Hadron widths in mixed-phase matter

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We derive classically an expression for a hadron width in a two-phase region of hadron gas and quark-gluon plasma (QGP). The presence of QGP gives hadrons larger widths than they would have in a pure hadron gas. We find that the ϕ width observed in a central Au+Au collision at $\sqrt{s}=200$ GeV/nucleon is a few MeV greater than the width in a pure hadron gas. The part of observed hadron widths due to QGP is approximately proportional to $(dN/dy)^{-1/3}$.

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There is much interest in the physics of light vector mesons in ultrarelativistic nuclear collisions [1–4]. Experimental studies of the ρ may give a measure of the transition temperature to quark-gluon plasma (QGP) [2], T_c, while studies of the ϕ can also be used to determine T_c [3] as well as the duration of the transition and the temperature range over which the transition takes place [4]. However, no previous studies have considered the effects of QGP on hadron properties observed in ultrarelativistic nuclear collisions. These effects can be large, as much of the hadron signals come from the period during which hadronic matter and QGP coexist, and hadron properties in QGP are considerably different from those in hadronic matter.

In this paper, we derive an expression for a hadron width in a "static" mixed-phase region. This is a purely classical derivation, which does not include any quantum-mechanical effects. We then use this derivation to estimate the change in the observed ϕ width in a central Au+Au collision at $\sqrt{s}=200$ GeV/nucleon, and to discuss the effect of varying projectile and/or target size and collision energy on observed hadron widths. We use standard high energy conventions, $\hbar=c=k_B=1$, throughout the paper.

The hadron width in the mixed phase is

$$\Gamma^m = \frac{R^m}{n^m}, \qquad (1)$$

where R^m and n^m are respectively the hadron decay rate per unit four-volume and density per unit volume in the mixed phase. Assuming that hadrons decay but are not created in the QGP, the decay rate in the QGP is equal to $\mathscr{A} \times J \times (1-P_S)$, where $\mathscr{A}$ is the area of the hadronic matter—QGP interface, J is the rate per unit area per unit time for hadrons to cross into the QGP (i.e., the flux density), and P_S is the survival probability (the probability that a given hadron survives a passage through the QGP). For simplicity, we treat the mixed-phase matter as a gas of spherical QGP droplets in hadronic matter; this is reasonable for high energy collisions, as experimental data is always taken over the whole mixed-phase period, and the four-volume of QGP is much less than the four-volume of hadronic matter.

The first two quantities can just be written as

$$\mathscr{A}= \int_0^\infty dr \ \mathscr{P}(r) 4\pi r^2, \qquad (2)$$

$$J=\frac{\sigma}{(2\pi)^2}\int_0^\infty dk \ \frac{k^3}{E} f_B(E) \int_0^1 dx \ x. \qquad (3)$$

Here $\mathscr{P}(r) \ dr$ is the probability of finding a QGP droplet with radius between r and $r+dr$, σ is the number of internal degrees of freedom of the hadron, k and $E=(k^2+m^2)^{1/2}$ are the hadron momentum and energy respectively, m is the hadron mass, $f_B(E)$ is the Boltzmann distribution, and the hadron's angle of incidence to the interface is $\theta=\cos^{-1}x$. The hadron's velocity is k/E, and the distance the hadron must go to cross the QGP is $2rx$, so the survival probability is

$$P_S=\exp[- \alpha(k) \ rx], \qquad (4)$$

where

$$\alpha(k)=2E \ \gamma^q(k)/k, \qquad (5)$$

and $\gamma^q(k)$ is the QGP decay rate for a hadron of momentum k. We thus obtain

$$R^m=fn^h\Gamma^h+\frac{\sigma}{\pi V}\int_0^\infty dr \ r^2 \ \mathscr{P}(r)\int_0^\infty dk \ \frac{k^3}{E} f_B(E)$$

