A Methodology For Identifying Retrofit Energy Savings In Commercial Buildings

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ABSTRACT

Measured energy savings resulting from energy conservation retrofits in commercial buildings can be used to verify the success of the retrofits, determine the payment schedule for the retrofits, and guide the selection of future retrofits. This paper presents a structured methodology, developed for buildings in the Texas LoanSTAR program, for measuring retrofit savings in commercial buildings. This methodology identifies the pre-retrofit, construction and post-retrofit periods, normalizes energy savings for changing weather, accounts for missing energy consumption data, and quantifies the uncertainty associated with the measured savings. A case study from the Texas LoanSTAR program is presented as an example.

INTRODUCTION

Energy conservation retrofits of commercial buildings are typically initiated based on predictions of how much energy and money the retrofit will save. Predicted energy savings are generally calculated using the performance specifications of energy-using equipment and estimates of the physical characteristics and operating hours of the building. Frequently, several values necessary for these calculations, such as the operating hours of lights and electrical equipment, infiltration rates, solar loads, and outside-air flow rates for ventilating equipment are estimated using "engineering judgment". The calculation procedure or software may also make simplifying assumptions in order to reduce the complexity and time required for the calculations. Because of these factors, predicted savings often differ substantially from measured savings. In a study of over 1,700 building energy retrofits, fewer than one in six came within 20% of measured results (Greely et al. 1990).

Because of the potentially large discrepancy between predicted and measured savings, there is substantial interest in measuring energy savings. Measured energy savings resulting from energy conservation retrofits in commercial buildings can be used to verify the success of the retrofits, determine the payment schedule for the retrofits, and guide the selection of future retrofits. Measured savings, in contrast to predicted savings, can also benefit utilities that support energy conservation and demand side management programs.

The simplest method to measure energy savings is to directly compare pre-retrofit and post-retrofit energy use. However, varying weather conditions between the pre-retrofit

and post-retrofit periods can influence energy use and may obscure the change in energy use caused by a retrofit. To provide a more accurate measure of the energy saved by a retrofit, the effect of changing weather conditions on energy use should be removed. This is accomplished by developing a weather-dependent model of a building's pre-retrofit energy use. The building's pre-retrofit energy use can then be simulated under post-retrofit weather conditions and compared with post-retrofit energy use to determine savings.

The pre-retrofit model of energy use may be either an empirical (statistical) or a simulation model. Calibrated simulation models of pre-retrofit energy use (Katipamula and Claridge, 1991) can be used when pre-retrofit energy consumption data is limited, however, the uncertainty introduced by simulation models is difficult to assess. Statistical models of pre-retrofit energy consumption are almost always easier to develop than simulation models, and the uncertainty associated with the resulting savings can be calculated using accepted statistical procedures.

This paper describes a structured, statistical methodology to measure energy savings in commercial buildings. The methodology can be subdivided into six steps as shown in Figure 1. The next six sections of the paper describe these steps, followed by a case study example. This methodology is currently used by the Texas LoanSTAR (Loans to Save Taxes And Resources) Program (Claridge et al. 1991) as part of an effort to measure energy savings in state owned buildings.

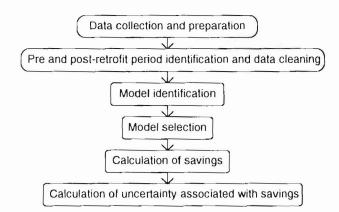


Figure 1. Flow chart of the methodology for measuring retrofit energy savings in commercial buildings.

DATA COLLECTION AND PREPARATION

A statistical pre-retrofit model requires energy consumption data from before a retrofit is installed. These data provide baseline information about how much energy the building consumes in it's pre-retrofit condition and are essential if savings are to be measured. The methodology described here uses daily energy consumption data which has been summed from hourly data. Daily energy consumption data may be available from a building's energy management and control system (EMCS) with energy trending capabilities (Claridge et al. 1992). In buildings without a trending EMCS. an independent data acquisition system may have to be installed in order to acquire daily energy consumption data. In several cases, the continuous metering of energy use in commercial buildings has saved many times the cost of the data acquisition system by identifying inefficient operating and control procedures (Haberl and Claridge, 1987, Haberl and Vajda, 1988, MacDonald et al. 1989, Kissock et al. 1991).

To provide the best measure of energy savings, the energy use of each type of equipment being retrofitted should be separately metered. For example, if constant-volume air handlers are being converted to variable-air-volume air handlers, the total electricity used by the air handlers should be metered. This can often be accomplished by metering the electricity used by a distribution panel (often called a motor-control-center) which distributes electricity to all of the air handlers in the building.

If the energy savings generated by lighting retrofits are to be exactly measured, then all of the electrical feeds to the lighting fixtures must be identified and metered. In practice, this is often difficult and expensive since lighting feeds are usually distributed throughout a building. A less expensive method to determine savings generated by a lighting retrofit is to meter the whole-building electricity use and air handler electricity use. The difference between these two channels is lighting and equipment (LE) electricity use. Comparing a short period of LE electricity use immediately before and immediately after a lighting retrofit will yield a good estimate of the electricity saved by the retrofit. Over a longer period of time, however, this estimate of savings from the lighting retrofit may become less accurate as other electrical equipment is added to or removed from the building.

If a retrofit is expected to reduce heating and cooling energy use, these channels should also be metered. In buildings where heating and cooling are generated on-site, metering the energy supplied to the heating and cooling equipment is sufficient. If the building subscribes to district heating and/or cooling, then whole-building heating and/or cooling can be measured by metering heating and cooling energy as it enters or leaves the building. For more information about sub-metering energy use in buildings see O'Neal et al. 1990 and Boecker et al. 1992.

Average daily outdoor air temperature is used as the primary indicator of environmental conditions that affect

weather.

Other methods, most notably the Princeton Scorekeeping Method (Fels, 1986), have used monthly billing data to normalize savings for changing

building energy use. Outdoor air temperature data may be available from a building's EMCS. If not, the National Weather Service provides minimum and maximum daily temperatures for most U.S. cities from which the average daily temperature can be calculated.

