Atom localization via phase and amplitude control of the driving field

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Control of amplitude and phase of the driving field in an atom-field interaction leads towards the strong line narrowing and quenching in the spontaneous emission spectrum. We exploit this fact for the atom localization scheme and achieve a much better spatial resolution in the conditional position probability distribution of the atom. Most importantly the quenching in the spontaneous emission manifests itself in reducing the periodicity in the conditional position probability distribution and hence the uncertainty in a particular position measurement of the single atom by a factor of 2.

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I. INTRODUCTION

In recent years, a considerable attention is given to the development of various techniques based on the optical methods to measure the position of the atom. This is due to the fact that the optical methods provide better spatial resolution in position measurement of the atom. For example, the concept of optical virtual slits is proposed to localize the atom within the subwavelength domain of the optical field. In this scheme the atom interacts with a standing-wave field and during its interaction a phase shift is introduced in the field. The measurement of this phase shift then gives the position information of the atom [1].

In a related experiment an encoding method, in which the atom's position is encoded with the excitation amplitude of two long-lived electronic states, is used to localize the atom in an array of virtual slits [2]. This experiment gives the possibility of producing narrow localization structures with widths below $\lambda/20$. In another idea, the phase quadrature measurement is investigated for the case when transverse motion of the atom cannot be neglected [3,4]. In these schemes, the method of continuous homodyne detection, which is based on the Monte Carlo technique [5], is employed to monitor the transverse motion of the atom. In another related scheme, in which the atom localization is proposed by using Ramsey interferometry, it is shown that the use of a coherent-state cavity field is better than the classical field to get a higher resolution in position information of the atom [6].

Earlier, some experiments based on the resonance imaging methods are proposed for the precision position measurement of the moving atoms [7]. In these experiments information about the position of the atom is obtained with a spatial resolution of 1.7 μm when a magnetic-field gradient is used [8]. This spatial resolution is then enhanced to 200 nm when instead of magnetic-field gradient a light shift gradient is used [9,10].

More recently, some methods based on the detection of the spontaneously emitted photon during the interaction of an atom with the classical standing-wave field [11–17] are proposed. These methods are based on the fact that the frequency of the spontaneously emitted photon, due to its direct relation with the position-dependent Rabi frequency of the atom, gives the position information of the atom. The main advantage of these schemes over the previous schemes is that one gets the position information of the atom in real time within the subwavelength domain of the optical field and with a high spatial resolution.

However, there is a main drawback in all of the schemes in which the atom interacts with the standing-wave field, namely, the periodicity of the standing-wave field yields four, equally probable, different positions of the atom in a unit wavelength domain of the optical field when a spontaneously emitted photon is detected. Therefore, for a single required frequency measurement, the probability of finding the atom at the particular position is 1/4. Keeping in mind the simplicity of these techniques [11–17], the main question, therefore, arises; is there any way to reduce this uncertainty in measuring a particular position of the atom? In this paper we address this question and show that we can reduce the periodicity in the conditional position probability distribution by employing the phase and amplitude of the driving field.

The idea of phase-controlled fluorescence is originated by Bais, Yodth, and Mossberg [18]. They experimentally studied the resonance fluorescence during phase-controlled transient excitation and modified the resonance fluorescence spectrum using phase-controlled optical fields. Recent studies show that ultranarrow spectral lines with a selective and total cancellation of the fluorescence decay can be achieved by controlling the relative phase and amplitudes of the driving fields [19,20]. This strong line narrowing and selective cancellation of the fluorescence decay can play a strong role in enhancing the efficiency of the atom localization schemes. Here we suggest that the control of the phase associated with the classical standing-wave driving field reduces the periodicity in the position probability distribution by a factor of 2. This occurs due to the quenching of the spontaneous emission by controlling the phase associated with the classical standing-wave driving field. Therefore, certain choice of the phase associated with the classical standing-wave driving field increases the probability of localizing the atom at a particular position in a single required frequency measurement by a factor of 2. We also observe that control of the amplitudes of the driving field exhibits a strong line narrow-

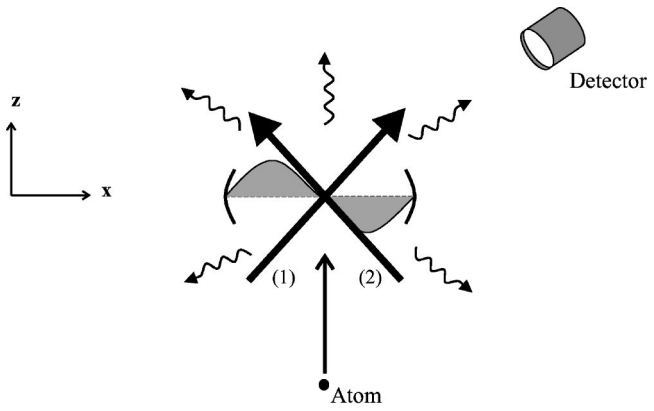

FIG. 1. An atom is moving along z direction and passing through an interaction region. There are three classical fields interacting with the atom one of which is a standing-wave field aligned in the x axis. While the traveling-wave fields (1) and (2) are aligned in such a way that these are mutually perpendicular. The atom radiates spontaneously and the frequency of the spontaneously emitted photon is measured by a detector.

ing that yields a better spatial resolution in position measurement of the single atom.

