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Risk premia and European debt crisis: CDS-hedging, Ponzi public finance and a dynamic approach to sovereign default

PhD Thesis

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Introduction

"When the crisis came, the serious limitations of existing economic and financial models immediately became apparent. Arbitrage broke down in many market segments, as markets froze and market participants were gripped by panic. Macro models failed to predict the crisis and seemed incapable of explaining what was happening to the economy in a convincing manner. As a policy-maker during the crisis, I found the available models of limited help. In fact, I would go further: in the face of the crisis, we felt abandoned by conventional tools." [108]

These words, extracted from former ECB President Jean-Claude Trichet's *Opening Address* at the ECB Central banking conference at the end of 2010, date back to two years after the unfold of global financial crisis across the *European Monetary Union* (EMU). Europe was, and still is, in constant need of improving its macroeconomic and financial stability indicators.

The European Union, through Maastricht's Treaty [46], imposed restrictions both on governments accounting and financial variables. The *Maastricht convergence criteria* provide a cap to debt level (60%) and primary deficit (3%), measured as fractions of nominal output on yearly basis.

Furthermore, the difference between each member state and the average over the three lowest inflations countries of both long term interest rates and inflation rates must not exceed 2% and 1.5% level, respectively. This criteria trace the boundary of a *stability region* the uniqueness of which had been investigated [94] early before 2008: multiple stability regions would originate multiple equilibria.

In particular, the impact on the growth of the economy of fiscal policies directed to Maastricht convergence has been recently object of sharp debate.

The existence of a meaningful debt/gdp threshold beyond which target economy suffers from recession was established in [98], and harshly criticised [59]; an interesting study on convergence patterns across the Union is [25].

The outspread of the crisis caused all countries across the Euro-area to fail in abiding by Maastricht's caps on debt; several member states largely exceeded also the floor on primary balance. Low inflation made of German long-term yield a meaningful benchmark across European interest rates, despite the rise in debt/gdp level.

The spread *vis-à-vis* German yield thus became a benchmark *risk* measure for Euro-zone economies, driving financial market sentiment.

The massive use of yields spreads as a monitoring instrument for EMU members economic policies demanded for a rigorous analysis of its evolution over time [36].

It is interesting to deduce the *exhogenous* determinants of yields spreads, in order to measure the reaction of financial market to shifts in its fundamentals.

This allows in turn to determine what are the *risk factors* driving the price of European obligations on such different patterns across countries.

The level of yields on financial markets commonly reflects risk premia over a risk free benchmark. A brief review of current literature, see [5], permits to separate the determinants of yields into common and country-specific factors: among the latters, *deterioration in creditworthiness* is obviously included.
The difference between the target country and Germany's long term yields cancels common factors and, under the assumption that Germany is a *risk-free benchmark*[1], discloses *country specific risk premia*.
The first objective of this thesis is to compare yields spreads with the measure of *credit risk premia* supplied by credit derivatives market through the prices of *Credit Default Swaps* (CDS). This allows to separate the amount of country-specific risk premia embedded in yields spreads which is attributable to credit risk from that deserving different explanations.
A similar approach to corporate entities can be found in [13]: results indicates that bond spreads over a risk-free rate within a CAPM [105] framework performs better than structural models or external ratings in predicting default.
The approach is strictly statistical, but the exhogenous factors which are considered are not suitable to sovereign borrowers. The yields which are normally considered risk-free rates are local government bonds, but the same reasoning applied to German yields in the EMU might be coarsed.
Several studies (see [34] and references within) suggest that, especially after 2010, yield spreads in Europe were mainly driven by negative market sentiments.
It is thus worth to compare yields spreads and CDS prices restricting the discussion to financial markets alone. A possible approach is to jointly model bonds and CDS prices within an exhogenous factor model: an example is [48].
The approach pursued here is different in that it remains steadily locked to financial modeling also in the methodologies.
Market yields are not directly tradeable: they are implied using the most liquid fraction of sovereign bonds by assuming the existence of a term structure for target country's obligations market and applying a numerical interpolation procedure [1].
The Nelson Siegel model [89] assumes the existence of an implicit *term-structure* for the obligation market, which can be used to discount *deterministic* future cash flows. Standard non-arbitrage assumptions provide this cash flow to be equal to the bond price as determined by market transactions.
The term structure is retrieved by minimizing the (weighted) sum of square distances between (model dependent) future cash flow and spot market price of each relevant obligation.

[1] Yields are considered to be more informative than alternative risk-free rates (such as Eonia or Euribor) since German debt market is accessible to a larger share of investors.

A yield curve is implied for each of the EMU members, and the 10-year rates are then considered[2], which correspond to the yield of a theoretical *zero coupon bond* (*zcb*) lasting ten years from trade date.
Hence, yields spreads *vis-à-vis* Germany correspond to the difference in the yield to maturity between target country and Germany's 10-year bonds paying no coupons. This difference is the risk premium that the market demands to exchange a long position from a German bond to target country's obligation.
Note that no exhogenous factors are included: under the assumption that a *zcb* is available for the 10-year maturity, yields spread is the percentage of face value which is earned when purchasing a distressed bond *selling-short* a German one.
Two questions arise straightforwardly: the first is whether Germany can be retained a reliable *risk-free* benchmark. The second is whether the spread effectively measures credit risk of target borrower, or if it embeds additional risk premia attributable to sources which are not included in such a modeling framework.
In order to answer both of them, we need an alternative method to extract credit risk premia out of financial market: the idea is to use credit derivatives.
The *CDS spread* is the cost of buying the right to swap a distressed obligation for face value in case of credit events affecting the issuer: this coupon is paid every quarter and quoted on a yearly basis, see [22].
The determinants of CDS spreads across Europe were recently investigated by the *International Monetary Fund* (IMF) [58]. Results pointed out a strong link between macroeconomic fundamentals[3] and CDS spreads from mid-2009 to the end of 2012, with a subsequent decrease due to postive outlooks towards European markets.
The process of price formation of CDS (traded on the market) in a pure financial framework is considered to be informative on the perception of credit risk, as the two counterparties agree upon the cost of swapping it.
Before 2009, the assumption of CDS prices being actually informative on credit risk was however questionable. CDS contracts were normally traded *over the counter* (OTC) and specifically tailored on the peculiar needs of the counterparties.
OTC transactions had been a major shortcoming of the financial industry during latest crisis: within such a market "*..there was no price discovery process and in turn no easy and definitive way to value the securities. The failure of the price discovery process aggravated the problems at banks and other financial firms .... by making it more difficult...reporting requirements on the value of their securities and derivatives positions.*" [40].

[2]It has also been proved, that even larger spreads are registered with *mid-term* (5-years) and *mid-short-term* (2 years) maturities, see [36].

[3]The problem of mixing data with different frequencies arises when merging CDS market with the macroeconomic conditions of the sovereign borrower [58]

In particular, CDS contracts fell short of rigorously defining several fundamental agreements entangled in the contract, such as relevant credit events or settlement method of defaulted obligations.
An early attempt devoted to resolve such issues is dated 2003: the *International Swaps and Derivatives Association* (ISDA) proposed a set of rules [62] to which parties were invited to agree so as to take off contracts misspecifications.
Additional shortcomings of such market came from the use of credit derivatives as speculative instruments. Allowing naked CDS positions and free entrance in the contracts resulted in a cumulate purchase of protection outdoing the aggregated face value of the underlying obligations [114].
Since the purpose is to compare CDS-implied credit risk premia with yield spreads, it is also important to rely on the same techniques and modeling assumptions when computing both of them. The methodology to imply yield curves is rather standard across both private data provider and regulatory authorities [1].
The ideal would be to dispose of those same techniques in a wider market, where combined positions in CDS and obligations, with hedging purposes, are tradeable. Yields-to-maturities of such positions, bearing the cost of hedging, can be subtracted from *naked* yields to retrieve the *credit risk premium* associated to target obligation.
Standardization of credit derivatives market [82] followed the outburst of the crisis in the US, and interested European credit market from the second half of 2009.
Rules for *standard CDS contracts* included *establishment of regulating authorities*, definition of *credit events*, hardwiring of *post-default auctions* intended to price and settle defaulted obligations and introduction of an *electronic trading platform* backed by a *central counterparty* [83].
The most important features of new contracts rely in the cash flow they provide. Namely, a (spot) *upfront* payment to enter into the transaction plus a *fixed standard CDS spread* to be paid on a set of (future) *standard payment dates* up to the chosen *standard maturity* of the contract.
The introduction of this new scheme for CDS cash flow assimilated derivatives contracts to their underlying asset: standard CDS are directly comparable to bonds, as composed by (negative) *deterministic future coupons* plus a payment to be made at inception (*spot price of credit risk*).
In the combined *CDS-bond market*, hedging each obligation with a CDS contract results thus in a *synthetic portfolio* with higher initial price and lower future coupons with respect to the correspondent naked obligation.
It is then possible to imply a *CDS-bearing yield curve* by assuming the existence of a term structure of such portfolios, and using the same interpolation techniques.
The difference between naked and CDS-bearing term structures defines the *term structure of credit risk premia.*

The term structure implied in this way is strictly country specific: this allows, on the one hand, to verify whether the assumption of Germany being credit risk free is confirmed; on the other, it permits a direct comparison among 10-years credit risk premium out of the implied term structure and 10-years yields spreads.
If Germany is one of the countries to be benched, an alternative benchmark rate must be included in the analysis: coherently with CDS-pricing models, we select the *Euribor* yield curve as bootstrapped from spot and swap Euribor rates [85].
Yields of hedged portfolios are *actually* obtainable[4] by trading in the CDS-bond market, hence the term structure of credit risk premia evolves in time *under the physical measure*, as built up with the sole use of market quotes [31].
Furthermore, the set of hedging portfolios induce *country-specific* and *credit-risk free* yield curves. Hedging strategies should in principle project naked yields on a common *credit-risk free pattern*, where portfolios yields should differ from each other by residuals attributable to white noise disturbances.
Nonstationarity or systematicity of differences among country specific *CDS-bearing yields* might reveal the existence of *arbitrages* on the aggregated European sovereign CDS-bond market. The eventual shortcomings in hedging strategies can in turn be attributed to *additional risk premia* provided either in the CDS contracts or in the bonds, which are generated by different sources of risk.
A relevant stream of literature focused on European yield curves in latest years, often agreeing upon the fact that factors different from credit risk were affecting Euro-zone countries obligations markets [99].
As an example, we quote [87], where *liquidity risk* is considered a major source of additional premia: the *regime switching* modeling framework adopted allows for Markov models gathering together credit *and* liquidity-related variables.
Similarly, the *safe haven phenomena* which determined the *excess liquidity* of German obligations market is the principal determinant of yields spreads in crisis [36].
In 2011, Jean-Claude Trichet, on behalf of the European Central Bank, stated the *"intellectual challenges to financial stability analysis"* which Europe was going to face, discussing new objectives of economic research in the Monetary Union.
He criticised a pure statistical approach as it *"..does not assume very high levels of rationality of economic agents, strong tendency towards equilibrium situations or universal efficiency of financial markets.."* [109]. Specifically, one of the challenges: *"..concerns how financial instability interacts with the macroeconomy"*[*Ibd.*], and researchers were invited to *"..integrate more realistic characterisations of financial systems in macroeconomic models and to capture the relevant nonlinearities that are so typical for the unfolding of financial crises..."*[*Ibd.*].

[4]Provided target investor is allowed to easy access the CDS-bond market, which is the case for large investment institutions holding most of European debt [35].

The mutual impact of macroeconomy and the term structure of interest rates has been widely investigated from a statistical perspective: an interesting overview is provided in [115]. A relatively recent review [76] argues that the *expected future values* of macroeconomic variables, as measured by international surveys on economic forecasts, are likely to affect yields more than their current values.
The explicit inclusion of macroeconomic variables in statistical yields models traces back instead to the seminal ideas in [12] and [39], refined in [38].
This approach jointly models macroeconomic and financial variables in *factor models* [53] for yield curves. A recent investigation in this sense [93] provides for a *structural* joint VAR modeling to be identified and estimated.
The idea is to express all variables according to short-term and long-term latent factors and proceed to statistical estimation, so as to retrieve a complete model for macroeconomic and financial (*macrofinancial*) variables.
Another possibility is to model financial variables in a Markov-chain regime switching model [54], where macroeconomic variables induces a change in the *status* of the economy as soon as they exceed estimated thresholds.
Despite the wide range of different approaches, none of the aforementioned is strictly based on economic principles. Econometrics is used to model *a priori* the possible net of relationships among variables, and not only as the mean to get to conclusions.
The second objective of this work is to follow the guidances of ECB and, backed by economic theory, create a *macrofinancial scoring system* by constructing a *physical default probability measure* in the spirit of [87].
Macroeconomic variables are expected to improve market-based distress measures by providing for additional sources of risk which might explain the unexpected widening of spreads across the Euro-area [36].
Liquidity of cross-countries obligation market appears to be a major source of *surprise effects* in yields spreads, because excess demand of German obligations caused a rapid decrease of the correspondent yields, with a consequent increasing of spreads. On the contrary, Greece experienced default when no investor was willing to purchase Greek debt anymore.
Demand for sovereign obligations is the relevant concept to be analyzed: when a country is facing financial distress and is forced to issue more debt, *the new debt supply may demand for additional risk premia*, which in turn depend on the *risk attitude* of investors towards target obligations.
We consider each sovereign obligations market as a *single-good market*, where the *quantity* supplied is public debt and the correspondent *price* is the (average) yield on debt: a similar approach is pursued in [55]. In order to preliminarly assess our intuitions, we start by modeling a *linear demand function*.

Demand functions are generally unobservable, especially when dealing with public finance [19]: the first approach to macrofinancial modeling will be straightforward. Namely, feedback effects among yields and public debt are estimated within a bivariate *vector auto regression* (VAR).
The first issue to be tackled is *which* yields are to be used. The ECB supplies quarterly levels of interest paid on debt, out of which an *internal* yield can be retrieved using custom accounting techniques [49]. Another candidate is current market yield, with maturity equal to *debt duration*, averaged across Europe.
A statistical comparison among such yields is pursued, and the impact of debt/gdp level on both these interest rates is inferred: meaningful coefficients will be rough estimates of investors' risk aversion towards target obligations market.
An interesting issue is suggested by the evolution of yields and debt market of Portugal and Ireland: more precisely, a high level of debt *variation* (*debt speed*) was registered in such countries as soon as yields spreads dramatically widened.
In a demand/supply framework, the role of debt speed is easily recognizable [102]: if any distressed sovereign borrower is forced to issue more debt in a given period, he must either gather new investors or demanding further trustworthiness from the existing ones, by offering a higher yield in order to match the demand curve.
The main shortcoming of such a direct approach is the implicit assumption of risk aversion being constant across pre and post crisis periods, which is clearly unrealistic [104]. Risk appetites are hidden variables containing investors' expectations on future creditworthiness as well as forecasts on eventual additional risks [76].
Discovering the underlyings determinants of such risk appetites in a macrofinancial context is simply another way to formulate the initial problem.
In line with the speech [116] of the *Federal Reserve* (Fed) current president Janet Yellen, and inspired by the seminal ideas in [42], we advocate the background supplied by H. Mynsky's with his *Financial Instability Hypothesis* (FIH) [86].
Particularly, once the concept of *loan* is broadened to *debt/gdp ratio*, Minsky's classification of borrowers is considered. A *Ponzi* borrower within a generic loan agreement is defined as the investor who, within a period between two interest payments dates, is not able to repay neither a fraction of these interests, and is forced to augment his debt level to face his duties of debtor.
Ponzi schemes in public debt had already been investigated in [21], but from a macroeconomic perspective only. Here, according to Minsky's original definition, we take the *positive part* of debt speed over a quarter as representant of the amount of Ponzi investment (which we briefly refer to as *actual Ponziness*) that target borrower is demanding within that period. A possible consequence of perpetuated Ponzi borrowing might be a liquidity drain up of the correspondent obligations market [42].

The idea is to monitor eventual Ponzi schemes of target sovereign borrower, measured with respect to the whole financial market. Actual Ponziness is thus transformed into *market* Ponziness by substituting the internal yield underlying public debt with the yield quoted by financial markets.
Market Ponziness (or, briefly, the *Ponzi score*) is retained to be a relevant indicator of financial distress which serves our purposes: namely, it embeds macroeconomic and financial variables, it is economically founded, and also country specific.
Alternatively, a second *score* can also be constructed by taking differences among target country and Germany's market Ponziness, in the same spirit of market yields spreads (*spreaded Ponzi score*).
The first issue to tackle is to ensure that the construction of such scorings is not redundant, in the sense that they embed different signals from those included in yields or yields spreads, respectively.[5]
In order to validate the two Ponzi indexes using econometric techniques, we must firstly infer the statistical properties that two custom sovereign risk measures share with each other. Comparisons among yields spreads and *CDS implied* default probabilities is investigated in [60] and [80], as well as in the aforementioned [87].
Here, we chose to compare yield spreads to *default intensities*[6] in a pure econometric framework. The estimated feedback effects retrieved from statistics will be considered relevant in assessing the linkages among the two.
The (positive) scores induce two *physical* default probability measures that can be compared with CDS-implied default pobability by fitting bivariate VAR models.
In case the two implied measures share similar properties to that shown by yields spreads and default intensities, then the scoring can be considered meaningful.
The econometric analysis will also be a way to monitor how much of the variability in credit risk perception can be measured using a combined macrofinancial set of variables, and also to see the distance between financial distress measures and a score built up with a relatively small set of variables.
Since statistical analysis will be the mean to drag out conclusions, and guess possible answers to the questions we asked, it is worth to disclose the methodology that is used to gather informations from data.
The VAR approach which is pursued is object of critcisms as its *endogenous* nature is often considered *atheoretical* and noncausal [32]. Particularly, the determinants underlying the variables under analysis will not be exhogenously modeled here.
We start to observe that comparing different portfolio yields means to compare statistical hedging techniques, thus a pure mathematical approach is fully justified.

[5]If this was the case, it could be possible to assert that inclusion of macroeconomic variables improves distress measures with respect to market yields.

[6]As implied through the ISDA CDS Standard Model, see appendix C.

Concerning yields and macroeconomic variables, the VAR approach is adopted as the relationships are conditional expectations in a theoretical model. The aim is to estimate a latent variable which is deemed to embed all the information that any *complementary* set of exhogenous variables is able to supply.
As credit scores are compared, CDS-implied default probabilities and yields spreads will always be considered as meaningful benchmarks, and their determinants not exhogenously modeled.
Their mutual relationship will be useful in order to collect common features of risk measures to be used as the basis for comparing default probabilities.
The variables will be typically nonstationary: the mutual feedback effects among them can be splitted into short-run effects, including both feedback effects on lagged differences and residual correlation, and long-run effects, measured by eventual cointegrating relations. When the comparison deals with portfolio yields, the higher data frequency suggests short-run analysis to be the most relevant. Cointegrating relations will instead acquire major importance when a score is implied at quarterly frequency on a larger time window.
The thesis is divided in three chapters plus a brief conclusion, and three appendices. Chapter 1 introduces the set of econometric instruments which are needed in order to proceed to statistical analysis. Univariate processes are presented in section 1.1; useful results concerning both lag number selection procedure and unit-root tests are provided in section 1.2. Vector processes are defined in section 1.3: the approach to lag selection procedure is different because of the singularity of asymptotics estimators of covariance matrices in case of cointegrated processes. Specifically, we use the *lag-augmented procedure* defined in [41] so as to retrieve an algorithm similar to the univariate case. Johansen's framework for cointegrated VARs is presented in section 1.4, and a two-stage parameter estimation is reported in section 1.5. Section 1.6 summarizes the general procedure, while section 1.7 discloses custom statistics for checking whiteness of residuals.
Chapter 2 is divided in two parts: section 2.1 shortly discusses the standardization process which interested the credit derivatives market, focusing on CDS contracts. The discussion is based on [28], which in turn explores regulatory and technical documentation of credit derivatives market, extracting the pricing model directly out of contractual clauses. The second part deals with the construction of CDS-bearing yield curves, and is based on the ideas explored in [27]: a brief revision of interpolation methods is presented in section 2.2, while econometric results are gathered in section 2.3. Specifically, subsection 2.3.1 concerns Germany and its role of credit-risk-free benchmark; subsection 2.3.2 compares yields spreads and CDS-implied credit risk premia, while subsection 2.3.3 compares the two synthetic credit risk free yields and analyzes the basis.

Chapter 3 introduces some stylized facts about the evolution of accounting variables in Europe since the introduction of Euro (section 3.1). Section 3.2 presents the theoretical demand/supply framework and describes the dataset which will be used. Section 3.3 proceeds with a direct econometric approach.
It starts with yields analysis (subsection 3.3.1) and proceeds to measure mutual impacts of yields and debt level (subsection 3.3.2) and of yields (yields spreads) and debt speed (subsection 3.3.3).
The last part revises and enhances the ideas in [29]: the problem of time variating risk appetites in a demand/supply framework is introduced in section 3.4, where the construction of the two *scores* is also presented, and econometrically discussed in section 3.5. A comparison among scores and yields (yields spreads) is presented in subsection 3.5.1, while subsection 3.5.2 investigates the relevant properties that two custom distress measures share with each other. A direct comparison among scoring and default intensities is pursued in subsection 3.5.3, while subsection 3.5.4 summarizes results.
After the conclusions, the thesis presents three appendices collecting statistical tables as well as useful technical results which will be repeatedly used.
Appendix A, B and C contains statistical tables, econometric tools and financial tools respectively: all references to equation, sections, tables and figures starting with a capital letter refers to the correspondent appendix.

1 Econometric modeling

Econometric models offer a wide range of different approaches devoted to withdraw informations out of data: the choice is to lean towards the endogenous[7] parametric approach of vector autoregressive moving average (VARMA) processes.
The approach is maintained in both chapters 2 and 3, independently from the class of dataset examined: it is preferable to rely on the same techniques so that conlcusions out of observations are drawn using a uniform methodology.
The strategy is to assume a distribution (often gaussian) for stochastic innovations in time, determine the degree of self-forecastability of the time series and construct a model based on a parameters set which is retrieved out of data; construction of the parameters estimators, together with their interpretation and siginificance, are the object of this chapter. A theoretical model which is fitted to data will be also called *data generating process (dgp)*; cross-country analysis will not be performed in a strict statistical sense: the country in exam will be considered as fixed, hence results are presented avoiding additional indexations.
The topics discussed in the next two chapters are grounded on the econometric analysis of time series sampled at different frequencies, hence time-steps among observations will be specified therein, together with sample sizes.
A moving average process will also be performed *ex-ante* so as to smoothen time series: the *observations* will be the smoothened data.
Any n-dimensional vector process under analysis is built up by stacking target n variables into a matrix:

$$[Y^1 \dots Y^n] \in \mathbb{R}^{T \times n}$$

being T the (common) size of datasets and, given any matrix Θ, being Θ' its transpose; each row $(Y^1_t \dots Y^n_t)$ will be the t-th observation coming out of the *vector* dgp Y. Before a vector model is built up, it is necessary to step back in order to discuss single variables modeling, which will be useful in addressing multivariate estimation: this will be the object of section 1.1 and 1.2, while section 1.3 to 1.5 will deal with vector analysis.
Section 1.6 summarizes the estimation procedure while section 1.7 concludes the chapter by focusing on the peculiar interpretation which will be given to statistical results throughout this work.
Basic tools of univariate statistics are not reported here[8]; main results on multivariate processes are instead summarized in appendix B.

[7] The epiteth *endogenous* refers to the dynamics underlying such processes, where no exhogenous explanatory variable is provided; given an initial condition, trajectories evolves according to the same process' history plus a conditionally independent innovation.

[8] Refer to [56] and [57] for an exhaustive treatment.

1.1 Univariate data generating processes

Let y be a univariate dgp and assume that it can be written as:

$$y = x + d^* \tag{1.1.1}$$

where the *support process* x represents the purely stochastic part of y and d^* is a *deterministic trend component*; the most general deterministic function provided[9] in the present work will be linear (affine):

$$d_t^* = d_0^* + d_1^* t \tag{1.1.2}$$

Statistical properties clearly regards the stochastic part only, thus definitions will be given for x and extended to y with little abuse of terminology.
Define the *difference operator* Δ so that $\Delta\zeta_t = \zeta_t - \zeta_{t-1} = (1 - L)\zeta_t$ for any process ζ, where L is the usual lag operator, defined so that $L^N\zeta_t = \zeta_{t-N}$, and 1 is to be intended as the identity operator.

1.1.1 ARMA processes: general features

Definition 1.1.1. *The (one-dimensional) dgp process x is an ARMA(p, p') process if defined by:*

$$\gamma(L)x_t = \phi(L)u_t \quad , \quad t \geq 1 \tag{1.1.3}$$

where u_t is a (one-dimensional) gaussian white noise process (definition B.1.1), $\gamma(L) := 1 - \gamma_1 L \cdots - \gamma_p L^p$ is the AR part, $\phi(L) := 1 + \Phi_1 L \cdots + \Phi_{p'} L^{p'}$ is the MA part, with $\gamma_i, \phi_h \in \mathbb{R}$; in particular, we call AR(p) and MA(p′) the subfamily of models obtained by setting either $\phi(L) \equiv 1$ or $\gamma(L) \equiv 1$, respectively. Obviously, $\max\{p, p'\} > 1$.

The choice here is to limit the analyses to the AR(p) subclass of models so as to maintain conditional independence of the process from the whole history of innovations ([78], chapter 12). The assumption $x \sim AR(p)$ can be combined with (1.1.2):

$$\begin{aligned} x_t =& \gamma_1 x_{t-1} + \cdots + \gamma_p x_{t-p} + u_t \\ d_t^* =& d_0^* + d_1^* t \end{aligned} \tag{1.1.4}$$

so as to define the dgp y out of equations (1.1.4) using (1.1.1). This implies:

$$y_t = \gamma_1 y_{t-1} + \ldots \gamma_p y_{t-p} + d_0 + d_1 t + u_t \tag{1.1.5}$$

[9] Specific single-variables analysis concerning deterministic terms will not be performed: testing deterministic terms in each component can be misleading in a multivariate context, when cointegrating relation arise ([78], p. 244).

where d_0, d_1 are suitable parameters retrieved from (1.1.4) with little algebra. Equation (1.1.5) can be rewritten in vector form as:

$$y_t = [\boldsymbol{\gamma}\ \mathbf{d}] \cdot [\ y_{t-1} \dots y_{t-p}\ 1\ t\ ]' + u_t \tag{1.1.6}$$

where $\boldsymbol{\gamma} = [\gamma_1 \dots \gamma_p]$ and $\mathbf{d} = [d_0\ d_1]$; rewriting y in VAR-operator form:

$$(1 - \gamma_1 L - \dots - \gamma_p L^p) y_t = d_t + u_t \tag{1.1.7}$$

allows to define the *characteristic polynomial* of the process:

$$(1 - \gamma_1 z - \dots - \gamma_p z^p) \tag{1.1.8}$$

as the polynomial associated to the operator on the left side of (1.1.7).
All roots of (1.1.8) are assumed to lie on or outside the complex unit circle[10]; an ARMA process x admitting real roots *strictly* outside the unit circle is called *stable*, in symbols, $x \in \mathcal{I}(0)$. The definition of *q-integrated* processes comes straightforward.

Definition 1.1.2. *A univariate ARMA process x is called integrated of order q, in symbols $x \in \mathcal{I}(q)$, if $\Delta^q x$ is stable or, equivalently, $\Delta^q x \in \mathcal{I}(0)$, and $\Delta^{q-1} x$ is not.*

1.2 AR processes: lag numbers and unit roots

This section is not devoted to parameter estimation, which will be the object of multivariate analysis; the task here is to design procedures aimed to determine two features of single-valued time series.
The first one is to determine a correct number of lags for univariate time series, so that given any vector process $Y = (Y^1 \dots Y^n)$ a number of lag $\hat{p}^j$ can be estimated for any of the components Y^j.
The second procedure is instead devoted to infer the eventual presence of *unit roots*: two popular approaches (see [57], p. 435) are used when dealing with non-stationary processes. Namely, trend stationary processes are those data generating processes which become stationary after a linear deterministic time-trend is subtracted, while *unit-root* processes will be those admitting $z = 1$ as a root of (1.1.8).
Before approaching multivariate modeling, both the number of lags and unit-roots must be settled for any of the components: since the number of lags is independent from the presence of unit roots ([78], Proposition 8.1, p. 326), it will be the first to be retrieved.

[10] If the true model provided, for example, real roots *inside* the unit circle, then we would be fitting models where true variances diverge with exponential rates: such models would however be unrealistic for most economic data ([78], p. 242)

1.2.1 Determining the number of lags

A first modeling restriction is the decision of selecting a uniform[11] (with respect to each component j of the vector) upper bound $\bar{p} \in \mathbb{N}$: this represents the maximum number of lags provided by each single-variable model; notice that, as a consequence of uniformity, $\bar{p}$ will also be the maximum number of lags for the vector process Y. The estimated number of lags $\hat{p}$ is retrieved for each component using an iterative procedure (see [57], p. 530); a simple hypothesis testing is provided within each of the iterations. The aim is to shorten the process' memory in the true model so as to diminish the number of lags as much as possible and reduce the number of parameters to be estimated.
The starting condition is $\hat{p}_1 = \bar{p}$: for $h = 1 \ldots \bar{p} - 1$, the h-th iteration tests:

$$H_0 : y \sim AR(p_h) \quad , \; \gamma_{p_h} = 0 \quad vs \quad H_1 : y \sim AR(p_h);$$

where the null is equivalent to $y \sim AR(p_h - 1)$; using equation (1.1.6), the null at the h-th iteration can be rewritten as:

$$H_0 : [R^{(h)} \; 0_{1\times 2}] \cdot [\gamma_1 \; \ldots \; \gamma_{p_h} \; \mathbf{d}]' = 0 \tag{1.2.1}$$

where $R^{(h)} = [0 \ldots 0 \; 1] \in \mathbb{R}^{1\times p_h}$. Iterations are made up of two steps, which are repeated until the number of lags has been estimated: here, T is the sample size.

- **Step 1**: Compute the OLS estimator of $\boldsymbol{\gamma}$ in (1.1.6) and call $\hat{u} \in \mathbb{R}^{T\times 1}$ the vector of realized residuals; compute the restricted OLS estimation (see [56], pp. 157-163), with restrictions defined by (1.2.1), and define $\hat{u}_R$ the vector of realized residuals in the restricted model.

- **Step 2**: Under H_0, the test statistic:

$$\mathbf{F}^{(h)} = (T - p_h - 1) \cdot \frac{(\hat{u}_R'\hat{u}_R - \hat{u}'\hat{u})}{\hat{u}'\hat{u}} \sim \mathbf{F}(1, T - p_h - 1) \tag{1.2.2}$$

  has an $\mathbf{F}$-distribution with $(1, T - p_h - 1)$ degrees of freedom (see [57], pp. 205-207), being $T - p_h$ the effective sample size under the alternative H_1.
  If the null is accepted, then $p_{h+1} = p_h - 1$ and we repeat the procedure; else, the estimated number of lags is $\hat{p} = p_h$.

At the end of the procedure, an estimated number of lags $\hat{p}$ is available for each component of the vector process Y under exam; we keep on writing p so as to lighten notations, but we stress that single-variables numbers of lags are assumed to be known at this stage.

[11] The bound is assumed to be uniform so as not to privilege any of the components in terms of self-explainability and source of informations within multivariate context.

1.2.2 Testing for unit roots

The issue of unit-roots is fundamental for hypothesis testing on estimated parameters, as well as for their interpretation; a wide range of different approaches can be pursued, leading to different test-statistics: an interesting survey is [106].
The most common are the Phillip Perron (PP) [96] and the (Augmented) Dickey Fuller (ADF) [37] tests: the former is based on a statistical adjustment for serial correlation in a simple AR(1) model, while the latter adds lags to the autoregression in the test statistic itself (details can be found in [57], chapter 17).
Since data samples will always be statistically *small* as compared to the number of observations, we follow the indication in [103] and proceed with ADF test procedure; three different tests will be performed:

- **Test 1** (T1) :

$$H_0: \quad y_t = y_{t-1} + c_1 \Delta y_{t-1} + \cdots + c_{p-1} \Delta y_{t-p+1} + u_t$$
$$H_1: \quad y_t = c_0 y_{t-1} + c_1 \Delta y_{t-1} + \cdots + c_{p-1} \Delta y_{t-p+1} + u_t \ , \quad c_0 < 1$$

- **Test 2** (T2) :

$$H_0: \quad y_t = d_0 + y_{t-1} + c_1 \Delta y_{t-1} + \cdots + c_{p-1} \Delta y_{t-p+1} + u_t$$
$$H_1: \quad y_t = d_0 + c_0 y_{t-1} + c_1 \Delta y_{t-1} + \cdots + c_{p-1} \Delta y_{t-p+1} + u_t \ , \quad c_0 < 1$$

- **Test 3** (T3) :

$$H_0: \quad y_t = d_0 + y_{t-1} + c_1 \Delta y_{t-1} + \cdots + c_{p-1} \Delta y_{t-p+1} + u_t$$
$$H_1: \quad y_t = d_0 + d_1 t + c_0 y_{t-1} + c_1 \Delta y_{t-1} + \cdots + c_{p-1} \Delta y_{t-p+1} + u_t \ , \quad c_0 < 1$$

Each test provide a true unit root in the null. Tests T1 and T2 provides the alternative AR model with $c_0 < 1$; the alternative under T3 is a trend stationary process. The test statistic $\mathbf{Z}_{ADF}$ has a distribution which depends on the test being performed: its explicit form is not reported it here, being it custom in most of the econometric literature (see for example [57], pp. 523-25).
Furthermore, the asymptotic distribution of $\mathbf{Z}_{ADF}$ is non-standard, hence its value has to be compared to specific p-values which were originally tabulated in [37]; here, however, those reported in [57] (Table B.7, p. 764) are used.
It could happen to infer mixed responses from the three tests, hence discussing results on a case-by-case basis will be opportune.
Next section collects multivariate econometrics results: each of the components of vector Y is now equipped with an estimated lags number, and the eventual presence of any unit root in them has been determined.

1.3 Multivariate data generating processes

Modeling vector processes goes along the lines of univariate processes, although several features originate only in such a multivariate context, thus a peculiar analysis will be devoted to this case.
Let $Y \in \mathbb{R}^n$ be a vector dgp: similarly to the univariate case, equation (1.1.1), set:

$$Y = X + D^* \tag{1.3.1}$$

where the vector support process X again represents the pure stochastic part and D^* is the deterministic component, which again will be affine:

$$D_t^* = D_0^* + tD_1^* \in \mathbb{R}^n.$$

Statistical properties of process Y, are defined to be those of its pure stochastic part X: definitions will concern this latter, and naturally extended to Y.

1.3.1 VARMA processes: general features

Definition 1.3.1. *The process X is a n-*dimensional VARMA(p, p') *process if defined by:*

$$\Gamma(L)X_t = \Phi(L)U_t \quad , \quad t \geq 1 \tag{1.3.2}$$

where U_t is a n-dimensional gaussian white noise process (definition B.1.1) $\Gamma(L) := I_n - \Gamma_1 L \cdots - \Gamma_p L^p$ is the AR part, $\Phi(L) := I_n + \Phi_1 L \cdots + \Phi_{p'} L^{p'}$ is the MA part (with $\Gamma_i, \Phi_h \in \mathbb{R}^{n \times n}$) and I_n is the $n \times n$ identity operator; in particular, we call VAR(p) and VMA(p′) processes the subfamily of models obtained by setting $\Phi(L) \equiv I$ or $\Gamma(L) \equiv I$, respectively. Obviously, $\max\{p, p'\} > 1$.

Results on matrix operators $\Theta(L)$ can be stated in terms of their correspondent polynomials matrices $\Theta(z)$; observe that $z \in \mathbb{C}$ is a univariate variable because the lag operator works equivalently on any of the components; given two generic $X, U \in \mathbb{R}^n$, consider:

$$\Gamma(z)X = \Phi(z)U$$

left multiplication by $\Gamma(z)^{adj}$ yields

$$[\det \Gamma(z)]X = \Gamma(z)^{adj}\Phi(z)U \tag{1.3.3}$$

The *characteristic polynomial* of X is now $\det \Gamma(z)$: a process X is thus said to admit unit roots if $\det \Gamma(1) = 0$, which motivates the definition that follows.

Definition 1.3.2. *A* VARMA(p, q) *process* X *is called* stable *if*

$$\det \Gamma(z) = \det(I_n - z\Gamma_1 - \cdots - z^p\Gamma_p) \neq 0 \qquad \forall z \in \mathbb{C} : |z| \leq 1 \tag{1.3.4}$$

that is, all roots lie strictly outside the unit circle.

It will always be *assumed* that the stability condition (1.3.4) holds for true parameters Γ_i when $|z| < 1$, as mentioned in previous section.
Let $q \leq p$ be the *exact* number of unit roots of $\det \Gamma(z)$; using basic algebraic results, we can rewrite (1.3.3) as:

$$[\Psi(z)(1 - z)^q]X = \Gamma(z)^{adj}\Phi(z)U \tag{1.3.5}$$

with $\Psi(1) \neq 0$. We can exploit again duality between polynomial matrices and lag polynomials, and rewrite (1.3.5) in terms of matrix operators; this yields:

$$[\Psi(L)(1 - L)^q]X_t = \Gamma(L)^{adj}\Phi(L)U_t \tag{1.3.6}$$

It is worth to stress that the operator on the left side is *diagonal*, and acts equivalently on any single component of X_t, so that for $j = 1 \ldots n$:

$$[\Psi(L)]\Delta^q X_t^j = \left(\Gamma(L)^{adj}\Phi(L)\right)^j U_t \tag{1.3.7}$$

and the operator $\Psi(L)$ is scalar.
Since U is a (gaussian) white noise, direct inspection shows that $\left(\Gamma(L)^{adj}\Phi(L)\right)^j U_t$ is a (gaussian) white noise process too, according to definition B.1.1: hence, (1.3.7) is the ARMA form of $\Delta^q X_j$ with characteristic polynomial $\Psi(L)$, for any $j = 1 \ldots n$. Moreover, since $\Psi(1) \neq 0$, the ARMA process $\Delta^q X_j$ is stable for any j, in the sense of definition 1.1.2: this motivates the following.

Definition 1.3.3. *A* VARMA(p, p') *process* X *is called* integrated of order q, *in symbols* $X \in \mathcal{I}(q)$, *if* $\Delta^q X = (1 - L)^q X$ *is stable while* $\Delta^{q-1} X$ *is not.*

Let q be the (estimated) integration order of vector process X and q^j the (estimated) integration order of the j-th component X^j, so that $\Delta^{q_j} X^j \in \mathcal{I}(0)$, for each j; if $q_\infty := \max_{j=1 \ldots n} q^j$, then $q_\infty \leq q$ because $\Delta^q X^j \in \mathcal{I}(0)$ for each j, as previously observed. The integration order of each component X^j is determined by iterated application of unit root tests of section 1.2.2 to the process and its higher order differences: the estimated $q^j \geq 0$ is the first positive integer for which the tests unanimously rejects the null.
Most of the time series under analysis will show $q^j \leq 1$: eventual exception will be specifically discussed when necessary. Hence $q \leq 1$ is assumed, so that any VAR process $X \in \mathcal{I}(1)$ if $X^j \in \mathcal{I}(1)$ for at least one j.

1.4 VAR processes: cointegrating relations

The modeling framework will be that of VARMA$(p, 0)$≡VAR(p) processes, as in the single variable case, and for similar reasons; thus $\Phi(L) \equiv I$ and $p > 1$.
Theoretical underlyings which are presented here will be limited to those needed for $\mathcal{I}(1)$ random vectors[12]: as previously mentioned; expliciting the lag operator yields the VAR(p) form:

$$X_t = \Gamma_1 X_{t-1} + \cdots + \Gamma_p X_{t-p} + U_t \tag{1.4.1}$$

Another representation, which is very useful in the framework of integrated VAR(p) processes, is the *vector error-correction* VEC$(p-1)$ representation:

$$\Delta X_t = C_0 X_{t-1} + C_1 \Delta X_{t-1} + \cdots + C_{p-1} \Delta X_{t-p+1} + U_t. \tag{1.4.2}$$

where $C_0 := I_n - \Gamma_1 - \ldots \Gamma_p$, $C_{p-1} = -\Gamma_p$ and $C_j = -\Gamma_{j+1} \cdots - \Gamma_p$.
The reason for the introduction of this second representation is the concept of *cointegration*, pioneered by Engle and Granger [45]: here it will be introduced as in Lütkepohl ([78], chapter 6), who in turns follows the general lines of Johansen's formalization [74].

Definition 1.4.1. *The integrated n-dimensional VAR(p) process $X \in \mathcal{I}(q)$ is said to be* cointegrated *if there exits $\beta \in \mathbb{R}^n$ and $q' \in \mathbb{N}^+$ such that $\beta' X_t \in \mathcal{I}(q - q')$; β is called a* cointegrating vector *for X.*

This definition is slightly uncommon in the sense that it does not explicitely require that *all* components of X are $\mathcal{I}(q)$; for example, if $X \in \mathcal{I}(1)$ and $X^j \in \mathcal{I}(0)$, then setting $\beta = e_j$ to the j-th vector of the canonical basis, $e_j' X_t = X^j$ is a cointegrating relation itself according to defintion 1.4.1.
The advantage lies in simplification of terminology and notations; single case of study will isolate eventual *spurious*[13] cointegrating relations ([78], p. 246).

Definition 1.4.2. *The rank $r = \operatorname{rk} C_0$ of C_0 is called* cointegration rank *of X and C_0 is called* cointegration *(or* cointegrating*) matrix.*

Observe that $X \in \mathcal{I}(0)$ if and only if $r = n$; assume $X \in \mathcal{I}(1)$; then $\Delta X \in \mathcal{I}(0)$ so, since U is white noise, $C_0 X_{t-1} \in \mathcal{I}(0)$; in particular, the rows C_{0i} are cointegrating vectors (definition 1.4.1), thus justifying definition 1.4.2.
In this case, the assumption $X \in \mathcal{I}(1)$ implies that the cointegrating matrix has reduced rank $0 \leq r < n$.

[12] Johansen [74] explores in detail the case of $\mathcal{I}(q)$ vector autoregressive models up to $q = 2$, which he considers to provide a sufficiently broad class of dgp for most of the cases of interest.

[13] It is of particular importance to remove spurious relations especially when constructing EGLS estimators, because of the normalization procedure, as will be shown in next section.

In order to complete the underlying building blocks, and define the dgp Y, an affine deterministic term is introduced, ; the VEC$(p-1)$ form of Y is:

$$\Delta Y_t = C_0 Y_{t-1} + C_1 \Delta Y_{t-1} + \cdots + C_{p-1} \Delta Y_{t-p+1} + D_0 + D_1 t + U_t. \tag{1.4.3}$$

with

$$D_0 = C_0(D_1^* - D_0^*) + (I - C_1 - \cdots - C_{p-1})D_1^* \ ,$$

and

$$D_1 = -C_0 D_1^*$$

The peculiar form for deterministic component must be estimated together with the cointegration rank, so as to identify the best-fit model subfamily, as cointegration tests are very sensitive to the form chosen for D.
In this sense, it is useful to present a further representation of Y which will be useful in estimating different cointegrated models, corresponding to different restrictions on the deterministic component.
Namely, assume that the cointegrating rank $r = \operatorname{rk} C_0$ is known: C_0 can thus be written as $C_0 = AB'$, being A, B two $n \times r$ matrices of full column rank.
The r-dimensional column span of B is called *cointegrating basis*; equation (1.4.3) can be rewritten as:

$$\Delta Y_t = D_0 + tD_1 + AB'Y_{t-1} + C_1 \Delta Y_{t-1} + \cdots + C_{p-1} \Delta Y_{t-p+1} + U_t. \tag{1.4.4}$$

Now projecting D on A allows to decompose:

$$\begin{array}{rcl} D_0 & = & AD_0^{\parallel} + A^{\perp} D_0^{\perp} \\ D_1 & = & AD_1^{\parallel} + A^{\perp} D_1^{\perp} \end{array} \tag{1.4.5}$$

being $A^{\perp} \in \mathbb{R}^{n \times (n-r)}$ an *orthogonal complement* of the column span[14] of A, where $D_{(\cdot)}^{\parallel} = (A'A)^{-1} A' D_{(\cdot)} \in \mathbb{R}^r$ and $D_{(\cdot)}^{\perp} = (A^{\perp\prime} A^{\perp})^{-1} A^{\perp\prime} D_{(\cdot)} \in \mathbb{R}^{n-r}$ are the projections of $D_{(\cdot)}$ on A and $A^{\perp}$, respectively; we can thus rewrite (1.4.4) as:

$$\Delta Y_t = [AB' \ D_0^{co} \ D_1^{co}] \begin{bmatrix} Y_{t-1} \\ 1 \\ t \end{bmatrix} + [C_1 \ldots C_{p-1} \ D_0^{ex} \ D_1^{ex}] \begin{bmatrix} \Delta Y_{t-1} \\ \vdots \\ \Delta Y_{t-p+1} \\ 1 \\ t \end{bmatrix} + U_t \tag{1.4.6}$$

[14]For $\Theta \in \mathbb{R}^{n \times r}$ with $\operatorname{rk}(\Theta) = r < n$, the orthogonal complement is any $\Theta^{\perp} \in \mathbb{R}^{n \times (n-r)}$: $\Theta' \Theta^{\perp} = 0$.

	H	H^*	H_1	H_1^*	H_2
d_t^{co}	[ ]	$[t]$	[ ]	$[1\ t]$	[ ]
d_t^{ex}	$[1\ t]$	[1]	[1]	[ ]	[ ]

Table 1.1: Deterministic terms in identified $\hat{H} \in \mathcal{H}$ (Lütkepohl [78], p. 256)

where:

$$\begin{aligned} D_t^{co} =& A^{\parallel} D_0^{\parallel} + tA^{\parallel} D_1^{\parallel} = D_0^{co} + tD_1^{co} \\ D_t^{ex} =& A^{\perp} D_0^{\perp} + tA^{\perp} D_1^{\perp} = D_0^{ex} + tD_1^{ex} \end{aligned}$$

Collecting the T observations the matrix form of the model is obtained:

$$\Delta \mathbf{Y} = [C_0^+ \quad \mathbf{C}^+] \begin{bmatrix} \mathbf{Y}^+ \\ \Delta \mathbf{Z}^+ \end{bmatrix} + \mathbf{U} \ , \tag{1.4.7}$$

where:

$$\Delta \mathbf{Y} = [\Delta Y_1 \dots \Delta Y_T] \in \mathbb{R}^{n \times T} \quad \mathbf{Y}^+ = \begin{bmatrix} Y_0 & \dots & Y_{T-1} \\ d_0^{co} & \dots & d_{T-1}^{co} \end{bmatrix} \in \mathbb{R}^{K \times T} \quad ,$$

$$\Delta \mathbf{Z}^+ = \begin{bmatrix} \Delta Z_0 & \dots & \Delta Z_{T-1} \\ d_0^{ex} & \dots & d_{T-1}^{ex} \end{bmatrix} = \begin{bmatrix} \Delta Y_0 & \dots & \Delta Y_{T-1} \\ \vdots & \vdots & \vdots \\ \Delta Y_{-p+2} & \dots & \Delta Y_{T-p+1} \\ d_0^{ex} & \dots & d_{T-1}^{ex} \end{bmatrix} \in \mathbb{R}^{J \times T} \tag{1.4.8}$$

$$\mathbf{U} = [U_1 \quad \dots \quad U_T] \in \mathbb{R}^{n \times T}$$

$$C_0^+ = [C_0 \quad D_0^{co} \quad D_1^{co}] \in \mathbb{R}^{n \times K} \qquad \mathbf{C}^+ = [C_1 \dots C_p \quad D_0^{ex} \quad D_1^{ex}] \in \mathbb{R}^{n \times J}$$

with $d_t^{(\cdot)}$ chosen from table 1.1 according to the specific model $\hat{H}$ presented in next section. An empty component in any $d_t^{(\cdot)}$ is to be intended as the correspondent deterministic component being not present.
In this sense, the dimension $\dim(d^{(\cdot)})$ will be equal to the number of elements appearing therein, which allows to explicit:

$$\begin{aligned} K &= K(n, \hat{H}) = \dim(d^{co}) + n \\ J &= J(n, p, \hat{H}) = \dim(d^{ex}) + n(p-1). \end{aligned}$$

Furthermore, $\{Y_{-p+1} \dots Y_0\}$ will be the vector collecting p initial conditions.

1.5 VAR processes estimation

The notation introduced deserves a further comment: it is evident that LS estimation out of equation (1.4.7) does not distinguish model H, H^* and H_1^* (table 1.1) as long as restricted LS estimation is not explicitelty performed.
If instead ML estimation is used, the estimation technique provides cointegrating basis to be *concentrated-out* first and projected on ΔZ^+, and (1.4.8) is the correct formulation to be used (see B.3.1) .
The procedure devoted to lag number estimation will be presented in the beginning, as again its results are independent from true cointegration order r ([78], p. 326).
The *in-chapter* discussion will be strictly limited to what is necessary to disclose the procedure that will be used when dealing with data analysis; detailed description of both the estimators and their asymptotic distributons are provided in appendix B.

1.5.1 Estimating the number of lags p

As observed in [78] (Proposition 8.1, p. 326, which follows Paulsen [95]), a consistent estimate $\hat{p}$ of the total number of lags can be retrieved using standard information criteria, even in the presence of unit roots.
A relatively[15] short dataset requests however much more precision than the one provided by either the Akaike (AIC) or Bayesian (BIC) Information Criteria : it would be preferrable to dispose of a procedure similar to that of the univariate case. Namely, considering the VAR form 1.4.1, a sequence of *portmanteau* tests in the form:

$$H_0 : \Gamma_j = 0 \quad vs \quad H_1 : \Gamma_j \neq 0$$

should be performed for any j going from some upper-bound value down to 1.
Andrews [10] shows that, defining $\mathbf{\Gamma} = [\Gamma_1 \dots \Gamma_p]$ and given a generic matrix of coefficients restrictions $R \in \mathbb{R}^{K \times n^2 p}$ with full row rank K, the test statitistic $\boldsymbol{\lambda}_R$ used for general tests of the form:

$$H_0 : R\,\mathrm{vec}(\mathbf{\Gamma}) = 0 \quad vs \quad H_1 : R\,\mathrm{vec}(\mathbf{\Gamma}) \neq 0$$

might cause distortions in refusing the null or viceversa.
Indeed (see observations B.2.2 and B.5.1), the asymptotic distribution of LS/ML estimators is gaussian but with a singular covariance matrix; hence the restricted estimator does not lead to standard **F**-tests like univariate cases, because relevant Wald statistics have not a standard **F**-distribution.