PRE AND POST-RETROFIT PERIOD IDENTIFICATION AND DATA CLEANING

Preferably, a full year of pre-retrofit data should be collected to account for seasonal effects on energy use. The pre-retrofit period should extend until retrofit construction influences the energy consumption of the building. If a detailed schedule of construction activities is available, then determining when a retrofit becomes operational is trivial. In some cases, however, exact retrofit construction dates are difficult to obtain. In these cases, the end of the pre-retrofit period and the beginning of the post-retrofit period can often be determined by inspecting energy consumption data and searching for discontinuities in the energy consumption patterns. For example, when constant-volume air handlers are converted to variable-air-volume air handlers, the time-series plot of air handler electricity use changes from a nearly constant signal to a signal with small discontinuities (during construction) and then to a variable signal when the variableair-volume system comes on line (Kissock et al. 1992).

In practice, the construction and commissioning of a retrofit may take a period of weeks or even months. During this period, energy use may be different than in the pre-retrofit period and different than the energy use after the retrofit is fully operable. We call this period the "construction period" and calculate the energy saved or the additional energy consumed during this period. In our experience, most buildings save energy during the construction period. In a few cases, energy use actually increases during the construction period. We try to identify the practices that cause the increased energy use and to inform the installers of the retrofit of these practices so that they can be minimized in the future. In any case, we regularly include energy savings (or "negative" savings) during the construction period in the total energy saved by the retrofit.

Depending on the number of channels being monitored, ensuring the quality of the data may be a formidable task. Meters must be correctly calibrated when installed and recalibrated at frequent intervals to avoid "drift" in the signal. Detecting bad data can also be difficult. There are several pieces of software which can automate parts of the quality control task, however, an extensive discussion of quality control procedures is beyond the scope of this paper. As a first step, producing time series and relational plots of the data and comparing the values of measured data with expected values can identify many instances of bad data. Energy consumption data which is incorrect or highly questionable should be removed from the data set in order to improve the reliability of the results. We perform the tasks of period identification and some quality control procedures using a data browsing software (Lantern, 1990) that quickly makes time-series and relational graphs (see Figure 4 for example).

MODEL IDENTIFICATION

Next, statistical models of the pre-retrofit energy use of each type of energy influenced by the retrofit should be developed. The functional form of each model is suggested by our physical understanding of how a particular type of energy use should vary. For example, constant-volume air handler electricity consumption is independent of weather conditions, but may vary with the operating schedule of the air handlers. Therefore, constant-volume air handler electricity use can be modeled as the mean electricity use during each operational period. If the air handlers are shut-down on weekends, then separate weekday and weekend models are appropriate.

Lights and equipment (LE) electricity use is also reasonably independent of the weather² and can be modeled as mean values of weekday and weekend LE electricity use. Mean models of energy use are called one-parameter models because only one parameter, the mean, is determined statistically.

Occasionally, it is not clear whether air handler or LE electricity use vary sufficiently during different operational periods to justify a separate energy use model for each period. In these cases, separate models of energy use for each operational period and for the entire period are developed. A statistical procedure called a t-test is then administered during the Model Selection procedure to determine whether the use of separate models for each operational period is statistically justified.

Changes in the quantities of heating and cooling energy use are primarily determined by changing weather, internal loads (heat generated by electrical equipment and people), and the operating schedule of the air handlers. Since the operating schedule of the air handlers is typically the same as the schedule of occupancy (and internal loads), the influence of both the air handler schedule and varying internal loads on heating and cooling can be accounted for by separating the data into bins which correspond to this schedule. For most commercial buildings, separating the data into weekday and weekend bins will account for the effects on thermal energy use of changing internal loads and air handler shut-off. In the Model Selection section, a statistical procedure called an F-test is administered to determine whether separating the data into weekday and weekend bins is statistically justified.

Once the effects of changing internal loads and air handler operating schedules are accounted for by separating the data into bins, the effect of changing weather on energy use must be considered. In our experience, which is mainly with institutional buildings in Texas supplied with district heating and cooling, daily heating and cooling energy use are adequately correlated with the average daily dry-bulb temperature of the outside air (Kissock et al. 1992). Numerous other studies (for example Fels, 1986, Fels et al. 1991, Schrock and Claridge, 1989) have also documented the

correlation between heating and cooling energy use and average outdoor temperature. The effects of other weather related parameters such as humidity and solar radiation on building energy use are currently being investigated (Ruch et al. 1991, Wu et al. 1992), but appear to be less important predictors of thermal energy use than outside air temperature.

Two functional forms of the relationship between thermal energy use and outside air temperature are regressed on each bin of cooling and heating energy use. The first and simplest is the linear relation:

$$E = \alpha_1 + \alpha_2 * T \tag{1}$$

where E is thermal energy use, T is outside air dry-bulb temperature, and α_1 and α_2 are regression coefficients. This relation is suggested by the steady state conduction and convection equations in which heat transfer varies linearly with temperature.³ This functional form is called a two-parameter model since two parameters, α_1 and α_2 , are determined by regression.

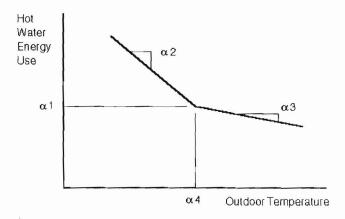
The second functional form is a four-parameter changepoint model (Ruch and Claridge, 1991) of the form:

$$E = \alpha_1 + \alpha_2 * (T - \alpha_4) \qquad T <= \alpha_4 E = \alpha_1 + \alpha_3 * (T - \alpha_4) \qquad T > \alpha_4$$
 (2)

where α_1 is the energy consumption at the change-point temperature, α_2 and α_3 are the low and high temperature slopes, and α_4 is the change-point temperature. This model describes a relationship between energy use and temperature in which energy use varies linearly with temperature in each of the low-temperature and high-temperature regions; however, the relationship (slope) is different in each temperature region (Figure 2). There are several physical processes which may initiate change-point behavior in commercial buildings, however, the description of these processes is beyond the scope of this paper and will be addressed in future work.

