USE OF SPATIAL ARCHETYPES FOR OPTIMIZED ENERGY PERFORMANCE

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ABSTRACT

Energy conscious building design has always been a goal for consultants, architects and researchers. Due to the increase in energy consumption, optimal building environmental performance has gained a more significant role in the design process. This paper describes a methodology that facilitates thermal building performance optimization and whose successful use will allow more architects to get involved in the process even during the early design stages.

The aim of the research described in this paper is the development of a methodology that enables the categorization of built spaces with similar thermal behavior, namely spatial archetypes. These spatial archetypes are created based on realistic architectural design problems where various built forms can be represented by one unique archetype, which takes into account basic design standards. More specifically, these standards include furniture, building, ASHRAE (American Society of Heating Refrigerating and Air-Conditioning Engineers) and geometric standards.

Architects can use these archetypes in designing complex buildings, such as, institutional buildings, hospitals, hotels or dorms. However, this paper will not attempt to derive a unique or universal method for classification.

INTRODUCTION

Classification, Standardization, Archetypes

Currently, it is quite evident that building simulations require specialized knowledge in order for the typical architect to use them. The main reason for that is that the tools used to simulate objects as complex as a building are difficult to use. Generalizations about buildings are not easy to make. However, quite often these are easier on built forms (Martin and March, 1975).

It is often the case that within a building, two or more built forms share common properties or functionality. The same concept can be applied to thermal building behavior. It is believed that creating an archetype paradigm on built forms with respect to their thermal behavior is possible, and can provide “ready-made” solutions for repetitive geometries.

In the field of architecture there are many examples of built forms that share properties or functionality. These examples had a different application of standardization, mass-production and pre-fabricated housing. Throughout these examples the meaning of *module* changed or enhanced its identity. Initially, it was used as a measurement of components like doors and wall panels. Then, it became a spatial unit, for example a kitchen or a living room, and finally, a measurement unit so that spaces would follow common dimensions. It went from a small measurement unit to more complex when functionality and other properties of space were taken into account. The main goal behind all examples was mass-production that would reduce cost and time in the construction of a building.

The methodology proposed, will follow the main idea behind modularity and will focus on creating a paradigm of built form archetypes relevant to thermal performance in buildings. The optimum building behavior will then be sought by designing these built forms in an effort to facilitate energy conscious design.

Before proceeding, it would be useful to specify what is the difference, if any, between module, archetype and classification. According to the Oxford dictionary, *module,* is a small-scale plan or design of something (Oxford English Dictionary on Line). An *archetype,* is the original pattern or model from which copies are made; a prototype. According to the same source, *classification* is the action of classifying or arranging in classes, according to common characteristics or affinities. In the building design field, labeling an architectural form with a common term such as *bedroom* or *window* means that this specific form represents a member of a class. The important questions to be asked are related to the classes that must be recognized and their characterization and labeling in the design of thermally efficient buildings.

The classes of forms introduced in this paper will signify those building spaces that share properties that define their thermal behavior (e.g., number of windows or wall properties). In complex buildings that such classes appear repetitively, creating a prototype of those would help economize. Each of these prototypes will represent a *spatial archetype*.

Groups of such spatial archetypes will be created in an effort to classify spaces that are frequently required to be thermally optimized in buildings, for example, by minimizing energy loads and obtaining thermal comfort at the same time. In mechanical engineering each of these classes is usually referred to as a *product*. Since this methodology is borrowing techniques from mechanical engineering field, the build forms will often be referred to as products.

Products

One could consider built forms like those on Table 1.

Table 1. Products in buildings

P_A	Dorm Room
P_B	Hospital Room
P_C	Motel Room
P_D	Office Room

These built forms can be expressed with simple mathematical models, used to represent the required design performance. To proceed, it is necessary to make some assumptions both to simplify the problem and to handle it mathematically. Minimum energy consumption is realized by minimizing the surface area of a building. Thus, spherical buildings could be considered as a good idea. However, most buildings for many practical reasons have a rectangular shape and therefore it can be assumed that most built forms will be considered rectangular.

Built forms have common characteristics (Table 2). Each of these characteristics is expressed with design variables. To create the mathematical model finding the mathematical relationships between different variables that affect the thermal behavior of the space under study must be found. These relationships will be based on standards, as they exist today in the building industry. Activity will also be assessed as it affects the thermal behavior of the built forms.

