Optimization of Multi-Stack Exhaust Systems -New System Design Application

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Abstract

Both analytical and numerical optimization methods have been developed to optimize multi-stack exhaust systems for application in new system designs. In these systems the combined airflow capacity of the stacks equals the maximum laboratory exhaust airflow. The theoretical analysis indicates that the optimized design uses as little as 50% of the design fan power annually. This paper presents the system models, the optimization methods, and describes appropriate applications.

Introduction

Laboratory exhaust airflow rates vary significantly when variable air volume fume hoods are used. Even in constant volume fume hoods the actual lab exhaust airflow rate often differs from the design rate. There are a number of reasons for this. For example, some of the fume hoods are often turned off when they are not used for relatively long periods of time. Although the laboratory exhaust airflow rate varies significantly, the constant volume exhaust fan system is designed to maintain stack exit velocity [ASHRAE 1999]. A recent study indicates that the constant volume exhaust system consumes the same amount or a greater amount of fan energy when the laboratory exhaust airflow is less than the exhaust system design airflow [Wang & Liu, 2001].

A number of researchers [Moyer and Dungan 1987, Rabiah and Wellenbach 1993, and Varley 1993] studied the potential fan power savings available through downsizing the exhaust system. The research showed that the maximum operation time is 5.5 hours per day for all types of fume hoods, and that during working hours, only 20% of the fume hoods are used simultaneously. If the fume hoods are not all used simultaneously, the system could be downsized by an overall system usage factor. However, design engineers are reluctant to adopt a usage factor because of the worry that full-load operation might occur at some time in the life of the facility. Ideally, the exhaust systems should be able to have an energy efficient design for typical use, but also the capacity for full load use.

To reduce exhaust fan energy consumption, Wang and Liu [2001] developed the Energy Efficient Single Stack Exhaust Fan System (E^3S^3F). An version of the E^3S^3F modulates a make-up air damper to maintain the required static pressure at the inlet of the fan while the modulation damper maintains the required static pressure at the fume hood. The potential fan power savings can be up to 15% compared to the conventional design. Another version of E^3S^3F adds a variable speed drive (VSD) to the fan, and a static pressure sensor to the inlet of the stack. In this system the controller modulates the make-up air damper to maintain the fume hood's static pressure, and modulates the fan speed to maintain the static pressure at the stack inlet. The fan power savings with this configuration can be up to 60%.

Because the stack size is constant, all of the E^3S^3F versions still require significant make-up air in order to maintain constant stack exit velocity. To reduce the make-up air flow rate, Wang et al [2002] developed a multi-stack exhaust system. For retrofitting existing systems, one or two smaller stacks and an airflow station are added to the E^3S^3F. Any given stack is activated only based on the building exhaust airflow rate. The annual fan energy savings is up to 40% for a typical exhaust system. The optimal design procedure has been developed for the existing system retrofits.

For new system designs the aggregated stack capacities can be set equal to the design capacity. This design reduces the size and the initial cost of the system. This paper presents the optimization principals and models for a new system design, the optimization procedure, and the applications.

System Model and Optimization Principals

The multi-stack system consists of a bundle of stacks, an exhaust fan, a make-up air damper, and two static pressure sensors. The sensors are located at the fume hood and at the stack inlet (see Fig.1).

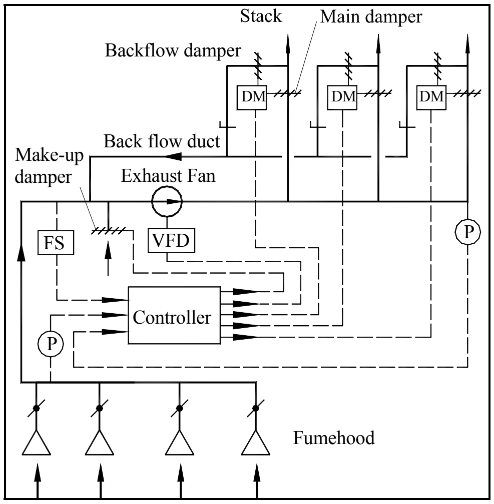

Figure 1: *Schematic diagram of the multi-stack exhaust system*

The bundle of stacks may consist of two or three stacks. More than three is not necessary. Each stack can be turned on/off by a control damper.