[15] Relative with respect to the number of parameters to be estimated.

Dolado and Lütkepohl [41] tackled this issue by proving the following proposition, which is presented here as in [78] (Proposition 7.8, p. 319): it applies to VAR(p) processes in level form, obtained by combining (1.3.1) and (1.4.1).

Proposition 1.5.1. *Let $Y \in \mathcal{I}(1)$, $Y = X + D$ be a n-dimensional $VAR(p)$ process, with X in VAR form (1.4.1); let $\hat{\mathbf{\Gamma}} = [\hat{\Gamma}_1 \dots \hat{\Gamma}_p]$ be the unrestricted[16] LS estimator of $\mathbf{\Gamma} = [\Gamma_1 \dots \Gamma_p]$. Let*

$$\begin{aligned} \mathbf{\Gamma}_{(-i)} &= [\Gamma_1 \dots \Gamma_{i-1}\ \Gamma_{i+1} \dots \Gamma_p] \\ \hat{\mathbf{\Gamma}}_{(-i)} &= [\hat{\Gamma}_1 \dots \hat{\Gamma}_{i-1}\ \hat{\Gamma}_{i+1} \dots \hat{\Gamma}_p] \end{aligned}$$

be the matrices obtained by removing Γ_i and $\hat{\Gamma}_i$ from $\mathbf{\Gamma}$ and $\hat{\mathbf{\Gamma}}$, respectively. Then:

$$\sqrt{T}\Big(\operatorname{vec}(\hat{\mathbf{\Gamma}}_{(-i)}) - \operatorname{vec}(\mathbf{\Gamma}_{(-i)}) \Big) \xrightarrow{d} \mathcal{N}\big(0, \Sigma_{(-i)}\big) \tag{1.5.1}$$

where $\Sigma_{(-i)} \in \mathbb{R}^{n^2(p-1)\times n^2(p-1)}$ is non-singular; a consistent estimator is:

$$\hat{\Sigma}_{(-i)} = T[(\mathbf{Y}_{(-i)}\mathbf{Y}'_{(-i)})^{-1}]_{(11)} \otimes \hat{\Sigma}_U \tag{1.5.2}$$

where $\hat{\Sigma}_U$ is the LS estimator of Σ_U, $[(\mathbf{Y}_{(-i)}\mathbf{Y}'_{(-i)})^{-1}]_{(11)}$ is the north-west $\mathbb{R}^{n(p-1)\times n(p-1)}$ minor of $[(\mathbf{Y}_{(-i)}\mathbf{Y}'_{(-i)})^{-1}] \in \mathbb{R}^{np\times np}$ and $\mathbf{Y}_{(-i)} = [Y_0^{(-i)} \dots Y_{T-1}^{(-i)}]$, being

$$Y_t^{(-i)} = (\Delta_{i-1}Y_{t-1} \dots \Delta_{i-p}Y_{t-p})' \in \mathbb{R}^{np\times 1} \quad , \quad t = 0 \dots T-1$$

and $\Delta_j Y_t := Y_t - Y_{t-j}$ for any $j \in \mathbb{Z}$.

This proposition is useful in this framework in view of the following:

Corollary 1.5.1. *In the framework of proposition 1.5.1, a test statistic for the null of the form:*

$$H_0 : R\operatorname{vec}(\mathbf{\Gamma}_{(-i)}) = 0$$

being $R \in \mathbb{R}^{K\times n^2 p}$ a full row rank matrix, can be defined as:

$$\boldsymbol{\lambda}_R = T\hat{\mathbf{\Gamma}}'_{(-i)}R'(R\hat{\Sigma}_{(-i)}R')^{-1}R\hat{\mathbf{\Gamma}}_{(-i)} \tag{1.5.3}$$

and $\boldsymbol{\lambda}_R \xrightarrow{d} \boldsymbol{\chi}^2(n^2)$.

[16] Results are independent from the form of the deterministic component ([78], p. 300), hence no restriction is needed; the estimators $\hat{\Gamma}_i$ can easily be obtained from $\hat{C}_i$, see observation B.2.1.

In practice, the corollary provides to estimate $\mathbf{\Gamma}$ in a VAR$(p+1)$ model, with the underlying prior that $\Gamma_{p+1} = 0$; the null $\Gamma_p = 0$ is then tested using the first p estimated matrices, since standard asymptotic distribution can be retrieved whenever at least one of the matrices Γ_j is not restricted under the null.
Boundaries for minimum/maximum number of lags to be tested are retrieved out of one dimensional tests by defining:

$$\hat{p}_\infty^- = \min\{\hat{p}^j : j = 1 \dots n\} \quad , \quad \hat{p}_\infty^+ = \max\{\hat{p}^j : j = 1 \dots n\} \tag{1.5.4}$$

the maximum number of lags *estimated* among single components of vector $Y \in \mathbb{R}^n$.
A sequence of tests can be performed to estimate $\hat{p}$ in the vector process; the initial condition is $\hat{p}_0 := \hat{p}_\infty^+$, and the h-th test, $h = 0, 1 \dots \hat{p}_\infty^-$, tracks three steps:

- **Step 1**: Estimate the VAR$(\hat{p}_h + 1)$ model via LS (Proposition B.2.1 and observation B.2.1) to retrieve an estimator for the AR matrices $\hat{\mathbf{\Gamma}}$.

- **Step 2**: Define $\hat{\mathbf{\Gamma}}_{-(\hat{p}_h+1)}$ the estimator collecting the first $\hat{p}_h$ autoregressive matrices, as described in Proposition 1.5.1, and $R = [\mathbf{0} \quad I_n]$, where $\mathbf{0}$ is the zero-matrix of $\mathbb{R}^{n^2 \times n^2(\hat{p}_h - 1)}$.

- **Step 3**: Test the null $H_0 : \quad R\,\mathrm{vec}(\hat{\mathbf{\Gamma}}_{-(\hat{p}_h+1)}) = 0$, using the estimator $\boldsymbol{\lambda}_R$ as defined in (1.5.3) with $\hat{\mathbf{\Gamma}}_{(-i)} = \hat{\mathbf{\Gamma}}_{-(\hat{p}_h+1)}$; by proposition 1.5.1, $\boldsymbol{\lambda}_R \xrightarrow{d} \boldsymbol{\chi}^2(n^2)$.
  If the null is accepted, we set $\hat{p}_{h+1} = \hat{p}_h - 1$ and go back to step 1; else, we set $\hat{p} = \hat{p}_h$, and the procedure ends.

At the end of the procedure, an estimated number of lags $\hat{p}$ is retrieved, which serves as a first proxy to be used when performing residual analysis: this point will be clearified in section 1.5.4; next section describes the cointegrated VAR framework of Johansen [72] and defines procedures to estimate the cointegration rank r.

1.5.2 Estimating the cointegration rank

Differently from lags number estimation, test results on cointegrating rank *are not independent from the form of deterministic terms which are included.*
The general framework is that of equation (1.3.1): restrictions to deterministic terms have already been introduced in table 1.1 and will be justified in this section.
The null hypothesis in cointegration tests does always assume that cointegrating rank is equal to some r, so it is worth to work under equation (1.4.6), that is, decomposing $C_0 = AB'$ and projecting D on A; restrictions on projections will characterize different models.

$H(0) \subset$	$\dots$	$H(r)$	$\dots \subset H(n)$
$\cup$		$\cup$	$\cup$
$H^*(0) \subset$	$\dots$	$H^*(r)$	$\dots \subset H^*(n)$
$\vert\vert$		$\cup$	$\cup$
$H_1(0) \subset$	$\dots$	$H_1(r)$	$\dots \subset H_1(n)$
$\cup$		$\cup$	$\cup$
$H_1^*(0) \subset$	$\dots$	$H_1^*(r)$	$\dots \subset H_1^*(n)$
$\vert\vert$		$\cup$	$\cup$
$H_2(0) \subset$	$\dots$	$H_2(r)$	$\dots \subset H_2(n)$

Table 1.2: Relations between the possible n-dimensional VAR(p) processes when the vector is $\mathcal{I}(1)$. (Johansen, [74], Table 5.1, p. 82)

Johansen ([74], pp. 79-80) distinguishes five different possible submodels in the $\mathcal{I}(1)$ framework: cointregration tests wll be constructed by combining within the null the assumption of cointegrating rank equal to r and additional restrictions. A set of submodels, each corresponding to a specific null, can thus be constructed:

- $H(r): \quad \text{rk}(C_0) = r;$
- $H^*(r): \quad \text{rk}(C_0) = r, \;\; A^{\perp} D_1^{\perp} = 0;$
- $H_1(r): \quad \text{rk}(C_0) = r, \;\; D_1 = 0;$
- $H_1^*(r): \quad \text{rk}(C_0) = r, \;\; A^{\perp} D_0^{\perp} = 0, \; D_1 = 0;$
- $H_2(r): \quad \text{rk}(C_0) = r, \;\; D_0 = 0, \;\; D_1 = 0.$

Table 1.2 shows the mutual relationship between the various models defined by previous restrictions. Fixing a row provides a sequence of nested models, which has the same form in the deterministic terms; fixing a column provides a set of nested models with the same cointegrating rank r and number of restrictions increasing as the column is scrolled upwards.
Johansen ([74], pp. 89-100) provide a set of test statistics based on ML estimation (proposition B.3.1), and the procedure is independent[17] from D_t.
Defining $L(\tilde{H}_0|\tilde{H}_1)$ the likelihood ratio test statistic for generic $\tilde{H}_0$ in $\tilde{H}_1$, with the null $\tilde{H}_0$ nested inside an alternative $\tilde{H}_1$, we can collect Johansen's result in the two propositions that follows.

[17] Obviously, the form of the matrices S_{ij} depends on the specific VAR ([78], p. 299), as well as the eigenvalues λ_i.

Proposition 1.5.2. *Let $Y \in \mathcal{I}(1)$ be a n-dimensional VAR(p) process in the form* (1.4.4)*; let λ_i be the eigenvalues as determined by the ML estimation procedure in any of the different models presented in table 1.2 ; then, for any $\tilde{H} \in \mathcal{H} = \{H, H^*, H_1, H_1^*, H_2\}$ the likelihood ratio test statistic $-2\log L(\tilde{H}(r)|\tilde{H}(n))$ is:*

$$-T\sum_{i=r+1}^{n}\log(1-\lambda_i) \xrightarrow{d} \text{tr}\left[\int_0^1 (dW)\tilde{F}'\left(\int_0^1 \tilde{F}\tilde{F}'\right)^{-1}\int_0^1 \tilde{F}(dW)'\right] \tag{1.5.5}$$

where W is a standard $n-r$ dimensional Brownian motion and $\tilde{F}$ is a $n-r$ dimensional random vector which distribution depends on the specific $\tilde{H}$; the statistic (1.5.5) *is commonly referred to as* trace statistic.

Proposition 1.5.3. *Let $Y \in \mathcal{I}(1)$ be a n-dimensional VAR(p) process in the form* (1.4.4) *and consider two consecutive column nested models of the form (H, H^*) or (H_1, H_1^*). Let λ_i, λ_i^* be the eigenvalues in the trace statistic* (1.5.5) *out of the ML estimation of each of the two models, respectively; the likelihood ratio test statistic:*

$$-2\log L(H^*(r)|H(r)) = T\sum_{i=r+1}^{n}\log\left(\frac{1-\lambda_i^*}{1-\lambda_i}\right) \xrightarrow{d} \boldsymbol{\chi}^2(n-r) \tag{1.5.6}$$

and the same holds for $-2\log L(H_1^*(r)|H_1(r))$.

Next proposition solves instead the issues related to different models in each column of table 1.2, collecting the results in [74](Theorem 11.3, p. 162).

Proposition 1.5.4. *Let $Y \in \mathcal{I}(1)$ be a n-dimensional VAR(p) process in the form* (1.4.4) *and consider two consecutive column nested models of the form (H_1, H^*) or (H_2, H_1^*). Let λ_i, λ_i^* be the eigenvalues in the trace statistic* (1.5.5) *out of the ML estimation of each of the two models, respectively; the likelihood ratio test statistic:*

$$-2\log L(H_1(r)|H^*(r)) = T\sum_{i=1}^{r}\log\left(\frac{1-\lambda_i}{1-\lambda_i^*}\right) \xrightarrow{d} \boldsymbol{\chi}^2(r) \tag{1.5.7}$$

and the same results hold for $-2\log L(H_2(r)|H_1^*(r))$.

Results from previous propositions can be rewritten in terms of the trace statistic (1.5.5), through simple algebraic manipulation.
The distributions of $\tilde{F}$ in (1.5.5) is always non-standard, whatever $\tilde{H}$ is tested: a formal definition is given in Johansen ([74], Theorem 11.1, p. 156), and distributions are also tabulated therein ([74], pp. 214-216).

Despite statistics are available so that any model can be tested with respect to those lying upwards, downwards and left or right in table 1.2, testing two non-nested models (in *diagonal* positions) is not possible; this implies also that if two different models induce two different cointegration ranks, there is no formal way to decide *ex-ante* which of them fits best and, so neither which the correct r is.
Johansen [73] suggests an iterative procedure aimed to jointly test cointegration rank and the form of deterministic terms, which prioritizes vertical movements in the table: indeed, in the last resort, a model must be prioritized *by choice* (see [3]) since no total order is available in two-dimensions.
Here no selection procedure will be performed: a prior model will instead be chosen according to some preliminary stylized facts the analysis is expected to match.
First of all, no deterministic trend is expected within the cointegrating relation and no quadratic trend is expected in the data, thus excluding models H and H^*; a constant is instead envisaged within the cointegrating relation, thus model H_2 is to be excluded too; these prior exclusions will be justified in section 1.7.
Two possible models, H_1 and H_1^* are left, the latter nested into the former: proposition 1.5.3 supplies the correct test statistic to be applied so as to discriminate between these two models; such tests refuses the null of model H_1^* only in four cases among the over one-hundred models which are explored within chapters 2 and 3 thus assuming the prior $\hat{H} = H_1^*$ does not bring forth heavy restrictions.
This choice enables the iterate use of proposition 1.5.3, in testing the null $H_1^*(r)$ against the alternative $H_1^*(n)$, with $r = 0 \dots n-1$.
The section that follows presents a two-stage parameter estimation which is devoted to vector processes exhibiting a cointegrating rank $\hat{r} > 0$ only: indeed, if $\hat{r} = 0$, then $A = B = 0$, hence model H_1^* reduces to a simple regression[18] in differences:

$$\Delta \mathbf{Y} = \mathbf{C} \Delta \mathbf{Z} + \mathbf{U}$$

which is estimated by standatd LS, as it involves stationary variables only.
Furthermore, it is worth to stress (see section B.4). that spurious cointegrating relations (where $Y^j \in \mathcal{I}(0)$ for at least one j) must be taken off in advance at this level; whenever the null of unit root is refused by all tests in 1.1 for Y^j, cointegrating rank will be priorly settled to $\hat{r} = 0$.
It could be argued that a true cointegrating relation may arise among the other variables: in this work however, only two-dimensional models will be analyzed, hence the prior $\hat{r} = 0$ becomes fundamental not to run up against modeling shortcomings.

[18]Observe that with model H_1^* no constant is provided outside the cointegrating relations.

1.5.3 Two-stage parameter estimation

This section presents the procedure used to estimate the parameter set of the cointegrated VAR when cointegration rank is estimated to be $\hat{r} > 0$; the selected model is $\hat{H} = \hat{H}_1^*$ but general notations will be kept along for the sake of completeness. This procedure is particularly useful as it can be adopted for testing hypothesis on the cointegration basis separately from the other parameters; moreover in small samples EGLS is preferrable to ML estimation, see [23]; details concerning the construction of the estimators are instead provided in section B.4.
As aforementioned (section 1.5), if the cointegrating rank is known to be r then the cointegrating matrix C_0 can be written as $C_0 = AB'$, where $A, B \in \mathbb{R}^{n \times r}$ and $\text{rk}(A) = \text{rk}(B) = r$; in order to identify the decomposition, a custom choice, equation (B.4.1), is *normalization* :

$$C_0^+ = A \cdot [I_r \quad B'_{n-r} \quad D_0^{\|} \quad D_1^{\|}] = A \cdot [I_r \quad B^{+'}_{n-r}] \tag{1.5.8}$$

so that equation (1.4.7) can be rewritten[19] as:

$$\Delta\mathbf{Y} = [A[I_r \quad B^{+'}_{n-r}] \quad \mathbf{C}^+] \begin{bmatrix} \mathbf{Y}^+ \\ \Delta\mathbf{Z}^+ \end{bmatrix} + \mathbf{U} \tag{1.5.9}$$

with $B^+_{n-r} \in \mathbb{R}^{(K-r)\times r}$; in this way, we can exploit superconsistency of the estimators of the cointegrating matrix and retrieve proxies of the true parameters of the model following a two stage procedure (further details are provided in section B.4).

- **Step 1**: Retrieve ML estimators $[\hat{C}_0^+ \quad \hat{\mathbf{C}}^+]$ of the parameters underlying the identified model $\hat{H}$ as well as of the white noise covariance matrix $\hat{\Sigma}_U$ (section B.3). Under the assumption of cointegrating rank r and of a normalized model in the form (1.5.9), we can use the first r columns of $\hat{C}_0^+$ as an estimator $\hat{A}$ and construct the normalized EGLS estimator $\hat{\hat{B}}^+_{n-r}$ defined in (B.4.3).

- **Step 2**: Superconsistency of $\hat{\hat{B}}^+_{n-r}$ allows to substitute it in (1.5.9) treating it as it was thet rue parameter, and to estimate the parameters left by ordinary LS. The exact form of the estimators $[\hat{\hat{A}} \quad \hat{\hat{\mathbf{C}}}^+]$ is given in equation (B.5.8).

The two step procedure allows to measure parameters significance separately for the long and short run dynamics of the model: standard asymptotic distributions can be retrieved out of linear transforms of the estimators, yielding meaningful **t**-ratios, which will be presented alongside the estimates.

[19] Observe that this would not be possible if any of the components was stationary.

Consider the normalized EGLS estimator in vectorized form $\text{vec}\left(\hat{\hat{B}}^+_{n-r}\right) \in \mathbb{R}^{r(K-r)\times 1}$ as retrieved out of the first step of parameters estimation procedure.
Proposition B.5.2 shows the asymptotic distribution of the (vectorized) estimator to be $\mathcal{N}(0, \Sigma^*)$ for some Σ^*, retrieved out of equation (B.5.3); hence:

$$\frac{\text{vec}\left(\hat{\hat{B}}^+_{n-r}\right)_i}{\Sigma^*_{ii}} \sim \mathbf{t}(T - rK + r^2) \quad , \quad i = 1 \dots r(K-r) \tag{1.5.10}$$

being $\mathbf{t}(N)$ a student-$\mathbf{t}$ distribution with N degrees of freedom.
Similarly, by corollary B.5.1, the EGLS estimator of the short-run parameters $\text{vec}\left([\hat{\hat{A}} \quad \hat{\hat{\mathbf{C}}}^+]\right) \in \mathbb{R}^{n(J+r)}$ has an asymptotic gaussian distribution $\mathcal{N}(0, \Sigma_{A,\mathbf{C}})$, where $\Sigma_{A,\mathbf{C}}$ is defined in (B.5.6). Hence:

$$\frac{\text{vec}\left([\hat{\hat{A}} \quad \hat{\hat{\mathbf{C}}}^+]\right)_i}{(\Sigma_{A,\mathbf{C}})_{ii}} \sim \mathbf{t}(T - nJ - rn) \quad , \quad i = 1 \dots n(J+r) \tag{1.5.11}$$

1.5.4 Whiteness of residuals and lags refinement

Last step is to inspect whether the white noise assumption for the innovations $\mathbf{U}$ is satisfied a fortiori, by checking whether estimated autocovariance matrices are significantly different from zero. Define the residuals[20]:

$$\hat{\mathbf{U}} = \Delta\mathbf{Y} - [\hat{\hat{A}}\hat{\hat{B}}^+ \quad \hat{\hat{\mathbf{C}}}^+] \begin{bmatrix} \mathbf{Y}_+ \\ \Delta\mathbf{Z}_+ \end{bmatrix} \tag{1.5.12}$$

being $\hat{\hat{A}}, \hat{\hat{B}}^+_{n-r}, \hat{\hat{\mathbf{C}}}^+$ set up as in (B.4.3) and (B.5.8); the estimated autocovariance matrices are defined as:

$$\hat{\Omega}_i := \frac{1}{T}\sum_{t=i+1}^{T} \hat{U}_t \hat{U}'_{t-i} \quad , \quad i = 0 \dots h \tag{1.5.13}$$

where h is chosen depending on T, as it will be clear in what follows.[21]
Lütkepohl ([78], Lemma 8.1, p. 346) shows that the asymptotic distribution of the estimated autocovariance matrix is the same whether single or two-step procedure is used, again because of the superconsistency of $\hat{\hat{B}}^+_{n-r}$.
As a maximum number of lags h is selected, define:

$$\hat{\mathbf{\Omega}}_h = [\hat{\Omega}_1 \quad \dots \quad \hat{\Omega}_h] \in \mathbb{R}^{n \times nh} \tag{1.5.14}$$

A first *portmanteau* test statistic can be constructed so as to test the null:

$$H_0 : \mathbf{\Omega}_h = 0 \qquad vs \qquad H_0^* : \mathbf{\Omega}_h \neq 0 \tag{1.5.15}$$

[20]In case of cointegrated VAR, $\hat{\mathbf{U}} = \hat{\hat{\mathbf{U}}}$, the residuals out of the two-step procedure. In case of pure VAR in differences ($A = B = 0$), residuals comes out of LS estimation.
[21]Clearly, setting $i = 0$, we retrieve the estimated autocovariance matrix $\hat{\Omega}_0 = \hat{\Sigma}_U$.

As relatively small datasets are examined in this work, we use the corrected test statistic:

$$\boldsymbol{\lambda}_W(h) := T^2 \sum_{i=1}^{h} (T-i)^{-1} \operatorname{tr}\left(\hat{\Omega}_i^{'}\hat{\Omega}_0^{-1}\hat{\Omega}_i\hat{\Omega}_0^{-1}\right) \tag{1.5.16}$$

Under H_0, for[22] large T *and large* h, $Q_h \approx \boldsymbol{\chi}^2\big(n^2h - n^2(p-1) - nr\big)$; h must then be chosen large enough so as to ensure a well defined $\boldsymbol{\chi}^2$ distribution.
Particularly the distribution of the test statistic (1.5.16) might not be well defined for small values of h, and thus a correct test statistic for low order autocorrelation would not be available. It is thus convenient to define a second statistic based on a Lagrange Multiplier (LM) test. First of all, we rewrite Ω_i as:

$$\Omega_i = U\mathcal{G}_iU' = \sum_{t=i+1}^{T} u_t u_{t-i}^{'} \quad i = 1\dots h$$

where $\mathcal{G}_i \in \mathbb{R}^{T\times T}$ is conveniently defined (see [78], p. 158), stack the matrices in $\boldsymbol{\mathcal{G}} = (\mathcal{G}_1 \dots \mathcal{G}_h) \in \mathbb{R}^{T\times Th}$ and define $\boldsymbol{\mathcal{U}} := (I_h \otimes U)\boldsymbol{\mathcal{G}}$.
The estimated quantities $\hat{\Omega}_i$ and $\hat{\boldsymbol{\mathcal{U}}}$ are defined by substituting $\hat{U}$ to U; notice also that (1.5.14) can be rewritten as $\hat{\boldsymbol{\Omega}}_h = \hat{U}\boldsymbol{\mathcal{G}}(I_h \otimes \hat{U}')$.
Consider the *auxiliary regression model*:

$$\hat{U} = \boldsymbol{\mathcal{C}}\mathcal{Z} + \boldsymbol{\mathcal{D}}\hat{\boldsymbol{\mathcal{U}}} + \mathcal{E} \tag{1.5.17}$$

where $\boldsymbol{\mathcal{E}} = [\mathcal{E}_1 \dots \mathcal{E}_n] \in \mathbb{R}^{n\times T}$ collects the trajectory of a white noise and the regressors $[\mathcal{Z}\ \hat{\boldsymbol{\mathcal{U}}}]$ are defined from:

$$\mathcal{Z} = [\mathbf{Y}^+ \Delta\mathbf{Z}^+]' \qquad \boldsymbol{\mathcal{C}} = [C_0^+ \quad \mathbf{C}^+] \qquad \boldsymbol{\mathcal{D}} := [\mathcal{D}_1 \dots \mathcal{D}_h] \in \mathbb{R}^{n\times nh}$$

hence $\hat{U} = \Delta\mathbf{Y} - \hat{\boldsymbol{\mathcal{C}}}\mathcal{Z}$ are the usual estimated residuals of the LS estimator (B.2.5) (see (1.4.7)). Each column of matrix equation (1.5.17) can thus be rewritten as:

$$\hat{U}_t = \boldsymbol{\mathcal{C}}\mathcal{Z}_t + \mathcal{D}_1\hat{U}_{t-1} + \dots + \mathcal{D}_h\hat{U}_{t-h} + \mathcal{E}_t \quad , \quad t = 1\dots T$$

The aim is to retrieve a consistent statistic out of the portmanteau test:

$$H_0 : \boldsymbol{\mathcal{D}} = 0 \quad vs \quad H_1 : \mathcal{D}_j \neq 0 \ \ \exists j = 1\dots h \tag{1.5.18}$$

using the Lagrange multiplier principle. The idea (see [78], p. 696) is to consider an estimator of $[\boldsymbol{\mathcal{C}} \ \ \boldsymbol{\mathcal{D}}]$ in (1.5.17) restricted under the null in (1.5.18):

$$\hat{U} = \boldsymbol{\mathcal{C}}\mathcal{Z} + \mathcal{E}$$

Ordinary LS estimation implies:

$$\boldsymbol{\mathcal{C}}_R = (\mathcal{Z}\mathcal{Z}')^{-1}\mathcal{Z}'\hat{U} = (\mathcal{Z}\mathcal{Z}')^{-1}\mathcal{Z}'(\Delta\mathbf{Y} - \hat{\boldsymbol{\mathcal{C}}}\mathcal{Z}) \tag{1.5.19}$$

[22] Differently from the stationary case, a VAR model with unit roots yields a $\boldsymbol{\chi}^2$ asymptotic distribution whose degrees of freedom depend on the cointegrating rank too.

The first order conditions used to retrieve the estimators imply ([78], p. 696):

$$\mathcal{Z}'(\Delta\mathbf{Y} - \hat{\boldsymbol{\mathcal{C}}}\mathcal{Z}) = 0. \tag{1.5.20}$$

Hence, combining (1.5.19) and (1.5.20), and under the null $\boldsymbol{\mathcal{D}} = 0$, the model (1.5.17) yields $\hat{U} = \mathcal{E}$, that is, residuals are white noise. Hence the null in (1.5.18) tests exactly the white noise assumption of $\hat{U}$; the available test statistic is ([78], p. 172):

$$\boldsymbol{\lambda}_{LM}(h) = \mathrm{vec}\,(\hat{U}\hat{\mathcal{U}}')' \cdot \left([\hat{\mathcal{U}}\hat{\mathcal{U}}' - \hat{\mathcal{U}}\mathcal{Z}'(\mathcal{Z}\mathcal{Z}')^{-1}\mathcal{Z}\hat{\mathcal{U}}']^{-1} \otimes I_n\right) \cdot \mathrm{vec}\,(\hat{U}\hat{\mathcal{U}}') \tag{1.5.21}$$

that can be retrieved out of the original model using the LS estimated residuals $\hat{U}$ and substituting $\mathcal{Z}$ according to (1.4.8).
The following proposition ([78], Proposition 4.8, p. 173) defines the asymptotics of the test stastistic:

Proposition 1.5.5. *The test statistic* (1.5.21) *has an asymptotic* $\boldsymbol{\chi}^2(hn^2)$ *distribution.*

We will perform both this latter test for any $h = 1 \dots \bar{h}$, being $\bar{h}$ a maximum number of backwards lags selected in advance, as well as test (1.5.16) from the first $h \leq \bar{h}$ for which the distribution is well defined up to $\bar{h}$.
The analysis of residuals is useful to refine the estimated number of lags out of the two-step procedure in 1.5.1; in case any of the two statistics rejects the null in (1.5.15) or (1.5.18) for any of the h, then an additional lag is provided in the model and estimation of cointegrating rank, basis and short-run parameters is repeated, until the respective nulls are accepted for any test and any h.

1.6 VAR estimation procedure: summary

We conclude this section by summarizing the estimation procedure pursued in any of the vector models. The significance level of test statitstics is set to 5%.

1. Take raw data out of the provider and smoothen them with the correspondent moving average of each component Y^j of $Y = (Y^1 \dots Y^n)$.

2. Estimate each process' number of lags $\hat{p}^j$ according to the procedure described in 1.1 and define $\hat{p}^-_\infty$ and $\hat{p}^+_\infty$ according to (1.5.4).

3. Perform ADF test for unit-roots for each of the processes with number of lags equal to the estimated $\hat{p}^j$, and estimate their integration orders q^j.

4. Construct the vector process Z and estimate the number of lags $\hat{p}$ with the restriction $\hat{p}^-_\infty \leq \hat{p} \leq \hat{p}^+_\infty$

5. Estimate[23] the cointegrating rank $\hat{r}$ with prior $\hat{H} = H_1^*$ (table 1.2) and number of lags $\hat{p}$. If $\hat{r} = 0$, proceed with LS estimation (B.2.5) with $C_0^+ = 0$ and go to step 8, else proceed. Condifence level is set to 10% in cointegrating tests.

6. Estimate model in the form 1.4.7 by LS (equation (B.2.5)) and retrieve a first estimate $\hat{A}$ and $\hat{C}^+$.

7. Construct the normalized EGLS estimator $\hat{\hat{B}}_{n-r}$ through the procedure in 1.5.3, and retrieve the two-stage estimators $\hat{\hat{A}}$ and $\hat{\hat{C}}^+$

8. Use estimated parameters to compute residuals $\hat{U}$ and test for autocorrelation of residuals using test statistics (1.5.16) and (1.5.21) for $h = 1, \ldots \bar{h}$.
   If both tests fail in refusing the null for any h, then we have a model for Z; else, we augment $\hat{p}$ in step 4 and repeat the procedure.

1.7 Concluding remarks

It is clear that the statistical procedure which is used to infer relationships among observations is a choice among a large number of possible alternatives.
It is thus worth to justify this choice by commenting the way that economic and financial conclusions are taken out of statistics in the next chapters. The aim of this section is to distinguish the results which are considered relevant to drag out conclusions, and to legitimize leaving others out of discussion.
The observations in chapter 2 collects portfolio yields; chapter 3 focus on credit risk scores involving macroeconomic and financial variables. Concerning the former, one purpose is to isolate credit-risk premia through statistical hedging strategies and compare them to the canonical spread *vis-à-vis* Germany. The other is comparing the elements of the resulting set of credit-risk free portfolios, as mutual comparison might reveal eventual market shortcomings.
The first part of chapter 3 deals with standard econometrics while the second is instead devoted to the construction of physical default probability measures. These are compared to standard CDS-implied default probabilities, to measure the portion of market-implied creditworthiness which is explained by a relatively small set of macroeconomic and financial variables
The common drawback of endogenous approaches is that they cloak the underlying determinants of the processes.

[23]If $n = 2$ and any $Y^j \in \mathcal{I}(0)$, then the cointegration rank is *set* to $\hat{r} = 0$.

When comparison involve custom benchmarks, these latter will be considered reliable, and their underlying determinants will not be explored. This reasoning applies both to chapter 2 and the second half of chapter 3. The econometric analysis which is pursued in the first part of chapter 3 is rather standard, but the coefficients to be inferred came out of a theoretical model and their determinants are not investigated.
The amount of feedback effects among the components of any vector process might help to explain risk determinants without explicitly defining them.
This also clearifies the choice to prioritize model H_1^* (and H_1) among Johansen's alternatives; a quadratic trend is excluded from data (model H) so as to be coherent with single-variables unit root tests models under the alternative.
A trend in the cointegrating relation (model H^*) is also to be excluded: if the purpose is to construct processes replicating the in-crisis behavior of custom credit risk measures, their behavior must not be trended in equilibrium.
A constant is instead allowed in the cointegrating relation (hence H_2 is excluded), because a constant in the cointegrating relation is the minimum number of parameters to be included so as to take into account different measurement units[24].
A rigorous statistical analysis of each of the models, would be necessary to exhaust the discussion, but this would result in a huge amount of data analysis which would move the discussion away from the scope of present work.
The amount of statistical results which is presented here is retained to be a good trade-off among econometrics and its interpretation: on the one hand, it is sufficient to validate conclusions, on the other it remains not the core of the thesis.
The choice is to present as much intuitions as possible, and expose the minimum number of results needed to validate them.
In this sense, no multiple restriction tests on models parameter will be performed. Any parameter will be retained to be significant and discussed each time that single-valued **t**-statistics out of 1.5.10 and 1.5.11 are greater than 2.[25]
Furhtermore, the first AR matrix C_1 is presented only, and comments on short term dynamics will be limited to this latter; major interest is devoted to *short-term feedback effects*, while diagonal elements of C_1 (the *pure autoregressive component* of each single process) will be almost overlooked.
Time series in each respective chapter are sampled at different frequencies (weekly and quarterly, respectively) and on different time windows (3.5 versus 13 years). Hence, despite the same approach is pursued, a different relevance must be given to the outcomes of the econometric analysis across the two chapters.

[24]See [3] and references within.

[25]The value which yields roughly the 5% p-value for tw-tailed student-**t** tests with degrees of freedom between 40 and 200 is roughly 2.

Short-run analysis is very important in chapter 2: as the time frequency is higher and the time window shorter. The effectiveness of hedging strategies and the possible arbitrages portfolios sharing the same risk factors have to be measured locally. Bivariate models with highly correlated innovations will be indicators of the hedging strategies being effective, as the combined position presents the same source of randomness as one of the two components.
Cointegrating relations are instead not expected here. The short time window covered by weekly-frequencies datasets in chapter 3 delineate a perfect framework for local analysis but prevents from drawing conlclusion on the long-run.
Quoting Johansen ([74], p. 41): *"...the long-run relations...are relations between the variables in the economy, as described by the statistical model, which show themselves in the adjustment behaviour of the agents in the sense that the agents try to force the variables back towards the attractor set defined by the relations"*.
Cointegrating relations are equivalent to underlying economics relationships which lingers in time, from which the two variables are moved and towards which they tend, thus justifying the epiteth *structural*.
Cointegrating basis will thus be given major importance in chapter 3, as containing information on *long-term* relationship among vectors components.
First of all, following Johansen's interpretation, within a larger time window it should in principle be more likely that cointegrating relations arise.
Secondly, credit risk measures embed *markets expectations on future creditworthiness* of target borrower. A long-term equilibrium among two different credit risk measures is thus interpretable as the information sets lying behind each respective model are sharing the same *long run expectations* on target borrower.
Concerning short-run analysis in chapter 3, values for matrix C_1 and covariance matrix Σ are as well reported here, for the sake of completeness, but retained of minor relevance.
Short-run significant feedback effects among mixed macroeconomic and financial variables at this frequency are less expectable and less meaningful becuase of the relatively small size T of the observation sets.[26]

[26] The innovation processes should be modeled in a *mixed-frequency* framework, so that VARs can be retained significant at slower (quarterly) frequencies.

2 Sovereign yields and credit risk premia

Financial markets registered early announces of the burst of *debt crisis* in Europe before the general cross-country rise in debt level, when a uniform increase in *credit risk perception* affected the whole Monetary Union.
Bond yields spreads of Euro-area countries *vis-à-vis* Germany became a major indicator of member states financial state of health.
Yields spreads are assumed to embed country-specific risk premia, while Germany is used as a risk-free benchmark: the difference is commonly computed between long term (10-years) interest rates.
Credit conditions remained stable from 2000 up to 2008: significant nonzero spreads across different countries during this period were limited, and normally attributed to negligible country-specific factors. [36]. Sovereign bonds yields remained globally aligned one with the other, and also with European benchmark money-markets.
In 2010, upward shifts of sovereign yields in many distressed countries raised German obligations to the role of preferred investments across Europe.
Demand for German obligations exceeded normal levels because the country was perceived as the most reliable credit-risk free borrower across the Euro-market.
Observing the rising importance of spreads magnitude in recent years, that severely affected policy making too, the reliability of such a number when used to reckon European countries creditworthiness is to be investigated.
The use of yield spreads as a proxy for credit risk relies on the assumptions that all country-specific yields embed a credit-risk premium, which is defined as the excess rate with respect to German one, perceived as credit risk-free.
Nonetheless, yields can be statistically explained using a wide range of different factors [36]; this suggests that credit risk could be only one of the determinants of Euro-area spreads. A second issue to be enquired is thus whether each specific long-term sovereign rate includes other premia: by *other*, we mean any premium that is neither directly attributable to credit risk nor to systemic movements in the EMU (and thus still remains country-specific).
It is common across literature to model sovereign yields according to three different latent factors: a *credit risk premium*, a *liquidity risk premium* and an *international risk factor*, see [79]. Observe that the latter is a common across any sovereign yield in the EMU, while the formers are strictly country specific.
Figure 2.1(a) presents the underlying decomposition of sovereign yields across the Euro-zone that will serve as a building block throughout this chapter, namely:

$$y = \pi + L + b \tag{2.1}$$

where π is credit-risk premium, L is a common Euro-area benchmark and b collects the (eventual) additional risk premia [27].

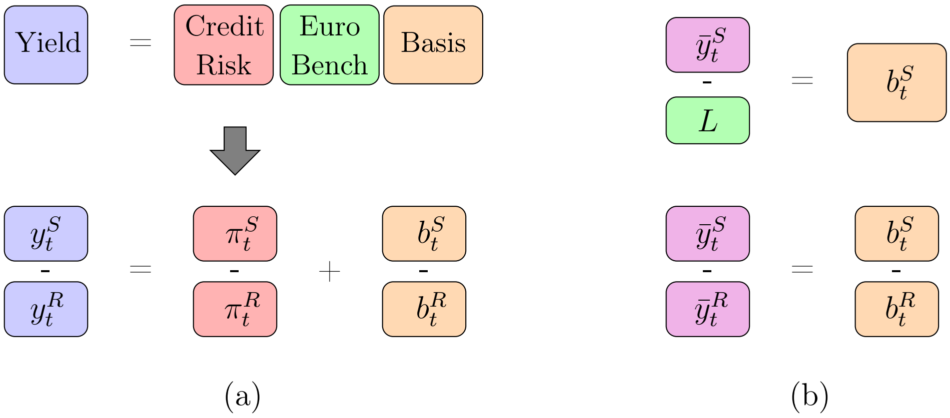


Figure 2.1: (a): European yields determinants and yields spreads; (b): CDS-bearing yield spread on Euribor yields the basis (a) while CDS-bearing spreads between different countries yields basis difference in (b) should be zero if $b^S \equiv b^R$.

This decomposition is different from that proposed in [79] in at least two aspects. The first one, is that the international risk factor[27] has been substituted by an *ad-hoc* European benchmark rate. It would be faulty to assume that this latter embeds international risk factors, as the general decrease of risk-free rates in Europe coincides with the harshest crisis moments for peripheral EMU countries.
This will not be a major shortcoming however when yields spreads are considered, as European yields common factors should cancel.
The second is that the country-specific component b is not directly attributed to liquidity risk at the moment. The idea is to assume that yields spreads reflect credit risk premium only, observe the eventual discrepancies among European portfolios where credit risk is hedged, and reckon the determinants of such misalignements. Liquidity risk will obviously considered as one possible candidate.
Following decomposition (2.1), we assume that differencing yields on Germany corresponds to assume that π^{DE} and b^{DE} are negligible.
The first issue to be tackled is to spin off within any other sovereign yield the portion which is attributable to credit risk *per-se*, without using Germany as benchmark, so as to isolate the portion b deserving different explanations.
In order to decouple these components, the choice was to use credit derivatives: the basic vanilla instrument is *Credit Default Swap* (CDS).
Briefly, CDS contracts *swaps* (up to a certain maturity) losses due to eventual *credit events* (section 2.1) with a periodic premium (the CDS spread), which value is actively traded on the market. The protection *buyer* pays the premium each quarter up to any credit event.

[27] In [11], yields covary importantly with financial measures of global risk: the *Volatily Index* (VIX) is then chosen as proxy, similarly to [36].

If this latter occurs, the contract *triggers* and the (protection) *seller* reimburses the difference between face value and post-default price.[28]

CDS were thus created to *hedge* target obligations from *credit risk*: CDS-based hedging strategies exploits the countermonotonic movements of CDS spreads and bond yields. As an example, consider a case of *credit crunch*: any combined long position on CDS and bond benefits from the rise in the derivative position and is affected by the rise in yields. Market efficiency implies that the difference of these two variations is zero, thus hedging is complete; a rather detailed survey upon statistical hedging techniques built up with credit derivatives is [31].

We define *CDS-bearing bond* a combined long position on bond and correspondent (in terms of maturity) CDS, and let $\bar{y}$ be the correspondent yield-to-maturity (*CDS bearing yield*) of such portfolio.

If the hedging is perfect, the resulting $\bar{y}$ is released from credit risk premium: the difference $y - \bar{y} = \pi$ represents exactly the credit-risk premium comparable to spread *vis-à-vis* Germany. In this way, we can also test the choice of benchmarking over Germany can be tested through the computation of π^{DE}.

Taking differences $\bar{y} - L$ over a risk-free rate L reveals the existence of additional country-specific risky factors. Furthermore, given two different countries A and A', the difference $\bar{y}^A - \bar{y}^{A'}$ allows to infer eventual misalignements independently from the chosen benchmark L.

This chapter investigates such issues: beyond Germany (DE), the risky countries under exam are Italy (IT) and Spain (ES).

The first step is to retrieve portfolio yields, which is a typical issue in fixed-income analysis [26]: fixed future cash flows are combined with current price to retrieve a continuosly compounded yield-to-maturity, using absence of arbitrage.

Term structures are implied using yields to different maturities, and interpolating a set of time-continuous functions, which in turn are described through a relatively limited parameter set: details are provided in section C.1.

It is worth to stress that, by setting $y - \bar{y} = \pi$, we implicitly assume that CDS-hedging is perfect, that is, $\bar{y}^A$ is a credit-risk free yield for any country A.

In this sense, the standardization process [82] which interested European CDS market in late 2009 [83], provides a correct and computationally efficient framework to pursue our objectives. Before 2009, credit derivatives were specifically tailored to counterparties writing them, and traded OTC, thus the assumption that their prices contained information on credit risk only could be questioned.

The *ISDA CDS-Bang* standardization process was aimed to regulate the CDS market which suffered from (and was cause of) several shortcomings during latest financial crisis.

[28] Details can be found in section C.2.

New contract features include the introduction of regulating authorities: a *Central Clearing House* (CCH), aimed at reducing both counterparty and liquidity risk, and a *Determination Committee*, endowed with the power of taking binding decisions on credit events deemed to trigger any of the relevant credit derivatives.
This new *standardized* market provides changes in specific contract features too: standard CDS have standard payment dates, standard maturities and fixed coupons. Moreover, an *upfront payment* made at inception is introduced: this *spot price of credit risk*, floating according with the microstructure of this new market.
The characteristics of new cash flows can be exploited so that custom fixed-income methodologies can be used for market-implied term structures. The presence of a CCH is assumed to justiffy this approach in considering CDS-spreads not to contain any counterparty or liquidity risk premium, thus reflecting credit risk *only*.
The process b in (2.1) is named after *basis* and deserves further comments: indeed, imperfections in hedging strategies based on CDS already captured the attention of central banks and regulators.
Particularly, given any obligation, the *CDS-bond basis* can be defined as the difference between the CDS spread and the *asset swap spread* (ASW) (see [35] and [48]).
This latter, quoted on the market, is the difference between the obligation's yield to maturity and a benchmark rate, often the Euribor rate itself. If CDS spread S is equal to credit risk premium π then:

$$\pi - ASW = \pi - (y - L) = L - (y - \pi) = L - \bar{y} = -b$$

and b is the CDS-bond basis as in [35] and [118], except for the change in sign.
The choice was however to maintain the minus sign to stress the interpretation of the basis as an additional risk premium to be paid in distress times, and will still be addressed to as the CDS-bond basis.
The term structure of interest rates $\{y(t,T)\}_{T\geq t}$ is implied for any of the countries together with the correspondent CDS-bearing term structure $\{\bar{y}(t,T)\}_{T\geq t}$.
The money market $L(t,T)$ offer spot rates only when $T \leq 1$ year: rates with longer maturities are bootstrapped using swap rates, see section C.3.1.
Equation (2.1) is considered with $T = 10$ years, so $y_t = y(t,10)$, $L_t = L(t,10)$ and $\bar{y}(t,10) = \bar{y}_t$; this implies that $\pi_t = \pi(t,10)$ and $b_t = b(t,10)$.
Standardization of CDS market across Europe became effective in July 2009, but six months are left to the market to digest new features: the sample period, with weekly frequency, starts in January, 2010 and end up in June, 2013.
The chapter is organized as follows: section 2.1 provides for a description of the aforementioned standardization process which underlies the construction of the term structure experiments, out of [28]. The description is rather detailed but it will be useful on the one hand to justify the methodology used in the hedging procedure, and on the other to guess possible determinants of the basis itself.

Section 2.2 provides for a brief description of the Nelson-Siegel [89] framework for implying yield curves: section 2.2.1 specializes that to CDS-bearing yields.
Section 2.3 presents the econometric results based on the procedure described in section 1.6: the role of Germany is tested in subsection 2.3.1, by computation of π^{DE} and direct comparison with L. Subsection 2.3.2 compares credit risk premia π and yield spreads *vis-à-vis* Germany, and the performances of hedging strategies are monitored by comparing $\bar{y}$ with L for distressed countries. The comparison of synthetic credit risk free yields is disclosed in subsection 2.3.3, while results are commented and collected in subsection (2.3.4).

2.1 CDS and the crisis: a standardized market

The credit derivatives market experienced rapid growth in the last decade, attracting several types of investors. It also became the subject of intense debate, involving financial economists, institutions and regulators, as well as a large share of the public, especially after recent financial turmoil.
The birth of this market can be traced back to the early nineties, when the first vanilla instruments (CDS), were created with the purpose of hedging credit risk exposures to a given *reference entity*.
The buyer pays a periodic coupon in order to receive protection against deteriorations in the creditworthiness of such entity which might cause permanent impairments to the value of its obligations. When a *credit event* affects the reference entity, the protection seller bears the financial loss of the buyer and partially refunds him up to a certain *notional* amount $\wp^*$ (usually the obligation face value) of the *reference obligations* issued by that entity[29].
CDS contract could be likened to a traditional insurance contract: there are, however, at least two relevant differences. Firstly, stopping premium payments is typically sufficient to unwind an insurance contract, while, as for most of derivatives, closing out a CDS position means to sign another CDS contract and taking the opposite side of the trade. Secondly, there is no need for the buyer to *actually* hold the obligations on which the CDS is written: he could be willing to take purely speculative positions and trade *naked* CDS [90].
Notwithstanding speculation, CDS are attractive as hedging instruments too: when competition increased across markets in the nineties, causing a relevant number of bankruptcies (for example, Enron and Worldcom), banks were forced to monitor and manage their credit portfolios more actively [114].

[29]Notice that reference obligation of a given entity includes a wide range of obligations; before standardization, no limits were imposed to relevant obligations for a transaction, as long as the buyer found a counterparty willing to accept them.

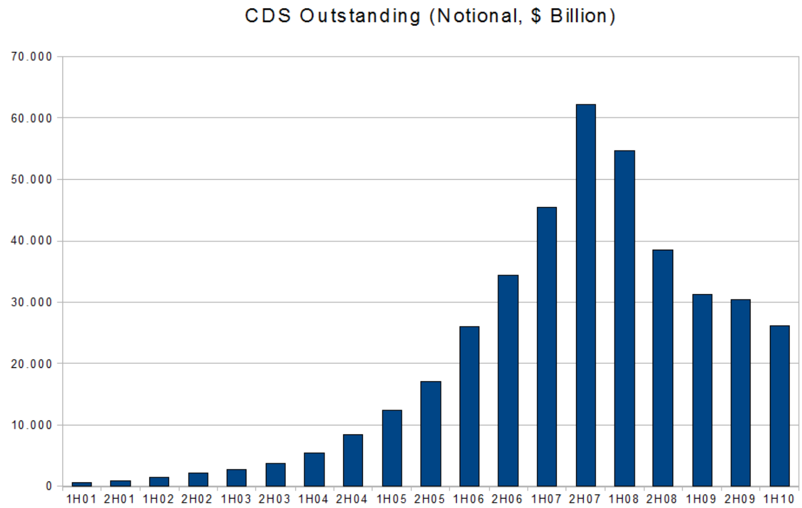


Figure 2.2: Semiannual evolution of CDS market, 2001-2010. (*Source: BIS* [113])

CDS allowed credit risk to be managed separately from loans: by buying CDS protection, banks could mitigate the risk profile of their portfolios without altering their compositions.

As an example, consider a small commercial bank who wants to hedge credit risk of a corporate borrower towards whom the bank is already largely exposed; by transferring this credit risk to another bank through a CDS, the bank can keep lending to this customer, but avoids excessive concentration risk, reduces the resources committed to the borrower and frees capital for different investments.