Table 2. Common characteristics of building products

C_1	Windows
C_2	Doors
C_3	Ducts
C_4	Wall Materials
C_5	Wall Thickness
C_6	Room Height
C_7	Wall properties (interior, exterior)

When dealing with optimization, the relationships among these variables are described through constraints, while optimizing for a specific objective function. The optimal design problem formulation for a product has the following form:

$$\min_{x^p} : f^p(x^p) \qquad (1)$$

subject to:
$$g^p(x^p) \leq 0$$
$$h^p(x^p) = 0$$

where f^p is the objective function of the product *p* with respect to a vector of design variables x^p. This product is subjected to specific requirements where h^p represents the vector of equality constraints and g^p the vector of inequality constraints with respect to a vector of design variables x^p.

This effort of creating a classification paradigm of built forms should not be confused with an automatic design method for producing plans. Rather it is an effort to provide a new methodology for thermal building performance optimization using spatial archetypes. In optimization problems, more than one objectives could be examined (multi-objective problems). The approach followed here will try to minimize the compromise made while considering optimal solutions for two or more built forms that share some of their characteristics instead of optimizing them independently.

For example, a dorm room and an office room are both architectural spaces of the same class. Assumptions are made that they both have a window on one wall and a door on the opposite wall, that they both have rectangular shapes, and that both rooms could share material properties. In the case of the dorm room one design objective is to reduce heating and cooling loads, whereas in the office room it might be to increase thermal comfort. Other objectives also exist, but for this problem formulation, only these two will be considered. Reducing heating and cooling loads can result in thermal discomfort and so these competing objectives will be addressed using the method of multiobjective optimization.

METHODOLOGY

The method used is based on the product family design. This design is based on a multiobjective problem formulation and the aim is to evaluate the compromise between two or more competing objectives in architectural design spaces.

Before going any further, it would be useful to give an explanation to the commonly used terms in the product family discourse. A *component* is defined as the smallest element that is part of the assembly and is represented by a set of design variables. A *product* is an artifact that is compiled from components. A *model* is a mathematical representation of a product where a vector of variables is the input and a vector or responses is the output. A *product platform* is the set of all elements that are common in a set of products. A *product family* is the set of products that share a product platform.

Multiobjective Methods

Nelson et al. introduced a multicriteria optimization formulation for product family design and is based on the idea of maximizing product performance with respect to product design variables subjected to product requirements and a commonality constraint (Nelson et al. 2001). The commonality constraint defines the design variables that two or more products will be sharing. The two products based on the equation 1 have individual mathematical formulations as follows:

Product A

$$\min_{x^{p_1}} : f^{p_1}(x^{p_1})$$
$$\text{subject to: } \begin{array}{l} h^{p_1}(x^{p_1}) = 0 \\ g^{p_1}(x^{p_1}) \le 0 \end{array} \qquad (2)$$

Product B

$$\min_{x^{p_2}} : f^{p_2}(x^{p_2})$$
$$\text{subject to: } \begin{array}{l} h^{p_2}(x^{p_2}) = 0 \\ g^{p_2}(x^{p_2}) \le 0 \end{array}$$

where f^{p_1} and f^{p_2} is the objective function for each product respectively. Similarly, g^{p_1} and g^{p_2} are their inequality constraints and h^{p_1} and h^{p_2} their equality constraints with respect to a vector of their input variables x^{p_1} and x^{p_2}. The idea is that these individual optimization problems could share some components and therefore some variables, and could be seen as one multiobjective design optimization problem that includes an equality constraint defining the shared components.

Based on equation 2, Nelson et al., used the following model for the multiobjective formulation:

$$\min_{x=[x^{p_1},x^{p_2}]} : w_1 f^{p_1}(x^{p_1}) + w_2 f^{p_2}(x^{p_2}) \qquad (3)$$

$$\text{subject to: } \begin{array}{l} g^{p_1}(x^{p_1}) \le 0 \\ g^{p_2}(x^{p_1}) \le 0 \\ h^{p_1}(x^{p_1}) = 0 \\ h^{p_2}(x^{p_2}) = 0 \\ x_i^{p_1} = x_j^{p_2} \end{array} \qquad \begin{array}{l} \forall p_1, p_2 \in P \\ p_1 < p_2 \\ (i, j) \in S^{p_1 p_2} \end{array}$$

where w_1, w_2 are the usual weights in the scalar substitution function, and x_i is the vector of the design variables that correspond to the shared components for each product. The last equation is the commonality constraint, which represents the variables that the two projects are sharing. $S^{p_1 p_2}$ consists of the index pairs of elements that are shared between the two products p_1 and p_2.

