_q(\vec{r}, \vec{p}, t)}{\partial t} + \vec{v}_q \cdot \nabla_{\vec{r}} f_q(\vec{r}, \vec{p}, t) - \nabla_{\vec{r}} U_q \cdot \nabla_{\vec{p}} f_q(\vec{r}, \vec{p}, t) = 0 \quad (46)$$

in terms of the Wigner function for particle type $q = n, p, e$

$$f_q(\vec{r}, \vec{p}, t) = \frac{1}{(2\pi)^3} \sum_i \int \phi_{qi} \left(\vec{r} - \frac{\vec{s}}{2}, t \right) \times \phi_{qi}^* \left(\vec{r} + \frac{\vec{s}}{2}, t \right) e^{i\vec{p}\cdot\vec{s}} d^3s, \quad (47)$$

where ϕ_{qi} is the wave function of i th particle of type q . In Eq. (46), $\vec{v}_q = \vec{p}/(m_q^* + p^2)^{1/2}$ denotes the particle velocity and $m_e^* = m_e$.

To study the density fluctuation caused by a collective mode with frequency ω and wavevector \vec{k} in the nuclear matter, we follow the standard procedure [30] by writing

$$f_q(\vec{r}, \vec{p}, t) = f_q^0(\vec{p}) + \delta f_q(\vec{r}, \vec{p}, t) \quad (48)$$

with

$$\delta f_q(\vec{r}, \vec{p}, t) = \delta \tilde{f}_q(\vec{p}) e^{-i\omega t + i\vec{k}\cdot\vec{r}}. \quad (49)$$

As the momentum dependence of the Wigner function is mainly from $f_q^0(\vec{p})$, we have $\nabla_{\vec{p}} f_q(\vec{r}, \vec{p}, t) \approx \nabla_{\vec{p}} f_q^0(\vec{p})$ and can thus rewrite the Vlasov equation [Eq. (46)] as

$$-i\omega \delta f_q + \vec{v}_q \cdot (i\vec{k}) \delta f_q - \frac{\partial f_q^0}{\partial \epsilon_q} \nabla_{\vec{p}} \epsilon_q \cdot \left(\sum_{q'} \frac{\delta U_q}{\delta \rho_{q'}} \nabla_{\vec{r}} \rho_{q'} \right) = 0, \quad (50)$$

where ϵ_q is the single-particle energy. Using $\nabla_{\vec{p}} \epsilon_q = \vec{v}_q$ and writing $\rho_q(\vec{r}, t) = \rho_q^0 + \delta \rho_q(\vec{r}, t)$ with

$$\delta \rho_q(\vec{r}, t) = \frac{2}{(2\pi)^3} \int \delta f_q(\vec{r}, \vec{p}, t) d^3p, \quad (51)$$

which has the same time and spatial dependence as $\delta f_q(\vec{r}, \vec{p}, t)$, Eq. (50) can be rewritten as

$$\delta f_q = -\frac{\partial f_q^0}{\partial \epsilon_q} \left(\sum_{q'} \frac{\delta U_q}{\delta \rho_{q'}} \delta \rho_{q'} \right) \frac{\vec{k} \cdot \vec{v}_q}{\omega - \vec{k} \cdot \vec{v}_q}. \quad (52)$$

By substituting the above equation into Eq. (51), we obtain in the low-temperature limit

$$\delta \rho_q \approx \frac{1}{2\pi^2} \int_0^{p_q^F} \left(-\frac{\partial f_q^0}{\partial \epsilon_q} \right) p^2 dp \int_{-1}^1 \frac{\cos \theta d(\cos \theta)}{s_q - \cos \theta} \times \left(\sum_{q'} \frac{\delta U_q}{\delta \rho_{q'}} \delta \rho_{q'} \right), \quad (53)$$

with $s_q = \omega/kv_q^F$ and $v_q^F = p_q^F/(m_q^* + p_q^F)^{1/2}$ being the Fermi velocity. Carrying out the angular integration leads to the usual Lindhard function

$$L_q = \int_{-1}^1 \frac{\cos \theta d(\cos \theta)}{s_q - \cos \theta} = -2 + s_q \ln \left(\frac{s_q + 1}{s_q - 1} \right). \quad (54)$$