CDS were originally highly tailored to the needs of the counterparties, which were free to privately agree on any of the clauses in these contracts; among them, the *payment dates* for the buyer's premium or the different types of *credit events* leading to the *triggering* of the contract (the activation of the seller's payment) and the post-default settlement of the contract.

Particular mention deserves the latter of these clauses: choosing different settlement methods is indeed strictly connected to the determination of the post-default value R of the reference obligation.

Cash settlement provides a reimbursement equal to the difference between the par value and the post-default value of the buyer's position. Since forecasting this *recovery rate* R was rather challenging for both counterparties, 73% of CDS contracts had *Physical Settlement* as a settlement clause until 2005 [114].

Physical settlement eliminates the problem of determining the final price R: indeed, it provides for physical delivery of the reference obligation for a face value of $\wp^*$ in exchange for the seller's post-default payment.

Figure 2.2 shows the impressive increase in trading volume of CDS on a global scale, measured as the aggregated outstanding notional of any outstanding contract, which reached its peak of nearly $60 trillions in the second half of 2007.
As observed in [113], the subsequent decline was not due to a decrease in the appeal of the CDS market, but rather to *trade compression* procedures aimed to reduce the outstanding notional.[30]
Despite the benefits stemming from risk management and hedging procedures, the crisis revealed several structural and operational shortcomings of credit derivatives market. In particular, because of the OTC nature of this latter, relevant information about the *real* credit risk borne by protection sellers was partly concealed, preventing regulators from collecting complete information on existing trades. [91]
Moreover, the bilateral nature of CDS contracts exposes them to *counterparty risk*, that is, the risk that one of the two parties does not fulfill its obligations. [9]
Counterparty risk is not independent from credit risk; as an example, consider a rise in credit risk of the reference entity: this deterioration in creditworthiness weakens the seller of protection, by increasing the likelihood that he will be asked to pay, and increases counterparty risk for the buyer.[31]
The growth of the CDS market required the creation of a framework of greater legal certainty, capable of reducing the number of disputes and facilitating supervision by market authorities. The main obstacle was the highly tailored nature of different contracts, self-assessed by the parties to each transaction.
Standardizing credit derivatives market was therefore considered a necessary step towards a better regulation of the CDS market: a first attempt in this direction was made by the *International Swap and Derivatives Association*[32](ISDA) through the *2003 Credit Derivatives Definition* [62].
The financial turmoil started in mid-2007 brought the CDS market to the attention of regulators again, while many weaknesses of the financial system were exacerbating a widespread impact due to the interconnections across different markets.
Several regulatory statements included guidances for the CDS market: among them, the most influential was the President's Working Group[33] (PWG)'s *Policy Statement on Financial Markets* [91], dated 2008.

[30] Any CDS, since bilaterally traded, is also double counted with this method so gross notional reflects past trades but provides little information on real credit risk bore by a dealer in this market, see again [114].

[31] There is also an indirect connection between credit and counterparty risk, due to the posting of collaterals [114]; recent studies [15] introduced also the concept of *wrong way risk* where the interaction between the two is reversed.

[32] ISDA is a private international association collecting more than 800 members including, among others, dealers, issuers and law firms.

[33] The PWG was originally established to respond to the "Black Monday"of 1987, and gathers together several key representatives of US financial institutions.

The main purpose of the document was to analyze the causes of the financial crisis, and to provide recommendations in order to "*..take the steps necessary to mitigate systemic risk, restore investor confidence, and facilitate stable economic growth*". Severe shortcomings in risk-management practices were revealed: these practices caused significant losses and balance-sheet pressures, contributed to the tightening of lending terms and negatively impacted economic growth too.
Among them, the inaccuracy and untimeliness of trade data submission, lack of robust procedures for the resolution of trade matching errors, operational problems (counterparties miscommunication, backlogs of unconfirmed trades) and uncertainty in post-default settlement.
In order to tackle these issues, the PWG proposed to create an infrastructure endowed with decisional power in any of the processing events over the lifetime of such contracts, so as to ensure transparency and coordination in determining relevant credit events for any transaction in the market. This infrastructure should also be responsible for determining a post-default value acknowledged by any investor. A precise ratification of relevant credit events was thus to be introduced: furthermore, it was deemed necesessary to reach also a certain degree of standardization of clauses which were formerly tailored to the needs of each pair of counterparties, in order to facilitate an electronic processing similar to that of an exchange board.
A response to PWG's guidances was the introduction by ISDA of two supplements to [62], namely the *March 2009 Supplement* [63] , also known as the "*CDS Big Bang*" followed by the *July 2009 Supplement* [64], named "*CDS Small Bang*".
More than 2000 market participants, including banks as well as institutional investors, voluntarily adhered to *Big Bang* protocol, see [9].
These latter, despite some changes to be globally applied, were specifically addressed to North American corporate contracts: the following *Small Bang* protocol was drawn to introduce the same amendments for European corporate and sovereign CDS, as well as to deal with the problem of *credit restructuring.*
The main novelties introduced by the *Big Bang* contract are summarized as:

- introduction of *Determination Committees*;
- introduction of an electronic *Central Clearing House*;
- introduction of an *Auction Settlement Method*;
- restrictions on *Restructuring* conventions;
- creation of a *Credit Event Backstop Date*;
- introduction of a *First Full Coupon* clause;
- introduction of a *Fixed Coupon* plus *Upfront* fees.

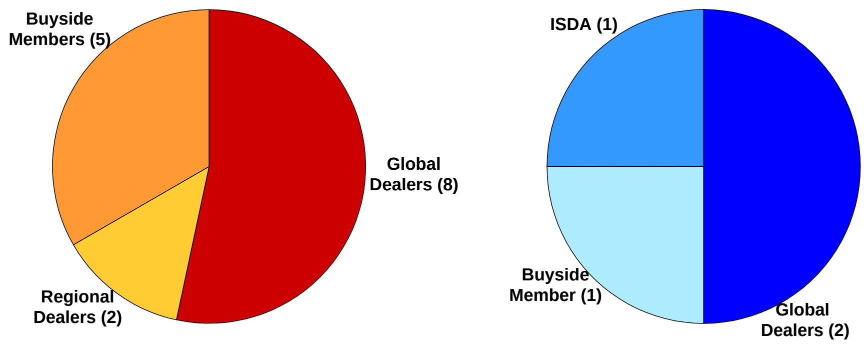


Figure 2.3: DCs composition: Voting and Non-voting members. (*Source: Markit* [82])

The *Small Bang* brought about these changes to European contracts; among the novelties of this second protocol, the most important was the hardwiring of an auction mechanism for restructuring credit event.
Standardization of CDS contracts[34] is also a straightforward way to net out a large number of opposite positions with the same features (entity, maturity..), offsetting the cash flows that these positions generate.
Netting out opposite positions can significantly reduce payment volumes so that cash shortages are less likely to cause a default. The total outstanding notional, as measured after netting out positions, should also give a more efficient measure of global exposures to credit derivatives market.
This section focuses on this standardization process, exploring its main features, with particular interest for the standardization of coupons and payment dates, and the introduction of upfront payments in CDS contracts, which are the cornestores of the CDS-bearing yields framework (section 2.2).
Before 2009, counterparties priced a CDS contract by agreeing on the annual coupon of the CDS itself (the *spread* S); after the introduction of a standard coupon in 2009, counterparties started to price a CDS contract by agreeing on the upfront payment, which represents the expected discounted value of the difference between the coupon that would have been agreed upon in the old regime and the new one.
The upfront correspondent to any given spread must be determined in the same way by all market participants: in order to accomplish this task, ISDA developed a toolkit, the *ISDA CDS Standard Model*, that provides a one-to-one mapping of these quotations (upfront and conventional spread), based on standard no-arbitrage principles; details are presented in C.2.1.

[34] Note that standardization applies to the whole credit derivatives market; here however, we will deal only with single name vanilla instruments.

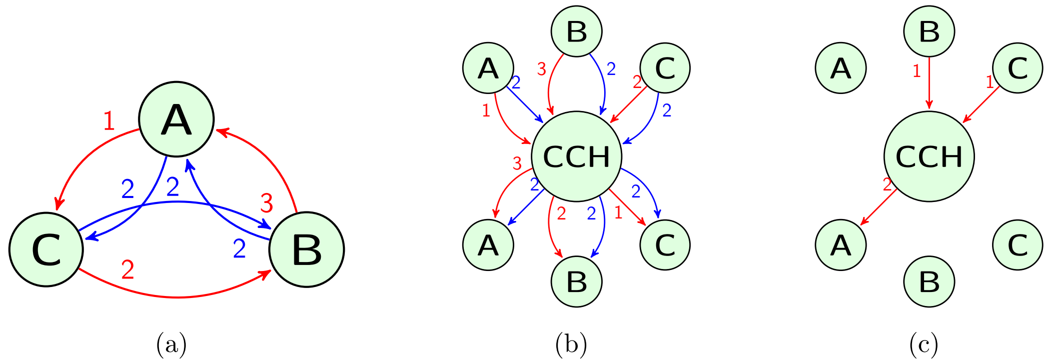


Figure 2.4: Trade compression through CCH: A, B, C are dealers and arrows points to the one buying protection for a notional equal to the number over it. Different colors refers to different reference entities. Figure 2.4(a) shows a network of bilateral transactions processed to a central counterparty in 2.4(b) and then compressed 2.4(c). The outstanding notional is reduced from 24 to 8. (*Source: Markit* [82])

2.1.1 Credit Derivatives Determination Committees

The *Big Bang* introduced five *Determination Committees* (DCs), one for each relevant region[35], and each of them having responsibilities with respect to that region.
Rules to determine the composition of any DC are explained in detail in [82]: the final composition of each of the five is shown in Figure 2.3.
DCs are mainly addressed to harmonize industry and avoid misinterpretations as any credit event affects the reference entity. According to the 2003 rules, the potential occurrence of a credit event was determined through a private *notice* delivered from one party to the other: this of course creates often disputes, both on relevance and timing of that event. It could be also the case that triggering a CDS in any of these bilateral transactions led other backlogged transactions to demand for payment, creating unpredictable reactions in the market.
The *Big Bang* intended to rule out these problems introducing a simpler principle: any of the ISDA members, with the sponsor of a DC member, is endowed with the right to request the DC to be conveyed in order to take a decision on whether and when a specific credit event for the transaction occurred. [63]
The *DC credit event/No credit event Announcement* is the day ISDA effectively takes its binding decision on target credit event: in case this latter is announced, it will be the DC again to decide the terms of the post-default *Auction* (see section 2.1.3) and the set of *Deliverable Obligations*.

[35]Namely the Americas, Asia-ex Japan, Australia-New Zealand, EMEA (Europe,Middle-East and Africa) and Japan itslef.

The presence of a determination committee is also fundamental in order to introduce a Central Clearing House (section 2.1.2): it standardizes the occurrence of the credit event and draws out the rules to determine the Final Price, so that any of the positions of the CCH referred to the same reference obligation and credit event will be dealt with according to the same rules.

2.1.2 Central Clearing House

The introduction of a dedicated central counterparty in the credit derivatives market was a further step towards counterparty risk reduction [82].
Common feature to achieve this goal was the idea of *trade compression*, that is, reducing the number of redundant contracts.
This was achieved through private operators collecting multilateral information from the network of counterparties. These operators, maintaining the same risk profile of each of the participants' positions, proposed then a renewed set of trades that becomes compulsory for each of the parties agreeing to it.
Before the *CDS Bangs*, the CDS market exhibited a complicated network of bilateral transactions, each of them possibly providing different clauses that were to be mathematically translated in a huge amount of stand-alone variables interacting in the same network, see Figure 2.4(a).
The introduction of multilateral agreements urged coordination of investors: the *Central Clearing House* is an improvement for the netting procedure, see figures 2.4(b) and 2.4(c), and it also substitutes bilateral counterparty risk with the risk of its own failure. The drawback of such a market is that market participants are forced to abandon the opportunity of meeting their demands for specific products[36].

2.1.3 Auction Settlement

Early in the life of CDS contracts, most of defaulted obligations were settled according to physical settlement, in order to avoid forecasting post-default values when marking the contract to the market. This system was coherent with the use of CDS as hedging instruments, but the growing interest of speculators in this market enhanced the number of entities for which the outstanding notional of CDS surpassed the outstanding debt they referred to [9].
When a credit event occurred, speculators on buy-side were forced to purchase obligations to be delivered in order to settle the transaction and thus receive payment from the CDS contract, creating artificial price pressures and distorting the market.

[36]Moreover, clearing houses are not particularly efficient with illiquid financial products, as is precisely the case with customized derivatives, see [107] for a deeper analysis of the trade-off between centralexchanges and OTC markets in this context.

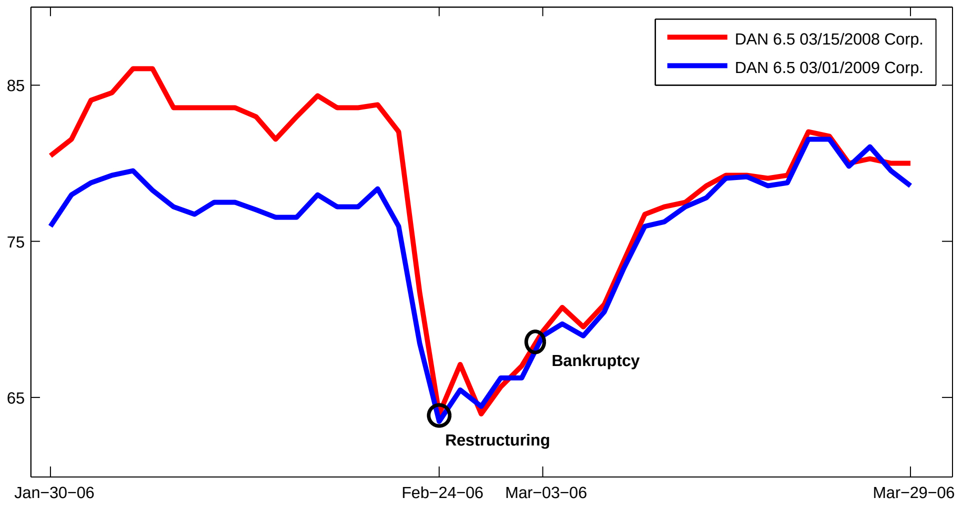


Figure 2.5: *Short squeeze* of Dana Corporation bonds at the turn of default. On February 24, Dana starts debt restructuring and on March 3 filed under Chapter 11 for Bankruptcy. After that, rush to buy bonds resulted in a sudden price rising of a yet defaulted obligation. (*Source: Amadei et al. [9]*)

An interesting example is the *short squeeze* of the price of *Dana Corporation* bonds which followed its bankruptcy in March 2006, see figure 2.5.
Cash Settlement was introduced in order to avoid these issues: payments owed to buyers due to triggered CDS did not request any delivery. The problem to be faced was to find a mechanism to set a transparent and trustworthy final price R that the whole market could use.
Following the recommendations of the PWG and in the same standardization spirit, *Auction Settlement* is introduced in the *Big Bang* supplement.
As pointed out in [82], several auctions had been hold also before the *CDS Bangs*: their weakness was that participants were requested to sign separate protocols to adhere to any of the auctions, which was not particularly efficient compared to the hardwiring of an auction methodology into standard contracts.
The main benefit of holding an auction is to set a market price to be used to set *all* trades across the market. Physical delivery at different times could expose the buyer-side counterparties to further profit or loss due to the investors scrambling to buy bonds, even if those same buyers' positions remained flat.[37]

[37]Physical settlement was maintained in standard CDS as a *fallback settlement method* in case the DC, due to an insufficient number of dealers willing to trade defaulted obligations, decides not to hold any auction.

2.1.4 Restructuring Clause Convention

Among credit events, restructuring is the most difficult to define: it refers to changes in the covenants of reference obligations, like delaying or change in the currency composition of payments.
Restructuring used to be considered a credit event only if *related to deterioration of credit worthiness of the borrower*, a relationship yet difficult to be determined.
It could happen, for example, that these changes are not disadvantageous for the buyer, and yet he finds profit in triggering the CDS, or viceversa.
As an example (see [83] and [92]), in 2000 Conseco Finance restructured its bank debt to include new guarantees and increased coupons: these changes were not disadvantageous for the holders of these obligations, and this fact was reflected by their price which was almost unaffected. However, some banks on buyer-side of transactions triggered the CDS, delivering cheaper longer dated bonds and receiving this almost-par value for their restructured bonds.
According to ISDA [62], four different types of restructuring clauses can be specified: *Old Restructuring* (*Old R*), *Modified Restructuring* (*Mod R*), *Modified - Modified Restructuring* (*Mod-Mod R*) and *No Restructuring* (*No R*). The differences among the first three focus mainly on maturity/transferability of deliverable obligations; the last name speaks for itself.
Trading contracts with restructuring obviously demand for additional premia: before 2009, CDS on *North American Investment Grade* typically traded with *Mod R* while *North American High Yield* traded with *No-R*; most European contracts instead provided *Mod-Mod R* clause.
The *Big Bang* ensured DCs the authority to hold auctions to settle contracts after either a failure to pay or a bankruptcy event. It however prohibits from authorizing auctions to settle trades after restructuring events: under the US jurisdiction, many restructuring scenarios are filed as bankruptcy under "*Chapter 11*".
On the contrary European jurisdiction separates bankruptcy and restructuring in a much more sharped way: as a result approximately 96% of European CDS contracts used to trade with Mod-Mod R [82].
ISDA decided to include restructuring among the significant credit events in Europe, yet still an auction mechanism was to be designed distinctively for such events, and the CDS Small Bang is addressed exactly to tease out this problem.
Briefly, the problem was that the combination of maturity limitation of deliverable obligations and maturity of CDS could require a too large number of different auctions to settle all contracts, hardening price discovery and increasing operational risk as well as mispricing between one auction and another. We will not enter into details, which are examined again in [82].

In order to deal with these issues, DCs were granted with the power to aggregate deliverable obligations into *maturity buckets*, and to set auctions only for those buckets, so that multiple auctions are allowed, but their number is restrained.
DC can also decide not to hold any auction for a given bucket[38] if it is likely that the auction will be conducted on illiquid credits, or be redundant across different buckets. If no auction is held for a given maturity bucket, a *movement option* is provided: the buyer can move to the closest following bucket for which an auction is being held and the seller can move to the 30 years maturity bucket.
The last clause dealing with restructuring is the so called *Use it or Lose it* feature: in case of restructuring, a triggering deadline of five business days following the publication of the final list of deliverable obligations is established; this is aimed to prevent protection buyers not to trigger a CDS even if the reference obligation is traded below par value, in order to wait a subsequent event and get a higher payout.
As a result of both *CDS Bangs*, North American CDS provides *No R* clause while European Corporate and Sovereign trade with *Mod-Mod R*.

2.1.5 Credit Event Backstop Date

Before the *CDS Bangs*, CDS Protection started in most of the cases the first calendar day after the trade date; the introduction of a lookback period for credit events was deemed to be mandatory in order to reduce backlog of trades and allow the DC to announce an eventual a credit event without affecting the market with the time they spend in taking any decision.
In order to clarify this point, we follow the example in [82]: assume an investor enters into a short position on a CDS. One week later, in order to offset this position, he enters a long position on a CDS with the same exact features of previous one.
Assume that DC has been convened within these two transaction dates and later on they decide that a relevant credit event occurred during this rather small time interval. The two positions are not truly offset, since the investor must reimburse the buyer because of first transaction but gets no money from the second, because the relevant credit event is timed before this latter, see figure 2.6(a).
In order eliminate this *residual stub risk* a *credit event backstop date* was introduced. Let t be the *trade date*: the *Big Bang* provides that the effective date for protection to be be the backstop date $t - 60$, that is the trade date itself minus sixty calendar days, see figure 2.6(b).

[38]Unless the so called *300/5* criterion applies, that is, if five or more dealer members of the DC are involved into more than 300 transactions triggered by that credit event.

2.1.6 Payment dates and Full First Coupon

As far as 2003, the dates when the buyer paid the coupon to the seller were specifically agreed by the two parties: the *Big Bang* explicitly standardized the CDS dates[39] defining them to be the 20th of March, June, September and December, business adjusted, which are also the standard days of contracts maturities.
The first payment date is the first of these days following $t+1$: for example, trading a CDS with Maturity $T = 5$ years in January 2013 means the 20th of March 2013 will be the first payment date and the 20th of March 2018 will be the maturity.
Observe that payments are postponed on the buyer side: the coupon he pays the seller on each payment date for the protection offered in *previous* period. If trade date t is set up off a standard payment date, the amount and date for the uncovered period must be refunded to the buyer. In order to determine the accrued amount, it is sufficient to count the number of days in this time stub and compute the correspondent fraction of the annual payment S.
It is left to be established when this payment is due. Before the *Big Bang*, the procedure, established by the 2003 definitions [62], was yet rather tricky.
The payments schedule depended on whether the trade date occurred before or within 30 days from first payment date. In the first case the payment was accrued within a *short stub period* going from t to the first payment date T_1 following t, and made on T_1; else, the payment was accrued on a *long stub period*, going from t to the second payment date T_2, and made on this latter date, figure 2.7.
This mechanism clearly jars the request for standardisation, and mantains the complications in payments offsetting because discounted future cash flows are a discontinuous function of trade date.
The proposed solution was to introduce a *Full First Coupon Payment*: the payment is due on T_1 and accrued on the whole period elapsing from previous standard CDS date T_0. In order to compensate the buyer for the unprotected days he paid for, a *Riskless Accrued Premium* $AP_t = S \cdot AP01_t = S \cdot (t - T_0)$ is then[40] owed by the seller at inception, seefigure 2.8(b).

2.1.7 Standardized Coupon

The last relevant issues concern the CDS spread, which was to be standardized so as to match as many contract as possible in the central clearing procedure and facilitate trade compression, see figure 2.8(a).

[39] Although it was already common practice, to blend positions in CDS and bond market.

[40] Here S indicates generically CDS spread, since this clause is not necessarely subordinated to standard coupons conventions whether the contract is traded OTC. Observe that, on the seller's side, this is equivalent to the accrued coupon of an obligation that is received when purchased between two payment dates.

It was typical to trade CDS according to their annual coupon S, quoted in basis points per year. Since late 2008, it became common practice to trade CDS for highly distressed/high yields names, with a fixed coupon (typically 500 basis points) and an attached *upfront payment* to be made at the time of inception t.
It is clear (see [100]) that a fixed coupon coupled with an upfront payment facilitates unwinds of contracts, as only a cash adjustment is needed to exchange a CDS *when a fixed cash flow would income in the future*, and the source of variability is that of the term structure used to discount cash flows.
Upfront payments make it easier for sellers to deal with early default that triggered relevant quantities of contracts, thus reducing counterparty risk by *pumping liquidity* in the market itself. Moreover, this trading convention prevents speculators to enter the market of CDS, as attracted by zero entry costs.
The *Big Bang* standardizes coupon for North American contracts, either to 100 bp or 500 bp: the choice of the latter value was due to common market practices for highly volatile names while the former was chosen to let the most number of names to trade greater or equal than par with respect to previous quoted spread.
In this way the market avoids to request a huge amount of payments *from* the seller to the buyer in order to match any of the transactions, which would be the case if a value lower than average was chosen as standard. The *Small Bang* addressed these issues to European corporate and western sovereign CDS, but leaves a wider range o f standard coupons[41] with respect to correspondent American contracts.
Even if upfront payment becomes in this way the real metric for CDS market, most dealers are still quoting the CDS spread rather than their upfront.
These *conventional*, or *par*, spreads are the values that sets to zero the expected cash flow of a CDS trading without any upfront payment. Such a computation requires a model that marks the CDS contract to the market.
It is important to stress that a *unique methodology* must be set for the whole market to convert spreads to upfront payments.
ISDA developed an algorithm, the *ISDA CDS Standard Model*[42], which allows to convert quotes with a standardized algorithm, in line with the purpose of the *Bangs*.
An interesting question arise from such new contracts: modeling hedging strategies that use CDS requires indeed the analysis of their real cash flow, which *is not the one provided by the par spread.*
Next section explores yield curves modeling framework and shows the crucial role of this new payments schedule when the CDS-bearing term structure is defined.

[41] Namely 25bp, 100bp, 300bp, 500bp, 750bp and 1000bp; a reason for introducing such different coupons is the cautiousness of customers to trade with large upfront points, see [83].

[42] Markit is currently the administrator of this code, providing support and mantainance, as well as further development: the code is available with open source license at *http://www.cdsmodel.com.*

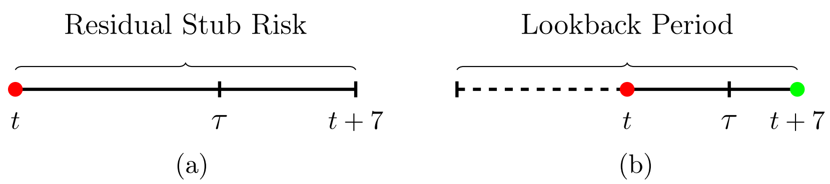


Figure 2.6: Protection before 2.6(a) and 2.6(b) after the *CDS Big Bang*. Red (green) dots shows the triggered short (long) positions. (*Source: Markit* [82])

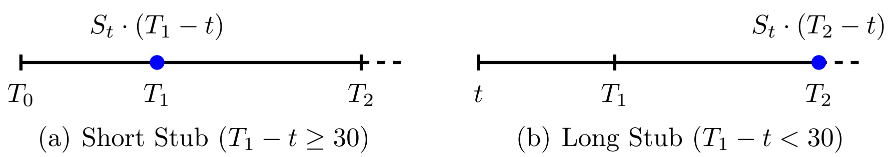


Figure 2.7: Cash flow of buyer's first payment before the *CDS Big Bang* in case of Short Stub 2.7(a) and Long Stub 2.7(b). Blue dots point out payment dates while S is the annual coupon rate of the contract. (*Source: Markit* [82])

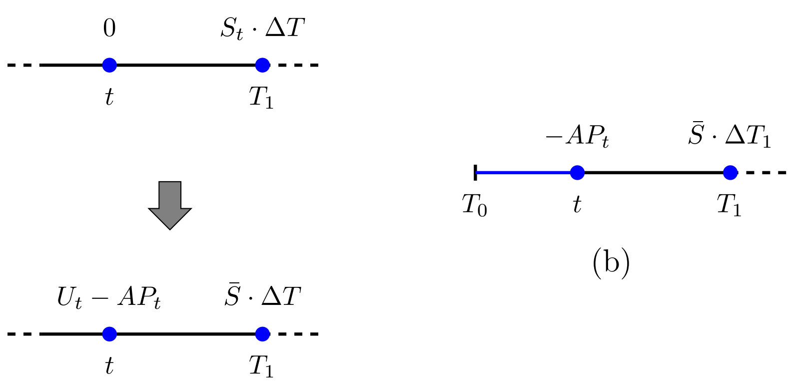


Figure 2.8: Transition from floating spread S_t to floating upfront U_t plus fixed spread $\bar{S}$, 2.8(a). Cash flow of buyer's first payment after *CDS Big Bang*, with full first coupon. The minus sign refers to positive cash flows *towards* the buyer 2.8(b). (*Source: Markit* [82])

2.2 Modeling the term structure of interest rates

Several interpolation methods can be used to extract a benchmark term structure from government bond market[43]. All of them rely on common assumptions inducing similar features. At time t, it is assumed that target country A admits a *term structure* of its obligations market: that is, a *zero-coupon-bond* (zcb) exists for any maturity $T > t$, so that it is possible to define a time-indexed set of maps:

$$\{P(t, \cdot) : T \mapsto P(t, T), T \geq t\}_{t \geq 0} \tag{2.2.1}$$

being $P(t, T)$ the price at t of a zcb paying 1 at T.
Consider a generic (coupon) obligation, with maturity T_K: standard non-arbitrage assumptions imply that current bond *dirty*[44] price $\wp(t)$, observed on the market, must be equal to the sum of the obligation's discounted future cash flow:

$$\wp(t) = C \cdot \sum_{k=1}^{K} P(t, T_k) + \wp^* \cdot P(t, T_K), \tag{2.2.2}$$

where $\mathcal{T} = \{T_k : k = 1 \dots K\}$ is the set of future payment dates, C is the bond's coupon and $\wp^*$ is the *face value* paid at maturity T_K. Each of the zcb prices can be used to retrieve the correspondent *yield-to-maturity*:

$$y(t, T) = -\frac{\log(P(t, T))}{T - t} \qquad \forall \;\; T > t \;. \tag{2.2.3}$$

and this latter is in turn expressed as a function of the set of parameters θ_t.
In this way, $y(t, T) = y(\theta_t, T)$ so that, inverting (2.2.3), the term structure at time t can be expressed as a function of those same parameters $\{P(t, T)\} = \{P(\theta_t, T)\}$.
Assuming face value is the same for any obligation, we can write:

$$\wp(t) = \wp(y(t, \mathcal{T}), C) = \wp(y(\theta_t, \mathcal{T}), C) = \wp(\theta_t, \mathcal{T}, C) \tag{2.2.4}$$

where with $y(t, \mathcal{T})$ we indicate the unknown function $y(t, \cdot)$ evaluated on the grid defined by the fixed time schedule of target obligation.
Observe that $\mathcal{T}$ and $\mathcal{C}$ are known at t, hence θ_t remains the only variable because $\wp = \wp(\theta_t, \mathcal{T}, \mathcal{C})$ does not depend on time explicitely.
Figure 2.9 outlines the reasoning behind this procedure: given any obligation, the sum of current *clean* price and accrued coupon (both *floating*, as daily marked to market) is expressed as a combination of *fixed* future cash flows on a *fixed* time schedule. Market price $\wp(t)$ is then a function of $\wp(\theta_t, \mathcal{T}, \mathcal{C})$, defined in (2.2.4).

[43] See for example [88] for a rather exhaustive review.

[44] Bearing accrued coupon.

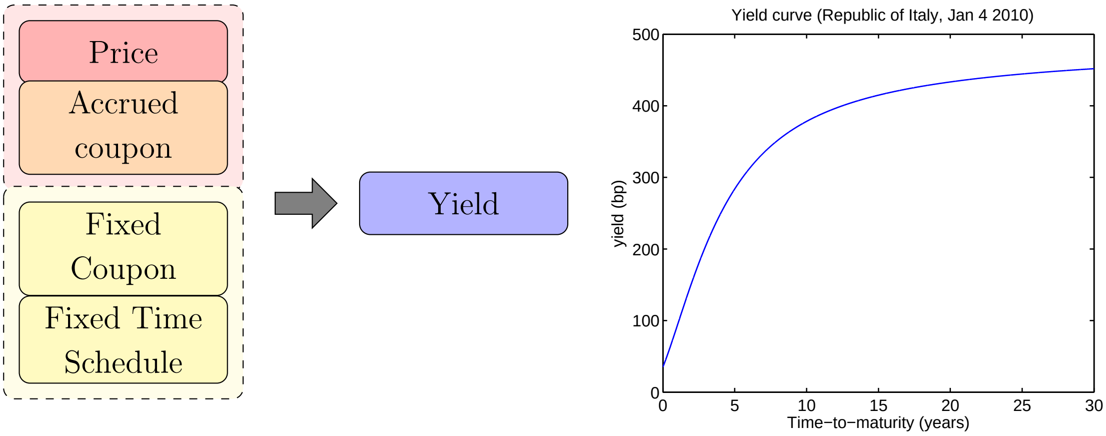


Figure 2.9: Yield curve: for each obligation, combine floating quantities (red) with fixed ones (yellow) and use interpolation techniques to retrieve a yield curve.

Now consider at time t the set of the J_A obligations relevant for country A. Specifically, define the set of *dirty prices* $\{\wp_j^A(t)\}_{j=1}^{J_A}$, each with maturity T_{Kj}^A, coupon payment dates grid $\mathcal{T}_j^A = \{T_{kj}^A\}$ and fixed coupon $\{C_j^A\}$. Country A will be fixed, so the index is dropped in order to lighten notations.
The advantage of (2.2.4) is that it allows to write the price of each obligation as a function of the same parameter vector θ_t. We collect such prices in the vector:

$$\boldsymbol{\wp}(\theta_t, \mathcal{T}, \mathcal{C}) = (\wp_1(\theta_t, \mathcal{T}_1, C_1) \cdots \wp_J(\theta_t, \mathcal{T}_J, C_J)) = \boldsymbol{\wp}(\theta_t) \tag{2.2.5}$$

where $\mathcal{C} = \{C_1 \dots C_J\}$ and $\mathcal{T} = \{\mathcal{T}_1\} \cup \dots \{\mathcal{T}_J\}$ are the set of aggregated coupons and relevant maturities for the obligations set, and the last equality comes from the fact that these latter are fixed at t. The *best-fit parameter vector* θ_t^* is defined as the solution to the weighted least squares problem:

$$\theta_t^* = \underset{\theta_t}{\text{argmin}} \;\; (\boldsymbol{\wp}_t - \boldsymbol{\wp}(\theta_t))' W(\theta_t)(\boldsymbol{\wp}_t - \boldsymbol{\wp}(\theta_t)) \tag{2.2.6}$$

where $\boldsymbol{\wp}_t = (\wp_1(t) \dots \wp_J(t))$ is the vector of observed prices at trade date t and the weighting matrix $W(\theta_t)$ is chosen so that it refines interpolation in those time intervals where most of the relevant bonds maturities are concentrated.
A set of maps of the form $T \mapsto \wp(\theta_t^*, T)$ is dynamically implied for any t using the sequence of best-fit parameters vector $\{\theta_t^*\}$, determined iterating this method in t.
The form of $\wp(\theta_t, T)$ typically depends on the interpolating functions: here we chose Nelson-Siegel method [89], as their algorithm and following refinements are the most used among central banks [1].

It is worth to stress that *no default probabilities are taken into account when computing yields*. It is commonly assumed that the obligations market prices reflect any risk premium attributable to credit events that are likely to occur in the future. In this setting, θ_t will be a four parameter vector

$$\theta_t = (\beta_{0t}, \beta_{1t}, \beta_{2t}, \beta_{3t})$$

which allows in turn to express yields as:

$$y(t,T) = y(\theta_t, T) = \beta_{0t} + \beta_{1t} \cdot g_{1t}(\beta_{3t}, T) + \beta_{2t} \cdot g_{2t}(\beta_{3t}, T)$$

where g_{it} are defined in section C.1, where a detailed description of the method is also provided; it is common to refer to the parameters respectively as *level*, *slope*, *curvature* and *scale* of the yield curve.
The reasons behind this terminology are beyond the aim of this paper, although names are helpful in capturing different forms of shape changes of the curve along time: a detailed discussion can be found in [39].
A typical shape is shown in figure 2.9: as mentioned earlier, the focus is on $T = 10$ years maturity, hence y_t will simply indicate $y(\theta_t^*, 10)$, where for any *fixed* t the vector θ_t^* is defined in (2.2.6).

2.2.1 Standard CDS contracts: CDS-bearing yields

As mentioned in previous section, standardization of credit derivatives provides the correct framework to perform our tests using market data. The summary provided in section 2.1 concerning the standardization process is sufficient[45] to disclose it.
The main assumption validating the arguments which will follow is rather a stress test for standard market.
More precisely, it is assumed that the whole market trade on a platform monitored by a CCH (section 2.1.2), constituted by big dealers having easy access to credit market which are effectively the major owners of European sovereign debt.
It is assumed that counterparty risk is now sufficiently scattered among them so that *standard CDS-contracts are unresponsive to counterparty risk.*
Furthermore, it is also assumed that the CCH injects sufficient liquidity in the market so that *positions on any of the derivative contracts for any reference entity are not charged with liquidity premia.* Hence, CDS *prices credit risk only.*
It is clear that ascribing these assumptions to the introduction of the standard market would be strained: as imperfections will come out from the hedging strategy (section 2.3), a possible solution could be dropping any of these two hypotheses. This could also suggest possible patterns for future market enhancements.

[45] A rigorous description of the standardiztion process is available: the interested reader can consult the official documentation [63] and [64].

The changes in the contracts cash flow, payment dates and coupons are instead fundamental building blocks of the set up underlying the techniques which are used to construct the *hedged* yield curve.
Figure 2.8(a) shows the changes in CDS coupons with the introduction of the standard coupon $\bar{S}$. The market switches free-entrance contracts with floating spreads to contracts providing a floating entrance fee and a fixed future payment.
As aforementioned, the introduction of upfront payments contributed to discourage purely speculative long naked CDS positions, because of the initial cost.
This enforces our choice to perform this analysis in the perspective of a hedger, as this type of investor should be the most attracted by standard market improvements.
From a technical point of view, whether future coupons were priced daily, we would obtain future cash flows varying in time, and this would add another source of randomness and inaccuracy in the statistical results.
Upfront payment is immediately[46] due to the counterparty and can be interpreted as the *minimum premium demanded by the market to transfer credit risk*.
This premium is settled by imposing an additional percentage of the obligation's face value to be attached to the initial price together with a set of (standard) fixed coupons to be paid at future (standard) dates.
The first full coupon clause introduced in subsection 2.1.6 provides the buyer to pay full protection for a fixed accrual period (one quarter) between standard dates.
He is then reimboursed of the accrued premium $AP_t = \bar{S} \cdot (T - t)$, see figure 2.8 (b), which repays him for the credit risk protection period he did not benefit from.
The premium is then subtracted from upfront price to determine the *cash settlement amount* $u(t) = U_t - AP_t$.
Credit derivatives market platforms continue however to quote CDS at time t in terms of the time-varying spread S_t. ISDA provides a standard mechanism [84] to convert quoted spread to upfront payment: section C.2 provide a rigorous description of their algorithm.
It is worth to stress that the mechanism is *uniquely* defined for each participant in the standard market, so that there's no subjectivity in converting S_t to U_t.
As long as a unique pricing mechanism is used by any dealer to imply upfront payments from quoted spread, we have a unique mapping from CDS quoted spread to upfront payments, so any discussion on correctedness of ISDA modeling framework is unnecessary here.
Any investor willing to enter into a CDS position observes market quotes for buying protection and determines the same *spot price* for credit risk $u(t)$, playing the role of dirty bond price $p(t)$.

[46]Ignoring small effects coming from settlement dates and day count conventions.

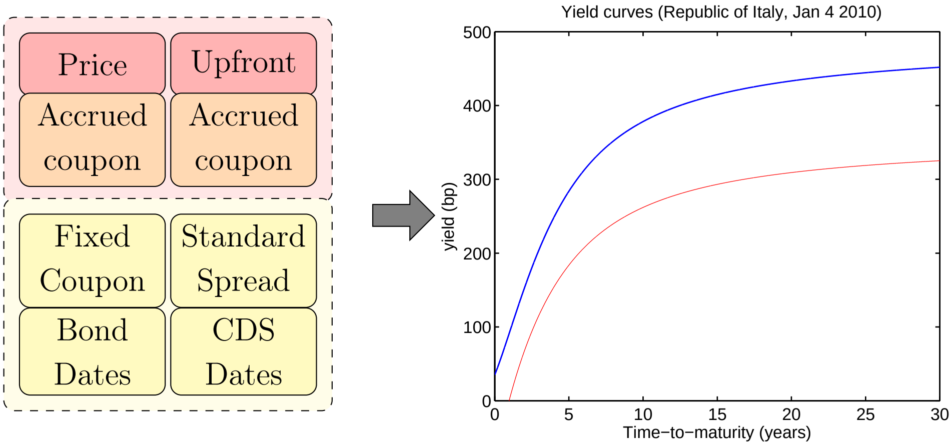


Figure 2.10: CDS-bearing yield curve: standardization allows to rely exactly on the same methodology commonly used for custom yields. Spot floating variables (red) are combined with future fixed ones (yellow) and the yield curve is retrieved using the same interpolation techniques.

It is clear how this standardization process calls for the use of fixed-income tools. The cash flow of new CDS contracts includes a cash amount to be immediately paid, computed combining a spot price and the accrual of a coupon, and a future cash flow which is deterministic on the whole set of subsequent trade dates, both in terms of payment dates and amounts (figure 2.10).

The cash flow of the derivative is now aligned to that of its underlying. It can be argued that future CDS payments are contingent on default, hence default probabilities should be taken into account as source of randomness in future cash flows, and used in pricing equations.

However, as mentioned before, that same assumption lies behind any of the standard methods used to imply yield spreads *vis-à-vis* Germany: the existence of a term structure reflecting credit risk within the correspondent yields is postulated, and differences with German yields are then taken exactly to isolate this component.

Consider now a *credit risk hedging strategy*, obtained by combining a long position on the obligation with a long position on a standard CDS contract: we define *CDS-bearing bond* such a hedging portfolio for target obligation.

The payoff of this portfolio is that of a more costly bond in terms of initial price that will also repay the owner with lower coupons, due to the (quarterly) payment of CDS standard spread. In order to build up the CDS-bearing yield curve, further details must be specified about the construction of the hedging strategy.

First of all, the choice of maturity of the CDS contract: given any obligation contributing to the *naked* yield curve construction, a standard CDS is purchased and chosen *the way an infinitely risk averse investor would.*

Standardized contracts are traded with only ten different standard maturities, while at trade date t the set of obligations residual lives $\{T_{kj}\}$ is generally larger. The choice is to hedge the long bond position with the CDS expiring on the *first standard maturity which is able to cover the whole residual life of the bond.*
This latter is an example of shortcomings stemming from standardized CDS market: for example a large premium must be paid for early expiring obligations, which must be covered up to the minimum standard available maturity, namely six months. Credit derivatives could thus be less attractive at short maturities, and investors might prefer to enter into specifically tailored over-the-counter transactions [47].
The marked-to-market value of the position at time t is given by the sum of the obligation price and the upfront considering accrued coupons for both of them:

$$\wp(t) + u(t)$$

As for naked bond positions, standard non-arbitrage principles set this value equal to its discounted future cash flow.
Assume that there exists a term structure of such portfolios $\{\bar{P}(t,T)\}$, where $P(t,T)$ is the price of a zcb maturing at t which is protected by a *standard* CDS contract with maturity T. In a non standard-market, this assumption adds nothing new to those underlying the construction of naked yield curve, since whether the zcb exists, a specifically tailored CDS with maturity T can be settled.
In a standard market, such assumption is a bit more strained, so again eventual hedging imperfections could be solved by dropping it.
The discount curve $\bar{P}(t,T)$ differs from $P(t,T)$ in that credit risk is completely hedged through a standard contract[48]. It can be thus used as a discount factor for coupon portfolios where credit-risk is not present; the equivalent of equation (C.1.2) is:

$$\wp(t) + u(t) = C \cdot \sum_{T \in \mathcal{T}} \bar{P}(t,T) + \wp^* \cdot \bar{P}(t,T_K) - S \cdot \sum_{\bar{T} \in \bar{\mathcal{T}}} \bar{P}(t,\bar{T}) \qquad (2.2.7)$$

where $\bar{\mathcal{T}}$ is the set of the correspondent CDS payment dates, $\mathcal{T}$ is the set of target bond payment dates, including the obligation's maturity T_K, and C and S are the fixed bond and CDS coupon, respectively.
Applying to each obligation the same reasoning which is used to retrieve custom yield curve, the same procedure can be used to imply a *CDS-bearing yield curve* $\bar{y}(\bar{\theta}_t^*, \cdot)$ for any t; details are provided in section C.1.1.

[47]This however does not affect the analysis when long term rates are considered.
[48]Observe that the difference $P(t,T) - \bar{P}(t,T)$ is by definition the theoretical cost of standard CDS-protection for maturity T.

Obviously, equation (2.2.7) can be written in terms of quoted spread S_t as

$$\wp(t) = C \cdot \sum_{T \in \mathcal{T}} \bar{\bar{P}}(t,T) + \wp^* \cdot \bar{\bar{P}}(t,T_K) - S_t \cdot \sum_{\bar{T} \in \bar{\mathcal{T}}} \bar{\bar{P}}(t,\bar{T}) \tag{2.2.8}$$

but the implied term structure $\{\bar{\bar{P}}(t,T)\}_{t \geq 0}$ will be different from $\{\bar{P}(t,T)\}_{t \geq 0}$. Indeed, see section C.2, the upfront payment satisfies the equality[49]:

$$u(t) + \bar{S} \sum_{\bar{T} \in \bar{\mathcal{T}}} L(t,\bar{T}) \mathbb{Q}(\tau > \bar{T}) = S_t \sum_{\bar{T} \in \bar{\mathcal{T}}} L(t,\bar{T}) \mathbb{Q}(\tau > \bar{T}) \tag{2.2.9}$$

where $\mathbb{Q}(\tau > \bar{T})$ is the *risk-neutral default probability* and $L(t,T)$ is the zero discount factor implied through Euribor rates, as provided by the ISDA CDS Standard model. Hence (2.2.8) and (2.2.9) are equivalent, that is $P(t,\cdot) \equiv \bar{\bar{P}}(t,\cdot)$, only if $\bar{P}(t,\cdot) \equiv L(t,\cdot)\mathbb{Q}(t,\cdot)$, which cannot be as $\bar{P}(t,T)$ is credit risk free.
Again, the 10-years CDS-bearing yield $\bar{y}(\bar{\theta}_t^*, 10) = \bar{y}_t$ is considered, so that, given a generic country A, it is possible to imply out of market observations two time series $\{y_t^A\}$ and $\{\bar{y}_t^A\}$. Under the assumption that counterparty and liquidity risk are negligible, the difference between the yields of naked and hedged portfolios is due to credit risk only.
We define the *credit-risk (yield) premium* for country A at time t as

$$\pi_t^A = y_t^A - \bar{y}_t^A. \tag{2.2.10}$$

Notice that, as this new payments schedule is composed of an additional cost to bear at inception plus a sequence of negative coupons, $\bar{y}_t$ will be lower than y_t for any t, so π_t is a nonnegative process by construction.
Moreover, the yield $\bar{y}_t$ is a *country-specific credit-risk free yield*, constructed within the money and credit derivatives market of target country only.
It is thus interesting to measure that same credit risk premium π_t^{DE} for Germany, to capture eventual movements in the perceptions of Germany being a risk-free country. The central role of this country can be verified also by comparing German yields to a credit-risk free curve. Euribor rate is chosen as the benchmark credit-risk free term structure, so as to be coherent with CDS pricing and conversion methods[50].
The resulting 10-years Euribor rate $L_t = L(t, 10)$ is considered; the first econometric results thus concern the VAR processes (y^{DE}, L), $(\bar{y}^{DE}, L)$ and $(y^{DE}, \bar{y}_t^{DE})$.
The credit risk premium π^A can be directly compared to spread *vis-à-vis* Germany $s_t = y_t^A - y_t^{DE}$ to reveal eventual mismatches possibly related to the choice of a benchmark wich is not, in principle, credit-risk free.

[49] Accrual at default is not considered, yet it does not affect our argument.
[50] See section C.3.1.

Hence, the VAR models (π^{IT}, s^{IT}) (π^{ES}, s^{ES}) are explored too. Since the resulting yields $\bar{y}^A$ are in principle credit risk free yields, it is worth to examine the relationships with L_t, so the models $(\bar{y}^{IT}, L)$ and $(\bar{y}^{ES}, L)$ will be discussed. The CDS-bond basis:

$$b_t^A = \bar{y}_t^A - L_t \tag{2.2.11}$$

is interpreted as the *additional risk premium* which is paid by an investor purchasing a hedged bond position financing himself at Euribor.
This analysis could also be influenced by the fact that Euribor is not the correct credit-risk free benchmark. it is thus worth to examine the VAR models $(\bar{y}_t^{IT}, \bar{y}_t^{ES})$ and (b^{IT}, b^{ES}). The CDS-bearing yield difference:

$$\bar{y}_t^{IT} - \bar{y}_t^{ES} \tag{2.2.12}$$

is independent from L. Trended or systematic deviation from zero of (2.2.12) invalidates any attempt of modeling the basis as white noise discrepancy, disclosing *structural imperfections of the CDS-bond market.*

2.3 Econometric analysis

Data are sampled at daily frequencies from January 2, 2010 to June 21, 2013, this latter being the latest CDS standard date for the first half of 2013. Bond prices are taken from Bloomberg database, while CDS spreads and Euribor rates are supplied by Markit. The ISDA CDS Standard model is applied to quoted spread to convert them into upfront payments. Both naked and CDS-bearing yields are computed using the methods presented in section C.1.
We imply the processes y and $\bar{y}$ for each of the three countries and the Euribor 10-years par-rate L, and proceed to compute the yields spread s by taking differences of naked yields on y^{DE}, as well as the risk premia π and the basis b using equations (C.2.7) and (2.2.11). After this procedure is completed, for any of the processes under analysis we take weekly averages and select end-of-week data, so that we are left with a sample of $T = 177$ observations covering from 2010 to 2013 (first half).
Following the estimation procedure resumed in section 1.6, the first step is to select the number of lags p for each single-valued process under analysis: values of the test statistic (1.2.2) for testing one-lag reduction within a p lag framework for $p = \bar{p} \ldots 2$ are reported in tables A.1 and A.2. Table 2.1 collects the result, having selected a maximum number of lags $\bar{p} = 4$ uniformly across processes.
The risk premium and the basis are the process with shortest memory: only b^{DE} requires $p = 2$ lags; a result enforcing the efficiency of the hedging portfolio is that the combined information of yields and CDS explains the variability of π, releasing him from dragging additional self-explanatory lags.

	IT	ES	DE	EUR
y	3	2	2	–
$\bar{y}$	2	1	2	–
π	1	1	1	–
b	1	1	2	–
s	2	2	–	–
L	–	–	–	2

Table 2.1: Unidimensional number of lags $\hat{p}^j$, estimated through the step-procedure described in section 1.1, with $\hat{p}^j \leq 4$ by construction.