Fellini et al., took this idea further and developed a methodology for making commonality decisions while controlling individual performance losses (Fellini et al., 2000). An optimal design problem was formulated to choose product components to be shared without exceeding a user-specified performance loss tolerance. This allowed the designer to have some control over the trade-offs. An optimal product family design was obtained for several performance losses while maximizing the parts that were shared, the commonality.

Furthermore, Fellini et al., proposed a methodology for selecting the product platform using information obtained from the individual optimization of the product variants (Fellini, 2003). A sharing penalty vector (SPV) was proposed that considered individual optimal designs and sensitivities of functional requirements. Based on that SPV design decisions were made for the commonality. The product family was then designed optimally with respect to the chosen platform.

Pareto Set Theory

Nelson et al., also suggested a more generic formula for family product platforms as a multi-objective formulation:

$$\min_{x^p=[x^{p1},x^{p2}\ldots]} : f^p(x^p) \qquad (4)$$

$$\text{subject to: } \begin{array}{ll} g^p(x^p) \le 0 & \forall p,q \in P \\ h^p(x^p) = 0 & p < q \\ x_i^p = x_j^q & (i,j) \in S^{pq} \end{array}$$

where f^p is the objective function of the design variables $x = [x^{p_1}, x^{p_2}, \ldots]$ for the entire product family *P*. The family constraints are represented by the vectors of g^p and h^p. Finally the commonality constraints are expressed with the equality $x_i^p = x_j^q$. Different combinations of commonality can be tested, each time giving a different platform. In the case where $S^{pq} = 0$ meaning that no sharing exists between the products, defines the *null platform*. On the other hand when all variables are shared between the products, that defines the *total platform*.

The solution of model (4) for a specific S^{pq} set forms a Pareto set, defined such that for each point of the Pareto set it is not possible to improve the objective function of one product without making the other worse. Each point in the Pareto curve is a solution of the multi-objective design problem.

It is possible to bound the area of the Pareto solutions. For the model described by (3), where *P*={p_1,p_2} and p_1=A and p_2=B, the null platform is represented by (f_A°, f_B°), meaning the solution of each objective separately without sharing any components. Similarly, $(f_A^\bullet, f_B^\bullet)$ represents the utopia point, the optimal quantities for the platform with common parts. This can be obtained by setting weights w_1, w_2 to $\{0,1\}$ and $\{1,0\}$. This point is the best possible point and therefore the aim is to get closer to that point. These three possible designs bound the Pareto set; the utopia point $(f_A^\bullet, f_B^\bullet)$, $(f_A^\bullet, f_B)$ and the $(f_A, f_B^\bullet)$ points (Figure 1).

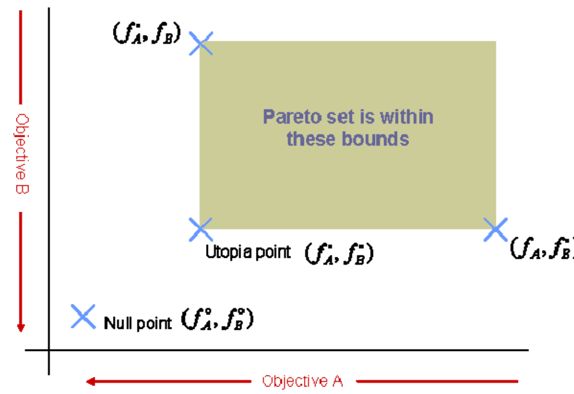


Figure 1. Graph of the Pareto region

Usually the null platform is a better design that the utopia point and from their Euclidean distance, the designer can make assessments for the feasibility of the model. The platforms that share more components will have worse designs. So in Figure 2 platform 2 is sharing more components between products A and B than platform 1 and that is why that design is further away from the utopia point.

