USING FOURIER SERIES TO MODEL HOURLY ENERGY USE IN COMMERCIAL BUILDINGS

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Energy Systems Laboratory Technical Report # ESL-TR-93/11-01

November 1993

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ABSTRACT

Fourier series analysis is eminently suitable for modeling strongly periodic data. Weather independent energy use such as lighting and equipment load in commercial buildings is strongly periodic and is thus appropriate for Fourier series treatment. Weather dependent energy use such as cooling energy use and heating energy use is dependent both on internal load and weather variables. Both the driving forces have strong periodicity and consequently, weather dependent energy use is equally suitable for Fourier series analysis.

The procedure for modeling hourly energy use has two steps. Mean daily energy use is different during working weekdays, working weekends, holidays and Christmas due to major change in mode of operation or scheduling. The first step is to do Day-typing to remove such effects. The second step is to develop Fourier series model using the forward selection procedure of Statistical Analysis System (SAS) program. While the model for weather independent energy use is developed from a Fourier series with the hour of the day as the independent variable, the model for weather dependent energy use is developed from a set of variables that has both Fourier frequencies and products of Fourier frequencies and weather variables, e.g., ambient temperature, out door specific humidity and horizontal solar flux.

Two other existing regression approaches for modeling hourly energy use that give very high prediction accuracy are (1) individual hourly approach and (2) aggregated hourly approach. Several case studies have been made to test the power of Fourier series technique in comparison with the existing regression techniques. The prediction accuracy of Fourier series model is found very close to that of individual hourly and aggregated hourly approaches, in all the cases. However, a smooth functional form of model for each day-type obtained in Fourier series approach gives much better insight to the load profile. This, method, therefore, will be very useful for diagnostic purposes.

In short, the new approach of modeling hourly energy use adapting Fourier series, is suitable for retrofit savings analysis as well as diagnostics.

ACKNOWLEDGEMENTS

This research is funded by the Texas State Energy Conservation Office of the Intergovernmental Division of the General Services Commission (State Agencies Program) as part of the LoanSTAR Monitoring and Analysis Program. We gratefully acknowledge usefull discussions with Kelly Kissock, Srinivas Katipamula, S. Thamilseran and Jeff Haberl. The assistance from Jingrong Wang in processing the data speeded up the work.

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NOMENCLATURE

a_0	A constant
a_k	Coefficient of kth sine frequency of a Fourier series
b_k	Coefficient of kth cosine frequency of a Fourier series
CH_x	xth frequency cosine term of Fourier series for annual cycle
CH_y	yth frequency cosine term of Fourier series for diurnal cycle
C(p)	Mallow's coefficient of parameters
C. V.	Coefficient of Variation (%)
d	Day of year, 1 on 1st January and 365 (366 for leap years) on 31st December
E	Hourly energy use (kWh/hr for weather independent energy use, MMBtu/hr for weather dependnet energy use)
$\overline{E}$	Mean hourly energy use (kWh/hr for weather independent energy use, MMBtu/hr for weather dependnet energy use)
$\hat{E}$	Predicted hourly energy use (kWh/hr for weather independent energy use, MMBtu/hr for weather dependnet energy use)
F	Ratio of post-retrofit Christmas mean energy use to post-retrofit working weekdays mean energy use
F(x)	A function of independent variable x
h	Hour of day
I	Horizontal solar flux (W/m^2)
k	Frequency of a Fourier series representation
p	Number of parameters of a linear model
P_k	Period of a Fourier series representation with frequency = k
RMSE	Root mean square error
SHx	xth frequency sine term of Fourier series for annual cycle
SHy	yth frequency sine term of Fourier series for diurnal cycle
S1SHy	Product of yth frequency sine term of Fourier series and specific humidity difference
S1CHy	Product of yth frequency cosine term of Fourier series and specific humidity difference
SL1SHy	Product of yth frequency sine term of Fourier series and horizontal solar flux
SL1CHy	Product of yth frequency cosine term of Fourier series and horizontal solar flux
T	Outdoor temperature, degree Fahrenhiet
T1SHy	Product of yth frequency sine term of Fourier series and outdoor temperature
T1CHy	Product of yth frequency cosine term of Fourier series and outdoor temperature
W	Specific humidity (lb/lb of dry air)
W^+	Difference between actual outdoor specific humidity and specific humidity at 55 degree Fahrenhiet dew point temperature (0.0092)
x	Frequency of annual cycle
y	Frequency of diurnal cycle

Greek Symbols

α	Coefficient of Fourier series sine frequencies

β	Coefficient of Fourier series cosine frequencies
γ	Coefficient of product of sine frequencies and outdoor temperature in weather dependent energy use model
δ	Coefficient of product of cosine frequencies and outdoor temperature in weather dependent energy use model
ε	Random error of a linear model
ϕ	Coefficient of product of sine frequencies and humidity difference in weather dependent energy use model
ψ	Coefficient of product of cosine frequencies and humidity difference in weather dependent energy use model
$\mu(h)$	A function of hour of day
η	Coefficient of product of sine frequencies and horizontal solar flux in weather dependent energy use model
ζ	Coefficient of product of sine frequencies and horizontal solar flux in weather dependent energy use model

Subscripts

b	Aggregated hourly bin
d	Day of year
h	Hour of day
i	Day-type
k	kth frequency term
n	nth frequency term
x	xth frequency term
y	yth frequency term

CHAPTER I
INTRODUCTION

Energy conservation programs can help delay construction of new power generation, transmission and distribution facilities and, by that way, provide energy to the customers at a competitive price. About 13% of total energy use in the U.S.A. being in the commercial buildings (EIA, 1986), a tangible reduction in energy use can be achieved by evaluating the present building systems, implementing proper retrofitting and improving operation and maintenance. Energy retrofits in 1700 buildings are reported to have achieved a median annual savings of 18% of whole building energy usage, with a median payback time of 3.1 years; according to a recent study (Greely et al., 1990).

Failure to measure retrofit savings will hinder adoption of efficiency measures in buildings. The motivations for Texas LoanSTAR Monitoring and Analysis Program (MAP) program are to determine energy savings accurately and to suggest improvement in operation and maintenance (Turner, 1990). LoanSTAR has adopted and developed energy use models of following types: (i) one-variable regression models using two parameter, three parameter and four parameter sub-categories, (ii) multiple linear regression analysis (MRA), (iii) principal component analysis (PCA) and (iv) calibrated models for different applications. The two, three and four parameter models to predict daily energy use are currently being used to determine retrofit energy savings (Kissock et al., 1992). These daily models give reasonably accurate prediction of savings but are inherently deficient in detecting schedule changes, maintenance and operational problems, etc. Hourly models are much more suitable for such applications. Models using MRA suffer from multicollinearity effects among regressor variables, but techniques such as PCA do not seem to be entirely satisfactory (Wu et al. 1992). The Calibrated model approach is used when pre-retrofit data set is so short that regression analysis is unsuitable (Katipamula & Claridge, 1992).

Energy use in commercial buildings are strongly periodic. Fourier series is a powerful tool for analyzing such periodic data. The present work develops a general linear regression modeling procedure to perform retrofit savings analysis at the hourly level, by adapting Fourier series in combination with weather variables (ambient temperature, humidity and solar radiation).

This thesis has eight chapters : (1) Chapter I contains introduction, (2) Chapter II contains the literature review, (3) theory and basis of Fourier series to model hourly energy use in commercial buildings is presented in Chapter III, (4) the modeling procedure for weather independent energy use, case studies and comparison with other multiple linear regression approaches are described in Chapter IV, (5) Chapter V describes a modeling procedure for weather dependent energy use, presents a case study and compares the results with those of individual hourly multiple linear regression approach, (6) identifying of important Fourier frequencies and characterizing diurnal load shapes is described in Chapter VI, (7) effect of short data set on monthly prediction accuracy is addressed in Chapter VII and (8) Conclusions and future directions are presented in Chapter VIII.

CHAPTER II
LITERATURE REVIEW

DEMAND SIDE MANAGEMENT PROGRAMS - ITS OBJECTIVES

Demand Side Management (DSM) programs for energy conservation and management in commercial buildings may have macro or micro objectives. Macro-objectives may be to implement retrofits in a large number of buildings, determine overall savings and report the results in a suitable form to the appropriate agencies. This calls for defining populations to sample, collecting and processing weather data and utility bills, analyzing data to determine savings. DSM programs conducted by utilities and state Public Utility Commissions (PUC) are macro-objective oriented. Monthly consumption data are normally used to determine monthly energy savings. Three modeling approaches that are often adopted are (i) difference and ratio estimation, (ii) statistically adjusted engineering estimation and (iii) multivariate modeling (Hirst et al., 1991).

The objectives of energy conservation programs at the micro level are to determine retrofit savings for individual buildings, study diurnal load profiles, identify possible operation and maintenance problems, and, generally, provide better understanding and insight into how energy use in commercial buildings can be decreased. These objectives require careful monitoring and analysis of energy use at the daily and the hourly level for each building. The Texas LoanSTAR program (Claridge et al., 1991) is concerned with these micro objectives of a DSM program. A state-wide energy Monitoring and Analysis Program (MAP) began in 1989 as a part of this program. The major objectives were: (i) verifying energy and dollar savings of the retrofits, (ii) reducing energy cost by identifying operational and maintenance improvements, (iii) improving retrofit selection in future phases of the LoanSTAR program and (iv) initiating a data base of energy use in institutional and commercial buildings in Texas.

EXISTING MODELING TECHNIQUES

The Texas LoanSTAR MAP program emphasizes selection and development of analyzing techniques. The one-parameter (outdoor dry-bulb temperature) regression model approach to determine retrofit savings currently uses one of the following sub-model types: two parameter, three parameter or four parameter models (Ruch et al. 1991). These models are often applied separately for weekdays and weekends at the daily level (Kissock et al. 1992). Other techniques such as Principal Component Analysis (PCA) (Ruch, et. al. 1991, Reddy & Claridge 1993), Improved Measures for Goodness of fit and Calibrated System models (Katipamula & Claridge1992), are also being developed. Since modeling hourly energy use is useful in both retrofit savings analysis savings and diagnostics, one of the current issues is to concentrate in this area.

Different approaches that are suitable for retrofit savings analysis at the hourly level are shown in Figure 1. As shown, these can be divided into three approaches: calibrated models, Artificial Neural Network (ANN) (Krieder et al. 1991) and regression models. The calibrated simulation approach needs information concerning operation of the mechanical systems of the buildings and is currently considered tedious. Among them, the simplified modeling approach described by Katipamula and Claridge (1992) is simpler to calibrate and is used by the LoanSTAR MAP to identify retrofit savings when no pre-retrofit data are available. However, the simulated cooling load for Zachry Engineering Center (Texas A&M University) showed a poor fit to the measured cooling load at the hourly level for VAV system (Figure 11, Katipamula et al. 1992).

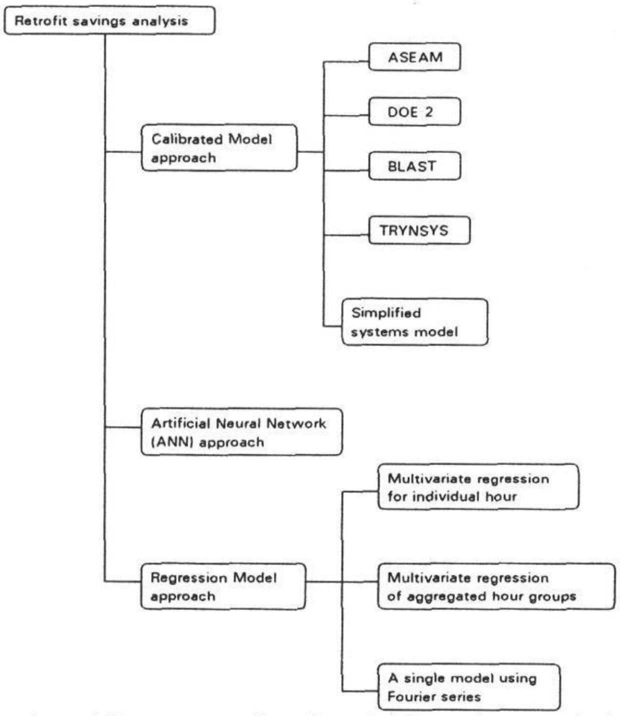


Figure 2.1 Different modeling approaches for retrofit savings analysis at the hourly level

A recent development in modeling techniques is the application of ANN. ANNs are simplified mathematical models of biological neural networks. An ANN is a massive, parallel, dynamic system of interconnected, interacting parts and learns by example. The network is trained using the back propagation method. This method predicts energy use with high accuracy but a challenge for this method is to reduce the substantial time taken for training the network. Also, selection of number of nodes and layers of the network depends heavily on experience; it is currently more of an art than a science (Anstett et al. 1992).

When adequate length of data (three to four months atleast) is available, the general linear regression approaches seem to be the most attractive because of simplicity. Also, such approaches take less time and computer memory. A separate multivariate general linear regression model for every hour and for each day type is one approach. The functional form of this model is of the following form:

$$E_{h,i} = a_{h,i} + b_{h,i} T_{h,i} + c_{h,i} (SH_{h,i} - SH_{55})^{+} + d_{h,i} S_{h,i}, \quad i = 1 \quad \text{to} \quad k \tag{2.1}$$

where a, b, c, d are regression coefficients, T is outdoor dry-bulb temperature, SH is specific humidity and S is horizontal solar flux. The subscript h stands for the hours of the day and i stands for the day type. SH_{55} is the specific humidity at 55 degree F, which is generally assumed to be the average cold coil surface temperature of the HVAC system. The choice of this variable is based on the idea that latent load is removed only when dew point temperature of the air stream passing over the cooling coil is above cooling coil surface temperature. This method was used to model energy uses in several LoanSTAR buildings (Srinivas et al., 1993). The results showed high prediction accuracy. However, identifying building behavioral patterns become difficult from the models developed with this approach.

Another approach of multivariate regression is aggregating the hours of the day that have similar pattern of energy use and performing a regression for each aggregated group. The regression model is then as follows:

$$E_{h,i} = a_{b,i} + b_{b,i} T_{h,i} + c_{b,i} (SH_{h,i} - SH_{55})^{+} + d_{b,i} S_{h,i}, \quad i = 1 \quad \text{to} \quad k \tag{2.2}$$

where b stands for every aggregated bin. Such models applied to a few LoanSTAR buildings show slightly less prediction accuracy of this method as compared to the individual hour modeling approach. However, the aggregated hourly approach gives better insight into the changes in operational pattern and weather dependence of energy use in a building.

FOURIER SERIES APPROACH

Diurnal variations of weather variables, operational changes of equipment, scheduling changes, etc., are so periodic that they can be handled more conveniently with a smooth function that eliminates the necessity for creating hourly bins. A functional form of a model is also useful for further mathematical analysis, for example, dynamic analysis of building energy use. A Fourier series is a combination of sine and cosine waves of several frequencies and can represent a shape that repeats periodically. Seem and Braun (1991) tried to model hourly cooling energy use with Fourier series. However, no weather variable was considered and the change in mode of operation was ignored. The set of harmonics had a maximum of 168 hours' period including both weekdays and weekends in a single model. The model gave a poor fit to the data. An improved methodology, therefore, has been developed in the present work which has the following distinct features: (i) it uses Fourier series in conjunction with weather variables to model diurnal profiles of energy use, (ii) day-typing is done to account for the differences between weekday, weekend and holiday behavior and (iii) daily (twentyfour hours) and annual (365 days) cycles are used. The new approach gives better fit to the data and the objective of a single model determining savings as well as yielding physical insight into energy use in buildings is fulfilled.

CHAPTER III
SUITABILITY OF FOURIER SERIES ANALYSIS OF HOURLY ENERGY USE IN COMMERCIAL BUILDINGS

INTRODUCTION

Fourier analysis is one of the most important tools of mathematics and mathematical physics. It originally arose from the problems involving vibrating cords that were being studied by Leonard Euler and Daniel Bernoulli in the 18th century. The subject was named after Jean-Baptist-Joseph Fourier, whose work on the representation of mathematical functions was published in his *Theorie analytique de la chaleur* (The Analytical Theory of Heat) in 1822.

The present context is to analyze hourly energy use such as lighting and equipment energy use, electricity use by air handling units, cooling energy use, heating energy use, etc., in commercial buildings which are inherently periodic in nature. The profiles of energy uses repeat themselves in the diurnal cycle due to fixed scheduling, as well as over the annual cycle due to seasonal variation. These energy uses, are therefore, well-suited for Fourier series treatment.

A Fourier series representation of a function F(x) is (Graybill, 1976)

$$F(x) = \frac{a_0}{2} + \sum_{k=1}^{n} [a_k \sin kx + b_k \cos kx] \tag{3.1}$$

where the finite sum from k = 1 to n of the weighted trigonometric terms is called a trigonometric polynomial of order n (or less, if both a_n and b_n are zero). A trigonometric polynomial of order n has 2n+1 coefficients a_1, b_1,.....,a_n, b_n and it can be shown that if 2n+1 distinct points are fixed in the interval $0 \leq x \leq 2\pi$, there is always a unique trigonometric polynomial of order n or less that takes prescribed values at these points. This idea led to the development of Fourier series modeling of hourly energy use in commercial buildings.

In this chapter, the presence of periodicity in the energy use is illustrated with examples and the theory of Fourier series is discussed. The following section discusses the annual and diurnal periodicity in the data.

PERIODICITY IN THE DATA

As mentioned earlier in this chapter, both weather independent and weather dependent energy use in commercial buildings show periodicity. The daily mean energy use may vary over a year in a building. For example, in institutional buildings, energy use is different during spring, summer and fall semesters, semester breaks, Christmas, etc. Besides, the energy use keeps increasing slowly from the beginning of a semester and decreases gradually during the end. This primarily affects the lighting and equipment (LE) energy use or whole building electricity (WBELEC) use, which (internal load) in turn affects cooling and heating energy use, cooling and heating energy use. Such effect, known as seasonal effect, leads to the presence of periodicity in the annual pattern of energy use. Figure 3.1 shows a time series plot of WBELEC energy use during 1992 in a large university building (Zachry Engineering Center) in Texas A&M University campus, while Figure 3.2 shows LE energy use during 1992 in Business building in University of Texas at Arlington (UTA). We note in both the plots, increases in energy consumption at the beginning of spring (January), summer (May end) and fall semesters (August end) and drops in energy use at the semester ends. Energy use also drops during spring break and Christmas in these sites. These two examples illustrate the presence of annual periodicity in weather independent energy use.

Time series plots of cooling energy use in Zachry Engineering Center and in Business building are shown in Figure 3.3 and Figure 3.4 respectively. Though cooling energy use is

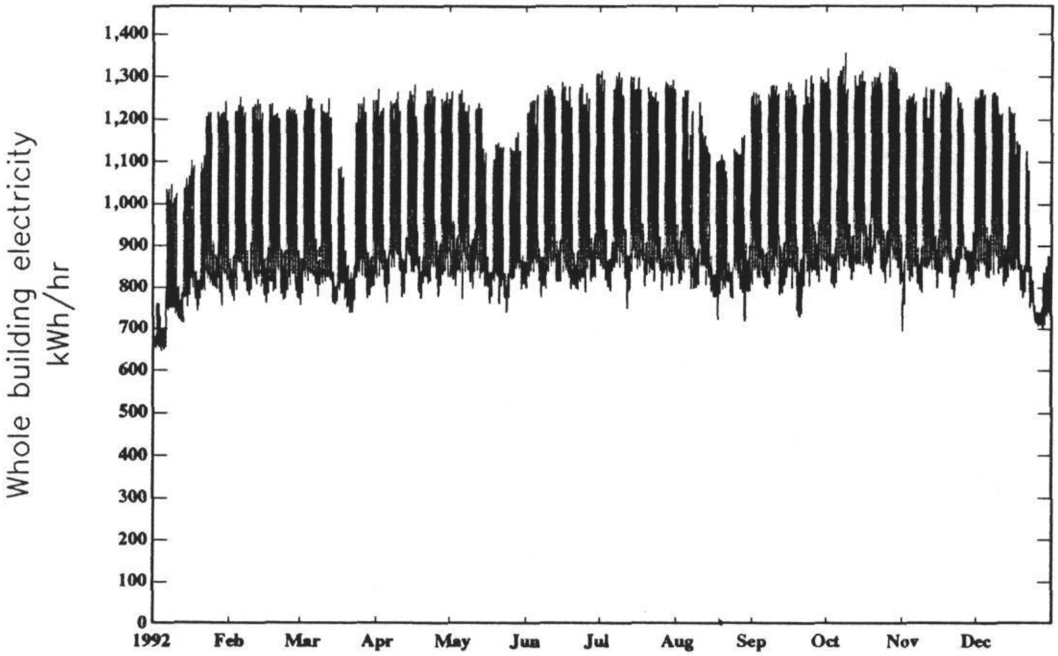


Figure 3.1 Whole building electricity use in Zachry Engineering Center for 1992. The plot shows the presence of seasonal effect on energy use.

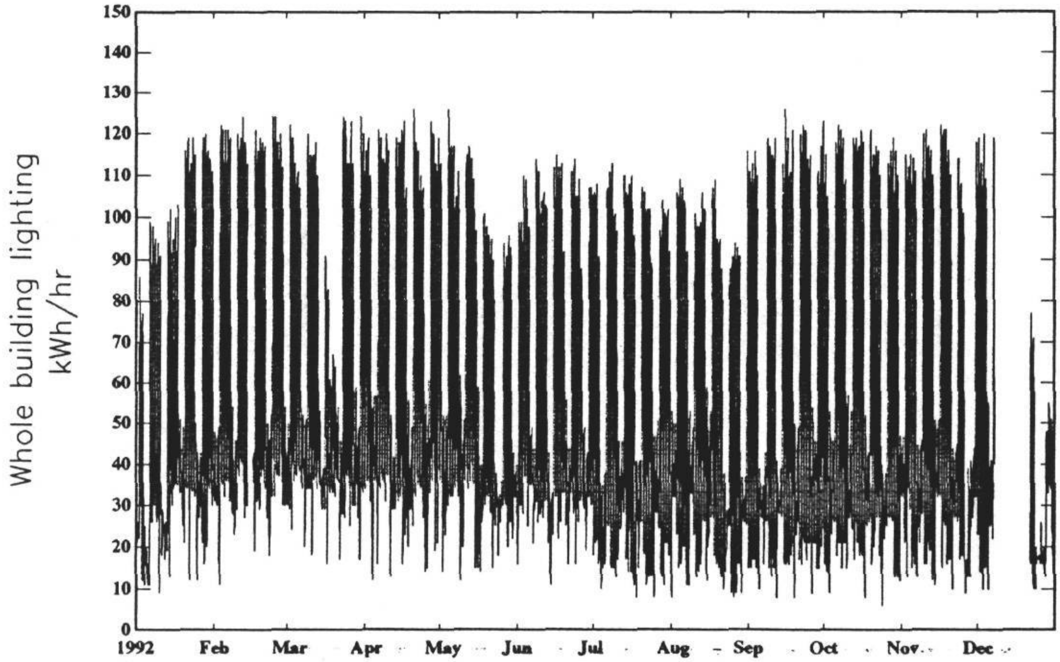


Figure3.2 Lighting energy use in Business building in University of Texas at Arlington in 1992. The plot shows the presence of seasonal effect on energy use.

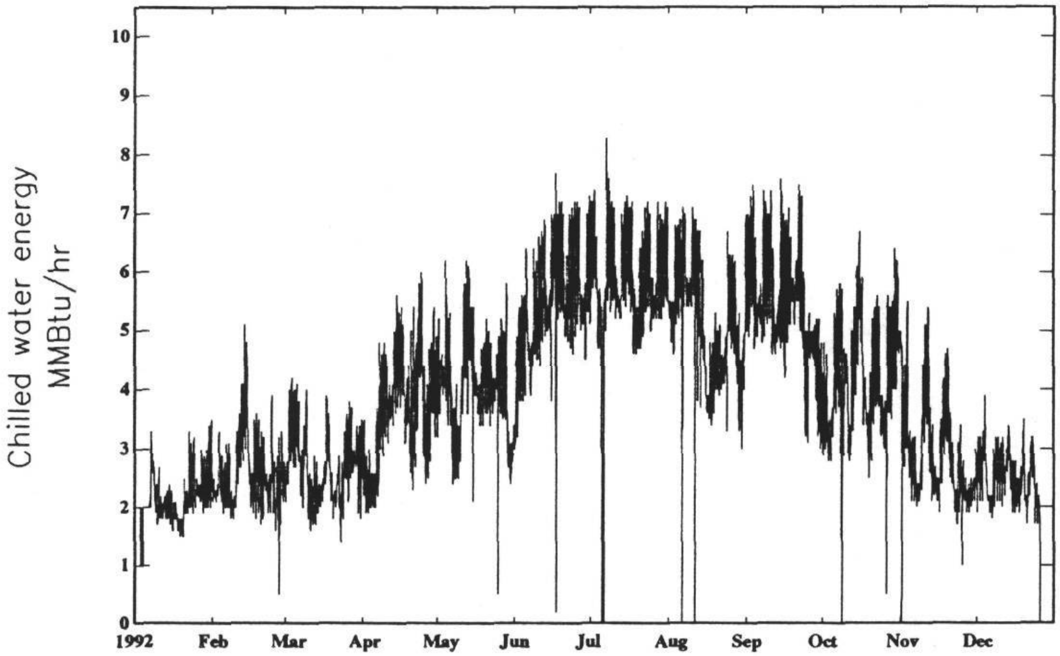


Figure3.3 Cooling energy use in Zachry Engineering Center in 1992. The plot shows the presence of seasonal effect on energy use.

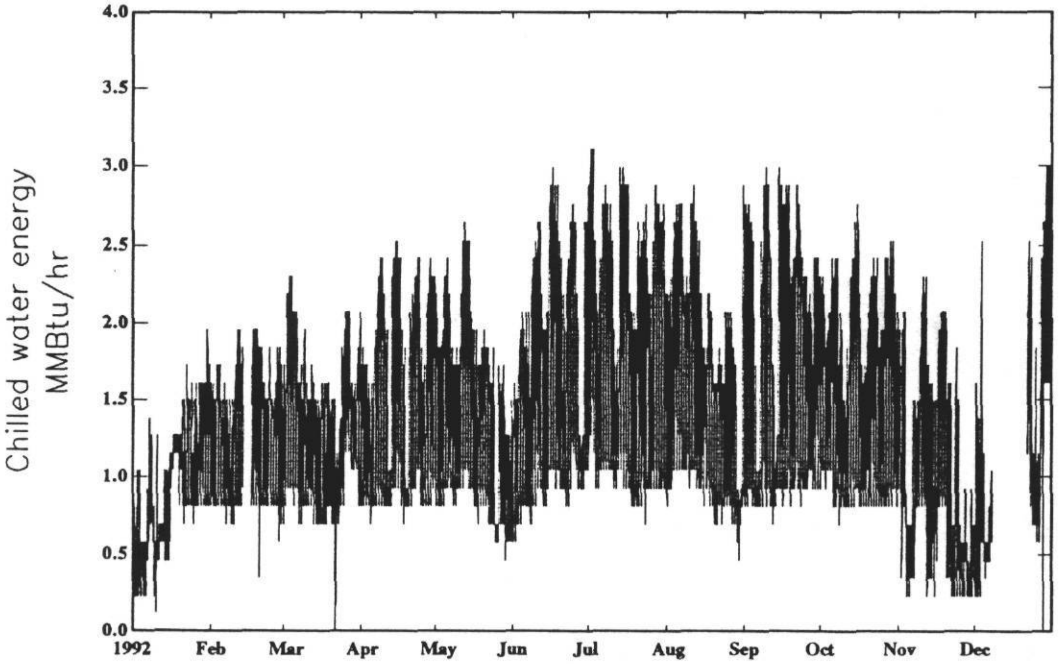


Figure3.4 Cooling energy use in Business building in University of Texas at Arlington in 1992. The plot shows the presence of seasonal effect on energy use.

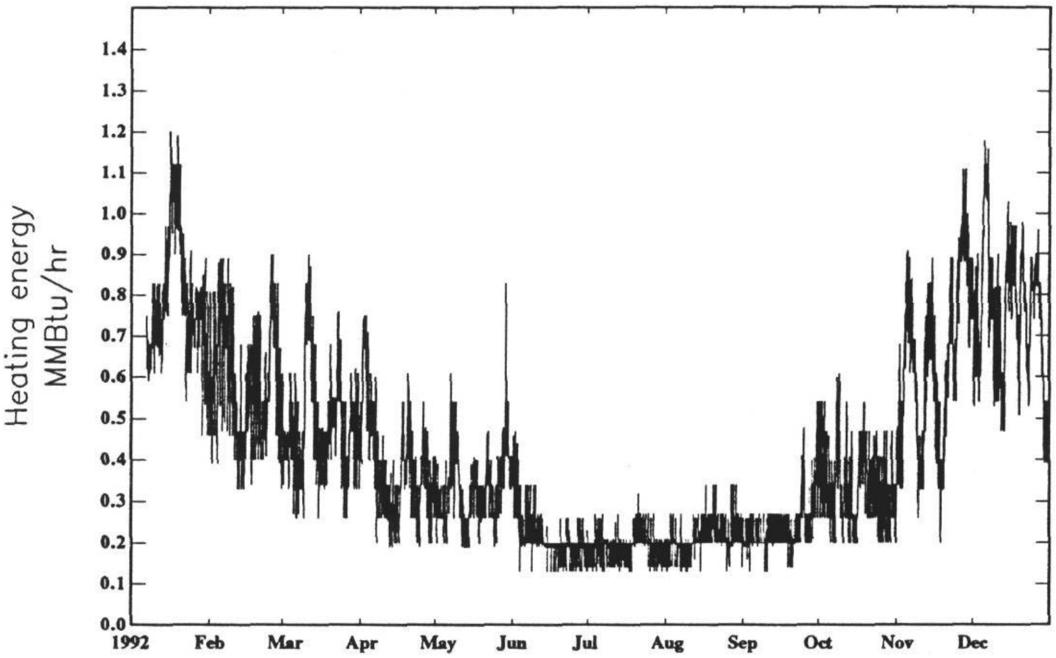


Figure3.5 Heating energy use in Burdine building in University of Texas at Austin in 1992. The plot shows the presence of seasonal effect on energy use.

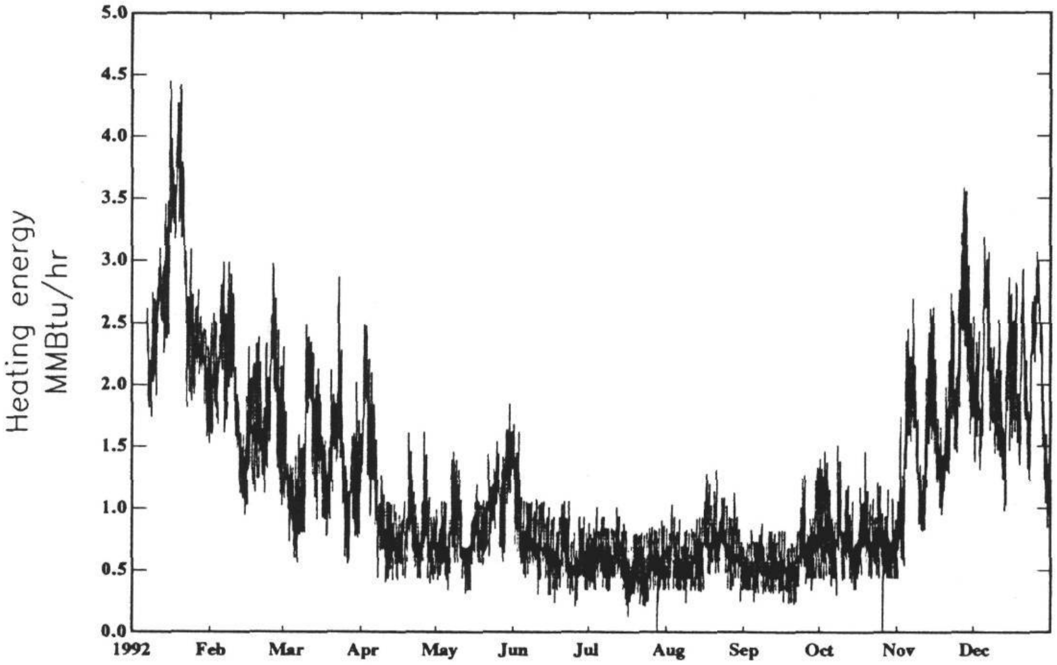


Figure3.6 Heating energy use in P. C. Library in University of Texas at Austin in 1992. The plot shows the presence of seasonal effect on energy use.

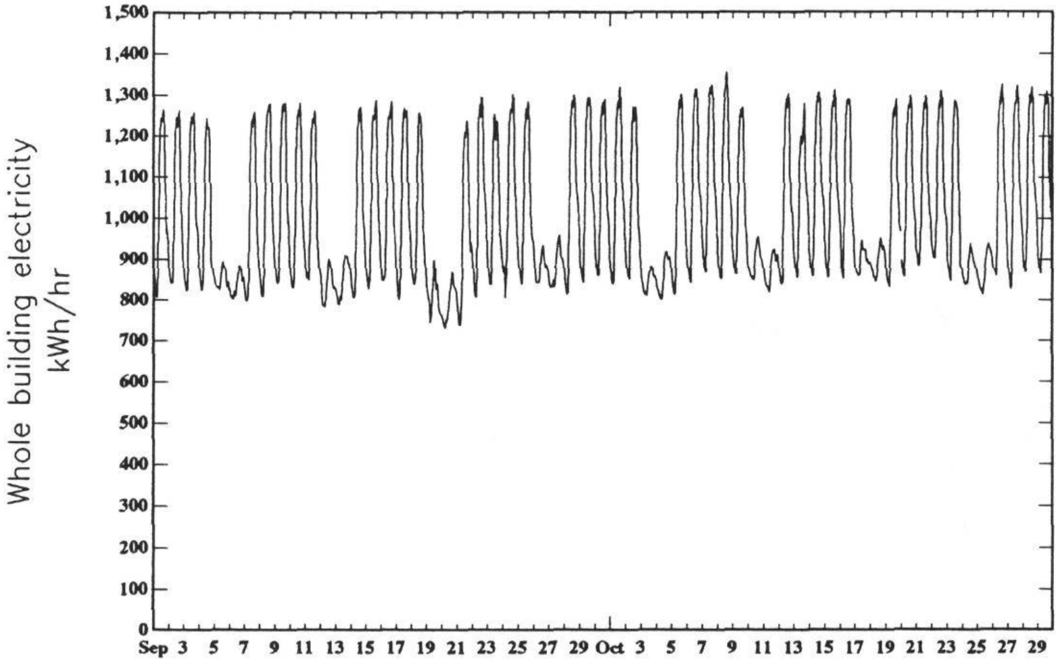


Figure3.7 A time series plot of whole building electricity use in September'92 and October'92 in Zachry Engineering Center that shows diurnal periodicity.