The capacity of a particular combination of stacks depends on the stacks' design capacities. The stack combination capacity series $\{z_j\}(j=1,2,\cdots,n)$ is selected from the single stack design capacity series $\{y_i\}(i=1,2,\cdots,m)$. These series are arranged in order of increasing capacity:

$$y_i \le y_{i+1}(i=1,2,\cdots,m-1) \quad (1a)$$

$$z_j \le z_{j+1}(j=1,2,\cdots,n-1) \quad (1b)$$

The total stack combination number n, which precludes the zero combination, z_0, is calculated from the stack number, m.

$$n=\sum_{i=1}^{m} C_i^m = 2^m - 1 \quad (2)$$

In order to minimize the system size and initial cost, the sum of the stack capacities is set equal to the exhaust system design airflow.

$$z_n = \sum_{i=1}^{m} y_i = 100\% \quad (3)$$

For a two-stack system, the total number of stack combinations is 3 and the stack combination design airflow series is expressed as:

$$\{z_j\}(j=1,2,3)=\{y_1, y_2, y_1+y_2=1\} \quad (4)$$

For a three-stack system, the total number of stack combinations is 7 and the calculation is classified into one of two situations.

For $y_1 + y_2 \le 0.5$

$$\{z_j\}(j=1,2,\cdots,7)=\{y_1, y_2, y_1+y_2, y_3, y_1+y_3, y_2+y_3, y_1+y_2+y_3=1\} \quad (5a)$$

And for $y_1 + y_2 \ge 0.5$

$$\{z_j\}(j=1,2,\cdots,7)=\{y_1, y_2, y_3, y_1+y_2, y_1+y_3, y_2+y_3, y_1+y_2+y_3=1\} \quad (5c)$$

The status (on/off) of each stack depends on the laboratory exhaust airflow and the stack combination's design capacity. If the laboratory exhaust airflow is between the capacities of stack combinations *i* and *j*, all stacks in combination j can be turned on and the rest of the stacks turned off (assuming stack combination j has a greater capacity than stack combination *i*).

The exhaust airflow can be determined using the following equation (assuming $z_0 = 0$):

$$\overline{Q} = z_j(j=1,2,\cdots,n) \text{ when } z_{j-1} < \overline{Q}_h \le z_j \quad (6)$$

where

$\overline{Q} = Q/Q_d$

$\overline{Q}_h = Q_h/Q_d$

The airflows may be different in different parts of the system when the make-up air damper is open or partially open. The duct from the fume hoods to the make-up air duct will only have the flow from the fume hoods Q_h. Then the make-up air is added, and the total, Q, flows through the fan up to the stack exit.

To maintain the safety of workers on the roof there is a backflow system, which includes a main damper in the stack and a backflow duct with a damper. When the stack is shut off, the main damper is closed and the backflow damper is opened. The toxic air, which can leak through the main damper, is returned back to the fan inlet through the backflow duct. Because the airflow through the backflow duct is small, it is ignored in determining the fan flow.

The backflow system may not be required if iris dampers are used. The iris damper has minimal or no intrusion to the airflow when it is full open. When it is closed, it has negligible air leakage.

The fan provides enough head to overcome the duct resistance and to provide the dynamic head of the exhaust air. The fan head is expressed by:

$$H = \Delta P_s + \Delta P_{\mathrm{d}} + \Delta P_2 + \Delta P_3 \quad (7)$$

The controller modulates the fan speed to maintain the static pressure set point at the inlet of the stack, and modulates the make-up air damper to maintain the static pressure set point at the fume hood. It is assumed that these static pressure set points are constant. A reset schedule could be integrated if necessary. The pressure loss in the duct is a function of the airflow through each duct section. The dynamic head difference is relatively small and is ignored in this study. Thus the relative fan head can be expressed with the design parameters and the actual airflows.