In the low-temperature limit, the momentum integration can be approximately evaluated as

$$X_q = \frac{1}{2\pi^2} \int_0^{p_q^F} \left(-\frac{\partial f_q^0}{\partial \epsilon_q} \right) p^2 dp \approx \frac{p_q^F m_q^*}{2\pi^2} \left[1 - \frac{\pi^2}{24} \left(\frac{T}{\epsilon_q^F} \right)^2 \right] \quad (55)$$

for $q = n, p$, with $\epsilon_q^F \approx p_q^F/2m_q^*$ and

$$X_e \approx \frac{\mu_e^2}{2\pi^2} \left[1 + \frac{\pi^2}{3} \left(\frac{T}{\mu_e} \right)^2 \right] \quad (56)$$

for electrons, where $\mu_e \approx p_e^F$ is the electron chemical potential.

For the factor $\sum_{q'} \frac{\delta U_q}{\delta \rho_{q'}} \delta \rho_{q'}$ in Eq. (53), there are the local contribution δU_n^0 , the direct contribution δU_n^D from the finite-range interaction, and the gradient contribution δU_n^∇ from the finite-range exchange interaction using the density matrix expansion. In the case of neutrons, they are given by

$$\delta U_n^0 = \frac{\partial U_n^0}{\partial \rho_n} \delta \rho_n + \frac{\partial U_n^0}{\partial \rho_p} \delta \rho_p, \quad (57)$$

$$\delta U_n^D = \left(W + \frac{B}{2} - H - \frac{M}{2} \right) \frac{4\pi}{k^2 + \mu^2} \delta \rho_n + \left(W + \frac{B}{2} \right) \frac{4\pi}{k^2 + \mu^2} \delta \rho_p, \quad (58)$$

$$\delta U_n^\nabla = -k^2 (G_n^{nn2} \delta \rho_n + G_n^{pp2} \delta \rho_p), \quad (59)$$

after neglecting higher-order terms in $\delta \rho_q$. In the long-wavelength case, the finite-range direct contribution can be rewritten as

$$\delta U_n^D \approx \frac{\partial U_n^{D0}}{\partial \rho_n} \delta \rho_n + \frac{\partial U_n^{D0}}{\partial \rho_p} \delta \rho_p - k^2 (G_n^{nnD} \delta \rho_n + G_n^{ppD} \delta \rho_p) \quad (60)$$

with

$$\frac{\partial U_n^{D0}}{\partial \rho_n} = \frac{4\pi}{\mu^2} \left(W + \frac{B}{2} - H - \frac{M}{2} \right), \quad (61)$$

$$\frac{\partial U_n^{D0}}{\partial \rho_p} = \frac{4\pi}{\mu^2} \left(W + \frac{B}{2} \right), \quad (62)$$

$$G_n^{nnD} = \frac{4\pi}{\mu^4} \left(W + \frac{B}{2} - H - \frac{M}{2} \right), \quad (63)$$

$$G_n^{ppD} = \frac{4\pi}{\mu^4} \left(W + \frac{B}{2} \right). \quad (64)$$

For protons, there are additional direct and exchange Coulomb contributions given, respectively, by

$$\delta U_p^{CD} = \frac{4\pi e^2}{k^2} (\delta \rho_p - \delta \rho_e), \quad (65)$$

$$\delta U_p^{CE} = -\frac{1}{3} e^2 \left(\frac{3}{\pi} \right)^{1/3} \rho_p^{-2/3} \delta \rho_p. \quad (66)$$

For electrons, there are only direct and exchange Coulomb contributions to $\sum_{q'} \frac{\delta U_q}{\delta \rho_{q'}} \delta \rho_{q'}$.

After linearizing the Vlasov equation, the collective density fluctuation $\delta\rho_q$ then satisfies the equation