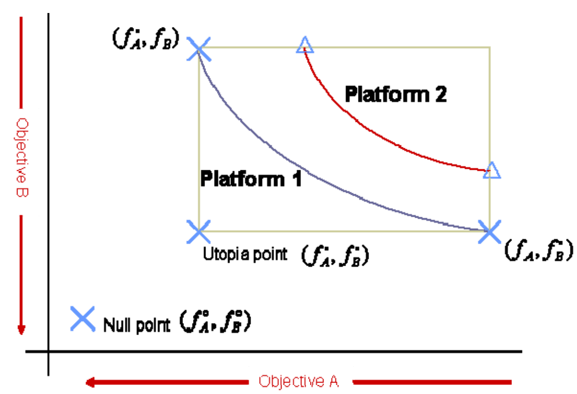


Figure 2. Two different Platforms shown on the Pareto region

Nelson et al. suggested six steps to use this methodology:

1. Identification of the commonality set,
2. Formulation of the multicriteria problem,

3. Determination of the points necessary to bound the Pareto Set (Figure 1),
4. Determination of the cost of commonality based on the distance between null and utopia points,
5. Calculation of the Pareto set for each combination of common parts, namely product platform
6. Select the best platform based on criteria posed by the designer

Computing the Pareto Set: The Constraint Method

A common way to solve Pareto problems, which was used in this paper, is called the "constraint method." One objective is minimized while constraining the remaining objective to be less than given upper bound values. The problem formulation can be defined as,

$$\begin{aligned} &\min_{x^p=[x^{p_1},x^{p_2}]} f_{p_1}(x^p) \\ & && \forall p_1, p_2 \in P \\ &g^p(x^p) \le 0 && p_1 < p_2 \\ &h^p(x^p) \le 0 \\ &f^{p_2}(x^p) \le z && i, j \in S^{p_1 p_2} \\ &x_i^{p_1} = x_j^{p_2} \end{aligned} \quad (5)$$

where z is the upper bound set for the objective values. The solution to the problem formulated is one Pareto optimal point. More Pareto optimal points can be generated by adjusting the value for the upper bounds z of the constraint objectives. The bound values are obtained by solving several single objective function problems. This problem formulation will be used in the demonstration study that follows.

DEMONSTRATION STUDY

To use this methodology a demonstration study was conducted. Two built forms were examined; an office room and a dorm room, both with similar geometry, materials and orientation.

Dorm Room

The dorm room with a rectangular shape was considered product A.

Design Variables

The variables for this problem are shown on Table 3. They were dimension variables of the room and windows areas Figure 3.

Table 3. Design variables for Product A

PRODUCT A	
x_{Ar}	X dimension of the room floor
y_{Ar}	Y dimension of the room floor
z_{Ar}	Z dimension of the room (height)
x_{Af}	Width of fenestration area
y_{Af}	Height of fenestration area

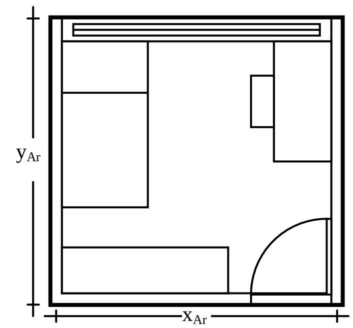


Figure 3 Product A: Dorm room

Objective Function

In this study, heating and cooling load calculations were used as the design objective

$$f_A = L_h + L_c, \quad (6)$$

where L_h and L_c are the heating and cooling load respectively. These functions were computed using the simulation software Energy Plus (Crawley et al, 2000) and were dependent on all design variables. As the variable values increased so did the total loads of the room.

Constraints

The constraints were divided into building standards, requirements for furniture functionality, ASHRAE standards and geometrical requirements.

Building Standards

1. Total Area: The total area of the floor of the room should be at least 70 square feet, as posed by the National Building Code Compliance Manual (Parish, 1998). In addition, each of the width and length of the room plan dimensions should not be less than 7 feet. This meant that:

$$\begin{aligned} &g_{A1}: 70 - x_{Ar} y_{Ar} \le 0 \\ &g_{A2}: 7 - x_{Ar} \le 0 \\ &g_{A3}: 7 - y_{Ar} \le 0 \end{aligned} \quad (7)$$

2. Ceiling Height: The ceiling height of the room was required to be less than 15 feet and more than 7 feet high.

$$g_{A4} : z_{Ar} - 15 \le 0 \qquad (8)$$
$$g_{A5} : 7 - z_{Ar} \le 0$$

3. Window area: The minimum window area for natural ventilation and natural lighting according to the national building code compliance was 4 square feet. Additionally operable exterior openings should occupy at least 4% of the total floor area for natural ventilation. Similarly, the openings should occupy at least 8% of the total floor area for obtaining natural lighting.

$$g_{A6} : 4 - x_{Af} y_{Af} \le 0$$
$$g_{A7} : 0.04 x_{Ar} y_{Ar} - x_{Af} y_{Af} \le 0 \qquad (9)$$
$$g_{A8} : 0.08 x_{Ar} y_{Ar} - x_{Af} y_{Af} \le 0$$