$$\overline{H} = x_1 + x_2 \cdot \overline{Q}_{\mathrm{h}}^{\,2} + x_3 \cdot \overline{Q}^{\,2} \quad (8)$$

where:

$$\overline{H} = H / H_{\mathrm{d}}$$

$$x_1 = \Delta P_{s,d} / H_{\mathrm{d}}$$

$$x_2 = \Delta P_{2,\mathrm{d}} / H_{\mathrm{d}}$$

$$x_3 = \Delta P_{3,\mathrm{d}} / H_{\mathrm{d}}$$

The fan power depends on both the fan airflow and head. The relative fan power (the ratio of fan power to design fan power) is generally expressed as:

$$\overline{W} = \overline{W}(\overline{H}, \overline{Q}) \quad (9)$$

where:

$$\overline{W} = W / W_{\mathrm{d}}$$

The laboratory exhaust airflow variation can be expressed using a laboratory exhaust airflow time distribution or time density. The time distribution T_k is defined as the number of operating hours in each range of relative airflow rate from $\overline{Q}_{h,k-1}$ to $\overline{Q}_{h,k}$, where $\overline{Q}_{h,0} = 0$ and $\overline{Q}_{h,l} = 1$. The time density $f(\overline{Q}_h)$ is defined as the number of operating hours coinciding with each category of relative exhaust airflow rate from $\overline{Q}_{h,0}$ to $\overline{Q}_{h,l}$. The time density and time distribution can be substituted for one another using the approximate relationship in Eqs.(10a) and (10b).

$$f(\overline{Q}_{h,ave,k}) \approx \frac{T_k}{\overline{Q}_{h,k} - \overline{Q}_{h,k-1}} \quad (10a)$$

$$T_k = \int_{\overline{Q}_{h,k-1}}^{\overline{Q}_{h,k}} f(\overline{Q}_h) d\overline{Q}_h \approx f(\overline{Q}_{h,ave,k})(\overline{Q}_{h,k-1} - \overline{Q}_{h,k}) \quad (10b)$$

where

$$\overline{Q}_{h,ave,k} = 0.5(\overline{Q}_{h,k} + \overline{Q}_{h,k-1})$$

The annual relative fan energy consumption can be calculated using Eqs.(11a) and (11b).

$$\overline{E} = \frac{1}{T}\int_0^1 \overline{W} \cdot f(\overline{Q}_h) \cdot d\overline{Q}_h = \int_0^1 \overline{W} \cdot \bar{f}(\overline{Q}_h) \cdot d\overline{Q}_h \quad (11a)$$

$$\overline{E} = \frac{1}{T}\sum_{k=1}^{l} \overline{W}_{ave,k} \cdot T_k = \sum_{k=1}^{l} \overline{W}_{ave,k} \cdot \overline{T}_k \quad (11b)$$

where

$$\overline{W}_{ave,k} = 0.5(\overline{W}_{\overline{Q}_{h,k}} + W_{\overline{Q}_{h,k-1}})$$

$$\bar{f}(\overline{Q}_h) = \frac{f(\overline{Q}_h)}{T}$$

$$\overline{T}_k = \frac{T_k}{T}$$

Optimization

The purpose of the system optimization is to identify the optimal stack capacities or diameters to minimize the annual fan energy consumption. Both analytical and numerical methods can be developed based on Eqs.(11a) and (11b).

Analytical method

If the fan efficiency is treated as a constant and the time density can be expressed as a polynomial equation, then the annual fan energy consumption can be deduced from Eq.(11a).

$$\overline{E} = \sum_{k=0}^{l}\sum_{j=1}^{n} a_k \int_{z_{j-1}}^{z_j} (x_1 z_j + x_2 \overline{Q}_{\mathrm{h}}^{\,2} z_j + x_3 z_j^3) \overline{Q}_h^k d\overline{Q}_h \quad (12)$$

$$\bar{f}(\overline{Q}_h) = \sum_{k=0}^{l} a_k \overline{Q}_h^k \quad (13)$$

The partial derivative of the expression for annual fan energy consumption with the stack capacities can be expressed as:

$$\frac{\partial \overline{E}}{\partial y_i} = \sum_{j=1}^{n}\sum_{k=0}^{l} a_k \{\frac{\partial z_j}{\partial y_i}[(\frac{k+4}{k+3}x_2 + \frac{k+4}{k+1}x_3) z_j^{k+3} + \frac{k+2}{k+1}x_1 z_j^{k+1} - (\frac{x_1 + 3x_3 z_j^2}{k+1} + \frac{x_2 z_{j-1}^2}{k+3}) z_{j-1}^{k+1})] - \frac{\partial z_{j-1}}{\partial y_i}[(x_1 + x_2 z_{j-1}^2) z_j z_{j-1}^k]\} = 0 (i = 1, 2, \cdots, n-1) \quad (14)$$