² LE electricity use in commercial buildings has been tested for seasonal variation by regressing LE against outdoor air temperature. In all of the cases that we have tested, the regression coefficient for the slope is insignificant, indicating that LE electricity use in commercial buildings is not significantly determined by seasonal effects.

 $^{^3}$ The steady state conduction equation for heat transfer across a solid medium is $Q=UA\Delta T$ where Q is the rate of heat transfer, U is the overall conductance of the medium, A is the cross-sectional area of the medium and ΔT is the temperature difference across the medium. The steady state convection equation for heat transfer from a solid to a fluid is $Q{=}hA\Delta T$, where h is the convection coefficient, ΔT is the temperature difference between the solid and the fluid and Q and A are the same as in the conduction equation. In both of these equations, Q varies linearly with temperature.



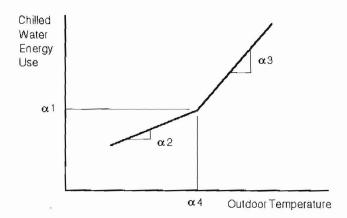


Figure 2. Examples of four-parameter change-point models for hot water and chilled water energy use.

MODEL SELECTION

At the end of the Model Development procedure, several models of energy use (weekday, weekend, all-day, and one, two, and four-parameter models)⁴ may be available for each type of energy use affected by the retrofit. In the Model Selection procedure, the best pre-retrofit model for each type of energy use is selected. To accomplish this, models which are unnecessary, such as separate weekday and weekend models which are nearly identical, or physically inconsistent, such as models with a negative cooling slope are eliminated. Then, the remaining model that bests fits the data is selected as the best model of that type of energy use.

Testing If Weekday And Weekend Models Are Identical

If the coefficients of weekday and weekend models are very different, it is probable that weekday energy use is different from weekend energy use. In this case, it is appropriate to consider separate weekday and weekend models. If the weekday and weekend model coefficients are nearly identical, then it is probable that no difference between weekday and weekend energy use actually exists and that the building's energy use can be accurately represented by a single model which includes both weekdays and weekends. The use of separate weekday and weekend models in this case is unnecessary and misleading. Statistical procedures such as a t-test (for one-parameter models) or an F-test (for multiple parameter models) can determine if separate weekday and weekend models or if a single all-day model of energy use is appropriate.

For one-parameter mean models, the t-test is the appropriate test. The t-test procedure is as follows. The sample standard deviation of weekday energy use (Box et al. 1978, pg. 76) is:

$$sd_{wd} = \left[\frac{\sum_{d=1}^{n_{wd}} (E_d - \overline{E}_{wd})^2}{(n_{wd} - 1)} \right]^{\frac{1}{2}}$$
 (3)

where E_d is daily weekday energy use, \overline{E}_{wd} is the mean daily weekday energy use, and n_{wd} is the number of weekdays in the sample. The sample standard deviation of weekend energy use is found by substituting values of weekend energy use and the number of weekend days into Equation 3. The combined sample standard deviation of the weekday and weekend energy use (Box et al. 1978, pg. 76) is:

$$sd_{c} = \left[\frac{(n_{wd} - 1)(sd_{wd})^{2} + (n_{wc} - 1)(sd_{we})^{2}}{(n_{wd} + n_{ws} - 2)} \right]^{\frac{1}{2}}.$$
 (4)

The t-statistic, t₀, is defined (Box et al. 1978, pg. 76)

$$t_{o} = \frac{(\overline{E}_{wd} - \overline{E}_{we}) - (M_{wd} - M_{we})}{sd_{c}\sqrt{\frac{1}{n_{wd}} + \frac{1}{n_{we}}}}$$
(5)

where M_{wd} and M_{wc} are the unknown values of the true weekday and weekend population means. If the implicit assumption of random sampling holds true, then t_0 is distributed as the well known t-distribution.

The t-distribution can be used to test any hypothesized difference in population means. We are interested in testing the hypothesis that weekday and weekend energy use are identical. If this hypothesis is true, then separate weekday and weekend models are unnecessary and energy use can be more succinctly described by the use of a single all-day model. The mathematical formulation of this hypothesis is known as the null hypothesis and is $M_{wd} = M_{wc}$. To test this hypothesis, we

⁴ A three parameter model, such as the models used by the Princeton Scorekeeping Method's Heating Only and Cooling Only procedures (Fels, 1986), is a special case of the more general four-parameter model where either the low-temperature slope or the high-temperature slope is constrained to zero.

assume that it is true and so M_{wd} - M_{we} = 0 in Equation 5. The degrees of freedom (df) associated with t_0 (Box et al. 1978, pg. 76) are:

$$df = n_{wd} + n_{wc} - 2 . (6)$$

The probability that the null hypothesis is true, i.e., that energy use is actually the same on weekdays and on weekends, is determined by referring t₀ to a t-table with *df* degrees of freedom. T-tables are available in any standard statistical text such as Box et al. 1978 or Neter et al. 1989.

If the probability that weekday and weekend energy use is actually the same is less than 0.05, we conclude that separate weekday and weekend models are appropriate. If the probability that weekday and weekend energy use is actually the same is greater than 0.05, we conclude that the weekday/weekend models may have come from the same population and we reject the weekday/weekend models. This decision criteria rejects separate weekday and weekend models unless there is strong evidence that weekday and weekend energy use are indeed different.

For two and four-parameter models, the F-test⁵ (Neter et al. 1989, pgs. 87-100, 368) is the appropriate statistical test to determine if separate weekday and weekend models are needed. The use of the F-test to determine if separate weekday and weekend models are needed for the case of simple linear models is described below. The first step is to combine weekday and weekend models into a single regression model using an indicator variable, I. The combined weekday and weekend regression model for daily energy use is:

$$\hat{E}_{d} = \beta_{1} + \beta_{2} T_{d} + \beta_{3} I + \beta_{4} T_{d} I \tag{7}$$

where \hat{E}_d is daily energy use predicted by the model, T_d is the average daily outdoor air dry-bulb temperature, 1 is the indicator variable, and β_1 , β_2 , β_3 , and β_4 are regression coefficients.