$$\times \int_0^1 dx \ x \ [1-e^{- \alpha(k)rx}], \qquad (6)$$

$$=fn^h\Gamma^h+\frac{\sigma}{\pi V}\int_0^\infty dr \ r^2 \ \mathscr{P}(r)$$

$$\times \int_0^\infty dk \ \frac{k^3}{E} f_B(E) \ g[\alpha(k)r], \qquad (7)$$

where V is the volume of hot matter, f is the fraction of matter in the hadronic phase,

$$n^h=\frac{\sigma}{2\pi^2}\int_0^\infty dk \ k^2 f_B(E) \qquad (8)$$

is the hadron density in hadronic matter, and

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$$g(z) = \frac{1}{2} - \frac{1}{z^2} [1 - (1+z)e^{-z}].$$ (9)

The QGP contribution to the ϕ density, n^m, is equal to $\mathscr{A} \times J \times \bar{t} / V$, where $\bar{t}$ is the mean time a given hadron spends in the QGP before either decaying or escaping. The survival probability after time t is $\exp[-\gamma^q(k)t]$, so we find

$$\bar{t} = \frac{1}{\gamma^q(k)} [1 - e^{-\alpha(k)rx}].$$ (10)

Combining this with the expressions for $\mathscr{A}$ and J above, we obtain

$$n^m = fn^h + \frac{\sigma}{\pi V} \int_0^\infty dr \; r^2 \; \mathscr{P}(r) \int_0^\infty dk \; \frac{k^3}{E} f_B(E)$$

$$\times \int_0^1 dx \; x \; \frac{1}{\gamma^q(k)} [1 - e^{-\alpha(k)rx}],$$ (11)

$$= fn^h + \frac{\sigma}{\pi V} \int_0^\infty dr \; r^2 \; \mathscr{P}(r)$$

$$\times \int_0^\infty dk \; \frac{k^3}{E} f_B(E) \; \frac{1}{\gamma^q(k)} \; g[\alpha(k)r].$$ (12)

If the decay rate in QGP is small [$\gamma^q \ll k/(rE)$ for thermal momenta], we expand the exponentials and find

$$R^m = fn^h \Gamma^h + \frac{2\sigma}{3\pi V} \int_0^\infty dr \; r^3 \; \mathscr{P}(r)$$

$$\times \int_0^\infty dk \; k^2 \; f_B(E) \; \gamma^q(k),$$ (13)

$$= n^h [f\Gamma^h + (1-f)\Gamma^q],$$ (14)

$$n^m = n^h,$$ (15)

where

$$\Gamma^q = \frac{\int_0^\infty dk \; k^2 \; f_B(E) \; \gamma^q(k)}{\int_0^\infty dk \; k^2 \; f_B(E)}$$ (16)

is the thermally averaged hadron decay rate in QGP. In deriving these results, we used the normalization condition

$$\int_0^\infty dr \; \mathscr{P}(r) \; \frac{4}{3} \pi r^3 = (1-f) \; V.$$ (17)

Dividing the decay rate by the density, we obtain the simple expression

$$\Gamma^m = f\Gamma^h + (1-f)\Gamma^q.$$ (18)

If the decay rate is large, the expression for the width is not so simple. In the limit $\gamma^q(k) \to \infty$, we obtain

$$R^m = fn^h \Gamma^h + \frac{\sigma}{2\pi V} \int_0^\infty dr \; r^2 \; \mathscr{P}(r) \int_0^\infty dk \; \frac{k^3}{E} f_B(E),$$ (19)

$$n^m = fn^h,$$ (20)

$$\Gamma^m = \Gamma^h + \frac{\sigma}{2\pi f V n^h} \int_0^\infty dr \; r^2 \; \mathscr{P}(r) \int_0^\infty dk \; \frac{k^3}{E} f_B(E).$$ (21)

Note that, in this case, the width depends on the details of the mixed phase structure through $\mathscr{P}(r)$, and not just on the fraction of QGP that is present.