$$C^f(\delta\rho_n, \delta\rho_p, \delta\rho_e)^T = 0, \quad (67)$$

$$C^f = \begin{pmatrix} X_n L_n \left(\frac{\partial U_n^0}{\partial \rho_n} + \frac{\partial U_n^{D0}}{\partial \rho_n} \right) - 1 & X_n L_n \left(\frac{\partial U_n^0}{\partial \rho_p} + \frac{\partial U_n^{D0}}{\partial \rho_p} \right) & 0 \\ X_p L_p \left(\frac{\partial U_p^0}{\partial \rho_n} + \frac{\partial U_p^{D0}}{\partial \rho_n} \right) & X_p L_p \left(\frac{\partial U_p^0}{\partial \rho_p} + \frac{\partial U_p^{D0}}{\partial \rho_p} \right) - 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} - k^2 \begin{pmatrix} X_n L_n (G_n^{nnD} + G_n^{nn2}) & X_n L_n (G_n^{ppD} + G_n^{pp2}) & 0 \\ X_p L_p (G_p^{nnD} + G_p^{nn2}) & X_p L_p (G_p^{ppD} + G_p^{pp2}) & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & X_p L_p \left(\frac{4\pi e^2}{k^2} + \frac{\delta U_p^{CE}}{\delta \rho_p} \right) & -\frac{4\pi e^2}{k^2} X_p L_p \\ 0 & -\frac{4\pi e^2}{k^2} X_e L_e & X_e L_e \left(\frac{4\pi e^2}{k^2} + \frac{\delta U_e^{CE}}{\delta \rho_e} \right) \end{pmatrix}. \quad (68)$$

We note that the three terms in the above equation are due to, respectively, the bulk, the density gradient, and the Coulomb contribution.

C. The transition density in neutron stars

Nontrivial solutions of Eq. (67) are obtained when $|C^f| = 0$, which also determines the dispersion relation $\omega(k)$ of the collective density fluctuation. The transition density in a neutron star is the density at which the collective density fluctuation would grow exponentially, resulting in the instability of the neutron star matter, and this happens when the frequency ω becomes imaginary. To determine the condition for this to occur, we let $s_q = -i v_q$ ($v_q > 0$) and rewrite the Lindhard function as $L_q = -2 + 2v_q \arctan(1/v_q)$. Since the values of L_q are $-2 < L_q < 0$, the critical values $L_n = L_p = L_e = -2$, corresponding to $v_q = 0$, then determine the spinodal boundary of the system when they are substituted into $|C^f| = 0$. Expanding the nucleon and electron densities at low temperatures according to

$$\rho_e \approx \frac{\mu_e^3}{3\pi^2} \left[1 + \pi^2 \left(\frac{T}{\mu_e} \right)^2 \right], \quad (69)$$

$$\rho_q \approx \frac{(2m_q^* \epsilon_q^F)^{3/2}}{3\pi^2} \left[1 + \frac{\pi^2}{8} \left(\frac{T}{\epsilon_q^F} \right)^2 \right], \quad q = n, p, \quad (70)$$

we obtain from Eqs. (55) and (56) the following relations:

$$1/2 X_e = \partial \mu_e / \partial \rho_e, \quad (71)$$

$$1/2 X_q = \partial \epsilon_q^F / \partial \rho_q. \quad (72)$$

The bulk contribution in Eq. (68) in the low-temperature limit can thus be rewritten as

$$-8 \begin{pmatrix} X_n (\partial \mu_n / \partial \rho_n) & X_n (\partial \mu_n / \partial \rho_p) & 0 \\ X_p (\partial \mu_p / \partial \rho_n) & X_p (\partial \mu_p / \partial \rho_p) & 0 \\ 0 & 0 & X_e (\partial \mu_e / \partial \rho_e) \end{pmatrix}. \quad (73)$$

Denoting the effective density-gradient coefficients as

$$D_{nn} = -G_n^{nnD} - G_n^{nn2}, \quad (74)$$

$$D_{np} = -G_n^{ppD} - G_n^{pp2}, \quad (75)$$

$$D_{pn} = -G_p^{nnD} - G_p^{nn2}, \quad (76)$$

$$D_{pp} = -G_p^{ppD} - G_p^{pp2}, \quad (77)$$

we then obtain the same expressions for determining the spinodal boundary of npe matter as in the so-called dynamical approach [15,31–33], if the Coulomb exchange terms are neglected. In particular, in the long-wavelength limit, the spinodal boundary is determined by the vanishing point of

$$V_{\text{dyn}}(k) = \frac{|C^f|_{L_q=-2}}{X_n X_q X_e (\partial \mu_n / \partial \rho_n)} \approx V_0 + \beta k^2 + \frac{4\pi e^2}{k^2 + k_{\text{TF}}^2}, \quad (78)$$

where

$$V_0 = \frac{\partial \mu_p}{\partial \rho_p} - \frac{(\partial \mu_p / \partial \rho_n)^2}{\partial \mu_n / \partial \rho_n} + \frac{\delta U_p^{CE}}{\delta \rho_p}, \quad (79)$$