The indicator variable is defined to be 1 for weekdays and 0 for weekends. For weekdays, l=1 and daily energy use is given by:

$$\hat{E}_d = (\beta_1 + \beta_3) + (\beta_2 + \beta_3)T_d. \tag{8}$$

For weekends, I=0 and daily energy use is given by:

$$\hat{E}_{d} = (\beta_{1}) + (\beta_{2})T_{d}. \tag{9}$$

The F-test will test the null hypothesis, Ho: $\beta_3 = \beta_4 = 0$, against the alternative hypothesis, Ha: not both $\beta_3 = \beta_4 = 0$. The null hypothesis is accepted when both weekday and weekend energy use is adequately modeled by Equation 9 and separate weekday and weekend models are unnecessary.

To test this hypothesis, the sum of squared error for the "full" model, SSE(F), and the sum of squared error for the "reduced" model, SSE(R), must be calculated. The sum of squared error (Neter et al. 1989) is:

$$SSE = \sum_{d=1}^{n} (E_d - \hat{E}_d)^2$$
 (10)

where E_d is daily measured energy use, \hat{E}_d is the daily energy use predicted by the model, and n is the number of observations of daily energy use in the sample. To calculate SSE(F) use the full model, Equation 7, to calculate \hat{E}_d in Equation 10. The "reduced" model is the model that results when the conditions given by the null hypothesis are enforced and is given by Equation 9. To calculate SSE(R) use Equation 9 to calculate \hat{E}_d in Equation 10. The number of degrees of freedom associated with the full model, df_F , and with the reduced model, df_R , are:

$$df_F = n - 4 \tag{11}$$

$$df_{\nu} = n - 2 \tag{12}$$

where n is the number of observations of daily energy use data that are used in the regression models.

The general linear F-Test (Neter et al. 1989, pg. 99) is:

$$F_o = \frac{SSE(R) - SSE(F)}{df_R - df_F} \div \frac{SSE(F)}{df_F}$$
 (13)

The probability that the null hypothesis is true, i.e. that separate weekday and weekend models are essentially identical, can be found by referring F_0 to the F-distribution with $df_R - df_F$ numerator degrees of freedom and df_F denominator degrees of freedom. Tables of the F-distribution are found in any statistics text such as Box et al. 1978 or Neter et al. 1989. Our decision criterion is to use separate weekday and weekend models if the probability that the weekday and weekend models are identical (i.e., that Ho is true) is less than 0.05. Like the decision criterion used for the t-test of one parameter models, this decision criteria rejects separate weekday and weekend models unless there is strong evidence that weekday and weekend energy use are different.

Rejection Of Physically Inconsistent Models

We also reject models which are not consistent with our physical understanding of energy use, such as a model which

⁵ The F-test is also referered to as A three-parameter model, such as used by the Princeton Scorekeeping Method (Fels, 1986), is a special case of the more general four-parameter model where either the low-temperature slope or the high-temperature slope is constrained to zero, the generalized linear test.

predicts decreasing chilled water use with increasing temperature. Physically inconsistent models usually arise because of small pre-retrofit data sets which may misrepresent the actual energy use patterns of a building.

Selecting The "Best" Remaining Model

Finally, the remaining model with the best fit to the data is selected as the best model. The coefficient of variation of the standard deviation (CV-SD) is used for one-parameter models to determine goodness-of-fit. The coefficient of variation of the root mean square error (CV-RMSE) is used for two and four-parameter models to determine goodness-of-fit. The CV-SD and the CV-RMSE (SAS, 1990) are:

$$CV-SD = 100 * \frac{\left[\sum_{d=1}^{n} (E_d - \overline{E})^2 / (n-1)\right]^{\frac{1}{2}}}{\overline{E}}$$
 (14)

CV-RMSE =
$$100 * \frac{\left[\sum_{d=1}^{n} (E_d - \hat{E}_d)^2 / (n-p)\right]^{\frac{1}{2}}}{\overline{E}}$$
 (15)

where $\mathbf{E}_{\mathbf{d}}$ is energy use on any day d, \overline{E} is the mean daily energy use in the pre-retrofit period, \hat{E}_d is the daily energy use predicted by the pre-retrofit model, n is the number of days in the pre-retrofit period, and p is the number of regression parameters in the model. The CV-SD and CV-RMSE are non-dimensional measures of how well a model fits the data; low values represent good fits. The remaining model with the lowest coefficient of variation is selected as the best model of pre-retrofit energy use.

CALCULATING SAVINGS

Retrofit energy savings (S) are calculated by subtracting measured energy use in the post-retrofit period (M) from the energy use predicted by the pre-retrofit model (P). The energy saved on any day (d) in the post-retrofit period is:

$$S_d = P_d - M_d. \tag{16}$$

The total savings (S_1) during any period of m days in the post-retrofit period is the sum of the individual daily savings:

$$S_{t} = \sum_{d=1}^{m} S_{d}$$
 (17)

This method of determining energy savings compares energy use predicted by a pre-retrofit model to measured energy use from the post-retrofit period. It is also possible to determine savings by comparing the energy use predicted by a pre-retrofit model to the energy use predicted by a model of post-retrofit energy use. Each method has advantages and disadvantages. The advantage of comparing the energy use

predicted by a pre-retrofit model with measured energy use from the post-retrofit period is that this method avoids the additional uncertainty introduced by using a model to estimate post-retrofit energy consumption. Hence, this method provides a more precise measure of the energy saved during the post-retrofit period.