The basis b, in the relevant (see section 2.3.1) cases of Italy and Spain, should in principle be zero: a performing (statistical) hedging strategy is that which reduces the difference between the two risk-free rates to white noise.
Particularly, self-memory of the process is expected to be short, and these preliminary results seem to point in that direction. The most frequent estimated values among all processes is $\hat{p} = 2$, with the exception of naked yields for Italy for which $\hat{p} = 3$. The time series of y^{IT} show indeed two different peaks, debt-crisis in Italy itself at first and the side effect of Spanish crisis later, which are hard to be properly fitted without allowing for a higher number of lags (figure 2.13).
Tables in A.2 collects p-values from conducting Augmented Dickey Fuller tests, for all relevant processes and countries. The results show that the null of unit-root is not refused by any test on any of the processes, hence each component of the VAR under study is an $\mathcal{I}(1)$ process in a strict sense. In particular, the basis b^{IT} and b^{ES} are integrated processes.
Next section explores the statistical features of benchmark European rates, and reveals the properties of credit risk hedging strategies for Germany.
As explained in chapter 2, the estimation procedure provides for any vector model to be firstly estimated with the number of lags estimated as in (1.5.1), resumed in table A.4. Subsequently, residuals are computed and whiteness is verified.
The residuals presented in table A.7 are collected after a first complete estimation of the model: whether any of the statistics $\boldsymbol{\lambda}_{LM}(h)$ or $\boldsymbol{\lambda}_W(h)$ defined in section 1.5.4 rejects the null of no autocorrelation for any of the $h \leq \bar{h} := 4$, the number of lags is augmented by 1 and the procedure repeated. A second iteration of the procedure is sufficient to fully whithen residuals in all cases requiring so at a first stage.
The number of lags reported in table 2.2, 2.3 and 2.4 is the final one, which is selected to estimate all statistics and parameters presented either in this section and in tables A.5 and A.6. The VARs which required lag augmentation to whiten residuals are evidenced in each table. The unit of measure for yields is basis points.

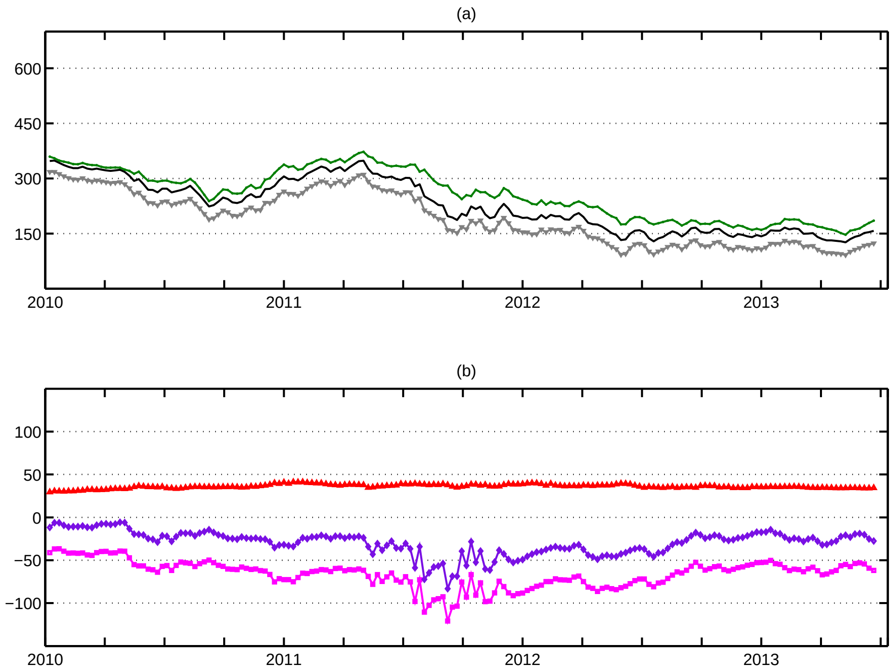


Figure 2.11: (a): Naked yield (black), CDS-bearing yield (grey-dotted) for Germany, and Euribor (green-triangled) rate ; (b) Credit risk premium (red-squared), CDS-bond basis (pink-squared) for Germany and $y^{DE} - L$ (purple-squared) spread. (*Source: Bloomberg, Markit and author's computations.*)

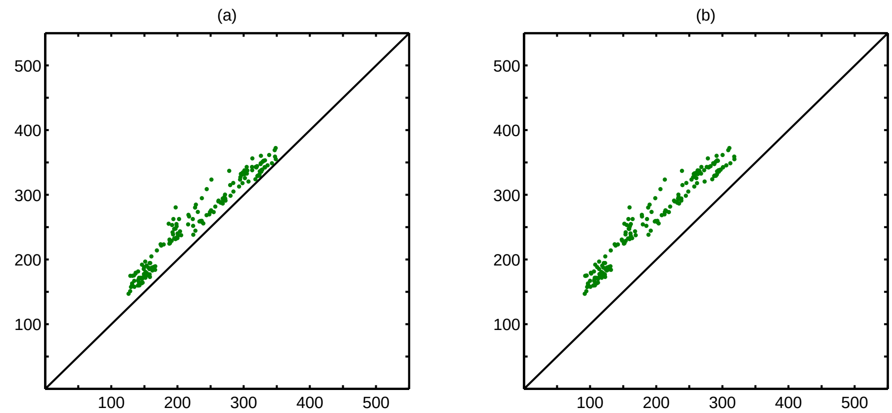


Figure 2.12: (a) Scatter plot of Euribor rate and naked yield for Germany. (b) Scatter plot of Euribor rate and naked yield for Germany. (*Source: Bloomberg, Markit and author's computations.*)

	p	r
(y^{DE}, L)	2	0
$(\bar{y}^{DE}, L)$	2	0
$(y^{DE}, \bar{y}^{DE})$	2	0

Table 2.2: Estimated cointegrating order and number of lags $(\hat{r}, \hat{p})$ used in parameters estimations; $\hat{p}$ is selected according to the procedure described in section 1.5.1. No corrections are needed to assess residuals whiteness.

2.3.1 Risk-free yields

Figure 2.11 shows the time-series evolution of both naked and CDS-bearing yields for Germany and Euribor rate, as well as the processes implied by differencing such yields; figure 2.12 shows the scatter plot, with Euribor rate on the horizontal axis and naked (left) and CDS-bearing (right) German yields on the vertical axis. It is evident the downward trending of all rates ending up in late 2012, after the turmoil caused by the Spanish banking sector crisis.
All the three rates moves together on a relatively similar pattern; by exhamining the credit risk premium π^{DE}, we find an almost constant process with average $\mathbb{E}(\pi^{DE}) =$ 36 bp, and standard error $\sigma(\pi^{DE}) = 2$ bp. The average difference $\mathbb{E}(y^{DE} - L) = -30$ bp, but with $\sigma(\pi^{DE}) = 15$ bp. We can however state that it is likely for this difference to be negative on average, which shows how financing at Euribor to buy German obligations is not convenient. This is due to the market preventing easy access to German debt, which demand was constantly increasing[51].
Table 2.2 shows the selected number of lags for these VARs as well as the estimated cointegration rank. It could be expectable to find cointegrating relations appearing among the VAR components (see [48] and [35]), but this is not the case. Recall that the general interpretation of cointegrating relations is that of long-run equilibria. The time series are weekly quotes within a forty-two months window, thus long-run equilibria may be not perceivable with this frequency in this relatively short stub.
Table A.6 shows the short-run analysis for $(y^{DE}, \bar{y}^{DE})$, (y^{DE}, L) and $(\bar{y}^{DE}, L)$; with only two lags and no cointegrating relations in a H_1^* framework (see section 1.5.2), a generic bivariate VECM in the equation system form becomes:

$$\begin{cases} \Delta z_t^1 &= C_1(1,1) \cdot \Delta z_{t-1}^1 + C_1(1,2) \cdot \Delta z_{t-1}^2 + U_t^1 \\ \Delta z_t^2 &= C_1(2,1) \cdot \Delta z_{t-1}^1 + C_1(2,2) \cdot \Delta z_{t-1}^2 + U_t^2 \end{cases} \tag{2.3.1}$$

where U_t is a gaussian white noise process with covariance matrix Σ.

[51] The flight to credit quality may lead to the existence of a significant *convenience yield*, see [75].

	p	r
(π^{IT}, s^{IT})	2^{*}	1
(π^{ES}, s^{ES})	2	1
$(\bar{y}^{IT}, L)$	2	0
$(\bar{y}^{ES}, L)$	2	0

Table 2.3: Estimated cointegrating order and number of lags $(\hat{r}, \hat{p})$ used in parameters estimations; p is selected according to the procedure described in section 1.5.1. Values with (*) provide an additional lag to whithen the residuals.

Observe that in case the correlation coefficient $\rho(U^1, U^2) = \rho$ is equal to 1, then the components U^1 and U^2 are linearly dependent, thus the source of randomness in the two equation is the same.
Computing differences among different yields correspond to combine a long position and a short position in the two respective portfolios. In case the two portfolios are linearly dependent, the combination of the two positions has the same variability of one of them, enforcing the good performance of hedging strategy.
The case $(y^{DE}, \bar{y}^{DE})$ is the easiest to deal with: the correlation ρ is estimated to be 1 while $\hat{C}_1$ is not significant. The two processes can be modeled as two random walks sharing the same innovation process.
The position of Germany as a preferred investment in the European sovereign market makes the analyzes of (y^{DE}, L) and $(\bar{y}^{DE}, L)$ very simple too. In light of the relationship between y^{DE} and $\bar{y}^{DE}$, the discussion can be limited to (y^{DE}, L) only, as $(\bar{y}^{DE}, L)$ will share with it similar statistical properties.
The correlation coefficient is larger than 0.9, and significant short-run bidirectional feedback effects $C_1(1, 2) \approx 0.33$ and $C_1(2, 1) \approx 0.45$ are registered.
In particular, since $C_1(2, 1) > C_1(1, 2)$, variations of German yields Δy^{DE} have greater impact on Euribor variations than the other way around.
It is also interesting to observe that the correlation between innovations is larger in this model with respect to $(\bar{y}^{DE}, L)$. This implies that naked German yields explain more of Euribor variability when credit risk is not hedged.
The basis $b^{DE} = \bar{y}^{DE} - L = y^{DE} - L - \pi^{DE}$ (figure 2.11b) is always negative, -60 bp on average. This result is coherent with [48] and [35], where the difference between CDS and ASW (which is equivalent to $-b$) is positive on average.
The pattern of Euribor moves away from that of German rates between 2010 and 2012; both these facts are again due to yields decrease related to the *flight-to-quality* phenomenon.

	$\hat{B}_1$		$\hat{D}^{\Vert}$	$\hat{A}$	
(π^{IT}, s^{IT})	1.000	-0.687	-0.007	-0.123	0.064
		(0.045)	(0.001)	(0.021)	(0.031)
(π^{ES}, s^{ES})	1.000	-0.625	-0.008	-0.167	0.029
		(0.060)	(0.002)	(0.078)	(0.011)

Table 2.4: Cointegrating vectors (π, s): EGLS estimation of the normalized cointegrating basis $[1 \quad B_1]$, the constant within the cointegrating relations $D^{\Vert}$ and the error-correction *speeds* A; correspondent standard errors in brackets.

2.3.2 CDS-bearing credit risk premia and yields spreads

We turn now to compare the country specific risk premium π^{IT} and π^{ES} with spread *vis-à-vis* German obligations of the respective naked yields s^{IT} and s^{ES}.
Table 2.3 reports the number of lags selected, namely $p = 2$, for the two vector processes (π^{IT}, s^{IT}) and (π^{ES}, s^{ES}): lag augmentation was necessary for the former because a single lag induces residuals autocorrelation when $h = 3, 4$.
A similar analysis is conducted in [48], using weekly data but covering a larger time window (2006-2010) and allowing for models time breaks in September 2008, before the crisis affected Europe. In the first period, no cointegrating relation involving CDS and bond spreads is present: conversely, the ocurring the crisis in Europe tightened the linkages between the two markets, and long-run equilibria arise.
The latent variable is credit risk, so it is interesting to investigate which of the two components contribute most in its price discovery process. The existence of cointegration between CDS and bond spreads implies that at least one market has to contribute to price discovery and the other has to adjust [48].
This in turn is determined by the error-correction speeds vector A: if both components are significant, then both markets contribute to mutual prices discovery.
Alternatively, if spreads are likely to move first, then A_1 will be significant and negative, while a positive significant A_2 implies that π contribute more to the price formation of s than the other way around.
Figure 2.13(a) shows the time series evolution: in both cases, the two components share a common pattern. Cointegrating relations appear even in this short time window, and the cointegration basis are reported in table 2.4.
In particular, $\pi^{IT} = 0.69 s^{IT} + 70\text{bp}$ and $\pi^{ES} = 0.62 s^{ES} + 80\text{bp}$; hence, the credit-risk premium which is paid when hedging with CDS contracts is composed by a fixed sunk cost plus approximatively 60% of the spread *vis-à-vis* Germany.

This decomposition is interesting because it shows how purchasing protection from the credit market is always costly, even when spreads are zero.
Results deviate from those in [48] for what concerns the error-correction speeds. The difference in the parameters sizes is probably due to the fact that cointegrating relations are estimated using daily data, differently from this work, where weekly frequency is maintained.
A difference which is instead worth to mention is that our analysis implies that spreads anticipates the price formation of CDS risk premia, while the converse happens on daily data from 2008 to 2010. The rush to buy German obligations, started in 2010, makes of the spreads the leading indicator of financial distress of risk measures. Short-run analysis (table A.6) reveals significant pure autoregressive components, and significant impact of Δs_{t-1} on $\Delta\pi_t$ are registered, again confirming the central role of Germany in European debt market.
Correlation in each country specific VAR process is approximatively 0.8, thus also the *local* (short-run) innovations of the processes are higly dependent.
It is notable that in both the cases of Italy and Spain the spread tends to overprice the implied credit-risk premium in times of crisis; the average excess is 20 bp in Italy and 40 bp in Spain, computed between half 2011 and half 2013. The excess demand for German obligations contributed to lowering y^{DE}, and the spread reflect this fact by demanding a higher premium with respect to country specific ones. Notice that the average difference $\mathbb{E}(\pi - s)$ on the whole sample period results in approximatively 1 bp for both countries, thus the spread is also underpricing credit risk in normal times.

2.3.3 Hedging imperfections: the CDS-bond basis

Figure 2.14(a) shows the evolution of CDS-bearing yields compared to the Euribor rate; the analysis in section 2.3.1 suggests that similar result would come out if any of the two German yields would be substituted to L.
The number of selected lags for both processes is $p = 2$ (see table 2.3), and no cointegrating relations arise. The effectiveness of the hedging strategy is evident: it projects the time series of naked yields on the pattern followed by the Euribor rate. The average difference $\mathbb{E}(b) = \mathbb{E}(\bar{y} - L)$ is negative, approximatively -50 bp for Italy and -27 bp for Spain on the whole sample period.
This means that financing at Euribor a CDS-hedging strategy would have resulted in average losses on the time stub considered: this result is coherent with [35], [36] as well as [48].
Short-run analysis (table A.6) reveal a significant pure autoregressive effects on the Euribor and a high correlation among the CDS-bearing portfolios $\bar{y}^{IT}$ and $\bar{y}^{ES}$ and the Euribor rates innovations, namely $\rho = 0.93$ and $\rho = 0.88$, respectively.

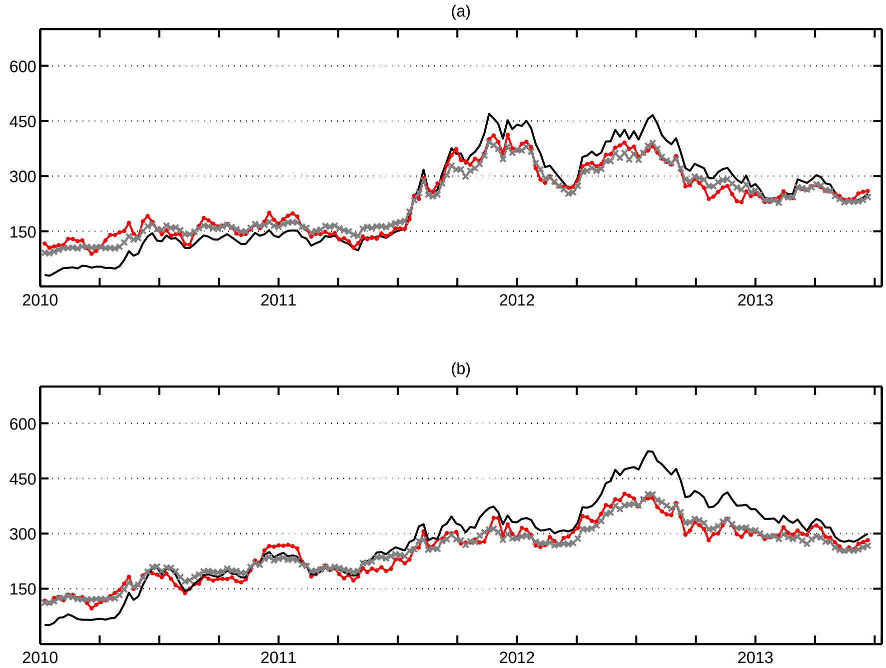


Figure 2.13: 10y yield spread *vis-à-vis* Germany (black), credit risk premium π (red-dotted) and cointegrating relations $\pi = B_1 s + D_1^{\parallel}$ for Italy, figure 2.13(a), and Spain 2.13(b) (*Source: Bloomberg, Markit and author's computations*).

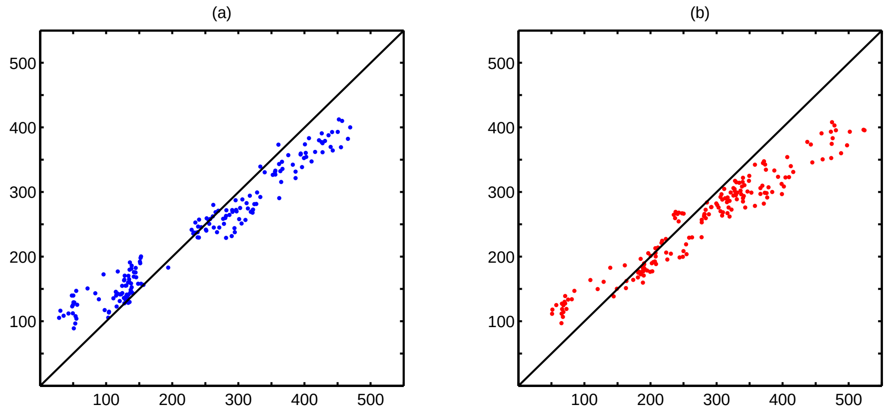


Figure 2.14: Scatter plot of credit risk premium π and spread *vis-à-vis* Germany for Italy (a) and Spain (b). (*Source: Bloomberg, Markit and author's computations.*)

	p	r
(y^{IT}, y^{ES})	3^*	0
$(\bar{y}^{IT}, \bar{y}^{ES})$	2	0
(b^{IT}, b^{ES})	2^*	0

Table 2.5: Estimated cointegrating order and number of lags $(\hat{r}, \hat{p})$ used in parameters estimations; p is selected according to the procedure described in section 1.5.1. Values with (*) provide an additional lag to whithen the residuals.

Concerning the matrix C_1, only diagonal elements are significant in both cases. It is also interesting to compare Italian and Spanish yields before comparing between each other the synthetic credit-risk free positions and the basis b for both countries. The estimated number of lags for all processes (y^{IT}, y^{ES}), $(\bar{y}^{IT}, \bar{y}^{ES})$ and (b^{IT}, b^{ES}) are reported in table 2.5; no cointegrating relations arise.
The VAR process (y^{IT}, y^{ES}) requires an additional lag with respect to the others in order to whithen residuals: a longer memory is required to properly fit two processes with different peaks, corresponding to side-effects of the crisis hitting the two countries at different times.
The average spread $E(y^{IT} - y^{ES})$ among the two countries is -38 bp, hence Spain embed a higher risk premium in its yields in the period considered.
Short run analysis reveal $\rho = 0.91$: the high number of lags in the VECM form leaves a matrix of residuals which is close to singular.
The unique significant feedback effect among the two processes is $C_1(1,2) \approx 0.31$, thus Spanish lagged yields differences impacts Italian yields differences. This is a direct consequence of Italy experiencing Spanish crisis and not vice-versa.
It is clear from the scatter plot that the variability with respect to the Euribor (horizontal axis) of both CDS-bearing yields $\bar{y}_t$ diminishes, figure 2.16(a), with respect to the correspondent scatter plot which compares L to naked yields y 2.16(b). Correlation of innovations however decreases when comparing the two CDS-bearing yield spreads through the VAR process $(\bar{y}^{IT}, \bar{y}^{ES})$, namely $\rho = 0.66$.
This is due to the combined position of two obligations and two CDS contracts within the vector process evolution, which amplify the sources of randomness as dealing with the aggregate process.
No other relevant short-run feedback effect is measured, hence the two processes $\bar{y}$ evolve as a vector random walk with relatively high correlated disturbances.
This is a signal of hedging performing well on each country specific market, resulting in two portfolios unaffecting each other through their respective short-run memories.
The analysis of the process (b^{IT}, b^{ES}) and their difference $b^{IT} - b^{ES}$, figure 2.15(b), reveals interesting features of the basis which may help to explain its origin.

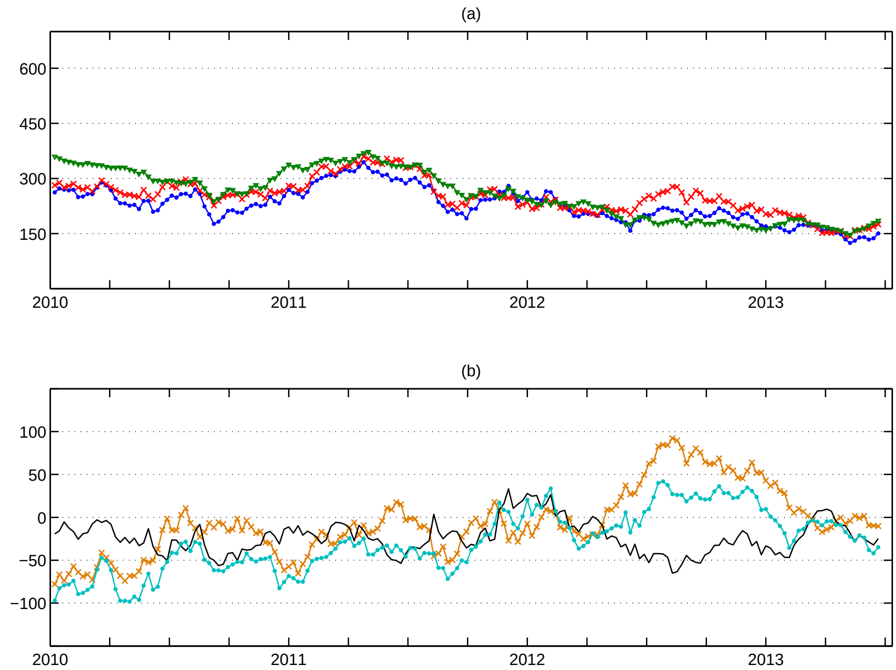


Figure 2.15: CDS-bearing yield for Italy (blue-dotted) and Spain (red-crossed) compared to the Euribor (green-triangled) rate, figure 2.15(a); CDS-bond basis for Italy (light-blue-dotted), Spain (orange-crossed) and CDS-bearing spread $\bar{y}^{IT} - \bar{y}^{ES} = b^{IT} - b^{ES}$, 2.15(b). (*Source: Bloomberg, Markit and author's computations.*)

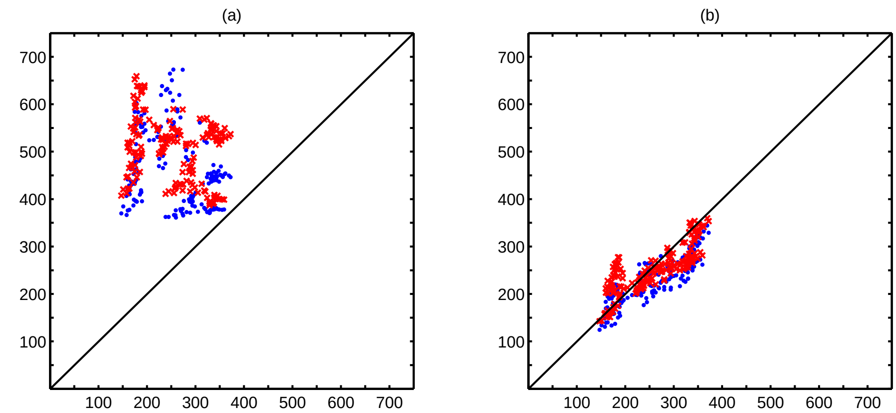


Figure 2.16: Scatter plot of naked yield for Italy (blue) and Spain (red) versus Euribor, figure 2.16(a); scatter plot of CDS-bearing yield for taly (blue) and Spain (red) versus Euribor, 2.16(b). (*Source: Bloomberg, Markit and author's computations.*)

The situation is similar to that of the VAR process $(\bar{y}^{IT}, \bar{y}^{ES})$: no short-run feedback effects are registered and the correlation among innovations is even lower $(\rho = 0.45)$. An additional lag is also required with respect to a first estimation to whithen residuals when $h = 3$ (table A.7).
Again, we have a vector random walk: the components of the vector of innovations show a non-zero correlation, but the estimated covriance matrix is far from being singular. As aforementioned, the difference $b^{IT} - b^{ES}$ is negative on average, but it systematically point in the direction of the country suffering the highest financial distress among the two at that moment.
Particularly, if the basis spread is interpreted as the payoff of buying a CDS-hedged Italian obligation and sell short a similarly hedged Spanish one, we have $b^{IT} - b^{ES} > \mathbb{E}(b^{IT} - b^{ES})$, with highest positive peak of -6 bp in late 2011, when Italy was suffering its most difficult moment, and $b^{IT} - b^{ES} < 0$ in mid 2012, with lowest peak of -50 bp, during the Spanish banking crisis.
The basis difference reacts even to the brief turmoil which followed political elections in Italy dated March 2013, increasing from the average negative value to a positive one, with highest peak of 28 bp.
As the two portfolios in principle are fully hedged against credit risk, the basis should be zero in both cases: this obviously implies that also the difference between the respective basis must be zero, up to statistical discrepancies.
The portfolios provide instead for a useful tool to monitor non-credit related risks in this aggregated market *as excess risk appears exactly when macrofinancial stability of target country teeters.*

2.3.4 Econometric analysis: further comments

The first outcome of econometric analysis is that market perceives credit risk to be absent in Germany. German naked bond yield y^{DE} is the primary source of fluctuation in the Euro money market, able to significantly impact the Euribor rate too, with which it shares a similar pattern.
Credit risk as measured with CDS-hedging does not sensibly variate with respect to its average value of 36 bp; it is different from zero only because any contract has a non zero price on the market, but the hedging strategy is not needed.
Hedging is considered to be relevant when correlation among residuals is high, so that the two portfolios yields share a unique source of randomness.
As dealing with German yields, CDS are not needed: the correlation is already high in (y^{DE}, L) and the average negative difference y^{DE} shows the market discouraging investors willing to finance at Euribor long positions on German yields.
This same rush to quality is the cause of the spread *vis-à-vis* Germany overpricing CDS-implied credit risk π in moments of major distress.

The hedging strategy allowing for the construction of the two credit-risk free yields $\bar{y}$ is effective in projecting all rates on a common *credit-risk free* pattern, the same already shared by y^{DE} and L.
The basis, defined here as the difference between the hedged portfolio and Euribor, is however an integrated process, and shows a *systematic* discrepancy arising when major distress is experienced by the country under exam.
It could be argued that this basis depends on the specific benchmark L chosen to measure the credit risk free rate in Europe, and is merely a *theoretical* arbitrage. The Euribor rate is not directly accessible to every dealer as well as the CDS-one, and restrictions may not coincide, so that the combined position is difficult to achieve in practice.
However, even if the analysis is limited to the CDS-bond market, the difference $b^{IT} - b^{ES}$, reveals movements from the (negative) average basis in the direction of the country suffering major distress at that moment.
We thus interpret the basis as an additional risk premium: even when obligations are hedged using CDS contracts, the combined positions still embed a higher risk premia when combined macroeconomic and financial distress affect target country.
Next chapter explores in detail the consequences of macroeconomic distress for a larger set of Euro-area countries, to measure the impact that public economy has on financial markets and viceversa.
Sovereign yields are combined with public accounting variables to analyze the portion of variability in country credit risk (as measured by the implied default probabilities) which are explained by unhealthy public accounting management.

3 Macro-financial risk measures

The effects of global financial crisis in Europe were not restricted to financial markets alone: releasing the global network of financial transactions from public economy by assuming no feedback effects among the two would be partial and unrealistic.
Advanced economies in the Euro-area suffered both recession and deterioration in public finance: the lowering of primary surplus led in turn to a rise in debt level, even higher if measured as a fraction of nominal output.
The analysis is limited here to eight state members: Belgium (BE), Germany (DE), Spain (ES), France (FR), Ireland (IE), Italy (IT), Netherlands (NL) and Portugal (PT). Greece (GR) has been excluded as the significance[52] of a rigorous econometric analysis of this case might be questionable, but it will however be used as a benchmark from the bottom up.

3.1 The crisis across EMU: stylized facts

The crisis unfolded in the EMU approximatively in 2008: the first alarming signals came with the uncovering, between 2008 and 2009, of high structural deficit in Greek public accounting. The deficit/gdp ratio rapidly boosted below any level expected by the market, reaching in 2010 downward peaks of nearly -10% on a yearly basis, while debt/gdp ratio had reached 130%.
In April 2010 Greece was no more able to borrow from the market: Greek government was granted a first bailout loan[53], on May 2, 2010. Roughly, one fourth came from the IMF while the rest was injected by member states and the ECB.
In mid-2011 it was evident that efforts in this direction were insufficient; a second rescue package [111] was finally endorsed on February 21, 2012, regulating one of the largest sovereign debt restructuring in history [2].
On March 9, 2012, the *EMEA* determination committee [69] recognized a credit event occurred in Greece, so that also CDS triggered (see chapter 3): Greece will thus be considered to have defaulted in $2012 : Q1$, so the last relevant observations time for this country will be $2011 : Q4$.
The Greek case is therefore an example of high deficit and recession, followed by a reaction of market interest rates, which ultimately led to a default on public debt; no other default occurred among other Euro-area countries, although some of them suffered heavy downgrades due to both specific and idiosyncratic events. Glaring examples are the cases of Ireland and Portugal.

[52]The chain of structural reforms in Greece which followed the bailout programs might be misleading when interpreting the mutual effects of macroeconomic and financial variables.

[53]See [30] for details.

The former was hit when the government reacted to the bursting of the housing bubble, rescuing distressed banks' bondholders by rising public debt. The latter simply paid the cost of years-long poor government expenditure management.
Spain and Italy are the biggest harshly distressed economies among the EMU: Spain suffered from a sudden increase in public debt from 2010 to 2013 (about 30% on aggregate) to finance banking institutions in wretched conditions, while Italy beholded a strong recession on top of a multi-year debt/gdp ratio above 100%.
France and Belgium experienced just a single downgrade, but differ in their respective economies sizes: furthermore, Belgium is representative for a highly-rated country with a debt/gdp floating around 100%.
Finally, Germany and Netherlands are included as examples of countries that kept unchanged their rating across the crisis. The latter experienced several[54] financial institutions default while the former was considered, and still is, a risk-free haven for Euro-area investments.
The unfold of financial crisis showed *"...the serious limitations of existing economic and financial models..."* [108]: ECB and state members thus looked for efficient methods devoted to track financial distress, so that suitable countermeasures can be set up with as larger anticipation as possible.
Conventional stability measures provide EMU members to monitor financial stability by controlling several macroeconomic indicators: Maastricht treaty [46] and the revisions which followed [47], provide a *cap* for debt/gdp ratio (60%) and a *floor* (-3%) for primary surplus/gdp.
Financial markets measure creditworthiness using a wide range of market implied indicators, including risk premia over a benchmark (spread *vis-à-vis* Germany), derivatives (CDS models) and implicit rating (scoring-based).[55]
The idea is to solve this duality by constructing a *score* which gathers signals from both sources. The minimal amount of information needed incorporates yields (spreads *vis-à-vis* Germany), capturing market credit risk, and public accounting variables, conveying government's policies. The score must be specific for each country but retrieved with a common methodology across the whole EMU.
Section 3.2 briefly describes the theoretical underlyings together with the dataset and defines variables dynamics, while section 3.3 presents the results out of a direct econometric approach. Section 3.4 aims to present the framework in which to construct the scoring method; section 3.5 collects econometrics results out of direct statistical comparison of credit risk scores with custom credit-risk measures.

[54] More precisely, *DSB* bank defaulted in 2009 while other troubled financial institutions were bailed out with the help of foreign investors and Dutch government, for example *ABN-AMRO*.

[55] In this sense, it is worth to mention the pioneering [7] and recent [8] work of Altman on scoring models, sharing their conceptual background with the approach pursued in this work.

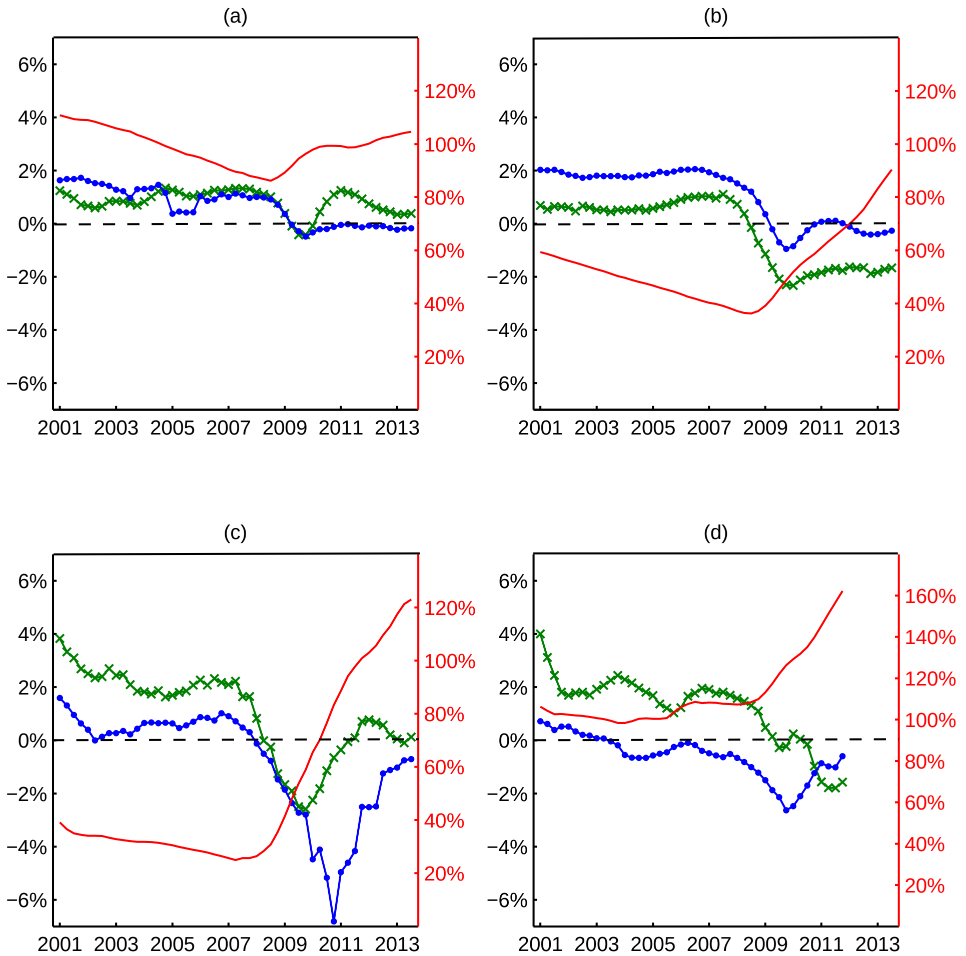


Figure 3.1: Gdp-growth rate g_t (green-crossed, left-axis), primary surplus/gdp n_t (blue-dotted, left-axis) and debt/gdp ratio x_t (red-straight, right-axis) for Belgium (a), Spain (b), Ireland (c) and Greece (d); black-dotted line is zero level for left axis. (*Source: ECB and author computations*)

3.2 Supply of public debt and demand for risk premia

The arguments which will be used in order to retrieve a country specific scoring system requires some underlying assumptions, which in a first moment subordinate statistical modeling to economic intuition.

The idea is to consider each country's specific sovereign debt market as a *single-good*[56] *market* in a demand/supply framework, being the quantity supplied the amount of debt issued and its price the *yield-to-maturity*[57] y of the aggregate debt burden.

The choice of y will be discussed in next section: it will be sufficient at this stage to consider it as a rate of return on public debt *which includes credit-risk premia* as settled by financial markets in the obligations prices formation processes.

[56]No distinctions relative to maturity of debt will be considered throughout the discussion.

[57]Formally, yields are *inverse* prices, as increasing yields corresponds to decreasing prices when relevant obligations are traded at the same face-value.

Greek default is interpreted here as the result of the lack of demand for its debt: bailout loans served the purpose of matching, through an *artificially-inflated demand*, the additional supply of debt that Greece was forced to issue.
As soon as liquidity completely dried up, and regulators considered not worthy to fill demand-gaps anymore, default became unavoidable.
Let x be the debt/gdp ratio: fix target country, and define an *exhogenous* model:

$$y_t = \boldsymbol{\omega}(x_t) \tag{3.2.1}$$

with the underlying assumption that the one-period demand function $\boldsymbol{\omega}$ is able to fully determine y_t given x_t, see figure 3.2(a).
It is preferrable to use debt/gdp rather than (the logarithm of) debt level for at least two reasons: the first is that it easily allows for cross-country comparison.
The second deals instead with modeling assumptions: the willingness of lenders to borrow, cloaked within $\boldsymbol{\omega}$, must award a good use of public debt.
Among countries sharing the same y, the risk premium is inversely proportional with respect to growth of their respective economy, hence debt/gdp is the relevant variable. A linear model[58] can always be obtained by first order expansion:

$$y_t = \omega x_t \tag{3.2.2}$$

where the constant coefficient $\omega > 0$ is the *risk aversion coefficient* with respect to target country's obligations, considered as an aggregate single-good.
For any process Z, the *natural filtration* for Z is the $\boldsymbol{\sigma}$-algebra generated by process Z up to time t, and will be indicated as $\mathcal{F}_t^{\boldsymbol{Z}} = \boldsymbol{\sigma}(Z_s : 0 \leq s \leq t)$.
Assume that each country's sovereign yield is composed by the credit risk premium charged to the borrower π over a risk free rate:

$$y_t = L_t + \pi_t \tag{3.2.3}$$

being L_t a common EMU-benchmark rate and π_t a country-specific credit risk premium; if the benchmark rate is not conditionally expected to variate over $[t-1, t]$ for any t, the *(conditional) expected variation* at $t-1$:

$$\mathbb{E}(\Delta y_t | \mathcal{F}_{t-1}^{\boldsymbol{x}}) = \mathbb{E}(\Delta \pi_t | \mathcal{F}_{t-1}^{\boldsymbol{x}}) \tag{3.2.4}$$

is equal to the expected variation of credit risk premia in that same period.
In times of crisis, financial distress combined with recession economy makes easy to predict a rise in debt/gdp over the next period.

[58] A more realistic model would provide for a constant $y = y_0 + \omega x_t$, so that when risk-aversion is zero the yield is equal to the risk-free yield. The constant is not explicited here as it is not relevant when describing this modeling framework.

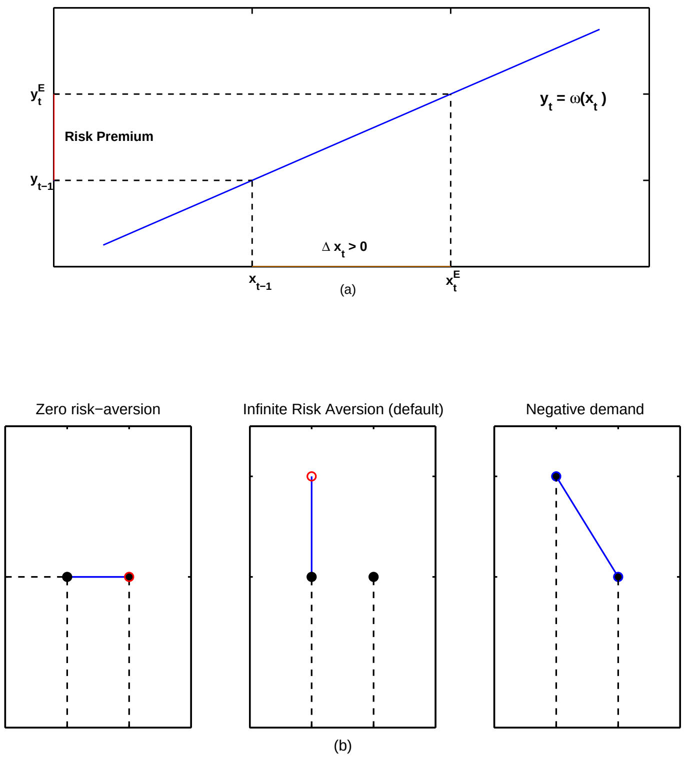


Figure 3.2: (a): One-period linear demand function $y_t = \omega(x_t)$. (b): Different types of risk attitude in a one-period curve.

Consider then a government for which x_t is $\mathcal{F}_t^{\boldsymbol{x}}$ predictable, that is, the *one-period supply function* is predictable and totally inelastic in the time window $[t-1, t]$. The credit-risk premium π_t is expected to increase proportionally to ω when debt/gdp level increase: combine (3.2.2) and 3.2.4 to obtain:

$$\mathbb{E}(\Delta\pi_t | \mathcal{F}_{t-1}^{\boldsymbol{x}}) = \mathbb{E}(\omega\Delta x_t | \mathcal{F}_{t-1}^{\boldsymbol{x}}) = \omega\Delta x_t \tag{3.2.5}$$

which shows how ω captures the reactions of lenders to new debt issuing in terms of credit-risk premia variations.
Forecast procedures in such framework will obviously suffer from the restrictive assumption that the only determinant of credit risk is x.
Equation (3.2.5) can yet be useful to capture any *slow-frequency amplifying effect* on π (or y) attributable to x only.
Indeed, (3.2.5) could be combined with a *faster* (e.g. daily sampled) model for market-implied credit risk in order to improve the underlying dgp model.[59]
Since the purpose here is to create a macro/financial-based *credit score*, also market data must be considered at quarterly frequency in (3.2.5), in order to permit the standard VAR approach.

[59] In this sense, a possible direction could be to fit *mixed data-sampling* (MIDAS) models (see [44] and the interesting application in [51]).

Figure 3.2 (b) shows three different shapes: a perfectly elastic demand (a), a perfectly inelastic demand (b) and a *'negative'* (downward sloping) demand (c).
The first case ($\omega = 0$) shows the market perceiving the borrower as creditworthy: whatever the quantity of new debt is demanded, the interest rate does not change.
The second case ($\omega = +\infty$) instead represents *default*. The perfectly elastic demand results in the unmatching of the curves which makes impossible for the government to rise debt/gdp level, whatever the interest rate he offered.
The case of negative demand ($\omega < 0$) is also included: it is clear that ω is expected to be positive, but a negative value is aimed to include *flight-to-quality* phenomena, such as that which interested Germany starting in 2010. That is, despite the unwillingness of government to issue more debt, demand constantly increased, while interest rates decreased, even down to negative values for some maturities.
It is worth to observe that, formally, there's no way to switch with continuity from models with positive to models with negative ω. This will not be considered a major shortcoming as default is *de facto* a limiting case.
Furthermore, negative demand function is a rather uncommon and case-specific situation, so we allow this exception to mathematical harmonization.
A straightforward method to imply a credit-risk score is to deem ω to be the relevant measure of creditworthiness. Consider (3.2.2) within a statistical framework: it could be sufficient to measure ω using the aforementioned econometric techniques, and chart them according to this output to construct a *dynamic credit score.*
It is clear that the observation y_t and x_t are simultaneous, thus any statistical approach to equation (3.2.2) is partial, as feedback effects of y over x shall be of interest too. The VAR approach which will be adopted will permit to measure both signals at the same time.
Before discussing the econometric properties of this *direct* approach (section 3.3), we explore the dataset supplied by the ECB *Statistical Data Warehouse* (SDW), together with the underlyings dynamics of the macroeconomic variables of interest.

3.2.1 The dataset: debt dynamics in practice

The SDW supplies publicly available quarterly statistics for any memeber state within the EMU. The official documentation provides an exact equation for debt level in each of the countries. Namely,

$$D_t = D_{t-1} + I_t - N_t + M_t \tag{3.2.6}$$

and the relevant processes are defined as:

- D_t : total government consolidated debt.
- N_t : government *primary* surplus
- I_t : interest payable (accrued between $t-1$ and t)
- M_t : deficit-debt adjustment (DDA)

see [110] (Table 1A, p. 11 and Table 2A, p. 39) or [16] (p. 11).
Several modification have been approved on government accounting methods, finally endorsed[60] in a Council Regulation: a new methodology, the ESA2010 replaced former ESA95 in defining rules for debt computation and accounting under the Excessive Deficit Procedure (EDP).[61].
ESA2010 provides debt to be evaluated at face value, while ESA95 defines debt level as that which *"..reflects the sum of funds originally advanced, plus any subsequent advances, less any repayments, plus any accrued interest* ([16], p. 383); ESA95 time series are thus to be used in order to match (3.2.6).
The DDA reconciles change in debt $D_t - D_{t-1}$ with (gross) surplus $N_t - I_t$ by taking into account adjustments due to statistical discordances (between members central banks and the ECB) and to peculiar accounting-based discrepancies. Details are provided in [110] (pp. 34-37).
The interest payable is defined as the difference between primary surplus and the surplus itself: following [49], we can define an implicit interest rate $\tilde{y}_t$:

$$I_t = \tilde{y}_t \cdot D_{t-1} \tag{3.2.7}$$

which will be also addressed to as internal or accounting rate. The dynamics of public accounting are then retrieved weighting all variables with nominal output level G_t. Notice that gdp ratios of primary deficit (as well as DDA) are commonly weighted by the gdp level over a quarter G_t^*, while debt/gdp at time t is defined as the ratio between current level of debt and the *aggregated* gdp-level over latest year, namely $G_t = \sum_{k=0}^{3} G_{t-k}^*$. The choice was to follow the second convention: define

$$g_t = \frac{G_t - G_{t-1}}{G_{t-1}} \tag{3.2.8}$$

the *quarterly* growth rate of *yearly aggregated* nominal output, divide (3.2.6) by G_t and combine (3.2.7) and (3.2.8) with (3.2.6) to obtain:

[60]Commission Regulation (EU) No 220/2014. ESA stands for *European System of Accounts*.

[61]This latter takes place in case a government is in persistent deficit excess with respect to the provided floor and *"provides for the necessary steps to be taken-.. (which)..could ultimately lead to imposing sanctions on the country concerned."* (ECB, *www.ecb.eu*)

$$\frac{D_t}{G_t} = (1+\tilde{y}_t)\frac{D_{t-1}}{G_t} - \frac{N_t}{G_t} + \frac{M_t}{G_t}. \tag{3.2.9}$$

Equation (3.2.9) can in turn be rewritten at first order by exploiting Taylor expansion $\frac{1}{1\pm x} \approx 1 \mp x$ and removing cross-products among variables. This yields:

$$\Delta x_t = (\tilde{y}_t - g_t)x_{t-1} - n_t + m_t. \tag{3.2.10}$$

where we defined:[62]

$$n_t := \frac{N_t}{G_t} \ , \ m_t := \frac{M_t}{G_t} \ , \ x_t := \frac{D_t}{G_t}$$

and equation (3.2.10) remains an identity[63].

The quantity $(\tilde{y}_t - g_t)x_{t-1}$ measures the *snowball effect* ([49], p. 11), that is, the self-reinforcing of debt accumulation due to the difference between the cost of borrowing and the growth rate the country is able to achieve.[64] Further comments are necessary for what concerns the implied interest rate $\tilde{y}_t$.

Eurostat rules state that I_t is accrued according to the *debtor approach*, that is, the yield on each debt instrument which is issued is *".. the* cost of borrowing *as observed at the time the instrument is created. As a consequence, interest must be accrued using the market rate (yield-to-maturity) or the contractual rate available at inception of the instrument.."*([16], pp. 86).

Moreover, the methodologies to imply single-instruments yield-to-maturity is custom ([16], pp. 353-354) and is the same which is used in the construction of the yield curve (appendix B). Each debt instrument can thus be considered as a financial operation whose yield is implied by assuming that the market is arbitrage-free (equation (C.1.2)).

Debt instruments serve also the purpose of implying a sovereign yield curve, as showed in chapter 2; in this case, considering the total set of issued instruments as an aggregated single financial operation, the yield at time t of such portfolio will be $y(t, T^*)$, being T^* the *duration* of the compound portfolio.

The SDW do not offer observations[65] on government debt duration; figure 3.3 shows the average (residual) debt maturity across countries, and the average of this latter over the periods 2001-2013 and 1995-2013, respectively.

An average duration of 4.3 years concerning this eight-countries panel is obtained using datasets of the *Organisation for Economic Cooperation and Development* (OECD), although observations are often sparse, and lasts in the year 2010.

---

[62] n_t and m_t are roughly 1/4 of correspondent *quarterly* surplus/gdp (DDA/gdp) defined in [14].

[63] Discrepancies arise due to Taylor approximation: market data induce a maximum error of 7 bp with respect to the true Δx_t, thus (3.2.10) will still be considered an identity.

[64] Notice that (3.2.10) does not change even if *real* interest rate and output are considered.

[65] An interesting review on debt size and composition across the EMU can be found in [77].