$$\beta = D_{pp} + (D_{np} + D_{pn})\zeta + D_{nn}\zeta^2, \quad (80)$$

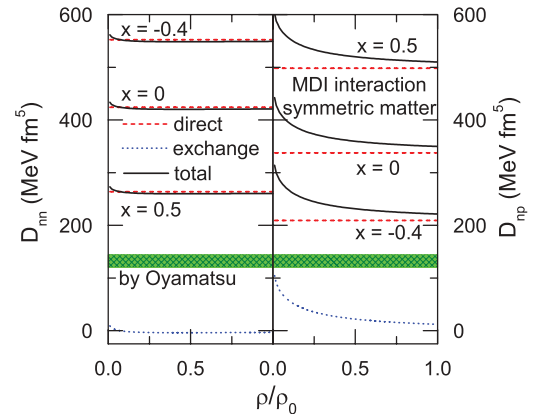


FIG. 2. (Color online) Density dependence of effective density-gradient coefficients from different values of x for symmetric nuclear matter. The finite-range direct and exchange contributions are compared, and the value extracted by Oyamatsu [34] is also shown.

$$\zeta = -\frac{\partial\mu_p/\partial\rho_n}{\partial\mu_n/\partial\rho_n}, \quad (81)$$

$$k_{\text{TF}}^2 = \frac{4\pi e^2}{(\partial\mu_e/\partial\rho_e + \delta U_e^{\text{CE}}/\delta\rho_e)}. \quad (82)$$

To find the upper limit of the spinodal boundary for all possible wave vectors, we first find the minimum value of V_{dyn} given by

$$V_{\text{dyn}} = V_0 + 2(4\pi e^2\beta)^{1/2} - \beta k_{\text{TF}}^2, \quad (83)$$

which occurs when $k^2 = (4\pi e^2/\beta)^{1/2} - k_{\text{TF}}^2$. The density that makes Eq. (83) vanish then determines the spinodal boundary in the neutron star matter or the transition density at the inner edge of neutron star crusts.

In Fig. 2, we show the density dependence of effective density-gradient coefficients from different values of x for symmetric nuclear matter. It is seen that the contribution from the finite-range direct interaction is much larger than that from the finite-range exchange interaction. Also, D_{nn} decreases while D_{np} increases with increasing value of x . Furthermore, $D_{nn} = D_{pp}$ and $D_{np} = D_{pn}$ for symmetric nuclear matter, whereas all four D 's have different values for asymmetric nuclear matter. The density-gradient coefficients extracted from the MDI interaction are, however, much larger than the value of $132 \pm 12 \text{ MeV fm}^5$ shown in the figure that was used by Oyamatsu [34] to fit the nuclear radii and is also consistent with the average value from different Skyrme interactions. We note that the large density-gradient coefficients in the present study are due to our assumption that all of the momentum dependence in the MDI interaction comes from the finite-range exchange term.

In Fig. 3, we compare the transition density and pressure in this study with those from previous calculations [15,21] that use the value of 132 MeV fm^5 from Ref. [34] for the density-gradient coefficients and neglect the exchange Coulomb interaction. It is seen that the present results are smaller than previous ones for neutron star temperatures $T = 0$ and 1 MeV. We note that including the exchange Coulomb term for protons slightly increases ρ_t while the

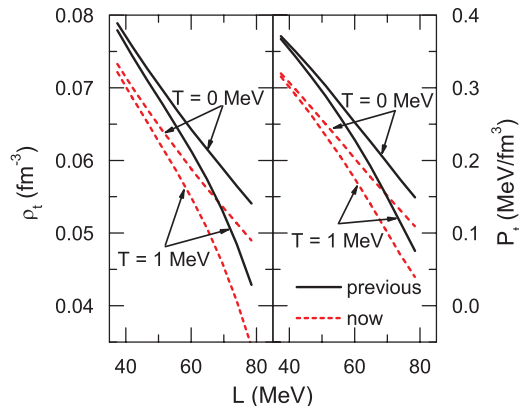


FIG. 3. (Color online) The transition density ρ_t and pressure P_t at the inner edge of neutron star crust as functions of the slope parameter L for $T = 0$ and 1 MeV. Previous results [15,21] based on density-gradient terms from Ref. [34] are also shown for comparison.