II. MODEL AND EQUATIONS

We consider an atom, moving in the z direction with a center-of-mass wave function $f(x)$, passes through the classical standing-wave field, which is aligned along the x axis (see Fig. 1). The atom has an higher energy level $|a\rangle$ and a lower energy level $|d\rangle$. The atomic decay takes place from the level $|a\rangle$ to $|d\rangle$ at a rate Γ due to the interaction of the atom with the reservoir modes. The decaying level $|a\rangle$ is coupled to another energy level $|b\rangle$ and further the level $|b\rangle$ is coupled to a level $|c\rangle$ via classical fields. The corresponding Rabi frequencies are Ω_1 and Ω_2, respectively. At the same time the decaying level $|a\rangle$ is also coupled to the level $|c\rangle$ via a classical standing-wave field of frequency ν and having a relative phase φ. The corresponding Rabi frequency is Ω_3 (see Fig. 2).

The atom during its motion in the z direction passes through the classical standing-wave field. The interaction between the atom and the classical standing-wave field is position dependent and hence the corresponding Rabi frequency of the atom is defined as $\Omega_3(x)=\Omega_3 \sin(\kappa x)$, where $\Omega_3(x)$ is the position-dependent Rabi frequency and κ is the wave vector of the standing-wave field that is defined as $\kappa = 2\pi/\lambda$.

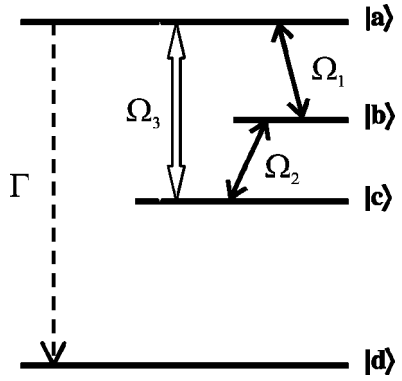

FIG. 2. The energy-level diagram of the atom. The atomic decay takes place only from level $|a\rangle$ to the level $|d\rangle$.

Here we consider that the atom is moving with high enough velocity that its interaction with the driving fields does not effect its motion along the z direction and, therefore, we may treat its motion in the z direction classically. We also assume that interaction time of the atom with the standing-wave field and hence the Rabi frequency is sufficiently small so that the center-of-mass position of the atom along the standing wave does not change during the interaction time and thus we may neglect the kinetic-energy term of the atom in the interaction Hamiltonian under the Raman-Nath approximation. Therefore, the resulting interaction Hamiltonian for the resonant atom-field system can be written as

$$\begin{aligned}\mathcal{V}=&\hbar[\Omega_1 e^{ikx\cos\theta_1}|a\rangle\langle b|+\Omega_1 e^{-ikx\cos\theta_1}|b\rangle\langle a|]\\&+\hbar[\Omega_2 e^{ikx\cos\theta_2}|b\rangle\langle c|+\Omega_2 e^{-ikx\cos\theta_2}|c\rangle\langle b|]\\&+\hbar[\Omega_3\sin(\kappa x)|a\rangle\langle c|e^{i\varphi}+\Omega_3\sin(\kappa x)|c\rangle\langle a|e^{-i\varphi}]\\&+\hbar\left[\sum_{\mathbf{k}}(g_{\mathbf{k}}(x)e^{i\delta_k t}|a\rangle\langle d|b_{\mathbf{k}}+g^*_{\mathbf{k}}(x)e^{-i\delta_k t}|d\rangle\langle a|b^\dagger_{\mathbf{k}})\right].\end{aligned} \tag{1}$$

Here $g_{\mathbf{k}}(x)$ is the coupling constant associated with the spontaneous emission of the photon and $b_{\mathbf{k}}$ and $b^\dagger_{\mathbf{k}}$ are the annihilation and creation operators corresponding to the reservoir modes $\mathbf{k}$. The detuning parameter $\delta_{\mathbf{k}}$ is defined as

$$\delta_{\mathbf{k}}=\omega_{ad}-\nu_k, \tag{2}$$

where ω_{ad} is the atomic transition frequency between the energy levels $|a\rangle$ and $|d\rangle$ and ν_k is the frequency of the reservoir mode $\mathbf{k}$.

The atom-field state vector for the complete system can be written as

$$\begin{aligned}|\Psi(x;t)\rangle=\int dx f(x)|x\rangle\Big[&C_{a,0}(x;t)|a,0\rangle+C_{b,0}(x;t)|b,0\rangle\\&+C_{c,0}(x;t)|c,0\rangle+\sum_{\mathbf{k}}C_{d,1_{\mathbf{k}}}(x;t)|d,1_{\mathbf{k}}\rangle\Big],\end{aligned} \tag{3}$$

where the probability amplitudes $C_{i,0}(x;t)$ $(i=a,b,c)$ represent the state of the atom when there is no spontaneously emitted photon present in the reservoir mode $\mathbf{k}$ and $C_{d,1_{\mathbf{k}}}(x;t)$ is the probability amplitude that shows that at time t the atom is in the level $|d\rangle$ with one photon emitted spontaneously in the reservoir mode $\mathbf{k}$.

Our atom localization scheme is based on the fact that the spontaneously emitted photon carries the information about the position of the atom due to the dependence of its frequency on the position-dependent Rabi frequency of the atom associated with the classical standing-wave field [11–16]. Therefore, the position measurement of the atom is conditioned on the detection of the spontaneously emitted pho-

ton. The conditional position probability distribution $W(x;t|d,1_{\mathbf{k}})$ is thus defined as the probability of finding the atom at position x in the standing-wave field given that a spontaneously emitted photon is detected at time t in the reservoir mode of wave vector $\mathbf{k}$. This conditional position probability distribution can be determined by taking the appropriate projection over the atom-field state vector (3) and thus we obtain

$$W(x)\equiv W(x;t|d,1_{\mathbf{k}})=\mathcal{F}(x;t|d,1_{\mathbf{k}})|f(x)|^2, \tag{4}$$

with the filter function defined as

$$\mathcal{F}(x;t|d,1_{\mathbf{k}})\equiv|\mathcal{M}|^2|C_{d,1_{\mathbf{k}}}(x;t)|^2. \tag{5}$$

This shows that the conditional position probability distribution follows from the probability amplitude $C_{d,1_{\mathbf{k}}}$ and we need to determine this probability amplitude.