The disadvantage of this method is that savings depend on the weather during the post-retrofit period. An unusually cold post-retrofit winter, for example, may increase steam savings if the retrofit reduced steam use more during cold weather than during warm weather. The savings during "average" weather conditions can be determined by comparing the energy use predicted by a pre-retrofit model with the energy use predicted by a post-retrofit model if "average" weather data is used in the models. The Princeton Scorekeeping Method (Fels, 1986) uses this method to compute the Normalized Annual Consumption of buildings based on approximately ten years of measured daily temperature data.

Occasionally, energy consumption data from the postretrofit period are unavailable due to equipment failure or normal maintenance of the metering equipment. In these cases, a statistical model of post-retrofit energy use is developed using the same procedures described earlier for preretrofit models.⁶ Post-retrofit energy use can then be predicted and substituted for M_d in Equation 16 to estimate energy savings.

CALCULATING THE UNCERTAINTY OF SAVINGS

It is important to assign a measure of uncertainty to the calculated savings. This enables users of the calculated savings to know how confident they can be of the reported results. There are essentially three types of error (or uncertainty) which can influence the reported savings: extrapolation error, systematic error, and random error. Only random error can be quantified using statistical procedures, however, knowledge of the nature and causes of extrapolation and systematic errors can enable the practitioner to minimize or avoid these types of errors.

The pre-retrofit models of thermal energy use described above are regression models which describe the relationship between thermal energy use and outdoor air temperature over the range of outdoor temperatures present in the pre-retrofit data set. If the pre-retrofit period is less than a full year, outdoor temperatures from the missing season(s) may not be represented in the data set. In these cases, daily outdoor air temperatures during the post-retrofit period may be well outside of the range of the daily temperatures encountered during the pre-retrofit period, requiring extrapolation of the pre-retrofit model outside of the range of temperatures for which it was developed. This extrapolation of the pre-retrofit model requires an assumption that energy use will continue to increase (or decrease) linearly into the new temperature region, an assumption which may or may not be correct.

⁶ The post-retrofit model may or may not have the same functional form as the pre-retrofit model.

The simplest way to avoid model extrapolation error is to ensure that the pre-retrofit period extends for a full year, or at the minimum for a six-month period which includes both hot summer days and cold winter days. In situations where this is not possible, the modeler should inspect the range of temperatures over which the pre-retrofit models are developed and note the temperatures which will require that the model be extrapolated. The modeler can then report that an assumption of linearity was made during these periods.

Systematic error occurs when values are incorrectly reported in a consistent manner, such as when a sensor consistently reports values which are 10% greater than the true value. In practice this type of error can be minimized by selecting high quality metering equipment and installing it correctly, but it can never be completely eliminated. Experience gained during the Texas LoanSTAR program can suggest some procedures to minimize systematic error. One of the most important lessons learned is that for the purpose of calculating retrofit savings, the continuity of metered data is more important than the precision of the data. For example, the procedure of replacing a sensor may introduce an unwanted source of systematic error into the savings calculations if the sensors are not exactly calibrated with each other. Uncalibrated sensor change-outs frequently introduce a discontinuity into the data which makes an accurate comparison of pre and post-sensor-replacement impossible and thus renders a part of the data set useless. To minimize such problems, sensor replacement should be limited to those cases when it is absolutely necessary. Whenever a sensor must be replaced the new sensor should be pre-calibrated before it is installed, and the field sensor should be postcalibrated as soon as it is removed from the field. In this way, a measure of the difference in the values reported by the sensors can be determined and the new (or old) data can be adjusted to preserve the continuity of the data set. For more extensive discussions of the calibration procedures used by the Texas LoanSTAR program see (Robinson et al. 1992, O'Neal et al. 1992, Boecker et al. 1992).

Even with a resolute effort to minimize systematic error, some will always be present. The statistical determination of the uncertainty associated with savings that follows does not account for systematic error and so underestimates the true uncertainty of savings. Engineering judgment should therefore be applied in the interpretation of all statistical determinations of uncertainty, especially when the practitioner has reason to believe that significant systematic error is present in the data.

There are two components of random error, which is the random fluctuation of data about an "expected" or "true" value. The first component is random *measurement* error and is associated with the precision of sensors and metering equipment. The magnitude of the possible random measurement error is usually specified by the manufacturer of a sensor, often as a percentage of the sensor reading. The second component is random *model* error, which is the deviation of individual observations from the value predicted by a model. This magnitude of this type of error is a measure of how well a model "fits" the data. Both components of

random error can be quantified using the procedure that follows.

The energy savings and associated uncertainty on any day (d) in the post-retrofit period can be written as:

$$(S_d \pm \varepsilon_{sd}) = (P_d \pm \varepsilon_{pd}) - (M_d \pm \varepsilon_{md})$$
 (18)

where S_d is the energy <u>saved</u>, P_d is the energy use <u>predicted</u> by the pre-retrofit model, M_d is the energy use <u>measured</u> during the post-retrofit period, and ε is the random error associated with each parameter.

Figure 3 gives a graphical example of Equation 18 for the case of a two-parameter model of chilled water energy use. The daily chilled water energy use predicted by the preretrofit model for a given ambient temperature is shown as the point P_d . The uncertainty associated with predicting P_d is shown as a set of prediction uncertainty bands of width ϵ_{pd} . A model that fits the data well will have narrow prediction uncertainty bands. Measured energy use from the post-retrofit period, M_d , and the uncertainty of the measurement, ϵ_{md} , are also shown. The energy saved is represented by the vertical distance between P_d and M_d . The uncertainty associated with the energy saved is found by combining the prediction and measurement uncertainties,

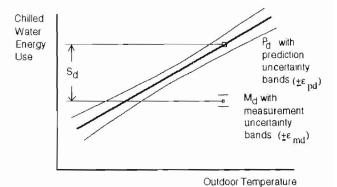


Figure 3. Example of predicted energy use (P_d) , measured energy use (M_d) , and energy saved (S_d) with associated uncertainty bands $(\pm \varepsilon_{pd} \text{ and } \pm \varepsilon_{md})$. The uncertainty of the energy saved is the root sum of squares of the prediction and measurement uncertainty.