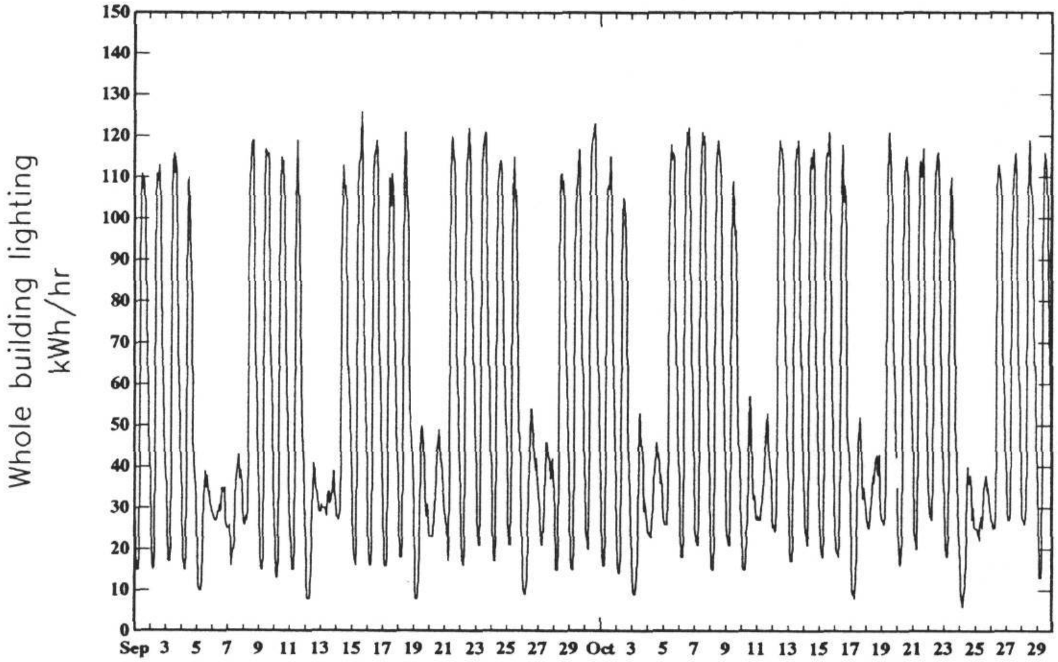


Figure3.8 A time series plot of lighting energy use in Burdine building at UT Austin in September'92 and October'92 that shows diurnal periodicity.

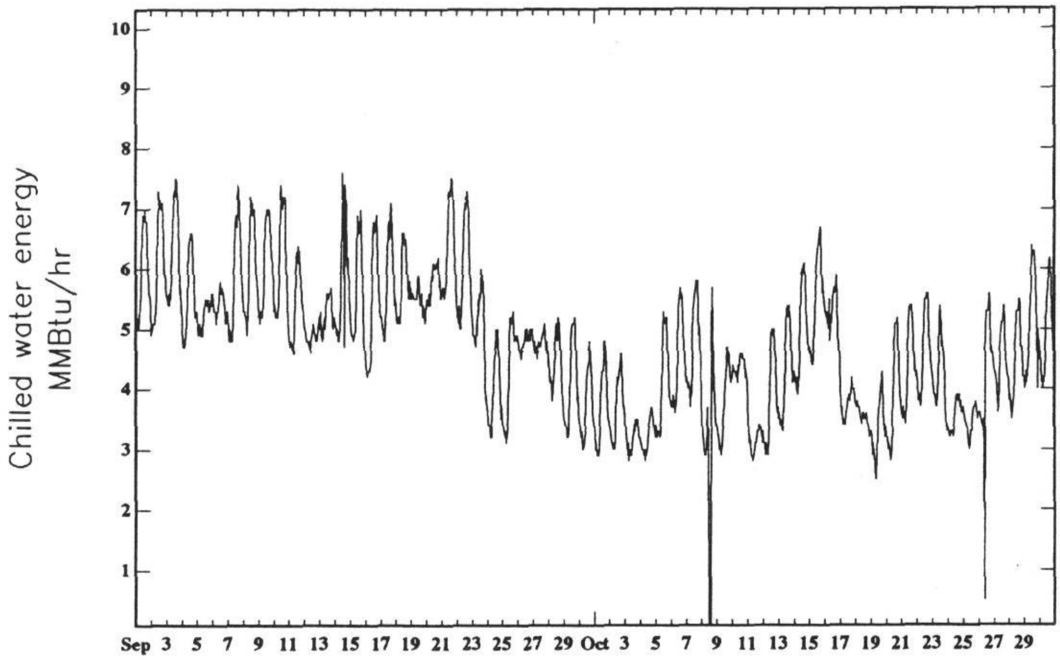


Figure3.9 A time series plot of cooling energy use in September'92 and October'92 in Zachry Engineering Center that shows diurnal periodicity.

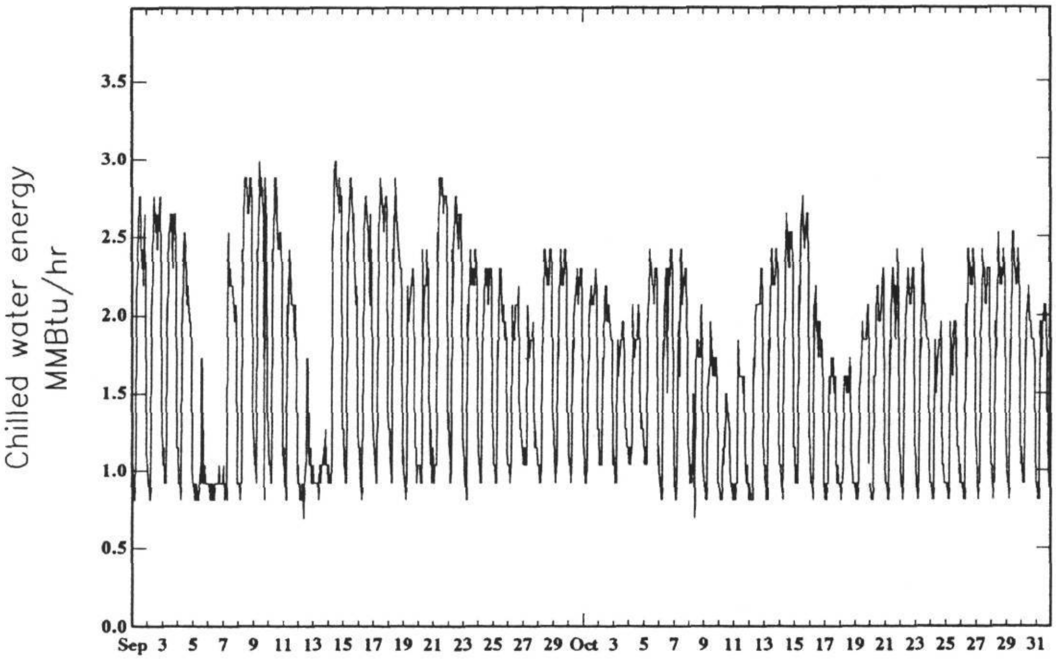


Figure3.10 A time series plot of cooling energy use in Burdine building at UT Austin in September'92 and October'92 that shows diurnal periodicity.

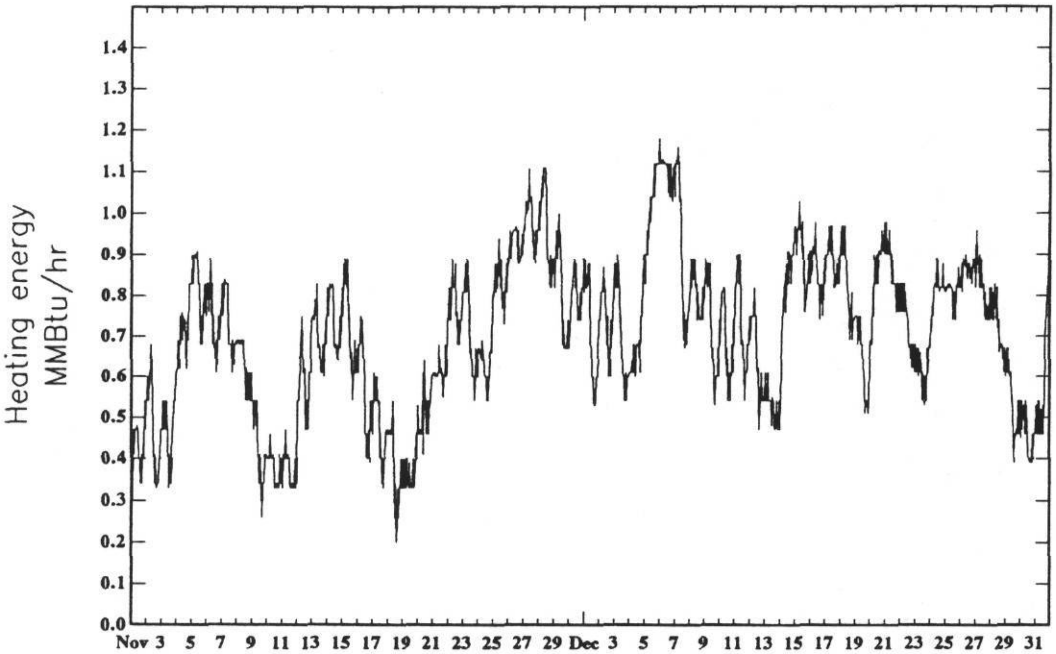


Figure3.11 A time series plot of heating energy use in Burdine building at UT Austin in November'92 and December'92 that shows diurnal periodicity.

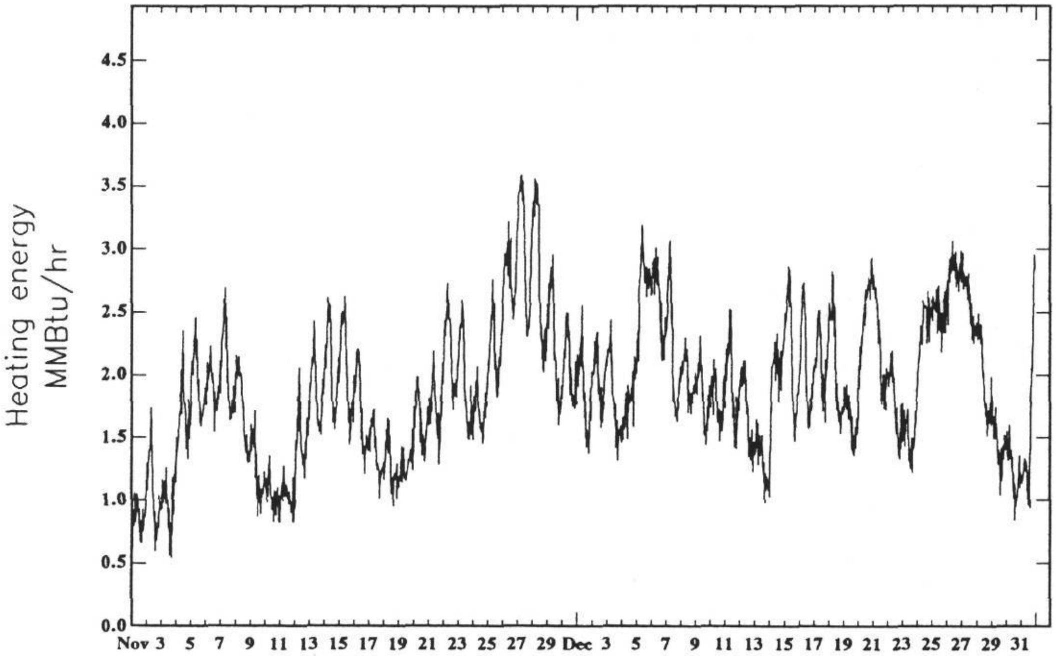


Figure3.12 A time series plot of heating energy use in P. C. Library at UT Austin in November'92 and December'92 that shows diurnal periodicity.

largely weather dependent, the seasonal effect is observable in these time series plots. The seasonal effect is obvious from the time series plots of heating energy use in Burdine and P. C. Library buildings, both located in University of Texas at Austin (UT) (Figure 3.5 and Figure 3.6).

Apart from the annual periodicity discussed in the preceeding paragraph, the pattern of weather independent energy use such as internal load in a building repeats itself over a day due to the fixed operating schedule of the building. This diurnal periodicity is illustrated by the time series plots of the weather independent energy uses during September'92 and October'92 in Zachry Engineering Center and Business building (Figure 3.7 and Figure 3.8). It is observed that during the weekdays, energy use starts increasing at around 8 a.m., drops a little during lunch time and again drops sharply in the evening. However, the patterns are different during weekdays and weekends. This is accounted for by day-typing which is discussed in Chapter IV. Figures 3.9 to 3.12 show time series plots of cooling and heating energy uses during September and October '92. As mentioned previously, these energy uses are weather dependent and the plots show diurnal periodicity with varying amplitude due to the weather effect.

The following section describes how annual and diurnal periodicities can be accounted for in the Fourier series model of energy use in commercial buildings.

FOURIER SERIES MODELING-ACCOUNTING FOR PERIODICITIES

A general representation of a linear model of energy use is as follows:

$$E = \mu(h) + \varepsilon \qquad (3.2)$$

where E is the energy use, h is the independent variable and ε is the random error. In a commercial building, the variation of E with hour of day h is periodic. $\mu(h)$ can, therefore, be represented by a Fourier series.

$$E_h = \beta_0 + \sum_{k=1}^{l}\left[\alpha_k \sin\left(\frac{2\pi}{P_k}h\right) + \beta_k \cos\left(\frac{2\pi}{P_k}h\right)\right] + \varepsilon_h \qquad -\infty < h < +\infty \qquad (3.3)$$

where E_h is the energy use at hour h, a_k and b_k are the coefficients of kth sine and cosine frequencies and P_k is the period for k^{th} frequency. An upper limit on the number of frequencies that can be chosen is l < (h/2-1) since otherwise, the number of parameters will be greater than the number of observations. When data has both annual and diurnal periodicity, one Fourier series each for the annual cycle and the diurnal cycle appears in the model:

$$E_{d,h} = \beta + \sum_{x=1}^{n}\left[\alpha_x \sin\left(\frac{2\pi d}{P_x}\right) + \beta_x \cos\left(\frac{2\pi d}{P_x}\right)\right] + \sum_{y=1}^{l}\left[\gamma_y \sin\left(\frac{2\pi h}{P_y}\right) + \delta_y \cos\left(\frac{2\pi h}{P_y}\right)\right] + \varepsilon_{d,h} \qquad (3.4)$$

where first Fourier series with subscript x represents the annual periodicity and second Fourier series with subscript y represents the diurnal periodicity in the right hand side.

A suitable independent variable needs to be chosen for representing the annual variation of energy use in a Fourier series. Three options were considered : (i) day as the variable, (ii) week as the variable and (iii) month as the variable. One year (1992) data of weather independent energy use in two institutional buildings (Zachry Engineering Center (ZEC), Business building(BUS)) were taken and the daily mean energy use was fitted with all these three options. The R-square and Coefficient of Variation (C.V.) values of the models with three aforesaid options are summarized in Table 3.1. As can be seen from Table 3.1, using day as the variable gives significantly higher R-square and lower C.V. when compared with the other two models. The independent variable was thus chosen as the day of the year.

TABLE 3.1

Comparison of R-square and C. V. Between the Fourier Series Models Using (a) Month, (b) Week and (c) Day as the Independent Variable, to Account for Annual Periodicity

Independent Variable	Day-type	R-square		C.V.(RMSE), %	
		ZEC	BUS	ZEC	BUS
Month	Weekdays	0.45	0.20	4.87	13.7
	Weekends	0.43	0.33	3.65	14.7
Week	Weekdays	0.13	0.11	6.13	14.4
	Weekends	0.15	0.04	4.45	17.7
Day	Weekdays	0.57	0.37	4.32	12.2
	Weekends	0.56	0.39	3.21	14.1

However,the hour of the day h is the independent variable to represent the diurnal cycle. Since there are 24 hours in a day, one needs to choose the independent variables from the first twelve frequencies.

The model equation, therefore, takes the following form :

$$\hat{E}_{d,h} = \beta + \sum_{x=1}^{182}\left[\alpha_x \sin\left(\frac{2\pi}{P_x}d\right) + \beta_x \cos\left(\frac{2\pi}{P_x}d\right)\right] + \sum_{a=1}^{11}\left[\gamma_y \sin\left(\frac{2\pi}{P_y}h\right) + \delta_y \cos\left(\frac{2\pi}{P_y}h\right)\right] \qquad (3.5)$$

where the periods are defined as follows :

$$P_x = \frac{365}{x}, \qquad x = 1,2,3,\ldots,182. \qquad\qquad P_y = \frac{24}{y}, \qquad y = 1,2,3,\ldots,11.$$

We note from equation 3.5 that a maximum of first 183 frequencies can be used to account for the annual periodicity. d is the day of the year having value 1 on January 01 and 365 on December 31. However, in a leap year, 31st December is the 366th day and the longest period changes to 366.

SUMMARY

In this chapter, we first pointed out that Fourier series is a powerful tool for analyzing periodic data. Energy uses in commercial buildings are largely periodic and can, therefore, be modeled by the Fourier series approach. The presence of annual periodicity due to the seasonal effect, and diurnal periodicity due to the fixed operating schedule have been illustrated with monitored data from several buildings. How these periodicities can be accounted for in the model, have been shown. While the choice of hour of the day as the independent variable was obvious for the diurnal cycle, the day of the year was found to be the best choice for the annual cycle.

In the following chapter, the modeling procedure for weather independent energy use in commercial buildings is described and illustrated with examples.

CHAPTER IV
MODELING WEATHER INDEPENDENT HOURLY ENERGY USE

INTRODUCTION

The previous chapter discussed the presence of periodicity in the different energy uses in commercial buildings. Also, the theory and the basis of Fourier series modeling were described. However, diurnal pattern of energy use is different on different days, for example, working weekdays, weekends, holidays and Christmas. Day-typing, therefore, needs to be done to remove this effect before Fourier series model can be developed. The present chapter elaborates the modeling procedure for weather independent energy use. Frequently, a model needs to be developed from data spanning for less than a year. Using a Fourier series to account for annual periodicity in the model is not possible in such cases. Consequently, the variation of daily mean energy use due to the seasonal effect needs to be determined separately. A method to do this is presented.

The modeling procedure is illustrated with a detailed case study. The case study also contains an interpretation of the frequencies of the Fourier series model and of the residual analysis. The results of the models developed for energy uses in some LoanSTAR buildings are summarized. Besides, the results of Fourier series model have been compared with that of individual hourly approach and aggregated hourly approach using an example.

MODELING PROCEDURE

Weather independent energy use pattern is dependent upon the mode of operation. The first step in the modeling procedure is to do day-typing according to energy use to account for the major changes in mode of operation. Once day-typing is done, modeling can be done using Fourier series for each day-type separately. Day typing and Modeling are discussed in the following sub-sections.

Day-typing

The day-typing procedure adopted here is termed as Calendar method. The steps of this day-typing method are shown in the flowchart in Figure 4.1. In many LoanSTAR sites, which are generally institutional buildings, mean energy use is observed to be different on weekdays, weekends, semester breaks, spring break, holidays (Memorial day, Labor day, Thanksgiving etc.) and Christmas. However, slow decrease in energy use at the end of the semesters and again a smooth increase at the beginning of the semesters are well represented by the annual frequency of Fourier series model. This will be illustrated later in this chapter. The hourly data from which the model needs to be developed, is, therefore, divided into weekdays, weekends, holidays and Christmas group. Once this is done, weekdays group is divided further into working days and weekdays during spring break. The next step is to perform Duncan's multiple range test (Ott, 1988) to check if statistically significant difference between the means of the various groups exists. Groups with mean that are statistically indifferent are combined together to form one day type. In this way, the final day-types are identified.

Modeling

The modeling procedure for weather independent energy use in commercial buildings is shown in Figure 4.2. To start with, we rewrite the model equation 3.5 as follows:

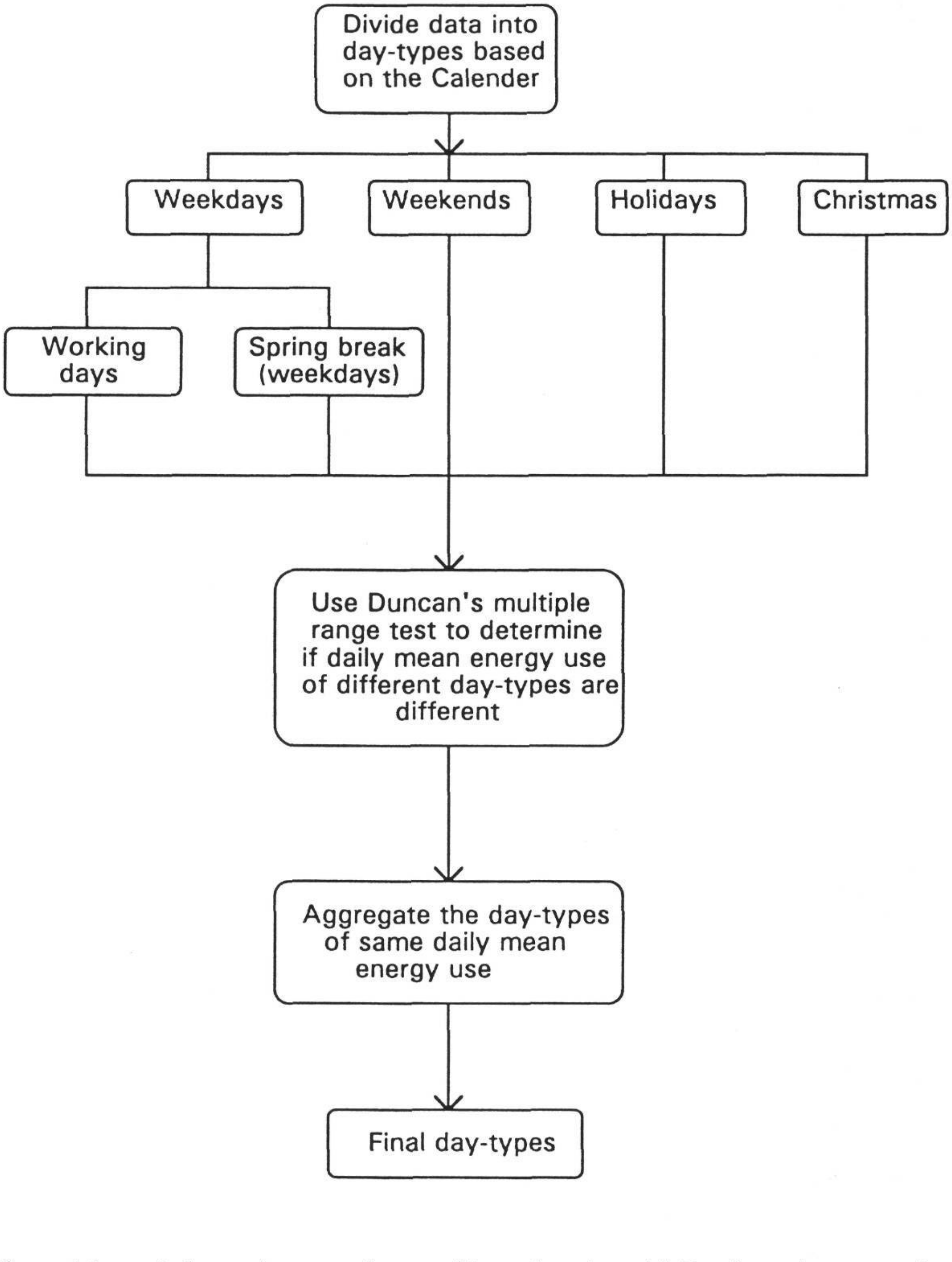


Figure 4.1 A day-typing procedure used in conjunction with Fourier series approach to model hourly energy use in commercial buildings.

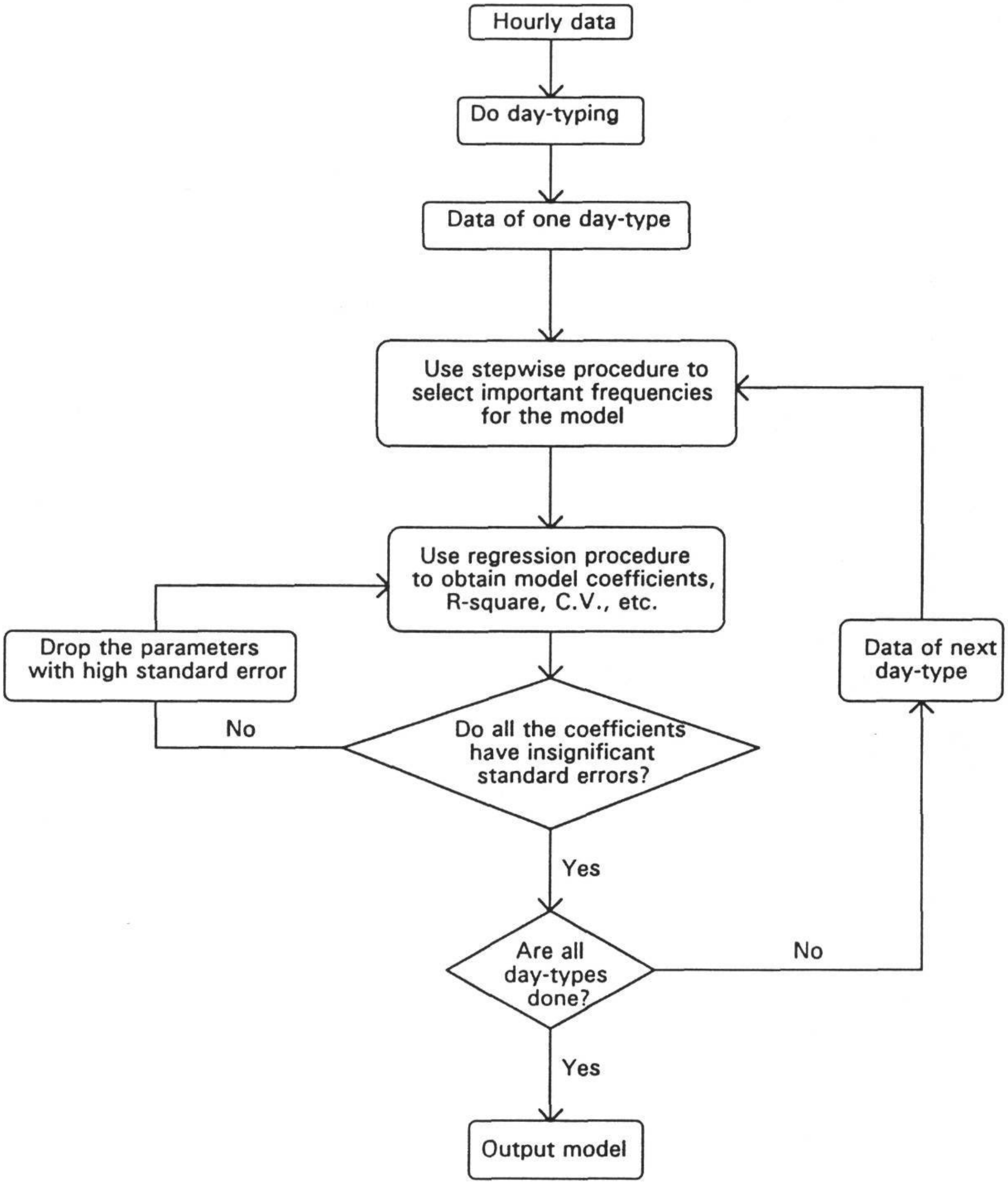


Figure 4.2 The modeling procedure adopted in the present Fourier series approach to model hourly energy use in commercial buildings.

$$E_{d,h} = \beta + \sum_{x=1}^{182}\left[\alpha_x \sin\frac{2\pi x}{365}d + \beta_x \cos\frac{2\pi x}{365}d\right] + \sum_{y=1}^{11}\left[\gamma_y \sin\frac{2\pi y}{24}h + \delta_y \cos\frac{2\pi y}{24}h\right] \quad (4.1)$$

We note from the above equation that the Fourier series for the annual cycle can contain up to 182 frequencies. However, in several cases, only the first five frequencies in the model are adequate to obtain a good model. We have, therefore, limited our model to the first five frequencies only. Higher frequencies can, of course, be used if a particular case demands.

For a particular day-type, hourly data is used and stepwise forward selection in Statistical Analysis System (SAS) program performed to select the significant independent variables from the set of Fourier frequencies. Mallow's C(p) $\approx$ p for minimum p, the number of independent parameters including the intercept, is used as the criteria for such selection. Once this is done, regression is performed using the selected variables. However, the model may contain parameters with significant standard error. In that case, such parameters are dropped from the model. A level of significance (p-value) of 10% is used as the criteria. The procedure needs to be repeated for other day types. Finally, a set of models for all day types is obtained.

It is worth mentioning here that the above procedure, when applied to model energy use, can identify a large number of terms for the final model. Higher frequency terms sometimes may have negligible partial R-square and may improve C.V. of the model marginally. In such cases, the higher frequency terms may be dropped for simplicity.

ACCOUNTING SEASONAL VARIATION FOR SHORT DATA SETS

The model equation 4.1 has Fourier series for both annual cycle and daily cycle. When data is available for one year, using this equation accounts for the seasonal variation reasonably. However, if data is available for a short period, for example four or five months, a model developed using equation 4.1 will predict unrealistically for the rest of the periods of the year. The Fourier series for the annual cycle, therefore, can no more be used. The equation reduces to

$$E_{i,h} = \beta_i + \sum_{y=1}^{11}\left[\gamma_{i,y} \sin\frac{2\pi y}{24}h + \delta_{i,y} \cos\frac{2\pi y}{24}h\right] \quad (4.2)$$

Rigorous day-typing needs to be done to remove seasonal effect from the data in such cases. The subscript i stands for the day-type in the above equation. Moreover, the pre-retrofit data may not contain all the day-types. Developing a pattern of energy use for those day-types becomes necessary. This is done by using the post-retrofit data. To explain how this is done, let us assume that we have data for working weekdays in the pre-retrofit data and that we do not have Christmas data for the pre-retrofit period. Now, the mean energy use during pre-retrofit working weekdays needs to be determined. Also, the ratio of hourly mean energy use during Christmas to that on working weekdays during the post-retrofit period is found. The energy use at each hour on working weekdays in the pre-retrofit period is then multiplied by the factor.

$$F = \frac{\overline{E}_{post,Christmas}}{\overline{E}_{post,\,working\ weekdays}},$$

$$E_{h,\,pre,Christmas} = F \times E_{h,\,pre,\,working\ weekdays} \quad (4.3)$$

The hourly mean energy consumption can then be used to develop the Fourier series model for pre-retrofit Christmas period.

APPLICATION TO MONITORED DATA

The present Fourier series approach has been applied to model monitored data collected from various LoanSTAR sites. One example is presented here to illustrate the modeling procedure and discuss the model results. In addition, the contribution of annual and daily frequencies to the final model are analyzed and, as stated in the introduction section of this chapter, residual analysis has been performed to check the assumptions of linear regression models.

Model Development

The Whole building electricity (WBELEC) consumption in 1992 in a large institutional building (Zachry Engineering Center, Texas A&M University) has been chosen for the case study. Figure 3.1 is replotted in Figure 4.3 to show the time series plot of WBELEC use in Zachry Engineering Center (ZEC) in 1992. The plot shows a slow change in energy use during the beginning and the end of the semesters, low energy use during spring break. Energy use during Thanksgiving and 4th July look similar to those on weekdays. Also, the period ranging from Christmas to first seven days of the year has the lowest average energy consumption. However, the following day-types were obtained :

(1) Working weekdays group,
(2) Weekends, 4th July, Thanksgiving (26th November to 29th November) and weekends during spring break, referred to as Working weekends & Thanksgiving group here after,
(3) Weekdays during spring break (16th March to20th March), referred to as spring break group here-after,
(4) 1st January to 7th January, 23rd December to 31st December, referred to as Christmas group here after.

Fourier series model was developed for each day-type mentioned above. To illustrate the selection procedure of independent variables, a summary of forward selection procedure for working weekday group is presented in Table 4.1. It can be noted that C(p) decreases with the increase in number of parameters and becomes close to number of parameters at step 22. All the parameters have insignificant standard error. However, the number of parameters will be large if 22 variables are selected. R-square increases with increase in number of parameters but partial R-square of higher frequency terms are negligible. One might, therefore, want to drop the higher frequency terms (6 and above) with a little sacrifice of prediction accuracy of the model, for simplicity. The final set of parameters that is retained for the model is as follows:

Annual frequencies : SD1, SD2, SD4, SD5, CD1, CD2, CD3, CD5.
Daily frequencies : SH1, SH2, SH3, SH4, SH5, CH1, CH2, CH3, CH4, CH5.