Geometric standards

Some geometric constraints were related to window size not exceeding the wall size. A 1-inch (0.083 feet) margin around the window ensured the proper construction of such window area

$$g_{A9} : x_{Af} - x_{Ar} + 0.083 \le 0$$
$$g_{A10} : y_{Af} - z_{Ar} + 0.083 \le 0 \qquad (10)$$

ASHRAE Standards

1. Thermal Transmittance Value (U_o value): Any residential building that is heated and mechanically cooled should have a combined thermal transmittance value for the gross area of exterior walls not exceeding the value of 0.26 in Detroit, where the heating degree-days are 6569. (ASHRAE, 2001)

$$g_{A11} : (U_{Aw} A_{Aw} + U_{Af} A_{Af}) - 0.26 x_{Ar} z_{Ar} \le 0 \ , \qquad (11)$$

where U_{Aw} is the thermal transmittance of all elements of the opaque wall area A_{Aw} and U_{Af} is the thermal transmittance of the window area (fenestration) A_{Af}. These areas are expressed in terms of the design variables through the geometric relations:

$$h_{A1} : A_{Aw} = x_{Ar} z_{Ar} - A_{Af}$$

$$h_{A2} : A_{Af} = x_{Af} y_{Af} \qquad (12)$$

The U values taken from ASHRAE are shown below.

Table 4. U values for the wall and window materials

U_{Aw}	Transmittance value of the wall (0.082 btu/ft^{2}F*h or 0.014 W/m^{2}K)
U_{Af}	Transmittance value of the windows (0.49 btu/ft^{2}F*h or 0.086 W/m^{2}K)

Therefore equation 11 can be rewritten as

$$g_{A11} : (0.082 A_{Aw} + 0.49 A_{Af}) - 0.26 x_{Ar} z_{Ar} \le 0 \qquad (13)$$

Furniture Requirements

1. Total floor area: For a floor area to accommodate the furniture for a dorm, three different values were given (De Chiara, 2001). Minimum required area was 90 square feet, best suggested was 110 square feet and generous 120 square feet. Only the upper and lower limits were used, letting the optimization problem determine the best values for the present problem:

$$g_{A12} : 90 - x_{Ar} y_{Ar} \le 0$$
$$g_{A13} : x_{Ar} y_{Ar} - 120 \le 0 \qquad (14)$$

Problem Formulation (Design model)

The problem as described above had 5 variables, 13 inequality constraints and 2 equality constraints. After a model simplification by eliminating the equality constraints and eliminating inequality constraints that are redundant the model formulation was simplified:

min: $f = L_h + L_c$

subject to:

$$g_{A2} : 7 - x_{Ar} \le 0$$
$$g_{A3} : 7 - y_{Ar} \le 0$$
$$g_{A4} : z_{Ar} - 15 \le 0$$
$$g_{A5} : 7 - z_{Ar} \le 0$$
$$g_{A6} : 4 - x_{Af} y_{Af} \le 0$$
$$g_{A8} : 0.08 x_{Ar} y_{Ar} - x_{Af} y_{Af} \le 0 \qquad (15)$$
$$g_{A9} : x_{Af} - x_{Ar} + 0.083 \le 0$$
$$g_{A10} : y_{Af} - z_{Ar} + 0.083 \le 0$$

$$g_{A11}: [0.082(x_{Aw}z_{Ar} - x_{Af}y_{Af}) + 0.49x_{Af}y_{Af}] - 0.26x_{Ar}z_{Ar} \leq 0$$
$$g_{A12}: 90 - x_{Ar}y_{Ar} \leq 0$$
$$g_{A13}: x_{Ar}y_{Ar} - 120 \leq 0$$

Because Energy Plus uses SI units, the model 15 was transformed on those units to be used in the simulations.

Office room

Product B is an office room with a rectangular shape.

Design Variables

The variables for this problem are shown on Table 5. They were dimension variables of the floor and windows areas, same as Product A (Figure 4).