The required analytical expression for the probability amplitude $C_{d,1_{\mathbf{k}}}$ can be found by solving the Schrödinger wave equation and by using the interaction picture Hamiltonian (1). We then obtain the rate equations for the corresponding probability amplitudes as follows:

$$\dot{C}_{a,0}(t)=-i\Big[\Omega_1 e^{ikx\cos\theta_1}C_{b,0}(t)+\Omega_3\sin(\kappa x)C_{c,0}(t)e^{i\varphi}+\sum_{\mathbf{k}}g_{\mathbf{k}}(x)e^{i\delta_{\mathbf{k}}t}C_{d,1_{\mathbf{k}}}(t)\Big], \tag{6}$$

$$\dot{C}_{b,0}(t)=-i[\Omega_1 e^{-ikx\cos\theta_1}C_{a,0}(t)+\Omega_2 e^{ikx\cos\theta_2}C_{c,0}(t)], \tag{7}$$

$$\dot{C}_{c,0}(t)=-i[\Omega_2 e^{-ikx\cos\theta_2}C_{b,0}(t)+\Omega_3\sin(\kappa x)C_{a,0}(t)e^{-i\varphi}], \tag{8}$$

$$\dot{C}_{d,1_{\mathbf{k}}}(t)=-ig^*_{\mathbf{k}}(x)e^{-i\delta_k t}C_{a,0}(t). \tag{9}$$

On integrating Eq. (9) we obtain

$$C_{d,1_{\mathbf{k}}}(t)=-ig^*_{\mathbf{k}}(x)\int_0^t dt' e^{-i\delta_k t'}C_{a,0}(t'), \tag{10}$$

which on substituting in Eq. (6) and performing the Weisskopf-Wigner approximation, for the initial condition $C_{d,1_{\mathbf{k}}}(0)=0$, gives

$$\dot{C}_{a,0}(t)=-i[\Omega_1 e^{ikx\cos\theta_1}C_{b,0}(t)+\Omega_3\sin(\kappa x)C_{c,0}(t)e^{i\varphi}]-\frac{\Gamma}{2}C_{a,0}(t). \tag{11}$$

Here Γ is the atomic decay rate from the level $|a\rangle$ to the level $|d\rangle$. On taking the Laplace transform of the equations of motion for the probability amplitudes we get a new set of equations as follows:

$$\left(s+\frac{\Gamma}{2}\right)C_{a,0}(s)+i\Omega_1 e^{ikx\cos\theta_1}C_{b,0}(s)+i\Omega_3\sin(\kappa x)e^{i\varphi}C_{c,0}(s)=0, \tag{12}$$

$$i\Omega_1 e^{-ikx\cos\theta_1}C_{a,0}(s)+sC_{b,0}(s)+i\Omega_2 e^{ikx\cos\theta_2}C_{c,0}(s)=1, \tag{13}$$

$$i\Omega_3\sin(\kappa x)e^{-i\varphi}C_{a,0}(s)+i\Omega_2 e^{-ikx\cos\theta_2}C_{b,0}(s)+sC_{c,0}(s)=0, \tag{14}$$

$$C_{d,1_{\mathbf{k}}}(t)=-ig^*_{\mathbf{k}}C_{a,0}(s). \tag{15}$$

In writing these equations we consider that the atom is prepared initially in the level $|b\rangle$, so that only $C_{b,0}(0)=1$. The new set of coupled equations (12)–(14) can be solved easily using Cramer's rule and we thus obtain the solution of the Laplace transform of the probability amplitude $C_{a,0}(t)$ as

$$C_{a,0}(s)=\frac{a_2}{D}, \tag{16}$$

where

$$a_2=-is\Omega_1 e^{ikx\cos\theta_1}-\Omega_2\Omega_3\sin(\kappa x)e^{i\varphi-ikx\cos\theta_2}, \tag{17}$$

$$D=s^3+s^2\frac{\Gamma}{2}+s[\Omega_1^2+\Omega_2^2+\Omega_3^2\sin^2(\kappa x)]+\frac{\Gamma}{2}\Omega_2^2-2i\sin(\kappa x)\Omega_1\Omega_2\Omega_3\cos(\varphi-kx\cos\theta_1-kx\cos\theta_2). \tag{18}$$

The solution for the probability amplitude $C_{d,1_{\mathbf{k}}}(t)$ can be found directly by just replacing s with its value $i\delta_{\mathbf{k}}$ and, therefore, we get $C_{d,1_{\mathbf{k}}}(t)$ for large time limit when the interaction time is much larger than the atomic decay rate Γ, i.e., $\Gamma t\gg 1$ as

$$C_{d,1_{\mathbf{k}}}(t\to\infty)=-ig^*_{\mathbf{k}}(x)\left[\frac{\delta_{\mathbf{k}}\Omega_1 e^{ikx\cos\theta_1}-\Omega_2\Omega_3\sin(\kappa x)e^{i\varphi-ikx\cos\theta_2}}{D}\right], \tag{19}$$

with

$$D=-\frac{\Gamma}{2}(\delta_{\mathbf{k}}^2-\Omega_2^2)+i[\delta_{\mathbf{k}}\sin^2(\kappa x)\Omega_3^2-2\Omega_1\Omega_2\Omega_3\sin(\kappa x)\cos\varphi+\delta_{\mathbf{k}}(\Omega_1^2+\Omega_2^2-\delta_{\mathbf{k}}^2)], \tag{20}$$

where $\theta_1=\pi/4$ and $\theta_2=\pi/2+\theta_1$.