Regression is performed using the above set of parameters to determine coefficients, R-square, C.V., etc., of the model. A similar procedure is followed to get the model equations of other day-types. The model results are summarized in Table 4.2. The results show very low C.V. in all the day-types. However, R-square is low for working weekends and Thanksgiving group and Christmas group. This can be explained from the expression of R-square as given below :

$$R^2 = 1 - \frac{\sum\left(E - \hat{E}\right)^2}{\sum\left(E - \bar{E}\right)^2} \tag{4.4}$$

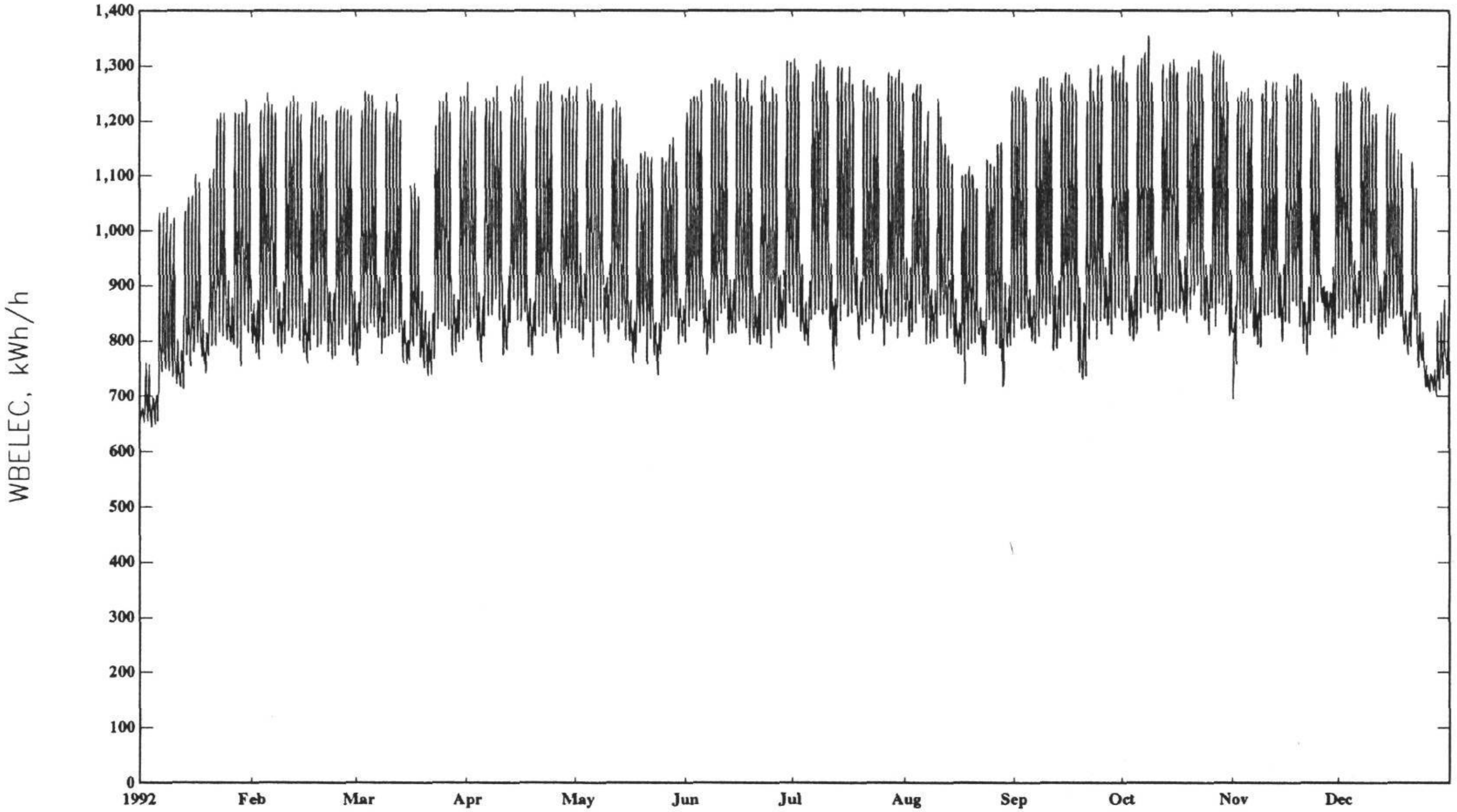


Figure 4.3 Time series plot of Whole Building Electricity (WBELEC) consumption in Zachry Engineering Center at Texas A&M University.

TABLE 4.1

Summary of Forward Selection Procedure for Whole Building Electricity Use During Working Weekdays in Zachry Engineering Center, Texas A&M University. Period : Post-retrofit, one year (1992).

Step	Variable entered	Partial R-square	Model R-square	C(p)	Prob>F
1	CH1	0.5496	0.5496	21661.7	0.0001
2	SH1	0.2347	0.7843	7327.9	0.0001
3	CH2	0.0385	0.8228	4978.2	0.0001
4	SH4	0.0117	0.8346	4263.8	0.0001
5	CD3	0.0093	0.8439	3696.8	0.0001
6	SD1	0.0083	0.8522	3190.5	0.0001
7	SD5	0.0085	0.8607	2675.8	0.0001
8	CD5	0.0073	0.8680	2230.2	0.0001
9	SD2	0.0062	0.8742	1851.0	0.0001
10	SH3	0.0060	0.8802	1486.9	0.0001
11	CD2	0.0057	0.8859	1139.0	0.0001
12	CD1	0.0060	0.8920	771.6	0.0001
13	SH2	0.0035	0.8955	560.8	0.0001
14	CH3	0.0032	0.8987	364.6	0.0001
15	SD4	0.0030	0.9017	185.3	0.0001
16	CH4	0.0016	0.9033	88.0	0.0001
17	SH5	0.0003	0.9036	72.9	0.0001
18	CH10	0.0002	0.9038	60.0	0.0001
19	CH5	0.0002	0.9041	47.4	0.0001
20	SH6	0.0002	0.9043	34.9	0.0001
21	CH9	0.0002	0.9045	25.7	0.0001
22	SH9	0.0001	0.9046	24.3	0.0001
23	SH10	0.0001	0.9046	23.1	0.0001
24	CH7	0.0000	0.9047	22.6	0.0001

We note from the above expression that R-square gives a comparison between the mean model and the actual model. If the actual model is a one-parameter mean model, the predicted energy use $\hat{E}$ will be equal to the mean energy use $\overline{E}$ and, consequently, R-square will be zero. Scatter of hourly energy use over the mean on a day of working weekends, Thanksgiving group or Christmas group is less compared to that on a day of working weekday group. This is why R-square is less for the groups other than working weekdays.

Model Fit to Monitored Data

Time series plots of measured energy use, predicted energy use and residual for the month of April '92 is shown in Figure 4.4. Similar plots for the first and the second half of 1992 are shown in Figure 4.5 and Figure 4.6 respectively. Figure 4.4 shows a close fit of the model to the measured data. It can be noted that drop in energy use during lunch time (12 noon to 1p.m.) and peak energy use at around 3 p.m. have been well captured by the model. Figure 4.5 and Figure 4.6 show that the model fits quite well over the year.

Interpreting Annual Frequencies

The model for working weekdays has been chosen to illustate how Fourier series accounts for the annual variation of energy use in a building. Sine and cosine frequencies have been plotted separately to interpret how the final annual shape is developed. In Figure 4.7, cumulative plots of sine frequencies are shown. The plots show how the first frequency with a period of 366 days gets modified when higher frequencies are added. Similarly, the cumulative plots of cosine frequencies are shown in Figure 4.8. While the final sine plot in Figure 4.7 does not clearly correspond to the annual pattern of energy use, the final cosine plot in Figure 4.8 is quite close to the annual shape that is observed in the timeseries plot in Figure 4.3. The plot (Figure 4.8) shows a large dip between spring and summer semesters, also between summer and fall semesters. Besides, the drop of mean energy use in Christmas is captured by the final cumulative cosine plot.

In Figure 4.9, resultant sine, resultant cosine and the final annual cycle developed by the Fourier series model are shown. It can be noted that while the cosine waves give the overall shape of the cycle, the sine waves modify it to the final shape. This is because of considering zero phase lag in the model. With ninety degree phase lag, resultant sine wave would have given the overall shape of the cycle, instead of resultant cosine wave.

Interpreting Daily Frequencies

As in the case of annual frequency interpretation, the sine and the cosine frequencies have been plotted separately to illustrate how the final diurnal shape is obtained. The model of the working day group has been used to interpret the diurnal frequencies. Figure 4.10 shows how the shape of the sine wave changes when higher frequency waves are added to the first frequency. It can be noted that the fifth frequency has changed the shape very little. This justifies dropping the higher frequency terms from the model. However, the final sine wave does not resemble the diurnal load pattern.

Figure 4.11 shows the plots of cumulative cosine frequencies. When the higher frequencies are added, a bell shape develops slowly. This is close to the diurnal load pattern. Also, contribution of fifth frequency is negligible, justifying dropping the higher frequency terms from the model, as in the case of sine wave.

Figure 4.12 shows the plots of final sine wave, final cosine wave and the diurnal load shape represented by the Fourier frequencies. As mentioned in the earlier paragraph, the cosine wave is bell shaped. How the sine wave modifies the cosine wave to mimic the final load shape, can be noted in Figure 4.12. The drop in energy use during lunch time (12 noon to 1 p.m.), Peak

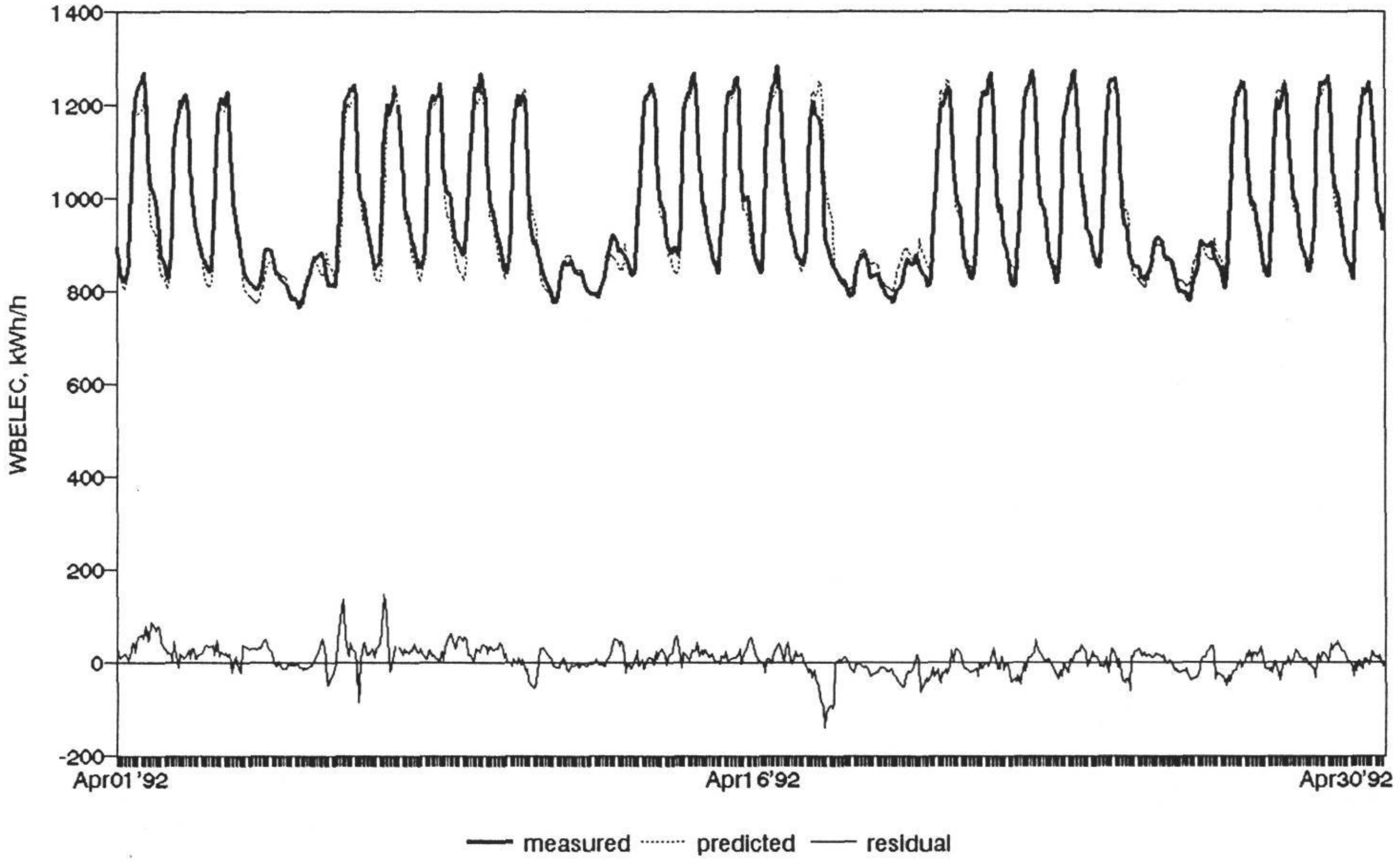


Figure 4.4 Time series plots of measured WBELEC, predicted WBELEC and residual in April, 1992. The figure shows a close fit of the Fourier series model to the measured energy use. The site is Zachry Engineering Center at Texas A&M University. The model has been developed from 1992 data.

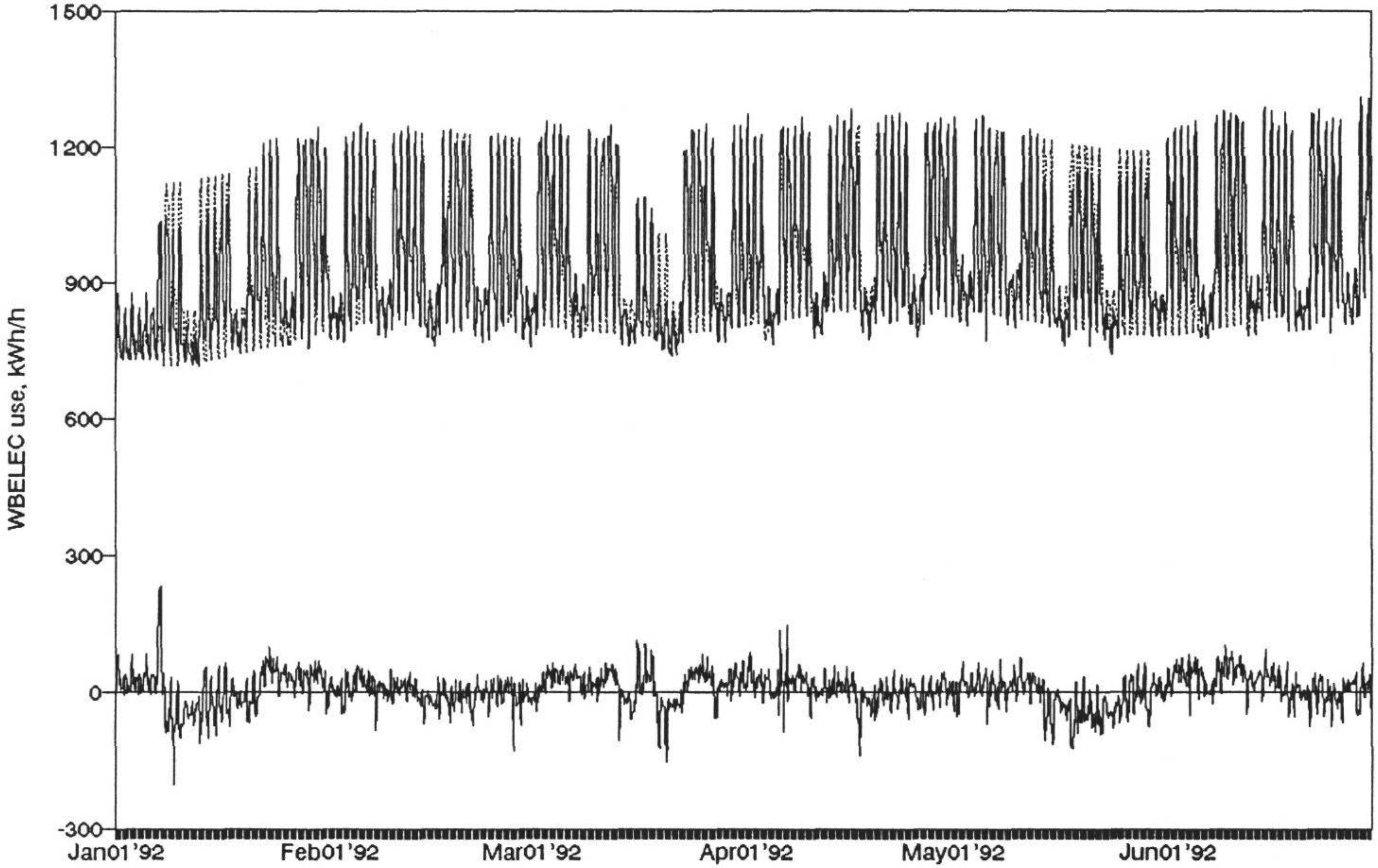


Figure 4.5 Time series plots of measured WBELEC, predicted WBELEC and residual from January '92 to June '92. The site is Zachry Engineering Center at Texas A&M University. The model has been developed from 1992 data.

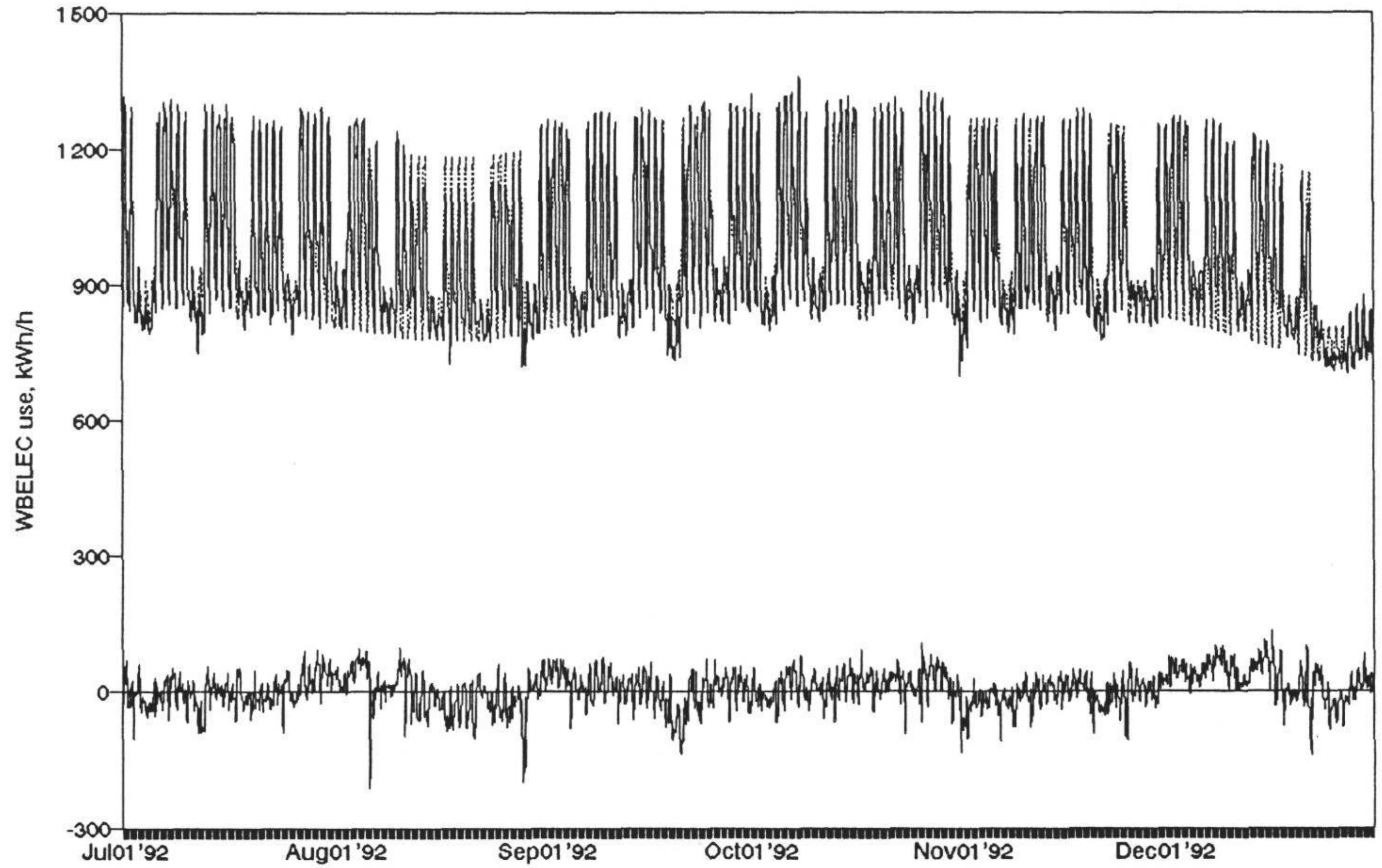


Figure 4.6 Time series plots of measured WBELEC, predicted WBELEC and residual from July '92 to December '92. The site is Zachry Engineering Center at Texas A&M University. The model has been developed from 1992 data.

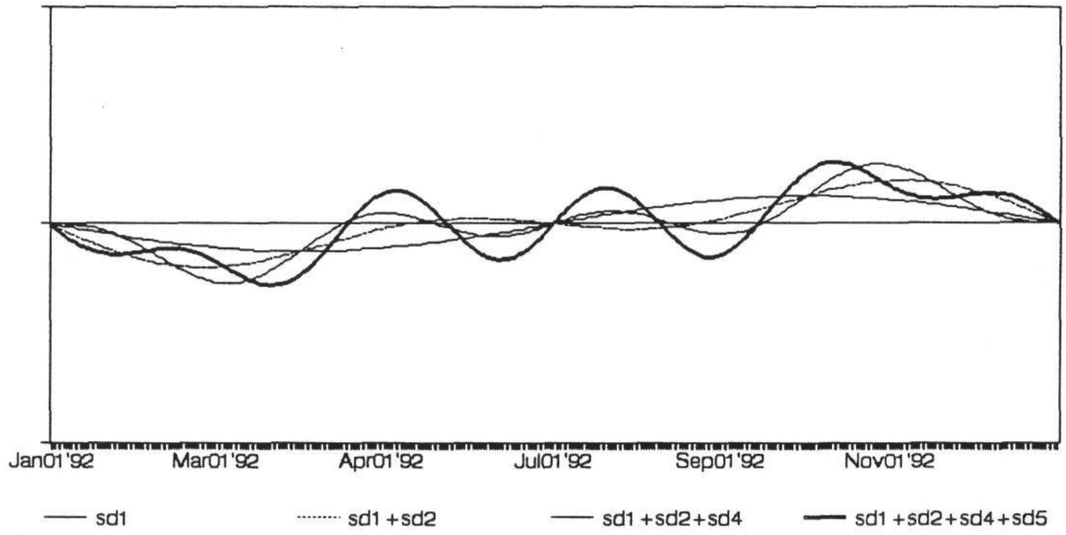


Figure 4.7 Cumulative plots of sine frequencies of the annual cycle of WBELEC model. Site: Zachry Engineering Center. Period: 1992.

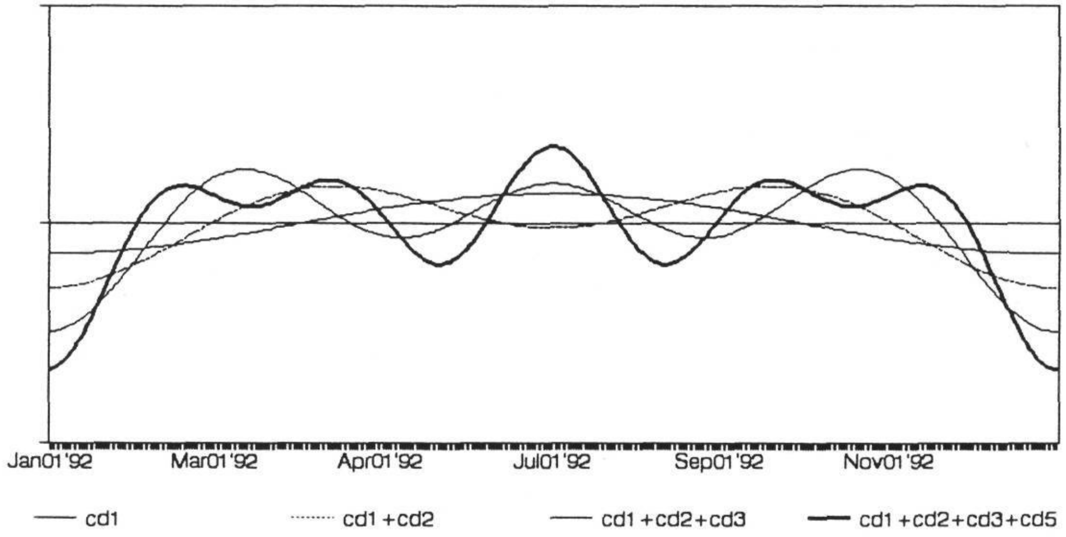


Figure 4.8 Cumulative plots of cosine frequencies of the annual cycle of WBELEC model. Site: Zachry Engineering Center. Period: 1992.

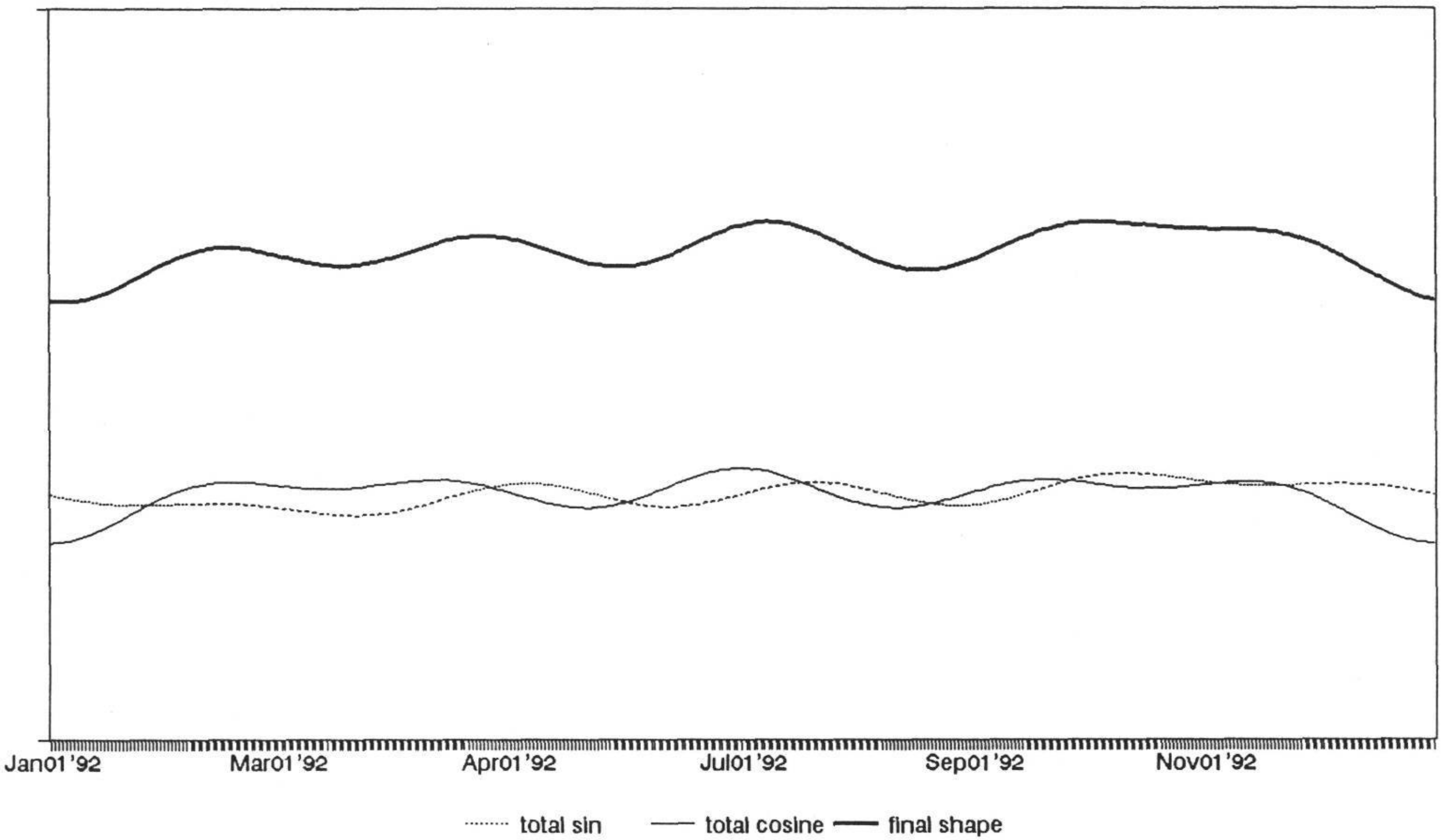


Figure 4.9 The plot shows how the sum of sine frequencies and the sum of cosine frequencies of the Fourier series model, when added, reproduces annual pattern of daily mean WBELEC consumption. The site is Zachry Engineering Center.

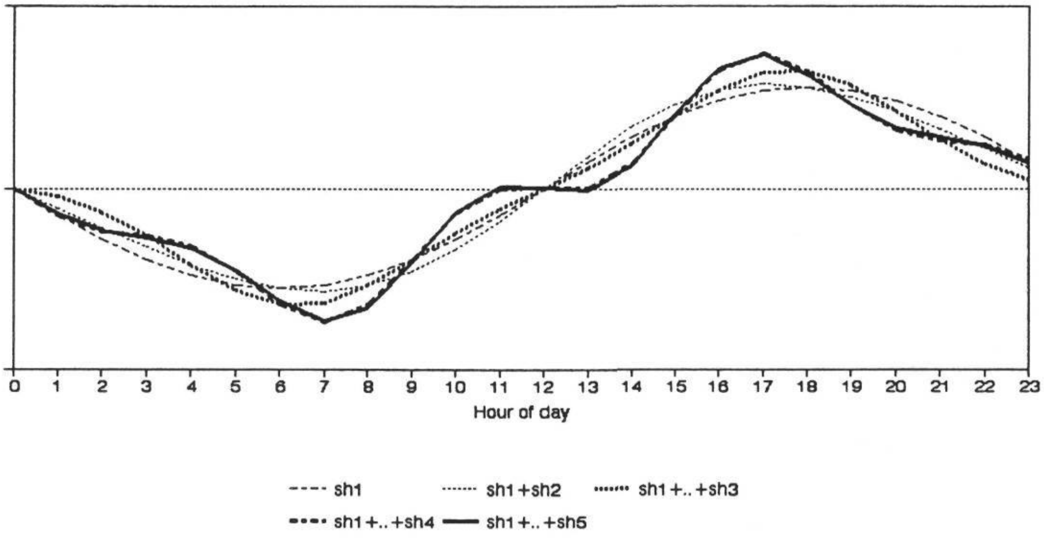


Figure 4.10 Cumulative plots of sine frequencies of the daily cycle of WBELEC model. Site: Zachry Engineering Center. Period: 1992.

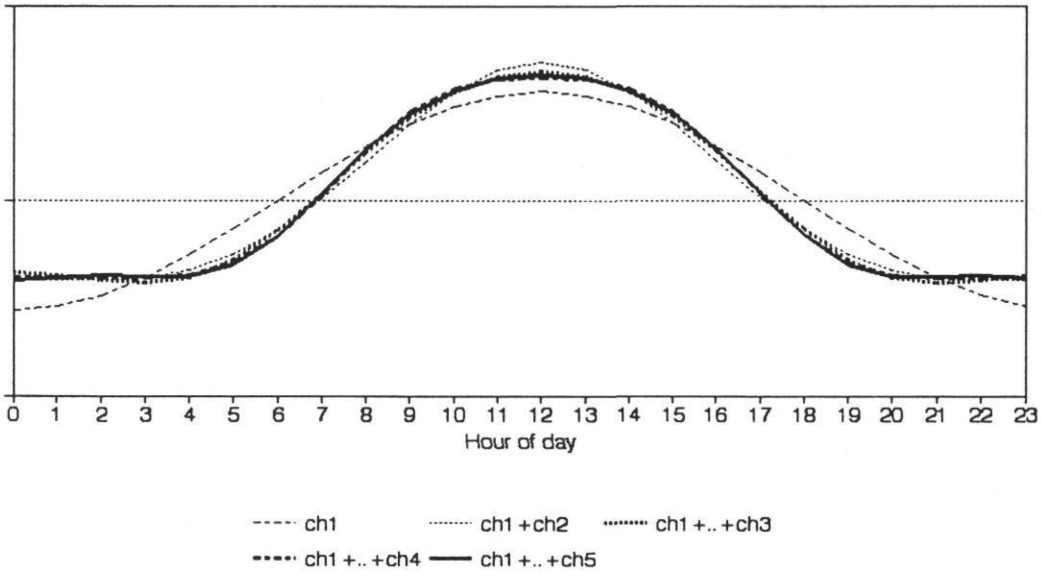


Figure 4.11 Cumulative plots of cosine frequencies of the daily cycle of WBELEC model. Site: Zachry Engineering Center. Period: 1992.