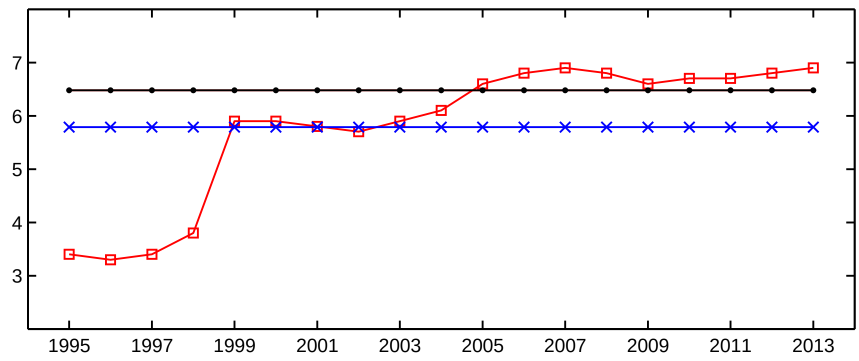


Figure 3.3: Time-evolution of average public debt maturity across Europe (red-squared); average from 1995 to 2013 is $T = 5.78$ years (blue-crossed) while average from 2001 is $T = 6.48$ (black-dotted). (*Source: ECB and authors computations*)

The choice is to compare $\tilde{y}_t$ with current market yield $y(t,T)$ and set $T = 5$, for at least two reasons. First of all, five years is the closest custom maturity to both the average debt maturity and its duration. Secondly, eventual movements in the slope and/or curvature of the term structure $y(t,T)$ (including *inversions*) are better captured with a mid-term rate than with a longer termed one, due to the flattening of the curve as T goes to infinite [39].

Figure 3.4 shows the time-evolution of $\tilde{y}_t$ and y_t for the countries under exam: market yield y_t reacts to the burst out of the crisis in reaching peaks of near 15% on a yearly basis (concerning Portugal and Ireland); time-evolution of $\tilde{y}_t$, on the contrary, is rather smooth, very similar across countries and almost not reacting to any of the crisis side-effects.

Before 2008, $\tilde{y}_t$ stays above market yields, a fact that could be attributed to the differences in debt maturity with respect to the choice $T = 5$. Observe that, in the case of Germany, market rate *always* stays lower than the internal rate, because a general lowering of interest rates.

Particularly, the internal rate $\tilde{y}_t$ does not embed credit risk premia but rather follows an arranged (hence highly predictable) pattern that remains similar throughout the whole countries panel.

Next section will present a *direct* econometric approach aimed to answer the questions presented thus far. First of all, the vector model $(y, \tilde{y})$ is fitted so as to unearth eventual common features of the two rates which could help out to settle the internal rate of return within financial markets rates.

Secondly, a VAR model for both the couples (y, x) and $(\tilde{y}, x)$ is set up, with the purpose of estimating ω out of equation (3.2.2), as well as the feeedback effects of rates on debt. It will be also useful to introduce another variable of interest, that is, debt variation (debt *speed*) Δx.

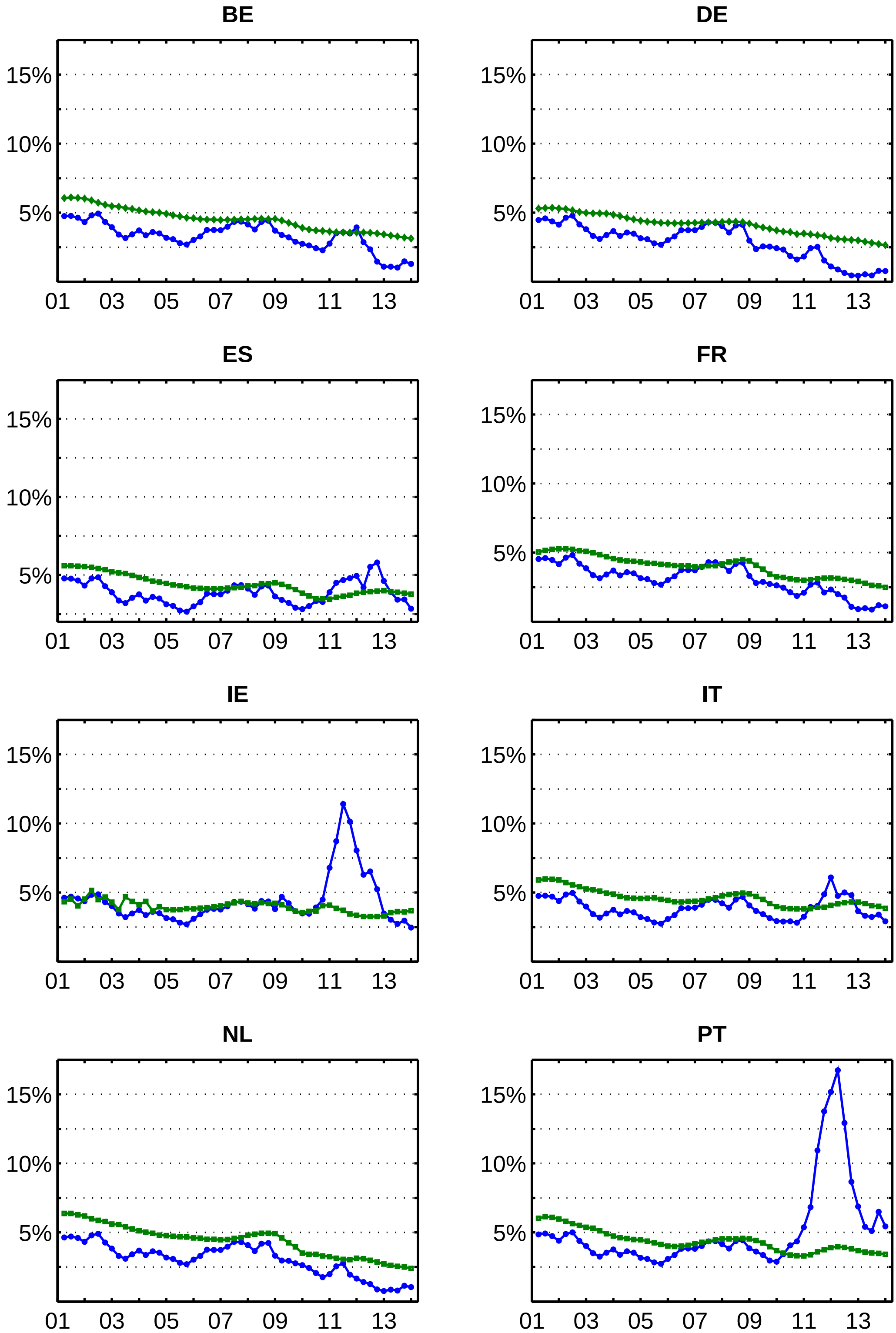


Figure 3.4: 5-years sovereign yield y_t (blue-dotted) and implied rate $\tilde{y}_t$ (green-squared) comparison, all data on a yearly basis (*Source: ECB, Bloomberg and authors computations*)

	BE	**DE**	**ES**	**FR**	**IE**	**IT**	**NL**	**PT**
y	2	3	1	1	2	4	2	3
$\tilde{y}$	2	3	3	2	4	2	2	2
x	2	2	3	3	2	3	2	2
s	2	–	4	2	2	4	1	3

Table 3.1: Unidimensional number of lags $\hat{p}^j$, estimated through the step-procedure described in section 1.1, $\hat{p}^j \leq 4$ by construction.

The reasoning behind comes from direct inspections of macroeconomic variables time evolution (figure 3.1). The examples of Portugal and Ireland, the latter showing a pre-crises consolidated debt 30%, suggest that a major source of increase in risk aversion (and credit risk premium) might be the speed at which this high level of debt is reached, rather than the level itself.
A VAR model of the form $(y, \Delta x)$ is thus inferred using the set of relevant observation. The idea is that debt variation could affect directly y in shifting upwards the slope of demand curve, as explained in section 3.3.
Moreover, associating to each country the speed Δx at which it gets into debt could be also seen as another possible way of scoring countries: that is, the faster it runs up into debt, the higher its credit risk premium (or yield, see (3.2.4)) variation.
It will thus be worth to discuss VAR models in the form $(s, \Delta x)$, where spread *vis-à-vis* Germany s is selected as benchmark (see chapter 2).

3.3 A direct econometric approach

Quarterly observations supplied by the SDW cover, at the moment, the time window 2000:Q1-2013:Q4; as mentioned, a four-quarter moving average is applied to available data to take out eventual season effects, which results in a sample period covering 2001:Q1-2013:Q4, for a total sample size of $T = 52$.
Market yields are instead taken from Bloomberg database: the dataset is composed of daily observations. In order to uniform the time scale to that of SDW macro-data, a quarterly moving average of *end-of-day* quotes is performed, and end-of-quarter observations are selected.
The choice of fitting two-dimensional vector models is appropriate, as the number of parameters to be estimated would be large with respect to T in case higher dimensional VAR were analyzed. Econometric results are presented model-by-model. Before analyzing vector models, single components properties are disclosed in line with the procedure in chapter 2.

Table 3.1 collects the estimated number of lags for each process and country, where maximum number of significant lags is set to $\bar{p} = 4$; test statistics (1.2.2) are presented in table (A.8). Most frequently, the criterion selects 2 lags.
Market interest rate y displays a longer memory in the case of Germany, Portugal ($\hat{p} = 3$) and Italy ($\hat{p} = 4$), while hypotheses testing does not refuse reduction to one lag in the case of Spain and France. The internal rate $\tilde{y}$ requires $\hat{p} = 4$ lags only in Ireland, because of the noisy behaviour of this time series between 2001 and 2004, due to alignement movements of the Irish economy to the new currency.
Debt/gdp evolution is similar across countries, showing a slightly upward linear trend all over the sample period, with a sudden shift in trend slope around 2009.
A linear trend is not provided in the dgp model, as the prior $\hat{H} = H_1^*$ provides no constant outside the cointegrating relation. Observe that even if trend was modeled, it should allow for a break in order to be significantly estimated: since no break is provided, x requires a high number of lags to be estimated.
Spread s almost lacks of memory in the case of healthy countries such as Netherlands while is very path dependent in distressed countries such as Spain, Italy (4) and Portugal (3) probably due to the unexpected spread-widening in-crisis.[66]
Tables (A.10) collect p-values from conducting unit root tests T1, T2 and T3: all ADF tests fail to reject the unit-root null, with the exceptions of $\tilde{y}$ in Belgium, France and Italy (T3 rejects H_0), and y in Italy (T1 rejects H_0).
Following definition 1.3.3, unit-root tests on processes in differences are performed so as to bound integration order from below: it turns out that each of the processes in each countries, shows stationarity in first differences, except for debt/gdp x.
The hypothesis $x \in \mathcal{I}(0)$ is rejected by all tests in all countries: moreover, in Spain, Ireland and Portugal, the same happens for Δx, hence the VAR processes (y, x) and $(\tilde{y}, x)$ for this countries are $\mathcal{I}(2)$ processes (definition 1.3.3) and will not be analyzed.
ADF tests does not give unilateral results instead on the integration order of Δx (except that it is smaller than 1) in other countries, hence vector models of the form (y, x) and $(\tilde{y}, x)$ will be fitted with the prior $x \in \mathcal{I}(1)$, and commented case by case.
Following the procedure resumed in 1.6, next step is to estimate the number of lags for the resulting vector processes: table A.12 in the appendix shows values of the statistics defined in (1.5.3) for testing one-lag reduction within a p lag framework for $p = \bar{p} \ldots 2$; eligible tests are those including values of $\bar{p}$ from a maximum $\hat{p}_\infty^+$ down to $\hat{p}_\infty^-$. The number of lags out of the estimation procedure described in 1.5.1 is resumed in table A.14; Johansen test is then performed, parameters are retrieved and the estimated residuals autocorrelation statistics are collected in table A.26.

[66] In order to avoid a large number of lags when modeling y for severely distressed countries, conditional heteroscedasticty effects should be considered even at quarterly frequencies.

	BE	DE	ES	FR	IE	IT	NL	PT
$(y, \tilde{y})$	$(0,2)$	$(1,2)$	$(0,3)$	$(0,2)$	$(0,2)$	$(0,3)$	$(1,2)$	$(0,3)$
(y, x)	$(0,2)$	$(1,2)$	–	$(0,3)^*$	–	$(0,3)$	$(0,2)$	–
$(\tilde{y}, x)$	$(1,2)$	$(0,3)^*$	–	$(0,3)^*$	–	$(1,2)$	$(1,2)$	–
$(y, \Delta x)$	$(0,2)$	$(0,2)$	$(0,2)$	$(0,2)$	$(0,2)$	$(0,2)$	$(0,2)$	$(0,3)$
$(s, \Delta x)$	$(0,2)^*$	–	$(0,2)$	$(0,2)$	$(0,2)$	$(0,2)$	$(0,2)$	$(0,3)$

Table 3.2: Estimated cointegrating order and number of lags $(\hat{r}, \hat{p})$ used in parameters estimations; p is selected according to the procedure described in section 1.5.1; models with (*) provide an additional lag to whithen residuals.

Such residuals are those collected after a first complete estimation of the model; whether any of the statistics $\boldsymbol{\lambda}_{LM}(h)$ or $\boldsymbol{\lambda}_{W}(h)$ defined in section 1.5.4 rejects the null of no autocorrelation for any of the $h \leq \bar{h} := 4$, the number of lags is augmented by 1 and the procedure repeated. A second iteration of the procedure is sufficient to fully whithen residuals in all cases which demanded so.
Table 3.2 collects the final results: the estimated number of lags floats between 2 and 3, and a longer memory is necessary again for Spain, Italy and Portugal as result of the long memory in y; Portugal needs also 3 lags to fit $(y, \Delta x)$ and $(s, \Delta x)$.
The interesting part for gleaning feedback effects among variable begins with the analysis of cointegration and parameters estimation, which will be discussed model by model: being the vectors bivariate, a maximum cointegrating rank of 1 is expected, although an eye must always be kept on unit-root tests.[67]

3.3.1 Market yield and internal rate

The first VAR model to fit compares the interest rates $(y, \tilde{y})$: a first look to figure 3.4 prevents from expecting strong linkages among the two rates.
Cointegrating relations arise hower in Germany and Netherlands (table 3.3): long-run equilibria relationship are $y^{DE} = 1.5\tilde{y}^{DE} - 3.6\%$ and $y^{NL} = 0.9\tilde{y}^{NL}$, having considered significant $D^{\parallel}$ only in Germany. It is not surprising to find market rates and internal rates on debt to share a common equilibrium for these countries.
The interpretation is a (perceived) healthy economy results in a market rate which is tied to debt-service (hence to public accounting) on the long-run.
We introduce here a measure of *dominance* among the two components of a bivariate cointegrated VAR. Namely, it is possible to infer which of them is likely to require the largest adjustment to reach the asymptotic steady state of the system.

[67]Stationarity of one between the two components yield however a $\mathcal{I}(1)$ model with a natural cointegrating relation, see definition 1.3.3 and the discussion that follows.

	$\hat{\hat{B}}_1$		$\hat{\hat{D}}^{\parallel}$	$\hat{\hat{A}}$	
DE	1.000	-1.547	0.036	-0.206	0.024
		(0.131)	(0.006)	(0.029)	(0.007)
NL	1.000	-0.884	0.008	-0.240	0.074
		(0.102)	(0.005)	(0.036)	(0.014)

Table 3.3: Cointegrating vectors $(y, \tilde{y})$: EGLS estimation of the normalized cointegrating basis $[1 \;\; B_1]$, the constant within the cointegrating relations $D^{\parallel}$ and the error-correction *speeds* A; correspondent standard errors in brackets.

The component which requires less adjustment thus *dominate* the other, in the sense that *the latter is moving towards the former*, which is closer to the long-run equilibria. The dominating component is then interpreted as the principal determinant of the long run equilibrium [48].
The *Gonzalo-Granger (GG) measure*, introduced in [52], is based on error correction speeds: whether both A_1 and A_2 are significant[68], the ratio $A_2/(A_2 - A_1)$ is close to 1 when Y^1 dominates, while the converse happens when such ratio is closer to 0.
The accounting rate $\tilde{y}$ determines a long-run equilibrum with market yields in countries with healthy balance accounting.
Furthermore, the error correction speeds reveal that, in both cases, the accounting rate is the principal component determining the steady state of the vector model.
No feedback effects are registered across other countries, except for Italy and Spain: roughly 5% of internal yield variation $\Delta\tilde{y}_t$ is explained by the lagged variation Δy_{t-1}, thus market yields are active part in the formation of debt service.
Pure autoregressive effects are significant for $\Delta\tilde{y}$ in any country: this variable is strictly macroeconomic, thus requires a longer memory, while the observations of its market counterpart are conditionally independent at quarterly frequencies.

3.3.2 Yields and public debt

The analysis of VAR models (y, x) and $(\tilde{y}, x)$ requires the foreward assumption that the null of unit root in Δx for any of the countries is refused. Formally, in order to homogenize models across countries, the alternative in T1 is accepted, that is, $\Delta x_t = c_0 \Delta x_{t-1} + c_1 \Delta^2 x_{t-1} + \ldots$ with $c_0 < 1$.
Again the univariate choice of the model is coherent with the prior H_1^* in vector modeling, providing for no constants in differences (no time trend in levels).
Vector models (y, x) supplies poor information: short-run effects are not significant and cointegrating rank $\hat{r}$ is different from zero in Germany only.

[68] If only one of the two speeds is significant, then we are back to the discussion of section 2.3.2.

	$\hat{\hat{B}}_1$		$\hat{\hat{D}}^{\parallel}$	$\hat{\hat{A}}$	
DE	1.000	0.303	-0.241	-0.041	-0.142
		(0.101)	(0.070)	(0.014)	(0.060)

Table 3.4: Cointegrating vectors (y, x): EGLS estimation of the normalized cointegrating basis $[1 \;\; B_1]$, the constant within the cointegrating relations $D^{\parallel}$ and the error-correction *speeds* A; correspondent standard errors in brackets.

The long-run equilibrium is $y^{DE} = -0.3x^{DE} + 0.2$: the sizes of cointegrating paraemters are questionable, but the fact that $\hat{B}_1 > 0$ is retained significant as exposing the aforementioned flight to quality phenomena *in the long-run*.
The error-correction speeds (table 3.4) reveal the dominance of rates on debt level, as it is expectable since the rate determines debt service.
Among short-run effects, only C_{22} is significant, yet not in all countries: long memory of x is recognizable in Belgium, Germany, Netherlands, France and Italy. Particularly, 5% confidence bandwiths induced by the estimator $\hat{C}_{22}$ include the unit root (table A.17), thus confirming the assumption $(y, x) \in \mathcal{I}(1)$ to be strained.
Statistical relevance of feedback effects between the two variables is thus globally unsatisfactory to drag conclusions.
Similar problems arise in the analysis of $(\tilde{y}, x)$. Cointegration arise in Belgium, Italy and Netherlands and the significant error-correction speed is $A_1 < 0$, suggesting debt determines the internal rate.
These poor statistical results are attributable to inaccurate estimates of integration orders of the vector processes. On the one hand, $\tilde{y}$ might be a trend stationary process (T3 refuses the null in Belgium and Italy), while on the other x could be an $\mathcal{I}(2)$ process. This partially invalidates also the significance of short-run feedbacks $(0 < \hat{C}_{12} < 1)$ of Δx_{t-1} on $\Delta \tilde{y}_t$ in these same countries.
The assumption $(\tilde{y}, x) \in \mathcal{I}(1)$ is less strained in France and Germany: pure AR coefficients are relatively far from 1 (FR) or not significant (DE), respectively. Germany shows $\hat{C}_{12} < 0 < \hat{C}_{21}$, with $|\hat{C}_{12}| = |\hat{C}_{21}| \approx 0.5$: the effect of debt/gdp variation on internal rate variation is negative (flight to quality) while positive variation of $\tilde{y}$ produces positive variation of x, as expectable.
Summing up, the mixed results out of Dickey-Fuller statistics suggest the assumption $x \in \mathcal{I}(1)$ to be inaccurate, so that cointegrating vectors might be faulty as retrieved out of incorrect dgp modeling underlyings.[69]
Attempts to directly measure ω in a VAR framework are thus nullified by the difference in the integration order of rates and debt/gdp.

[69] Recall that if the true VAR process is $\mathcal{I}(2)$, trace statistics are different (see [74], pp. 132-138) from the $\mathcal{I}(1)$. In this case, the use of cointegration tests for $\mathcal{I}(1)$ processes may not be informative.

	$\hat{\hat{B}}_1$		$\hat{\hat{D}}^{\parallel}$	$\hat{\hat{A}}$	
BE	1.000	1.108	-1.077	-0.031	0.069
		(0.019)	(0.018)	(0.002)	(0.460)
IT	1.000	2.721	-3.280	-0.091	0.227
		(0.015)	(0.016)	(0.006)	(0.290)
NL	1.000	-0.177	0.097	-0.086	0.111
		(0.012)	(0.007)	(0.004)	(0.405)

Table 3.5: Cointegrating vectors $(\tilde{y}, x)$: EGLS estimation of the normalized cointegrating basis $[1 \;\; B_1]$, the constant within the cointegrating relations $D^{\parallel}$ and the error-correction *speeds* A; correspondent standard errors in brackets.

3.3.3 Debt speed, yields and yields spreads

The vector processes $(y, \Delta x)$ and $(\tilde{y}, \Delta x)$, involve debt-speed (see figure 3.5). ADF tests accept the null of unit root for y and s for any country, except for y^{IT} (test T2 refuses the null): the prior assumption $y^{IT} \in \mathcal{I}(1)$ is explictly added, so that all models are homogeneously estimated in a $\mathcal{I}(1)$ framework.
Firstly, notice that no cointegrating vectors arise for any of the countries in any of the two VARs: for what concerns $(y, \Delta x)$, short-term dynamics move separately since significant coefficients belong to the diagonal of C_1 only.
The only relevant feedback effect is $C_{12}^{ES} \approx -0.35$: the minus sign is counterintuitive, but the p-value of the $\mathbf{t}$-statistic is very close to the 5% threshold, thus significance may be questionable.
The analysis of $(s, \Delta x)$ short-run dynamics reveals poor coefficient significance too: concerning the only exception is the impact of spread on debt-speed in Spain $C_{12}^{ES} \approx 0.28$, but again p-value is close to 5%..

3.3.4 A direct econometric approach: concluding remarks

The conclusions that can be drawn following a direct econometric approach are generally unsatisfactory. The comparison among interest rates $(y, \tilde{y})$ reveal structural connections among financial market and government accounting in the long run only in Germany and Netherlands.
If the VAR $(y^{DE}, \tilde{y}^{DE})$ VAR is fitted with prior $\hat{H} = H_2$ (no constant in the cointegrating relation), the estimated cointegrating rank is $r = 1$ with $B_1^{DE} \approx 1.1$, aligning to $[B_1^{NL} \; D^{\parallel}] \approx [0.9 \;\; \mathbf{0}_{1\times 2}]$.

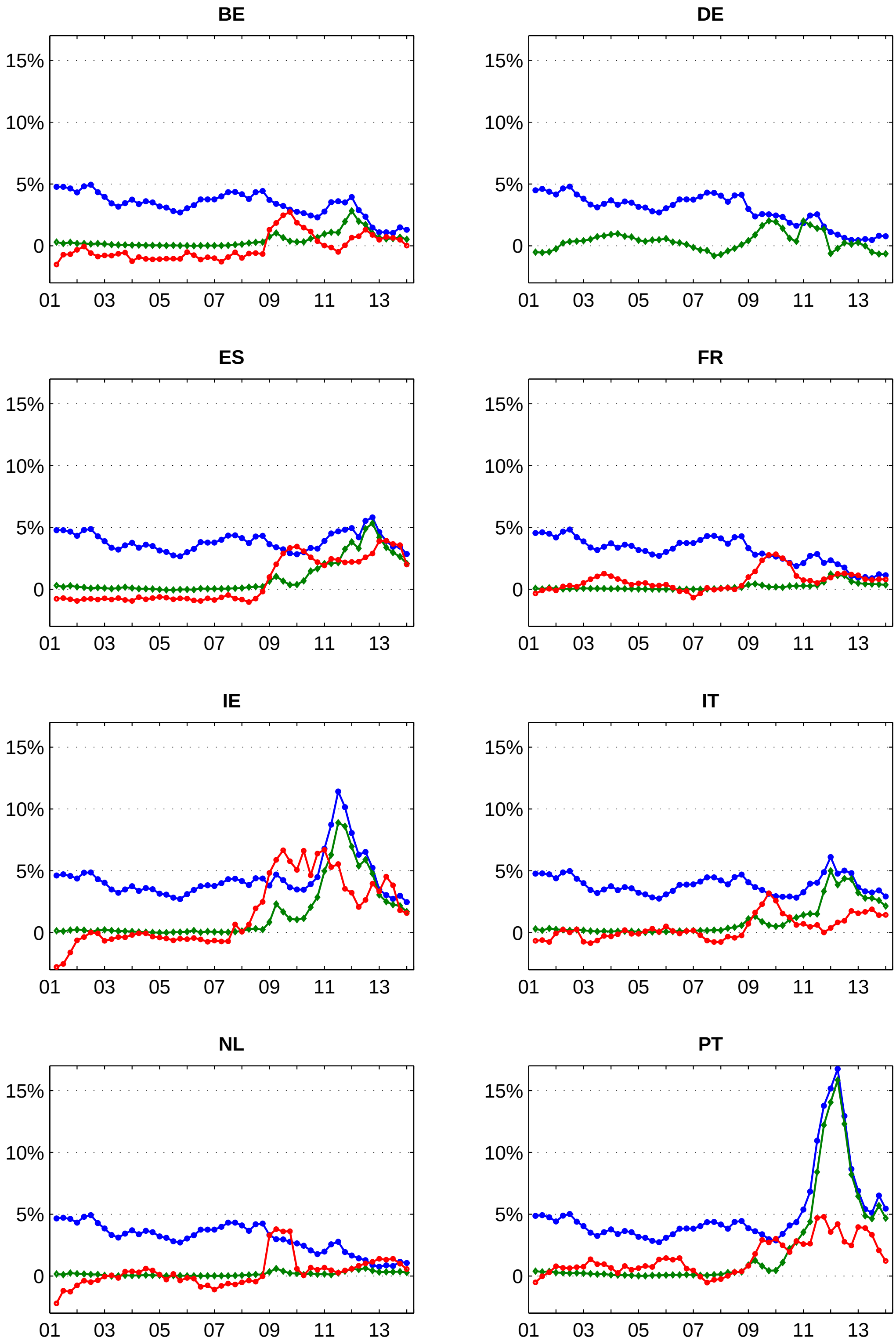


Figure 3.5: 5-y yields (blue-dotted) and 5-y yield spreads *vis-à-vis* Germany (red-dotted) versus debt speed (green-squared) (*Source: ECB, Bloomberg and authors computations*)

The VAR $(y, \tilde{y})$ instead scarcely displays feedback effects in distressed countries: the conclusion is that market risk premia are not beared by the internal-rate $\tilde{y}$. Statistics concerning any model where debt/gdp level x is present may be inaccurate as the true process x might belong to $\mathcal{I}(2)$.
The only result which is worth to mention concerns the signals of the flight-to-quality towards Germany, but it is clear that ω *out of* (3.2.2) *cannot be directly inferred.*
Finally, when switching to debt speed Δx, results are even poorest in terms of feedback effects, which were of most interest. Considering both $(y, \Delta x)$ and $(s, \Delta x)$, components of each of the two vectors are estimated to evolve as non-interacting AR processes, with slightly correlated residuals ($\rho \leq \rho^{PT} = 60\%$ in any country).
The linear model for debt/gdp and yields presented in section 3.2 does not find validation within the statistical framework.
Next section updates theoretical underlyings described in section 3.2, and presents an economically founded approach to *debt-crisis*, allowing to build up economically founded scores that can be compared with standard credit-risk indicators

3.4 Sovereign risk: a Minskian approach

Previous econometric analysis revealed that a demand/supply approach based on (3.2.2) produces relevant statistical results for healthy economies only.
Distressed countries displaying sensible increase in credit risk premia, as reflected by market yields, requires a different framework.
The reasons behind is that risk appetites of lenders towards EMU-countries debt market have not remained constant in the last decade.
Equation (3.2.1) should encompass a time-varying demand function, modeled by a time-varying operator $\boldsymbol{\omega} = \boldsymbol{\omega}_t$.
Assuming that the operator's functional form does not change over time and that it can be identified through a dynamic parameter ω_t in a product form, implies:

$$y_t = \boldsymbol{\omega}_t(x_t) := f(x_t, \omega_t) = \omega_t x_t \tag{3.4.1}$$

A one-period upward shift in risk aversion ($\Delta\omega_t > 0$) is able to steepen the line up, towards the the ultimate boundary, which is default. Taking the logarithms in (3.4.1), and defining:

$$\log \omega_t := \log y_t - \log x_t \tag{3.4.2}$$

it is possible to check if intuitions are confirmed. Figure 3.6 shows the evolution of the relative variation $\Delta \log \omega_t \approx \Delta\omega_t / \omega_{t-1}$ defined out of (3.4.1).

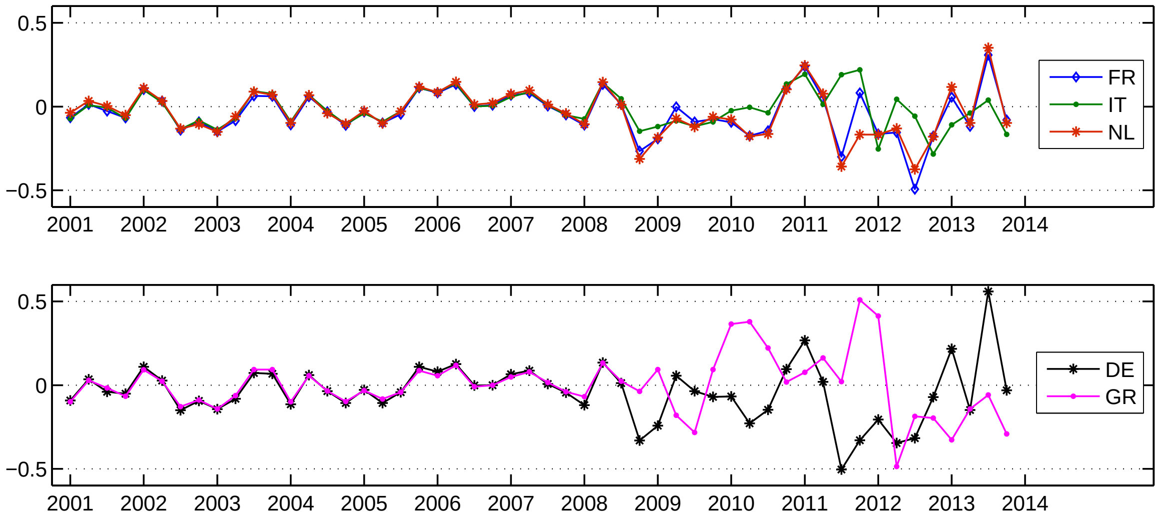


Figure 3.6: Quarterly relative variation $\Delta \log \omega_t \approx \Delta\omega_t/\omega_t$ comparison among France, Italy and the Netherlands (top) and among Germany and Greece (bottom) within the time stub 2001Q1:2013Q3 (*Source: ECB, Bloomberg and authors computations*)

A common cross-country pattern is recognizable up to 2008 approximatively: the burst of the crisis brought along the set of aforementioned side effects which allow to explicitly distinguish one country from the other.

Greece, for example, maintained a positive relative variation from 2010 on, with an upward peak reached soon before the second rescue package (February 2012). The downturn variation started in 2012, when credit restructuring removed pressures from Greek yields.

Conversely, during that same time-window, Germany shows the lowest peak together with France and Netherlands. Italy instead was suffering its worse distress period, thus the relative variation of ω is again positive therein.

These facts confirm the trivial intuition that risk appetites with respect to the EMU had a dynamic evolution along the past decade and invalidate any attempt to study a statistical model for (y, x) without time-variating risk aversion coefficients.

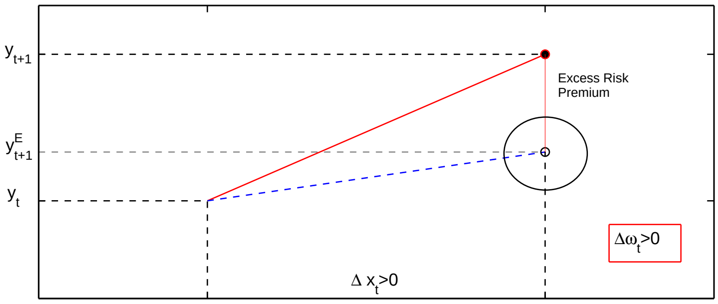
A dynamic risk aversion coefficient is able to model reactions of markets to the increasing financial distress.

Figure 3.7 (a) shows the one-period demand function when a shift in risk appetites $\Delta\omega_t > 0$ occurs: a similar approach can be found in [55].

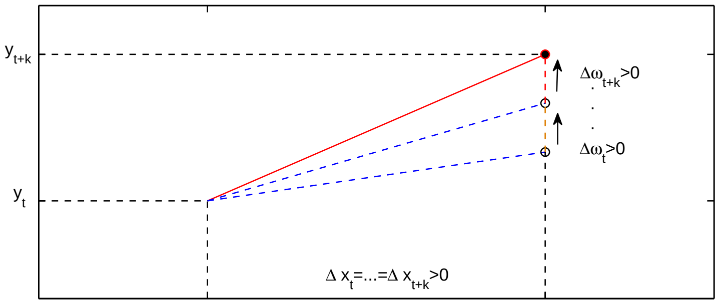
A static demand curve model (blue-dashed) would predict future yield to be y_t^E up to a certain confidence level (black circle): the aforementioned positive shift may push the realized interest rate $y_t > y_t^E$ outside that confidence region.

This *surprise effect* $y_t - y_t^E > 0$ measured by the difference between the realized and the expected rate results in an additional risk premium which *is not* $\mathcal{F}^{\mathbf{x}}_{t-1}$*-measurable.*

Figure 3.7(b) shows the effects of a sequence of positive debt variation under the assumption that each of them causes a positive shift in risk aversion.



(a)



(b)

Figure 3.7: One period (a) and multi-period (b) shifts in demand curve caused by a positive variation of ω_t in response to a positive debt variation Δx_t.

The cascade of upward slope shifts ultimatey leads country to default. Formally, define $\mathcal{F}^{\mathbf{x}}_t$ and $\mathcal{F}^{\boldsymbol{\omega}}_t$ the natural filtrations for processes x and ω; equation (3.4.1) implies $\mathcal{F}^{\mathbf{y}}_t \subseteq \boldsymbol{\sigma}(\mathcal{F}^{\mathbf{x}}_t \cup \mathcal{F}^{\boldsymbol{\omega}}_t) = \mathcal{F}_t$ and:

$$\Delta y_t = \omega_{t-1}\Delta x_t + x_t\Delta\omega_t \tag{3.4.3}$$

The exhogenous model for rates is now constituted by two components, namely debt/gdp level and risk aversion of market investors. Taking conditional expectations:

$$\mathbb{E}(\Delta y_t|\mathcal{F}_{t-1}) = \mathbb{E}(x_t\Delta\omega_t|\mathcal{F}_{t-1}) + \omega_{t-1}\mathbb{E}(\Delta x_t|\mathcal{F}_{t-1}) \tag{3.4.4}$$

the assumption that x_t is $\mathcal{F}^{\mathbf{x}}_t$ (hence $\mathcal{F}_t$) predictable implies:

$$\mathbb{E}(\Delta y_t|\mathcal{F}_{t-1}) = \omega_{t-1}\Delta x_t + x_t\mathbb{E}(\Delta\omega_t|\mathcal{F}_{t-1}) \tag{3.4.5}$$

The conditional expectation $\mathbb{E}(\Delta\omega_t|\mathcal{F}_{t-1})$ forecasts the *slope* at t of the demand curve in (3.4.1), and is needed for prediction even if future debt/gdp level is known. Previous equation does not add anything new: investors risk appetites ω, in a theoretical exhogenous model for market yields, embeds all the variability in y that is not explained by x, which is rather large, as proved in section 3.3.
Risk aversion ω is clearly unobservable: it contains general expectations on future creditworthiness, local (one-period) reactions to excessive $\Delta x > 0$ (lenders might demand additional risk premia to fulfill borrower's request) as well as contagion effects due to a general rise in risk aversion for EMU obligations market (see [36]).

Equation (3.4.5) sheds however light on the rather poor explanatory power of VAR modeling in (y, x) and $(y, \Delta x)$.
Time-varying ω nullifies estimation with constant coefficients. Furthermore, (3.4.5) explicitely model the impact on yields of debt speed as well as that of risk appetites variations, weighted by debt/gdp level.
As y and x (and their variations) are instead observable, it might be worth to exploit them in order to infer ω. This latter can be viewed as the informative variable on borrower's credit risk, and can in turn be compared to custom credit risk measures. A standard VAR approach to (3.4.5) is however impossible because of dynamic coefficients. The following sections present the construction of a score based on y and x which is able to capture investors' expectations on countries future creditworthiness. Subsection 3.4.1 discusses the underlying modeling assumption, tracing back to the seminal ideas of Minsky [86] in the framework of the basic model as presented in section 3.2. Section 3.5 presents econometric results out of comparison with standard credit risk measures, concluding the chapter.

3.4.1 The Ponzi-score

The reliance on economic theory serves the purpose of moving away from pure econometric modeling, which would require more advanced instruments than those supplied by the VAR framework.
The direction undertaken in this work is thus to construct a measure of credit risk based on economic theories on credit market in a demand/supply framework.
Namely, a growing interest of academics ([42] and references within) concerning the dynamics driving public debt was registered during this latest crisis following the pioneering ideas of H. Minsky [86] and his Financial Instability Hypothesis (FIH).
It is also worth to mention the speech of Janet Yellen, current president of the Federal Reserve (Fed), where Minsky's framework is contextualized to current crisis. The facility of markets to grant credit in the form of assets and derivatives purchasing was under question: *"..borrowers, lenders, and regulators (that) are lulled into complacency as asset prices rise. It was not so long ago..that many of us were trying to figure out why investors were demanding so little compensation for risk"* [116].
When lenders perceive global financial conditions to be healthy, borrowers are able to match demand for their debt under general compliance of regulators, which benefit from a liquid market. As confidence collapses, lenders dry up demand: the countereffect is to charge yields with higher risk premia so as to relaunch purchases. If at the same time real economy is in recession, borrowers may not be able to repay loans, due to low output and high deficit. Regulators, on their side, find themselves trapped between financial untrustworthiness and economic distress, disposing of monitoring instruments unable to predict sudden confidence shifts.

Particular interest will be devoted to Minsky's classification of loan agreements, where three kinds of borrowers are distinguishable: the hedger, the speculator and the *Ponzi*-borrower. The hedger repays at each payment date all accrued interest plus a fraction of principal, so that the outstanding debt diminishes in time. The speculator repays only the interests, keeping debt always at the same level, while the Ponzi borrower is not able to repay neither (a fraction of) the interests.
A Ponzi borrower, in a given period of time, is the one who has increased its debt level in that period: not only must he completely refinance the outstanding debt, but also issue new one not to default on interest payments.
He has to be able to find new lenders and/or persuade current ones to purchase more obligations, increasing in turn his credit exposure. This *deadly debt spiral* [42] will soon lead to default, that will occur when auctions will ultimately be diserted.
These ideas can be easily adapted to the demand-supply framework: broadening the concept of "loan" by replacing debt level with debt/gdp, a *government will be considered a Ponzi borrower within the time period* $[t-1, t]$ *if* $\Delta x_t > 0$.
The aim is to build up a scoring method which is able to recognize and monitor government Ponzi schemes, and evaluate their impact on countries crediworthiness perceptions (investors risk aversions).
The positive part of debt speed $\Delta x_t^+ = \max\{\Delta x_t, 0\}$ will be called the *actual Ponziness* of government, as it quarterly measures the size of government Ponzi scheme. Increasing debt supply may boost the requested risk premium and affect y, as shown by the example in figure 3.7(b), with time-observation of ω_t describing the path of market confidence.
A rather intuitive experiment is to ignore the size of Ponzi schemes and imply a score by counting the number of times the government is acting as a Ponzi borrower along the time window considered.
A set of thresholds determining rating migrations could then be easily implied by mapping target score to that of rating agencies, in the original spirit of Altman [7].
This approach suffers from the fact that similar signals were coming out of opposite situations, so that different countries would be labelled with the same scoring.
Along with the expansive phase which followed the collapse of Lehman between Q2:2009 and Q3:2010, ending up with the full onset of the Euro-Sovereign crisis, any of the countries under examination exhibits $\Delta x_t > 0$, and it thus becomes difficult to clearly distinguish them (a similar score would for example be attributed to Germany and Ireland).
It could be the case that quantifying the *size* of actual Ponziness is relevant for creditworthiness perception. Notice that the snowball effect in (3.2.10) is measured with respect to $\tilde{y}$, while it is clear that market yields are the main drivers of public debt demand and determine market liquidity.

Market yields perform better in reflecting critical conditions of the correspondent countries with respect to $\tilde{y}$ (figure 3.4) because they embed credit risk premia (section 3.3). Determining if a government is acting as a Ponzi borrower does not take into account external interventions such as bailout programmes, that allow the country to borrow at the *monitored* interest rate through extraordinary measures. Rewriting equation (3.2.10) as:

$$\Delta x_t = (y_t - g_t)x_{t-1} - n_t + m_t - (y_t - \tilde{y}_t)x_{t-1} \tag{3.4.6}$$

we define the process:

$$\phi_t := (y_t - g_t)x_{t-1} - n_t + m_t \tag{3.4.7}$$

by substituting the accounting rate with market rate equal to debt duration.
In other words, debt variation at time t is evaluated at market interest rate, thus measuring the speed of borrowing of target country considering global financial market as the lender: the higher ϕ, the lower the creditworthiness. In what follows, country A will be called a *Ponzi borrower* at time t whenever $\phi_t^A > 0$.
A closer look to (3.4.7) validates the introduction of ϕ: first of all, it contains Maastricht parameters x and n, and it includes both the growth rate of the economy and market yields within $y - g$.
Primary surplus directly affects the score, while *market snowball effect* $y_t - g_t$, being weighted by x_{t-1}, has direct (or amplified) effect in countries showing 100% of debt/gdp ratio (or greater). Combination of (3.4.6) and (3.4.7) yields:

$$\phi_t = \Delta x_t + (y_t - \tilde{y}_t)x_{t-1} \tag{3.4.8}$$

that is, ϕ_t is given by debt/gdp variation plus an additional penalty determined by the spread $y_t - \tilde{y}_t$, weighted by previous debt/gdp level.
The misalignement between y and $\tilde{y}$ could be used as a rough measure of credit risk, as it culminates during financial distress, and decreases after bailout programmes became effective. Another possible interpretation of $(y_t - \tilde{y}_t)x_{t-1}$ is the correction to debt variation indiced by market rates and attributable to external interventions.
Figure 3.8 shows the evolution of ϕ for our panel of countries: it is notable that from late 2008 on, all countries became Ponzi-borrowers due to the combined effect of recession, and rise in both credit risk and public deficits.
After 2009, the spread $\phi^{GR} - \phi^{DE}$ widens dramatically, and each country-specific ϕ lies between the two of them. An interesting exception is again that of Ireland, which underperforms Greece up to 2011, when the rescue of Irish banks was completed and the country returned to normal level of yields and deficit.

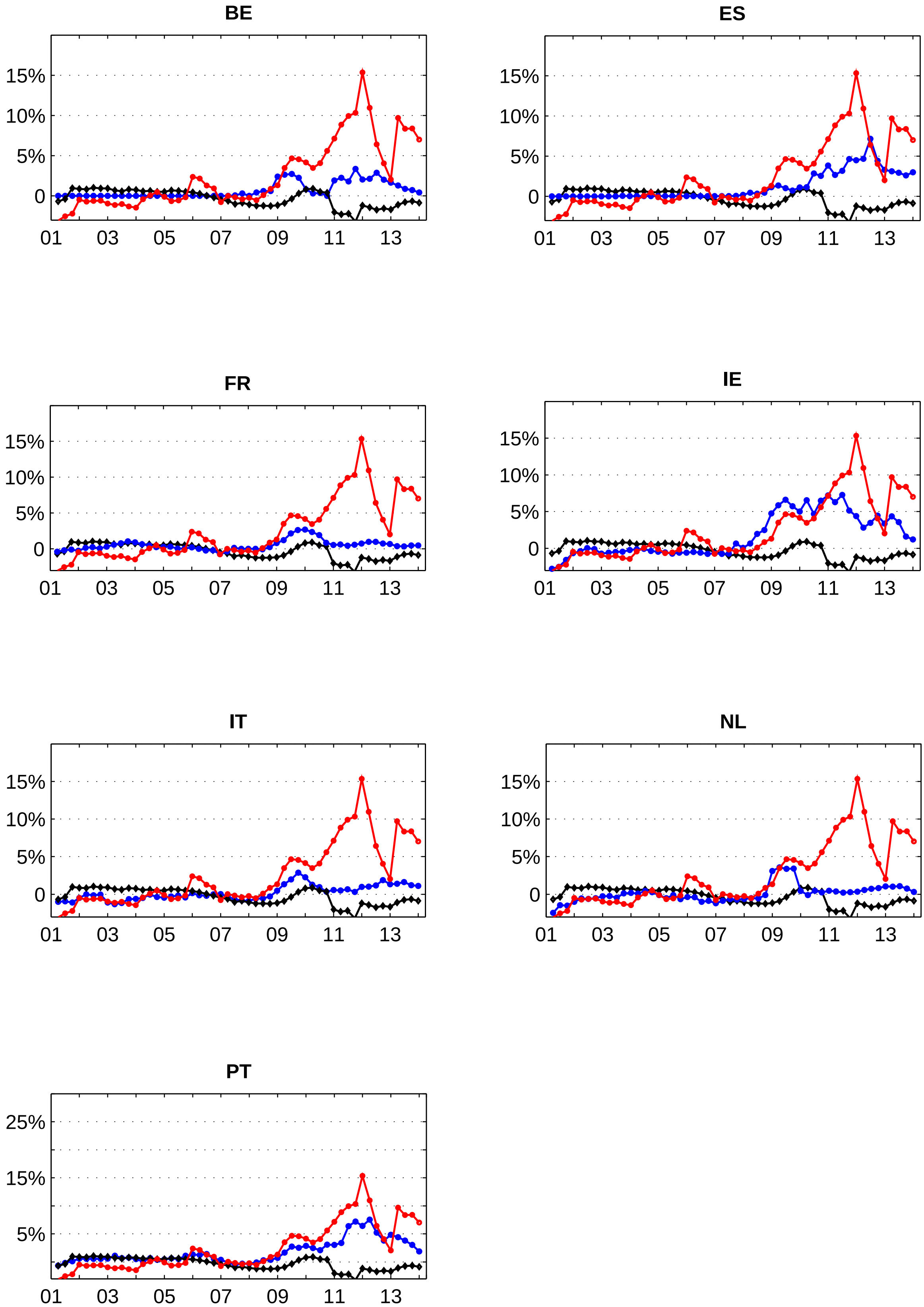


Figure 3.8: ϕ^A (blue-straight) compared to the correspondent indicator for Germany ϕ^{DE} (black-squared) and Greece ϕ^{GR} (red-squared).(*Source: ECB, Bloomberg and authors computations*)

The idea is to construct a score based on *market Ponziness*, defined as:

$$\phi_t^+ := \max\{\phi_t, 0\} \tag{3.4.9}$$

quantifying government's Ponzi scheme: we will refer to ϕ_t^+ as the *Ponzi score*.
The scoring system built up in this way is strictly country-specific but it does not encompass flight-to-quality cases, Since augmenting debt-level is always seen as negative (the higher the score the lower the creditworthiness).
It is thus worth to leave Germany out of this approach: relying on market's perception and on the general stable conditions of German's public economy, we define the *spreaded market Ponziness* as:

$$\begin{aligned} \phi_{*t} &= \phi_t - \phi_t^{DE} \\ &= (\Delta x_t - \Delta x_t^{DE}) + s_t x_{t-1} + (y_t^{DE} - \tilde{y}_t^{DE})(x_{t-1} - x_{t-1}^{DE}) + (\tilde{y}_t^{DE} - \tilde{y}_t)x_{t-1} \end{aligned} \tag{3.4.10}$$

If we further assume that the accounting rate is common for the whole monetary union, and also that the difference between such rate and German yield is neglectable, then (3.4.10) is approximated by:

$$\phi_{*t} \approx (\Delta x_t - \Delta x_t^{DE}) + s_t x_{t-1} \tag{3.4.11}$$

Thus ϕ_* is given by a nonlinear process in the spread as weighted by current debt level, plus the difference between target country and Germany's actual Ponziness.
The *spreaded Ponzi-score* can again be defined by taking the positive part in (3.4.11):

$$\phi_{*t}^+ := \max\{\phi_t - \phi_t^{DE}, 0\} \tag{3.4.12}$$

can be defined. In this way, the minimum score (maximum creditworthiness) is assigned to Germany by construction, which is *de facto* considered risk-free, coherently with the results presented in chapter 2.
This scoring system grants[70] to each country a certain degree of Ponziness, as long as it remains lower than that of Germany. This corresponds to assume that the combination of macroeconomic indicators and interest rates in Germany is such that the country is perceived as a risk-free issuer by the market.
Building up a scoring system is useful in order to extend a rating system (for example that supplied by rating agencies) which does not cover all entities.
Such techniques provide the new score to be constructed for all borrowers, so that a one-to-one mapping between the two curves can be interpolated using the entities providing both scores (see [7]).

[70] In the sense that an increase of ϕ_* does not impact market-implied creditworthiness.