The required probability of finding the atom in the level $|d\rangle$ with a spontaneously emitted photon of frequency ν_k corresponding to the reservoir mode $\mathbf{k}$ is then given by

$$|C_{d,1_{\mathbf{k}}}(t\to\infty)|^2=|G_{\mathbf{k}_0}|^2\left[\frac{\Omega_2^2\Omega_3^2[\sin(\kappa x)-L_1][\sin(\kappa x)-L_2]}{\Gamma^2/4(\delta_{\mathbf{k}}^2-\Omega_2^2)^2+\delta_{\mathbf{k}}^2\Omega_3^4[\sin(\kappa x)-R_1]^2[\sin(\kappa x)-R_2]^2}\right], \tag{21}$$

where

$$L_1=\frac{\delta_{\mathbf{k}}\Omega_1}{\Omega_2\Omega_3}(\cos\varphi+i\sin\varphi), \tag{22}$$

$$L_2=\frac{\delta_{\mathbf{k}}\Omega_1}{\Omega_2\Omega_3}(\cos\varphi-i\sin\varphi), \tag{23}$$

$$R_1=\frac{\Omega_1\Omega_2\cos\varphi+\sqrt{\Omega_1^2\Omega_2^2\cos^2\varphi-\delta_{\mathbf{k}}^2(\Omega_1^2+\Omega_2^2-\delta_{\mathbf{k}}^2)}}{\delta_{\mathbf{k}}\Omega_3} \tag{24}$$

and

$$R_2=\frac{\Omega_1\Omega_2\cos\varphi-\sqrt{\Omega_1^2\Omega_2^2\cos^2\varphi-\delta_{\mathbf{k}}^2(\Omega_1^2+\Omega_2^2-\delta_{\mathbf{k}}^2)}}{\delta_{\mathbf{k}}\Omega_3}. \tag{25}$$

Finally we obtain the filter function $\mathcal{F}(x)\equiv\mathcal{F}(x;t\to\infty|d,1_{\mathbf{k}})$ in the form

$$\mathcal{F}(x)=|\mathcal{N}|^2|G_{\mathbf{k}_0}|^2\left[\frac{\Omega_2^2\Omega_3^2[\sin(\kappa x)-L_1][\sin(\kappa x)-L_2]}{\Gamma^2/4(\delta_{\mathbf{k}}^2-\Omega_2^2)^2+\delta_{\mathbf{k}}^2\Omega_3^4[\sin(\kappa x)-R_1]^2[\sin(\kappa x)-R_2]^2}\right]. \tag{26}$$

III. ANALYSIS OF THE FILTER FUNCTION $\mathcal{F}(x)$

In this section we analyze our filter function $\mathcal{F}(x)$ for different experimental parameters and see on what conditions the conditional position probability distribution exhibits minimum atom localization peaks. We consider that the initial position distribution of the atom is a broad wave packet and the filter function $\mathcal{F}(x)$, therefore, directly gives the conditional position probability distribution [Eq. (4)]. In this conditional position probability distribution we get peak maxima at the normalized positions

$$\kappa x=\sin^{-1}(R_1)\pm n\pi, \tag{27}$$

and

$$\kappa x=\sin^{-1}(R_2)\pm n\pi, \tag{28}$$

where R_1 and R_2 are given by Eqs. (24) and (25) while n is an integer having values 0,1,2, ...

It was already discussed that our atom localization scheme requires the frequency measurement of the spontaneously emitted photon in some reservoir mode $\mathbf{k}$ and the expression (26) clearly shows this fact. From this expression we observe that the filter function $\mathcal{F}(x)$ has a direct relationship with the detuning parameter $\delta_{\mathbf{k}}$ which is proportional to the measured frequency ν_k of the spontaneously emitted photon [Eq. (2)]. Here, it is noticed that the filter function depends not only on the frequency of the spontaneously emitted photon but also on the amplitudes Ω_1, Ω_2, and Ω_3 of the traveling and standing-wave classical fields as well as on the phase φ associated with the classical standing-wave field [via Eqs. (22)–(25)].

In principle, the quantity that solely determines the conditional position probability distribution is the frequency of the spontaneously emitted photon. The question, however, is for what values of the frequency we get a maxima in the filter function? This can be easily answered by looking at the expression for filter function (26). It is obvious that we get peak maxima in the filter function whenever $\sin(\kappa x)=R_1$ or $\sin(\kappa x)=R_2$.