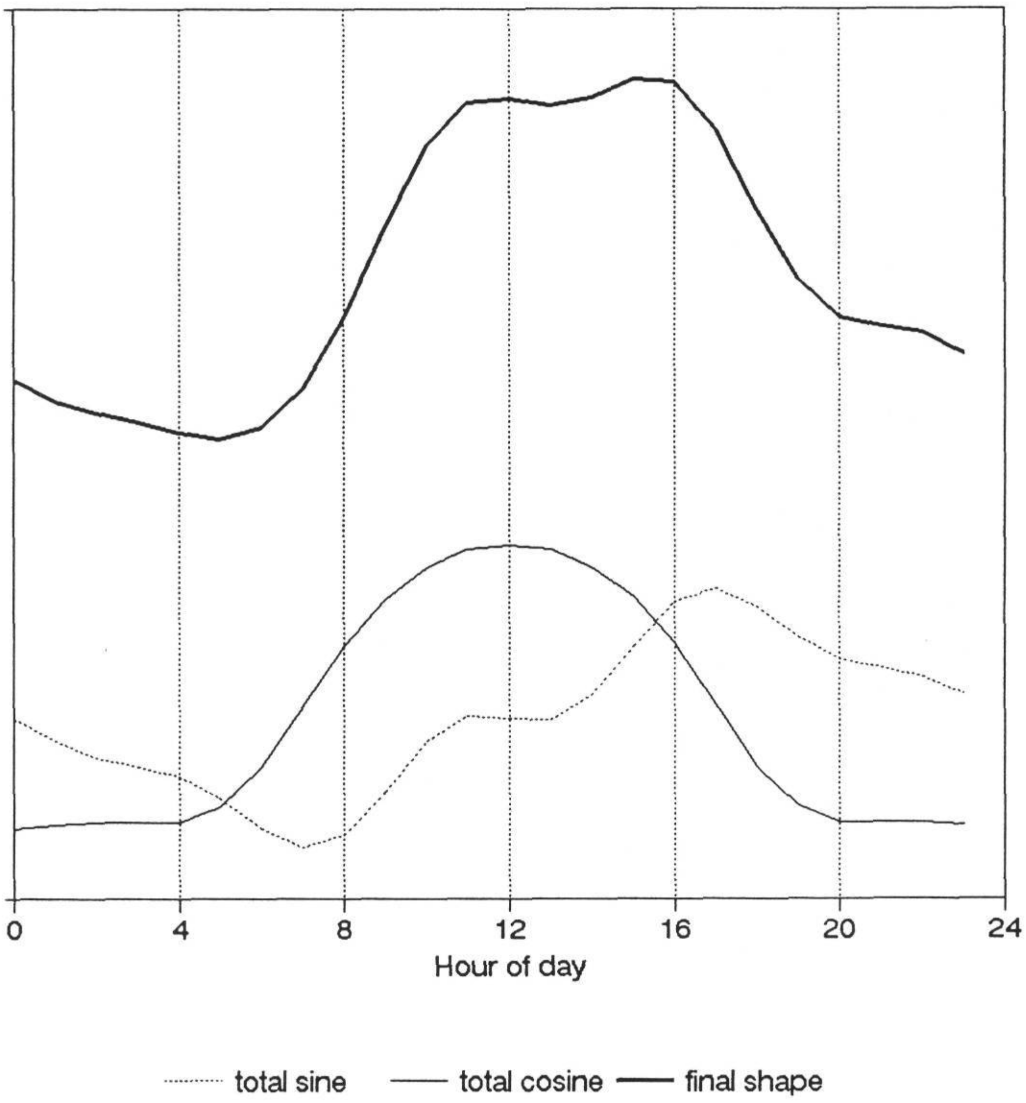


Figure 4.12 The figure shows how the diurnal load profile develops from the cumulative sine and cosine frequencies.

consumption between 3 p.m. 4 p.m, slower decrease in energy use after 8 p.m., etc., are well discernable by the Fourier series model.

RESIDUAL ANALYSIS

Three assumptions of linear regression modeling are (1) errors are normally distrbuted, (2) errors have a constant variance and (3) errors are independent. To check for normality, a histogram and a normal probability plot have been made. These are shown in Figure 4.13 and Figure 4.14 respectively. Both these plots show that the distribution of residual is approximately normal. The constant variance of residual was checked by plotting the residual against predicted energy use (Figure 4.15). The scatter plot in Figure 4.15 shows an approximately constant variance throughout the range of prediction, except for a outliers. These two assumptions, therefore, are validated.

However, presence of autocorrelation is noted in the residual (Figure 4.16) which shows a scatter plot of error against lagged error. This is because the model has been developed from time series data of energy use, which is inherently autocorrelated. Any purely deterministic model for time series data will have autocorrelation. Autocorrelation is found to reduce if higher frequencies are included in the model. However, in most of the cases Fourier model has low C.V.. The model may, therefore, be acceptable for practical purposes that does not demand high precision.

SUMMARY OF MODEL RESULTS FOR TWO LOANSTAR SITES

The Fourier series approach presented in this chapter was applied to many LoanSTAR sites to check its prediction accuracy. Models were developed for energy uses during both pre-retrofit and post-retrofit periods. Results obtained for two sites, namely Zachry Engineering Center at Texas A&M University and Record building at Texas Department of Health at Austin, are summarized in Table 4.2. As can be seen in this table, all the models have very low C.V. (less than 8%) and high R-square for almost all the day-types, except for the model during Christmas period in Record building. For the latter, the reason is that energy use during Christmas is less repititive or periodic, rather random due to the random changes in occupancy.

COMPARISON WITH OTHER MLR MODELS

In this section a particular case study has been presented to compare the prediction accuracy of the Fourier series modeling approach with two existing regression techniques: (a) individual hourly model and (b) aggregated hourly model. Let us briefly review these first.

Individual Hourly Method

A detailed day-typing of the hourly data for which the model needs to be developed, is done first to remove all seasonal effect from the data. Once day-typing is done, a mean model for each hour of each day-type is developed. If i number of day-types are present, then 24×i model equations are obtained. The general mathematical form of the model will be as follows:

$$E_{i,h} = \mu_{i,h} + \varepsilon_{i,h} \tag{4.5}$$

where subscript i stands for the day-type and h stands for the hour of the day.

Aggregated Hourly Method

This method is an extension of individual hourly model. The idea is to arrive at fewer coefficients and, at the same time, enable the model to provide insight into the load pattern. Generally, during working weekdays, energy use during the morning hours till about 8 a.m. are

```
                       Histogram                         #
 225+*                                                   5
    .*                                                   2
    .*                                                  21
  75+*********                                         725
    .************************************************ 4178
    .**********************************               2971
 -75+*******                                           615
    .*                                                  69
    .*                                                  14
-225+*                                                   5
     ----+----+----+----+----+----+----+----+----+---
     * may represent up to 88 counts
```

Figure 4.13 A histogram of residuals of Fourier series model. Site : Zachry Engineering Center, Texas A&M University. Period : 1992.

```
                   Normal Probability Plot
 225+                                                  *
    |                                                  *
    |                                                  *
  75+                                      +************
    |                      *****************
    |           *************+
 -75+++***********
    |***
    |*
-225+*
     +----+----+----+----+----+----+----+----+----+----+
         -2        -1         0        +1        +2
```

Figure 4.14 A normal probability plot of Fourier series model. Site : Zachry Engineering Center, Texas A&M University. Period : 1992.

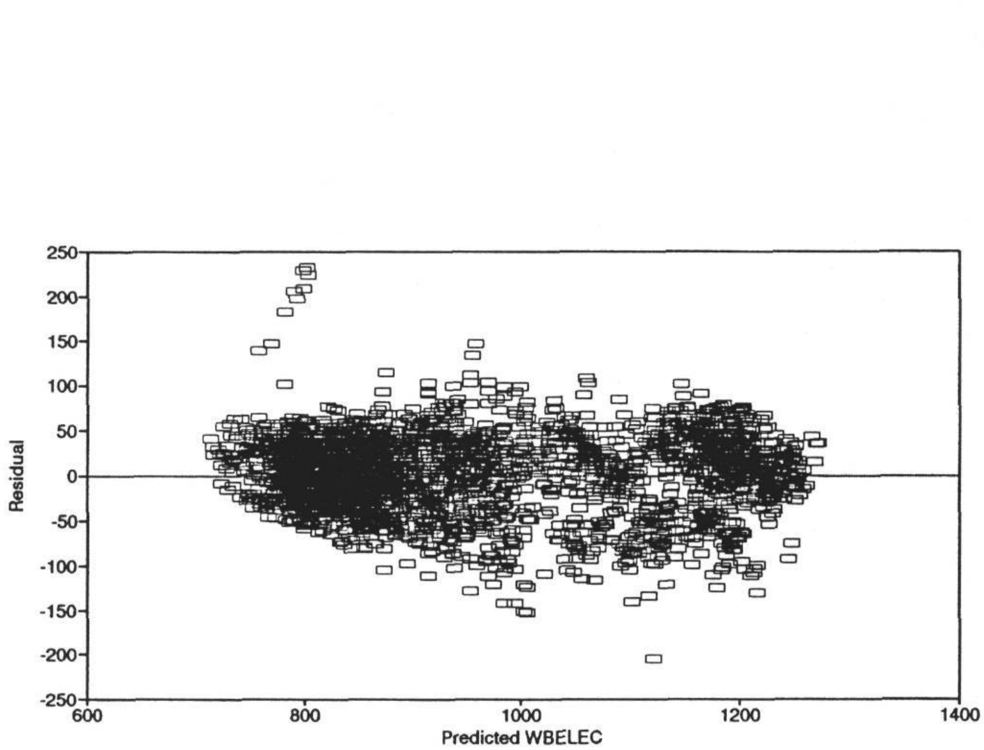


Figure 4.15 Residual vs. Predicted WBELEC plot of Fourier series model. Site: Zachry Engineering Center, Texas A&M University. Period : 1992.

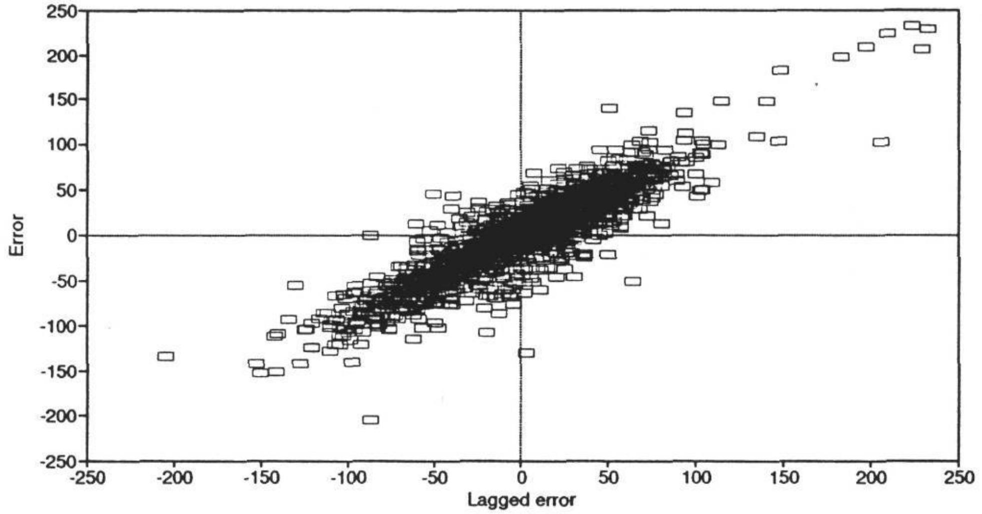


Figure 4.16 Error vs. lagged error plot of Fourier series model. Site: Zachry Engineering Center, Texas A&M University. Period : 1992.

TABLE 4.2

Fourier Model Results of Weather Independent Energy Use at Two LoanSTAR Sites

Building name	Location	Type of energy use	Pre-retrofit / Post-retrofit period	Duration	Day-type	Hourly mean energy use (kWh/h)	RMSE (kWh/h)	R-square	C. V. (RMSE), (%)
Zachry Engineering Center	Texas A&M University, College Station	LE	Pre-retrofit	Four months	Working weekdays	738	30.1	0.96	4.1
					Working weekends & Thanksgiving	568	29.0	0.64	5.1
					Christmas	398	33.4	0.87	8.4
		WBELEC	Post-retrofit	One year	Working weekdays	1010	49.9	0.90	4.9
					Working weekends & Thanksgiving	841	28.2	0.63	3.4
					Spring break	870	63.8	0.61	7.3
					Christmas	762	36.0	0.33	4.7
Record Building	Texas Dept. of Health, Austin	WBELEC	Pre-retrofit	One year	Working weekdays	65	9.2	0.93	14.3
					Working weekends & Thanksgiving	35	1.4	0.30	4.1
					Spring break	64	4.6	0.98	7.1
					Christmas	51	23.5	0.35	46.4
		WBELEC	Post-retrofit	Five months	Working weekdays	59	8.4	0.96	14.2
					Working weekends & Thanksgiving	23	1.5	0.12	6.5
					Christmas	54	22	0.70	40.3

approximately constant. At around 8 a.m. it may start increasing. Between 11 a.m. and 5 p.m., energy use may again remain approximately constant. The energy use may start decreasing after 5 p.m.. This is closely related to the occupancy pattern of the building. One can, therefore, aggregate the morning hours, from 11 a.m. to 5 p.m., etc., to get fewer model equations for each day-type, as compared to the individual hourly method. The general model equation can be represented as follows:

$$E_{i,j} = \beta_{i,j,0} + \beta_{i,j,1}\left(h - h_{i,j,s}\right) + \varepsilon_{i,j} \quad (4.6)$$

where subscripts i stands for the day-type, j for the aggregated hour group and s for the starting hour of the group. The slope $\beta_{i,j,1}$ is zero for the groups having approximately constant energy use. Duncan's multiple range test (Ott, 1988) is used to test the mean energy use at each hour of a day-type. Based on the result of the test, the hours with same trend of energy use are grouped together.

Modeling and Comparison

When the data has strong seasonal influence, both individual hourly method and aggregated hourly methods need detailed day-typing. However, if a short period, say a month, is chosen, the seasonal influence may be overlooked. Lighting energy use during October '89 in Zachry Engineering Center has been chosen and shown in Figure 4.17 as a time series plot.

Models were developed for weekdays and weekends using all three methods. The results of individual hourly model are summarized in Table 4.3. It has 24 coefficients each for weekdays and weekends. As can be seen from the table, R-square of the model are high and the C.V. are very low.

The results of aggregated hourly method are summarized in Table 4.4. There are three hour groups for weekdays and two groups for weekends. The R-square of mean models are zero for some hour groups, because of zero degree of freedom for those groups. However, C.V. of both weekday model and weekend are low.

Table 4.5 contains the results of comparing all three methods. The following points are noted while comparing these three methods:

(i) Fourier series method has the lowest C.V. and the aggregated hourly method the highest.

(ii) R-square for both weekdays and weekends are same for Fourier series and individual hourly approaches. This and other examples from LoanSTAR sites showed that Fourier series approach gives very high prediction accuracy, comparable to that of individual hourly approach.

(iii) Fourier series approach gives a single equation model with fewer coefficients, compared to the other methods.

(iv) When model is developed for one year period, Fourier series model does not require as rigorous day-typing as needed for individual or aggregated hourly approaches.

(v) Because Fourier series approach gives a single equation model for each day-type, it is able to characterize the load profile in a better way.

SUMMARY

In this chapter, we have discussed the Fourier series modeling procedure in detail. A case study has been presented to illustrate the use of the methodology. The case study contains interpretation of annual and diurnal frequencies of the Fourier series model which has given us an insight on how to relate different frequency terms of the model to the annual and the diurnal load pattern. Residual analysis shows that the errors are approximately normally distributed and have constant variance. However, the residuals exhibit autocorrelation which could be reduced

TABLE 4.3

Summary of Model Results Using Individual Hourly Method
Site : Zachry Engineering Center, Texas A&M University.
Period : October '89.

Hour of day	Weekdays			Weekends		
	Mean (kWh/h)	R-square	C. V. (%)	Mean (kWh/h)	R-square	C. V. (%)
0	623.1	0.96	3.9	576.5	0.61	4.2
1	594.6			563.7		
2	576.7			555.7		
3	567.1			550.9		
4	562.7			545.8		
5	560.3			542.4		
6	563.9			539.5		
7	597.5			538.7		
8	682.9			532.4		
9	828.9			532.7		
10	896.4			552.5		
11	925.4			571.3		
12	932.3			582.5		
13	921.0			592.5		
14	935.2			602.9		
15	948.7			609.0		
16	942.4			614.9		
17	906.7			616.9		
18	787.0			605.2		
19	720.6			597.3		
20	705.4			600.1		
21	698.9			604.7		
22	677.3			602.9		
23	649.2			694.2		

TABLE 4.4

Summary of Model Results Using Aggregated Hourly Method
Site: Zachry Engineering Center, Texas A&M University, Period : October '89

Day-type	Hour groups	Model (kWh/h)	R-square	C.V. (%)	Overall C.V. (%)
Weekdays	0 to 6 hours	578.3	0.00	6.0	4.7
	7 to 10 hours	595.0 + 104.3 × (h - 7)	0.92	4.5	
	11 to 17 hours	930.3	0.00	2.6	
	18 to 23 hours	765.4 - 23.6 × (h - 18)	0.53	5.4	
Weekends	0 to 12 hours	552.6	0.00	5.0	4.5
	13 to 23 hours	603.7	0.00	4.0	

TABLE 4.5

Comparison Between R-square and C. V. Values of Fourier Series, Individual Hourly and Aggregated Hourly Approaches

Method used	Weekdays		Weekends	
	R-square	C. V. (%)	R-square	C. V. (%)
Fourier Series Approach	0.96	3.9	0.61	4.0
Individual Hourly Approach	0.96	3.9	0.61	4.2
Aggregated Hourly Approach	----	4.7	----	4.5

by including higher frequency terms in the model. An example has been taken to compare the prediction accuracy of Fourier series approach with those of two other approaches : the individual hourly approach and the aggregated hourly approach. The results show that Fourier series approach gives very high prediction accuracy, comparable to that of individual hourly approach. However, the prediction accuracy of aggregated hourly approach is a little less compared to the other two methods.

In the next chapter, the methodology to model weather dependent energy use such as cooling energy use and heating energy use in commercial buildings has been discussed.

CHAPTER V
MODELING WEATHER DEPENDENT ENERGY USE

INTRODUCTION

Cooling energy use and heating energy use in a commercial building are periodic, also weather driven. A suitable model, therefore, needs to be developed that will incorporate the effect of periodicity and the weather variables. In this chapter, the Fourier series model for weather independent energy use has been modified by including the weather variables such as ambient temperature, outdoor specific humidity and horizontal solar radiation, to suit to the requirement of modeling weather dependent energy use. The modeling procedure, however, remains almost the same as in the case of modeling weather independent energy use. A case study has been presented which includes model development, summary of results and residual analysis. Also, a comparison of prediction accuracy is presented between Fourier series approach and individual hourly modeling approach.

MODEL EQUATION

The model equations for cooling energy use and heating energy use have been developed by combining the weather variables with Fourier series. While cooling energy consumption depends upon outdoor temperature, outdoor humidity ratio and horizontal solar flux, heating energy use does not depend upon the outdoor humidity ratio. We start with the development of model equation for cooling energy use and finally drop the specific humidity term from the equation for heating energy use.

Building load has two components: sensible and latent. The sensible heat gains are mainly due to the internal sensible heat load, the transmission and radiation gains through walls, roof and windows. Studies on modeling energy use in almost all LoanSTAR sites show that outdoor temperature and horizontal solar flux are the major drivers of sensible heat load. In a statistical model, the effect of internal load variation on cooling or heating energy consumption is also taken care to a large extent by these variables due to the presence of collinearity.

However, the latent heat gain in a building is due to the internal latent heat gain and the moisture content of the fresh air intake. When the mixture of fresh air and return air passes over the cooling coil, sensible heat of the air stream is removed at all temperature ranges but latent heat removal through condensation of moisture in the air takes place only when the dew point of mixed air is above the cooling coil surface temperature. The specific humidity of the mixed air depends upon the specific humidity of both fresh air and return air. While an exact accounting of this effect is difficult because we do not know the return air humidity ratio, various case studies have shown that the difference between the outdoor specific humidity and cooling coil surface temperature improves the model fit to the data considerably. The cooling coil surface temperature is not constant, it varies in the range of approximately 50 degree F to 60 degree F. The saturated specific humidity at the average of 50 degree F and 60 degree F, i.e., 55 degree F, is 0.0092 lb per lb of dry air. The variable chosen to account for the latent load is, therefore, as follows:

$$W^+ = (W - 0.0092)^+,$$

where W is the humidity ratio of outdoor air. When W is less than 0.0092, W^+ is considered to be zero. However, the sensible load of the building comes from internal load, outdoor temperature and horizontal solar flux. A general linear equation of cooling energy use at each individual hour of a day-type is:

$$E_h = a_h + b_h T_h + c_h W^+{}_h + d_h I_h \tag{5.1}$$

where E is the energy use, T is the outdoor temperature, W^+ is the humidity difference as explained earlier and I is the horizontal solar flux. Subscript h stands for a particular hour of day and is 0 to 23 for 24 hours of the day, starting from midnight. In the above equation, the coefficient c_h is set zero while modeling heating energy use.

Model for each individual hour of the day in a day-type was developed and the variations of the model parameters were studied for cooling energy use in various buildings. One such example is the chilled water energy use in Zachry Engineering Center at Texas A&M University during the period from September 01, '89 to December 22, '89. The cooling energy use on weekdays during this period was modeled for each hour and the normalized parameters were plotted against the hour of the day, as shown in Figure 5.1. Similar plots were made for heating energy use during the same period in Zachry Engineering Center (Figure 5.2). R-square and C.V. variations for cooling energy use and heating energy use models are shown in Figures 5.3a and 5.3b. The plots for whole building electricity use in Ward Memorial Hospital at Monahan, Texas during the period from February 01, '92 to March 31, '92 are shown in Figure 5.4. The whole building electricity consumption in Ward Memorial Hospital includes the chiller electricity use and, therefore, is weather dependent. However, all these plots show that coefficients a_h, b_h, c_h and d_h in equation 5.1 vary with the hour of the day. The pattern of such variation is different for different sites and energy uses. The variation of each of these coefficients with the hour of the day can be represented by a Fourier series. The model equation, therefore, takes the following form:

$$E_h = \beta + \sum_{n=1}^{11}\left[\alpha_n \sin\frac{2\pi n}{24}h + \beta_n \cos\frac{2\pi n}{24}h\right] + T_h \sum_{n=1}^{11}\left[\gamma_n \sin\frac{2\pi n}{24}h + \delta_n \cos\frac{2\pi n}{24}h\right]$$

$$+W_h^{+} \sum_{n=1}^{11}\left[\phi_n \sin\frac{2\pi n}{24}h + \psi_n \cos\frac{2\pi n}{24}h\right] + I_h \sum_{n=1}^{11}\left[\eta_n \sin\frac{2\pi n}{24}h + \zeta_n \cos\frac{2\pi n}{24}h\right] \qquad (5.2)$$

Equation 5.2 is the general Fourier series model equation for weather dependent energy use. However, the specific humidity terms do not appear in heating energy use models. We use the following symbols and rewrite equation 5.2:

$$SHn = \sin\frac{2\pi n}{24}h \qquad and \qquad CHn = \cos\frac{2\pi n}{24}h$$

$$T1SHn = T_h \sin\frac{2\pi n}{24}h \qquad and \qquad T1CHn = T_h \cos\frac{2\pi n}{24}h$$

$$W1SHn = W_h^{+} \sin\frac{2\pi n}{24}h \qquad and \qquad W1CHn = W_h^{+} \cos\frac{2\pi n}{24}h$$

$$I1SHn = I_h \sin\frac{2\pi n}{24}h \qquad and \qquad I1CHn = I_h \cos\frac{2\pi n}{24}h$$

$$E_h = \beta + \sum_{n=1}^{11}\left[\alpha_n SHn + \beta_n CHn\right] + \sum_{n=1}^{11}\left[\gamma_n T1SHn + \delta_n T1CHn\right]$$

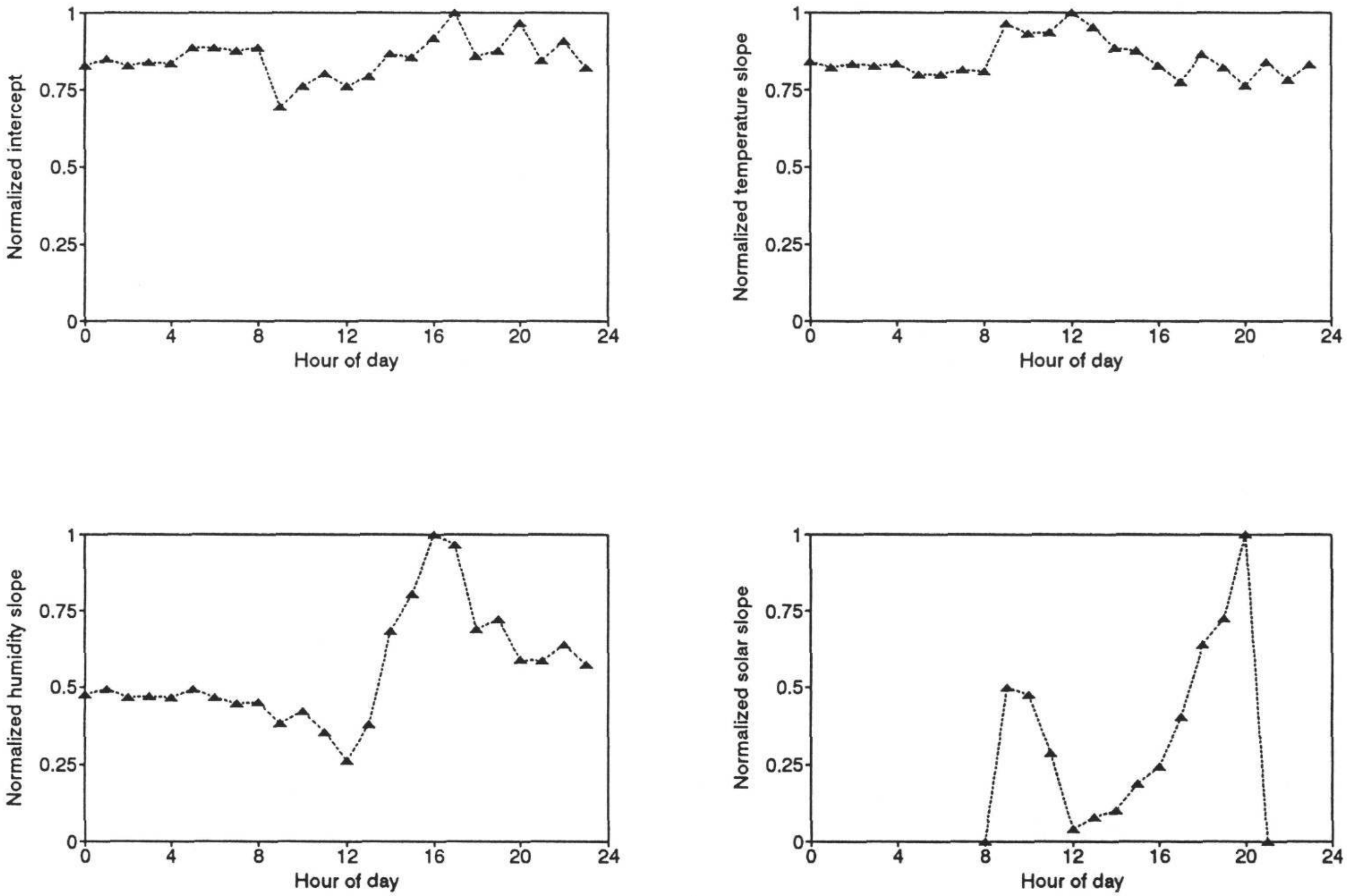


Figure 5.1 Individual hourly models for cooling energy use in Zachry Engineering Center was developed using temperature, humidity difference and horizontal solar flux as the variables. The above plots show the variations of normalized parameters with hour of day. The period is from 1st September '89 to 22nd December '89.

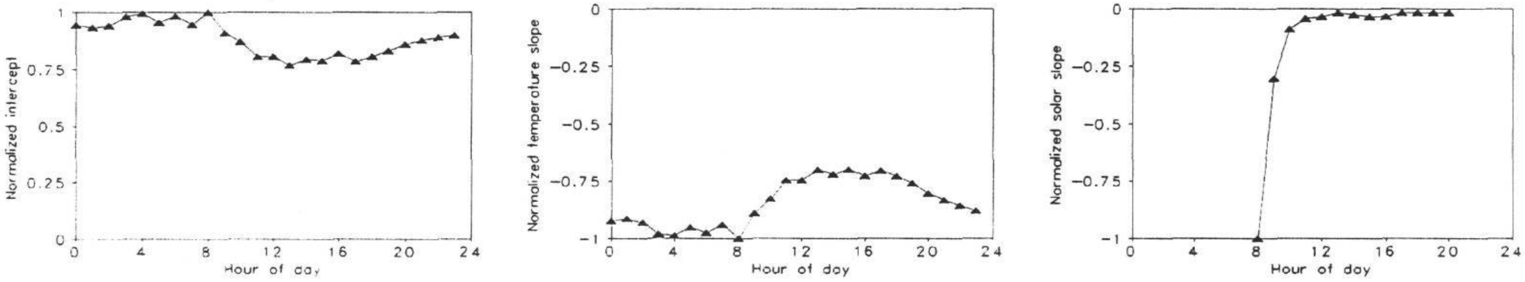


Figure 5.2 Individual hourly models for heating energy use in ZEC was developed using temperature and horizontal flux as the variables. The above plots show the variation of normalized parameters with hour of day. The period is from 1st September '89 to 22nd December '89.

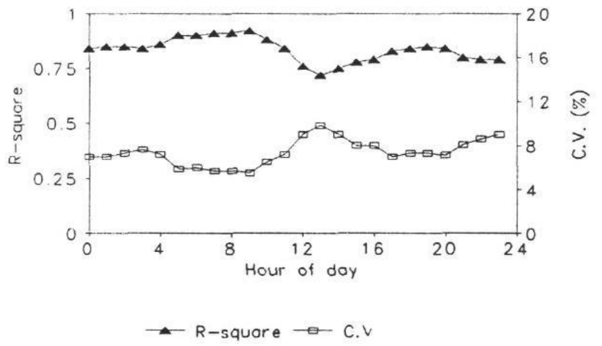


Figure 5.3a R-square and C.V. plot of individual hourly models of cooling energy use in ZEC.

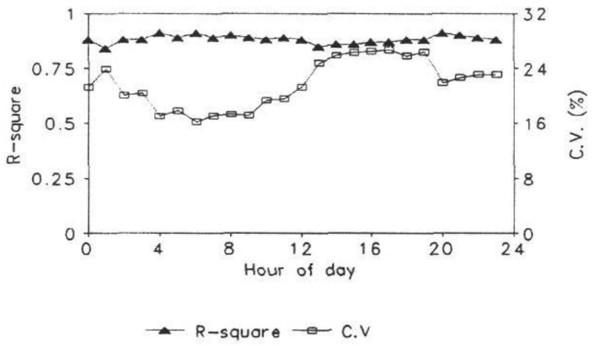


Figure 5.3b R-square and C.V. plot of individual hourly models of heating energy use in ZEC.

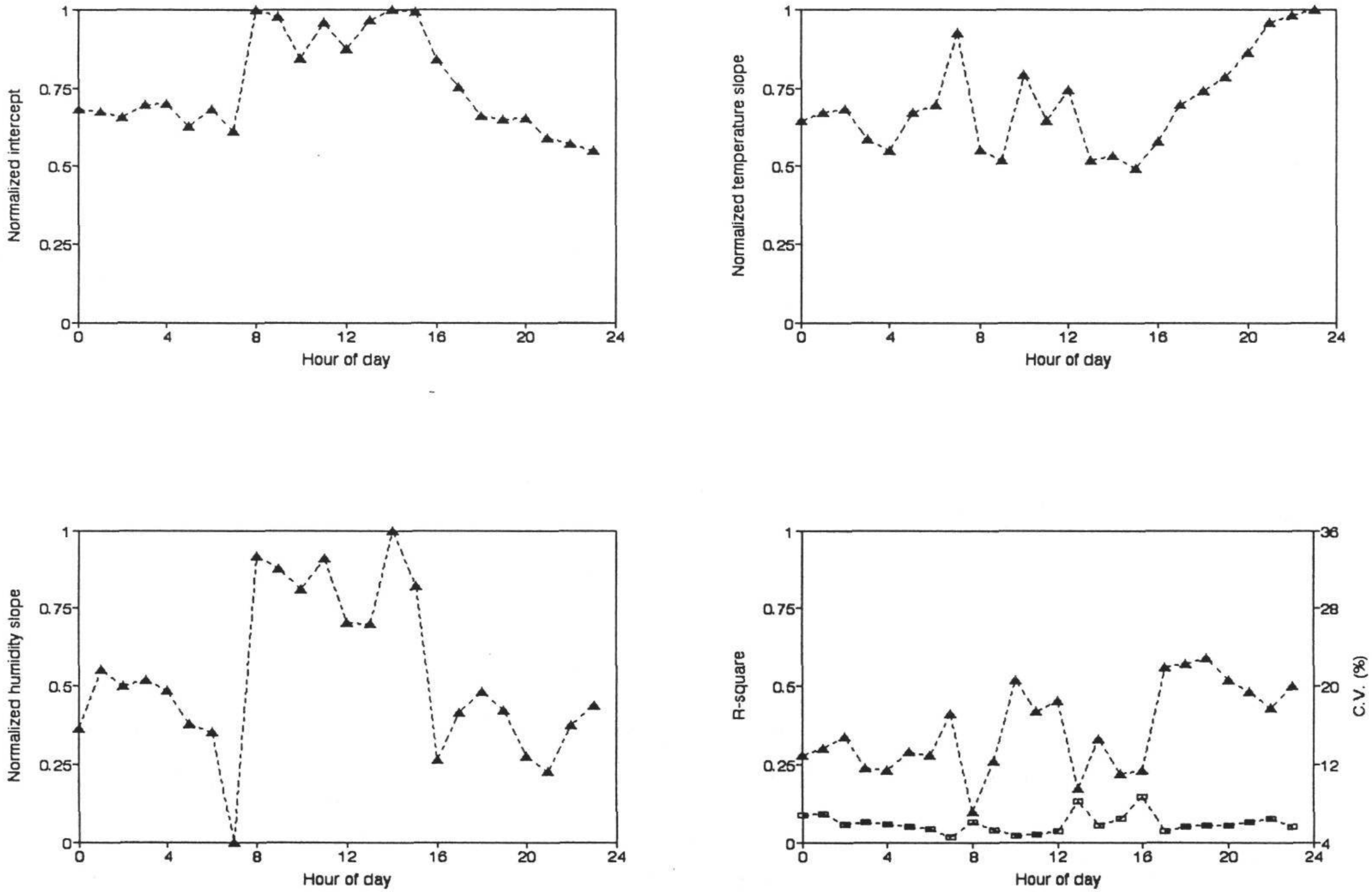


Figure 5.4 Individual hourly models for whole building electricity use in Ward Memorial Hospital, Monahan, was developed using temperature, humidity difference and horizontal flux as the variables. The above plots show the variations of normalized parameters, R^2 and C.V. with hour of day. The period is from 1st February '92 to 31st March '92.

$$+\sum_{n=1}^{11}\left[\phi_n W1SHn + \psi_n W1CHn\right] + \sum_{n=1}^{11}\left[\eta_n I1SHn + \zeta_n I1CHn\right] \tag{5.3}$$

The above equation is used to model chilled water energy use or chiller electricity use. The equation for modeling heating energy use is as follows:

$$E_h = \beta + \sum_{n=1}^{11}\left[\alpha_n SHn + \beta_n CHn\right] + \sum_{n=1}^{11}\left[\gamma_n T1SHn + \delta_n T1CHn\right]$$

$$+\sum_{n=1}^{11}\left[\eta_n I1SHn + \zeta_n I1CHn\right] \tag{5.4}$$

It can be noted that we have not considered any Fourier series to account for the annual variation in equations 5.3 and 5.4. However, the model has the weather variables, the annual variations of which are the main causes behind the seasonal effect present in weather dependent energy uses. The aforesaid equations, therefore are able to capture the seasonal effect even in the absence of annual frequencies.