Here we maintain the same approach in defining a uniform scoring methodology but with a different scope, as the panel of countries is commonly covered by all external ratings. The aim here is instead to construct a dynamic scoring which is able to capture eventual rating migrations with quarterly frequency.
The implied scoring system is mapped to custom credit-risk measures through VAR analysis to infer feedback effects between the score and the standard measure.
Among the most popular credit-risk measures in the EMU, we selected the (5-year) spread *vis-à-vis* the german bund s and the (5-year) CDS-implied default probability.
The term structure of CDS-implied default probabilities is retrieved out of CDS quotes through the ISDA CDS-Standard model (appendix C).
Particularly, ISDA methodology relies on the assumption that, for any trade date and standard maturity $(t, \bar{T}_j)$, the (conditional) probability of default *under the risk-neutral measure* occurring within the time window $[t, \bar{T}_j]$ at time t is:

$$Q(t, \bar{T}_j) = 1 - \exp(-\lambda_{\bar{T}_j} \cdot \bar{T}_j) \tag{3.4.13}$$

where default intensity is assumed to be *flat for any fixed maturity.*
Observe that implying λ by assuming a flat maturity is inconsistent from a theoretical point of view in that a different λ is implied for each different maturity.
This shortcoming invalidates pricing models when contract with different maturities are traded on the market at the same time (see [20]).
However, if the scope is to seek for relevant statistical relationships in terms of credit risk, and aggregated debt is considered to mature at five years, a more complicated[71] structure for the default intensity process is not essential.
The advantage is that we are able to imply a one-to-one mapping between default probability and default intensity out of (3.4.13). Setting $Q_t = Q(t, 5)$, the evolution of 5-year CDS-implied default probabilities yields[72]:

$$\lambda_t = -\log(1 - Q_t)/5 \tag{3.4.14}$$

where T is set to 5 years coherently with the assumed maturity (duration) of public debt.[73] It is left to understand how to quantify the extent to which this scoring method reflects market sentiment, that is, what are the characteristic we expect a sovereign scoring system to have so as to be considered a valid alternative.
Econometric analysis is again used as the mean to draw conclusions: as large consensus is received by both s and λ as distress indicators, a VAR model (λ, s) is analyzed, in order to benchmark the statistical features shared by these different credit risk metrics.

[71] Current practice is to assume a piecewise constant λ across standard maturities.

[72] Observe that first-order expansion of (3.4.14) implies $\lambda \approx Q/5$ for small λ.

[73] Notice that rules for computing default intensity provides λ_t to be always positive *whatever* the level of CDS-spread is observed, see [84].

	BE	DE	ES	FR	IE	IT	NL	PT
ϕ	4	4	2	2	4	4	1	4
ϕ_*	4	–	2	4	4	4	4	4
λ	1	1	1	1	2	1	2	2
ϕ^+	3	2	2	2	1	2	2	4
ϕ^+_*	4	–	2	4	1	2	4	4

Table 3.6: Unidimensional number of lags $\hat{p}^j$, estimated through the step-procedure described in section 1.1, $\hat{p}^j \leq 4$ by construction.

The following step is to seek those same properties when comparing q to the implied scores: namely, as both ϕ^+ and ϕ^+_* are positive for any t, it is convenient to analyze the VAR models (λ, ϕ^+) and (λ, ϕ^+_*).
Notice that, by defining a *Ponzi default probability* as an exponential term structure with default intensity equal to the score, we are *de facto* comparing two different default probability measures. It is worth to underline that default probabilities out of 3.4.13 are instead computed under the risk-neutral measure.
Before exploring the set of credit risk indicators, it is worth to analyze both (y, ϕ) and (s, ϕ_*): it could be argued that any of the scores does not add information with respect to yield or spread, respectively.
This would be the case whether these latter VAR models brought forth strong statistical dependence among their components. The inclusion of macroeconomic variables as explanatory factors must be justified by proving their structural relevance within each of the scoring systems.

3.5 Econometrics of the scoring systems

Table 3.6 collects the estimated number of lags for each univariate process and each country: again, the maximum number of significant lags is set to $\bar{p} = 4$. The values of test statistics (1.2.2) are presented in table (A.9).
Observe that one lag is selected for all countries but Ireland and Portugal ($\hat{p} = 2$): this rather short memory in 5-year default probabilities evolution is probably due to slow sample frequencies, inducing smooth time series.
On the other hand, a higher number of lags is required in order to fit ϕ and ϕ_*: this fact is not surprising considering the amount of different information they embed, and their intimate nonlinear nature.
When considering the positive parts of ϕ^+ and ϕ^+_*, respectively, the number of required lags decreases, as a large amount of observations before 2009 equals zero.

	BE	**DE**	**ES**	**FR**	**IE**	**IT**	**NL**	**PT**
(y, ϕ)	$(0, 2)$	$(0, 3)$	$(0, 2)$	$(0, 2)$	$(0, 2)$	$(0, 4)$	$(0, 2)$	$(0, 3)^*$
(s, ϕ_*)	$(0, 4)$	$-$	$(0, 3)^*$	$(0, 4)$	$(0, 2)$	$(0, 4)$	$(0, 4)$	$(0, 3)^*$
(λ, s)	$(0, 2)$	$-$	$(1, 4)^*$	$(0, 2)$	$(1, 2)$	$(0, 2)$	$(0, 2)$	$(1, 3)^*$
(λ, ϕ^+)	$(0, 3)$	$(0, 2)$	$(1, 2)$	$(0, 2)$	$(0, 2)$	$(0, 2)$	$(0, 3)^*$	$(1, 3)^*$
(λ, ϕ^+_*)	$(1, 4)$	$-$	$(1, 2)$	$(0, 4)$	$(0, 2)$	$(1, 4)$	$(0, 4)$	$(1, 3)^*$

Table 3.7: Estimated cointegrating order and number of lags $(\hat{r}, \hat{p})$ used in parameters estimations; p is selected according to the procedure described in section 1.5.1; models with $(^*)$ provide an additional lag to whithen residuals.

Tables A.11 collect p-values from conducting ADF tests on this second set of variables: results are mixed so it is worth to face them one by one.
First of all, test T1 rejects the null of unit root for process ϕ in Spain, Italy and the Netherlands. If combined with market yield, the process $(y, \phi) \in \mathcal{I}(1)$ as $y \in \mathcal{I}(1)$, but again care must be taken when performing cointegration tests.
The only questionable case is that of Italy, for which process y could be stationary too (test T2 refuses the null), so that a possible alternative model for (y^{IT}, ϕ^{IT}) could be an $\mathcal{I}(0)$ process. The choice was to consider $(y^{IT}, \phi^{IT}) \in \mathcal{I}(1)$ to compare all countries using the same integration order.[74]
Concerning ϕ_*, the null of unit-root is rejected by model T3 in Italy, in favor of the trend stationary model. It is not restrictive here not to consider this test because estimating with the prior $\hat{H} = H_1^*$ does not provide for linar trend in the VAR.
ADF tests for process q (or, equivalently, for process λ) do not reject the nulls of unit-root for any of the countries in the panel.
The assumption of unit root in ϕ^+ is rejected by both $T1$ and $T2$ in all countries except for Germany and Ireland. Furthermore, Italy shows rejection of unit root assumption for any test, hence $\phi^{+,IT} \in \mathcal{I}(0)$ although $(\lambda^{IT}, \phi^{+,IT}) \in \mathcal{I}(1)$ (definition 1.3.3): cointegration rank is set to zero in this case.
The same applies to $\phi_*^{+,BE}$ (rejections of T3) and $\phi_*^{+,NL}$ (all tests rejects unit-root): again the integration order of λ again allows to set $(\lambda, \phi_*^+) \in \mathcal{I}(1)$, but eventual cointegrating relations would be fallacious. In all these case, the cointegration order is again priorly set to $r = 0$.
Determining the lags of VAR models follows the same principles disclosed for previous observation sets: briefly, lags are chosen according to tables 3.7.
A first estimation of cointegrating basis and short-run parameters is performed, and in-sample residuals $\hat{U}$ are tested for autocorrelations.

[74]Statistical results out of a stationary VAR estimation of (y^{IT}, ϕ^{IT}) are however unsatisfactory.

	$\hat{\hat{B}}_1$		$\hat{\hat{D}}^{\parallel}$	$\hat{\hat{A}}$	
ES	1.000	-1.605	0.000	-0.442	-1.356
		(0.005)	(0.001)	(0.031)	(0.661)
IE	1.000	-1.282	0.003	-0.405	-0.383
		(0.008)	(0.001)	(0.034)	(0.311)
PT	1.000	-2.282	0.005	-0.236	-3.776
		(0.006)	(0.001)	(0.034)	(0.150)

Table 3.8: Cointegrating vectors (λ, s): EGLS estimation of the normalized cointegrating basis $[1 \quad B_1]$, the constant within the cointegrating relations $D^{\parallel}$ and the error-correction *speeds* A; correspondent standard errors in brackets.

Results out of these tests are collected in A.27 and lags are augmented by 1 in all those VARs exhibiting rejection of no-autocorrelation assumptions. The final number of lags is resumed in table 3.7, and each model is fitted using such $\hat{p}$'s.

3.5.1 Ponzi-scores: a new information set

The first step is thus to make sure that the scoring which were introduced provide different information with respect to yields and spread respectively. The idea is to fit VAR models (y, ϕ) and (s, ϕ_*) looking for eventual feedback effects. No cointegrating relations are present in any of the two models in all countries. Short-run feedback effects (tables A.21 and A.22) are not observed, with the exception of France and Netherlands where the estimated models are $\Delta s_t^{FR} \approx 0.11\Delta\phi_{*,t-1}^{FR} + u_t$ and $\Delta s_t^{NL} \approx 0.04\Delta\phi_{*,t-1}^{NL} + u_t$. The correlation among residuals is notable only in Portugal, both in (y^{PT}, ϕ^{+}), $\rho = 0.7$ and $(s^{PT}, \phi_*^{+,PT})$, $\rho = 0.6$.
C_{11} in (s, ϕ_*) is informative in all countries (but Netherlands), differently from C_{22}, which is never significant. The conclusion is that the implied scores embed different information sets with respect to market yields and spreads.
Feedback effects are not registered and the significance of diagonal elements out of models fitting point out separate dynamics for each of the two components.

3.5.2 Common features of credit-risk measures

Next step concerns VAR models of the form (λ, s), across countries: figure 3.9 shows the evolution of the two time series. The purpose of such comparisons is to enlighten the econometric features arising among two widely diffused credit-risk measure, so as to fix a set of criteria able to determine the relevance of a new scoring system.

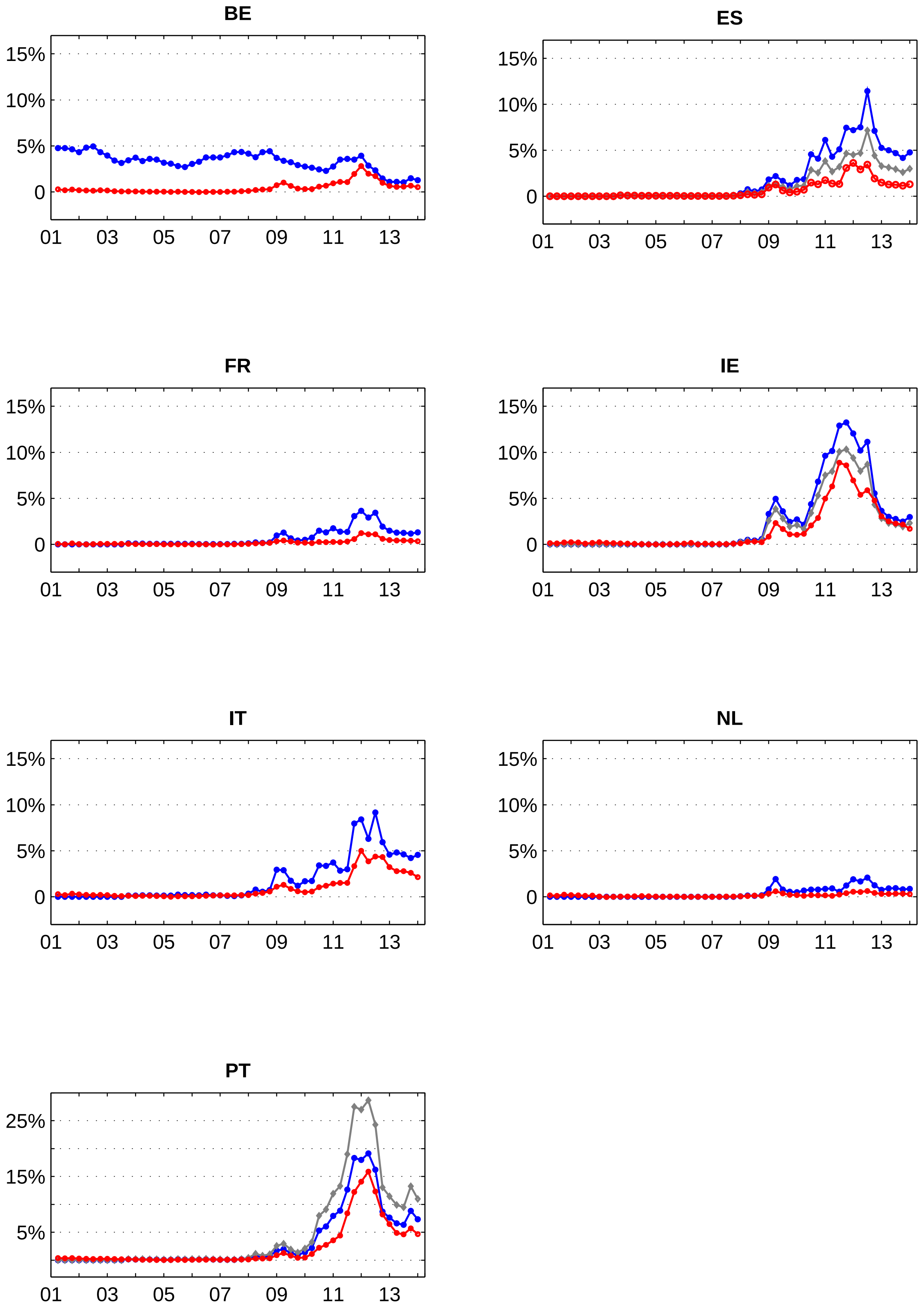


Figure 3.9: Default intensity (blue-dotted) and 5-years spread *vis-à-vis* Germany (red-dotted); eventual cointegrating relations $\lambda = \hat{\hat{B}}_1 s + \hat{\hat{D}}^{\parallel}$ are also shown (grey-squared). *(Source: ECB, Bloomberg and authors computations)*

Cointegrating relations arise in Spain, Ireland and Portugal, the most distressed countries within our panel: the long-run equilibria relationships yield $\lambda^{ES} \approx 1.6s^{ES}$, $\lambda^{IE} \approx 1.3s^{IE} + 0.3\%$ and $\lambda^{PT} \approx 2.3s^{PT} + 0.8\%$.
The GG measure implies that default intensity dominates the spread in Italy and Portugal, as the latter adjusts more slowly to the steady state. The sole significant error-correction speed in Ireland is $\hat{A}_1$, hence the situation is reversed, with default intensity reaching more slowly the steady state.
Short-run analysis (table A.23) reveals limited feedback effects, except for France and Ireland, where $\Delta\lambda_{t-1}$ impacts Δs_t with $C_{12} = 0.05$.
This result, particularly in Ireland, confirms the idea that yields spreads contain a richer information set with respect to credit derivatives prices. The shortcoming is the relatively large estimated short-run feedback effect $C_{21}^{PT} \approx 4.1$: a 1% variation of CDS-implied default probabilities would imply a rise in spread of 400 basis points.

3.5.3 The Ponzi-score(s) and credit risk

Consider now the vector processes (λ, ϕ^+) : figure 3.10 shows the evolution of these two processes between 2001 and 2013. The estimated cointegration rank is different from zero only in Spain and Portugal, and table 3.9 collects the cointegrating basis out of these two countries.
In particular, the long-run equilibria relationships are $\lambda^{ES} \approx 1.2\phi^{+,ES} + 3.5\%$ and $\lambda^{PT} \approx 2.5\phi^{+,PT} - 1.3\%$: using the first order approximation $Q \approx 5\lambda$, we deduce that a rise in the score $\Delta\phi^+ = 1\%$, for example, corresponds to a rise of 6% (ES) and 11.2% (PT) in the respective CDS-implied default probabilities, similarly to (λ, s).
This result is a signal pointing in the right direction: a higlhy distressed country shows a structural equilibrium between the Ponzi-score and market-implied Q.
It is also interesting to notice that, in Portugal, the Ponzi score dominates default intensity, while in Spain results are less explicit.
In the short-run (table A.24) feedback effects $\hat{C}_{12}$ are registered in all countries but they all differ in size and sign. The opposite effect $\hat{C}_{21}^{IT} \approx 0.01$ is registered in Italy only. These unaligned short-run effects may again be due the prior assumption $\phi^+ \in \mathcal{I}(1)$: coefficients estimation in the case of Belgium, Spain, Italy and the Netherlands may be biased ($\phi^+ \in \mathcal{I}(1)$ is refused by T_1 and T_2).
Figure 3.11 shows in turn the time evolution of (λ, ϕ_*^+) in the time window considered; by comparing these pictures with those collected in 3.10, an important property of both score processes is recognizable across countries.
The score becomes indeed significantly different from zero with approximatively two quarters of delay with respect to CDS-implied default probabilities.

	$\hat{\hat{B}}_1$		$\hat{\hat{D}}^{\parallel}$	$\hat{\hat{A}}$	
ES	1.000	-1.235	-0.035	-0.147	0.084
		(0.303)	(0.050)	(0.027)	(0.015)
PT	1.000	-2.501	0.013	-0.440	0.073
		(0.055)	(0.010)	(0.046)	(0.030)

Table 3.9: Cointegrating vectors (λ, ϕ^{+}): EGLS estimation of the normalized cointegrating basis $[1\ B_1]$, the constant within the cointegrating relations $D^{\parallel}$ and the error-correction *speeds* A; correspondent standard errors in brackets.

The lack of liquidity in sovereign CDS market, where CDS contracts before 2003 are not actively traded, sets implied default probability to zero[75].
The government becomes a Ponzi-borrower almost as soon as CDS-quotes became informative, hence, as soon as the risk of sovereing default in the EMU became a possible scenario in the view of financial markets.
This responds to eventual criticisms arguing that statistical results are biased because the time series of scores are zero in the first portion of the sample. Before the outspread of the crisis, credit risk in EMU countries was *actually* null (or perceived so), that is the reason why CDS contracts were not actively traded at those times.
In this sense, the fact that ϕ^{+}_{*} is equal to zero up to some (variating) observation time close to 2008 is considered as a good property of ϕ^{+}_{*}. The score activates as soon as CDS-default probabilities do, and each observation $(0, 0)$ is considered significant.
The scoring system as determined by ϕ^{+}_{*} is able to better capture variation in countries creditworthiness with respect to its country-specific counterparts ϕ^{+}.
The first result is that cointegrating relations arise in all countries but France, Netherlands[76] and Ireland, (table 3.10). Cointegrating relations would arise also in Ireland if dataset is limited to 2011:Q4, with a significant $\hat{B}_1^{IE} \approx 0.98$.
Results are mixed for what concerns the cointegrating constants $D^{\parallel}$, which are significant in Belgium and Portugal only. Cointegrating speeds are not informative on the principal determinant except for Belgium, where ϕ^{+}_{*} anticipates λ. Short-run dynamics are poorly informative too (table A.25): beyond each variable's specific autoregressive effects, short-term feedbacks are significant in Belgium and Italy ($\hat{C}_{12} < 0$) and the Netherlands ($\hat{C}_{12} > 0$). The negative sign in both Italy and Belgium could be interpreted as a tendency to reduce Ponziness as soon as default probabilites rise, so as to increase market confidence.

[75] Databases such as *Datastream* show no quotes, while specific financial provider such as *Bloomberg* provide them but the number of trades is not sufficient to consider prices as informative.
[76] The three ADF tests on $\phi^{+,NL}_{*}$ refuses the null of unit-root, hence we set $\hat{r} = 0$.

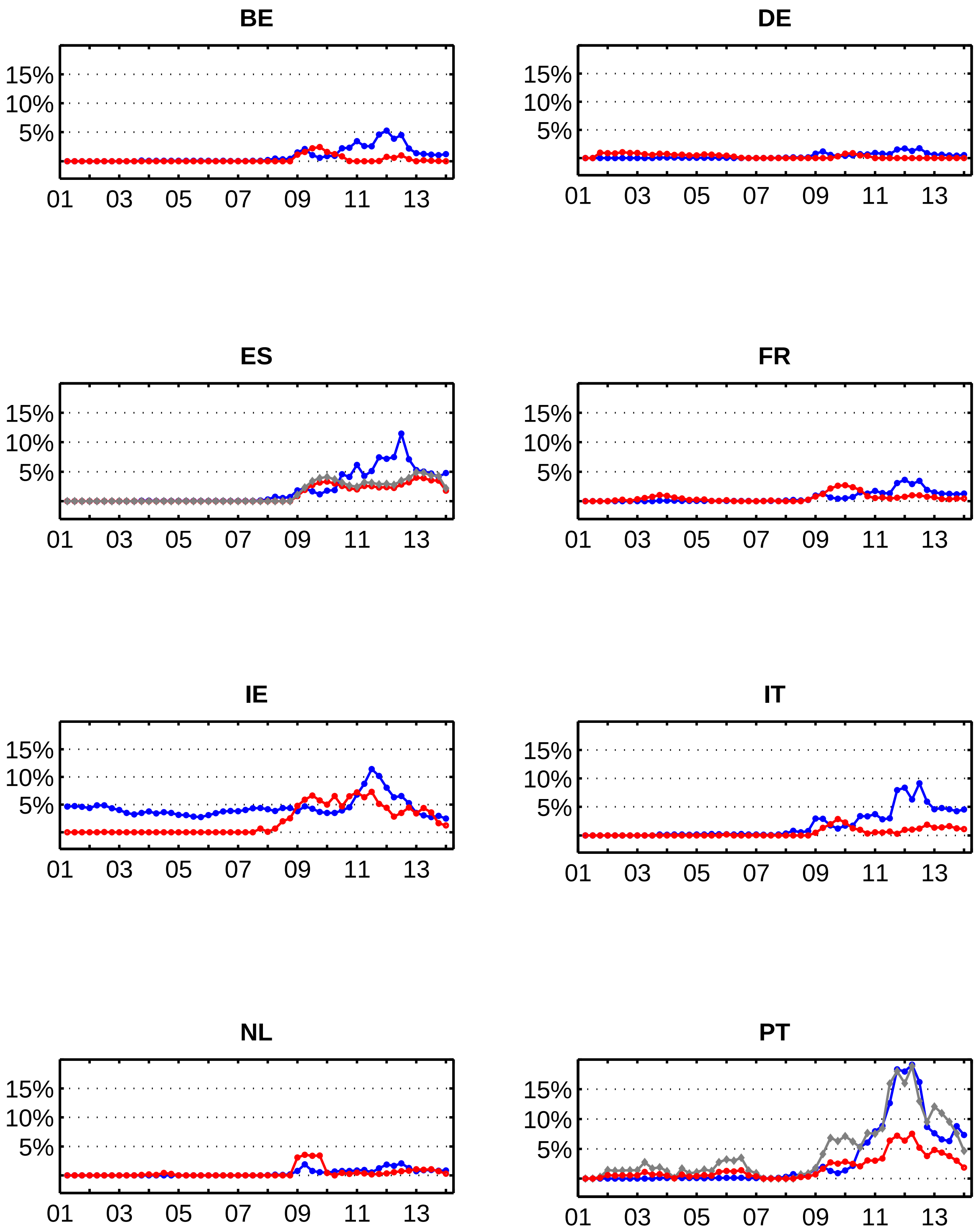


Figure 3.10: Default intensity (blue-dotted) and Ponzi score (red-dotted); eventual cointegrating relations $\lambda = \hat{\hat{B}}_1\phi^+ + \hat{\hat{D}}^{\|}$ are also shown (grey-squared). *(Source: ECB, Bloomberg and authors computations)*

	$\hat{\hat{B}}_1$		$\hat{\hat{D}}^{\parallel}$	$\hat{\hat{A}}$	
BE	1.000	-1.056	-0.002	-0.833	0.155
		(0.098)	(0.001)	(0.042)	(0.044)
ES	1.000	-1.165	-0.001	-0.223	0.206
		(0.116)	(0.003)	(0.046)	(0.029)
IT	1.000	-1.949	0.000	-0.142	0.286
		(0.067)	(0.000)	(0.058)	(0.027)
PT	1.000	-1.854	0.008	-0.655	0.410
		(0.039)	(0.001)	(0.059)	(0.073)

Table 3.10: Cointegrating vectors (λ, ϕ_*^+): EGLS estimation of the normalized cointegrating basis $[1 \quad B_1]$, the constant within the cointegrating relations $D^{\parallel}$ and the error-correction *speeds* A; correspondent standard errors in brackets.

It is notable that the steady state shows roughly double sensitivity in Portugal and Italy with respect to Belgium and Spain. This implies, for example, a decrease of 1% in n_t (see equation (3.4.6)) yields a 5% increase of default probabilities in these latter countries and of 10% in Portugal and Italy. The latter suffers also from an amplified snowball effect because of the high level of debt/gdp, similarly to Belgium. Such phenomenon is latter pronounced in Spain, where $x^{ES} < 100\%$ uniformly in t, and Portugal, where $x_t^{PT} \geq 100\%$ only for $t \geq 2011 : Q4$.

3.5.4 Econometrics of the scoring systems: concluding remarks

Between the two competing scores which had been defined, ϕ_*^+ is the more informative, and deserves further considerations. ϕ_*^+ is rolled every quarter, endowed with brand new macreoconomic and financial. Its value can be used as a default intensity, thus corresponds approximately to $5\hat{B}_1$-times physical default probabilities.
As aforementioned, it is typical to consider process λ (or Q) as containing the largest information set in terms of expectations on future creditworthiness of any issuer, including forecast on public accounting and market rates.
Countries displaying long-run equilibria with Q reveal structural relationships among current financial accounting and perceived creditworthiness. This relationship affects the most financially distressed countries only.
Public accounting and expectations on creditworthiness share a common equilibrium which is manifested on the long run. Cointegrating speeds shows the reaction of agents that, looking at current ϕ_*^+, change their expectations (as reflected by CDS prices) in order to move back to the asymptotic steady state.

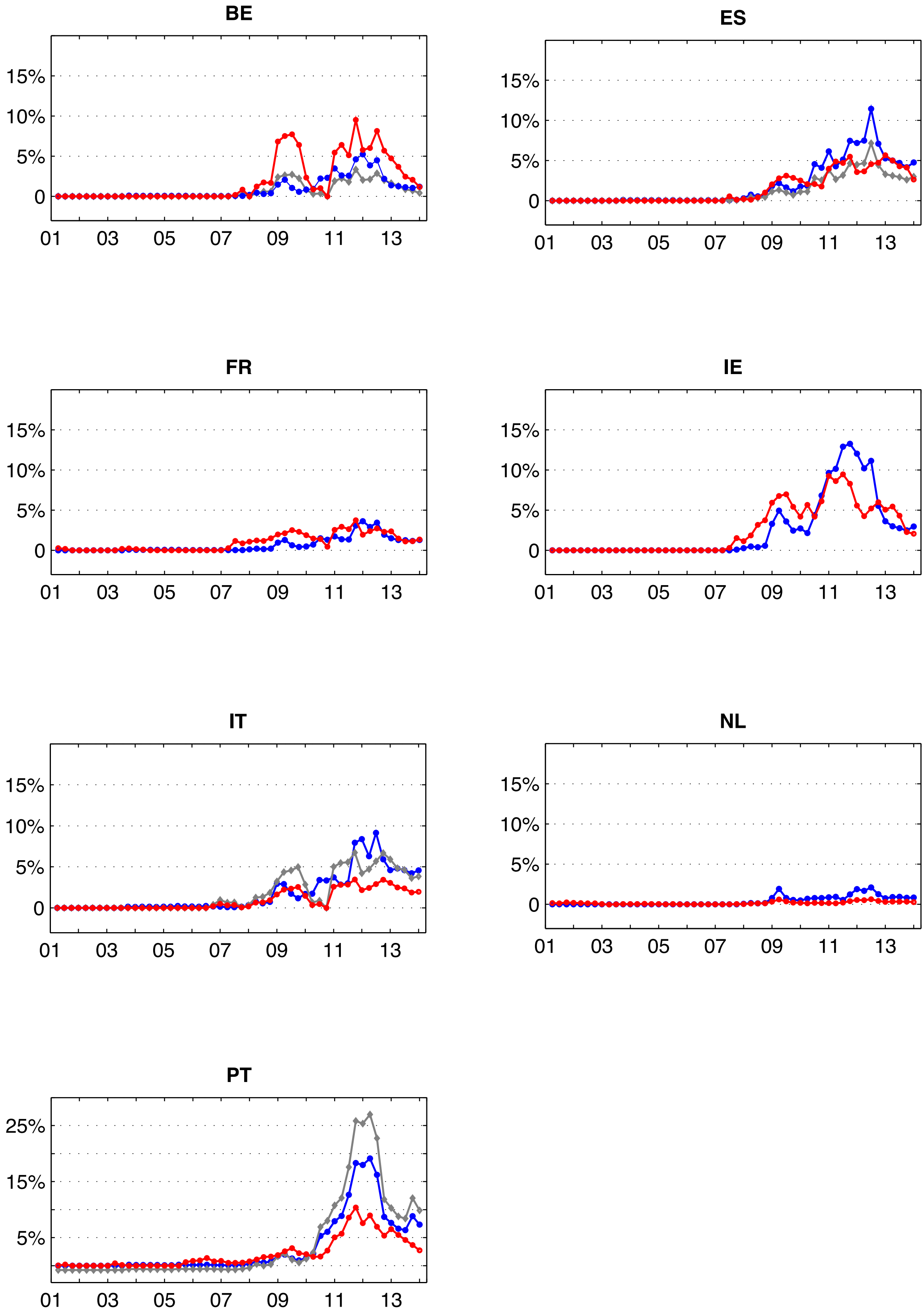


Figure 3.11: Default intensity (blue-dotted) and spreaded Ponzi score (red-dotted); eventual cointegrating relations $\lambda = \hat{\hat{B}}_1 \phi_*^+ + \hat{\hat{D}}^{\|}$ are also shown (grey-squared). *(Source: ECB, Bloomberg and authors computations)*

Conclusions

In the present work, the reliability of financial distress indicators across EMU countries originally inspired by Maastricht parameters is tested, and a new scoring method, supported by economic theory, is constructed by combining them.
The basis for comparison is credit derivatives market: information may be retrieved either from CDS prices or from model-based CDS-implied default probabilities.
Maastricht Treaty supplies monitoring systems based on macroeconomic and financial variables, but it is clear that the latter played a central role in directing economic policies across the Eurozone in the latest years.
The healthy situation of German's macro-financial conditions raised yields spreads *vis-à-vis* Germany to the role of custom indicator of distress for member states.
Particularly, yields spreads are commonly interpreted as *risk premia over a risk-free benchmark*. It is possible to test whether credit risk is the only determinant of yields spreads once an alternative measure of credit risk premia is available.
Standardization of credit derivatives market furnishes a perfect framework to assess credit risk premia through simple hedging strategies.
The enhancements of standard contracts rules, including central clearing, decisional committees and auction-based post-default settlement, are designed with the scope of cleaning CDS prices from risk factors different from credit risk.
Furthermore, the introduction of a bond-like structure for CDS contracts allows to integrate credit derivatives and obligations market (the *CDS-bond market*) through the fundamental introduction of a spot price for hedging credit risk.
A *CDS-bearing* portfolio is defined as a combined long position on a sovereign obligation and a standard CDS, expiring the first standard maturity which exceeds the residual bond life. Assuming the existence of a term structure of zero-coupon hedging portfolios, a country-specific *CDS-bearing yield curve* can be implied for any member state, using the same modeling assumptions underlying the construction of naked yield curves. The difference between naked and CDS-bearing yield curves:

$$\pi(t,T) = y(t,T) - \bar{y}(t,T)$$

determines for any trade date t the term structure of credit risk premia.
The use of custom yield curves modeling techniques allows to monitor comovements of CDS and obligations prices for any maturity using the parameter $\bar{\theta}^*$ out of Nelson-Siegel interpolation. Changes in level, slope, curvature and scale of country specific CDS-bearing yield curves can forecast cross-country arbitrage opportunities.
This framework allows to compare long-term credit risk premia $\pi_t = \pi(t, 10)$ to spreads *vis-à-vis* Germany without additional modeling assumptions.

Furthermore, it allows to imply credit-risk premia π^{DE} also for Germany: its time evolution confirms the market perception of a credit-risk free entity.
Credit risk premium is different from zero because CDS contracts must have a nonzero price, but the relative variance of π^{DE} is so small that it can almost be considered as constant on the selected time window.
A constant CDS premium certifies the illiquidity of German CDS market due to the unwillingness of investors to trade German credit risk: this confirms that, from a pure market perspective, credit risk is *de facto* not present.
The bivariate model $(y^{DE}, \bar{y}^{DE})$ shows perfect correlation $\rho = 1$ among the innovations components: hedging does not reduce naked yields variance because the two portfolios are already sharing the same source of randomness.
A high correlation ($\rho \geq 0.9$) is estimated among residuals out of (y^{DE}, L) and, consequently, between those out of $(\bar{y}^{DE}, L)$. The basis $b^{DE} = \bar{y}^{DE} - L$ is always negative: the market discourages the purchase of German bonds financed at Euribor rate, so as to deflate the flight towards German debt market's quality.
The absence of cointegrating relations in any of the three models is probably due the relatively high frequency of the dataset compared to the relatively short time examined. In the short run, German lagged yields differences have a positive impact on future Euribor rate differences, stressing the leading position of German bonds among European credit-risk free investments.
The computation of π^{IT} and π^{ES} permits the comparison with yields spreads *vis-à-vis* Germany $s^{IT} = y^{IT} - y^{DE}$ and $s^{ES} = y^{ES} - y^{DE}$, respectively.
Spreads overprice CDS-implied credit risk premia during the time of harshest distress of each respective countries, that is, the differences $(s^{IT} - \pi^{IT}) = (y^{IT} - y^{DE} - \pi^{IT})$ and $(y^{ES} - y^{DE} - \pi^{ES})$ are positive between 2011 and 2012 and in mid-2012, respectively.
Possible reasons for this misalignment additional country specific risk premia cloaked within yields levels, which are not directly attributable to credit risk.
A second explanation could be the excess demand for German debt: even if yields spreads in normal times measured exactly credit risk premia, the rush to buy German bonds resulting in an excess lowering of y^{DE} might have systematically risen the difference $(y - y^{DE} - \pi)$ to positive values in these years.
Moreover, since π underprices credit risk in normal times, purchasing a German bond before 2011 was more profitable than buying a peripheral Euro-country obligation and hedge credit risk by purchasing CDS protection: this could be in turn an *a priori* justification to the flight-to-quality phenomenon.
Cointegrating relations arise both in (s^{IT}, π^{IT}) and (s^{ES}, π^{ES}), despite frequency and time stub: the long run equilibrium provide credit risk premium to be roughly 60% of the spread plus a fixed cost of 70 basis points in both countries.

This *sunk cost* implies that hedging obligations positions through the credit derivatives market is always costly, even if s is zero. This fact is attributed to the purchase of a derivative contract which cannot have a zero price.
CDS-hedging *projects sovereign yields onto a credit-risk-free pattern*: the set of country-specific hedged portfolios together with the Euribor rate should in principle be representative of three credit risk free yields available on the market, thus we expect models in the form $(\bar{y}^{IT}, L)$, $(\bar{y}^{ES}, L)$ and $(\bar{y}^{IT}, \bar{y}^{ES})$ to share similar properties.
No cointegrating relations arise: the short-run analysis reveals a high correlation with Euribor rate ($\rho \approx 0.90$) for both $\bar{y}^{IT}$ and $\bar{y}^{ES}$, but a lower one among themselves ($\rho \approx 0.66$). This is probably due to the higher variability in the combination of the two hedged portfolios, since vector models residuals includes information stemming from two derivatives contracts and two obligations.
Although the pattern of $\bar{y}^{IT}$, $\bar{y}^{ES}$ and L is similar when looking at charts, the basis $b = \bar{y} - L$ is non stationary in both cases. Furthermore, b rises from average value when economic and financial conditions of target countries are severly distressed, thus suggesting a *systematic* nature. Both these facts exclude that the basis can be assimilated to a statistical discrepancy in the white noise family.
It is worth to underline that this conclusion is not related to the choice of the benchmark rate: by computing the difference $\bar{y}^{IT} - \bar{y}^{ES} = b^{IT} - b^{ES}$, the process which is obtained is negative on average, yet it exceeded systematically the mean from above (below) when Italy (Spain) was facing its harshest distress moments.
This argument applies also for relatively minor events, such as the turmoil which followed political election in Italy in March 2013.
The basis is thus interpreted as an *additional risk premium*, which is present even if credit risk is fully hedged. In principle, if credit risk was the only determinant of yields spreads, any systematic discrepancy between Italian and Spanish CDS-bearing portfolios yields is an *arbitrage opportunity.*
Sunk risk premia within such portfolios might be attributable to *counterparty risk* in the CDS contract, which is present notwithstanding the standardization process.
Alternatively, naked yields themselves might embed *liquidity risk premia* which are obviously not hedged using credit derivatives. Unexpected rise in yields level might be due to a drain of liquidity in distressed countries obligations market as a direct consequence of diffused *panic effects* on financial markets.
Liquidity of obligations markets is expected to be an important factor in determining sovereign yields levels. On the one hand, the rush to buy German obligations[77] resulted in a general decrease of the correspondent yields level.

[77]The *flight to quality phenomenon* induced a decreasing pattern of yields as a response to increasing supply of debt: between 2010 and 2012, the quarterly relative average debt variation is $dx/x \approx 1.5\%$ (DE), 0.8%(IT) 4.5%(ES), while $\Delta y/y = -6.5\%$ (DE), 8.25% (IT) and 6.35%(ES).

On the other, Greece experienced a total lack of demand for its obligations which ultimately caused default. In a standard *demand/supply* framework, countries sovereign obligations markets can be considered as *single-good markets* where yields are prices and demand is an increasing function of debt.
It is not retained a significant limitation to assume that the quantity being supplied is debt/gdp. Investors will be encouraged to purchase obligations in case the money which is lended is used profitably, hence the differences between yields and the growth rate of target economy are the relevant variables to be considered.
The panel of countries under analysis is widened to eight state members: Belgium, Germany, Spain, France, Ireland, Italy, Netherlands and Portugal.
The statistical analysis of a *linear demand function* requires an appropriate choice of yields in such context. Beyond market yield y, an alternative candidate is the *accounting rate* $\tilde{y}$, as implied through the interest quotes out of government balance accounting. The computation of y and $\tilde{y}$, with quarterly frequency, suggests the accounting rate not to be sensitive to market risk premia, and is thus retained of minor importance.
The two rates y and $\tilde{y}$ may be exchanged without affecting results only in highly rated countries, such as Germany and Netherlands. In both cases cointegrating relations reveal a long run equilibrium among the two rates, with cointegration coefficient of 1.5 (DE) and 0.9 (NL). Highly distressed countries such as Spain and Italy registered instead a significant impact of Δy_{t-1} on $\Delta\tilde{y}_t$: that is, the financial market impacts future accounting variables.
Any attempt of directly estimating the (linear) demand function in a VAR model involving any of the two yields and debt/gdp level x is nullified by the integration order of $x \in \mathcal{I}(2)$. This problem cannot be sidestepped in the case of Spain, Portugal and Ireland, as the three unit root tests do not refuse the null of unit root in Δx, therefore the assumptions $(y, x) \in \mathcal{I}(1)$ or $(\bar{y}, x) \in \mathcal{I}(1)$ are unrealistic.
Referring to other countries, no feedback effects and cointegrating relations are retrieved for processes of the form (y, x). The unique significant coefficient is the pure autoregressive component in debt differences, but confidence bandwiths for the estimators include unit roots: the short-run analysis may be then questionable too.
An exception is again Germany: $\tilde{y}^{DE}$ can be modeled as a $\mathcal{I}(1)$ process according to each of the three models, and, although parameter sizes are questionable, there's a negative impact of Δx_{t-1} on Δy_t, attributable to excess demand effects.
Considering the model $(\tilde{y}, x)$ cointegrating relations arise in Belgium, Italy and Netherlands, where also feedback effects of lagged speed on internal rate differences is registered. Such relations are of little interest as distorted from the fact that the assumption $\tilde{y} \in \mathcal{I}(1)$ is not unanimously accepted by the three tests.

The cases of Ireland and Portugal suggest that debt speed might be the correct variable affecting yields premia; this is also in line with this demand/supply approach as a government in recession which is issuing a large amount of debt in a short time period must encourage new investors by offering a higher yield to maturity.
The comparison of *debt speed* Δx does not produce meaningful long-run effects (no cointegrating relations) when combined either to naked yields y or to yields spreads s, with the partial exception of Spain, both in $(y, \Delta x)$ and $(s, \Delta x)$ where p-values for the impact of $\Delta^2 x_{t-1}^{ES}$ on Δy_t^{ES} are registered. The empirical evidence inspired by Portugal and Ireland do not find confirmation except for positive correlation of innovations components in Portugal, reaching 60%.
Positive debt speed inspired also the use of Minsky's classification of borrowers: broadening such definitions to *debt/gdp borrowers* a government is a Ponzi borrower when $\Delta x > 0$. The perpetuation of Ponzi schemes in public debt might shift investors risk aversion, up to default.
The idea is to build up a scoring methodology which is able to recognize and monitor eventual governments Ponzi schemes, and infer whether pursuing such schemes affects target country's creditworthiness as expressed by credit risk premia.
The simplest idea is to base the scoring methodology using *actual Ponziness* Δx^+ in order to measure the perpetuation of such schemes in times.
The unconditional expectation of the variables $\mathbb{1}_{\Delta x_t > 0}(t)$ on the time stub considered might be used to calibrate a set of thresholds and imply a rating measure by comparison with standard ratings.
The situation of government in crisis prevents from this approach, as almost any EMU countries has augmented his debt level during the crisis, but with different outcomes in terms of perceived creditworthiness.
The level of actual Ponziness $\Delta x_t^+ = \left[(\tilde{y}_t - g_t)x_{t-1} - n_t + m_t\right]^+$ does not offer satisfactory results when a direct VAR approach is pursued to compare it either to y_t and s_t. The reason is that the former is based on information coming from accounting variables only, while yields (yields spreads) are market variables, and the two information sets hardly interact at this time frequency.
The idea is to substitute market rates to accounting rates and define the process:

$$\phi_t = (y_t - g_t)x_{t-1} - n_t + m_t = \Delta x_t + (y_t - \tilde{y}_t)x_{t-1}$$

which is composed by actual Ponziness plus variation between market and internal rates, weighted by previous debt/gdp level.
The score ϕ combines Maastricht parameters through a *nonlinear function*: a possible limitation is that ϕ is not sensitive to inflation, as the nominal and real *snowball effects* are the same as resulting from the difference of yields and growth rates.

The impact of the snowball effect $y_t - g_t$ is mitigated or amplified depending on x_{t-1} being below or above 100%, while primary surplus has always a one-to-one impact on ϕ. A second score can be constructed by differencing on Germany:

$$\begin{aligned}\phi_{*t} &= \phi_t - \phi_t^{DE} \\ &= (\Delta x_t - \Delta x_t^{DE}) + s_t x_{t-1} + (y_t^{DE} - \tilde{y}_t^{DE})(x_{t-1} - x_{t-1}^{DE}) + (\tilde{y}_t^{DE} - \tilde{y}_t)x_{t-1} \\ &\approx (\Delta x_t - \Delta x_t^{DE}) + s_t x_{t-1}\end{aligned}$$

where last approximation comes from assuming that the accounting rate in Europe is constant across countries, and that $(y_t^{DE} - \tilde{y}_t^{DE})$ is small.
The first question was whether this processes embeds a different information set with repsect to their consitutives: particularly, it is necessary to verify whether yields (yields spreads) are not the main drivers of ϕ and ϕ^*, respectively.
The analysis of (y, ϕ) reveals no cointegrating relations and no short-run significant coefficients in the first case; correlation of stochastic innovations is high in Portugal only (70%). Models of the form (s, ϕ^*) present similar results, with a relatively high correlation among innovations estimated for Ireland (50%) and Portugal (60%), but remains lower than 30% in absolute value across the other countries.
Significant short-run feedback effects are retrievable in Spain and Netherlands, where score differences $\Delta\phi^*_{t-1}$ impacts future spread variation Δs_t.
Econometric results thus confirm that both processes are not fully determined by the respective underlying financial variables, that is, accounting variables play a significant role. Taking the positive part of such scores defines *market Ponziness* ϕ^+ and *spreaded market Ponziness* ϕ^+_* of target sovereign borrower.
The advantage of such scores is that they both *awake* (become different from zero) together with CDS-implied default probabilities, and increase comonotonically with them. In order to decide how to determine the significance of such scores as macrofinancial distress measures, the features of a VAR analysis of (λ, s) are retrieved, being λ the *default intensity* as implied by the CDS market.
Cointegrating relations arise in major distressed countries: approximating $Q \approx 5\lambda$ allows an easy description of this results in terms of default probabilities.
The *long-run equilibrium relationships* between yields spreads and default intensity implies $Q^{ES} = 8s^{ES}$, $Q^{IE} = 6.5s^{IE} + 0.3\%$ and $Q^{PT} = 11.5s^{PT} + 0.8\%$.
The presence of these equilibria are justifiable by observing that financial distress measures are based on *expectations* on future creditworthiness.
Hence, a VAR model exhibiting cointegrating relations indicates that expectations on future creditworthiness are linked in a long-run equilibrium.
When analyzing the models (λ, ϕ^+) and (λ, ϕ^+_*) cointegrating relations are thus considered of major importance.

Observe that, since both scores are non negative, a straightforward way to compare them to custom credit risk measure is to imply a *physical default probability* by using them as default intensities in a standard exponential framework.
Similarly to the case of λ and Q, the 5-years physical default probabilities are approximated multiplying the respective score by five.
Concerning ϕ^{+}, results are similar to those retrieved for yields spreads, namely $Q^{ES} = 6\phi^{+,ES} + 3,5\%$ and $Q^{PT} = 12.5s^{PT} - 1.3\%$.
A long-run equilibrium between accounting-based score ϕ and Q is interpreted as *current macorfinancial condition is affecting expectations on future creditworthiness in the long-run*.
An interesting feature is that ϕ^{+} is a country specific score, and does not need to be benchmarked on further countries. The disadvantage is the short-run analysis. Disturbances correlation is lower 40%(ES) and 60%(PT) and a rather counterintuitive short-run impact of $\Delta\phi^{+}_{t-1}$ on ΔQ_t is registered in Belgium, Spain and Italy: a possible explanation could be that the assumption $\phi^{+} \in \mathcal{I}(1)$ is a bit strained.
The VAR model (λ, ϕ^{+}_{*}) yields cointegrating relations in a larger number of countries: beyond Spain and Portugal, Belgium, Italy and also Ireland (if we limit the analysis to 2011:Q4) revealed the presence of long term equilibria. Cointegrating relations are expressed in terms of Q as: $Q^{BE} = 5.2\phi^{+,BE}_{*} - 0.2\%$, $Q^{ES} = 5.8\phi^{+,ES}_{*}$, $Q^{IT} = 9.5\phi^{+,IT}_{*}$ and $Q^{PT} = 9.0\phi^{+,PT}_{*} - 0.8\%$, respectively ($Q^{IE} = 4.5\phi^{+,IE}_{*}$ with limited dataset).
The presence of long-term equilibria is interpreted as a measure of interconnectedness among distress measures, thus the spreaded score ϕ^{+}_{*} performs *better* than ϕ.
Cointegrating relations among the two risk measures do not arise only in Netherlands and France. Mutual short-run feedback effects are of scarce significance across countries, except for residuals correlation ($\rho \geq 0.4$ across all countries).
The presence of cointegrating relations across any distressed country reveal that long-run determinants of CDS-based default probabilities are captured by such score, with accounting variables bringing forth their peculiar contribution.
A mixed-frequency approach to the construction of such scores might help to improve the performances of the scoring methods in explaining default probabilities implied by financial markets. Daily evolution of yields premia could be statistically combined with quarterly *shocks* attributed to the accounting variables of interest in order to retrieve a more sensitive risk indicator which evolves at market frequency.

Appendices

A Statistical tables

A.1 Statistical tables: Chapter 2

IT			
H_0	3|4	2|3	1|2
y	3.696	8.156	0.916
$\bar{y}$	0.114	0.288	4.640
π	0.773	0.554	1.656
b	0.142	0.225	3.048
s	8.171	2.048	4.462

ES			
H_0	3|4	2|3	1|2
y	0.360	0.278	8.209
$\bar{y}$	0.621	0.115	0.287
π	0.026	1.306	0.158
b	2.626	0.282	2.259
s	0.107	0.143	5.231

DE			
H_0	3|4	2|3	1|2
y	3.161	0.022	6.589
$\bar{y}$	2.737	0.002	6.769
π	1.519	1.175	0.001
b	3.589	1.439	39.10

EUR			
H_0	3|4	2|3	1|2
L	1.358	0.001	12.01

Table A.1: **Single variables lag selection**: Values of the $\mathbf{F}(1, T-p-1)$-statistic (1.2.2): degrees of freedom in testing $(p-1)|p$ correspond to the effective sample size under the alternative. The correspondent $\alpha = 5\%$ critical values are $\mathbf{F}(1, 172) = 3.897$, $\mathbf{F}(1, 173) = 3.896$ and $\mathbf{F}(1, 174) = 3.895$, respectively. Statistics are computed for each test, although the estimated numbers of lags p^j is selected as the first p for which $(p-1)|p$ is rejected.

IT			
H_0	T1	T2	T3
y	0.718	0.707	0.911
$\bar{y}$	0.620	0.677	0.602
π	0.714	0.543	0.785
b	0.094	0.319	0.342
s	0.683	0.527	0.809

ES			
H_0	T1	T2	T3
y	0.731	0.529	0.774
$\bar{y}$	0.640	0.749	0.694
π	0.750	0.371	0.545
b	0.055	0.389	0.609
s	0.755	0.324	0.752

DE			
H_0	T1	T2	T3
y	0.575	0.516	0.697
$\bar{y}$	0.539	0.465	0.687
π	0.736	0.073	0.173
b	0.705	0.143	0.446

EUR			
H_0	T1	T2	T3
L	0.617	0.646	0.849

Table A.2: **Unit-root tests** : p-values from Augmented Dickey Fuller hypotheses tests T1, T2 and T3 (section 1.1) for unit roots; **bold** numbers represent value under the significance level $\alpha = 0.05$. Critical values variate with estimated p as described in section 1.1. (ADF tests with $p \geq 6$ would be however affected by small sample bias.)