For a two-stack system, the optimal stack capacities can be determined by solving the following equation set.

$$\sum_{j=1}^{3}\sum_{k=0}^{l} a_k \{d_{j,1}[(\frac{k+4}{k+3}x_2+\frac{k+4}{k+1}x_3)z_j^{k+3}$$
$$+\frac{k+2}{k+1}x_1 z_j^{k+1}-(\frac{x_1+3x_3 z_j^2}{k+1}+\frac{x_2 z_{j-1}^2}{k+3})z_{j-1}^{k+1})]$$
$$-d_{j-1,1}[(x_1+x_2 z_{j-1}^2)z_j z_{j-1}^k]\}=0 \qquad (15)$$

$$z_0=0 \qquad (16)$$
$$z_1=y_1 \qquad (17)$$
$$z_2=y_2 \qquad (18)$$
$$z_3=y_1+y_2 \qquad (19)$$
$$y_1+y_2=1 \qquad (20)$$

where
$\{d_{j,1}\}(j=0,1,2,3)=\{0,1,-1,0\}$

Equation (15) is obtained by setting the derivative of the annual fan energy consumption to zero, and the five equations that follow it express the relationships between stack capacity and combination capacity, based on Eq.(4).

For a three-stack system, the optimal stack size can be determined using the following equation set.

$$\sum_{j=1}^{7}\sum_{k=0}^{l} a_k \{d_{j,1}[(\frac{k+4}{k+3}x_2+\frac{k+4}{k+1}x_3)z_j^{k+3}$$
$$+\frac{k+2}{k+1}x_1 z_j^{k+1}-(\frac{x_1+3x_3 z_j^2}{k+1}+\frac{x_2 z_{j-1}^2}{k+3})z_{j-1}^{k+1})]$$
$$-d_{j-1,1}[(x_1+x_2 z_{j-1}^2)z_j z_{j-1}^k]\}=0 \qquad (21a)$$

$$\sum_{j=1}^{7}\sum_{k=0}^{l} a_k \{d_{j,2}[(\frac{k+4}{k+3}x_2+\frac{k+4}{k+1}x_3)z_j^{k+3}$$
$$+\frac{k+2}{k+1}x_1 z_j^{k+1}-(\frac{x_1+3x_3 z_j^2}{k+1}+\frac{x_2 z_{j-1}^2}{k+3})z_{j-1}^{k+1})$$
$$-d_{j-1,2}[(x_1+x_2 z_{j-1}^2)z_j z_{j-1}^k]\}=0 \qquad (21b)$$

$$z_0=0 \qquad (22)$$
$$z_1=y_1 \qquad (23)$$
$$z_2=y_2 \qquad (24)$$
$$z_3=\begin{cases} y_1+y_2 & (y_1+y_2\le 0.5)\\ y_3 & (y_1+y_2\ge 0.5)\end{cases} \qquad (25)$$
$$z_4=\begin{cases} y_3 & (y_1+y_2\le 0.5)\\ y_1+y_2 & (y_1+y_2\ge 0.5)\end{cases} \qquad (26)$$
$$z_5=y_1+y_3 \qquad (27)$$
$$z_6=y_2+y_3 \qquad (28)$$
$$z_7=y_1+y_2+y_3 \qquad (29)$$
$$y_1+y_2+y_3=1 \qquad (30)$$

where, in area $y_1+y_2\le 0.5$:
$\{d_{j,1}\}(j=0,1,2,3,4,5,6,7)=\{0,1,0,1,-1,0,-1,0\}$
$\{d_{j,2}\}(j=0,1,2,3,4,5,6,7)=\{0,0,1,1,-1,-1,0,0\}$

and in area $y_1+y_2\ge 0.5$:
$\{d_{j,1}\}(j=0,1,2,3,4,5,6,7)=\{0,1,0,-1,1,0,-1,0\}$
$\{d_{j,2}\}(j=0,1,2,3,4,5,6,7)=\{0,0,1,-1,1,-1,0,0\}$

Equations (21a) and (21b) are obtained by setting the partial derivative of the annual fan energy consumption to zero, and the nine equations that follow it express the relationships between stack capacity and combination capacity, based on Eq.(5).