We now discuss the modeling procedure by using the above mentioned model equations in the following section.

MODELING PROCEDURE

The modeling procedure is same as in the case of weather independent energy use, described in the previous chapter. Day-typing is done first to remove the effect of major changes in operating schedule during weekends, holidays and Christmas. The weather independent energy use in the same building is used to do the day-typing. Once day-typing is done, models are developed for each of the day-types. However, the set from which the independent variables are chosen for the model, contain more number of parameters, as obvious from equations 5.3 and 5.4.

In the following section a case study is presented to illustrate the idea of Fourier series approach and discuss the results obtained.

APPLICATION TO MONITORED DATA

Application of the present Fourier series approach to weather dependent energy use is illustrated here with the example of cooling energy consumption in Zachry Engineering Center during the year 1992. The time series plot of cooling energy use is shown in Figure 3.3. From the plot, the outliers were first identified and removed. Doing day-typing is the next step. In this case we have only two day-types: weekdays and weekends. Holidays during spring break, Thanksgiving, etc., are included in the weekend group for simplicity. This can be done while modeling weather dependent energy use because weather is the dominant driver. Christmas has been excluded from the data.

The models for weekdays and weekends were developed by doing stepwise regression. The results are summarized in Table 5.1. It can be seen in Table 5.1 that standard error of coefficients are all within the acceptable limit (p value ≈ 0.05). R-square of both the models is 0.91. C.V. is also less; 11.4 for weekday model and 10.46 for weekend model.

Time series plots of cooling energy use, Fourier series model fit and residual are shown in Figures 5.5 and 5.6 for first and second half of the year. It can be noted that model fit well with the measured energy use, except at some points where unusual energy consumption is captured by the spikes in the residual pattern. A close view of how the model fits to January and February data is shown further in Figure 5.7. Residual is consistently low, reconfirming a good fit.

The residual was plotted against predicted energy use to check constant variance assumption of the model. This is shown in Figure 5.8. The plot shows a fairly constant variance throughout the range of prediction.

Figure 5.9 shows a scatter plot of error versus lagged error. It can be noted from the plot that the concentration of points is maximum around the origin and it slowly decreases towards negative and positive quadrants and ends at an approximately absolute value of one. The frequency distribution of error, therefore, is approximately normal. However, the plot shows the presence of auto correlation in the error. This is because the model is developed from auto correlated time series data, also there are assumptions involved, for example, we did not account for the internal latent load in the model. The auto correlation can be reduced by incorporating such effects in the model and including higher Fourier frequencies, if a particular application demands.

As mentioned earlier, the present Fourier series approach was also applied to energy uses to many LoanSTAR sites to verify its prediction accuracy. Some of the results are summarized in Table 5.2. As can be seen in Table 5.2, R-square in all the cases are high and C.V. obtained is low.

COMPARISON WITH INDIVIDUAL HOURLY METHOD

Comparing the Fourier series model results is of particular interest because the present approach is developed from individual hourly approach. Cooling energy use in Zachry Engineering Center from September 01 '89 to December 22 '89 was chosen for comparison. Equation 5.1 was used and model developed for each individual hour of weekdays and weekends. The R-squares and C.V. are summarized in Table 5.3. R-square is consistently high for all the hours and overall C.V. for weekdays and weekends are 7.5 % and 6.3 % respectively. The C.V. comparison of present Fourier series approach with individual hourly approach is shown in Table 5.4. It can be noted that they are very close. The Fourier series approach, therefore, has almost the same prediction accuracy as the individual hourly approach but has fewer coefficients involved.

SUMMARY

In this chapter, the modeling procedure for weather dependent energy use is discussed. This is illustrated further by a case study. The models developed in the example showed a good fit to the measured data. The results of several sites are summarized in a table. These results verify the suitability of Fourier series analysis of weather dependent energy consumption in commercial buildings. When compared to the results obtained using individual hourly approach, Fourier series approach showed equally good fit to the data. Now, it is important to investigate how well the present Fourier series model can predict energy use for a period other than that used to develop the model. Also, if data is available for a short period, what happens to its predicting ability needs to be seen. In the next chapter, we concentrate on these issues.

TABLE 5.1

Summary of Fourier Series Model for Cooling Energy Use in Zachry Engineering Center, Texas A&M University in 1992.

	Variable	Parameter estimate	Partial R-square	Prob > | T |	Model R-square	Model C.V. (%)
Weekdays	INTERCEPT	-0.676646		0.0001	0.91	11.43
	OATEMP	0.060440	0.8042	0.0001		
	SHDIFF	220.226594	0.0719	0.0001		
	T1CH1	-0.008279	0.0179	0.0001		
	S1SH1	-9.291279	0.0082	0.0310		
	CH1	0.347601	0.0024	0.0001		
	T1SH1	-0.010450	0.0010	0.0001		
	SH1	0.523899	0.0007	0.0001		
	T1CH2	0.000689	0.0006	0.0001		
	S1CH1	-23.054330	0.0001	0.0001		
Weekends	INTERCEPT	-0.336695		0.0001	0.91	10.46
	OATEMP	0.047930	0.1380	0.0001		
	SHDIFF	227.931758	0.7725	0.0001		
	SH1	0.413418	0.0010	0.0001		
	T1SH1	-0.005143	0.0016	0.0001		
	CH1	0.097832	0.0009	0.0001		
	S1SH1	-15.064530	0.0007	0.0001		

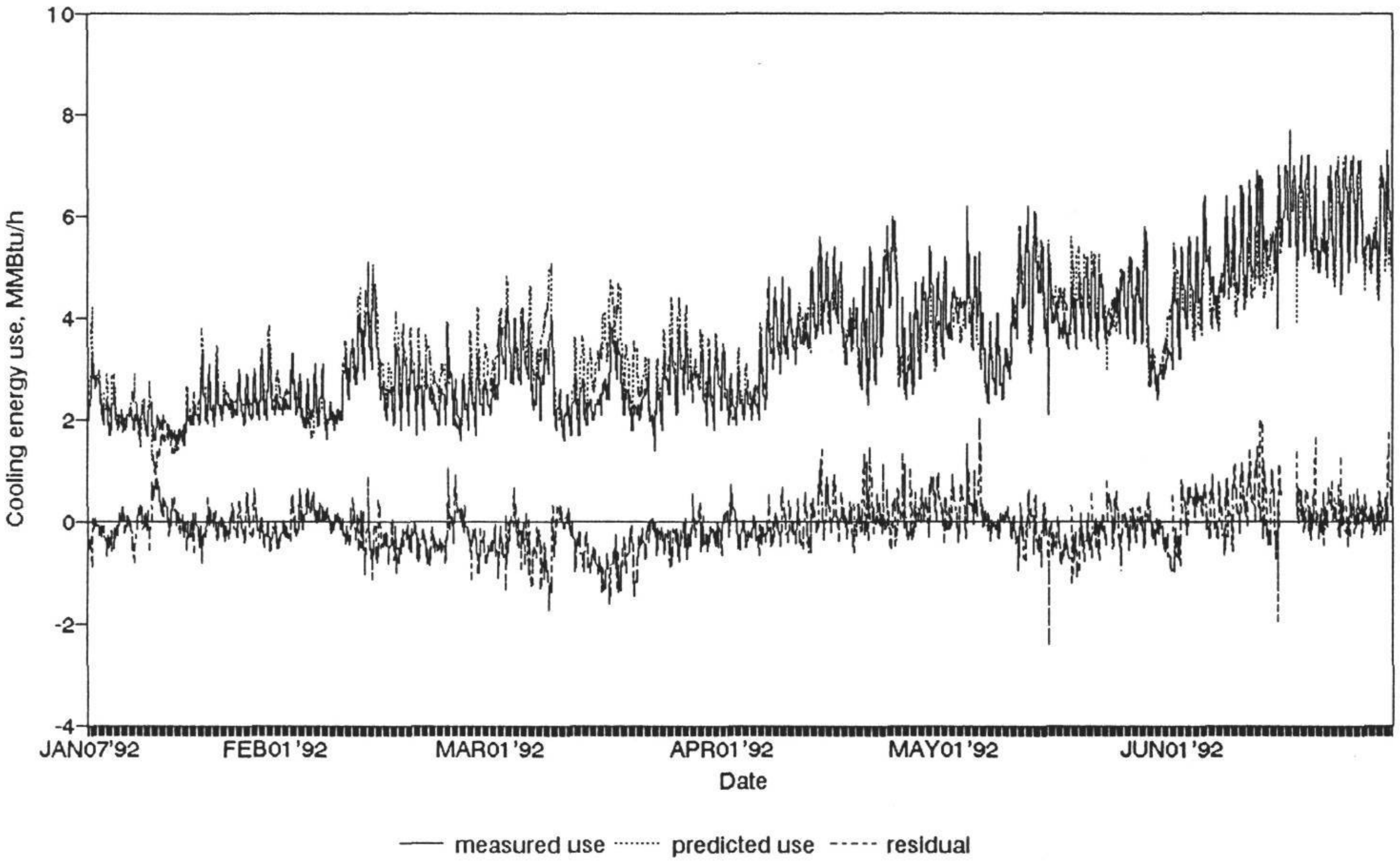


Figure 5.5 Time series plots of measured cooling energy use, predicted cooling energy use and residual in Zachry Engineering Center from January '92 to June '92.

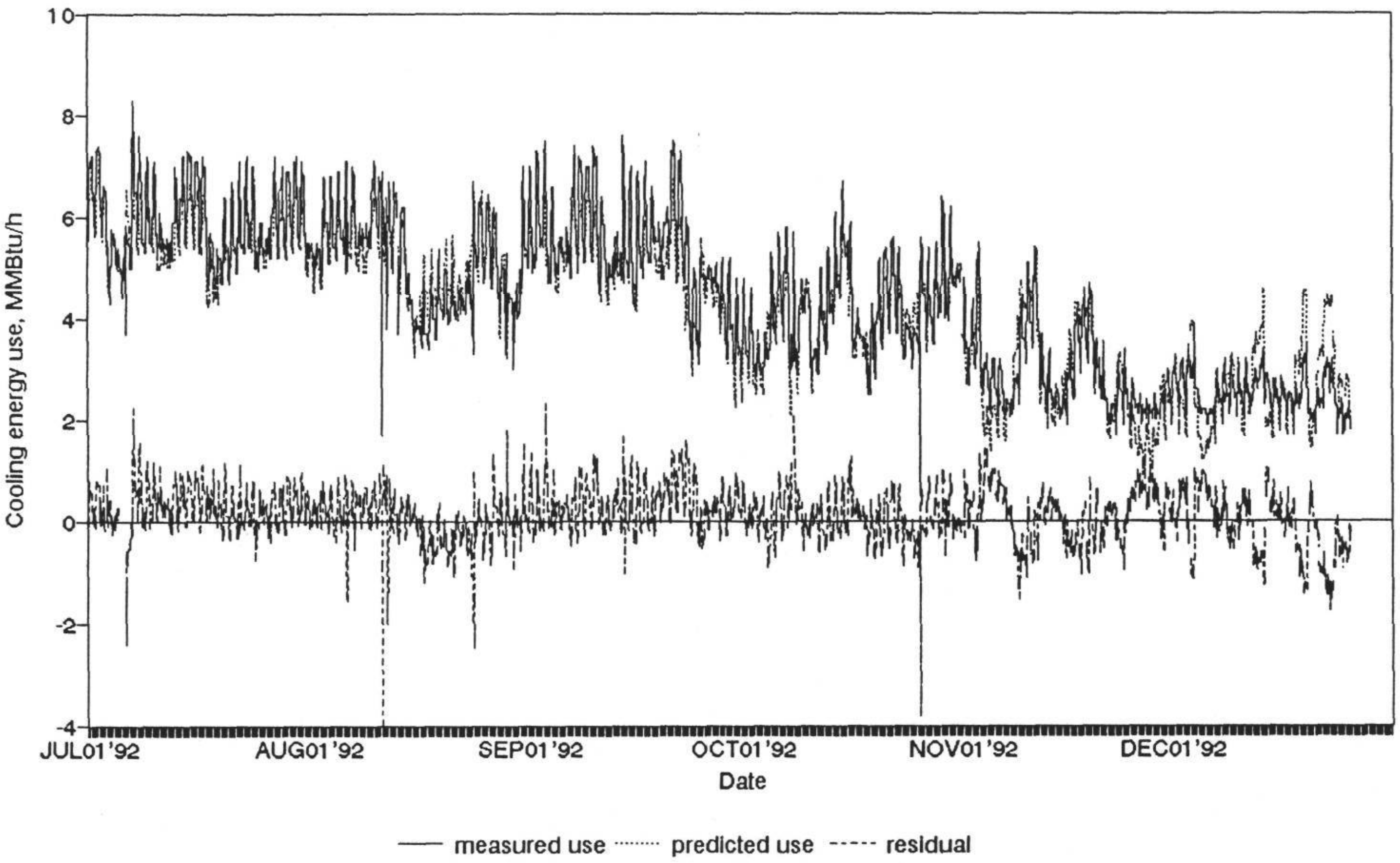


Figure 5.6 Time series plots of measured cooling energy use, predicted cooling energy use and residual in Zachry Engineering Center from July '92 to December '92.

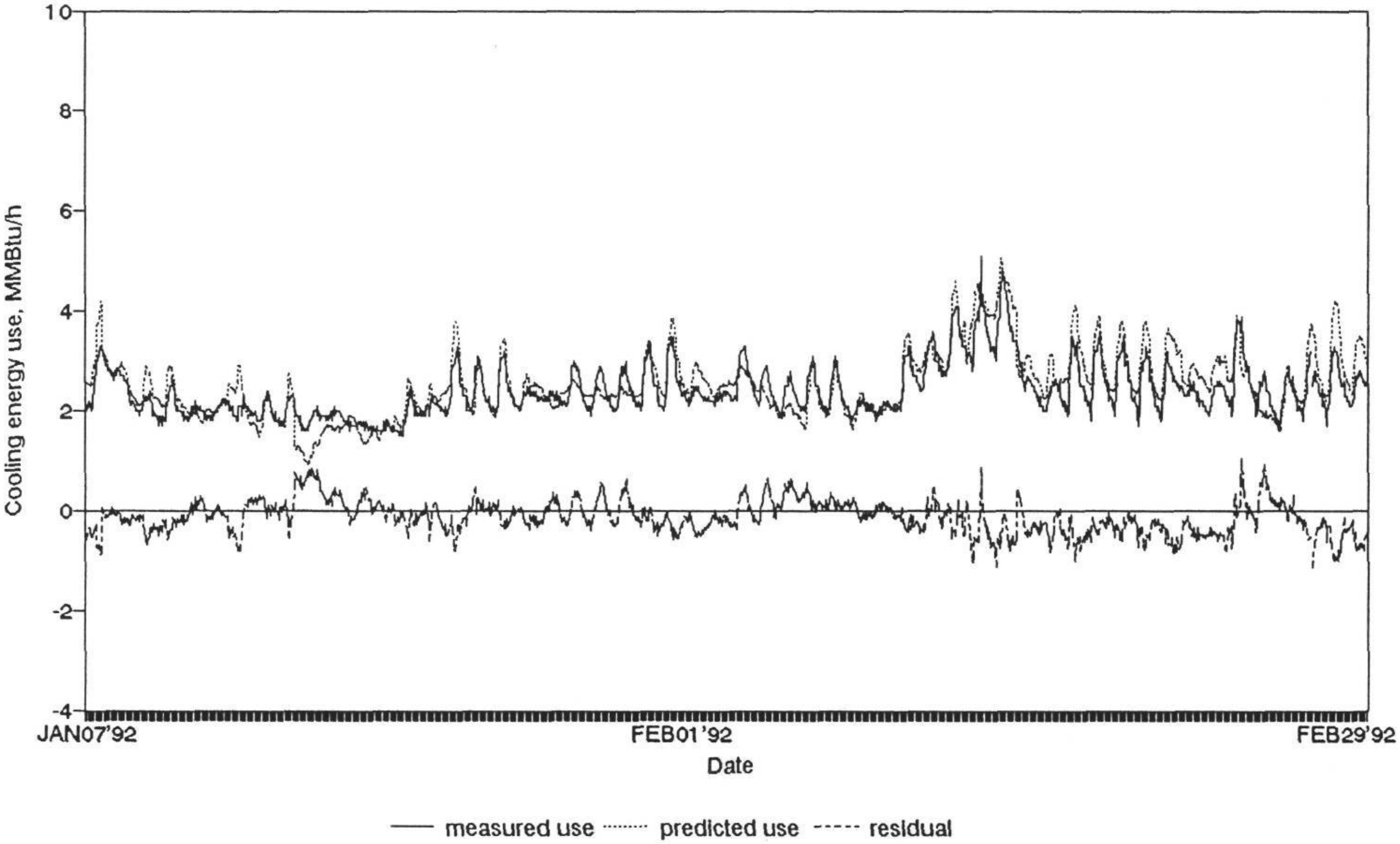


Figure 5.7 The above plot shows the Fourier series model fit to measured cooling energy use and residual in Zachry Engineering Center during January and February, 1992.

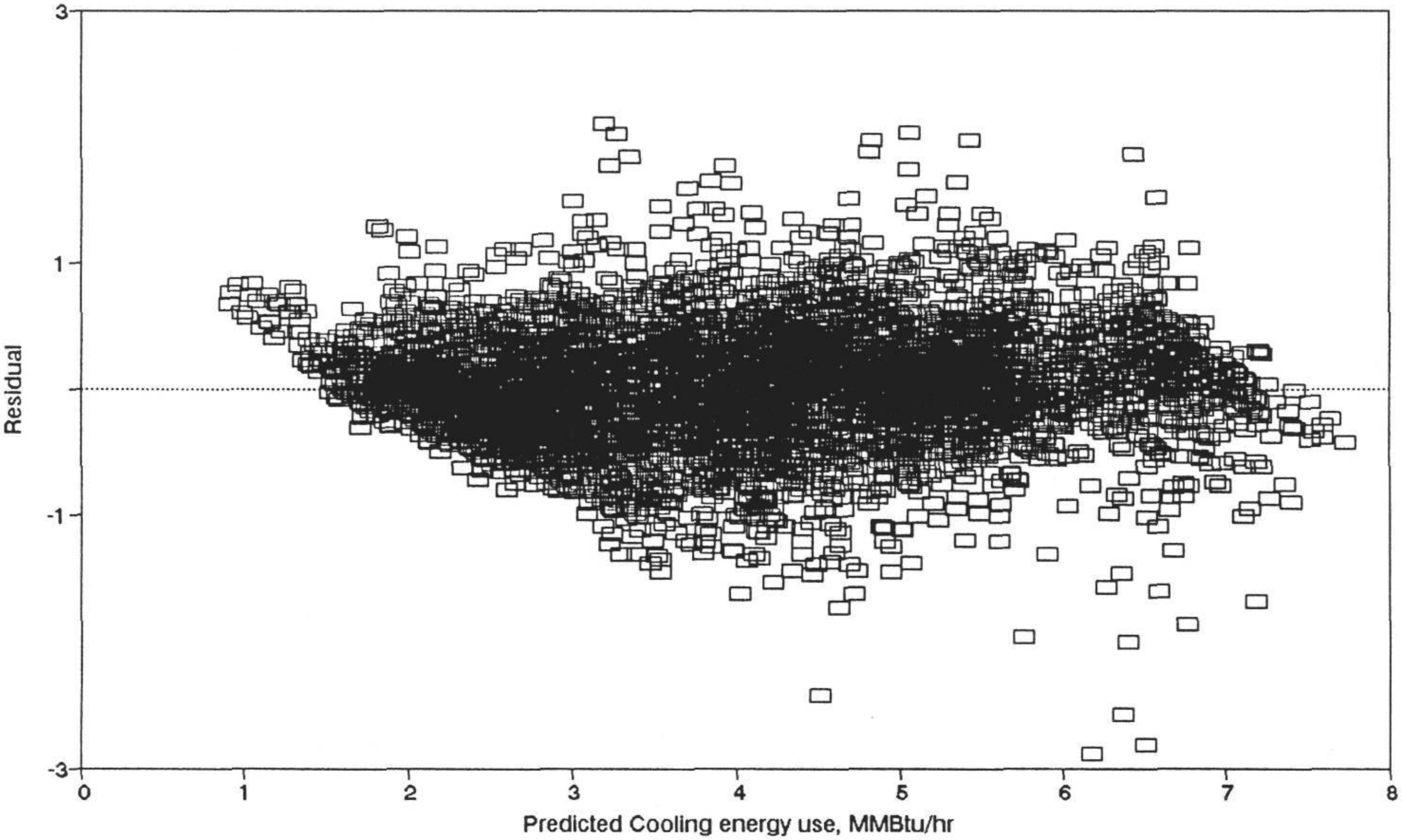


Figure 5.8 A scatter plot of residual vs. predicted cooling energy use in Zachry Engineering Center in 1992.

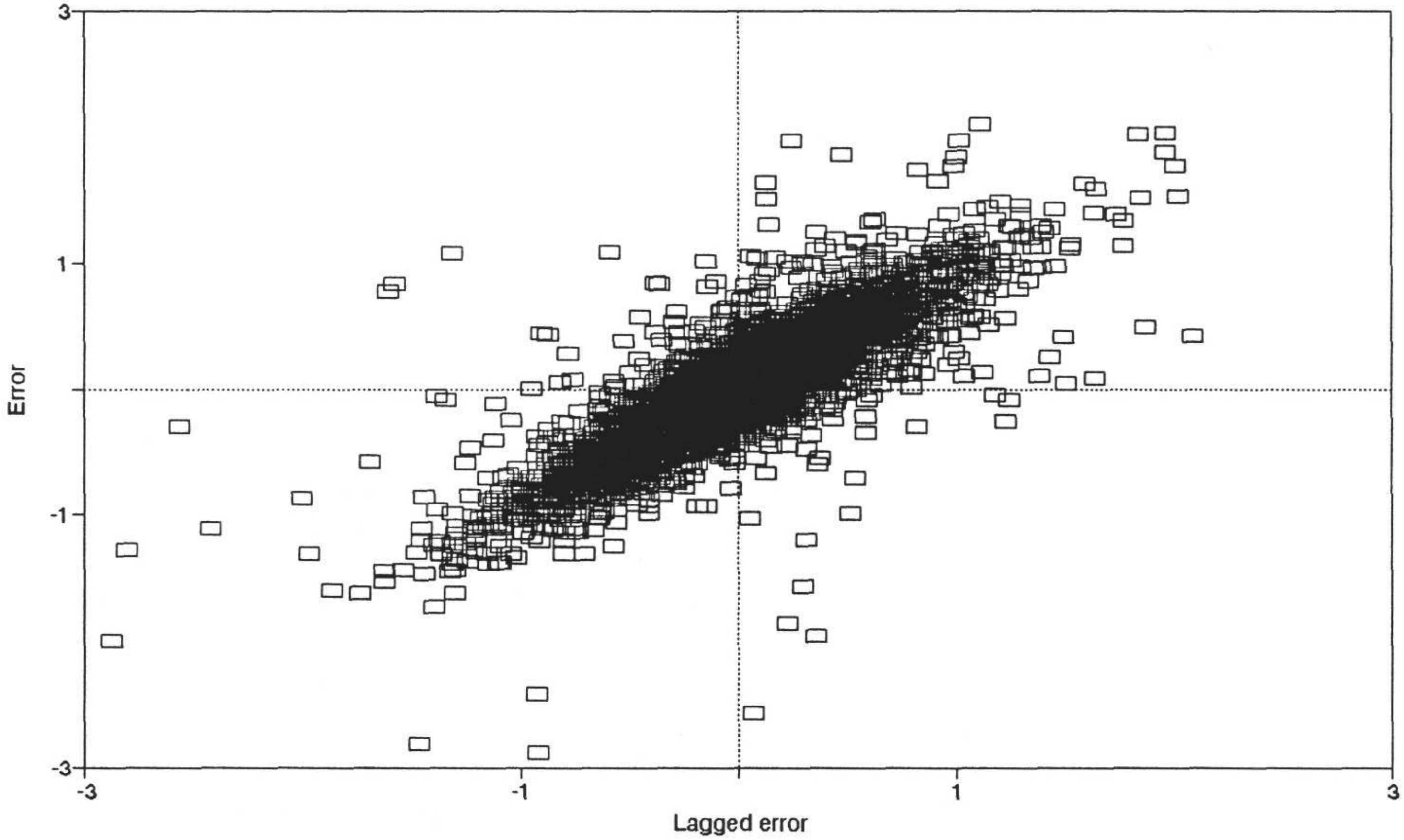


Figure 5.9 A scatter plot of error vs. lagged error for Fourier series model of cooling energy use in Zachry Engineering Center in 1992.

TABLE 5.2

Results of Fourier Series Models for Hourly Energy Use at Some LoanSTAR Sites

Site name	Site #	Type of energy use	Period	Day type	R^2	C.V. (rmse)	Dep. mean	RMSE
TDH	130	TWR & REC bldg. CW (MMBtu/hr)	Pre (02/16/91-08/12/92)	Weekdays	0.71	14.47	1.78	0.26
				Weekends	0.55	14.04	1.52	0.21
TDH	130	LAB & MAIN bldg.CW (MMBtu/hr)	Pre (02/16/91-08/12/92)	Weekdays	0.85	17.07	2.33	0.40
				Weekends	0.82	17.85	2.20	0.39
TDH	130	All buildings HW energy (MMBtu/hr)	Pre (02/16/91-08/12/92)	Weekdays	0.81	20.96	2.46	0.52
				Weekends	0.73	24.55	2.58	0.63
MCC	144	WBELEC (kWh/hr)	Pre (04/07/92 - 05/15/92)	Weekdays	0.90	12.80	210.1	26.88
				Weekends	0.93	8.58	140.4	12.06
WMH	145	WBELEC (kWh/hr)	Pre (02/01/92 - 03/31/92)	Weekdays	0.77	6.24	159.7	9.96
				Weekends	0.68	7.02	155.1	10.89
ZEC	001	CW (MMBtu/hr)	Post (01/07/92 - 12/31/92)	Weekdays	0.91	11.4	4.06	0.46
				Weekends	0.91	10.5	3.61	0.38
ZEC	001	CW (MMBtu/hr)	Pre (09/01/89 - 12/22/89)	Weekdays	0.87	7.7	5.30	0.41
				Weekends	0.91	6.3	5.10	0.32
ZEC	001	HW (MMBtu/hr)	Pre (09/01/89 - 12/22/89)	Weekdays	0.90	20.8	1.82	0.38
				Weekends	0.87	21.1	1.95	0.41

TABLE 5.3

Summary of Model Results Using Individual Hourly Approach for Cooling Energy Use in Zachry Engineering Center, Texas A&M University. Period : From September 01 '89 to December 22 '89.

	Weekdays			Weekends		
Hour of day	R-square	C. V. (%)	Overall C. V. (%)	R-square	C. V. (%)	Overall C. V. (%)
0	0.87	7.03	7.5	0.95	5.26	6.3
1	0.88	6.95		0.95	4.80	
2	0.88	7.29		0.94	5.36	
3	0.86	7.72		0.95	4.96	
4	0.88	7.32		0.95	5.05	
5	0.91	6.14		0.95	5.02	
6	0.91	6.22		0.94	5.58	
7	0.92	5.99		0.95	5.18	
8	0.91	6.22		0.95	5.54	
9	0.93	5.98		0.93	6.25	
10	0.90	6.82		0.88	8.36	
11	0.87	7.58		0.91	6.92	
12	0.81	8.95		0.91	6.68	
13	0.77	9.72		0.89	7.31	
14	0.80	8.95		0.87	7.60	
15	0.83	8.10		0.86	7.92	
16	0.83	8.00		0.88	7.18	
17	0.86	7.12		0.90	7.25	
18	0.87	7.00		0.90	6.63	
19	0.87	7.55		0.90	6.42	
20	0.87	7.34		0.90	6.28	
21	0.84	8.21		0.88	6.91	
22	0.83	8.59		0.93	5.41	
23	0.83	8.99		0.92	5.88	

TABLE 5.4

Comparison of C. V. Between Fourier Series Model and Individual Hourly Model for Cooling Energy Use in Zachry Engineering Center, Texas A&M University. Period : September 01 '89 to December 22 '89.

	Weekday C. V. (%)	Weekend C. V. (%)
Fourier Series Model	7.7	6.3
Individual Hourly Approach	7.5	6.3

CHAPTER VI
IDENTIFYING IMPORTANT DIURNAL FREQUENCIES AND COMPARING DIURNAL LOAD SHAPES OF DIFFERENT BUILDINGS

INTRODUCTION

Methodology of Fourier series modeling of weather independent and weather dependent hourly energy use is discussed in chapter IV. The frequencies that appear in the final model as the independent variables are selected by performing stepwise regression. For most of the buildings we have found the number of independent variables in the model is eight to ten. However, it is observed that including more than four or five independent variables does not improve R-square or C.V. appreciably. Identifying important diurnal frequencies as well as relating them to diurnal load shapes are presented in this chapter.

WEATHER INDEPENDENT ENERGY USE

Whole building electricity use or lighting energy use (when whole building electricity use is weather dependent) of eighteen buildings are considered for the study. To avoid the seasonal effect on variation of mean energy use, three months' periods is chosen at random for different sites. Mean load shapes during the selected periods for these sites are shown in Figures 6.1a through 6.1c. It can be noted that the mean load shapes of the buildings shown in Figures 6.1a and 6.1b are approximately the same except a little difference is observed in R. A. Steindam (RAS) building where, during the evening hours, energy use increases a little and drops after 9 p.m.. However, load shapes shown in Figure 6.1c have different patterns. One should be able to predict these differences from the Fourier frequencies that appear in the respective models.

Model Development and Identification of Important Frequencies

Fourier series models were developed for weather independent energy use on weekdays during the selected periods for all eighteen sites. The R-square and C. V. of the models are summarized in Table 6.1. In these models, the frequencies that had significant partial R-square were only selected. Including more than four or five (depending upon the load profile) frequencies, that have the highest partial R-squares, in the model, does not improve overall R-square and C. V. of the model appreciably. Three sites are chosen which are Zachry Engineering Center, Education building and RAS building to demonstrate this. R-square and C. V. plots of the models for these sites are shown in Figure 6.2. R-square is plotted against the frequencies chosen stepwise. So are the C. V.s. Saturation in both R-square and C. V. plots after adding fourth frequency can be noted.

Once the models were developed, all the coefficients of independent variables were normalized by dividing by the mean energy use. These coefficients are then plotted in two different ways. For each site, the normalized coefficients are plotted against the frequencies. These plots are shown in Figures 6.3a through 6.3c. Again, for each frequency, the coefficients are plotted against sites. These plots are shown in Figures 6.4a through 6.4c.

Normalized coefficients of the models maintain an approximately definite pattern in the sitewise plots. Moreover, some consistency can be observed in these plots when these are matched with the respective load shapes. There is less variation of magnitude of normalized coefficients for different frequencies when the energy use does not vary much from morning hours to noon (GAR, PAI and ZEC sites). Models for the buildings that have different load profiles, have different frequencies. Also, numbers of important frequencies are more (UTC, SIM, WIN, NUR and PCL).

In plots shown in Figures 6.4a through 6.4c, it can be observed that for most of the institutional buildings, four frequencies that are CH1, SH1, CH2 and SH4 have appeared in the models. SH5 and the frequencies 7 and above did never appear in the models. SH2, CH3, SH3, CH6 and SH6 appeared in the models when load shapes are distinctly different, as pointed out in the previous paragraph.

The most important conclusion that can be made from these plots is, therefore, for most of the institutional buildings, which have fairly the same load profile, the model can be developed from four frequencies : CH1, SH1, CH2 and SH4. In other words, we can visualize the load shape from the model frequencies. The models with more number of frequencies need careful check because this might say that there is an unusual pattern of energy use and corrective operational or maintenance measure may have to be taken (WIN; energy use is high during the evening hours, which may be reduced and energy saved).