H_0	3|4	2|3	1|2
(y^{DE}, L)	0.021	1.922	22.51
$(\bar{y}^{DE}, L)$	0.110	3.231	26.87
$(y^{DE}, \bar{y}^{DE})$	9.344	7.505	2.370
(π^{IT}, s^{IT})	4.579	4.727	**1.668**
(π^{ES}, s^{ES})	1.806	4.216	**37.98**
$(\bar{y}^{IT}, L)$	4.441	2.176	18.54
$(\bar{y}^{ES}, L)$	4.966	42.38	**12.93**
(y^{IT}, y^{ES})	8.527	**25.42**	25.02
$(\bar{y}^{IT}, \bar{y}^{ES})$	1.547	1.440	**4.578**
(b^{IT}, b^{ES})	2.613	2.411	6.267

Table A.3: **Vector lag selection**: values of the test statistic (1.5.3): the critical value at 5% level is $\boldsymbol{\chi}^2(n^2) = \boldsymbol{\chi}^2(4) = 9.488$ for any test of the form $p-1|p$; **bold** numbers report eligible tests, according to the limitations $\hat{p} : \hat{p}_\infty^- \leq \hat{p} \leq \hat{p}_\infty^+$. $\hat{p}$ is the first p such that $p-1|p$ is rejected; if no bold number is in a row then $\hat{p} = \hat{p}_\infty^- = \hat{p}_\infty^+$.

(y^{DE}, L)	2
$(\bar{y}^{DE}, L)$	2
$(y^{DE}, \bar{y}^{DE})$	2
(π^{IT}, s^{IT})	1
(π^{ES}, s^{ES})	2
$(\bar{y}^{IT}, L)$	2
$(\bar{y}^{ES}, L)$	2
(y^{IT}, y^{ES})	3
$(\bar{y}^{IT}, \bar{y}^{ES})$	2
(b^{IT}, b^{ES})	1

Table A.4: **VAR lag selection resume**: estimated number of lags out of the procedure described in 1.5.1, before eventual corrections due to residual analysis.

(y^{DE}, L)	0.364
$(\bar{y}^{DE}, L)$	0.415
$(y^{DE}, \bar{y}^{DE})$	0.300
(π^{IT}, s^{IT})	**0.057**
(π^{ES}, s^{ES})	**0.055**
$(\bar{y}^{IT}, L)$	0.510
$(\bar{y}^{ES}, L)$	0.773
(y^{IT}, y^{ES})	0.934
$(\bar{y}^{IT}, \bar{y}^{ES})$	0.328
(b^{IT}, b^{ES})	0.261

Table A.5: **Cointegration test statistic**: p-values for the cointegrating test statistics for integrated VAR(p) models, with p as in table 2.2, 2.3 and 2.5, respectively.

	$\hat{\hat{C}}_1(1,1)$	$\hat{\hat{C}}_1(1,2)$	$\hat{\hat{C}}_1(2,1)$	$\hat{\hat{C}}_1(2,2)$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\hat{\rho}$
(y^{DE}, L)	-0.008	**0.336**	**0.455**	-0.115	0.001	0.001	0.924
	(0.095)	(0.115)	(0.072)	(0.087)			
$(\bar{y}^{DE}, L)$	-0.010	**0.345**	**0.456**	-0.107	0.001	0.001	0.900
	(0.094)	(0.113)	(0.073)	(0.087)			
$(y^{DE}, \bar{y}^{DE})$	-0.042	0.219	-0.028	0.205	0.001	0.001	1.000
	(0.892)	(0.902)	(0.882)	(0.892)			
$(\pi^{IT}, s^{IT})^*$	**-0.092**	**0.249**	-0.011	**0.156**	0.002	0.002	0.782
	(0.022)	(0.036)	(0.033)	(0.035)			
$(\pi^{ES}, s^{ES})^*$	**-0.157**	**0.319**	-0.029	**0.205**	0.002	0.001	0.791
	(0.017)	(0.031)	(0.032)	(0.030)			
$(\bar{y}^{IT}, L)$	0.046	0.224	-0.044	**0.283**	0.001	0.001	0.936
	(0.094)	(0.144)	(0.060)	(0.092)			
$(\bar{y}^{ES}, L)$	0.013	0.013	-0.053	**0.285**	0.001	0.001	0.878
	(0.091)	(0.135)	(0.059)	(0.088)			
(y^{IT}, y^{ES})	-0.096	**0.316**	-0.147	**0.350**	0.002	0.001	0.914
	(0.103)	(0.136)	(0.076)	(0.101)			
$(\bar{y}^{IT}, \bar{y}^{ES})$	0.081	0.086	0.114	-0.058	0.001	0.001	0.666
	(0.098)	(0.101)	(0.095)	(0.098)			
(b^{IT}, b^{ES})	0.048	0.084	0.152	0.019	0.001	0.001	0.450
	(0.085)	(0.083)	(0.086)	(0.084)			

Table A.6: **Short-run dynamics:** estimated values of $\hat{C}_1$, relative standard errors (in brackets) and estimated correlation matrix of residuals; **bold** numbers indicates the significant coefficients; processes marked with (*) have estimated cointegrating rank $\hat{r} = 1$.

			$h=1$		$h=2$		$h=3$		$h=4$	
	r	p	$\boldsymbol{\lambda}_{LM}$	$\boldsymbol{\lambda}_W$	$\boldsymbol{\lambda}_{LM}$	$\boldsymbol{\lambda}_W$	$\boldsymbol{\lambda}_{LM}$	$\boldsymbol{\lambda}_W$	$\boldsymbol{\lambda}_{LM}$	$\boldsymbol{\lambda}_W$
(y^{DE}, L)	0	2	1	-	1	0.210	1	0.057	1	0.091
$(\bar{y}^{DE}, L)$	0	2	1	-	1	0.157	1	0.064	1	0.090
$(y^{DE}, \bar{y}^{DE})$	0	2	1	-	1	0.743	1	0.419	1	0.255
(π^{IT}, s^{IT})	1	1	1	0.984	1	0.168	1	**0.014**	1	**0.048**
(π^{ES}, s^{ES})	1	2	1	-	1	-	1	0.058	0.251	0.249
$(\bar{y}^{IT}, L)$	0	2	1	-	1	0.216	1	0.347	1	0.386
$(\bar{y}^{ES}, L)$	0	2	1	-	1	0.700	1	0.507	1	0.325
(y^{IT}, y^{ES})	0	2	1	-	1	**0.023**	1	**0.000**	1	**0.002**
$(\bar{y}^{IT}, \bar{y}^{ES})$	0	2	1	-	1	0.605	1	0.340	1	0.465
(b^{IT}, b^{ES})	0	1	1	-	1	0.285	1	**0.039**	1	0.629

Table A.7: **Analysis of residuals**: p-values out of test statistics (1.5.16) and (1.5.21), with $h = 1 \ldots 4$; **bold** numbers shows values under 5%. Residuals are estimated out of models with number of lags reported in tables A.4.

A.2 Statistical tables: Chapter 3

BE			
H_0	3|4	2|3	1|2
y	1.566	0.387	6.065
$\tilde{y}$	0.048	0.084	93.52
x	0.646	1.785	127.6
s	0.753	0.061	6.787

DE			
H_0	3|4	2|3	1|2
y	1.611	5.035	5.219
$\tilde{y}$	0.052	2.168	25.241
x	0.820	6.404	102.6
s	–	–	–

ES			
H_0	3|4	2|3	1|2
y	3.418	0.153	3.069
$\tilde{y}$	0.747	13.70	62.21
x	0.985	16.16	181.4
s	9.028	0.706	3.757

FR			
H_0	3|4	2|3	1|2
y	0.750	1.268	2.959
$\tilde{y}$	0.646	0.615	85.84
x	1.389	11.38	232.4
s	2.884	3.129	6.283

IE			
H_0	3|4	2|3	1|2
y	0.172	0.186	20.05
$\tilde{y}$	11.18	0.166	0.810
x	0.866	0.468	152.8
s	0.720	0.093	18.03

IT			
H_0	3|4	2|3	1|2
y	10.58	0.171	1.928
$\tilde{y}$	0.157	1.251	110.6
x	2.399	4.681	91.39
s	8.972	0.500	2.353

NL			
H_0	3|4	2|3	1|2
y	0.833	2.249	4.580
$\tilde{y}$	0.042	2.611	38.67
x	0.175	2.852	69.87
s	0.130	0.075	3.377

PT			
H_0	3|4	2|3	1|2
y	0.866	6.956	38.49
$\tilde{y}$	0.095	2.449	185.7
x	0.867	0.048	73.58
s	0.013	18.70	42.94

Table A.8: **Single variables lag selection (I)**: Values of the $\mathbf{F}(1, T-p-1)$-statistic (1.2.2); degrees of freedom in the test $(p-1)|p$ correspond to the effective sample size under the alternative. The correspondent $\alpha = 5\%$ critical values are $\mathbf{F}(1, 47) = 4.047$, $\mathbf{F}(1, 48) = 4.042$ and $\mathbf{F}(1, 49) = 4.038$, respectively. Statistics are computed for each test, although the estimated numbers of lags p^j is selected as the first p for which $(p-1)|p$ is rejected.

BE			
H_0	3|4	2|3	1|2
ϕ	6.727	1.563	2.439
ϕ_*	10.88	0.030	0.023
λ	1.738	0.183	0.530
ϕ^+	1.176	5.583	7.913
ϕ_*^+	18.50	1.610	0.257

DE			
H_0	3|4	2|3	1|2
ϕ	10.07	3.086	1.050
ϕ_*	–	–	–
λ	3.637	0.132	0.312
ϕ^+	2.723	0.067	4.752

ES			
H_0	3|4	2|3	1|2
ϕ	0.355	2.162	18.98
ϕ_*	0.214	0.419	5.796
λ	1.697	0.061	0.678
ϕ^+	0.487	1.529	19.03
ϕ_*^+	0.144	1.079	8.496

FR			
H_0	3|4	2|3	1|2
ϕ	2.455	0.699	17.90
ϕ_*	4.865	0.169	0.039
λ	1.416	0.123	0.904
ϕ^+	0.235	1.248	33.56
ϕ_*^+	5.555	0.309	0.738

IE			
H_0	3|4	2|3	1|2
ϕ	4.187	1.679	0.047
ϕ_*	4.389	0.000	2.700
λ	1.950	3.002	7.797
ϕ^+	3.047	1.891	0.048
ϕ_*^+	1.483	0.014	3.276

IT			
H_0	3|4	2|3	1|2
ϕ	5.111	3.479	6.034
ϕ_*	4.542	1.849	2.225
λ	3.734	1.113	0.019
ϕ^+	1.784	3.977	7.141
ϕ_*^+	0.565	2.902	5.114

NL			
H_0	3|4	2|3	1|2
ϕ	3.202	0.662	3.772
ϕ_*	7.959	3.165	2.923
λ	0.254	0.059	5.187
ϕ^+	2.925	1.099	4.335
ϕ_*^+	8.603	3.553	3.371

PT			
H_0	3|4	2|3	1|2
ϕ	10.17	0.078	1.262
ϕ_*	12.60	2.619	0.623
λ	0.933	1.369	12.02
ϕ^+	9.518	0.040	1.113
ϕ_*^+	13.20	2.783	0.856

Table A.9: **Single variables lag selection (II)**

BE			
H_0	T1	T2	T3
y	0.523	0.624	0.534
$\tilde{y}$	0.618	0.120	**0.034**
x	0.712	0.432	0.671
Δx	**0.017**	0.378	0.355
s	0.118	0.489	0.405

DE			
H_0	T1	T2	T3
y	0.530	0.771	0.775
$\tilde{y}$	0.617	0.179	0.663
x	0.762	0.532	0.395
Δx	**0.022**	0.198	0.473
s	–	–	–

ES			
H_0	T1	T2	T3
y	0.584	0.135	0.453
$\tilde{y}$	0.649	0.197	0.089
x	0.770	0.829	0.669
Δx	0.051	0.620	0.489
s	0.171	0.736	0.620

FR			
H_0	T1	T2	T3
y	0.520	0.676	0.595
$\tilde{y}$	0.598	0.495	**0.047**
x	0.801	0.346	0.790
Δx	**0.019**	**0.032**	0.052
s	0.053	0.243	0.052

IE			
H_0	T1	T2	T3
y	0.405	0.147	0.380
$\tilde{y}$	0.671	0.092	0.464
x	0.911	0.443	0.745
Δx	0.248	0.664	0.866
s	0.131	0.585	0.578

IT			
H_0	T1	T2	T3
y	0.578	**0.011**	0.051
$\tilde{y}$	0.626	0.122	**0.009**
x	0.760	0.575	0.563
Δx	**0.005**	0.199	**0.018**
s	0.466	0.814	0.591

NL			
H_0	T1	T2	T3
y	0.537	0.652	0.884
$\tilde{y}$	0.581	0.561	0.189
x	0.779	0.920	0.653
Δx	**0.003**	0.069	0.114
s	0.139	0.402	0.148

PT			
H_0	T1	T2	T3
y	0.221	0.167	0.177
$\tilde{y}$	0.623	0.321	0.129
x	0.915	0.697	0.841
Δx	0.421	0.638	0.737
s	0.082	0.502	0.379

Table A.10: **Unit-root tests (I)**: p-values from Augmented Dickey Fuller hypotheses tests T1, T2 and T3 (section 1.1) for unit roots, see table A.2.

BE			
H_0	T1	T2	T3
ϕ	0.164	0.788	0.828
ϕ_*	0.378	0.900	0.755
ϕ^+	**0.001**	**0.001**	0.065
ϕ^+_*	0.098	0.098	**0.009**
λ	0.198	0.637	0.618

DE			
H_0	T1	T2	T3
ϕ	0.130	0.719	0.148
ϕ_*	–	–	–
ϕ^+	0.113	0.113	0.492
ϕ^+_*	–	–	–
λ	0.188	0.551	0.321

ES			
H_0	T1	T2	T3
ϕ	**0.011**	0.460	0.296
ϕ_*	0.472	0.904	0.152
ϕ^+	**0.016**	**0.016**	0.195
ϕ^+_*	0.431	0.431	0.472
λ	0.592	0.847	0.688

FR			
H_0	T1	T2	T3
ϕ	**0.026**	0.112	0.252
ϕ_*	0.615	0.739	0.799
ϕ^+	**0.016**	**0.016**	0.085
ϕ^+_*	0.513	0.513	0.674
λ	0.278	0.667	0.458

IE			
H_0	T1	T2	T3
ϕ	0.412	0.635	0.913
ϕ_*	0.354	0.693	0.833
ϕ^+	0.362	0.362	0.954
ϕ^+_*	0.415	0.415	0.835
λ	0.122	0.542	0.427

IT			
H_0	T1	T2	T3
ϕ	**0.012**	0.199	0.040
ϕ_*	0.283	0.790	**0.017**
ϕ^+	**0.033**	**0.033**	**0.033**
ϕ^+_*	0.263	0.263	**0.036**
λ	0.503	0.777	0.429

NL			
H_0	T1	T2	T3
ϕ	**0.005**	0.095	0.158
ϕ_*	0.419	0.678	0.652
ϕ^+	**0.002**	**0.002**	0.078
ϕ^+_*	**0.028**	**0.028**	**0.020**
λ	0.148	0.494	0.074

PT			
H_0	T1	T2	T3
ϕ	0.494	0.786	0.885
ϕ_*	0.452	0.774	0.595
ϕ^+	0.403	0.403	0.809
ϕ^+_*	0.279	0.279	0.602
λ	0.132	0.588	0.356

Table A.11: **Unit-root tests (II)**

BE			
H_0	3|4	2|3	1|2
$(y, \tilde{y})$	11.66	8.193	0.161
(y, x)	1.208	1.171	11.06
$(\tilde{y}, x)$	0.281	0.026	1.109
$(y, \Delta x)$	1.158	1.405	**5.833**
$(s, \Delta x)$	12.08	8.824	**3.910**

DE			
H_0	3|4	2|3	1|2
$(y, \tilde{y})$	1.247	**0.971**	5.317
(y, x)	4.433	0.231	8.957
$(\tilde{y}, x)$	0.099	**12.55**	35.10
$(y, \Delta x)$	2.374	4.633	**17.55**
$(s, \Delta x)$	–	–	–

ES			
H_0	3|4	2|3	1|2
$(y, \tilde{y})$	5.851	**8.409**	**14.88**
(y, x)	–	–	–
$(\tilde{y}, x)$	–	–	–
$(y, \Delta x)$	5.228	2.554	**0.974**
$(s, \Delta x)$	**4.772**	**7.739**	2.944

FR			
H_0	3|4	2|3	1|2
$(y, \tilde{y})$	3.978	5.491	**20.39**
(y, x)	0.846	**3.343**	**41.74**
$(\tilde{y}, x)$	0.003	**0.097**	1.163
$(y, \Delta x)$	0.865	1.865	**4.908**
$(s, \Delta x)$	12.84	11.68	6.932

IE			
H_0	3|4	2|3	1|2
$(y, \tilde{y})$	**1.526**	**0.266**	2.783
(y, x)	–	–	–
$(\tilde{y}, x)$	–	–	–
$(y, \Delta x)$	7.688	0.197	**5.558**
$(s, \Delta x)$	8.395	0.704	**4.852**

IT			
H_0	3|4	2|3	1|2
$(y, \tilde{y})$	**3.719**	**22.64**	41.24
(y, x)	**4.805**	1.671	22.47
$(\tilde{y}, x)$	0.880	**0.023**	4.989
$(y, \Delta x)$	3.854	**4.359**	0.598
$(s, \Delta x)$	**6.987**	**6.769**	1.895

NL			
H_0	3|4	2|3	1|2
$(y, \tilde{y})$	1.119	4.719	3.662
(y, x)	0.876	0.031	19.10
$(\tilde{y}, x)$	0.260	0.286	2.229
$(y, \Delta x)$	4.376	6.808	**9.265**
$(s, \Delta x)$	8.705	27.60	9.994

PT			
H_0	3|4	2|3	1|2
$(y, \tilde{y})$	31.25	**22.90**	51.58
(y, x)	–	–	–
$(\tilde{y}, x)$	–	–	–
$(y, \Delta x)$	6.206	**9.493**	**9.950**
$(s, \Delta x)$	6.886	**29.35**	**5.151**

Table A.12: **Vector lag selection (I)**: Values of the test statistic (1.5.3): the critical value at 5% level is $\boldsymbol{\chi}^2(n^2) = \boldsymbol{\chi}^2(4) = 9.488$, see table A.3.

BE			
H_0	3|4	2|3	1|2
(y, ϕ)	**1.532**	**1.889**	4.129
(s, ϕ^*)	**12.17**	**9.502**	20.86
(λ, s)	1.026	5.210	**2.570**
(λ, ϕ^+)	1.622	**5.547**	**11.81**
(λ, ϕ^+_*)	**20.65**	**11.17**	**9.369**

DE			
H_0	3|4	2|3	1|2
(y, ϕ)	**5.092**	8.153	11.63
(s, ϕ^*)	–	–	–
(λ, s)	–	–	–
(λ, ϕ^+)	6.923	4.629	**18.66**
(λ, ϕ^+_*)	–	–	–

ES			
H_0	3|4	2|3	1|2
(y, ϕ)	5.253	2.982	**9.780**
(s, ϕ^*)	**4.631**	**9.589**	**8.277**
(λ, s)	**1.104**	**9.800**	**13.97**
(λ, ϕ^+)	7.469	19.19	**34.56**
(λ, ϕ^+_*)	8.140	32.59	9.677

FR			
H_0	3|4	2|3	1|2
(y, ϕ)	0.851	1.661	**4.910**
(s, ϕ^*)	**21.03**	**0.992**	27.90
(λ, s)	1.704	8.292	**0.262**
(λ, ϕ^+)	19.72	9.855	**8.051**
(λ, ϕ^+_*)	**17.94**	**4.610**	**8.760**

IE			
H_0	3|4	2|3	1|2
(y, ϕ)	**8.909**	**0.461**	8.422
(s, ϕ^*)	8.630	1.797	**9.651**
(λ, s)	0.276	0.432	7.522
(λ, ϕ^+)	0.944	0.162	**17.21**
(λ, ϕ^+_*)	2.185	1.306	**22.63**

IT			
H_0	3|4	2|3	1|2
(y, ϕ)	5.254	9.417	1.125
(s, ϕ^*)	**19.40**	**12.33**	8.303
(λ, s)	**0.703**	**6.820**	**15.53**
(λ, ϕ^+)	3.515	2.406	**12.46**
(λ, ϕ^+_*)	**21.07**	27.43	**2.592**

NL			
H_0	3|4	2|3	1|2
(y, ϕ)	5.460	6.031	**4.935**
(s, ϕ^*)	**15.75**	**20.03**	**19.51**
(λ, s)	0.381	0.779	**9.849**
(λ, ϕ^+)	10.28	20.345	**9.503**
(λ, ϕ^+_*)	**13.59**	**5.239**	**20.62**

PT			
H_0	3|4	2|3	1|2
(y, ϕ)	**6.128**	**4.976**	5.072
(s, ϕ^*)	**4.757**	**5.422**	31.26
(λ, s)	0.038	1.538	5.841
(λ, ϕ^+)	**7.179**	**6.048**	4.131
(λ, ϕ^+_*)	**5.043**	**0.902**	1.229

Table A.13: **Vector lag selection (II)**

	BE	DE	ES	FR	IE	IT	NL	PT
$(y, \tilde{y})$	2	2	3	2	2	3	2	3
(y, x)	2	2	–	2	–	3	2	–
$(\tilde{y}, x)$	2	2	–	2	–	2	2	–
$(y, \Delta x)$	2	2	2	2	2	2	2	3
$(s, \Delta x)$	2	–	2	2	2	2	2	3

	BE	DE	ES	FR	IE	IT	NL	PT
(y, ϕ)	2	3	2	2	2	4	2	2
(s, ϕ_*)	4	–	2	4	2	4	4	2
(λ, s)	2	–	3	2	2	2	2	2
(λ, ϕ^+)	3	2	2	2	2	2	2	2
(λ, ϕ^+_*)	4	–	2	4	2	4	4	2

Table A.14: **VAR lag selection resume**: estimated number of lags out of the procedure described in 1.5.1, before eventual corrections due to residual analysis.

	BE	DE	ES	FR	IE	IT	NL	PT
$(y, \tilde{y})$	0.106	**0.034**	0.218	0.358	0.287	0.358	**0.076**	0.332
(y, x)	0.308	**0.072**	–	0.119	–	0.470	0.523	–
$(\tilde{y}, x)$	**0.001**	0.834	–	0.226	–	**0.007**	**0.001**	–
$(y, \Delta x)$	0.330	0.557	0.426	0.340	0.340	0.267	0.280	0.192
$(s, \Delta x)$	0.474	–	0.447	0.382	0.279	0.603	0.158	0.229

	BE	DE	ES	FR	IE	IT	NL	PT
(y, ϕ)	0.401	0.444	0.423	0.341	0.326	0.145	0.299	0.192
(s, ϕ^*)	0.755	–	0.803	0.426	0.494	0.601	0.683	0.109
(λ, s)	0.210	–	**0.083**	0.279	**0.054**	0.931	0.561	**0.001**
(λ, ϕ^+)	0.571	0.535	**0.064**	0.266	0.170	0.576	0.173	**0.016**
(λ, ϕ^+_*)	**0.037**	–	**0.011**	0.644	0.253	**0.049**	0.223	**0.009**

Table A.15: **Cointegration test statistic**: p-values for the cointegrating test statistics for integrated VAR(p) models, with p as in tables 3.2 and 3.7, respectively.

	$\hat{\hat{C}}_1(1,1)$	$\hat{\hat{C}}_1(1,2)$	$\hat{\hat{C}}_1(2,1)$	$\hat{\hat{C}}_1(2,2)$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\hat{\rho}$
BE	0.246	0.355	0.021	**0.741**	0.004	0.000	0.016
	(0.136)	(0.891)	(0.013)	(0.088)			
DE*	**0.318**	**1.268**	0.011	**0.779**	0.003	0.000	0.299
	(0.004)	(0.052)	(0.246)	(0.033)			
ES	0.161	1.179	**0.049**	**0.431**	0.004	0.001	0.237
	(0.140)	(1.118)	(0.016)	(0.132)			
FR	0.180	-0.381	0.055	**0.701**	0.003	0.001	0.393
	(0.140)	(0.466)	(0.028)	(0.093)			
IE	**0.458**	0.656	-0.046	**-0.334**	0.007	0.003	0.202
	(0.121)	(0.357)	(0.044)	(0.131)			
IT	0.157	-1.417	**0.051**	**0.776**	0.004	0.001	0.338
	(0.144)	(1.111)	(0.020)	(0.151)			
NL*	**0.413**	**0.496**	0.052	**0.699**	0.003	0.001	0.086
	(0.009)	(0.053)	(0.124)	(0.032)			
PT	**0.610**	-0.354	0.012	**0.783**	0.010	0.001	0.400
	(0.125)	(1.399)	(0.007)	(0.082)			

Table A.16: **Short-run dynamics:** $(y, \tilde{y})$: estimated values of $\hat{C}_1$, relative standard errors (in brackets) and estimated correlation matrix of residuals, see table A.6.

	$\hat{\hat{C}}_1(1,1)$	$\hat{\hat{C}}_1(1,2)$	$\hat{\hat{C}}_1(2,1)$	$\hat{\hat{C}}_1(2,2)$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\hat{\rho}$
BE	0.210	-0.051	-0.266	**0.867**	0.004	0.004	-0.197
	(0.140)	(0.051)	(0.174)	(0.063)			
DE	0.279	-0.048	0.042	**0.841**	0.003	0.004	0.078
	(0.018)	(0.048)	(0.021)	(0.026)			
ES	–	–	–	–	–	–	–
	–	–	–	–			
FR	0.136	-0.273	-0.130	**1.336**	0.003	0.003	-0.168
	(0.132)	(0.143)	(0.110)	(0.117)			
IE	–	–	–	–	–	–	–
	–	–	–	–			
IT	0.058	-0.158	0.026	**1.158**	0.004	0.004	-0.251
	(0.144)	(0.144)	(0.137)	(0.137)			
NL	0.208	-0.009	-0.294	**0.782**	0.003	0.007	-0.339
	(0.142)	(0.040)	(0.299)	(0.085)			
PT	–	–	–	–	–	–	–
	–	–	–	–			

Table A.17: **Short-run dynamics:** (y, x):

	$\hat{\hat{C}}_1(1,1)$	$\hat{\hat{C}}_1(1,2)$	$\hat{\hat{C}}_1(2,1)$	$\hat{\hat{C}}_1(2,2)$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\hat{\rho}$
BE*	**0.350**	**0.841**	0.024	**0.871**	0.000	0.004	0.109
	(0.003)	(0.052)	(0.239)	(0.023)			
DE	1.561	**-0.476**	**0.506**	-0.033	0.000	0.004	0.298
	(1.021)	(0.129)	(0.136)	(0.017)			
ES	–	–	–	–	–	–	–
	–	–	–	–			
FR	0.236	-0.479	0.049	**0.748**	0.001	0.003	-0.331
	(0.146)	(0.744)	(0.154)	(0.030)			
IE	–	–	–	–	–	–	–
	–	–	–	–			
IT*	**0.170**	**0.566**	0.058	**0.831**	0.000	0.004	-0.357
	(0.004)	(0.053)	(0.221)	(0.029)			
NL*	**0.413**	**0.496**	0.052	**0.699**	0.000	0.007	0.116
	(0.099)	(0.053)	(0.124)	(0.031)			
PT	–	–	–	–	–	–	–
	–	–	–	–			

Table A.18: **Short-run dynamics:** $(\tilde{y}, x)$

	$\hat{\hat{C}}_1(1,1)$	$\hat{\hat{C}}_1(1,2)$	$\hat{\hat{C}}_1(2,1)$	$\hat{\hat{C}}_1(2,2)$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\hat{\rho}$
BE	0.240	-0.048	-0.114	0.153	0.004	0.005	-0.148
	(0.138)	(0.111)	(0.176)	(0.141)			
DE	0.217	0.114	-0.033	0.077	0.003	0.005	0.090
	(0.135)	(0.105)	(0.181)	(0.140)			
ES	0.088	**-0.354**	0.120	**0.645**	0.004	0.004	0.091
	(0.138)	(0.166)	(0.121)	(0.146)			
FR	0.134	-0.176	-0.071	**0.465**	0.003	0.003	-0.041
	(0.137)	(0.145)	(0.118)	(0.124)			
IE	**0.459**	0.067	-0.118	-0.039	0.007	0.009	0.163
	(0.125)	(0.116)	(0.150)	(0.140)			
IT	0.063	-0.144	0.065	0.233	0.004	0.004	-0.205
	(0.143)	(0.143)	(0.140)	(0.140)			
NL	0.223	0.009	0.034	0.143	0.010	0.006	0.466
	(0.142)	(0.066)	(0.311)	(0.145)			
PT	**0.781**	-0.146	-0.140	0.110	0.009	0.006	0.582
	(0.156)	(0.242)	(0.101)	(0.157)			

Table A.19: **Short-run dynamics:** $(y, \Delta x)$

	$\hat{\hat{C}}_1(1,1)$	$\hat{\hat{C}}_1(1,2)$	$\hat{\hat{C}}_1(2,1)$	$\hat{\hat{C}}_1(2,2)$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\hat{\rho}$
BE	0.291	-0.116	0.151	0.149	0.003	0.005	0.264
	(0.138)	(0.081)	(0.244)	(0.143)			
DE	–	–	–	–	–	–	–
	–	–	–	–			
ES	0.196	-0.240	**0.284**	**0.661**	0.004	0.003	0.114
	(0.138)	(0.160)	(0.120)	(0.138)			
FR	0.214	0.003	0.263	**0.465**	0.001	0.003	0.055
	(0.137)	(0.057)	(0.296)	(0.123)			
IE	**0.460**	0.045	-0.078	-0.039	0.007	0.009	0.200
	(0.125)	(0.103)	(0.171)	(0.141)			
IT	0.121	-0.254	0.195	0.238	0.004	0.004	-0.048
	(0.136)	(0.137)	(0.133)	(0.134)			
NL	0.021	0.042	1.235	0.090	0.001	0.007	0.322
	(0.140)	(0.017)	(1.186)	(0.145)			
PT	**0.850**	-0.134	-0.123	0.106	0.009	0.006	0.611
	(0.154)	(0.233)	(0.105)	(0.159)			

Table A.20: **Short-run dynamics:** $(s, \Delta x)$

	$\hat{\hat{C}}_1(1,1)$	$\hat{\hat{C}}_1(1,2)$	$\hat{\hat{C}}_1(2,1)$	$\hat{\hat{C}}_1(2,2)$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\hat{\rho}$
BE	0.251	-0.050	-0.094	0.135	0.004	0.005	0.043
	(0.135)	(0.112)	(0.170)	(0.140)			
DE	**0.319**	-0.185	-0.273	-0.004	0.003	0.005	0.012
	(0.125)	(0.098)	(0.223)	(0.140)			
ES	0.139	-0.359	0.040	**0.599**	0.004	0.004	0.258
	(0.135)	(0.187)	(0.123)	(0.151)			
FR	0.163	-0.170	-0.135	**0.455**	0.003	0.003	0.159
	(0.136)	(0.144)	(0.118)	(0.124)			
IE	**0.446**	0.056	0.023	-0.050	0.007	0.009	0.314
	(0.129)	(0.118)	(0.160)	(0.146)			
IT	0.119	-0.094	0.040	0.214	0.004	0.004	0.086
	(0.131)	(0.134)	(0.131)	(0.134)			
NL	0.221	0.009	0.042	0.139	0.003	0.007	-0.238
	(0.140)	(0.067)	(0.299)	(0.143)			
PT	**0.844**	-0.195	0.050	0.051	0.010	0.008	0.695
	(0.190)	(0.244)	(0.152)	(0.195)			

Table A.21: **Short-run dynamics:** (y, ϕ)

	$\hat{\hat{C}}_1(1,1)$	$\hat{\hat{C}}_1(1,2)$	$\hat{\hat{C}}_1(2,1)$	$\hat{\hat{C}}_1(2,2)$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\hat{\rho}$
BE	**0.401**	0.094	0.058	-0.160	0.002	0.006	0.290
	(0.132)	(0.055)	(0.352)	(0.146)			
DE	–	–	–	–	–	–	–
	–	–	–	–			
ES	**0.266**	0.061	0.022	0.091	0.003	0.039	0.002
	(0.133)	(0.087)	(0.211)	(0.138)			
FR	**0.354**	**0.108**	-0.432	-0.227	0.001	0.005	-0.253
	(0.107)	(0.026)	(0.588)	(0.141)			
IE	**0.390**	0.108	0.165	0.116	0.006	0.010	0.492
	(0.144)	(0.103)	(0.223)	(0.160)			
IT	**0.351**	0.045	-0.074	-0.071	0.004	0.006	0.109
	(0.119)	(0.082)	(0.198)	(0.136)			
NL	0.195	**0.036**	-0.031	0.083	0.001	0.008	0.214
	(0.135)	(0.012)	(1.575)	(0.138)			
PT	**0.981**	-0.183	0.212	-0.083	0.010	0.009	0.621
	(0.128)	(0.128)	(0.173)	(0.173)			

Table A.22: **Short-run dynamics:** (s, ϕ_*)

	$\hat{\hat{C}}_1(1,1)$	$\hat{\hat{C}}_1(1,2)$	$\hat{\hat{C}}_1(2,1)$	$\hat{\hat{C}}_1(2,2)$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\hat{\rho}$
BE	-0.073	0.039	-0.013	0.014	0.002	0.027	0.864
	(0.227)	(0.023)	(2.389)	(0.240)			
DE	–	–	–	–	–	–	–
	–	–	–	–			
ES*	**0.356**	0.028	0.218	-0.012	0.004	0.011	0.725
	(0.149)	(0.044)	(0.245)	0.090			
FR	-0.191	**0.038**	-1.004	0.076	0.001	0.038	0.497
	(0.188)	(0.013)	(2.944)	(0.210)			
IE*	0.121	**0.073**	-0.552	**0.423**	0.006	0.015	0.446
	(0.185)	(0.030)	(1.258)	(0.205)			
IT	0.120	0.000	7.601	-0.404	0.004	0.041	0.194
	(0.237)	(0.013)	(4.021)	(0.229)			
NL	-0.032	0.005	0.305	-0.110	0.001	0.041	0.687
	(0.180)	(0.004)	(8.785)	(0.183)			
PT*	**0.881**	-0.019	**4.104**	-0.180	0.006	0.026	0.476
	(0.178)	(0.043)	(0.637)	(0.153)			

Table A.23: **Short-run dynamics:** (λ, s)

	$\hat{\hat{C}}_1(1,1)$	$\hat{\hat{C}}_1(1,2)$	$\hat{\hat{C}}_1(2,1)$	$\hat{\hat{C}}_1(2,2)$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\hat{\rho}$
BE	0.058	**-0.629**	0.053	0.193	0.006	0.003	0.386
	(0.152)	(0.303)	(0.074)	(0.148)			
DE	-0.092	0.047	-0.099	0.070	0.003	0.002	-0.095
	(0.140)	(0.177)	(0.110)	(0.139)			
ES*	**-0.120**	**-0.187**	0.058	0.003	0.010	0.003	0.141
	(0.032)	(0.010)	(0.016)	(0.171)			
FR	0.012	-0.159	0.003	**0.567**	0.004	0.002	0.090
	(0.139)	(0.202)	(0.080)	(0.115)			
IE	**0.317**	-0.044	0.174	-0.122	0.012	0.008	0.142
	(0.136)	(0.199)	(0.095)	(0.139)			
IT	-0.223	**-1.136**	**0.121**	**0.300**	0.010	0.003	-0.038
	(0.132)	(0.427)	(0.039)	(0.128)			
NL	-0.068	**0.152**	0.293	0.060	0.003	0.006	0.270
	(0.128)	(0.065)	(0.265)	(0.134)			
PT*	**-0.440**	**0.370**	0.003	-0.120	0.014	0.008	0.590
	(0.046)	(0.027)	(0.073)	(0.057)			

Table A.24: **Short-run dynamics:** (λ, ϕ^{+})

	$\hat{\hat{C}}_1(1,1)$	$\hat{\hat{C}}_1(1,2)$	$\hat{\hat{C}}_1(2,1)$	$\hat{\hat{C}}_1(2,2)$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\hat{\rho}$
BE*	**0.280**	**-0.237**	0.046	**-0.137**	0.006	0.005	0.490
	(0.034)	(0.057)	(0.059)	(0.048)			
DE	–	–	–	–	–	–	–
	–	–	–	–			
ES*	**-0.223**	-0.020	0.205	-0.185	0.010	0.005	0.415
	(0.046)	(0.023)	(0.290)	(0.100)			
FR	0.155	-0.054	-0.288	-0.222	0.010	0.003	-0.012
	(0.113)	(0.104)	(0.154)	(0.141)			
IE	**0.310**	0.003	0.233	0.071	0.004	0.002	0.112
	(0.144)	(0.181)	(0.113)	(0.143)			
IT*	**0.090**	**-0.472**	-0.161	**0.251**	0.010	0.005	0.339
	(0.033)	(0.049)	(0.095)	(0.054)			
NL	0.074	**0.166**	-0.474	0.232	0.010	0.003	-0.138
	(0.130)	(0.046)	(0.400)	(0.141)			
PT*	0.323	-0.077	-0.027	**0.341**	0.014	0.009	0.772
	(0.038)	(0.064)	(0.077)	(0.049)			

Table A.25: **Short-run dynamics:** (λ, ϕ^{+}_{*})

			$h=1$		$h=2$		$h=3$		$h=4$	
		r p	$\boldsymbol{\lambda}_{LM}$	$\boldsymbol{\lambda}_{W}$	$\boldsymbol{\lambda}_{LM}$	$\boldsymbol{\lambda}_{W}$	$\boldsymbol{\lambda}_{LM}$	$\boldsymbol{\lambda}_{W}$	$\boldsymbol{\lambda}_{LM}$	$\boldsymbol{\lambda}_{W}$
BE	$(y, \tilde{y})$	0 2	1.000	–	1.000	0.914	1.000	0.836	1.000	0.781
	(y, x)	0 2	1.000	–	1.000	0.819	1.000	0.837	1.000	0.606
	$(\tilde{y}, x)$	1 2	1.000	–	1.000	–	1.000	–	1.000	0.059
	$(y, \Delta x)$	0 2	1.000	–	1.000	0.857	1.000	0.757	1.000	0.282
	$(s, \Delta x)$	0 2	1.000	–	1.000	0.501	1.000	0.405	1.000	0.153
DE	$(y, \tilde{y})$	1 2	1.000	–	1.000	–	1.000	–	1.000	0.397
	(y, x)	0 2	1.000	–	1.000	0.113	0.933	0.094	0.988	**0.003**
	$(\tilde{y}, x)$	0 2	1.000	–	0.999	0.615	0.662	0.512	0.991	**0.048**
	$(y, \Delta x)$	0 2	1.000	–	0.991	0.999	0.367	0.949	0.874	0.773
	$(s, \Delta x)$	– –	–	–	–	–	–	–	–	–
ES	$(y, \tilde{y})$	1 3	1.000	–	1.000	–	1.000	–	1.000	0.475
	(y, x)	– –	–	–	–	–	–	–	–	–
	$(\tilde{y}, x)$	– –	–	–	–	–	–	–	–	–
	$(y, \Delta x)$	0 2	1.000	–	1.000	0.922	1.000	0.917	1.000	0.306
	$(s, \Delta x)$	0 2	1.000	–	1.000	–	1.000	0.882	1.000	0.660
FR	$(y, \tilde{y})$	0 2	1.000	–	1.000	0.999	1.000	0.922	1.000	0.361
	(y, x)	0 2	1.000	–	1.000	**0.008**	1.000	0.018	1.000	**0.035**
	$(\tilde{y}, x)$	0 2	1.000	–	1.000	**0.002**	1.000	**0.002**	1.000	**0.000**
	$(y, \Delta x)$	0 2	1.000	–	1.000	0.885	1.000	0.893	1.000	0.564
	$(s, \Delta x)$	0 2	1.000	–	1.000	–	1.000	0.470	1.000	0.246
IE	$(y, \tilde{y})$	0 2	1.000	–	1.000	1.000	1.000	1.000	1.000	0.660
	(y, x)	– –	–	–	–	–	–	–	–	–
	$(\tilde{y}, x)$	– –	–	–	–	–	–	–	–	–
	$(y, \Delta x)$	0 2	1.000	–	1.000	0.999	1.000	0.967	1.000	0.742
	$(s, \Delta x)$	0 2	1.000	–	1.000	0.996	1.000	0.912	1.000	0.433
IT	$(y, \tilde{y})$	0 3	1.000	–	1.000	–	1.000	0.871	1.000	0.286
	(y, x)	0 3	1.000	–	1.000	0.835	1.000	0.550	1.000	0.430
	$(\tilde{y}, x)$	1 2	1.000	–	1.000	–	1.000	–	1.000	0.155
	$(y, \Delta x)$	0 2	1.000	–	1.000	–	1.000	0.929	1.000	0.663
	$(s, \Delta x)$	0 2	1.000	–	1.000	–	1.000	0.833	1.000	0.823
NL	$(y, \tilde{y})$	1 2	1.000	–	1.000	–	1.000	–	1.000	0.938
	(y, x)	0 2	1.000	–	1.000	0.876	1.000	0.928	1.000	0.311
	$(\tilde{y}, x)$	1 2	1.000	–	1.000	–	1.000	–	1.000	0.059
	$(y, \Delta x)$	0 2	1.000	–	1.000	0.982	1.000	0.998	1.000	0.454
	$(s, \Delta x)$	0 2	1.000	–	1.000	–	1.000	1.000	1.000	0.948
PT	$(y, \tilde{y})$	0 3	1.000	–	1.000	–	1.000	0.732	1.000	0.477
	(y, x)	– –	–	–	–	–	–	–	–	–
	$(\tilde{y}, x)$	– –	–	–	–	–	–	–	–	–
	$(y, \Delta x)$	0 2	1.000	–	1.000	–	1.000	–	1.000	0.105
	$(s, \Delta x)$	0 2	1.000	–	1.000	–	1.000	–	1.000	–

Table A.26: **Analysis of residuals (I)**: p-values out of test statistics (1.5.16) and (1.5.21), with $h = 1 \ldots 4$; **bold** numbers shows values under 5%. Residuals are estimated out of models with number of lags reported in tables A.14.

				$h=1$		$h=2$		$h=3$		$h=4$	
		r	p	$\boldsymbol{\lambda}_{LM}$	$\boldsymbol{\lambda}_{W}$	$\boldsymbol{\lambda}_{LM}$	$\boldsymbol{\lambda}_{W}$	$\boldsymbol{\lambda}_{LM}$	$\boldsymbol{\lambda}_{W}$	$\boldsymbol{\lambda}_{LM}$	$\boldsymbol{\lambda}_{W}$
BE	(y,ϕ)	0	2	1.000	–	1.000	0.855	1.000	0.752	1.000	0.269
	(s,ϕ^*)	0	4	1.000	0.993	1.000	0.465	1.000	0.052	1.000	**0.008**
	(λ,s)	0	2	1.000	–	1.000	–	0.997	–	1.000	0.785
	(λ,ϕ^+)	0	3	1.000	–	1.000	0.996	1.000	0.984	1.000	0.840
	(λ,ϕ^+_*)	0	4	1.000	–	0.998	0.442	0.258	**0.091**	0.894	**0.011**
DE	(y,ϕ)	0	3	1.000	–	1.000	–	1.000	0.938	1.000	0.397
	(s,ϕ^*)	–	–	–	–	–	–	–	–	–	–
	(λ,s)	–	–	–	–	–	–	–	–	–	–
	(λ,ϕ^+)	0	2	1.000	–	0.991	0.999	0.367	0.949	0.874	0.773
	(λ,ϕ^+_*)	–	–	–	–	–	–	–	–	–	–
ES	(y,ϕ)	0	2	1.000	–	1.000	0.925	1.000	0.913	1.000	0.335
	(s,ϕ^*)	0	2	1.000	–	1.000	0.113	0.933	**0.094**	0.988	**0.003**
	(λ,s)	1	3	1.000	–	0.999	–	0.662	0.512	0.991	**0.048**
	(λ,ϕ^+)	1	2	1.000	–	1.000	–	1.000	–	0.995	0.854
	(λ,ϕ^+_*)	1	2	1.000	–	1.000	–	1.000	–	0.960	0.177
FR	(y,ϕ)	0	2	1.000	–	1.000	0.881	1.000	0.904	1.000	0.593
	(s,ϕ^*)	0	4	1.000	–	1.000	–	1.000	–	1.000	**0.004**
	(λ,s)	0	2	1.000	–	1.000	0.953	1.000	0.185	1.000	0.100
	(λ,ϕ^+)	0	2	1.000	–	1.000	0.994	1.000	0.990	1.000	0.709
	(λ,ϕ^+_*)	0	4	0.999	–	**0.000**	–	–	–	–	0.001
IE	(y,ϕ)	0	2	1.000	–	1.000	0.998	1.000	0.941	1.000	0.745
	(s,ϕ^*)	0	2	1.000	–	1.000	1.000	1.000	1.000	1.000	0.565
	(λ,s)	1	2	1.000	–	1.000	–	1.000	–	1.000	0.985
	(λ,ϕ^+)	0	2	1.000	–	0.732	0.866	–	0.854	–	0.317
	(λ,ϕ^+_*)	0	2	1.000	–	1.000	0.949	0.922	0.942	0.162	0.167
IT	(y,ϕ)	0	4	1.000	–	1.000	1.000	1.000	0.894	1.000	0.629
	(s,ϕ^*)	0	4	1.000	–	1.000	–	1.000	–	1.000	**0.006**
	(λ,s)	0	2	1.000	–	0.996	0.254	1.000	0.365	1.000	0.118
	(λ,ϕ^+)	0	2	1.000	–	1.000	1.000	0.968	0.964	**0.000**	0.825
	(λ,ϕ^+_*)	1	4	1.000	–	1.000	–	**0.007**	–	**0.004**	**0.019**
NL	(y,ϕ)	0	2	1.000	–	1.000	0.982	1.000	0.998	1.000	0.448
	(s,ϕ^*)	0	4	1.000	–	1.000	–	1.000	–	1.000	0.363
	(λ,s)	0	2	1.000	–	1.000	0.878	0.971	0.965	1.000	0.978
	(λ,ϕ^+)	0	2	1.000	–	1.000	0.072	**0.045**	0.102	–	**0.005**
	(λ,ϕ^+_*)	0	4	1.000	–	0.993	–	**0.000**	–	**0.000**	**0.000**
PT	(y,ϕ)	0	2	1.000	–	1.000	0.992	1.000	0.866	1.000	**0.022**
	(s,ϕ^*)	0	2	1.000	–	0.997	**0.021**	0.997	**0.01**	1.000	**0.000**
	(λ,s)	1	2	0.365	–	0.498	–	**0.000**	–	**0.000**	**0.000**
	(λ,ϕ^+)	0	2	1.000	–	1.000	0.969	0.981	0.702	–	**0.000**
	(λ,ϕ^+_*)	1	2	1.000	–	1.000	–	1.000	–	1.000	**0.000**

Table A.27: **Analysis of residuals (II)**

B Econometric Toolbox

B.1 Basic definitions

Definition B.1.1. *A n-dimensional process U_t is called a* white noise *if U_t, U_s are independent, identically distributed n-dimensional random variables for which $\mathbb{E}(U_t) = 0$, $\mathbb{E}(U_t U_t^{'}) = \Sigma_U$ non-singular, and fourth moments are uniformly bounded in t; the process is called a* gaussian *white noise if $U_t \sim \mathcal{N}(0, \Sigma_U)$, any t.*

Static parametric analysis provide the data generating process to be modeled so that for any t:

$$Y_t = f(\Theta, Y_{t-1} \dots Y_0) + U_t$$

where f corresponds to the choice of the model, which in turn depends on a time-constant parameter matrix Θ, and U the random innovations.
The parameter set Θ is retrieved using the available data set which is considered to be generated by the dgp; given observations $\{Y_0 \dots Y_T\}$an *estimator*:

$$\hat{\Theta} = \hat{\Theta}(Y_0 \dots Y_T) = \hat{\Theta}_T$$

is a function of the observations which approximates the true parameters Θ depending both on the model chosen and on the fitting technique which is used; the estimator is itself a random variable which depends on the trajectory $Y_1 \dots Y_t$ as well as on the set of initial conditions Y_0.
Here, linear models are considered, hence the techniques will be custom *ordinary least squares* (OLS), *estimated generalized least squares* (EGLS) and *maximum likelihood* (ML). We do not present the ideas underlying each of them, and skip basic tools in random sequences different type of convergence, for which we refer to [56] and [57]; it is worth to present an additional definition:

Definition B.1.2. *We say that an estimator $\hat{\Theta}_T = \hat{\Theta}$ is* consistent *if*

$$\hat{\Theta}_T \xrightarrow{p} \Theta$$

that is, as the sample size T increases, the estimator Θ_T constructed with a T-sized observations set converges in probability *to the true parameter Θ.*

B.2 Unrestricted Least Squares Estimation

The estimators presented in this section concern processes with cointegration rank $r > 0$; in case of $\mathcal{I}(1)$ processes with $r = 0$, the model in difference is an ordinary regression model involving stationary variable and estimators are standard (see [57]). Consider a n-dimensional vector process $Y \in \mathcal{I}(1)$ in its VEC($p-1$) form (1.4.3), with unrestricted deterministic term: $D_t = D_0 + tD_1 \in \mathbb{R}^n$.
This corresponds to model H in table 1.2 or, equivalently, to choose $d^{ex} = [1\ t]'$ and $d^{co} = [\ ]$ in table 1.1, so that:

$$\Delta Y_t = [C_0 \ \ \mathbf{C}^+] \begin{bmatrix} Y_{t-1} \\ \Delta Y_{t-1} \\ \vdots \\ \Delta Y_{t-p+1} \\ d_t^{ex} \end{bmatrix} + U_t \quad , \tag{B.2.1}$$

$\mathbf{C}^+ = [C_1 \dots C_{p-1}\ D_0\ D_1] \in \mathbb{R}^{n \times J}$, $J = n(p-1) + 2$ and U_t is white noise; given a set of observations:

$$\mathcal{Y} = \{Y_{-p+1} \dots Y_0\} \cup \{Y_1 \dots Y_T\} \tag{B.2.2}$$

where the first subset contains p initial conditions[78] we can stack the data as:

$$\Delta \mathbf{Y} = [C_0 \ \ \mathbf{C}^+] \begin{bmatrix} \mathbf{Y}^+ \\ \Delta \mathbf{Z}^+ \end{bmatrix} + \mathbf{U} \tag{B.2.3}$$

see equation (1.4.8). The explicit form of the estimator comes straightforward out of the *vectorization* of equation (B.2.3); given any matrix $\Theta \in \mathbb{R}^{m \times n}$, we define:

$$\text{vec}(\Theta) = \begin{bmatrix} \Theta_1' \\ \vdots \\ \Theta_m' \end{bmatrix} \in \mathbb{R}^{mn \times 1}$$

the column vector obtained by stacking the transposes of the rows of Θ.
An exhaustive summary of the basic properties of this operator can be found, for example, in [78] (p. 663); these properties allow to rewrite (B.2.3):

$$\text{vec}(\Delta \mathbf{Y}) = \left(\begin{bmatrix} \mathbf{Y}^{+'} & \Delta \mathbf{Z}^{+'} \end{bmatrix} \otimes I_n \right) \begin{bmatrix} \text{vec}(C_0) \\ \text{vec}(\mathbf{C}^+) \end{bmatrix} + \text{vec}(\mathbf{U}) \tag{B.2.4}$$

which is now a standard LS model in $\mathbb{R}^{nT \times 1}$ with parameters vector $[\text{vec}(C_0)'\ \ \text{vec}(\mathbf{C}^+)']'$ in $\mathbb{R}^{n(n+J) \times 1}$ to be estimated.