Table 5. Design Variables for Product B

PRODUCT B	
x_{Br}	X dimension of the room floor
y_{Br}	Y dimension of the room floor
z_{Br}	Z dimension of the room (height)
x_{Bf}	Width of fenestration area
y_{Bf}	Height of fenestration area

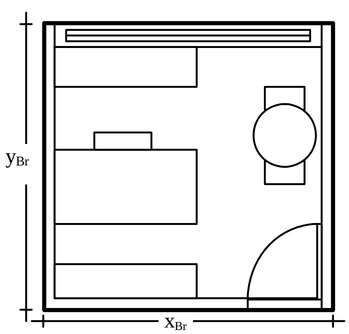


Figure 4. Product B: Office space

Objective Function

For this product, a different design objective was considered related to thermal comfort. The Predicted Mean Vote (PMV) was used. PMV is an indication for thermal comfort, and it varies from minus values to positive, with zero being the best. Positive values mean a hot uncomfortable feeling, and minus values a cold uncomfortable feeling.

$$f_B = \sqrt{PMV^2} \quad (16)$$

The PMV function was computed by the Energy Plus simulation program. While the room dimensions were decreasing, the absolute value of *PMV* was increasing. The effect of variable varying to the *PMV* was quite the opposite from the total loads. Therefore the objective functions were competing.

Constraints

The constraints were also divided into building standards, requirements for furniture functionality, ASHRAE standards and geometrical requirements.

Building Standards

1. Total Area: The total area of the floor of the room should be at least 70 square feet, as required by the National Building Code Compliance Manual (Parish, 1998). In addition, each of the width and length of the room plan dimensions should not be less than 8 feet. This meant that:

$$g_{B1}: 70 - x_{Br}y_{Br} \leq 0$$
$$g_{B2}: 8 - x_{Br} \leq 0 \quad (17)$$
$$g_{B3}: 8 - y_{Br} \leq 0$$

2. Ceiling Height: The ceiling height of the room was required to be less than 15 feet and more than 8 feet high.

$$g_{B4}: z_{Br} - 15 \leq 0$$
$$g_{B5}: 8 - z_{Br} \leq 0 \quad (18)$$

3. Window area: The minimum window area for natural ventilation and natural lighting according to the national building code compliance was 4 square feet. Additionally, operable exterior openings should occupy at least 4% of the total floor area into the yard or court for natural ventilation. Similarly, the openings should occupy at least 8% of the total floor area for obtaining natural lighting.

$$g_{B6}: 4 - x_{Bf}y_{Bf} \leq 0$$
$$g_{B7}: 0.04x_{Br}y_{Br} - x_{Bf}y_{Bf} \leq 0 \quad (19)$$
$$g_{B8}: 0.08x_{Br}y_{Br} - x_{Bf}y_{Bf} \leq 0$$

Geometric Standards

Some geometric constraints were related to window size not exceeding the wall size. A 1-inch margin (0.083 feet) around the window ensured the proper construction of such window

$$g_{B9}: x_{Bf} - x_{Br} + 0.083 \leq 0$$
$$g_{B10}: y_{Bf} - z_{Br} + 0.083 \leq 0 \qquad (20)$$

ASHRAE Standards

1. Thermal Transmittance Value (U_o value): Any building that is heated and mechanically cooled should have a combined thermal transmittance value for the gross area of exterior walls not exceeding the value of 0.315 in Detroit, where the heating degree days are 6569 and a building of more than 3 stories is considered. This value was taken from the ASHRAE. Thus,

$$g_{B11}: U_{Bw}A_{Bw} + U_{Bf}A_{Bf} - 0.315 x_{Br} z_{Br} \leq 0, \qquad (21)$$

where U_{Bw} is the thermal transmittance of all elements of the exterior opaque wall area A_{Bw} and U_{Bf} is the thermal transmittance of the window area (fenestration) A_{Bf}. These are expressed in terms of the design variables through the geometric relations:

$$h_{B1}: A_{Bw} = x_{Br} z_{Br} - A_{Bf}$$
$$h_{B2}: A_{Bf} = x_{Bf} y_{Bf} \qquad (22)$$

The *U* values taken from ASHRAE are shown below.