Numerical method

The optimized stack sizes can also be determined using a numerical method directly based on Eq.(11b). The numerical method can use the actual fan efficiency based on the fan curve. There is no need to express the time density with a polynomial equation.

Several options for the numerical method are available. After comparing different methods for this special application, the Hoole-Jeeves method was selected. The Hoole-Jeeves is a pattern search method. The optimization procedure is presented in Fig.2.

The independent stack capacities can be expressed as a vector $\mathbf{Y}=(y_1,y_2,\cdots,y_n)^T$, where n is the independent variable (1 for two-stack systems and 2 for three-stack systems). The basic procedure of the Hoole-Jeeves method is to find the resultant point $\mathbf{Y}^n$ from a start point $\mathbf{Y}$ with a step of length L and compared fan energy $\overline{E}$ at a compared point $\mathbf{Y}_c$. In procedure 1 the start point is the compared point. If $\overline{E}(y_1+L,y_2,\cdots,y_n)<\overline{E}(\mathbf{Y}_c)$, then $\mathbf{Y}^1=(y_1+L,y_2,\cdots,y_n)^T$. Otherwise, check $\overline{E}(y_1-L,y_2,\cdots,y_n)<\overline{E}(\mathbf{Y}_c)$. If the answer is Yes, then $\mathbf{Y}^1=(y_1-L,y_2,\cdots,y_k)^T$. Otherwise, $\mathbf{Y}^1=\mathbf{Y}$. Starting from $\mathbf{Y}^1$ the y_2-direction is tested in L steps. The result is $\mathbf{Y}^2$, and so on, until $\mathbf{Y}^n$ is obtained.

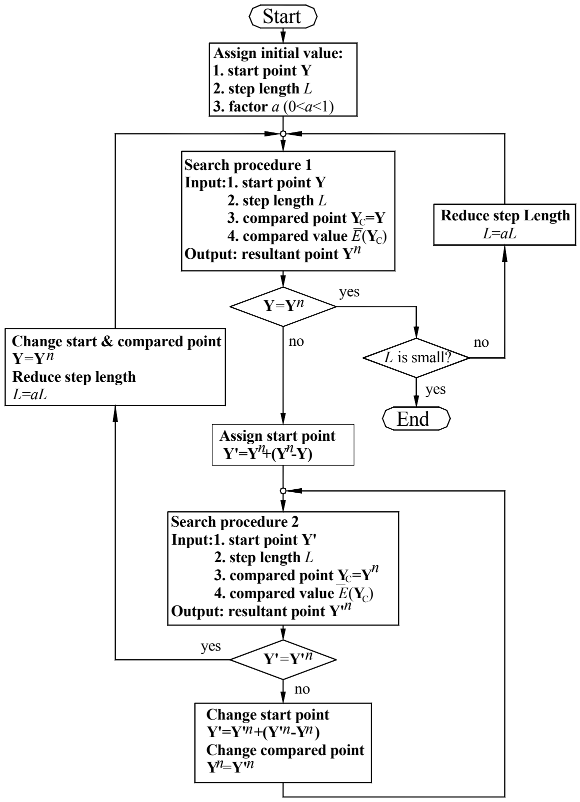

Figure 2: *Flowchart of Hoole-Jeeves method*

The start point ($\mathbf{Y}$), a step length (L), and a relaxation factor a ($0 < a < 1$) must be selected to initialize the calculation. The procedures are described in the flowchart (Fig.2). The optimal stack capacities are obtained when no new resultant point can be found and the step length L meets the required accuracy.