Unfortunately, the only way to calculate $\mathscr{P}(r)$ is to do a three-dimensional simulation with both nucleation and growth of droplets of hadronic phase in the QGP. This is because $\mathscr{P}(r)$ is really an inherently dynamical quantity, and goes to a δ function in a static mixed phase. If you begin with several droplets of one phase, the volume energy is independent of the number of droplets since total volume in the phase is conserved, but the surface energy grows with the number of droplets, so the free energy is minimized when all of the phase is in one large droplet. This can be seen from droplet nucleation and growth calculations [5]—if the droplet radius is larger than some critical radius, the droplet grows (forever, or until it runs out of matter), so any droplet distribution other than a single droplet is metastable.

If we specify the volume, V, then the droplet distribution is known:

$$\mathscr{P}(r) = \delta(r - r_d),$$ (22)

$$\frac{4\pi}{3} r_d^3 = (1-f)V.$$ (23)

We then evaluate the integrals over r to get

$$R^m = fn^h \Gamma^h + \frac{3\sigma(1-f)}{4\pi^2 r_d}$$

$$\times \int_0^\infty dk \; \frac{k^3}{E} f_B(E) \; g[\alpha(k)r_d],$$ (24)

$$n^m = fn^h + \frac{3\sigma(1-f)}{4\pi^2 r_d}$$

$$\times \int_0^\infty dk \; \frac{k^3}{E\gamma^q(k)} f_B(E) \; g[\alpha(k)r_d].$$ (25)

We now move to the specific case of the observed change in the ϕ width in an ultrarelativistic nuclear collision. The ϕ decay rate in the QGP, $\gamma^q(k)$, can be estimated by modeling the ϕ propagating through the QGP as an $s\bar{s}$ pair. We thus obtain

$$\gamma^q \approx 2\gamma_s,$$ (26)

where γ_s is the width of a thermal s (or $\bar{s}$), as we destroy the ϕ if either the s or $\bar{s}$ is scattered in the QGP.

Treating the propagating s and $\bar{s}$ as light quarks with thermal momenta, we find from Pisarski [6] that

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$$\gamma^q = 2\alpha_S C_f T_c \ln\left[\frac{1}{\alpha_S}\right], \tag{27}$$

independent of k. Here $\alpha_S = g^2/4\pi$, g is the strong coupling constant, $C_f = (N^2-1)/(2N)$ is the Casimir factor for the fermion representation, N is the number of colors, and the argument of the logarithm is an approximation to what one would get with a magnetic mass cutoff. Note that $\gamma^q(k)$ is four times the parton damping rate [the damping rate is for the wave function, but you square the wave function to get the probability density so the parton decay rate is twice the damping rate, and the ϕ decay rate is twice the parton decay rate]. With $N = 3$, $C_f = 4/3$, and $g \simeq 2$, we find

$$\gamma^q \simeq T_c. \tag{28}$$

To put this in a simple one-dimensional (no transverse expansion), boost-invariant, hydrodynamic collision simulation [7], we assume that we have an approximately massless gas so that τT^3 is conserved outside the mixed phase region, where τ is proper time. In the mixed phase region, the requirement of entropy conservation relates f and τ:

$$f(\tau) = \frac{r}{r-1}\left(\frac{\tau-\tau_q}{\tau}\right), \tag{29}$$

where r is the ratio of number of degrees of freedom in the QGP and hadronic phases, and τ_q is the proper time at which the period of two-phase coexistence begins.

During the early stages of the evolution, the behavior of $\mathscr{P}(r)$ is very complicated. However, the volume of hadronic matter is greatest near the end of the mixed-phase period, so most of the observed hadrons occur during the end of this stage. Late in the mixed phase, we can try to guess the droplet distribution. Because the droplets with similar velocities tend to merge, we assume that (i) there is only one droplet in any transverse slice, and (ii) the droplets are distributed uniformly in rapidity with distance y_d between droplets. Thus, we find

$$V(\tau) = A y_d \tau, \tag{30}$$

where A is the cross-sectional area of the hot matter.