Table 6. *U* values for wall and window materials

U_{Bw}	Transmittance value of the wall (0.082 btu/ft2F*h or 0.014 W/m2K)
U_{Bf}	Transmittance value of the windows (0.49 btu/ft2F*h or 0.086 W/m2K)

Therefore, equation 22 can be rewritten as

$$g_{B11}: 0.082 A_{Bw} + 0.49 A_{Bf} - 0.315 x_{Br} z_{Br} \leq 0 \qquad (23)$$

Furniture Requirements

1. Total floor area: For a floor area to accommodate the furniture for an office space three different values were given. The minimum required area was 100 square feet, the best suggested was 150 square feet and the generous 200 square feet (De Chiara, 2001). For this problem formulation only the upper and lower limits were used allowing the optimization to determine the best values for the present problem.

$$g_{B12}: 100 - x_{Br} y_{Br} \leq 0$$
$$g_{B13}: x_{Br} y_{Br} - 200 \leq 0 \qquad (24)$$

Problem Formulation (Design Model)

The problem as described above had 5 variables, 13 inequality constraints and 2 equality constraints. After model simplification by eliminating the equality constraints and the inequality constraints that were redundant the problem was simplified as follows:

min: $f_B = \sqrt{PMV}^2$

subject to:

$$g_{B2}: 8 - x_{Br} \leq 0$$
$$g_{B3}: 8 - y_{Br} \leq 0$$
$$g_{B4}: z_{Br} - 15 \leq 0$$
$$g_{B5}: 8 - z_{Br} \leq 0$$
$$g_{B6}: 4 - x_{Bf} y_{Bf} \leq 0$$
$$g_{B8}: 0.08 x_{Br} y_{Br} - x_{Bf} y_{Bf} \leq 0 \qquad (25)$$
$$g_{B9}: x_{Bf} - x_{Br} + 1 \leq 0$$
$$g_{B10}: y_{Bf} - z_{Br} + 1 \leq 0$$
$$g_{B11}: [0.082(x_{Br} z_{Br} - x_{Bf} y_{Bf}) + 0.49 x_{Bf} y_{Bf}] - 0.315 x_{Br} z_{Br} \leq 0$$
$$g_{B12}: 100 - x_{Br} y_{Br} \leq 0$$
$$g_{B13}: x_{Br} y_{Br} - 200 \leq 0$$

Again, since Energy Plus uses SI units, the constraints must be converted to these units.

Simulation

The models 15 and 25 were used as the two products that the formulation of Equation (5) required. However, the use of computer simulation was required. For the computation of both objective functions, Energy Plus was used as mentioned. Activity inside both rooms, HVAC systems and artificial lighting was taken into account by setting schedules.

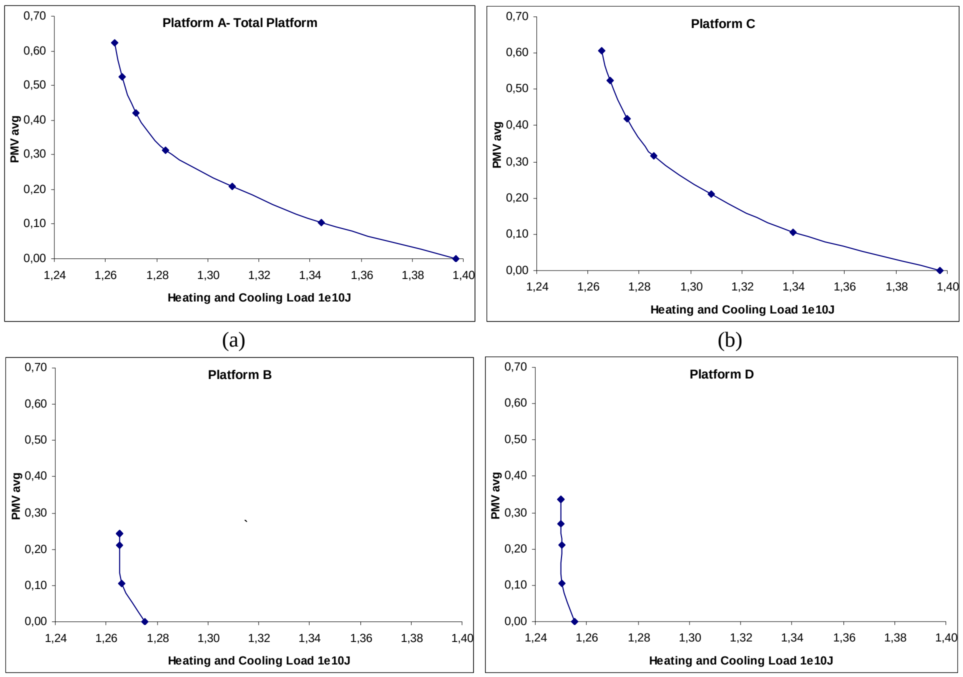
Matlab was used to implement the product family models as well as the optimization algorithm (Mathworks, 2002). For the optimization, the Sequential Quadratic Programming (SQP) algorithm was used (SQP is a gradient-based optimization algorithm).