larger density-gradient coefficients make ρ_t smaller. From the latest constraint $L = 58 \pm 18 \text{ MeV}$ on the density slope of the nuclear symmetry energy, the transition density and pressure are constrained within $0.050 < \rho_t < 0.071 \text{ fm}^{-3}$ and $0.12 < P_t < 0.31 \text{ MeV/fm}^3$ for $T = 0 \text{ MeV}$, and $0.038 < \rho_t < 0.070 \text{ fm}^{-3}$ and $0.06 < P_t < 0.30 \text{ MeV/fm}^3$ for $T = 1 \text{ MeV}$ in the present study. Although the transition density is smaller at fixed L compared to our previous results [15], which leads to a smaller crustal fraction of the moment of inertia for neutron stars and an even stricter constraint on the masses and radii of neutron stars, the upper limit values of ρ_t and P_t are larger because of the smaller values of L . The final constraint on the neutron star mass-radius (MR) relation is expected to be similar to that in the previous work [15].

V. SUMMARY

Assuming that the momentum dependence in the MDI interaction is entirely due to the finite-range exchange term in the nucleon-nucleon interaction, we have identified the NN interaction potential that underlies the MDI interaction, which has been extensively used in studying isospin effects in intermediate-energy heavy-ion collisions as well as the properties of hot nuclear matter and neutron star matter through the resulting potential energy density in infinite nuclear matter. Using the density matrix expansion, we have obtained an effective zero-range Skyrme-like interaction with density-dependent parameters from the finite-range exchange interaction. Compared to the density-gradient coefficients extracted by Oyamatsu [34] and the average value from different Skyrme interactions, the values from the MDI interaction are much larger. Since the momentum dependence in the MDI interaction could also come from the intrinsic momentum dependence in the elementary nucleon-nucleon interaction or be induced by the effects of short-range nucleon-nucleon correlations, we have thus overestimated the effect of the finite-range exchange term. As an application, the resulting interaction is used to determine the transition density and pressure at the inner edge of neutron star crusts based on the linearized Vlasov equation for the neutron star matter. Fortunately, the instability of neutron star matter is dominated by the bulk properties of nuclear matter; the large density-gradient coefficients due to the finite-range interaction do not much reduce the transition density and pressure in neutron stars.

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APPENDIX A: FROM THE NN INTERACTION TO THE POTENTIAL ENERGY DENSITY

The two-particle state $|ij\rangle$ in Eq. (3) contains the spatial, spin, and isospin parts as

$$|ij\rangle = |i_r i_\sigma i_\tau j_r j_\sigma j_\tau\rangle = |i_r j_r\rangle |i_\sigma j_\sigma\rangle |i_\tau j_\tau\rangle. \quad (\text{A1})$$

For the inner product of the spatial state, we have

$$\begin{aligned} \sum_{i,j} \langle i_r j_r | i_r j_r \rangle &= \sum_{i,j} \int \langle i_r j_r | \vec{r}_1 \vec{r}_2 \rangle \langle \vec{r}_1 \vec{r}_2 | i_r j_r \rangle d^3 r_1 d^3 r_2 \\ &= \sum_{i,j} \int \phi_i^*(\vec{r}_1) \phi_j^*(\vec{r}_2) \phi_i(\vec{r}_1) \phi_j(\vec{r}_2) d^3 r_1 d^3 r_2 \\ &= \int \rho(\vec{r}_1) \rho(\vec{r}_2) d^3 r_1 d^3 r_2 \end{aligned} \quad (\text{A2})$$

with $\phi_i(\vec{r}) = \langle \vec{r} | i_r \rangle$ being the space wave function for particle i and the density $\rho(\vec{r}) = \sum_i \phi_i^*(\vec{r}) \phi_i(\vec{r})$, and

$$\begin{aligned} \sum_{i,j} \langle i_r j_r | j_r i_r \rangle &= \sum_{i,j} \int \langle i_r j_r | \vec{r}_1 \vec{r}_2 \rangle \langle \vec{r}_1 \vec{r}_2 | j_r i_r \rangle d^3 r_1 d^3 r_2 \\ &= \sum_{i,j} \int \phi_i^*(\vec{r}_1) \phi_j^*(\vec{r}_2) \phi_j(\vec{r}_1) \phi_i(\vec{r}_2) d^3 r_1 d^3 r_2 \\ &= \int \rho(\vec{r}_1, \vec{r}_2) \rho(\vec{r}_2, \vec{r}_1) d^3 r_1 d^3 r_2 \end{aligned} \quad (\text{A3})$$

with the off-diagonal density $\rho(\vec{r}_1, \vec{r}_2) = \sum_i \phi_i^*(\vec{r}_1) \phi_i(\vec{r}_2)$, which reduces to the density when $\vec{r}_1 = \vec{r}_2$.