The savings uncertainty, ε_{Sd} , is found using the standard method of combining errors (Holman, 1978, pg. 45) as:

$$\varepsilon_{\text{sd}} = \left[(\varepsilon_{\text{pd}})^2 + (\varepsilon_{\text{md}})^2 \right]^{\frac{1}{2}}. \tag{19}$$

The measurement uncertainty, ϵ_{md} , is determined from the specifications of precision that usually accompany measurement sensors and is given by:

$$\varepsilon_{md} = \frac{(\% \text{ measurement uncertainty})}{100} \times M_d$$
 . (20)

The prediction uncertainty, ϵ_{pd} , is the uncertainty associated with predicting energy use using the pre-retrofit model. The prediction uncertainty includes the uncertainty associated with measuring pre-retrofit energy use (Neter et al. 1989, pg. 170-171). The prediction uncertainty, ϵ_{pd} , for a one-parameter mean model of energy use (E) is (White, 1980):

$$\epsilon_{\text{pd}} = t(1-\alpha/2, 1) \left[\frac{\sum_{d=1}^{n} (E_d - \overline{E})^2}{n(n-1)} \right]^{\frac{1}{2}}$$
(21)

The prediction uncertainty, ϵ_{pd} , for a two-parameter regression model of energy use (E) with temperature (T) as the independent variable (Neter et al. 1989) is:

$$\varepsilon_{\text{pd}} = t(1-\alpha/2, \text{ n-2}) \left[MSE \right]^{\frac{1}{2}} \left[1 + \frac{1}{n} + \frac{\left(T_d - \overline{T} \right)^2}{\sum_{d=1}^n \left(T_d - \overline{T} \right)^2} \right]^{\frac{1}{2}}$$
(22)

where:

MSE = Mean Square Error =
$$\frac{\sum_{d=1}^{n} (E_d - \hat{E})^2}{(n-2)}$$
 (23)

In Equations 21 and 22, $t(1-\alpha/2, n-p)$ is the t-statistic and is tabulated in any standard statistics text. The t-statistic is a function of the significance level (α) , the number of days in the pre-retrofit period (n), and the number of parameters in the model (p). The significance level (α) indicates the fraction of predictions that are likely to fall outside of the prediction uncertainty bands (Figure 3).

The uncertainty associated with the total savings over a period of m days is the root sum of squares of the uncertainty associated with each daily value of savings (Holman, 1978, pg. 45).

$$\varepsilon_{\text{St}} = \left[\sum_{d=1}^{m} (\varepsilon_{\text{sd}})^2 \right]^{\frac{1}{2}}$$
 (24)

For a one-parameter mean model, the uncertainty associated with the total savings over a period of m days can be calculated from Equations 19, 20, 21 and 24 to be:

$$\varepsilon_{\text{SI}} = \left[mt^2 \frac{\sum_{d=1}^{n} \left(E_d - \overline{E} \right)^2}{n(n-1)} + \sum_{d=1}^{m} \left(\varepsilon_{md} \right)^2 \right]^{\frac{1}{2}}$$
 (25)

For a two-parameter model of energy use as a function of outdoor temperature, the uncertainty associated with the total savings over a period of m days can be calculated from Equations 19, 20, 22, 23 and 24 to be:

$$\varepsilon_{\text{SL}} = t(1.a/2, \text{n-}2) \left[MSE \right]^{\frac{1}{2}} \left[m + \frac{m}{n} + \frac{\sum_{d=1}^{m} \left(T_d - \overline{T} \right)^2}{\sum_{d=1}^{n} \left(T_d - \overline{T} \right)^2} + \frac{\sum_{d=1}^{n} \left(\varepsilon_{md} \right)^2}{t^2 MSE} \right]^{\frac{1}{2}} (26)$$

For a four-parameter model, the uncertainty associated with the total savings can be estimated by applying Equation 26 to each of the two linear regions separately. The total uncertainty is then found by combining the low-temperature region uncertainty (ϵ_{Slt}) and the high-temperature region uncertainty (ϵ_{Slt}) as prescribed by Holman, 1978:

$$\varepsilon_{\text{SI}} = \left[(\varepsilon_{\text{SII}})^2 + (\varepsilon_{\text{ShI}})^2 \right]^{\frac{1}{2}} . \tag{27}$$

Equation 27 can also be used to calculate the uncertainty associated with the total savings from the uncertainty associated with weekday and weekend savings by substituting the uncertainty associated with weekday savings for ϵ_{slt} and the uncertainty associated with weekend savings for ϵ_{slt} .

$$\varepsilon_{\text{st}} = \left[(\varepsilon_{\text{wd}})^2 + (\varepsilon_{\text{we}})^2 \right]^{\frac{1}{2}} . \tag{28}$$

The relative uncertainty of total savings is the uncertainty associated with the total savings divided by the total savings.

Relative Uncertainty of Total Savings =
$$\frac{\varepsilon_g}{S_t}$$
 (29)

These formulations of the uncertainty associated with savings for the two and four-parameter models assume that the model residuals are uncorrelated. Because of the time-series nature of the energy use data used in the regression procedures, this assumption may not always be met. If model residuals are highly correlated, then the Mean Square Error (Equation 23) may under-estimate the true variance of the data about the model. In these cases, the data should be transformed to remove the auto correlation before being regressed against temperature (Neter et al. 1989) or this effect

⁷ Statistical methods to account for uncertainty in an independent variable, such as temperature, exist but are far too complex for most applications. Hence, most text books overlook this type of uncertainty.

⁸ The true uncertainty associated with savings calculated from a four-parameter model should include the uncertainty associated with determining the change-point temperature.

should be explicitly accounted for in the statistical equations of uncertainty. A future work will address this issue.

CASE STUDY EXAMPLE

In this example, air handler electricity and steam savings will be calculated for a building participating in the Texas LoanSTAR program. The example building is a 103,000 square foot university building containing classrooms, lecture halls, offices, and an auditorium. The primary retrofit to the building was the conversion of constant-volume air handling units to variable-air-volume air handling units.