Results

Four different cases of commonality were examined forming each time a different Pareto curve.

At first, all variables of the two products were shared, creating the Total Platform or Platform A (Figure 5-a). Then all variables were shared except the height and width of the fenestration area, Platform C, Figure 5-b). Heating and Cooling Loads were calculated in 10^{10} Joules and PMV was an average value with no unit as it provided an index indication of comfort. In both these curves it was obvious that there was a compromise between Heating and Cooling Loads and the PMV average.

The third commonality scenario that was tested, shared all the variables except the height of the room, Platform B (Figure 5-c). Finally, the last case that was tested was the products that shared all variables except the properties of the walls, meaning the length and width of the walls, Platform D (Figure 5-d). In these last cases it was noticed that the compromise is minimal (there was small curve). This means that these platforms while the PMV value was decreasing, there was a little increase in the total heating and cooling load of the room.



(a) (b)

(c) (d)

Figure 6. Total Platform (a), platform sharing all but window properties (b), platform sharing all but ceiling height (c), and platform sharing all but wall properties.

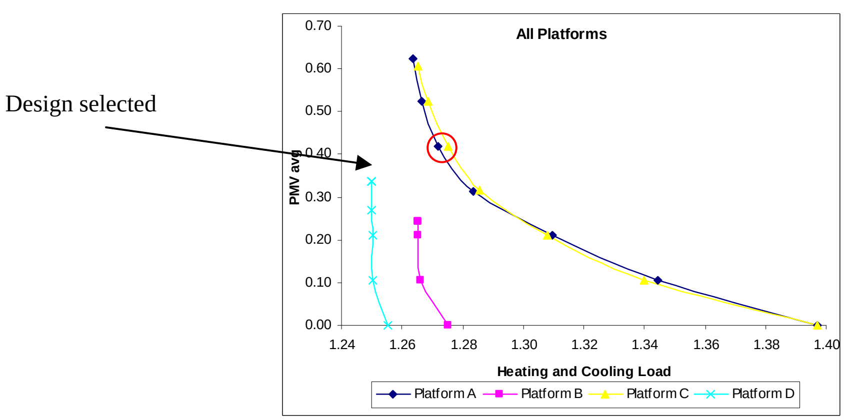


Figure 6. A plot of all platforms and the design selected

However, this is expected, since for heating and cooling it is necessary to minimize the room height, thus giving it the lower bound value; and for thermal comfort, it is necessary to maximize it, giving it the upper bound value. When the two products are not sharing the height variable, the compromise is minimal and the curve is small. Similarly, minimizing heating and cooling load requires small wall areas, whereas for optimum thermal comfort, the opposite is true. As such, the compromise in this case is also minimal and the curve small.

In Figure 6 all platforms are shown. All points in each of these lines are considered optimum for both minimum Heating and Cooling Loads, and for a thermal comfort close to zero. Selecting the best design was the next step according to Nelson et al. The first and most important criterion for such selections was to consider products that share the most variables, therefore select a point from the platform that had maximum commonality. However, at the same time, such design point should be close to the utopia point.

Other criteria for selecting the optimum point should be indicated by the designer. In this demonstration study, geometric criteria were posed. Maximum floor size, maximum fenestration area, and a constant height for both rooms were the geometric criteria that were considered. The design shown on a circle in Figure 6 was selected as the solution, the best design for both rooms. All the designs around that point were considered, and the one selected (Table 7) seemed to meet most of the criteria mentioned above.

Table 7. Design Point

x_r	y_r	z_r	x_{win}	y_{win}
3	**3.8**	**2.7**	**1**	**1**

CONCLUSIONS

Overall the demonstration study provided motivating results with regard to finding one solution for two different room types; a dorm room and an office room. The selected design point (Table 7) is the optimum for both products as it considered both objectives that were competing. This point therefore, comprises an archetype for such rooms where it takes into account their thermal behavior.

Following this methodology, many spatial archetypes can be created that take into account the thermal building performance. These archetypes can then be stored in a database of spaces that different conditions affecting thermal performance are exploited. As such, the practicing architect can make use of the archetypes without having to have much knowledge for the thermal phenomena. This selection will be based on material properties, dimensions and orientation.

This methodology can perform really well when the architect is in the early design stages. An analysis like the demonstration study can be done, and an archetype can be found. However, it would be more challenging to see how this can be applied in a real complex building and explore and existing situation.

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