For the case when $\Omega_1=\Omega_2=\Omega$ and $\varphi=\pi/2$ we get peak maxima when the detuning $\delta_{\mathbf{k}}$ satisfies the conditions

$$\delta_{\mathbf{k}}=\pm\sqrt{2\Omega^2+\Omega_3^2\sin^2(\kappa x)}. \tag{29}$$

Similarly for $\varphi=0$ and $\varphi=\pi$ we get peak maxima when the detuning $\delta_{\mathbf{k}}$ satisfies the conditions

$$\delta_{\mathbf{k}}=\frac{-\Omega_3\sin(\kappa x)\pm\sqrt{\Omega_3^2\sin^2(\kappa x)+8\Omega^2}}{2}, \tag{30}$$

and

$$\delta_{\mathbf{k}}=\frac{\Omega_3\sin(\kappa x)\pm\sqrt{\Omega_3^2\sin^2(\kappa x)+8\Omega^2}}{2}, \tag{31}$$

respectively. As the frequency of the spontaneously emitted photon is directly related to the detuning parameter $\delta_{\mathbf{k}}$ via relation (2), therefore, the atomic transition from the energy level $|a\rangle$ to the level $|d\rangle$ is accompanied by a spontaneously emitted photon of frequency

$$\nu_k=\omega_{ad}\mp\sqrt{2\Omega^2+\Omega_3^2\sin^2(\kappa x)}, \tag{32}$$

when $\Omega_1=\Omega_2=\Omega$ and $\varphi=\pi/2$ while the frequency of the spontaneously emitted photon corresponds to the relation

$$\nu_k=\omega_{ad}+\frac{\Omega_3\sin(\kappa x)\mp\sqrt{\Omega_3^2\sin^2(\kappa x)+8\Omega^2}}{2}, \quad (33)$$

for $\varphi=0$ and

$$\nu_k=\omega_{ad}-\frac{\Omega_3\sin(\kappa x)\mp\sqrt{\Omega_3^2\sin^2(\kappa x)+8\Omega^2}}{2}, \quad (34)$$

when $\varphi=\pi$. These relations clearly show that the position of the atom can be found easily once the spontaneously emitted photon is detected.

We also observe from Eq. (29) that when $\varphi=\pi/2$ we get four localization peaks in the conditional position probability distribution in a unit wavelength domain of the standing-wave field. The number of peaks reduces when $\varphi=0$ or $\varphi=\pi$ and we get two localization peaks in the conditional position probability distribution within a unit wavelength domain of the standing-wave driving field, see Eqs. (30) and (31). This is the main advantage in the present scheme because it doubles the probability of finding the atom at a given position for a particular frequency measurement.

A careful analysis of the expression (26) of the filter function $\mathcal{F}(x)$ shows that there are two roots, i.e., L_1 and L_2 of the function $\sin(\kappa x)$ in the numerator and two roots, i.e., R_1 and R_2 in the denominator. The root R_1 corresponds to the wavelength range $0\rightarrow\pi$ and the root R_2 corresponds to the wavelength range $-\pi\rightarrow 0$ in the unit wavelength domain of the standing-wave field. We observe that for a phase $\varphi=\pi/2$ associated with the classical standing-wave field the value of the roots R_1 and R_2 are different from the roots L_1 and L_2. In this case we expect four peaks in the conditional position probability distribution in a unit wavelength domain of the standing-wave field corresponding to the roots R_1 and R_2.

However, when the phase φ changes from $\pi/2$ to 0 or π the roots L_1 and L_2 become equal to R_1 or R_2, respectively. As a consequence we expect a quenching in the frequency measurement and two of the localization peaks vanish from one side of the unit wavelength domain of the standing-wave field depending on which root of $\sin(\kappa x)$ i.e., R_1 and R_2 in the denominator of the Eq. (26) becomes equal to L_1 and L_2.

A. Case I ($\varphi=0$)

For the case when phase φ associated with the classical standing-wave driving field is set to 0 and $\Omega_1=\Omega_2=\Omega$ the roots L_1 and L_2 of the sine function in the numerator become equal to the root R_1 of the sinusoidal function, i.e., $L_1=L_2=R_1=\delta_{\mathbf{k}}/\Omega_3$. As a result we expect a quenching in the spontaneous emission for $\sin(\kappa x)=R_1$ hence, the localization peaks in the wavelength range $0\rightarrow\pi$ vanish from the conditional position probability distribution and we are left with the localization peaks only in the wavelength range $-\pi\rightarrow 0$ at the normalized position $\kappa x=\sin^{-1}[(2\Omega^2-\delta_{\mathbf{k}}^2)/(\delta_{\mathbf{k}}\Omega_3)]$.

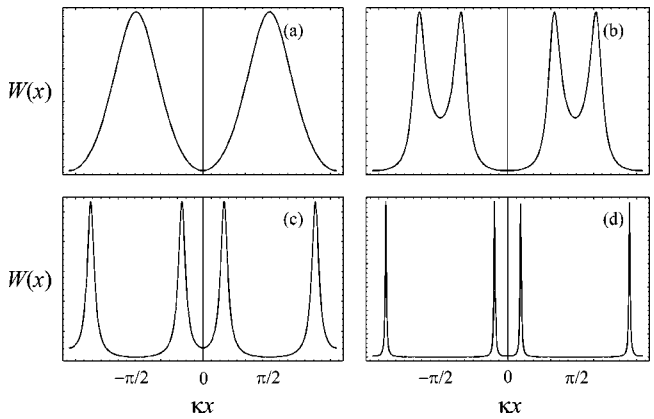

FIG. 3. The conditional position probability distribution $W(x)$ is plotted against the normalized position κx for $\Omega_1/\Gamma=\Omega_2/\Gamma=1$ and $\varphi=\pi/2$ when (a) $\delta_k/\Gamma=5$, and $\Omega_3/\Gamma=3$ (b) $\delta_k/\Gamma=3$, and $\Omega_3/\Gamma=3$ (c) $\delta_k/\Gamma=2$, and $\Omega_3/\Gamma=3$ (d) $\delta_k/\Gamma=5$, and $\Omega_3/\Gamma=16$.