For the zero-range term $v_0 = \frac{1}{6} t_3 (1 + x_3 P_\sigma) \rho^\alpha(\frac{\vec{r}_1 + \vec{r}_2}{2}) \delta(\vec{r}_1 - \vec{r}_2)$ in the MDI interaction, its direct contribution to the energy can be calculated using the fact that $P_\sigma = 1/2$ when considering the inner product of the spin state in spin saturated matter, and the result is

$$\begin{aligned} E_0^D &= \frac{1}{2} \sum_{i,j} \langle ij | v_0 | ij \rangle \\ &= \frac{t_3}{12} \int d^3 r_1 d^3 r_2 \rho(\vec{r}_1) \rho(\vec{r}_2) \rho^\alpha \left(\frac{\vec{r}_1 + \vec{r}_2}{2} \right) \\ &\quad \times \delta(\vec{r}_1 - \vec{r}_2) \left(1 + \frac{x_3}{2} \right) \\ &= \int d^3 r H_0^D(\vec{r}) \end{aligned} \quad (\text{A4})$$

with

$$H_0^D = \frac{t_3}{12} \left(1 + \frac{x_3}{2} \right) \rho^{\alpha+2}. \quad (\text{A5})$$

Its exchange contribution

$$E_0^E = \frac{1}{2} \sum_{i,j} \langle ij | v_0 (-P_r P_\sigma P_\tau) | ij \rangle \quad (\text{A6})$$

can be evaluated by using $(1 + x_3 P_\sigma)(-P_\sigma P_\tau) = -P_\sigma P_\tau - x_3 P_\tau$ and replacing P_τ by δ_{i_r, j_r} when considering the inner product of the isospin state, and the result is

$$\begin{aligned} E_0^E &= -\frac{t_3}{12} \int d^3 r_1 d^3 r_2 \rho(\vec{r}_1, \vec{r}_2) \rho(\vec{r}_2, \vec{r}_1) \rho^\alpha \left(\frac{\vec{r}_1 + \vec{r}_2}{2} \right) \\ &\quad \times \delta(\vec{r}_1 - \vec{r}_2) \left(\frac{1}{2} + x_3 \right) \frac{1 + \delta^2}{2} \\ &= \int d^3 r H_0^E(\vec{r}) \end{aligned} \quad (\text{A7})$$

with

$$H_0^E = -\frac{t_3}{12} \left(\frac{1}{2} + x_3 \right) \rho^{\alpha+2} \frac{1 + \delta^2}{2}, \quad (\text{A8})$$

and δ is the isospin asymmetry. The total contribution of the zero-range interaction to the potential energy density is thus

$$H_0 = H_0^D + H_0^E = \frac{t_3}{16} \rho^{\alpha+2} \left(1 - \frac{1 + 2x_3}{3} \delta^2 \right). \quad (\text{A9})$$

Comparing it with Eq. (1) leads to Eqs. (5)–(7).

For the contribution from the finite-range interaction in Eq. (2) to the energy, we use in the exchange term the relation $(W + B P_\sigma - H P_\tau - M P_\sigma P_\tau)(-P_\sigma P_\tau) = M + H P_\sigma - B P_\tau - W P_\sigma P_\tau$ to obtain the following total contribution to the potential energy:

$$\begin{aligned} E_r &= \frac{1}{2} \int d^3 r_1 d^3 r_2 \left\{ [\rho_n(\vec{r}_1) \rho_n(\vec{r}_2) + \rho_p(\vec{r}_1) \rho_p(\vec{r}_2)] \right. \\ &\quad \times \left(W + \frac{B}{2} - H - \frac{M}{2} \right) + 2 \rho_n(\vec{r}_1) \rho_p(\vec{r}_2) \left(W + \frac{B}{2} \right) \\ &\quad + [\rho_n(\vec{r}_1, \vec{r}_2) \rho_n(\vec{r}_2, \vec{r}_1) + \rho_p(\vec{r}_1, \vec{r}_2) \rho_p(\vec{r}_2, \vec{r}_1)] \\ &\quad \times \left(M + \frac{H}{2} - B - \frac{W}{2} \right) \\ &\quad \left. + 2 \rho_n(\vec{r}_1, \vec{r}_2) \rho_p(\vec{r}_2, \vec{r}_1) \left(M + \frac{H}{2} \right) \right\} \frac{e^{-\mu|\vec{r}_1 - \vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|}, \end{aligned} \quad (\text{A10})$$

with the first and second terms being the direct contribution, and the third and fourth terms being the exchange contribution.