Consistency in Load Profile from Season to Season

The consistency in the load profiles through out the year in the buildings were checked. In most of the cases the load profiles were consistent. Profiles of energy use in UTC, PCL and ZEC are shown as examples in Figures 6.5a through 6.5c. In ZEC, the profile is similar for all three periods chosen, with little variation in mean energy use from season to season. In UTC, the profiles are approximately same, however, variation in mean energy use from season to season exists. In PCL, the profile is different during February to April from 1 to 8 hours in the morning. This is because lights were not shut off in the morning during that period.

WEATHER DEPENDENT ENERGY USE

As in the case of weather independent energy use, weekday models of weather dependent energy use in eighteen different sites including institutional buildings, library, hospital and school buildings were developed using important frequencies only. The results are summarized in Table 6.2. For a better understanding, coefficients were normalized (divided by mean energy use), and plotted for each frequency, as shown in Figures 6.6a through 6.6d. The unit used is kBtu/hr for the energy use. The important conclusion that can be made from this analysis is that in most of the cases outdoor temperature has the highest partial R-square contribution to the model and up to second Fourier frequency and their product terms can adequately represent the hourly energy use pattern. Further study in this direction may be useful for a better characterization of weather dependent energy use in commercial buildings.

SUMMARY

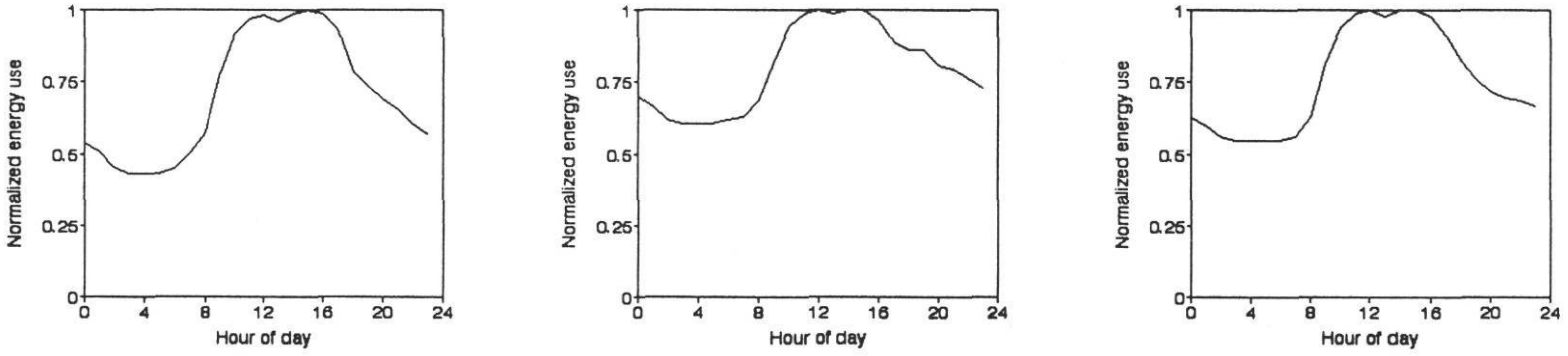
In this chapter, profiles of weather independent energy use and weather independent energy use have been studied, with more emphasis on weather independent energy use. A common pattern of weather independent energy use was observed in most of the institutional buildings and those could be characterized further by the Fourier frequencies in the model. While an effort is made to analyze weather dependent energy use profiles, much of work is needed in this direction for a clearer understanding.

In the next chapter, study predictive ability of weather dependent energy use is presented.

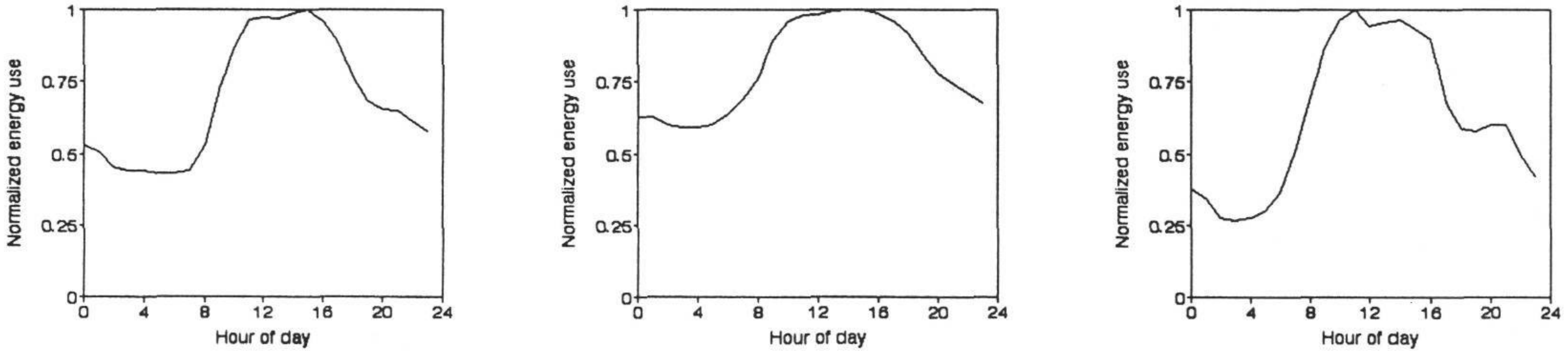
TABLE 6.1

Summary of R-square and C. V. of the Reduced Models Using Important Frequencies Only, for Weather Independent Energy Use at Eighteen Sites Including Institutional Buildings, Library and School Buildings.

Site	Type of energy use	Period (Weekdays)	Important frequencies	Model R-square	Model C.V. (%)
Education bldg.	WBELEC	Sep.'91 - Nov.'91	CH1, SH1, CH2, SH4	0.84	13.1
UTC bldg.	WBELEC	Sep.'92 - Nov.'92	CH1, SH1, CH2, SH2	0.76	13.2
P.C. Library bldg.	WBELEC	Jun.'92 - Jul.'92	SH1, CH1, CH2, SH3, CH4, SH6, SH2, SH4	0.75	14.8
Garrison bldg.	WBELEC	Jan.'91 - Mar.'91	CH1, SH1, CH2, SH4	0.76	10.0
Gearing bldg.	WBELEC	Feb.'92 - Apr.'92	CH1, SH1, CH2, SH4	0.83	10.3
Waggener bldg.	WBELEC	Sep.'92 - Nov.'92	CH1, SH1, CH2, SH4	0.87	11.4
Welch bldg.	WBELEC	Sep.'92 - Nov.'92	CH1, SH1, CH2, SH3, SH4	0.91	6.0
Burdine bldg.	WBELEC	Sep.'92 - Nov.'92	CH1, SH1, CH2, SH4	0.87	9.1
Nursing bldg.	WBELEC	Jun.'92 - Jul.'92	SH1, SH2, SH3, CH4, CH2, CH5, SH4, SH6	0.84	14.2
Winship bldg.	WBELEC	Sep.'92 - Nov.'92	SH1, CH1, CH2, SH3, SH2, SH4	0.80	10.3
R.A. Steindam bldg.	WBELEC	Feb.'93 - Apr.'93	SH1, CH1, CH2, SH2	0.80	20.2
Painter bldg.	WBELEC	Sep.'92 - Nov.'92	CH1, SH1, CH2, SH4	0.86	6.2
W.C. Hogg bldg.	WBELEC	Apr.'92 - Jun.'92	CH1, SH1, CH2, SH4	0.90	13.3
Zachry Engg. Center	WBELEC	Sep.'92 - Nov.'92	CH1,SH1, CH2, SH4	0.89	5.3
Business bldg.	Lighting	Sep.'92 - Nov.'92	CH1, SH1, CH2, SH4, SH3	0.86	21.9
Tower bldg., TDH	WBELEC	May'92 - Jul.'92	CH1, SH1, CH2, CH3, SH3, CH4, SH4	0.95	7.3
Sims School	Lighting	Sep.'92 - Nov.'92	CH1, SH1, CH2, SH2, SH4	0.80	26.1
Dunbar School	Lighting	Sep.'92 - Nov.'92	CH1, SH1, CH2, SH4	0.77	32.4

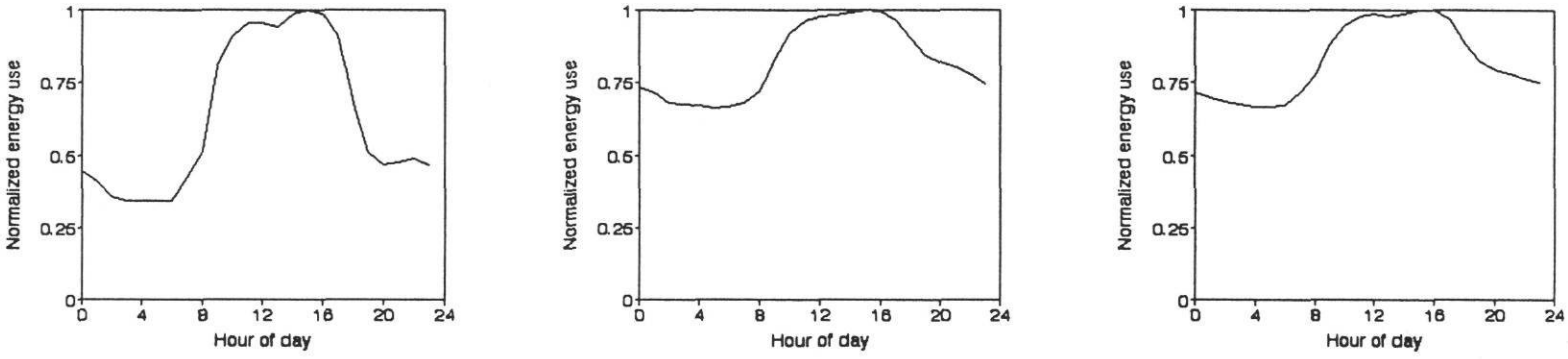


Education Building (EDB) Garrison Building (GAR) Gearing Building (GEA)



Waggener Hall (WAG) Welch Building (WEL) R. A. Steindam Building (RAS)

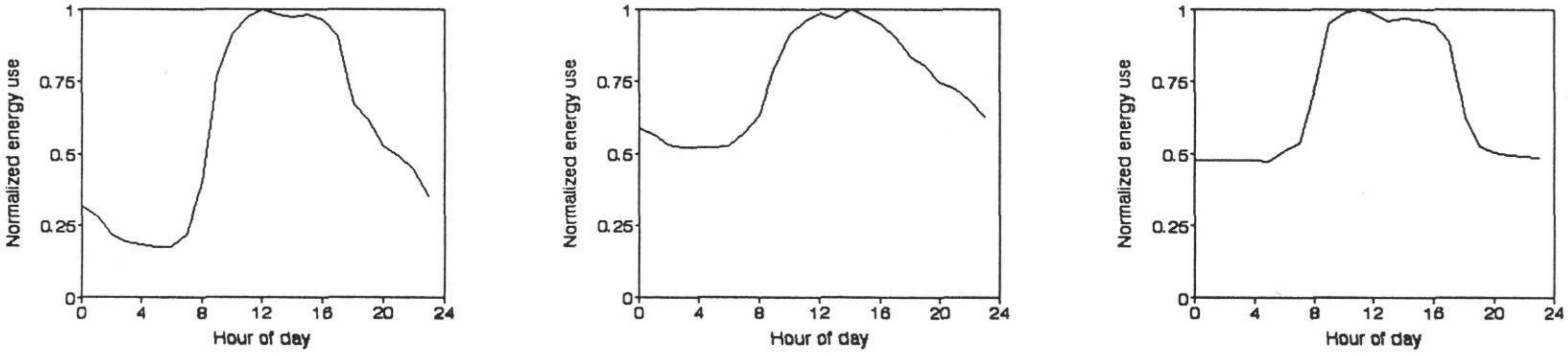
Figure 6.1a Normalized mean weather independent energy use profile at different buildings. Three months periods were chosen at random for different sites to generate these profiles.



W. C. Hogg Building (WCH)

Painter Building (PAI)

Zachry Engineering Center (ZEC)

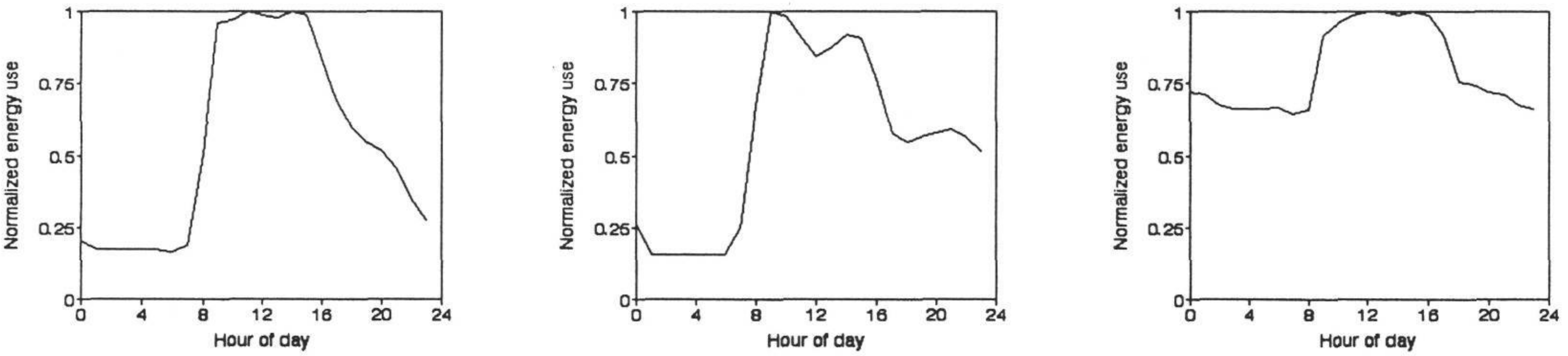


Business Building (BUS)

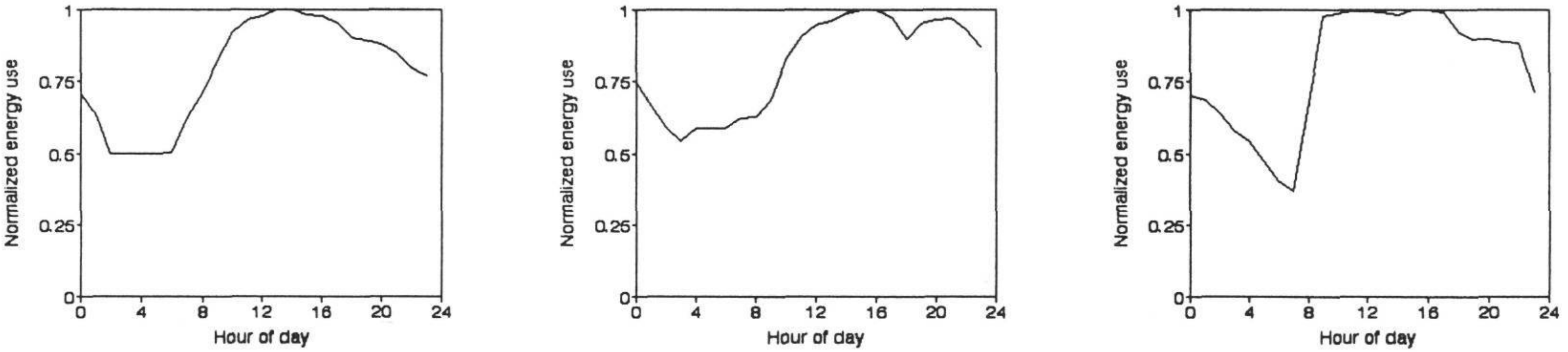
Burdine Building (BUR)

Texas Department of Health (TDH)

Figure 6.1b Normalized mean weather independent energy use profile at different buildings. Three months periods were chosen at random for different sites to generate these profiles.

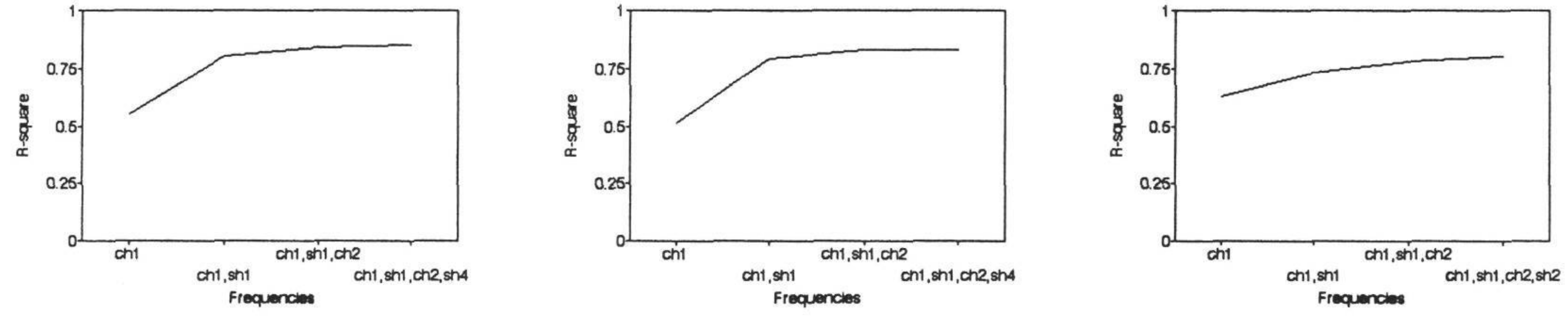


Dunbar Middle School (DMS) Sims Elimentary School (SIM) Nursing Building (NUR)



University Teaching Center (UTC) Winship Building (WIN) P. C. Library (PCL)

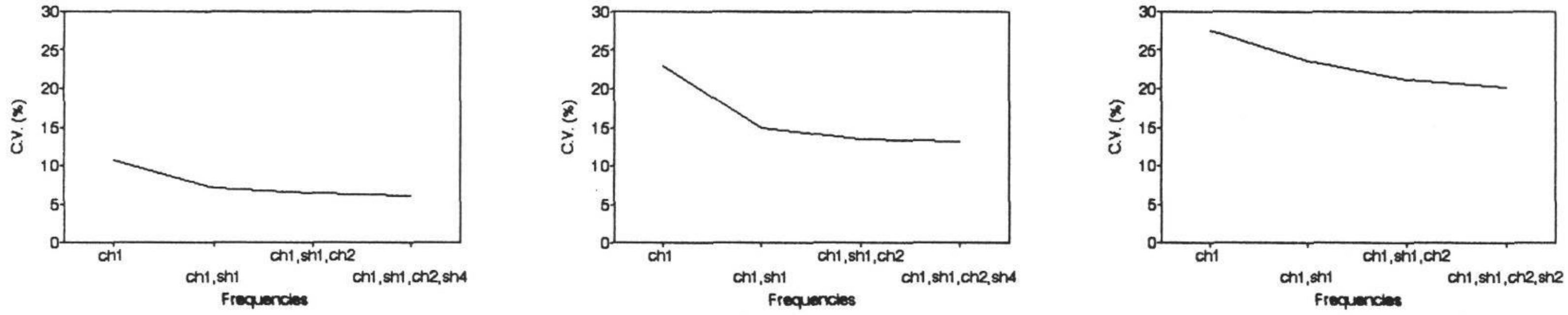
Figure 6.1c Normalized mean weather independent energy use profile at different buildings. Three months periods were chosen at random for different sites to generate these profiles.



ZEC

EDB

RAS

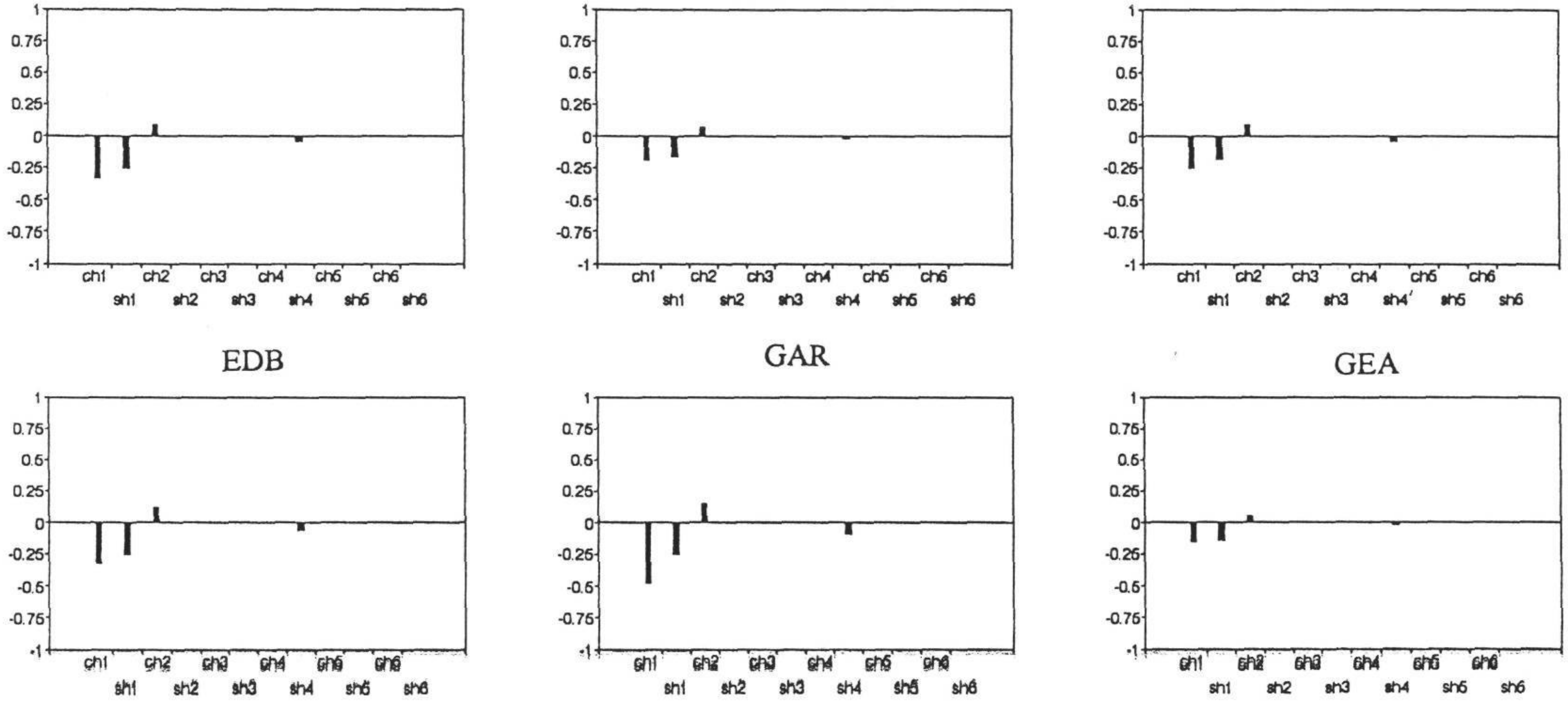


ZEC

EDB

RAS

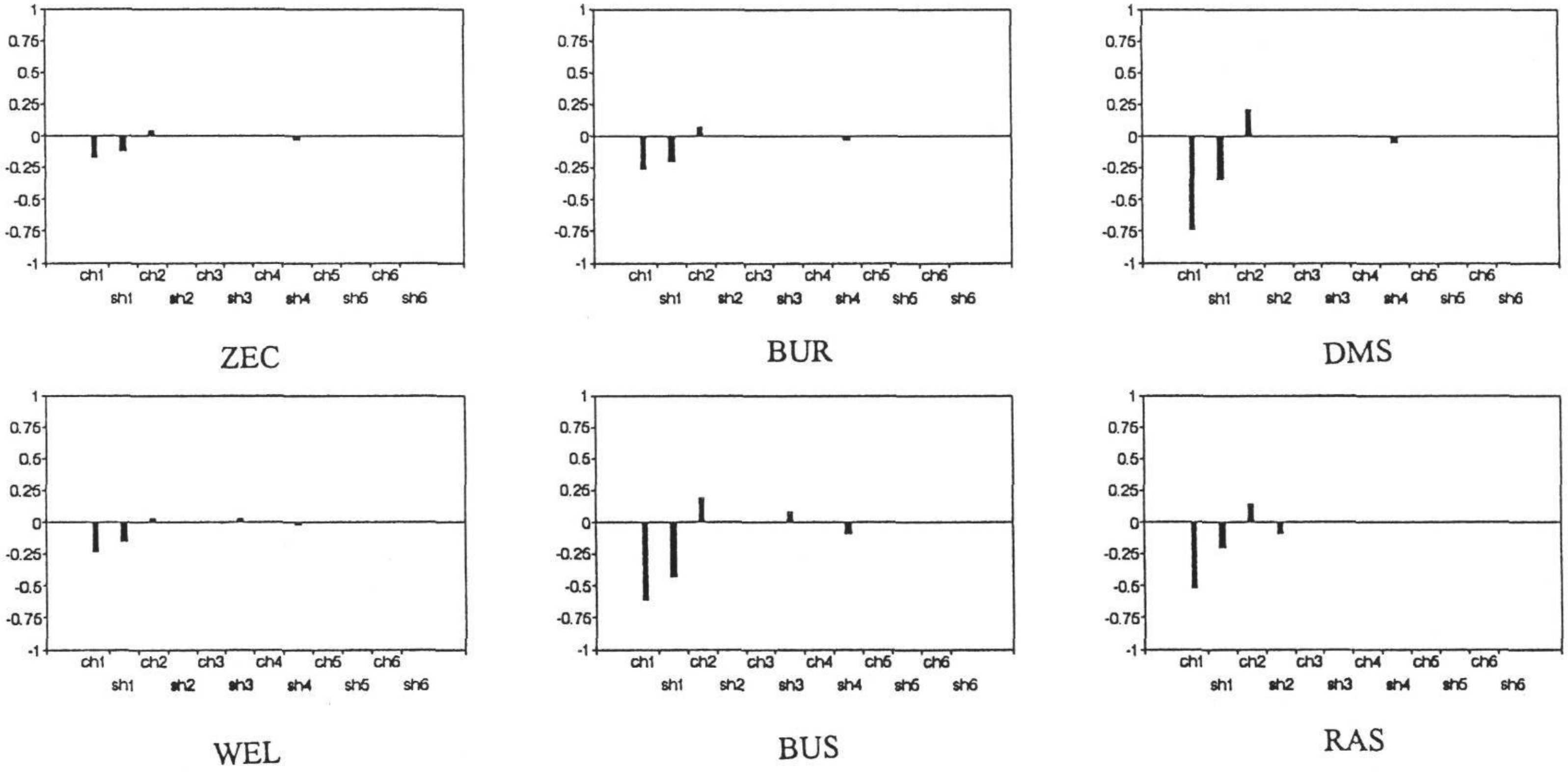
Figure 6.2 R-square and C.V. plots for three sites. The plots show that increasing number of frequencies in the model beyond step 4 will not improve R-square or C.V. appreciably.



EDB GAR GEA

WAG WCH PAI

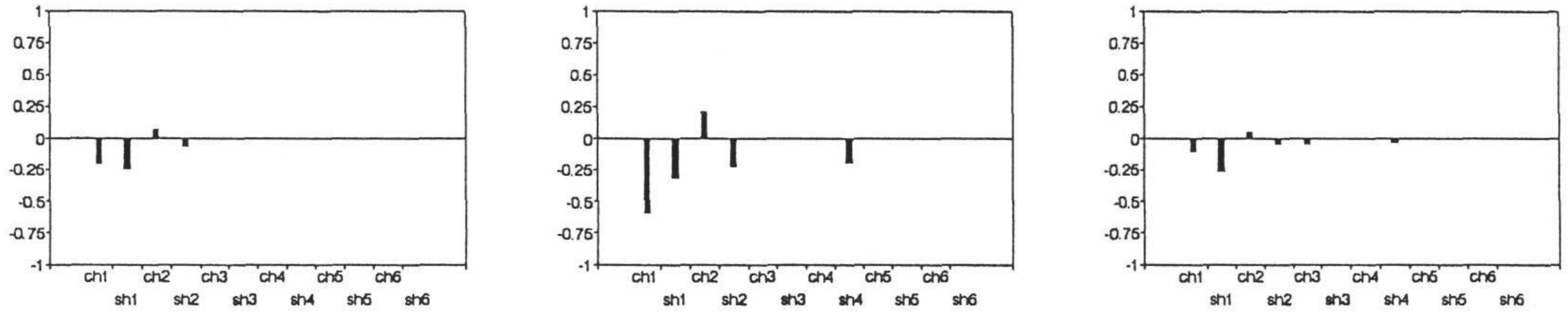
Figure 6.3a Normalized coefficient plot of Fourier series model of weather independent energy use at different sites. A common pattern may be observed for these sites.



ZEC BUR DMS

WEL BUS RAS

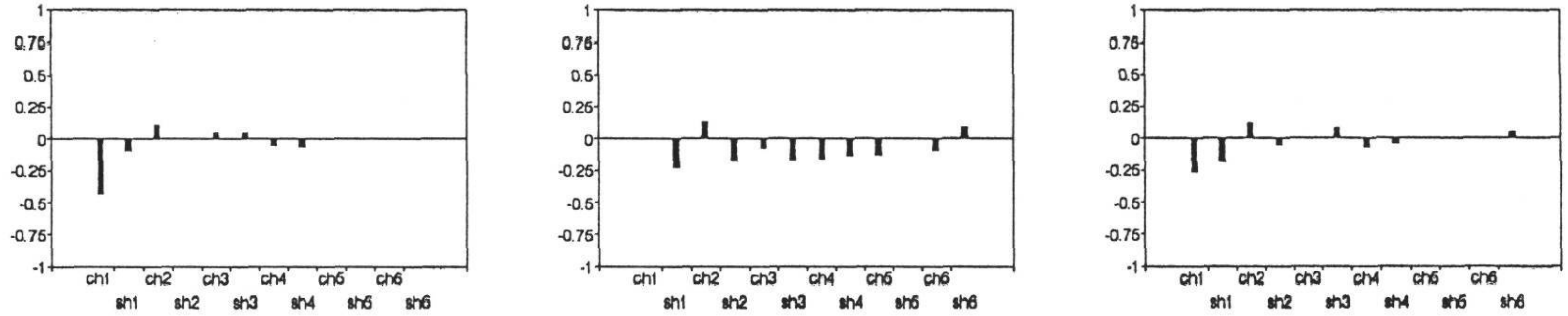
Figure 6.3b Normalized coefficient plot of Fourier series model of weather independent energy use at different sites. First three frequencies are found common for these sites



UTC

SIM

WIN

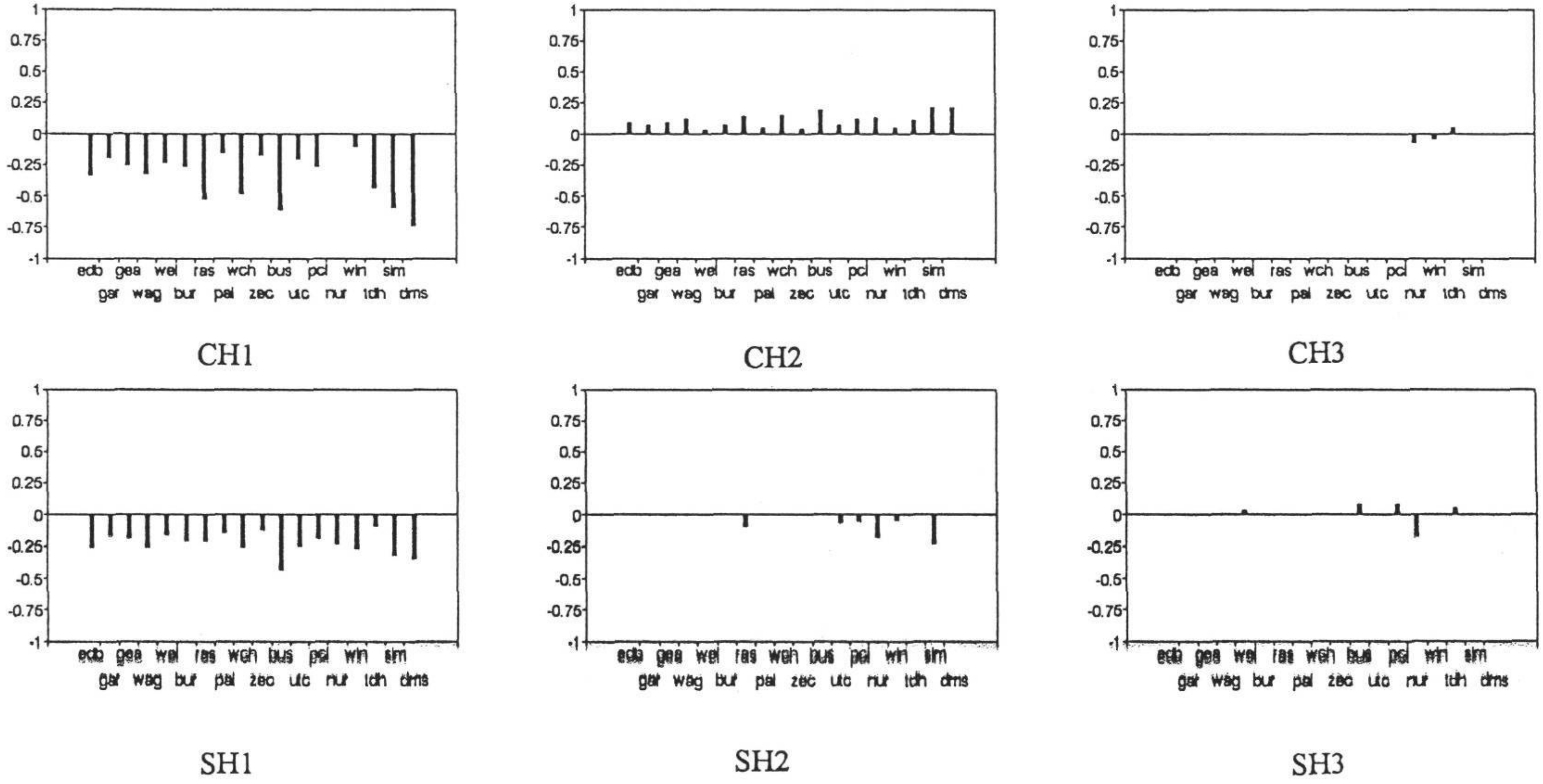


TDH

NUR

PCL

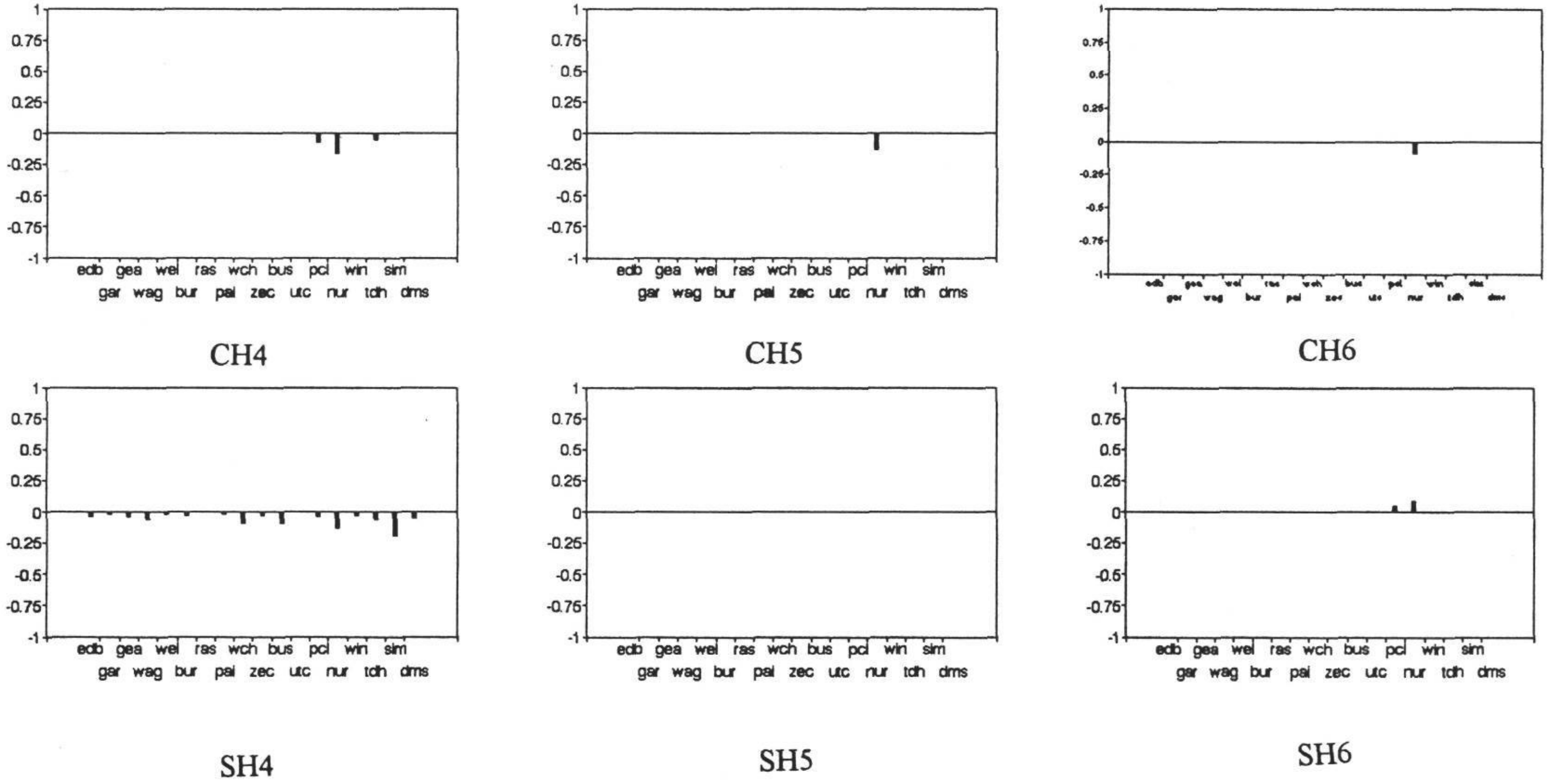
Figure 6.3c Normalized coefficient plot of Fourier series model of weather independent energy use at different sites.