B. Case II ($\varphi=\pi$)

In this case when the phase of the standing-wave field is set to π, i.e., $\varphi=\pi$ and $\Omega_1=\Omega_2=\Omega$ the root R_2 of the sinusoidal function in the denominator becomes equal to the roots L_1 and L_2, i.e., $L_1=L_2=R_2=-\delta_{\mathbf{k}}/\Omega_3$. In this case we expect a vice versa of the previous case ($\varphi=0$) and the quenching takes place in the wavelength range $-\pi\rightarrow 0$, i.e., for $\sin(\kappa x)=R_2$. Therefore, we still get two localization peaks in the conditional position probability distribution but in the range $0\rightarrow\pi$ of the unit wavelength domain of the standing-wave field at the normalized positions $\kappa x=\sin^{-1}[(\delta_{\mathbf{k}}^2-2\Omega^2)/(\delta_{\mathbf{k}}\Omega_3)]$.

IV. RESULTS AND DISCUSSION

In the earlier atom localization schemes, based on the frequency measurement of the spontaneously emitted photon [11–16], we observe that for a single frequency measurement the probability of finding the atom at a particular position is reduced to 1/4. This is due to the fact that the interaction of the atom with the standing-wave field introduces a periodicity in the system and we get four, equally probable, localization peaks in the unit wavelength domain of the classical standing-wave field for a single frequency measurement. However, the present scheme reduces the periodicity of the system due to the quenching of the spontaneous emission by controlling the phase of the standing-wave field. Therefore, we expect two localization peaks for a single frequency measurement instead of four peaks. This exhibits a possibility of increasing the probability of finding the atom at a particular position by a factor of 2 by controlling the phase of the driving field.

In Figs. 3(a)–3(c), we show the plots of the conditional position probability distribution $W(x)$ for different values of $\delta_{\mathbf{k}}$ ($\delta_{\mathbf{k}}/\Gamma=5$, $\delta_{\mathbf{k}}/\Gamma=3$, and $\delta_{\mathbf{k}}/\Gamma=2$) when $\Omega_1/\Gamma=\Omega_2/\Gamma=1$, $\Omega_3/\Gamma=3$, and $\varphi=\pi/2$. The dependence of the conditional position probability distribution on the detuning $\delta_{\mathbf{k}}$ is clear from these figures. When the detuning $\delta_{\mathbf{k}}/\Gamma=5$ the conditional position probability distribution follows the

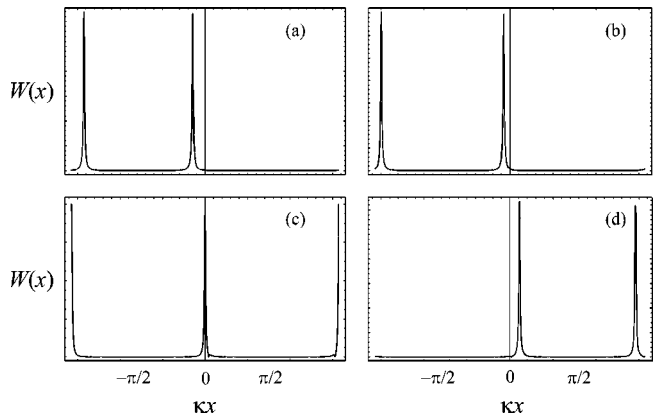

FIG. 4. $W(x)$ vs κx for $\Omega_1/\Gamma=\Omega_2/\Gamma=1$, $\Omega_3/\Gamma=16$, and $\varphi=0$ (a) $\delta_k/\Gamma=5$, (b) $\delta_k/\Gamma=3$, (c) $\delta_k/\Gamma=\sqrt{2}$, and (d) $\delta_k/\Gamma=0.5$.

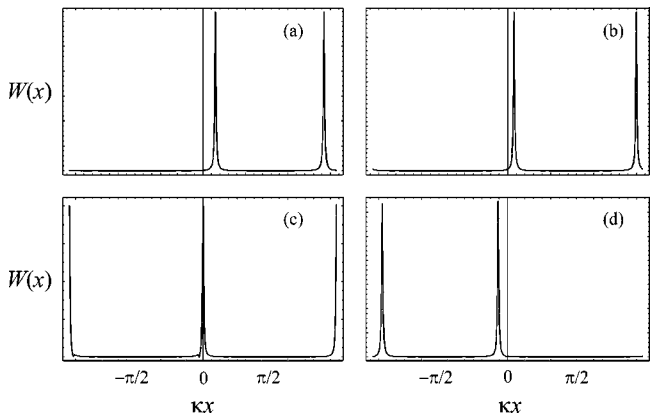

FIG. 5. $W(x)$ vs κx for $\Omega_1/\Gamma=\Omega_2/\Gamma=1$, $\Omega_3/\Gamma=16$, and $\varphi=\pi$ (a) $\delta_k/\Gamma=5$, (b) $\delta_k/\Gamma=3$, (c) $\delta_k/\Gamma=\sqrt{2}$, and (d) $\delta_k/\Gamma=0.5$.

standing-wave nature and we get peak maxima only at the antinodes of the standing wave. As $\delta_{\mathbf{k}}/\Gamma$ changes from 5 to 3 and then to 2, initial two peaks start splitting into four peaks, which then move away from the antinodes towards the nodes of the standing wave [see Figs. 3(a)–3(c)]. This clearly shows the strong correlation between the position of the atom and the frequency of the spontaneously emitted photon.