To obtain the energy density functional from the finite-range interaction, we introduce the coordinate transformation

$$\vec{r} = (\vec{r}_1 + \vec{r}_2)/2, \quad \vec{s} = \vec{r}_1 - \vec{r}_2. \quad (\text{A11})$$

For the direct contribution, we use the approximation of infinite nuclear matter with constant density and make use of the integral $\int d^3 s e^{-\mu s} / s = 4\pi/\mu^2$ to obtain

$$E_r^D = \int d^3 r H_r^D(\vec{r}) \quad (\text{A12})$$

with

$$H_r^D = \frac{2\pi}{\mu^2} \left(W + \frac{B}{2} - \frac{H}{2} - \frac{M}{4} \right) \rho^2 - \frac{2\pi}{\mu^2} \left(\frac{H}{2} + \frac{M}{4} \right) \rho^2 \delta^2. \quad (\text{A13})$$

For the exchange contribution, we express the density matrix in terms of the Fourier transform of the Wigner function, that is

$$\rho_\tau \left(\vec{r} + \frac{\vec{s}}{2}, \vec{r} - \frac{\vec{s}}{2} \right) = \int d^3 p f_\tau(\vec{r}, \vec{p}) e^{-i\vec{p} \cdot \vec{s}}. \quad (\text{A14})$$

Substituting Eq. (A14) into Eq. (A10) and evaluating the Fourier transform of the Yukawa interaction according to

$$\int d^3 s e^{-i(\vec{p} - \vec{p}') \cdot \vec{s}} \frac{e^{-\mu s}}{s} = \frac{4\pi}{\mu^2} \frac{1}{1 + (\vec{p} - \vec{p}')^2 / \mu^2}, \quad (\text{A15})$$

the exchange contribution of the finite-range interaction to the potential energy is then

$$E_r^E = \int d^3 r H_r^E(\vec{r}), \quad (\text{A16})$$

with

$$H_r^E(\vec{r}) = \frac{2\pi}{\mu^2} \int \frac{d^3 p d^3 p'}{1 + (\vec{p} - \vec{p}')^2 / \mu^2} \left[\left(M + \frac{H}{2} - B - \frac{W}{2} \right) \times [f_n(\vec{r}, \vec{p}) f_n(\vec{r}, \vec{p}') + f_p(\vec{r}, \vec{p}) f_p(\vec{r}, \vec{p}')] \right. \\ \left. + \left(M + \frac{H}{2} \right) 2 f_n(\vec{r}, \vec{p}) f_p(\vec{r}, \vec{p}') \right]. \quad (\text{A17})$$

Comparing Eqs. (A13) and (A17) with Eq. (1) then allows one to obtain Eqs. (8)–(12) for the parameters in the finite-range Yukawa potential in the MDI interaction.

APPENDIX B: DENSITY MATRIX EXPANSION OF THE FINITE-RANGE EXCHANGE INTERACTION

Here we follow exactly the same procedure for the density matrix expansion introduced in Ref. [19]. To obtain the energy density functional in Eq. (14) from the exchange contribution in Eq. (A10), we first use the coordinate transformation given by Eq. (A11) to express the density matrix as

$$\rho_\tau(\vec{r}_1, \vec{r}_2) = \rho_\tau \left(\vec{r} + \frac{\vec{s}}{2}, \vec{r} - \frac{\vec{s}}{2} \right) \\ = \sum_i \phi_{\tau i}^* \left(\vec{r} + \frac{\vec{s}}{2} \right) \phi_{\tau i} \left(\vec{r} - \frac{\vec{s}}{2} \right) \\ = e^{\frac{i}{2}(\nabla_1 - \nabla_2)} \sum_i \phi_{\tau i}^*(\vec{r}) \phi_{\tau i}(\vec{r}), \quad (\text{B1})$$

with $\nabla_{1(2)}$ acting on the first (second) term on the right, and $\phi_{\tau i}$ the spatial wave function of particle i with isospin τ . The angular average over the direction of \vec{s} is then

$$\frac{1}{4\pi} \int d\Omega_s \rho_\tau \left(\vec{r} + \frac{\vec{s}}{2}, \vec{r} - \frac{\vec{s}}{2} \right) \\ = \frac{\sinh \left[\frac{\vec{s}}{2} \cdot (\nabla_1 - \nabla_2) \right]}{\frac{\vec{s}}{2} \cdot (\nabla_1 - \nabla_2)} \sum_i \phi_{\tau i}^*(\vec{r}) \phi_{\tau i}(\vec{r}). \quad (\text{B2})$$