Data Preparation and Period Identification

Primary data preparation included summing hourly energy consumption data to daily data and collecting average daily temperature data from the National Weather Service. Next, the pre-retrofit and post-retrofit periods were identified. Since the retrofit construction schedule was unavailable, the preretrofit and post-retrofit periods were identified by inspecting a time-series plot of daily air handler electricity use for discontinuities that suggest when the conversion from constant-volume to variable-air volume air handlers occurred. In Figure 4, the pre-retrofit period was identified by the regular pattern of electricity use that extends to May 5, 1991. The pre-retrofit period is followed by a period of irregular and decreasing electricity use that extends until the end of May. The energy use during this period is characteristic of energy use during the construction and commissioning of the retrofit. We included this "construction period" in the post-retrofit period so that energy savings (or losses) incurred during construction and commissioning are included in the total of energy savings. The pre-retrofit period was then determined to be October 16, 1990 to May 5, 1991 and the post-retrofit period was determined to be May 6, 1991 to December 31, 1991.

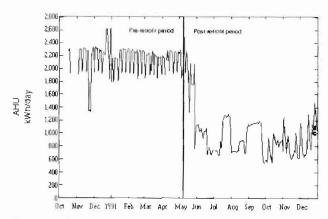


Figure 4. Time series plot of daily air-handler electricity use produced by data browsing software. The transition from preretrofit constant-volume air handler electricity use to postretrofit variable-air-volume electricity use is clearly evident.

Also apparent in Figure 4 is the unusual air handler electricity use that occurred during the Thanksgiving,

Christmas, and New Years Day holidays. Because the preretrofit period is less than a full year, these periods of unusual energy use would disproportionally influence the models of pre-retrofit energy use and so were excluded from the data used to develop the pre-retrofit models.

Model Development and Selection

The mean values of all-day, weekday, and weekend preretrofit air handler electricity use are collated in Table 1. The combined CV-SD of the weekday and weekend model is the weighted average of the weekday and weekend CV-SD and is calculated as:

$$(CV)_{combined} = \frac{n_{wd} (CV)_{wd} + n_{we} (CV)_{we}}{n_{wd} + n_{we}}$$
(30)

	n	AHU (kWh/day)	SD (kWh/day)	CV-SD (%)	Combined CV-SD (%)
All-day	177	2160	157	7.2	7.2
Weekday	127	2250	62.7	2.8	3.0
Weekend	50	1933	66.1	3.4	

Table 1. Decision table for air handler electricity use.

A t-test was administered to determine if separate weekday and weekend models were appropriate. From the values in Table 1 and Equations 4, 5, and 6:

$$sd_{c} = 51.3$$

 $t_{o} = 37.0$
 $df = 175$

From a t-table (Neter et al. 1989, pg. 630), the probability of the being greater than t_n was much less than 0.05, indicating that separate weekend and weekday models are appropriate. Because the combined CV-SD of the weekday and weekend models is less than the CV-SD of the all-day model, the weekday and weekend models were selected as the best models of pre-retrofit air handler electricity use.

Four possible models of steam use are collated in Table 2. The first two models are all-day models and the last two are weekday/weekend models of steam use. Models #1 and #3 are two-parameter (simple linear) models and Models #2 and #4 are four-parameter (change-point) models.

Model Type	n	α	α2	αγ	αμ	RMSE	CV- RMSE	Com- bined CV- RMSE
AD-1	179	67.6	820			2.64	14.8	14.8
AD-2	179	10.6	997	300	66.7	2.12	11.9	11.9
WD-3	130	69.8	849			2.51	13.9	14.3
WD-3	49	60.5	715			2.62	15.4	
WD-4	13()	11.0	-1.01	379	66.5	2.11	11.7	11.5
WD-4	49	6.67	877	.244	71.7	1.85	10.8	

Table 2. Model coefficients and inferential statistics for four possible pre-retrofit steam use models.

Model #4 is rejected because the high-temperature slope of the weekend model (α_3) is positive, which is inconsistent with our understanding of how steam use should vary with temperature. In this case, the particular sample of data available included a small number of data points that increased with temperature at the upper end of the temperature scale. In a larger sample, it is likely that α_3 would be negative.

An F-test was then administered to determine if the separate weekday and weekend models of Model #3 were appropriate. Table 3 shows the input parameters necessary for Equation 13 and the results of the F-test. The probability that Ho is true is found using a table of the F-distribution from Neter et al. 1989, pgs. 634-637. Since the probability of the null hypothesis (that the weekday and weekend models are identical) being true is less than 0.05, the null hypothesis is rejected. We conclude that separate models of weekday and weekend steam use are appropriate.

n	179	
SSE(R)	1348	
SSE(F)	1240	
F_0	7.62	
Pr(Ho is true)	Pr(Ho is true)<.001	

Table 3. F-test of steam use Model #3 to determine the appropriateness of using separate weekday and weekend models.

The weekday and weekend CV-RMSEs of Model #3 were combined using Equation 30 to produce a weighted average CV-RMSE of 14.3 %. The CV-RMSEs of Models #1, #2, and #3 were then compared to select the best pre-retrofit model. Model #2 (Figure 5) was selected since it has the lowest CV-RMSE.

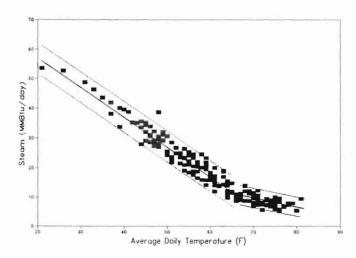


Figure 5. Four parameter pre-retrofit model of steam energy use and uncertainty bands shown with pre-retrofit steam energy use. $R^2 = .95$ and CV-RMSE = 11.9%.