However, the width of the localization peaks in the conditional position probability distribution is large and there is overlapping of the adjacent peaks as well, see Figs. 3(b) and 3(c). Hence we do not achieve best spatial resolution in atom localization for these cases. This is due to the fact that the coherence in the system plays a crucial role in the atom localization scheme and to get a minimum width with best spatial resolution in the position distribution we must have $\Omega_3/\Gamma \gg 1$. In Fig. 3(d), we observe that even for $\delta_{\mathbf{k}}/\Gamma=5$, for which we get worst localization in Fig. 3(a) when $\Omega_3/\Gamma=3$, we still get best localization peaks having smooth background and minimum widths when the amplitude Ω_3 of the standing-wave field is increased from 3Γ to 16Γ.

Next, we look at how the quenching in the spontaneous emission contributes in enhancing the precision position measurement of the single atom by controlling the phase of the standing-wave field? In Fig. 4, we show the plots of the conditional position probability distribution $W(x)$ for different values of $\delta_{\mathbf{k}}/\Gamma$ when the phase φ of the standing-wave field is set to 0 and $\Omega_1/\Gamma=\Omega_2/\Gamma=1$ while $\Omega_3/\Gamma=16$. We get a quenching of the spontaneous emission in the normalized position range $0\rightarrow\pi$ and we see that the atom localization peaks from the right side of the conditional position probability distribution completely vanish. Therefore, we have now two atom localization peaks instead of initial four peaks for a single frequency measurement of the spontaneously emitted photon. This shows a clear advantage over the earlier localization schemes [11–16] where we get four localization peaks in a unit wavelength domain for a single frequency measurement.

The position of the maxima in the conditional position probability distribution strongly depends on the value of the detuning $\delta_{\mathbf{k}}$. As the detuning $\delta_{\mathbf{k}}$ decreases the localization peaks shift towards the nodes of the standing wave and for $\delta_{\mathbf{k}}^2=2\Omega^2$ we get localization peaks at the nodes of the standing wave, see Figs. 4(a)–4(c). However, when $(2\Omega^2-\delta_{\mathbf{k}}^2)/(\delta_{\mathbf{k}}\Omega_3)>0$ the two localization peaks that are initially in the left side region of the conditional position probability distribution $(-\pi\rightarrow 0)$ completely shift to the right side region in the conditional position probability distribution $(0\rightarrow\pi)$, see Fig. 4(d). Figure 5 shows a completely inverse picture when the phase of the standing-wave field is set to π instead of 0.

It is already discussed that the line narrowing strongly depends on the amplitude of the Rabi frequency of the atom, i.e., Ω_3/Γ. Therefore, we get very narrow structure in the conditional position probability distribution for the higher values of Ω_3/Γ [see, Fig. 3(d), Fig. 4, and Fig. 5]. It is also noticed that the width of the localization peaks remains almost constant with the variation in the detuning parameter δ_k. In Fig. 6, we present the plot of the width w of the localization peaks Ω_3/Γ. We observe that the decrease in the width w of the localization peaks initially is very sharp but after a certain limit of the Ω_3/Γ, which is in our case equal to 8, it follows an asymptotic behavior and practically it remains constant for higher values of the ratio Ω_3/Γ.

V. CONCLUSION

In this paper we presented a scheme for the localization of the single atom. This scheme is based on the fact that the

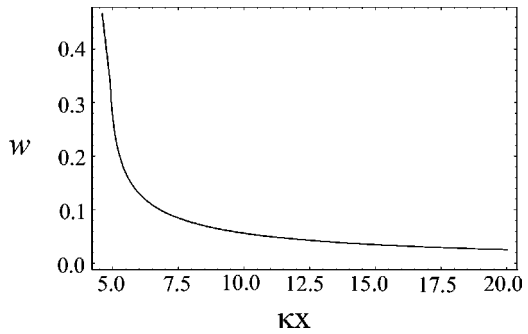

FIG. 6. Width $w=\kappa\Delta x$ vs Ω_3/Γ, when $\Omega_1/\Gamma=\Omega_2/\Gamma=1$, $\delta_k/\Gamma=5$, and $\varphi=\pi$.

spontaneously emitted photon carries the information about the position of the atom in a position-dependent interaction of the atom with the classical field. Therefore, measurement of the frequency of the spontaneously emitted photon localizes the atom in real time, i.e., when the atom is still in its passage through the driving field. This localization is done within the subwavelength domain of the optical field.

We demonstrated that the number of localization peaks in the conditional position probability distribution reduced to half as compared to the earlier related schemes. This is done by controlling the phase of the standing-wave field that quenched the spontaneous emission. This enhances the efficiency of the system and reduces the number of atoms required for a specific position measurement of the single atom by half.

Another important parameter to discuss, as far as atom localization is concerned, is the spatial resolution in the position distribution. It is already discussed that the amplitude of the driving field plays an important role in line narrowing and the filter function $\mathcal{F}(x)$, besides other parameters, strongly depends on the amplitudes of the driving fields. It is observed that the width of the atom localization peaks strongly depends on the amplitude of the standing-wave field. Therefore, it can be reduced considerably by controlling the amplitude of the standing-wave field. In the present scheme we observed very narrow localization peaks (Fig. 4 and Fig. 5) and a spatial resolution of the order of $\lambda/200$ is calculated for $\Omega_1/\Gamma=\Omega_2/\Gamma=1$ and $\Omega_3/\Gamma=16$.