Combining Eqs. (23), (29), and (30), we obtain

$$r_d(\tau) = \left[\frac{3A y_d}{4\pi(r-1)}(r\tau_q - \tau)\right]^{1/3}. \tag{31}$$

The observed width is then

$$\Gamma^m = \frac{r(r-1)^2\tau_q^2 n^h \Gamma^h + \dfrac{3\sigma}{2\pi^2}\displaystyle\int_{\tau_q}^{r\tau_q} d\tau \dfrac{r\tau_q - \tau}{r_d(\tau)}\displaystyle\int_0^\infty dk \dfrac{k^3}{E} f_B(E)\, g[\alpha(k)r_d(\tau)]}{r(r-1)^2\tau_q^2 n^h + \dfrac{3\sigma}{2\pi^2\gamma^q}\displaystyle\int_{\tau_q}^{r\tau_q} d\tau \dfrac{r\tau_q - \tau}{r_d(\tau)}\displaystyle\int_0^\infty dk \dfrac{k^3}{E} f_B(E)\, g[\alpha(k)r_d(\tau)]}. \tag{32}$$

Here we obtain respectively the numerator and denominator by integrating the decay rate due to the QGP droplets and the density over the four-volume of the mixed-phase matter. For a central Au+Au collision at $\sqrt{s} = 200$ GeV/nucleon, we take $A = 150$ fm^2, $r = 10$, and $\tau_q = 8$ (16) fm/c for $T_c = 190$ (150) MeV. For $T_c = 150$ MeV, we find $\Gamma^m - \Gamma^h = 1.1$ MeV with $y_d = 1$ and 2.3 MeV with $y_d = 0.1$, while for $T_c = 190$ MeV we find 1.5 and 3.1 MeV respectively. These values are smaller than the width in a hadronic gas (about 6 MeV at $T = 150$ MeV and 9 at $T = 190$ MeV [4]), but are still large enough to be significant.

We obtain a second estimate of $\gamma^q(k)$ from the damping rate in the heavy quark limit [6],

$$\gamma^q(k) = 2\alpha_S C_f T_c\left\{1 + \frac{k}{2E}\ln\left[\frac{4\pi^2(N+N_f/2)k^3}{3C_f^2\alpha_S E^3}\right]\right\}, \tag{33}$$

where N_f is the number of light fermion species. Taking $N = 3$, $C_f = 4/3$, $N_f = 3$ (u, d, s), and $g = 2$,

$$\gamma^q(k) = \frac{8}{3\pi} T_c\left[1 + \frac{3k}{2E}\ln\left(\frac{3\pi k}{2E}\right)\right]. \tag{34}$$

With this new expression for $\gamma^q(k)$, we find $\Gamma^m - \Gamma^h = 1.1$ MeV with $y_d = 1$ and 2.4 MeV with $y_d = 0.1$ for $T_c = 150$

MeV, while we find 1.5 and 3.3 MeV respectively for $T_c = 190$ MeV. Thus, our results are insensitive to our assumptions about γ^q.

In the limit $\gamma^q(k) \to \infty$, we find

$$\Gamma^m - \Gamma^h = \frac{9}{5} \frac{\bar{v}(T_c)}{r}\left[\frac{\pi}{6A\, y_d\, \tau_q}\right]^{1/3}, \tag{35}$$

$$\bar{v}(T_c) = \frac{\displaystyle\int_0^\infty dk\, \frac{k^3}{E} f_B(E)}{\displaystyle\int_0^\infty dk\, k^2 f_B(E)}. \tag{36}$$