Application

The primary independent parameters of the optimization are: the exhaust airflow time density or distribution, the pressure loss distribution, and the number of stacks. The optimization process and potential savings is demonstrated using the following example. The facility is a typical university laboratory building, that has five floors including one underground floor, and a total floor area of 9,300 m^2 (99,000 ft^2). The laboratories are located on the first, second and third floors and have approximately 100 variable air volume fume hoods. The design exhaust airflow is 28.3 m^3/s (60,000 CFM). Individual fume hood airflows vary from 60% to 100% based upon the sash position. The time density is shown as the thick line in Fig.3. The airflow varies from 30% to 100% of design airflow.

Equation (31) is the regression equation of the time density distribution. The regression of the time density is shown as the dash line in Fig.3. The sixth order regression has excellent accuracy for this airflow density distribution except in the low flow range.

$$\bar{f}(\overline{Q}_h) = -0.635 + 20.2\overline{Q}_h - 182\overline{Q}_h^2 + 643\overline{Q}_h^3 - 948\overline{Q}_h^4 + 599\overline{Q}_h^5 - 132\overline{Q}_h^6 \quad (31)$$

The time distribution in each airflow range (bin), shown in Table 1, is obtained from the time density. The bin time density is shown as the step curve in Fig.3.

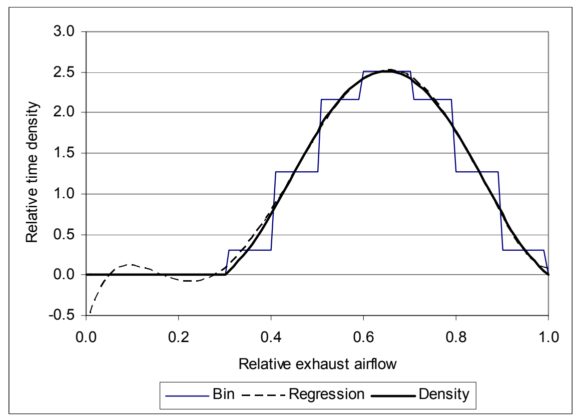

Figure 3*: Relative time density versus relative exhaust airflow*

The design fan head is 1,620 Pa (6.5" H_2O). The design static pressure at the fume hood is -375 Pa (-1.5" H_2O). The design static pressure loss is 75 Pa (0.3" H_2O). The design main duct pressure loss is 1,170 Pa (4.7" H_2O). Since the fan is close to the stack, the design duct pressure loss after the fan is approximated as zero. The fractions of pressure loss or pressure difference in each section are calculated based on the design condition:

$$x_1 = (0.3 + 1.5) / 6.5 = 0.28$$
$$x_2 = 4.7 / 6.5 = 0.72$$
$$x_3 = 0 / 6.5 = 0$$

Table 1: *Laboratory exhaust airflow time distribution and time density*

Airflow range $(\overline{Q}_{h,k-1}, \overline{Q}_{h,k})$	0.0-0.1	0.1-0.2	0.2-0.3	0.3-0.4	0.4-0.5	0.5-0.6	0.6-0.7	0.7-0.8	0.8-0.9	0.9-1.0
Relative Time Density $\bar{f}(\overline{Q}_h)$	0	0	0	0.308	1.267	2.169	2.511	2.169	1.267	0.308
Relative Time Distribution $\overline{T}_k$	0	0	0	0.0308	0.1267	0.2169	0.2511	0.2169	0.1267	0.0308

Table 2: *Optimal stack capacities and annual fan energy consumption*

System	Stack	Analytical method			Numerical method		
		Capacity (%)	Size In(mm)	Relative Fan energy (%)	Capacity (%)	Size In(mm)	Relative Fan energy (%)
Two Stack	1	25.5	30(760)	50	25.1	30(760)	50
	2	74.5	52(1320)		74.9	52(1320)	
Three Stack	1	15.0	23(580)	46	15.0	23(740)	46
	2	27.6	32(810)		27.9	32(990)	
	3	57.4	46(1170)		57.1	46(1420)	

The relative fan power and fan head curves are generated from the actual fan performance curve using the design fan airflow, design fan head and design fan power, as shown in Fig.4.