CH1 CH2 CH3

SH1 SH2 SH3

Figure 6.4a Normalized coefficient plots of different frequencies of weather independent energy use model for eighteen sites. CH1, CH2, and SH1 are seen to have appeared in all the models.



CH4 CH5 CH6

SH4 SH5 SH6

Figure 6.4b Normalized coefficient plots of different frequencies of weather independent energy use model for eighteen sites. SH4 is seen to have appeared in all the models.

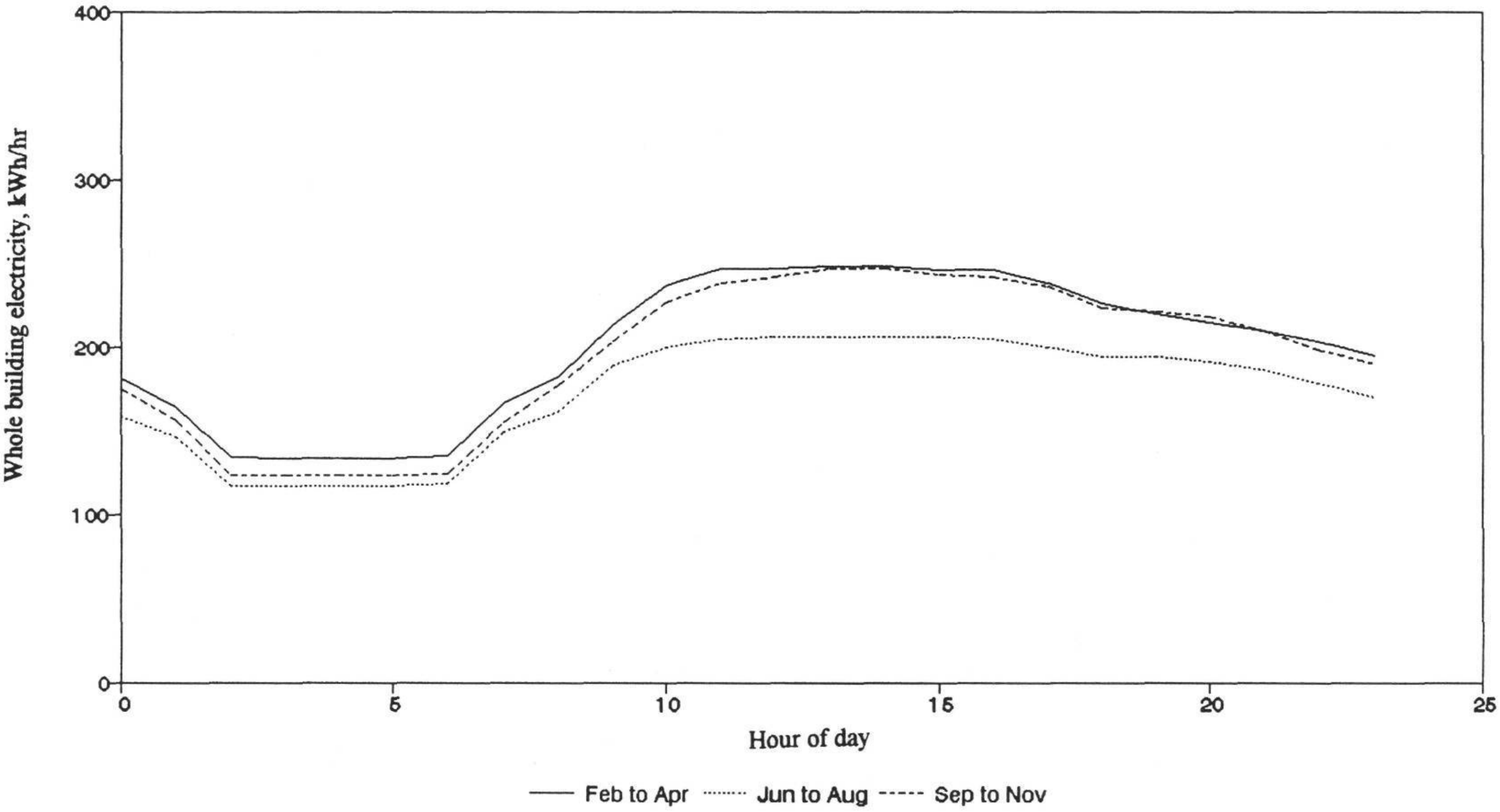


Figure 6.5a Mean weather independent energy use profile during three different three months periods of 1992 at UTC.

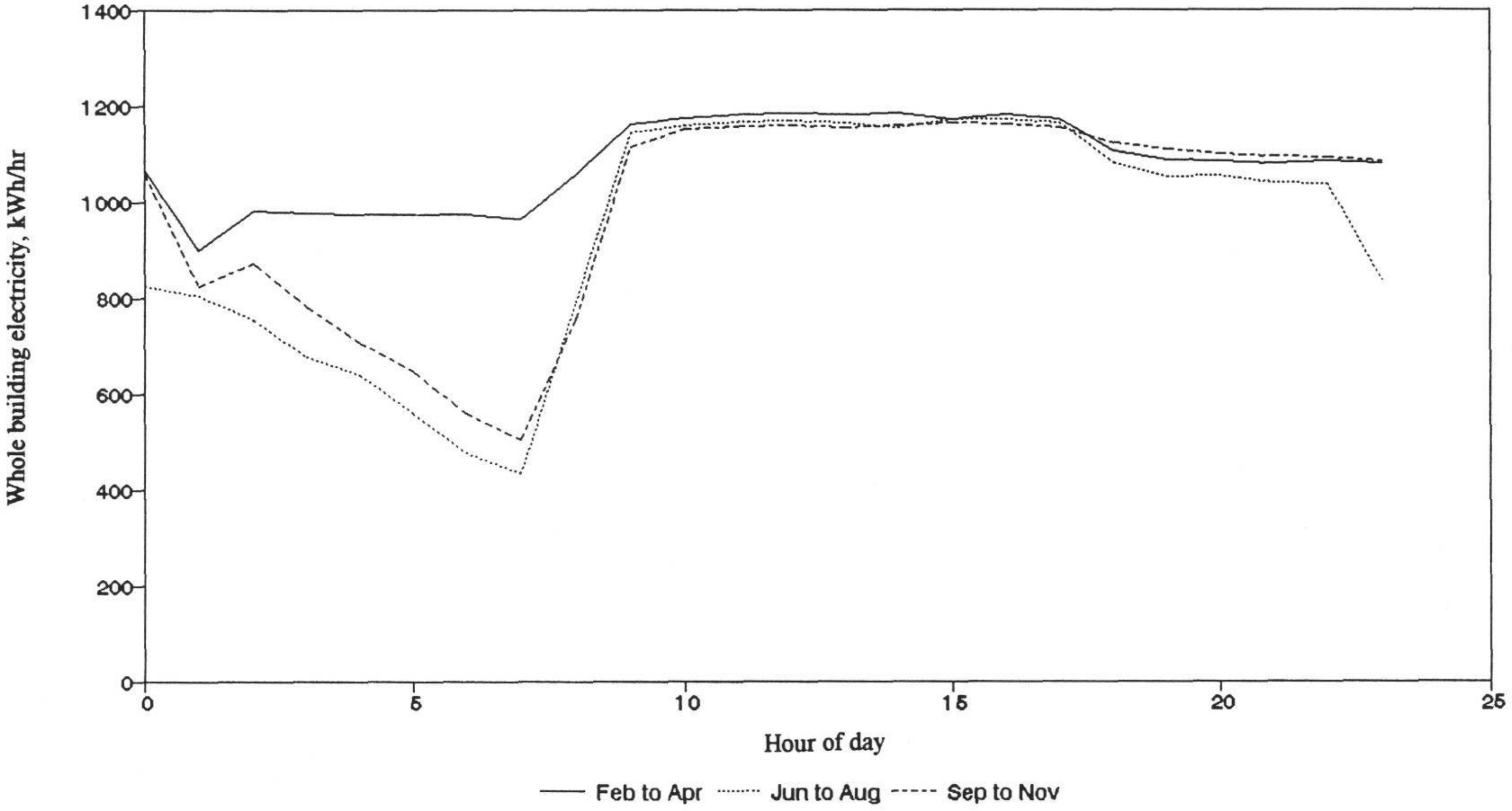


Figure 6.5b Mean weather independent energy use profile during three different three months periods of 1992 at PCL.

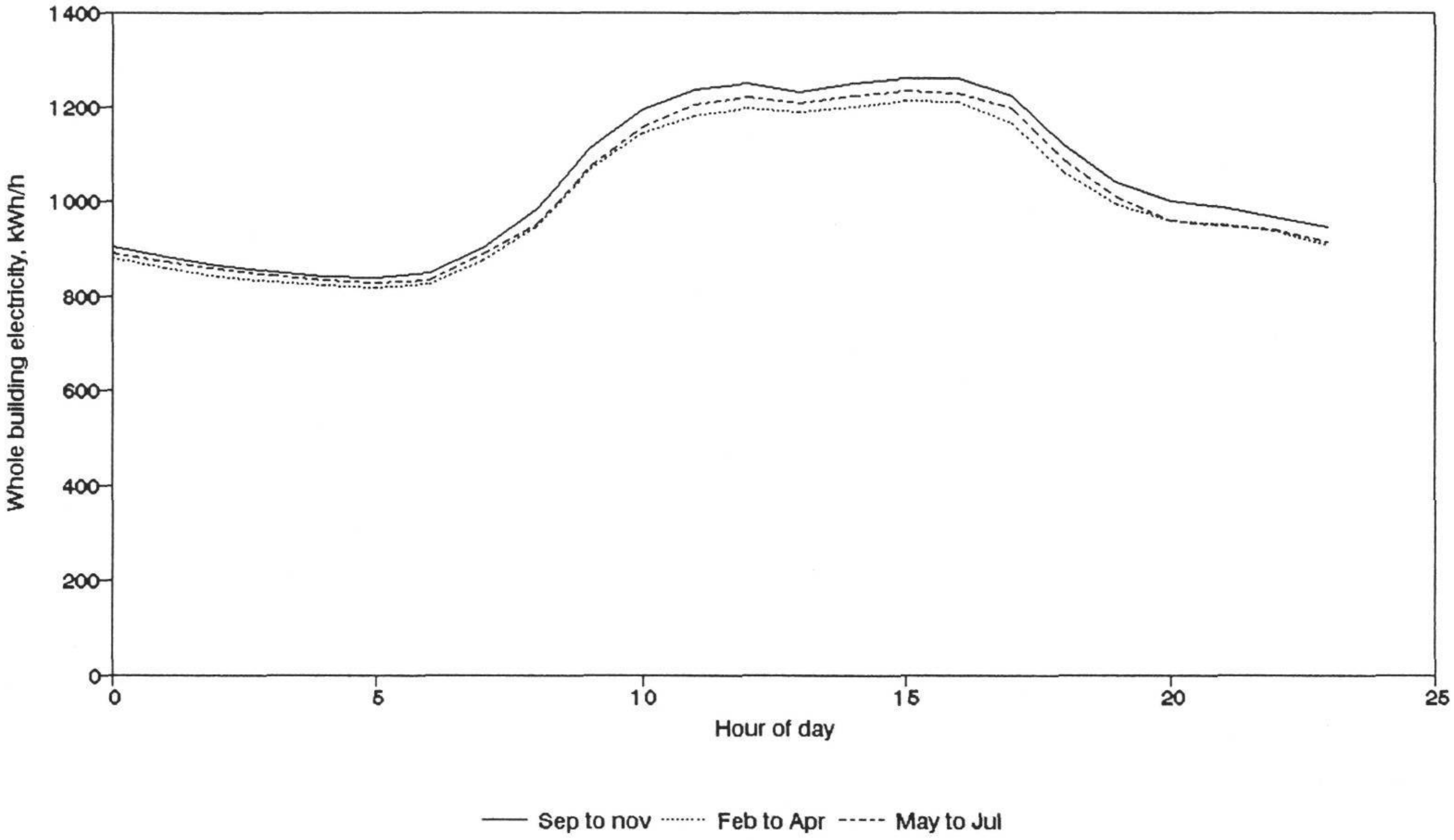
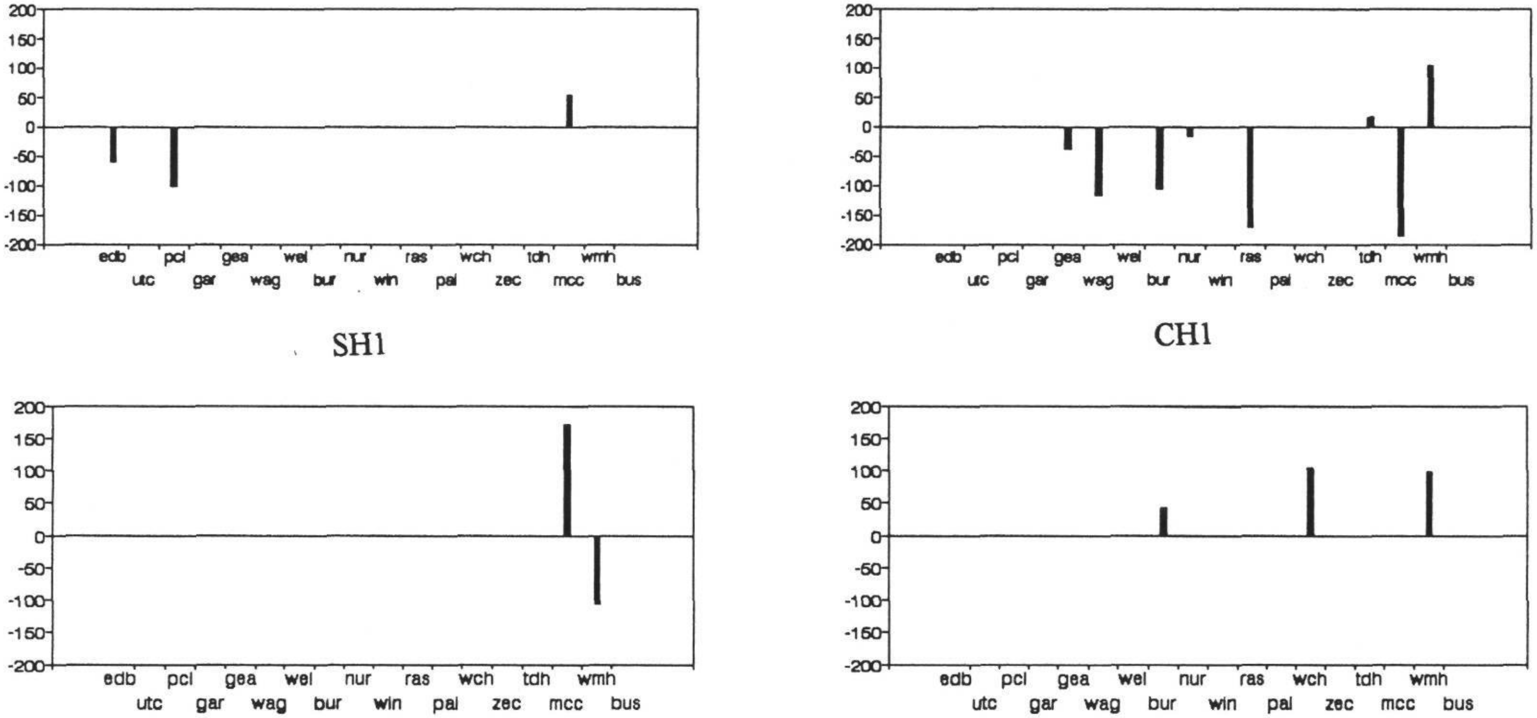


Figure 6.5c Mean weather independent energy use profile during three different three months periods of 1992 at ZEC.

TABLE 6.2

Summary of R-square and C. V. of the Reduced Models Using Important Frequencies Only, for Weather Dependent Energy Use at Eighteen Sites Including Institutional Buildings, Library, Hospital and School Buildings.

Site	Type of energy use	Period (Weekdays)	Important frequencies	Model R-square	Model C.V. (%)
Education bldg.	CW	Apr.'92 - Jun..'92	OATEMP, T1CH1, SH1, SL1SH2	0.81	15.4
UTC bldg.	CW	Sep.'92 - Nov.'92	OATEMP, T1CH1, SHDIFF, T1SH1	0.82	17.0
P.C. Library bldg.	CW	Jun.'92 - Aug.'92	OATEMP, T1SH1, SOLAR, SH1	0.43	9.0
Garrison bldg.	CW	Jun.'92 - Aug.'92	T1CH1, T1CH2, OATEMP, SHDIFF, SL1SH2	0.76	24.5
Gearing bldg.	CW	Apr.'92 - Jun.'92	OATEMP, CH1, SHDIFF, S1CH1	0.83	9.3
Waggener bldg.	CW	Jul.'92 - Sep.'92	OATEMP, CH1, SHDIFF, S1SH1	0.79	12.1
Welch bldg.	CW	Mar.'92 - May '92	SOLAR, OATEMP, T1SH1, T1CH1	0.91	17.4
Burdine bldg.	CW	Mar.'92 - May '92	OATEMP, CH1, S1CH1, CH2	0.70	13.8
Nursing bldg.	CW	Jun.'92 - Aug.'92	OATEMP, CH1, SL1SH2, SOLAR	0.76	10.8
Winship bldg.	CW	Jun.'92 - Aug.'92	OATEMP, SHDIFF, SL1SH1, T1CH4	0.78	10.6
R.A. Steindam bldg.	CW	Jun.'93 - Aug.'93	OATEMP, CH1, SHDIFF, T1SH2	0.83	10.1
Painter bldg.	CW	May '92 - Jul.'92	OATEMP, SHDIFF, T1SH1, S1SH1	0.79	7.9
W.C. Hogg bldg.	CW	Apr.'92 - Jun.'92	OATEMP, T1CH1, T1CH3, CH2	0.79	20.5
Zachry Engg. Center	CW	Sep.'92 - Nov.'92	OATEMP, SHDIFF, T1CH1, T1SH1	0.91	9.4
Business bldg.	CW	Jun.'92 - Aug.'92	OATEMP, SHDIFF, T1SH1, T1CH1	0.63	22.3
Lab & Main bldg., TDH	CW	May'92 - Jul.'92	OATEMP, CH1, T1CH1, T1SH1	0.60	12.7
Midland County Courthouse	WBELEC	Oct.'92 - Dec.'92	OATEMP, CH1, SH1, SH2	0.37	24.5
Ward Memorial Hospital	WBELEC	Sep.'92 - Nov.'92	OATEMP, CH1, SH2, CH2	0.23	23.4



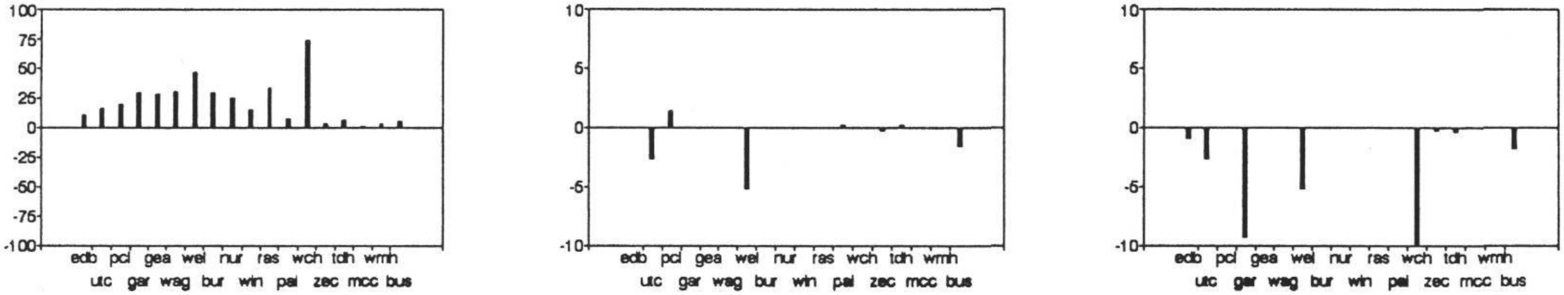
SH1

CH1

SH2

CH2

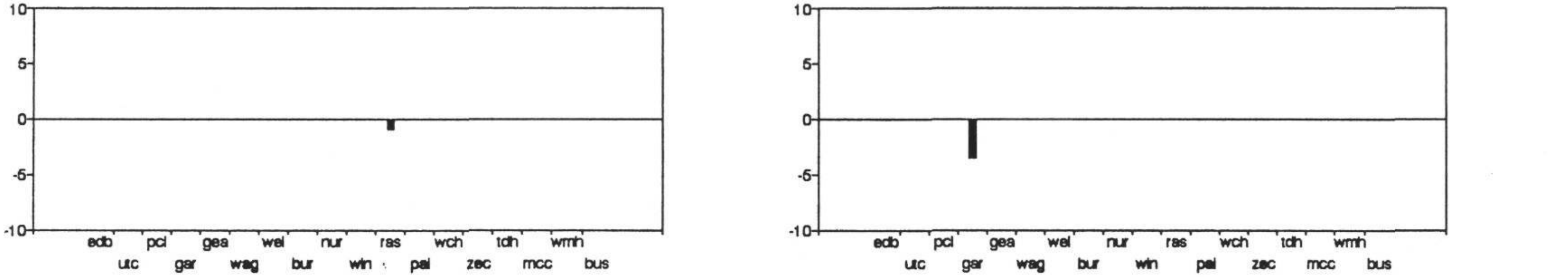
Figure 6.6a Normalized coefficient plots of different independent parameters of weather dependent energy use model for eighteen sites.



OATEMP

T1SH1

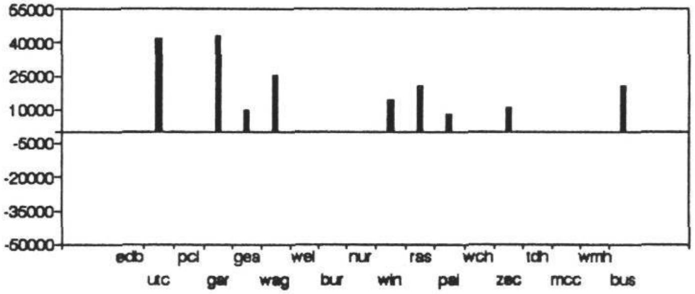
T1CH1



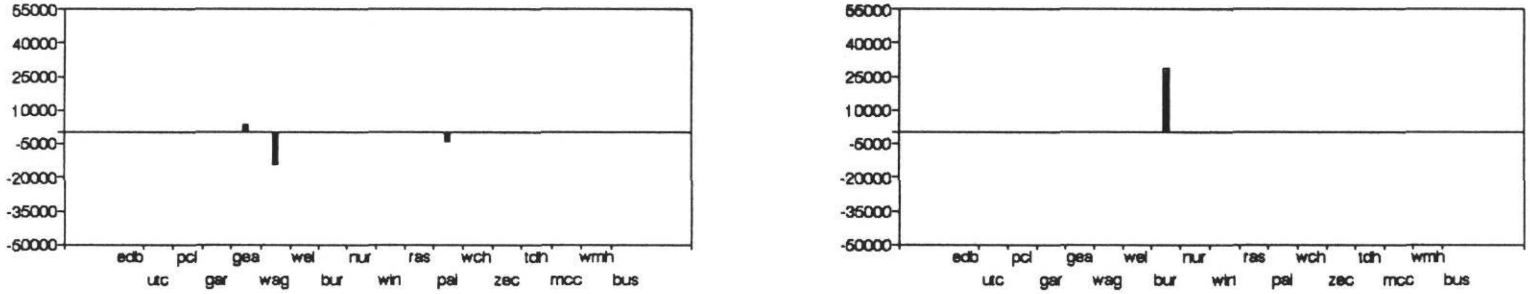
T1SH2

T1CH2

Figure 6.6b Normalized coefficient plots of different independent parameters of weather dependent energy use model for eighteen sites.



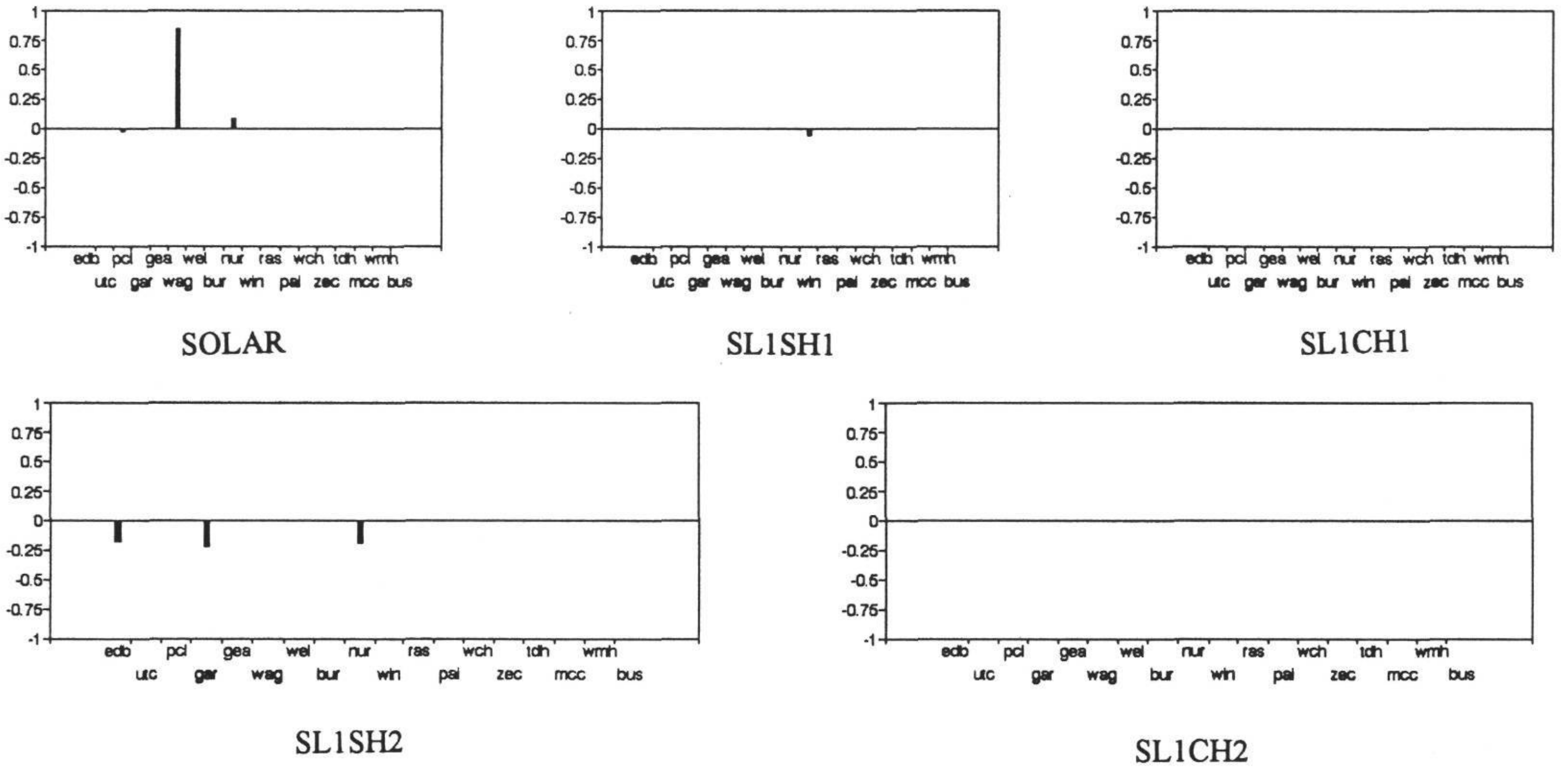
SHDIFF



S1SH1

S1CH1

Figure 6.6c Normalized coefficient plots of different independent parameters of weather dependent energy use model for eighteen sites.



SOLAR

SL1SH1

SL1CH1

SL1SH2

SL1CH2

Figure 6.6d Normalized coefficient plots of different independent parameters of weather dependent energy use model for eighteen sites.

CHAPTER VII

EFFECT OF SHORT DATA SET ON PREDICTION OF HOURLY ENERGY USE ON MONTHLY BASIS

INTRODUCTION

In the previous chapters, applicability of Fourier series to modeling hourly energy use in commercial buildings and methodologies of modeling weather independent as well as weather dependent energy uses were discussed. Also, important frequencies of Fourier series model were identified and discussed.

Very often, we do not have energy use data for a whole year. We may have data for one or two or three months. Models developed from such short data sets do not predict very well for other months of the year. Previous investigation (Kissock, Reddy, Fletcher and Claridge 1993) made certain conclusions on the annual prediction accuracy of models developed from short data sets. However, there is a need for looking at monthly prediction accuracy of such models to suggest any possible correction factor to the energy use prediction during the months when the prediction is off. In this chapter, some work in this direction is presented. Cooling energy use in two large institutional building (Zachry Engineering Center at Texas A&M University (ZEC) and Business building in UT Arlington(BUS)) in 1992 is chosen for the present study.

IDENTIFYING MONTH TYPE

Cooling energy use is predominantly driven by outdoor temperature. To identify month types, daily average outdoor temperature for the year 1992 has been used. The daily average outdoor temperature is smoothed by taking five point averages for twenty consecutive times. However, the choice of twenty times was arbitrary. The idea is to remove fluctuation to a reasonable extent so as to be able to arrive at some basis of grouping different periods of a year.