For $T_c = 150$ MeV, we obtain $\Gamma^m - \Gamma^h = 1.2$ MeV with $y_d = 1$ and 2.5 MeV with $y_d = 0.1$, while for $T_c = 190$ MeV we obtain 1.6 and 3.4 MeV respectively. We also calculate Γ^m as a function of total rapidity density (charged plus neutral), dN/dy, using the relation

$$3.6\, dN/dy = \frac{74\pi^2}{45} A\, \tau_q\, T_c^3 \tag{37}$$

(from entropy conservation) to obtain

$$\Gamma^m - \Gamma^h = \frac{\pi \, T_c \, \bar{v}(T_c)}{5 \, r} \left[\frac{111}{2 \, (dN/dy) \, y_d} \right]^{1/3}. \quad (38)$$

This last form is probably the most useful for experimenters, as dN/dy is readily measured, unlike A and τ_q.

Note that we do not include damping due to Debye screening, as the ϕ is already formed, unlike the case of J/ψ suppression [8]. As long as no constituents of the ϕ scatter, it should emerge from the QGP with its wave function intact. We also neglect the possibility of reflection from the hadronic matter—QGP interface. However, if we take the limit $\gamma^q \to \infty$ [Eq. (35)], our results are changed very little. Thus, adding effects that increase γ^q does not produce a large change in the ϕ width.

The main effect of changing the projectile or target is to change A. As Γ^m only depends on A through r_d, and r_d increases with increasing A, changing A is equivalent to changing r_d. For large $\gamma^q(k)$, which is the case for essentially any hadron in QGP, Γ^m decreases monotonically with increasing r_d and thus with increasing A or y_d. Therefore, Γ^m will be greater in collisions of smaller projectiles and/or targets.

Similarly, if collision energy is lowered then τ_q is reduced. This reduces r_d, thus increasing R^q. All other changes in Γ^m can be absorbed in the normalization of the integrals, so lowering the collision energy will increase Γ^m. Decreasing the projectile and/or target size also tends to decrease τ_q slightly, and this will increase hadron widths. Finally, raising T_c reduces τ_q, which is why $\Gamma^m - \Gamma^h$ is 30–40 % larger at $T_c = 190$ MeV than at $T_c = 150$ MeV.

The dependence of the width on beam energy, projectile or target sizes, r, and T_c can be explained simply. Because hadrons do not survive in the droplets, the number destroyed is approximately proportional to the area of the interface, A, while the number observed is proportional to the volume of the hadronic matter, V_h. Since $A \propto V_q^{2/3}$, where $V_q \propto y_d(dN/dy)/T_c^3$ is the volume of QGP at the start of the phase transition, and $V_h = r V_q$, this gives Γ

$\propto T_c [y_d(dN/dy)]^{-1/3}/r$. The remaining factor $\bar{v}(T_c)$ occurs because the flux across the interface must also be proportional to the mean hadron velocity. Because of this simple explanation, we expect that the qualitative behavior—the increase of Γ^m-Γ^h with decreasing projectile and/or target size, collision energy, or r and increasing T_c—will hold for any hadron.

The portion of the hadron width due to QGP can be observed experimentally using satellite vector meson peaks in the dilepton spectrum, which are a signal of a strong first-order hadronic phase transition [3]. These satellite peaks come from mesons that decay in the mixed phase, at fixed temperature, so the meson width in a hadron gas is constant. The increase in the width due to QGP droplets can be observed by comparing the widths in events with different values of dN/dy, as long as conditions are such that thermal equilibration should occur before the region of mixed-phase matter is formed. The portion of the width due to the QGP droplets should scale approximately as $(dN/dy)^{-1/3}$, following Eq. (38), since T_c and r do not vary between events and y_d probably does not vary much.

Note that this increased width is not an unambiguous signal for the presence of QGP. Suppose that the nature of the high-temperature phase is different (perhaps chiral symmetry is restored without deconfinement), but the hadron lifetimes are still short in this phase. In that case, the calculation would proceed just as above, and the effect of droplets of high-temperature phase on the hadron widths would be essentially unchanged. Thus, the observation of this increased width is a signal only for short hadron lifetimes in the high-temperature phase.

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