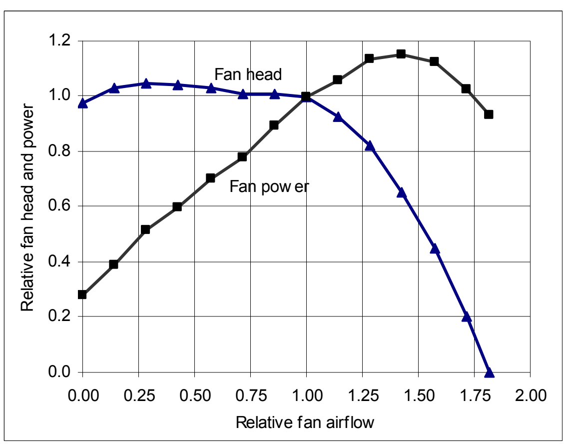

Figure 4: *Fan curve*

The optimized stack sizes have been determined using both the analytical and numerical methods. Table 2 summarizes the optimal stack capacities, stack sizes, and the annual average fan energy consumption. The stack size is calculated based on an exit velocity of 15 m/s (3,000 ft/min). Both methods produced the same stack sizes. It appears that the assumption of constant fan efficiency has little impact on the system optimization for this particular case. It also shows that the annual average fan power is 50% of the design fan power for the two-stack system, and 46% for the three-stack system.

Conclusions

Both analytical and numerical optimization methods have been developed for multi-stack exhaust systems in new system design applications, where the total capacity of the stacks equals the design exhaust airflow rate. When analytical method is used, the regression time density must have good accuracy.

The optimal stack sizes and the potential annual energy savings depend on the following parameters: the airflow distribution pattern and the main duct pressure loss ratios. The numerical results show that the annual average fan power consumption can be as low as 50% of the design fan power consumption for a typical laboratory exhaust system.

Nomenclature

a_k = Coefficient for the time density regression;

$\overline{E}$ = Relative annual fan energy consumption;

$f(\overline{Q}_h)$ = Time density versus relative laboratory exhaust airflow, hr;

$\bar{f}(\overline{Q}_h)$ = Relative time density versus relative laboratory exhaust airflow;

H = Fan head, Pa or in.wg;

H_d = Design fan head, Pa or in.wg;
$\overline{H}$ = Relative fan head;
n = Stack combination number;
m = Stack number; segment of airflow interval.
Q = Fan airflow, m^3/s or CFM;
Q_d = Design fan airflow or exhaust system design airflow, m^3/s or CFM;
Q_h = Laboratory exhaust airflow, m^3/s or CFM;
$\overline{Q}$ = Relative fan airflow;
$\overline{Q}_h$ = Relative laboratory exhaust airflow;
$\overline{Q}_{h,ave,k}$ = Average relative laboratory exhaust airflow in a range from $\overline{Q}_{h,k-1}$ to $\overline{Q}_{h,k}$;
T = Total operating hours, hr;
T_k = Operating time distribution in a range from $\overline{Q}_{h,k-1}$ to $\overline{Q}_{h,k}$, hr;
$\overline{T}_k$ = Relative time distribution in a range from $\overline{Q}_{h,k-1}$ to $\overline{Q}_{h,k}$;
W = Fan power, kW or hp;
W_d = Design fan power, kW or hp;
$\overline{W}$ = Relative fan power;
$\overline{W}_{ave,k}$ = Average relative fan power in a range from $\overline{Q}_{h,k-1}$ to $\overline{Q}_{h,k}$;
x_1 = Relative design static pressure difference at two static pressure sensors;
x_2 = Relative design pressure loss in the duct upstream of the fan;
x_3 = Relative design pressure loss the in duct downstream of the fan;
y_i = Stack design capacity series;
z_j = Stack combination design capacity series;
ΔP_2 = Duct pressure loss from the fume hood to the fan, Pa or in.wg;
$\Delta P_{2,d}$ = Design pressure loss in the duct upstream of the fan, Pa or in.wg;
ΔP_3 = Duct pressure loss from the fan to the stack, Pa or in.wg;
$\Delta P_{3,d}$ =Design pressure loss in duct downstream of the fan, Pa or in.wg;
ΔP_d = Dynamic head difference between the two static pressure sensors, Pa or in.wg;
ΔP_s = Static pressure difference between the two static pressure sensors, Pa or in.wg;
$\Delta P_{s,d}$ = Design static pressure difference between the two static pressure sensors, Pa or in.wg.

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