The time series plots of daily average temperature as well as the smoothed daily temperature in ZEC and BUS are shown in Figures 7.1 and 7.3 respectively. The smoothed temperature in both the cases are found to be in the range of 45 to 85 degree Fahrenheit (F). The entire range is then divided into four temperature bins: (1) 45 to 55 degree F, (2) 56 to 65 degree F, (3) 66 to 75 degree F and (4) 76 to 85 degree F. The months that fall approximately to same temperature bin are grouped together (Figures 7.2 and 7.4). The same groups have been obtained for both the sites, which are as follows:

(1a) January '92, (1b) December 01 '92 to December 13 '92,
(2a) February '92 and March '92, (2b) November '92,
(3a) April '92 and May '92, (3b) October '92 and
(4) June '92 through September '92.

MODELING AND PREDICTION

Four models for cooling energy use have been developed from (a) January '92, (b) February '92 and March '92, (c) April '92 and May '92 and (d) June '92 through September '92 data. Each of these models is applied to predict energy use for all other months of the year. The C.V.s are calculated for all the groups and plotted in Figures 7.5 and 7.6 for ZEC and BUS respectively. The plots show that C.V. is minimum when the model of the same period is used for prediction but higher when other models are used. Also, the increase can be observed to have a relationship with the difference between the mean temperature of the modeling period

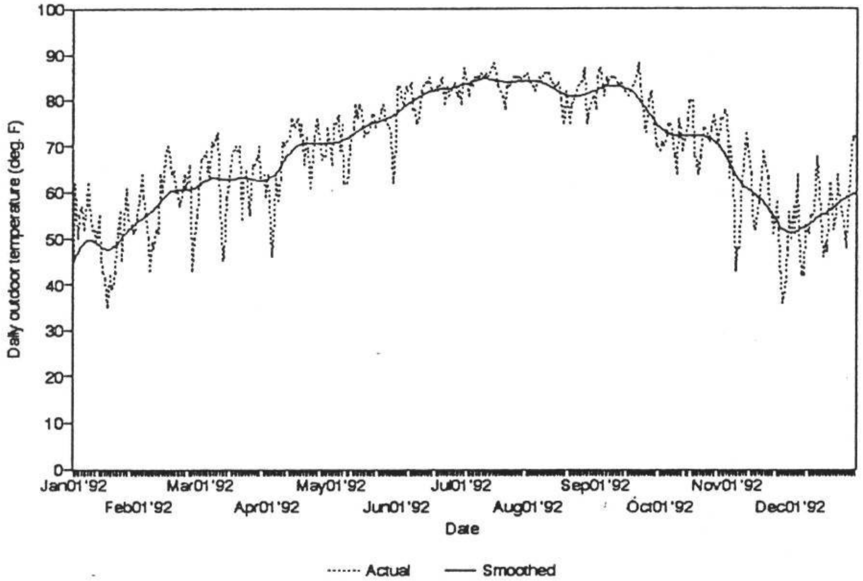


Figure 7.1 Time series plots of daily average outdoor temperature and smoothed temperature in Zachry Engineering Center in 1992.

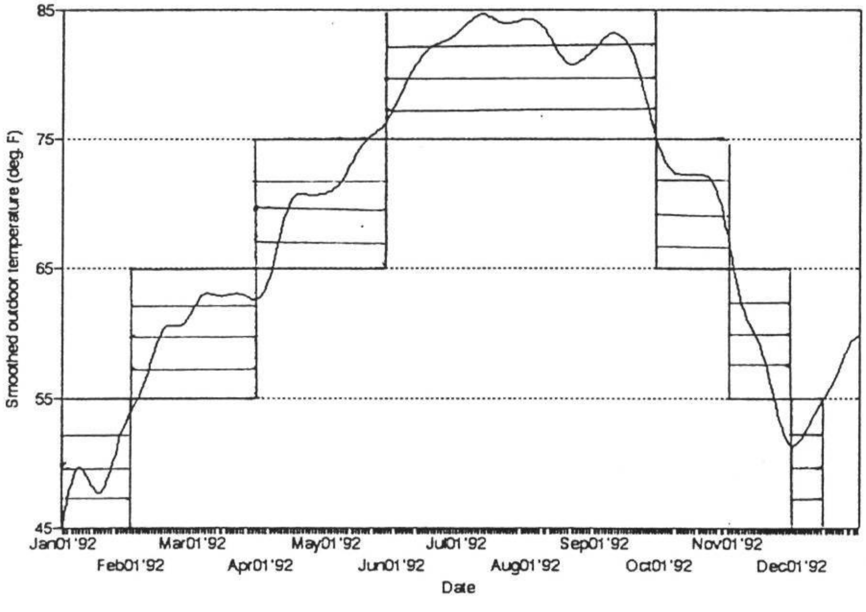


Figure 7.2 This plot shows the grouping of the months using the smoothed daily outdoor temperature.

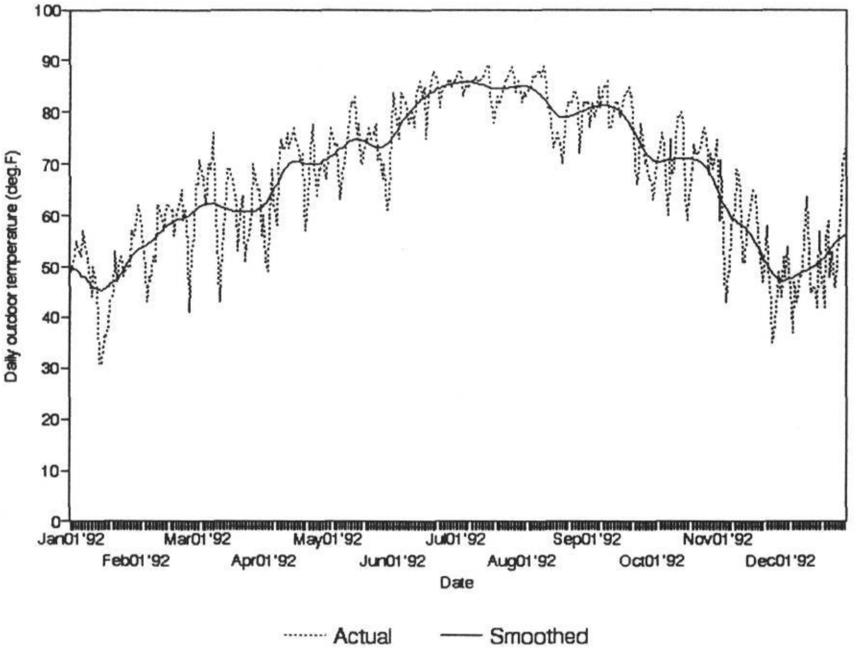


Figure 7.3 Time series plot of daily average outdoor temperature and smoothed temperature in Business building in 1992.

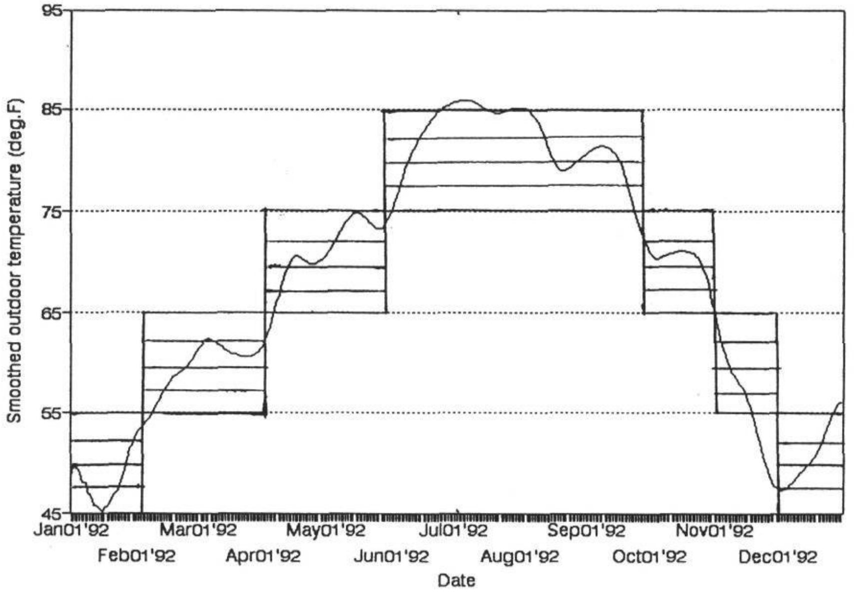


Figure 7.4 This plot shows the grouping of the months using the smoothed daily outdoor temperature for Business building in 1992.

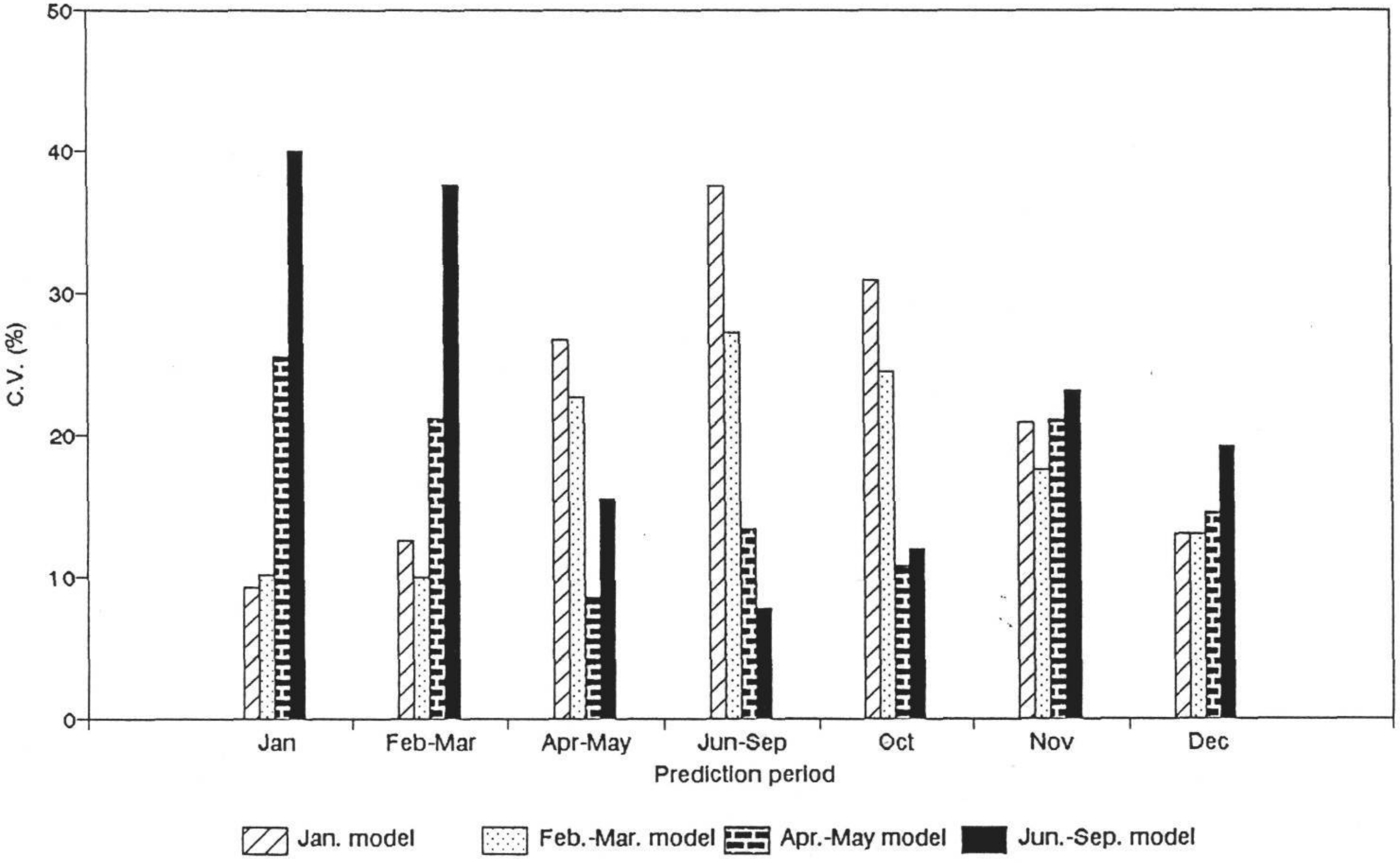


Figure 7.5 Bar chart of C.V.s of different periods using four different models. Site: Zachry Engineering Center.

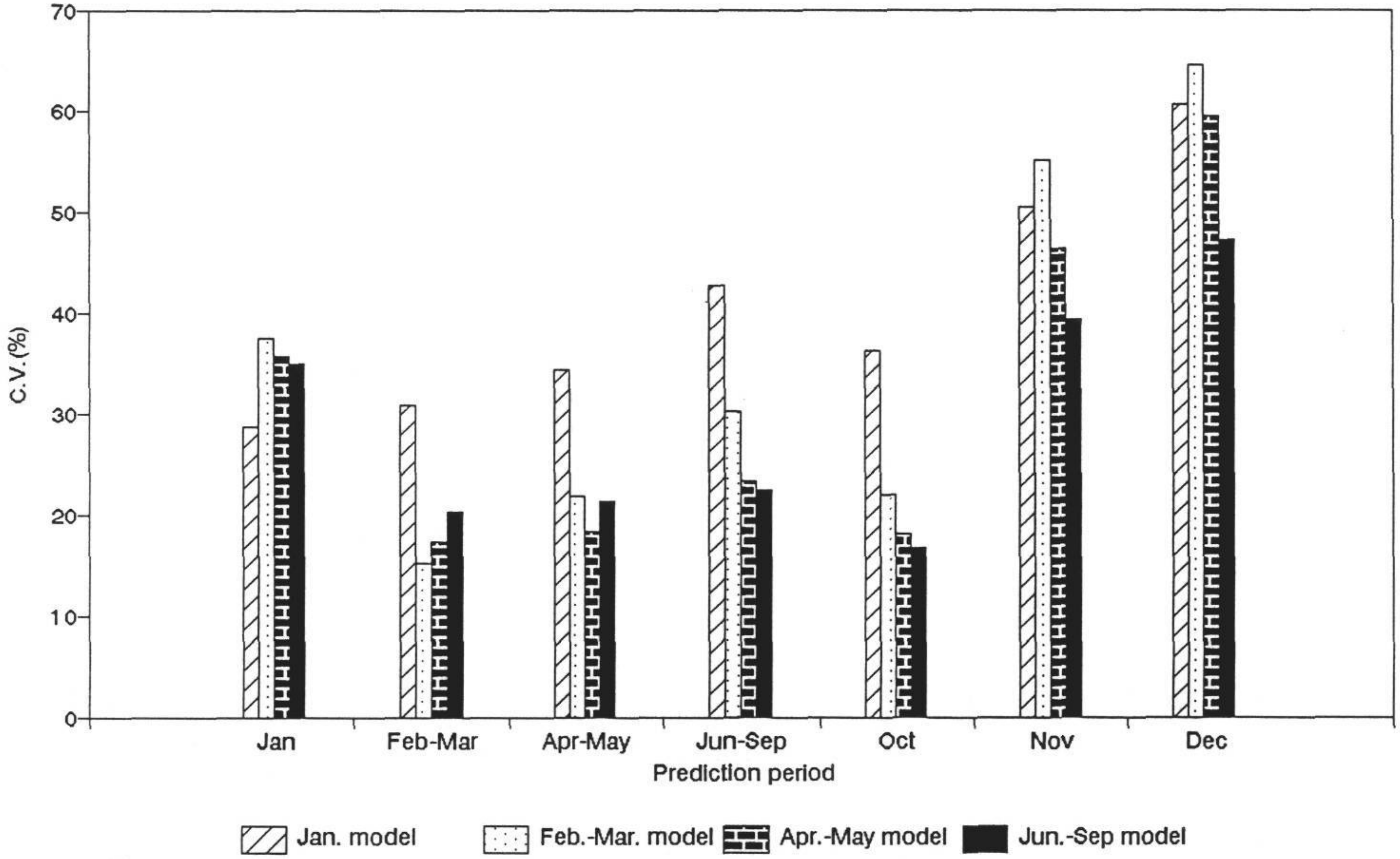


Figure 7.6 Bar chart of C.V.s of different periods using four different models. Site: Business building.

and prediction period. This led to the investigation of how average residual for each month behaves and whether any relationship between the average residuals and monthly average temperature or not. For a particular model, when average residuals for all the months are plotted against monthly average temperature, it shows a distinct relationship between these two. A 2-p linear fit is developed for each case by constraining the prediction to zero for the data period from which respective models are developed. The plots are shown in Figures 7.7 through 7.10 for ZEC and Figures 7.11 through 7.14 for BUS. The observations are discussed for each model in the following sub-sections:

January '92 Model

The monthly average residuals obtained for January model plotted against monthly average temperature are shown in Figure 7.7 (ZEC) and 7.11 (BUS). The monthly average temperatures are in the low range during January, February, March, November and December, in the high range during June, July, August and September and in the medium range during April, May and October. 2-p linear fit as shown in Figures 7.7 (ZEC) and 7.11 (BUS) show that relationship between average residual and monthly average temperature are very distinct.

February '92 Through March '92 Model

February through March '92 model is used to predict hourly energy use during other months and the average residual plotted against monthly average temperature, as shown in Figures 7.8 (ZEC) and 7.12 (BUS). 2-p model fit in Figure 7.8 (ZEC) shows a good fit for residuals for February onwards but under-predicts the residuals for January, November and December. However, In Figure 7.12 (BUS) the 2-p model represents the residual throughout the year very well. The relationship between average residual and monthly average temperature are again very distinct for both the examples.

April '92 Through May '92 Model

February through March '92 model is used to predict hourly energy use during other months and the average residual plotted against monthly average temperature, as shown in Figures 7.9 (ZEC) and 7.13 (BUS). The 2-p models represent the actual average residuals well in both the examples. However, the average residuals for ZEC are independent of average monthly temperature and are close to zero (Figure 7.9). In BUS (Figure 7.13), the residuals show temperature dependence throughout the range.

June '92 Through September '92 Model

June '92 through September '92 model is used to predict hourly energy use during other months and the average residual plotted against monthly average temperature, as shown in Figures 7.10 (ZEC) and 7.14 (BUS). In ZEC (Figure 7.10), the average residuals show temperature dependence, whereas in BUS, all the residual are close to zero (Figure 7.14).

SUMMARY AND CONCLUSION

The study on monthly prediction accuracy of short data sets presented in this chapter indicates that average temperature of the data set from which the model is developed, has a strong relationship with the increasing residuals of prediction for other periods of the year. However, the pattern of change in slope are not same for both the sites.

Monthly average temperature is a possible parameter that can be used to add correction to the prediction. However, the average residuals possibly will also depend on the other

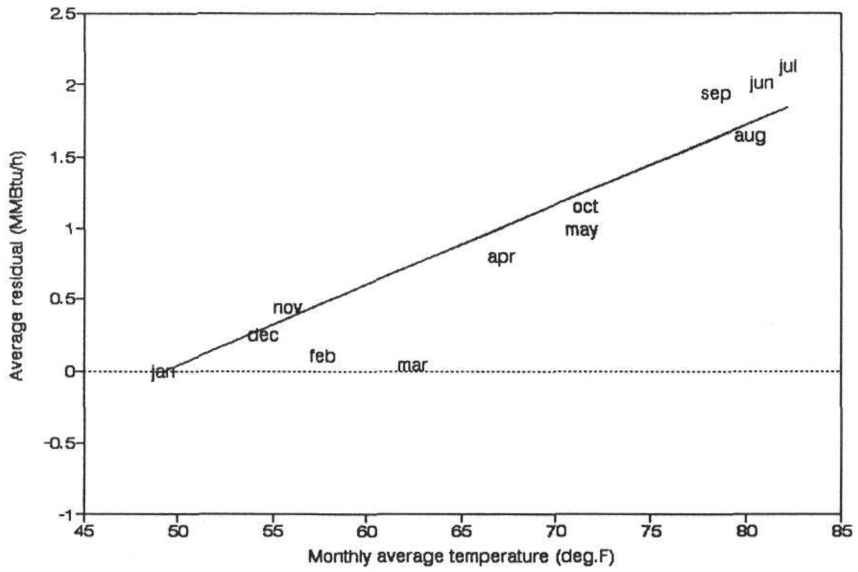


Figure 7.7 January model fit to actual residuals symbolized by the name of corresponding month. Site: ZEC.

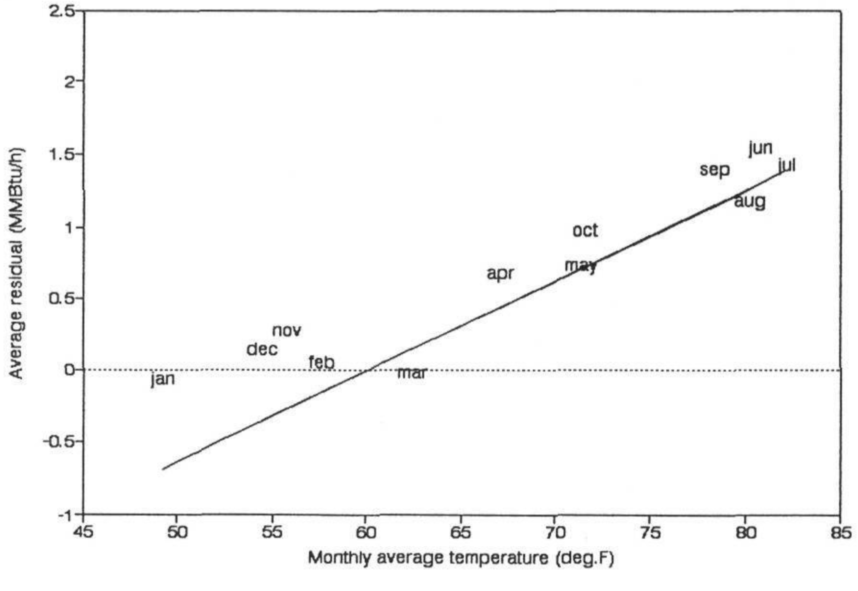


Figure 7.8 February-March model fit to actual residuals symbolized by the name of corresponding month. Site: ZEC.

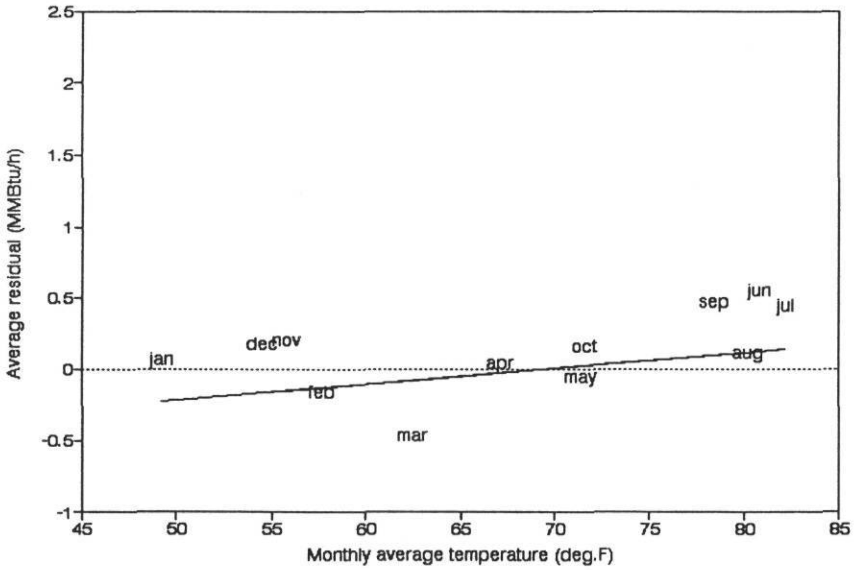


Figure 7.9 April-May model fit to actual residuals symbolized by the name of corresponding month. Site: ZEC.

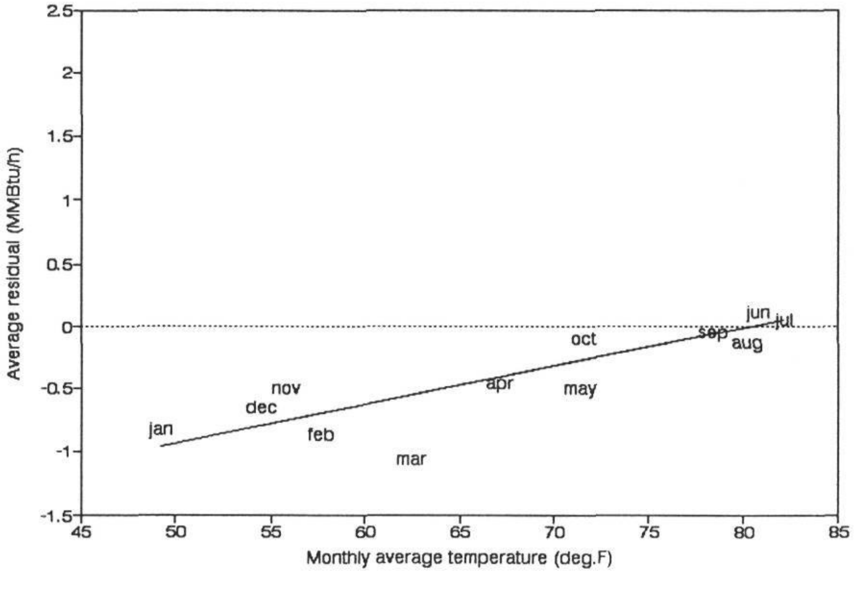


Figure 7.10 June-September model fit to actual residuals symbolized by the name of corresponding month. Site: ZEC.

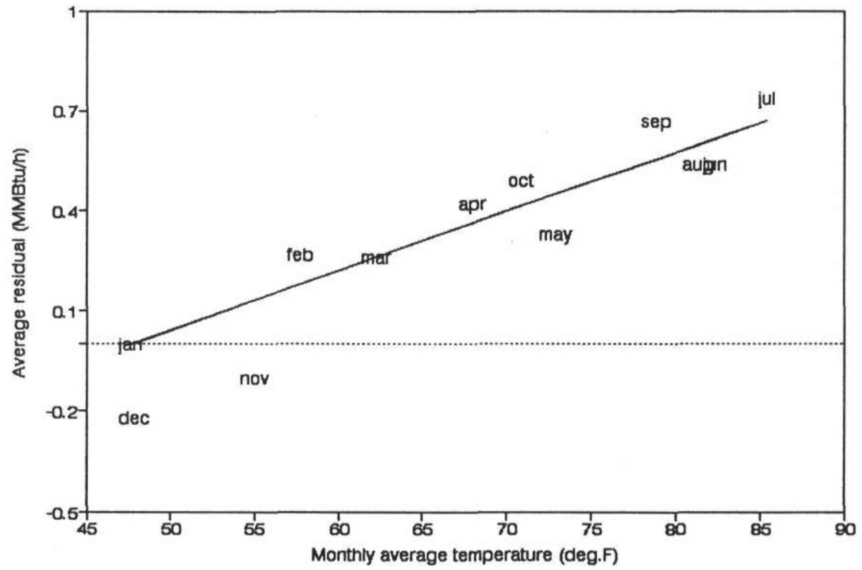


Figure 7.11 January model fit to actual residuals symbolized by the name of corresponding month. Site: BUS.

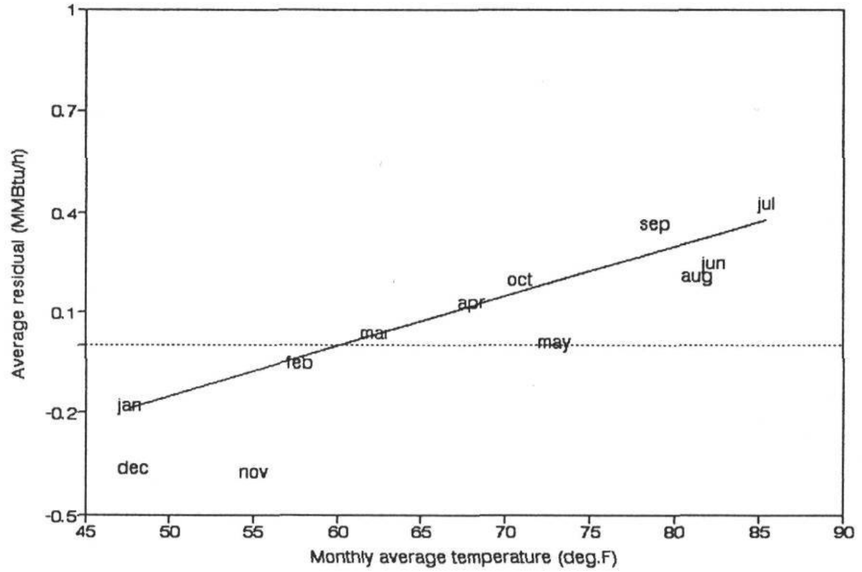


Figure 7.12 February-March model fit to actual residuals symbolized by the name of corresponding month. Site: BUS.

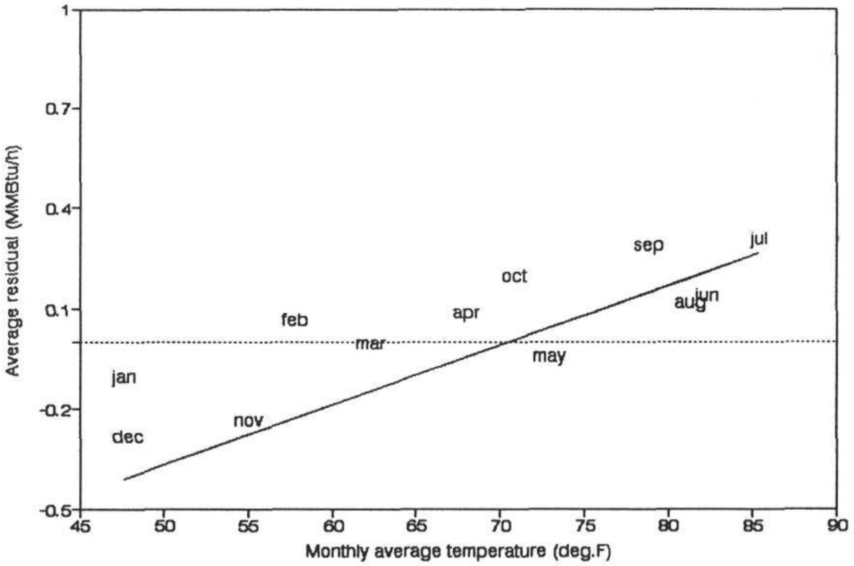


Figure 7.13 April-May model fit to actual residuals symbolized by the name of corresponding month. Site: BUS.

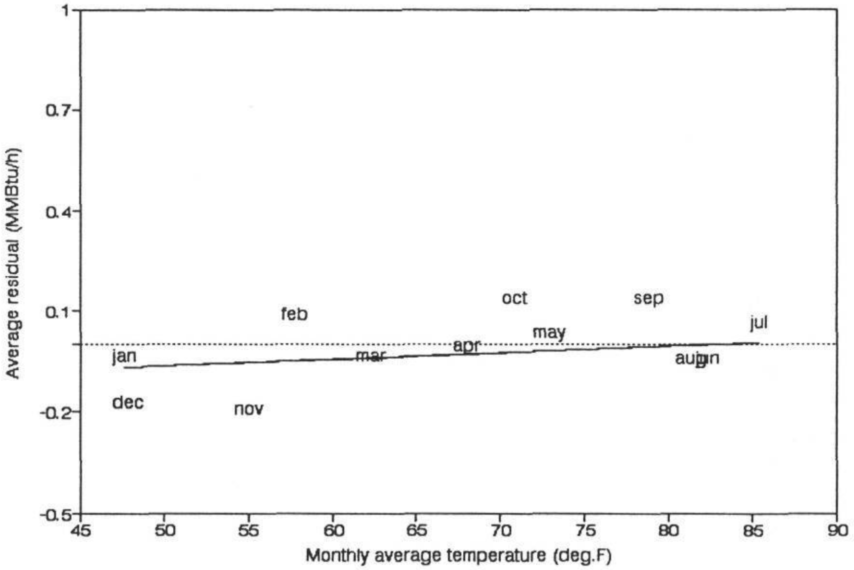


Figure 7.14 June-September model fit to actual residuals symbolized by the name of corresponding month. Site: BUS.

parameters, for example, mean energy use in a building, type of HVAC system, etc. Further study in this direction using monitored data from various buildings may help in finding a solution.

CHAPTER VIII
CONCLUSIONS AND FUTURE DIRECTIONS

CONCLUSIONS

Fourier series approach to modeling energy use has been studied earlier by other researchers (Braun et. al., 1987, Seem & Braun, 1991). However, the improvement in the model fit achieved by the approach presented here is due to the following reasons:

(i) A proper day-typing technique has been incorporated to remove the effect of major operational changes.
(ii) Separate model equations are suggested for weather independent and weather dependent energy use. While a simple Fourier series is used for modeling weather independent energy use, a combination of weather variables and Fourier frequencies is used for modeling weather dependent energy use.

The model results of energy use in different commercial buildings showed that this method gives a very close fit to the measured energy use, also only a few frequencies are adequate to characterize the energy use profiles. Individual hourly modeling approach, though have very high prediction accuracy, will have twenty four different models for each day-type. This Fourier series approach is superior in its involving only a few number of parameters, still achieving a very high prediction accuracy. Besides, this approach is suitable for load profile characterization. Because of such capability of the present approach, it suitable for retrofit savings analysis and diagnostics. When combined with stochastic approach, the present methodology may be a powerful tool for short term forecasting of energy use in commercial buildings.

FUTURE DIRECTIONS

Several areas that need more attention are as follows:

(i) A technique for forecasting may be developed by combining Fourier series with an auto regressive component (Braun et. al., 1987).
(ii) A further detailed study in the area of identifying important frequencies and relating them to the respective load profiles (both weather independent and weather dependent) will help better characterization of hourly energy use in commercial buildings.
(iii) Pursuing in the direction of finding a correction factor to improve the prediction accuracy of models developed from short data sets, as presented in chapter VII, will enable us to determine energy savings more accurately.
(iv) Fourier series approach may be useful for a better day-typing procedue. As only a few frequencies can adequately represent the operational pattern of the lighting and HVAC system of a building, these frequencies may be used to identify day-types.

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