Therefore, the proposed scheme suggests that the position information of the atom is greatly enhanced by controlling the phase and amplitude of the driving fields. It is also observed in the plots of conditional position probability distribution that we get a better signal to background ratio for a higher ratio of Ω_3/Γ. As a result we get best resolved peaks with strong line narrowing in the conditional position probability distribution for $\Omega_3/\Gamma=16$. The choice of this high ratio of the Rabi frequency to the atomic decay rate is achievable experimentally in the microwave [21] and optical regions [22]. It is also to be noticed that we must consider $\Omega_1=\Omega_2$, because a bichromatic field having different Rabi frequencies introduces an asymmetry in the sidebands of the spontaneous emission spectrum [23].

As a concluding remark we like to mention that abovementioned single atom position probability distribution is a conditioned probability distribution, which is conditioned on the measurement of the frequency and direction of the spontaneously emitted photon. An experiment using an atomic beam requires to measure the spontaneously emitted radiation of every atom that passes the field and due to isotropic nature of the spontaneous emission it is required to use 4π detectors. However, it is not necessary to find the position of each atom rather it is sufficient to select only those atoms whose spontaneous emitted radiation has definitely been detected by the detector.

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[1] P. Storey, M. Collet, and D. F. Walls, Phys. Rev. Lett. **68**, 472 (1992); M. A. M. Marte and P. Zoller, Appl. Phys. B: Photophys. Laser Chem. **54**, 477 (1992); P. Storey, M. Collet, and D. F. Walls, Phys. Rev. A **47**, 405 (1993); during atom-field interaction the phase shift can either be introduced in the atomic dipole, a measurement of this phase shift gives the position information of the atom, S. Kunze, G. Rempe, and M. Wilkins, Europhys. Lett. **27**, 115 (1994); M. Brune, S. Haroche, V. Lefevere, J. M. Raimond, and N. Zagury, Phys. Rev. Lett. **65**, 976 (1990).

[2] S. Kunze, K. Dieckmann, and G. Rempe, Phys. Rev. Lett. **78**, 2038 (1997).

[3] R. Quadt, M. Collett, and D. F. Walls, Phys. Rev. Lett. **74**, 351 (1995).

[4] A. M. Herkommer, H. J. Carmichael, and W. P. Schleich, Quantum Semiclassic. Opt. **8**, 189 (1996).

[5] H. J. Carmichael, *An Open Systems Approach to Quantum Optics* (Springer-Verlag, Berlin, 1993).

[6] F. L. Kien, G. Rempe, W. P. Schleich, and M. S. Zubairy, Phys. Rev. A **56**, 2972 (1997).

[7] J. E. Thomas, Phys. Rev. A **42**, 5652 (1990).

[8] K. D. Stokes, C. Schnurr, J. R. Gardner, M. Marable, G. R. Welch, and J. E. Thomas, Phys. Rev. Lett. **67**, 1997 (1991).

[9] J. R. Gardner, M. L. Marable, G. R. Welch, and J. E. Thomas, Phys. Rev. Lett. **70**, 3404 (1993).

[10] For a comprehensive report on existing schemes and some new suggested methods for precision position measurement of moving atoms in beams and traps, see J. E. Thomas and L. J. Wang, Phys. Rep. **262**, 311 (1995). This report also describes the applications of atom imaging to measure atomic interference patterns, and potential methods for correlated position measurement of multiple atoms; in a recent experiment parametric excitation and time resolved fluorescence detection is used to investigate the coherent control of center-of-mass wave packets for atoms confined in an optical lattice. In this experiment the temporal evolution for a Gaussian wave packet is reconstructed with a spatial resolution of $\lambda/6$, see P. Rudy, R. Ejnisman, and N. P. Bigelow, Phys. Rev. Lett. **78**, 4906 (1997).

[11] A. M. Herkommer, W. P. Schleich, and M. S. Zubairy, J. Mod. Opt. **44**, 2507 (1997).

[12] Sajid Qamar and M. S. Zubairy, in *Proceedings of the International Conference on Laser Physics and Quantum Optics*, edited by Z. Xu, S. Xie, S.-Y. Zhu, and M. O. Scully (Springer, Heidelberg, 2000), p. 307.

[13] Sajid Qamar, S.-Y. Zhu, and M. S. Zubairy, Opt. Commun. **176**, 409 (2000); presence of other relaxation channels in the system of a driven three level atom needs a time-dependent Autler-Townes microscopy (see, Refs. [15,16]).

[14] Sajid Qamar, S.-Y. Zhu, and M. S. Zubairy, Phys. Rev. A **61**, 063806 (2000).
[15] Sajid Qamar, Ph.D. thesis, Quaid-i-Azam University, 2000.
[16] Sajid Qamar and M. S. Zubairy (unpublished).
[17] E. Paspalakis and P. L. Knight, Phys. Rev. A **63**, 065802 (2001).
[18] Y. S. Bais, A. G. Yodth, and T. W. Mossberg, Phys. Lett. **211A**, 291 (1985).
[19] E. Paspalakis and P. L. Knight, Phys. Rev. Lett. **81**, 293 (1998).
[20] F. Ghafoor, S.-Y. Zhu, and M. S. Zubairy, Phys. Rev. A **62**, 013811 (2000).
[21] See, for example, G. Raithel, C. Wagner, H. Walther, L. M. Narducci, and M. O. Scully, in *Advances in Atomic, Molecular, and Optical Physics*, edited by P. Berman (Academic, New York, 1994).
[22] Q. A. Turchette, R. A. Thompson, and H. J. Kimble, Appl. Phys. B: Lasers Opt. **60**, S1 (1995); C. J. Hood, M. S. Chapman, T. W. Lynn, and H. J. Kimble, Phys. Rev. Lett. **80**, 4157 (1998).
[23] Z. Ficek and H. S. Freedhoff, Phys. Rev. A **48**, 3092 (1993).