Determining Savings

Air handler electricity use savings were determined using the mean weekday and weekend models in Table 1 and Equations 16 and 17. No data were missing from the postretrofit period and so a post-retrofit model is unnecessary. Table 4 shows a total savings of 274,384 kWh resulting from the VAV retrofit for the period from May 5, 1991 to December 31, 1991.

	Weekday	Weekend	Total
Predicted Use (kWh)	382,440	131,423	513,863
Measured Use (kWh)	171,298	68,190	239,488
Savings (kWh)	211,142	63,242	274,384

Table 4. Air handler electricity savings from May 5,1991 to December 31, 1991.

Model #2 from Table 2 is used to predict steam energy use (Figure 5). Since no post-retrofit data were missing, a post-retrofit model was unnecessary. The total savings are tabulated in Table 5 and are represented graphically in Figure 6.

Predicted Use (MMBtu)	2,412
Measured Use (MMBtu)	1,777
Savings (MMBtu)	635

Table 5. Steam savings for the period from May 5, 1991 to December 31, 1991.

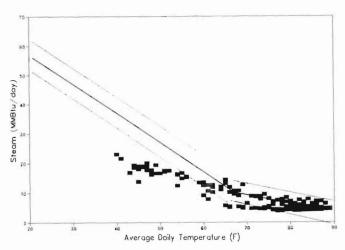


Figure 6. Pre-retrofit model of steam use and uncertainty bands shown with post-retrofit steam energy use. The vertical distance between the pre-retrofit model and a post-retrofit data point represents the amount of steam energy saved on a particular day.

Calculating the Uncertainty of Savings

Equation 25 was used to calculate the total error associated with weekday and weekend air handler electricity savings. Equation 28 was used to combine the weekday and weekend uncertainties to find the total uncertainty associated with the savings. Equation 29 was used to find the relative uncertainty of savings. The input parameters for Equation 25 and resulting uncertainty of the electricity savings are listed in Table 6.

Input Parameters and Results	Weekday	Weekend	Total
1)	127	50	177
m	170	68	238
α	0.05	0.05	
t(1-\alpha/2,n-1)	1.979	2.011	
$\sum_{d=1}^{n} (E_d - \overline{E})^2$	494.898	214,291	
$\sum_{d=1}^{m} \left(\varepsilon_{md} \right)^{2}$	19,794	7572	
Uncertainty of Savings	201	178	268
Total Savings	211,142	63,242	274,384
Relative Uncertainty	0.000952	.00281	0.000978

Table 6. Input parameters for Equations 25, 28, and 29 and the resulting uncertainty of the air handler electricity savings.

Input Parameters and Results	Low Temperature Region	High Temperature Region	Total
n	128	63	191
m	57	172	229
α	0.05	0.05	
t(1-α/2,n-2)	1.979	2.0	
MSE	6.36	2.78	
$\sum_{d=1}^{n} (T_d - \overline{T})^2$	9,977	10,894	
$\sum_{d=1}^{m} (T_{i,l} - \overline{T})^2$	3,482	78,172	
$\sum_{d=1}^{m} (\boldsymbol{\varepsilon}_{md})^2$	42.54	46.31	
Uncertainty of Savings	38.8	45.5	59.8
Total Savings	460	175	635
Relative Uncertainty	0.0845	0.260	0.0942

Table 7. Input parameters for Equations 23, 26, 27 and 28 and the resulting uncertainty of the steam savings.

Equations 23 and 26 were used to calculate the uncertainty of steam savings in each of the low-temperature and high-temperature regions of the four-parameter steam use model.

The input values necessary for Equation 26 and the resulting uncertainty of the savings are listed in Table 7. Equation 27 was used to combine the uncertainties from the low-temperature and high-temperature regions to find the total uncertainty associated with steam savings. Equation 29 was used to find the relative uncertainty of savings.

As was noted earlier, this statistical determination of uncertainty does not account for the possible presence of systematic error. Boecker et al. 1992 have identified a systematic bias in the readings of some flow meters used by the Texas LoanSTAR program. Because similar flow meters were used to determine the steam use in this example, a systematic bias may be present in the steam use data. Thus, this determination of the uncertainty associated with steam savings probably under-estimates the true uncertainty of the savings.

SUMMARY

This paper describes a statistical methodology to select pre-retrofit models of energy use, determine savings, and assign a measure of uncertainty to those savings. The use of one, two, and four-parameter models of energy use is described. The use of a t-test and a F-test to reject unnecessary weekday and weekend models and a decision criteria for selecting the best pre-retrofit model is described. The assignment of uncertainty to savings calculated using one, two, and four-parameter models is described. The procedure was demonstrated by developing and selecting pre-retrofit models of electricity and steam use, calculating savings, and assigning an uncertainty to those savings for a building participating in the Texas LoanSTAR program.

Future work will focus on refining the process of determining pre-retrofit and post-retrofit models. The effects of auto-correlated model residuals on model coefficients and uncertainty statistics will be specifically addressed. The physical interpretation of the functional forms of models is also under study. At present, several separate data processing and statistical tools are used to prepare the data and develop the pre-retrofit models. An integrated computer program which would automatically select the "best" pre-retrofit model is currently being developed. This integrated model development tool promises to reduce the time now required to develop energy use models.

The current procedures used to determine savings at LoanSTAR sites will also undergo revision. At present, a separate computer program is used to calculate savings at each site. In the future, a single program will be used to determine the savings at all LoanSTAR sites. The use of this "master" savings calculation program will reduce the time required for program maintenance and should improve the reliability of the results.

ACKNOWLEDGMENTS

The Texas LoanSTAR program is funded through the Texas Governor's Energy Management Center. Support for model development has been provided by the Energy

Research and Applications Program of the Texas Higher Education Coordinating Board through Project # 227.

Jeff Haberl provided valuable guidance throughout the work and made several helpful comments about the manuscript. Keith Boles, Robert Sparks, Rob Lopez, Dean Willis, Aamer Athar, Jinrong Wang and many others contributed their expertise and hard work to provided us with data and computer support. The comments of the anonymous reviewers of the paper were also appreciated and helped improve the final draft.

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