Using the Bessel-function expansion

$$\frac{\sinh(xy)}{xy} = \frac{1}{x} \sum_{n=0}^{+\infty} (4n+3) j_{2n+1}(x) Q_n(y^2) \quad (\text{B3})$$

with j_{2n+1} the $(2n+1)$ th-order spherical Bessel function and

$$Q_n(y^2) = \frac{1}{2^{2n+1}} \sum_{l=0}^n \frac{(4n+2-2l)! y^{2(n-l)}}{l!(2n+1-l)!(2n+1-2l)!}, \quad (\text{B4})$$

one obtains by identifying $x \sim k_\tau s$ and $y \sim (\nabla_1 - \nabla_2)/2k_\tau$ with k_τ being the Fermi momentum and keeping up to the third-order Bessel function the following result:

$$\frac{1}{4\pi} \int d\Omega_s \rho_\tau \left(\vec{r} + \frac{\vec{s}}{2}, \vec{r} - \frac{\vec{s}}{2} \right) \\ = \rho_{SL}(k_\tau s) \rho_\tau(\vec{r}) + g(k_\tau s) s^2 \\ \times \left[\frac{1}{4} \nabla^2 \rho_\tau(\vec{r}) - \tau_\tau(\vec{r}) + \frac{3}{5} k_\tau^2 \rho_\tau(\vec{r}) \right] \quad (\text{B5})$$

with $\rho_{SL}(k_\tau s) = \frac{3}{k_\tau s} j_1(k_\tau s)$, $g(k_\tau s) = \frac{35}{2(k_\tau s)^3} j_3(k_\tau s)$, and $\tau_\tau(\vec{r}) = \sum_i |\nabla \phi_{\tau i}(\vec{r})|^2$ being the kinetic energy density. By making the approximation

$$\frac{1}{4\pi} \int d\Omega_s \rho_\tau \left(\vec{r} + \frac{\vec{s}}{2}, \vec{r} - \frac{\vec{s}}{2} \right) \rho_{\tau'} \left(\vec{r} - \frac{\vec{s}}{2}, \vec{r} + \frac{\vec{s}}{2} \right) \\ \approx \rho_\tau(\vec{r}) \rho_{SL}(k_\tau s) \rho_{\tau'}(\vec{r}) \rho_{SL}(k_{\tau'} s) + \rho_\tau(\vec{r}) \rho_{SL}(k_\tau s) g(k_{\tau'} s) s^2 \\ \times \left[\frac{1}{4} \nabla^2 \rho_{\tau'}(\vec{r}) - \tau_{\tau'}(\vec{r}) + \frac{3}{5} k_{\tau'}^2 \rho_{\tau'}(\vec{r}) \right] \\ + \rho_{\tau'}(\vec{r}) \rho_{SL}(k_{\tau'} s) g(k_\tau s) s^2 \\ \times \left[\frac{1}{4} \nabla^2 \rho_\tau(\vec{r}) - \tau_\tau(\vec{r}) + \frac{3}{5} k_\tau^2 \rho_\tau(\vec{r}) \right], \quad (\text{B6})$$

and using the relation

$$F(\rho_n, \rho_p) \nabla^2 \rho_n = - \frac{\partial F(\rho_n, \rho_p)}{\partial \rho_n} (\nabla \rho_n)^2 \\ - \frac{\partial F(\rho_n, \rho_p)}{\partial \rho_p} \nabla \rho_n \cdot \nabla \rho_p. \quad (\text{B7})$$

we obtain

$$E_r^E = \int d^3 r H_{SL}^E(\vec{r}), \quad (\text{B8})$$

where $H_{SL}^E(\vec{r})$ is the Skymre-like zero-range energy density from the finite-range exchange interaction defined in Eq. (14).

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