[78] This implies that the effective sample size is T, which simplifies notations.

The estimators are then retrieved with little algebra: formal derivation can be found, for example, in [78] (section 7.2, pp. 286-305): here only the main results servicing consistent estimation are reported.

Proposition B.2.1. *Let $Y \in \mathcal{I}(1)$ be a n-dimensional $VAR(p)$ process in the form* (B.2.1) *that is assumed to generate the observations set $\mathcal{Y}$ as defined in* (B.2.2). *Let $\mathbf{Y}^+$, $\Delta\mathbf{Y}$, $\Delta\mathbf{Z}^+$ be as defined in* (B.2.3)*; then:*

$$[\hat{C}_0 \quad \hat{\mathbf{C}}^+] = \left[\Delta\mathbf{Y}\mathbf{Y}^{+'} \quad \Delta\mathbf{Y}\Delta\mathbf{Z}^{+'}\right] \begin{bmatrix} \mathbf{Y}^+\mathbf{Y}^{+'} & \mathbf{Y}^+\Delta\mathbf{Z}^{+'} \\ \Delta\mathbf{Z}^+\mathbf{Y}^{+'} & \Delta\mathbf{Z}^+\Delta\mathbf{Z}^{+'} \end{bmatrix}^{-1} \tag{B.2.5}$$

is the LS estimator[79] *of $[C_0 \quad \mathbf{C}^+]$; furthermore:*

$$\hat{\Sigma}_U := (T - np)^{-1} \cdot \hat{U}\hat{U}' \tag{B.2.6}$$

is a consistent estimator of the white noise covariance matrix, being

$$\hat{U} := \Delta\mathbf{Y} - \hat{C}_0\mathbf{Y}^+ - \hat{\mathbf{C}}^+\Delta\mathbf{Z}^+$$

the matrix of estimated residuals.

Observation B.2.1. *If the parameters of process $Y = X + D^*$ are estimated keeping X in level form* (1.4.1)*, then consistent estimators of parameters matrices Γ_i can be retrieved from $[\hat{C}_0 \quad \hat{\mathbf{C}}^+] = [\hat{C}_0 \ \hat{C}_1 \ \dots \ \hat{C}_{p-1} \ \hat{D}_0 \ \hat{D}_1]$ by reversing the algebraic procedure used to infer equation* (1.4.2)*, namely:*

$$\begin{aligned} \hat{\Gamma}_p &= -\hat{C}_{p-1} \\ \hat{\Gamma}_i &= \hat{C}_i - \hat{C}_{i-1} \quad , \quad 2 \le i \le p-1 \\ \hat{\Gamma}_1 &= I_n + \hat{C}_1 - \hat{C}_0 \end{aligned} \tag{B.2.7}$$

([78], Remark 4, p. 289)

Observation B.2.2. *The LS estimator $\hat{\mathbf{\Gamma}} = [\hat{\Gamma}_1 \dots \hat{\Gamma}_p]$ has a gaussian asymptotic distribution with a* singular *covariance matrix whenever the cointegration rank $r > 0$ ([78], Corollary 7.1.1, pp. 289-90).*

[79] Given any parameter matrix $\Theta \in \mathbb{R}^{m \times n}$, by writing that $\hat{\Theta}$ is an estimator for Θ we implicitly suggest that $\text{vec}(\hat{\Theta}) \in \mathbb{R}^{mn \times 1}$ is an estimator of $\text{vec}(\Theta) \in \mathbb{R}^{mn \times 1}$.

B.3 Maximum Likelihood (ML) estimation

Let $\hat{H} \in \mathcal{H}$ be an identified model among those in table 1.2, and assume the cointegration rank to be $0 < r < n$; equation (B.2.3) can be rewritten as:

$$\Delta\mathbf{Y} = [A \cdot B^{+'} \quad \mathbf{C}^{+}] \begin{bmatrix} \mathbf{Y}^{+} \\ \Delta\mathbf{Z}^{+} \end{bmatrix} + \mathbf{U} \ , \tag{B.3.1}$$

for any t, with $B^{+} = [B \quad D_0^{\parallel} \quad D_1^{\parallel}]$; the equation is specifically designed for each $\hat{H}$ according to the form of d_t^{ex} and d_t^{co} presented in table 1.1. Next proposition collects the results in [74] (chapter 6) while notations are those of [78] (p. 294).

Proposition B.3.1. *Let $Y \in \mathcal{I}(1)$ be a n-dimensional identified VAR(p) model $\hat{H}$, with cointegration rank $r : 0 < r < n$, so that, see* (1.4.6) *and* (B.3.1)*, the dgp is:*

$$\Delta Y_t = AB^{+'} \cdot Y_{t-1}^{+} + \mathbf{C}^{+} \cdot \Delta Z_{t-1}^{+} + U_t \tag{B.3.2}$$

where $A, B \in \mathbb{R}^{n\times r}$ with rank r, $B^{+} = [B \quad D_0^{\parallel} \quad D_1^{\parallel}]$, $Y_t^{+} = [Y_t \quad d_t^{co}]'$ and $\Delta Z_t^{+} = [\Delta Y_t \ldots \Delta Y_{t-p+1} \ d_t^{ex}]'$, and the correspondent d^{co}, d^{ex} are determined according to 1.1 as in section 1.5.2. Stack vectors into matrices as in (1.4.8)*, and define:*

$$\begin{aligned} M &:= I_T - \Delta\mathbf{Z}^{+'}(\Delta\mathbf{Z}^{+}\Delta\mathbf{Z}^{+'})^{-1}\Delta\mathbf{Z}^{+} \\ M_0 &:= \Delta\mathbf{Y}M \\ M_1 &:= \mathbf{Y}^{+}M \\ M_{ij} &:= M_i M_j' \qquad i,j \in \{0,1\} \end{aligned} \tag{B.3.3}$$

and let $\lambda_1 \geq \cdots \geq \lambda_n$ be the eigenvalues of $M_{11}^{-1/2} M_{10} M_{00}^{-1} M_{01} M_{11}^{-1/2}$, and $\{\mathbf{v}_1 \ldots \mathbf{v}_n\}$ the corresponding eigenvectors; the ML estimators of B, A and $\mathbf{C}^{+}$ are defined as:

$$\begin{aligned} \tilde{B}^{+'} = [\tilde{B} \ \tilde{D}_0^{\parallel} \ \tilde{D}_1^{\parallel}] &:= [\mathbf{v}_1 \ldots \mathbf{v}_r]' \cdot M_{11}^{-1/2} \\ \tilde{A} &:= M_{01}\tilde{B}^{+}(\tilde{B}^{+'} M_{11} \tilde{B}^{+})^{-1} \\ \tilde{\mathbf{C}}^{+} &:= (\Delta\mathbf{Y} - \tilde{A}\tilde{B}^{+'}\mathbf{Y}^{+})\Delta\mathbf{Z}^{+'}(\Delta\mathbf{Z}^{+'}\Delta\mathbf{Z}^{+'})^{-1} \end{aligned} \tag{B.3.4}$$

respectively, while the estimator of the innovations' covariance matrix Σ_U is:

$$\tilde{\Sigma}_U := (\Delta\mathbf{Y} - \tilde{A}\tilde{B}^{+'}\mathbf{Y}^{+} - \tilde{\mathbf{C}}^{+}\Delta\mathbf{Z}^{+})(\Delta\mathbf{Y} - \tilde{A}\tilde{B}^{+'}\mathbf{Y}^{+} - \tilde{\mathbf{C}}^{+}\Delta\mathbf{Z}^{+})'/T$$

and the maximum of the likelihood function is (omitting constant terms):

$$|M_{00}| \prod_{i=1}^{r} (1 - \lambda_i), \qquad \lambda_1 \geq \cdots \geq \lambda_r. \tag{B.3.5}$$

B.4 Normalized EGLS estimation

Assume that the VAR(p) model has cointegration rank $0 < r < n$; the decomposition of C_0 into the product AB' is not unique: given any nonsingular matrix Q and defined $A_* = AQ'$ and $B_* = BQ^{-1}$ we have $C_0 = AB' = A_* B_*'$.
However, if $\mathrm{rk}(B) = r$, then B has exactly r independent rows, and this allows for a convenient identification: it is easy to choose Q so that a unique cointegration matrix is identified in the *normalized* form:

$$B = \begin{bmatrix} I_r \\ B_{n-r} \end{bmatrix} \tag{B.4.1}$$

with $B_{n-r} \in \mathbb{R}^{(n-r)\times r}$, so that $B^{+'}_{n-r} = [B'_{n-r}\ D_0^{\|}\ D_1^{\|}] \in \mathbb{R}^{(K-r)\times r}$ and $K = n + \dim(d^{co})$.
Following [78] (pp. 291-294), for any fixed C_0^+, $\mathbf{C}^+$ is replaced in (B.2.3) by its LS estimator:

$$\hat{\mathbf{C}}^+ = (\Delta\mathbf{Z}^+\Delta\mathbf{Z}^{+'})^{-1}\Delta\mathbf{Z}^{+'}(\Delta\mathbf{Y} - C_0^+\mathbf{Y}^+)$$

so that (B.2.3) becomes:

$$M_0 = C_0^+ M_1 + \mathbf{U} = A[I_r\ \ B^{+'}_{n-r}]M_1 + \mathbf{U} \tag{B.4.2}$$

where M_0, M_1 are defined in proposition B.3.1; assume that any parameter but B^+_{n-r} is known, and partition (B.4.2) as:

$$M_1 = \begin{bmatrix} M_1^{(1)} \\ M_1^{(2)} \end{bmatrix} \quad M_1^{(1)} \in \mathbb{R}^{r\times r}, \quad M_1^{(2)} \in \mathbb{R}^{(K-r)\times r}$$

so as to obtain the *concentrated* least squares problem:

$$M_0 - AM_1^{(1)} = AB^{+'}_{n-r}M_1^{(2)} + \mathbf{U}$$

The *Generalized Least Squares* (GLS) estimator is then ([78], Chap. 7, p. 292):

$$\hat{B}^{+'}_{n-r} = (A'\Sigma_U^{-1}A)^{-1}A'\Sigma_U^{-1}(M_0 - AM_1^{(1)})M_1^{(2)'}(M_1^{(2)}M_1^{(2)'})^{-1}$$

is defined as the unique minimizer of the *generalized sum of squares* $\mathbf{U}'(I_T \otimes \Sigma_\mathbf{U}^{-1})\mathbf{U}$.
Notice that the same estimator would have been retrieved without *concentrating out* $\mathbf{C}^+$ first, as this latter has been replaced by its optimal value given *any* value of C_0^+.
The assumption that parameters other than B are known is however very restrictive: an *Estimated* GLS is thus constructed so as to drop this assumption.

Namely, given *consistent* estimators $\hat{A}$ and $\hat{\Sigma}_U$, we define the EGLS estimator

$$\hat{\hat{B}}^{+'}_{n-r} = (\hat{A}'\hat{\Sigma}_U^{-1}\hat{A})^{-1}\hat{A}'\hat{\Sigma}_U^{-1}(M_0 - \hat{A}M_1^{(1)})M_1^{(2)'}(M_1^{(2)}M_1^{(2)'})^{-1} \tag{B.4.3}$$

Next section explores asymptotic results for the parameters distribution that can be used to set up standard confidence intervals on the estimates.

B.5 Asymptotic properties of the estimators

The following proposition ([78], Proposition 7.1, p. 287) collects the results on asymptotic behaviour of LS estimators.

Proposition B.5.1. *Given an identified* VECM($p-1$) *model* $\hat{H} \in \mathcal{H}$, *the estimators* (B.2.5) *and* (B.2.6)*, as defined in Proposition B.2.1 and adapted according to equation* (B.3.1)*, are consistent estimators of the correspondent parameters; moreover:*

$$\sqrt{T}\,\mathrm{vec}\left([\hat{C}_0^+ \quad \hat{\mathbf{C}}^+] - [C_0^+ \quad \mathbf{C}^+]\right) \xrightarrow{d} \mathcal{N}(0, \Sigma^{co}) \tag{B.5.1}$$

and

$$\hat{\Sigma}^{co} := T \begin{bmatrix} \mathbf{Y}_+\mathbf{Y}'_+ & \mathbf{Y}_+\Delta\mathbf{Z}'_+ \\ \Delta\mathbf{Z}'_+\mathbf{Y}'_+ & \Delta\mathbf{Z}_+\Delta\mathbf{Z}'_+ \end{bmatrix}^{-1} \otimes \hat{\Sigma}_U \tag{B.5.2}$$

is a consistent estimator of Σ^{co}.

Observation B.5.1. *If* $Y \in \mathcal{I}(1)$*, which implies* $r < n$*, then the matrix* Σ^{co} *is singular. Hence, standard Wald tests on multiple parameter restrictions may not have the usual asymptotic* $\boldsymbol{\chi}^2(\cdot)$ *distribution. ([78], Remark 1, p. 287)*

Observation B.5.2. *The ML estimators* $\tilde{C}_0^+ = \tilde{A}\tilde{B}^+$ *and* $\tilde{\mathbf{C}}^+$ *as defined in* (B.3.4) *converge to the same asymptotic distribution* (B.5.1) *as the LS estimator.*
([78], Proposition 7.4, p. 296)

The estimator (B.5.1) globally converges at a rate $\sqrt{T}$, but distinguishing short from long-term dynamics reveals several interesting asymptotic properties.
In this sense, a central result is given in Ahn and Reisnel [4] and reported in [78] (Lemma 7.1, p. 271) in a slightly different form; we limit ourselves here to report the results which are strictly necessary to our analysis; the first result concerns the EGLS estimator $\hat{\hat{B}}^+_{n-r}$ and is provided in [78] (Proposition 7.2, p. 292).

Proposition B.5.2. *Let $\hat{\hat{B}}^+_{n-r}$ be the EGLS estimator of the normalized cointegration matrix*[80]*, as defined in* (B.4.3)*. We have:*

$$\text{vec}\left[(\hat{\hat{B}}^{+'}_{n-r} - B^{+'}_{n-r})\left(M_1^{(2)} M_1^{(2)'}\right)^{1/2}\right] \xrightarrow{d} \mathcal{N}\left(0, I_{(K-r)} \otimes \left(A'\Sigma_U^{-1}A\right)^{-1}\right) \qquad \text{(B.5.3)}$$

Moreover, the estimator converges at a rate T and hence is superconsistent.

The distribution of $\hat{\hat{B}}^{+'}_{n-r}$ is non-standard (see [4]), hence the result reported in proposition B.5.2 is very important for practical purposes, eventually allowing also for multiple restrictions Wald tests, as it will be clear in what follows.

Proposition B.5.3. *Assume that the normalized cointegration matrix B^+_{n-r} is known, so that the model in the form* (B.3.2) *can be rewritten as:*

$$\Delta\mathbf{Y} = A \cdot \mathbf{Y}_{++} + \mathbf{C}^+ \cdot \Delta\mathbf{Z}^+ + \mathbf{U} \qquad \text{(B.5.4)}$$

where $\mathbf{Y}_{++} = [I_r \ B^{+'}_{n-r}] \cdot \mathbf{Y}_+ = B^+ \cdot \mathbf{Y}_+$; the least squares estimator

$$[\hat{A} \quad \hat{\mathbf{C}}^+] = \left[\Delta\mathbf{Y}\mathbf{Y}'_{++} \quad \Delta\mathbf{Y}\Delta\mathbf{Z}'_+\right] \begin{bmatrix} \mathbf{Y}_{++}\mathbf{Y}'_{++} & \mathbf{Y}_{++}\Delta\mathbf{Z}'_+ \\ \Delta\mathbf{Z}'_+\mathbf{Y}'_{++} & \Delta\mathbf{Z}_+\Delta\mathbf{Z}'_+ \end{bmatrix}^{-1}$$

converges asymptotically to a non-singular gaussian distribution:

$$\sqrt{T}\,\text{vec}\left([\hat{A} \quad \hat{\mathbf{C}}^+] - [A \quad \mathbf{C}^+]\right) \xrightarrow{d} \mathcal{N}(0, \Sigma_{A,\mathbf{C}}) \qquad \text{(B.5.5)}$$

where

$$\hat{\Sigma}_{A,\mathbf{C}} = T \begin{bmatrix} \mathbf{Y}_{++}\mathbf{Y}'_{++} & \mathbf{Y}_{++}\Delta\mathbf{Z}'_+ \\ \Delta\mathbf{Z}'_+\mathbf{Y}'_{++} & \Delta\mathbf{Z}_+\Delta\mathbf{Z}'_+ \end{bmatrix}^{-1} \otimes \hat{\Sigma}_U \qquad \text{(B.5.6)}$$

is a consistent estimator of $\Sigma_{A,\mathbf{C}}$.

Note that the estimators of the parameters are consistent, converging asymptotically at a rate $\sqrt{T}$.
The fact that the EGLS estimator $\hat{\hat{B}}^+$ is superconsistent allows for two stage procedures: in a first step long-run relationship among variables are determined and then the short-run parameters are estimated by ordinary LS.

[80] We are abusing of terminology as $B^+_{n-r} = [B_{n-r} \ D_0^{\parallel} \ D_0^{\parallel}]$ while the normalized cointegration matrix is actually B_{n-r} ; notice also that $M_1^{(2)} = M_1^{(2)}(T)$.

Namely, consider the LS estimator of an identified model (B.5.1); if we further identfy also the cointegration matrix in the normalized form (B.4.1), then we can write

$$[\hat{C}_0^+ \quad \hat{\mathbf{C}}^+] = [\hat{A} \quad \hat{A}\hat{B}_{n-r}^+ \quad \hat{\mathbf{C}}^+] \tag{B.5.7}$$

so that the first r columns of $\hat{C}_0^+$ are a consistent estimator of A.
This estimator coupled with the estimator of the white noise covariance matrix $\hat{\Sigma}_U$ can be used to construct the EGLS estimator (B.4.3); superconsistency implies that the estimator we would obtain by substituting this latter to the true B_{n-r}^+ yields the same asymptotic distribution (B.5.5) we would obtain if the cointegration basis was known. This fact is formalized in the following result ([78], Remark 3, pp. 293-94).

Corollary B.5.1. *Let* $\hat{\hat{B}}^+$ *be the EGLS estimator* (B.4.3) *and consider:*

$$[\hat{\hat{A}} \quad \hat{\hat{\mathbf{C}}}^+] = \begin{bmatrix} \Delta\mathbf{Y}\hat{\mathbf{Y}}'_{++} & \Delta\mathbf{Y}\Delta\mathbf{Z}'_+ \end{bmatrix} \begin{bmatrix} \hat{\mathbf{Y}}_{++}\hat{\mathbf{Y}}'_{++} & \hat{\mathbf{Y}}_{++}\Delta\mathbf{Z}'_+ \\ \Delta\mathbf{Z}'_+\hat{\mathbf{Y}}'_{++} & \Delta\mathbf{Z}_+\Delta\mathbf{Z}'_+ \end{bmatrix}^{-1} \tag{B.5.8}$$

where $\hat{\mathbf{Y}}_{++} := [I_r \ \hat{\hat{B}}_{n-r}^{+'}] \cdot \mathbf{Y}_+$. *Then:*

$$\sqrt{T}\,\mathrm{vec}\left([\hat{\hat{A}} \quad \hat{\hat{\mathbf{C}}}^+] - [A \quad \mathbf{C}^+]\right) \xrightarrow{d} \mathcal{N}(0, \Sigma_{A,\mathbf{C}})$$

that is the estimator of the short-run parameters converges to the same distribution which would be obtained knowing the true B^+.

C Financial Toolbox

C.1 Nelson-Siegel method

Given a *zero-coupon bond (zcb)* quoted $P(t,T)$, the *yield-to-maturity*:

$$y(t,T) = -\frac{\log(P(t,T))}{T-t} \qquad \forall \;\; T > t \; . \tag{C.1.1}$$

is defined for any T as the log-return of the obligations over its lifetime. The yield curve $\{y(t,T) : T \geq t\}$ at time t is defined as the image of the function

$$T \mapsto y(t,T)$$

mapping $T \geq t$ to the yield of a zero-coupon-bond (*zcb*) with maturity T.
Since the market does not offer a zcb for any maturity T, the yield curve can be retrieved out of the most liquid fraction of the government bond market through a nonlinear interpolation method due to Nelson and Siegel [89].
The method requires, as inputs, current bonds prices together with their fixed coupons and payment dates, including reimbursment of face value at maturity.
It is worth to stress that *no default probabilities are taken into account as computing yields*: common practice is to assume that the obligations market prices embeds any risk premium attributable to eventual future credit events.
For a generic country A, we consider at time T the *dirty*[81] prices $\{\wp_j^A(t)\}_{j=1}^{J_A}$, each with maturity T_j^A, coupon payment dates $\{T_{kj}^A\}$ and fixed coupon $\{C_j^A\}$; consider the country to be fixed and drop index A for the sake of notation simplicity.
Let $P(t,T)$ denote the unknown *zcb* price at time t with maturity T; if the market is arbitrage free, then

$$\wp_j(t) = \sum_{k=1}^{K_j} C_j \cdot P(t,T_{kj}) + \wp^* \cdot P(t,T_{K_j,j}) \tag{C.1.2}$$

for any obligation $j \in \{1 \dots J\}$, where $\wp^*$ is the bond face value[82], which is assumed to be same for each obligation. Observe that left member of (C.1.2) is due to the bond issuer at inception while the right one is contingent on credit risk: future coupons will not be paid, for example, in case of default. In order to imply the yield curve, we interpolate the *zero-coupon curve* $\{P(t,T)\}$ and then use equation (2.2.3).

[81] The prices including the accrued payment on the bond's next coupon; the payment is zero only on any bond's coupon payment date.

[82] Again, for the sake of notation's simplicity, we simply write C_j but consider it as the annual coupon multiplied by the year fraction between two consecutive coupon dates.

Nelson-Siegel interpolation is based on the projection of zero-coupon price on a three-functions basis, which allows us to describe that curve in terms of a small parameters set; interpolation knots is the set of all coupon payments dates $\mathcal{T}$ for any of the obligations j, defined as:

$$\mathcal{T} = \{T_{kj} \;\; | \;\; k = 1 \dots \;\; K_j \;, \;\; j = 1 \dots J\}$$

The choice of the functions to be used is due to statistical analysis [89], which evidenced that most of the observed shapes in the zero-coupon curve were efficiently reproduced by setting:

$$\begin{aligned} y(t,T) =& \beta_{0t} + \beta_{1t} \left[\frac{\beta_{3t}}{T-t} \exp\left(-\frac{T-t}{\beta_{3t}} \right) \right] \\ &+ \beta_{2t} \left[\frac{\beta_{3t}}{T-t} \exp\left(-\frac{T-t}{\beta_{3t}} \right) - \exp\left(-\frac{T-t}{\beta_{3t}} \right) \right] \\ =& \beta_{0t} + \beta_{1t} \cdot g_1 \left(\frac{T-t}{\beta_{3t}} \right) + \beta_{2t} \cdot g_2 \left(\frac{T-t}{\beta_{3t}} \right) = y(\theta_t, T-t) \end{aligned} \tag{C.1.3}$$

for all $T \geq t$, where the parameter vector $\theta_t = (\beta_{0t}, \beta_{1t}, \beta_{2t}, \beta_{3t})$ has to be *dynamically* (for each t) implied from market quotes.
Equation (2.2.3) combined with (C.1.3) allows to write $P(t,T)$ as a function of θ_t given any $T \geq 0$. Hence, for each of the obligations j, the right member of (C.1.2) can be written as $p_j(\theta_t)$; the vector

$$\boldsymbol{\wp}(\theta_t) = \big(\wp_1(\theta_t) \dots \wp_J(\theta_t) \big)'$$

collects all model-implied prices of the J relevant obligations at time t, variating according to the parameter θ_t.
Define the vector $\boldsymbol{\wp}_t = (\wp_1(t) \dots \wp_J(t))$ the vector collecting the *spot* dirty prices of the J obligation at time t: in order to imply θ_t, the square distance between $\boldsymbol{\wp}(\theta_t)$ and $\boldsymbol{\wp}$ is computed with a correction embedded in a weighting matrix $W = W(\theta_t)$.
Here, following the guidances in [1], a diagonal weighting matrix

$$W(\theta_t) = \begin{pmatrix} w_1(\theta_t) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & w_J(\theta_t) \end{pmatrix} \tag{C.1.4}$$

is chosen, where each element $W_{jj}(\theta_t) = w_j(\theta_t)$ is defined by:

$$w_j(\theta_t) = \frac{\frac{1}{D_j(\theta_t)}}{\sum_{j=1}^{J} \frac{1}{D_l(\theta_t)}} \quad j = 1 \dots J \tag{C.1.5}$$

and is in turn a function of the *duration* D_j of the j-th obligation

$$D_j := \sum_{k=1}^{K_j} \left[P(t, T_{kj}) \cdot \left(\sum_{k=1}^{K_j} P(t, T_{kj}) \right)^{-1} \cdot (T_{kj} - t) \right] = D_j(\theta_t) \tag{C.1.6}$$

The best-fitting θ_t is defined as:

$$\theta_t^* = \text{argmin} \ \ (\wp_t - \wp(\theta_t))' \cdot W(\theta_t) \cdot (\wp_t - \wp(\theta_t)) \tag{C.1.7}$$

In this way, given a generic time grid $\{t_0, t_1 \dots t_n\}$, it is possible to imply the correspondent set of optimal parameters $(\theta_{t_0}^* \dots \theta_{t_n}^*)$, that in turn allow through (C.1.3) to retrieve the zero-coupon curve:

$$\left[\{y(t_0, T)\}_{T \geq t_0} \ , \ \dots \ \ \{y(t_n, T)\}_{T \geq t_n} \right] \tag{C.1.8}$$

for any $t_i \in \{t_0 \dots t_n\}$.

C.1.1 The CDS-bearing yield curve

This section extends the previously explained techniques to the case of CDS-bearing yield curve, that is, it permits to imply a term structure for synthethic hedging positions obtained by combining each bond with a CDS covering its whole lifetime. The method is essentialy the same as previous section, but it is worth to underline two shortcomings of hedging strategies which are immediate consequences of market standardization:

- Standard CDS contracts trade with notional $\wp^*$ equal to the face value of the underlying obligation; the CDS-bearing bond purchaser is not covered for the portion of dirty price exceeding this value.
  This is rarely a problem for zero coupon bonds, but longer-dated bonds with coupons exceeding those of more recently issued obligations of several basis points could be severely overpriced with respect to $\wp^*$.

- CDS are traded with standard maturities and standard scheduled payments; for any trade date t, the choice of the CDS is limited to a set of ten maturities covering a quarter plus thirty years at most[83].
  The choice was to hedge by selecting for each obligation the first available CDS maturity after bond's final payment date: if the residual life of the obligation exceeds the longest CDS maturity available, the 30 year contract is selected.

Notice again that such simplifications are not driven by computational needs but are instead direct consequence of market standardization: as soon as CDS contracts lost their specifically-tailored nature, investors will face such issues, and is obliged to enter into these *imperfect* hedges.
At time t, the relevant CDS-maturity for the j-th relevant obligations priced $\wp_j(t)$ is thus selected as:

$$\bar{T}_{H_j} = \min\{\bar{T}_h | \bar{T}_h \geq T_{K_j j}\}, \quad j = 1 \dots J$$

where $\{\bar{T}_h\}$ are the set of CDS standard payment dates which do not depend on the obligation; define:

$$\bar{\mathcal{T}} = \{\bar{T}_{hj} \ \mid \ h = 1 \dots H_j \ \ , \ \ j = 1 \dots J\}$$

the set of standard CDS dates for the whole set of relevant obligations, and assume that for any $T \in \mathcal{T} \cup \bar{\mathcal{T}}$ in the *complete* timegrid there exist a *zero-coupon CDS-bearing bond* priced $\bar{P}(t, T)$, and so a *CDS-bearing yield* $\bar{y}(t, T)$ out of (2.2.3).
Notice that, in a non-standard CDS market, this assumption is as restrictive as assuming the existence of a zcb for any $T \in \mathcal{T}$. Assume that for each t there exists a dynamic parameter vector $\bar{\theta}_t = (\bar{\beta}_{0t}, \bar{\beta}_{1t}, \bar{\beta}_{2t}, \bar{\beta}_{3t})$ such that $\bar{y}(t, T) = \bar{y}(\bar{\theta}_t, T - t)$ as in (C.1.3), *mutatis mutandis*. Current spot price of any hedged portfolio is:

$$\wp_j(t) + \wp^* \cdot U_j(t) - \bar{S} \cdot AP01(t) = \wp_j(t) + u_j(t) \quad j \in \{1 \dots J\}$$

where $u_j(t)$ is the *clean upfront*, obtaind by reducing $U_j(t)$ from the riskless accrued premium (see section 2.1.6). It is possible to use this new term structure $\{\bar{P}(t, T)\}$ as a credit risk free discount factor and combine it with the no-arbitrage principle to obtain the discounted future cash flow of each hedged obligation.

[83] If for example we choose a 30-year CDS on March 20th 2010, protection will last on June 20th 2040, thirty years later than first coupon payment, corresponding approximately to 30.25 years.

Namely, for any $j \in \{1 \dots J\}$ and t, the equality:

$$\wp_j(t) + u_j(t) = C_j \sum_{k=1}^{K_j} \bar{P}(t, T_{kj}) + \wp^* \cdot \bar{P}(t, T_{K_j}) - \bar{S} \cdot \sum_{h=1}^{H_j} \bar{P}(t, \bar{T}_{hj}) \qquad \text{(C.1.9)}$$

where $\bar{S}$ is CDS *yearly* standard spread.[84] Notice that, similarly to bond market, the left term of (C.1.9) is paid spot while the other is contingent on future credit events.
Proceeding as in previous section, we define $\bar{W}(\bar{\theta}_t)$ as in (C.1.4), with diagonal elements constructed as in (C.1.5) using (C.1.6) and $\bar{P}(t, T)$ replacing $P(t, T)$.
Setting:

$$\bar{\boldsymbol{\wp}}_t = (\wp_1(t) + u_1(t), \dots \wp_n(t) + u_n(t))$$

and $\bar{\boldsymbol{\wp}}(\bar{\theta}_t) = (\bar{\wp}_1(\bar{\theta}_t) \dots \bar{\wp}_J(\bar{\theta}_t))$ with

$$\bar{\wp}_j(\bar{\theta}_t) = C_j \sum_{k=1}^{K_j} \bar{P}(t, T_{kj}) + \wp^* \cdot \bar{P}(t, T_{jK_j}) - \bar{S} \cdot \sum_{h=1}^{H_j} \bar{P}(t, \bar{T}_{hj})$$

for any $j = 1 \dots J$, the relevant $\bar{\theta}_t^*$ is defined as in (C.1.7):

$$\bar{\theta}_t^* = \text{argmin} \;\; (\bar{\boldsymbol{\wp}}_t - \bar{\boldsymbol{\wp}}(\bar{\theta}_t))' \cdot \bar{W}(\bar{\theta}_t) \cdot (\bar{\boldsymbol{\wp}}_t - \bar{\boldsymbol{\wp}}(\bar{\theta}_t)) \qquad \text{(C.1.10)}$$

for any t, which allows to retrieve a *CDS-bearing zero-coupon curve* $\{\bar{y}(t, T)\}$ on the timegrid $\mathcal{T} \cup \bar{\mathcal{T}}$.

C.2 CDS pricing and contracts migration

We present here in detail how CDS contracts are marked to the market, which allows in turn to discuss the conversion mechanism that enables to switch from any CDS quoted spread to the correspondent upfront payment of the standard contract given the quoted spread S_t at time t.
First of all, a model is necessary for discounted future cash flows contingent on credit events; particularly, the present value of CDS contracts are obviously subordinated to determining default probabilities, which must be implied through market quotes. Pricing formulae for CDS contracts are direct consequence of the model which is chosen for default probabilities: in this section CDS pricing is explored in a general framework following [22], without any specific choice of such probabilities. A detailed analysis of the peculiar assumptions on such probabilities which underlie the ISDA *CDS Standard Model* will be provided in section C.3.

[84] Again, we consider it as the yearly spread implicitely multiplied by the year fraction occurring between two consecutive payment dates.

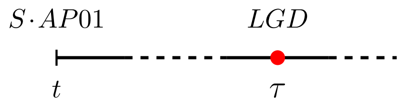


Figure C.1: Single name CDS: Cash flow of protection seller

The cash flow at trade date t of a CDS is contingent on a credit event eventually occurring at the *random time* τ before the maturity of the contract $\bar{T}_H$.
The *survival probability* at time t with respect to the *risk-neutral measure* $\mathbb{Q}$ is:

$$\mathbb{Q}(\tau \geq u) \tag{C.2.1}$$

representing the probability that τ occurs on or after time $u \geq t$.
Figure C.1 shows the complete schedule of payments for the protection seller, which takes into account the riskless accrued premium $S_t \cdot AP01(t)$ and the *Loss given default* $LGD = 1 - R$ that will be paid when any of the relevant credit event occurs. Here R is the *Recovery Rate*, which represents the post-credit event value of one unit of the defaulted obligations: at default, the protection buyer receives the obligations and repays face value, so that $N(1 - R) = N \cdot LGD$ is the amount to be reimbursed to the buyer for any obligation with face value N.
Let $\mathcal{L}_t(T)$ be the (possibly stochastic) risk-free discount factor at t for maturity T: the discounted future cash flow of the seller is the stochastic process:

$$\wp^* \big[S_t \cdot AP01(t) + LGD \cdot \mathcal{L}_t(\tau) \cdot \mathbb{1}_{[t,\bar{T}_H]}(\tau)\big] \tag{C.2.2}$$

where $\mathbb{1}_I(u)$ is the characteristic function, which is 1 if $u \in I$ and zero otherwise.
Figure C.2 shows the buyer's cash flow, which is composed by the *dirty* upfront payment U_t, the discounted sum of future CDS coupon payments and the *accrual at default*, which reimburses the seller for the time fraction that the contract would have still been protected after default.
Let $\mathcal{T} = \{\bar{T}_h\}_{h=1}^{H}$ be the set of standard dates, and define $\bar{T}_{h(\tau)}$ as the first payment date following τ, so that the discounted future cash flow of the buyer at time t is:

$$\wp^* \left[U_t + S_t \cdot \sum_{h=1}^{H} \mathcal{L}_t(\bar{T}_i) \cdot \Delta\bar{T}_i \cdot \mathbb{1}_{\tau \geq \bar{T}_i} + S_t \cdot \mathcal{L}_t(\tau) \cdot (\tau - \bar{T}_{h(\tau)-1}) \cdot \mathbb{1}_{[t,\bar{T}_H]}(\tau)\right] \tag{C.2.3}$$

Notice that spread-quoted contracts provide $U_t = 0$, while upfront-quoted contracts provide $S_t = \bar{S}$ and[85] $U_t \neq 0$, where $\bar{S}$ is standard spread.

[85] Obviously $U_t = 0$ if market quotation of the spread S_t is equal to the standard $\bar{S}$ *ex-ante*.

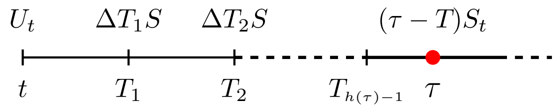


Figure C.2: Single name CDS: Cash Flow of protection Buyer

The no-arbitrage assumption implies that the *expected* discounted future cash flows under the pricing measure $\mathbb{Q}$ are equal.
Define $\wp^* \cdot \Pi_t$ the difference between (C.2.2) and (C.2.3), which corresponds to the marked-to-market value of the contract on the buyer's side.
Two assumptions are common to most credit derivatives models, namely:

- The discount factor process is independent of τ under $\mathbb{Q}$, so that for any u and I, $\mathbb{E}^{\mathbb{Q}}(\mathcal{L}_t(u)\mathbb{1}_I(\tau)) = \mathbb{E}^{\mathbb{Q}}(\mathcal{L}_t(u)) \cdot \mathbb{E}^{\mathbb{Q}}(\mathbb{1}_I(\tau)) = \bar{L}_t(u) \cdot \mathbb{Q}(\tau \in I)$.
- The recovery rate R is known, so that LGD is constant, thus non-stochastic.

Dropping one of these hypotheses (or both) is one of the addresses of current research; taking expectations with respect to $\mathbb{Q}$, we obtain the expected marked to market value:

$$\mathbb{E}^{\mathbb{Q}}(\Pi_t) = \bar{\Pi}_t = 0. \tag{C.2.4}$$

where the last equality comes from the aforementioned no-arbitrage assumption.
Define the *protection leg* $ProtLeg(t) = ProtLeg(R, \bar{L}_t(\cdot), \mathbb{Q})$ and the *premium leg* $PremLeg(t) = PremLeg(S, \bar{L}_t(\cdot), \mathbb{Q})$ the *unitary*[86] expectations under $\mathbb{Q}$ of (C.2.2) and (C.2.3) for non-standard contracts, respectively, so that (C.2.4) can be rewritten as:

$$ProtLeg(R, \bar{L}_t(\cdot), \mathbb{Q}) - PremLeg(S_t, \bar{L}_t(\cdot), \mathbb{Q}) + \big(S_t \cdot AP01(t) - U_t\big) = 0 \tag{C.2.5}$$

It is explicitly stressed that the equation depend on the selected term structure of *forward* riskless rates $\{\bar{L}_t(T)\}$ as well as on the model for the default probability $\mathbb{Q}$ in the risk-neutral world. It is custom ([22]) to rewrite the two lags as:

$$ProtLeg(R, \bar{L}_t(\cdot), \mathbb{Q}) = -LGD \cdot Prot01(\bar{L}_t(\cdot), \mathbb{Q}) \tag{C.2.6}$$

and

$$PremLeg(S_t, \bar{L}_t(\cdot), \mathbb{Q}) = S_t \cdot \big[Prem01(\bar{L}_t(\cdot), \mathbb{Q}) + ACC01(\bar{L}_t(\cdot), \mathbb{Q})\big] \tag{C.2.7}$$

[86]Divided by the face value $\wp^*$.

A little stochastic algebra allows to retrieve, under previous assumptions, explicit formulae for (C.2.8) and (C.2.7). Particularly, it can be shown that:

$$Prot01(\bar{L}_t(\cdot),\mathbb{Q}) = \int_t^{\bar{T}_h} \bar{L}_t(u) d\mathbb{Q}(\tau \geq u) \tag{C.2.8}$$

and

$$Prem01(\bar{L}_t(\cdot),\mathbb{Q}) = \sum_{h=1}^{H} \Delta\bar{T}_h \bar{L}_t(\bar{T}_h)\mathbb{Q}(\tau \geq \bar{T}_h) \tag{C.2.9}$$

$$\begin{aligned} ACC01(\bar{L}_t(\cdot),\mathbb{Q}) = & \int_t^{\bar{T}_1} \bar{L}_t(u)(u-\bar{T}_1) d\mathbb{Q}(\tau \geq u) \\ & + \sum_{h=2}^{H} \int_{\bar{T}_{h-1}}^{\bar{T}_h} \bar{L}_t(u)(u-\bar{T}_{h-1}) d\mathbb{Q}(\tau \geq u). \end{aligned} \tag{C.2.10}$$

C.2.1 Contract migrations: general framework and ISDA assumptions

Equation (C.2.4) allows to links the variables underlying the derivative into a nonlinear implicit equation:

$$\bar{\Pi}_t = \bar{\Pi}(t, \bar{\mathcal{T}}, \bar{L}_t(\cdot), \mathbb{Q}, R, S_t, U_t) = 0 \tag{C.2.11}$$

The explicit dependence on t is related to the accrued payments for both the obligation and the derivative; the timegrid $\bar{\mathcal{T}}$ is constant across standard contracts, although it distinguishes contracts with different maturities. A riskless term structure $\bar{L}_t(\cdot)$ has to be chosen so that it is independent from the entity's default; a model for $\mathbb{Q}$ is also to be chosen, while R is in principle the outcome of the post-default auction (see section 2.1.3) so it is also unknown at t.
One between the spread and upfront is instead observable in the market, and the problem is how to migrate from one to the other: it is clear that some variables must be settled, and we describe here the assumptions underlying ISDA Standard Model [84]. First of all, a standard contract is considered, with a given maturity $\bar{T}_H$; moreover, as aforementioned, it is typical to consider as fixed the recovery rate R, so that (C.2.11) can be simplified as:

$$\bar{\Pi}_t = \bar{\Pi}(\bar{L}_t(\cdot), \mathbb{Q}, S_t, U_t) = 0 \tag{C.2.12}$$

The relevant discount curve for European names $\bar{L}_t(T)$ is retrieved out of the Euribor market: spot rates are available up to one year, so a par-curve must be implied using swap rates and the absence of arbitrage; see section C.3.1 for details.

The ISDA CDS Standard model assumes that, at trade date t, the survival probabilities up to time $u \geq t$ follows an exponential distribution with *constant* parameter λ_H:

$$\mathbb{Q}(\tau \geq u) = \exp\big[-\lambda_H(u-t)\big]. \tag{C.2.13}$$

where λ_H is peculiar for maturity $\bar{T}_H$ only. This model is also known as *flat hazard curve*. The choice of modeling default intensities without either a term structure and a volatility generates theoretical controversies for which we refer to [20]; a glaring example is the unconsistency of the model across maturities. As soon as another contract with, for example, later maturity, $\bar{T}_{H*} > \bar{T}_H$ is traded on the market, then $\lambda(\bar{T}_{H*}) \neq \lambda(\bar{T}_H)$, hence the model is inconsistent on the whole time interval $[t, \bar{T}_H]$. This approach allows however to imply the whole term structure of the (risk-neutral) default probabilities using a simple parameter; more precisely, equation (C.2.12) becomes:

$$\bar{\Pi}(\lambda_t, S_t, U_t) = 0. \tag{C.2.14}$$

The conversion mechanism is based on the application of zero-search method for the nonlinear function in (C.2.14).

- From quoted spread S_t^* to upfront U_t^* plus standard coupon $\bar{S}$
  - Set $U_t = 0$ and $S_t = S_t^*$ (as observed on the market) in (C.2.14); solve $\bar{\Pi}(\lambda_t, S_t^*, 0) = 0$ and retrieve λ_t^*.
  - Set $S_t = \bar{S}$ and $\lambda_t = \lambda_t^*$ in (C.2.14); solve $\bar{\Pi}(\lambda_t^*, \bar{S}, U_t) = 0$ and retrieve the correspondent upfront U_t^*.
- From quoted upfront U_t^* plus standard coupon $\bar{S}$ to quoted spread S_t^*
  - Set $S_t = \bar{S}$ and $U_t = U_t^*$ (as observed on the market) in (C.2.14); solve $\bar{\Pi}(\lambda_t, \bar{S}, U_t^*) = 0$ and retrieve λ_t^*.
  - Set $U_t = 0$ and $\lambda_t = \lambda_t^*$ in (C.2.14); solve $\bar{\Pi}(\lambda_t^*, S_t, 0) = 0$ and retrieve the correspondent spread S_t^*.

Obviously, the flat curve assumption is totally unrealistic even within the standard market itself; current practice is to use a *piecewise constant default intensity* $\lambda_t(T)$ with knots $\bar{T} \in \bar{\mathcal{T}}$. At the moment, Markit has improved the CDS converter in a way such that this option is available; model consistency is preserved, although a stochastic component should be considered for the default intensity (see [31]).
Observe however that such framework has the advantage that it avoids the integrals to be approximated as a closed form is available under such assumptions.

C.3 ISDA-markit CDS Converter Specification

This section is rather technical and presents the exact procedure that the ISDA standard model uses to solve equation (C.2.14): section C.3.1 describes the sructure of $L_t(\cdot)$ out of the Euribor money market, while sections C.3.2 and C.3.3 deal with the computation of the derivative's payoff.

C.3.1 Interest Rate Curve

The construction of the risk-free term structure follow the guidelines in [85]: let trade date equal to t, business adjusted[87], and remove index t so as to lighten the notations. Standard CDS model uses Euribor rates to imply the discount curve $\bar{L}(\cdot)$. The conventional Euribor rates to be used are the ones locked at (business) time $t-1$, daily updated on ISDA's website, so that anyone is marking to the market the CDS contract using the same discount curve.
It is also assumed that the *spot date* for these rates is $t+2$ business adjusted: in this way, the discount factor for time t will be 1 at time $t+2$, which in turn implies that the discount factor will be slightly greater than one at time t.
Here, for the sake of simplicity, such distinctions will not be considered and we will assume that all payments are made at trade date t.
We derive the discount curve up to 1 year directly from Euribor Deposit Rates; we just have to consider the day count convention for these quotes, namely $ACT/360$. Euribor rates are available for $T = 1, 2, 3, 6, 9, 12$ months (deposit spot rate) as well as $T = 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, 20, 25$ year (swap rates). Define $\{j_k\}_{k=1}^{19}$ the succession of these time distances, so that

$$t_{j_k} = t + 2 + j_k$$

all dates *business adjusted following*.
The *daily count convention* (dcc) will be crucial in this section: define $d(t, u)$ the distance between times t and u measured with ACT/360 dcc, and set:

$$\Delta_{j_k} = d(t_{j_{k-1}}, t_{j_k}) \tag{C.3.1}$$

This sequence of dates defines a non-homogeneous timegrid

$$\mathcal{T}_1 = \{t_{j_k}\}_{k=0}^{19}$$

that will be the domain of the discount curve $\bar{L}(T) = \exp(-L(T)(T-t))$, where $L(T) = L(t, T)$ is the correspondent Euribor yield (as defined in chapter 3).

[87]Except for Japanese entities, no holidays are taken into account but Saturdays and Sundays

Let $\ell_t(t_{j_k})$, with $k \in 1 \dots 6$ be the conventional deposit rate at t for maturity t_{j_k}; set:

$$\bar{L}(t_{j_k}) = \frac{1}{1 + \ell(t_{j_k}) \cdot \Delta_{j_k}}.$$

A piecewise constant instantaneous forward rate is assumed, hence:

$$f(t_{j_k}) = f_{j_k} = -\frac{1}{\Delta_{j_k}} \cdot \log \frac{\bar{L}(t_{j_{k-1}})}{\bar{L}(t_{j_k})}, \quad k = 1 \dots 6 \tag{C.3.2}$$

where f_{j_k} is the forward rate for time interval $[t_{j_{k-1}}, t_{j_k})$.
In order to imply discount factors and forward rates for longer time periods, we use the Euribor Swap Rates $\ell(t_{j_k})$, $k = 7, \dots 19$.
The Swap Rate is an annual coupon which is paid every 6 months with day count convention 30/360; define $t_i = t{+}2{+}6i$, with $i = 0, 1 \dots$ months, and $\delta_i = d^{'}(t_{i-1}, t_i)$ the grid steps according to this convention.
For any maturity $I \in \{1, 2 \dots 25\}$, the par-swap relationship implies:

$$\sum_{i=1}^{I} \delta_i \ell(t_i) \bar{L}(t_i) = 1 \tag{C.3.3}$$

The discount curve is derived through an iterative process, assuming again a constant instantaneous forward rate between swap dates, that is, a log-linear interpolation of discount factors. For any $u \in (t_i, t_{i+1}]$

$$\bar{L}(u) = \exp\left(\frac{d(u, t_{i+1})}{d(t_i, t_{i+1})} f_i + \frac{d(t_i, u)}{d(t_i, t_{i+1})} f_{i+1} \right) \tag{C.3.4}$$

Under these assumption, the yield curve can be progressively bootstrapped from the market by adding one maturity at a time, use (C.3.4) to retrieve the following discount factor/forward rate and iterate the process until the whole grid is covered.[88] Iterating this process[89] leads to a complete set of forward rates $\{f_{j_k}\}$ evaluated on the grid $\mathcal{T}_1$, so that the piecewise constant rate $L(u)$ can be written as

$$L(u) = f_{j_1} \cdot \mathbb{1}_{[t_0, t_{j_1}]} + \sum_{k=1}^{19} f_{j_k} \cdot \mathbb{1}_{(t_{j_{k-1}}, t_{j_k}]}(u), \quad u \geq t$$

This leads to a closed form for the discount factor:

$$\bar{L}(u) = \exp\Bigg(- \sum_{k=1}^{19} f_{j_k} \cdot d(u \wedge t_{j_{k-1}}, u \wedge t_{j_k}) \Bigg), \quad u \geq t \tag{C.3.5}$$

with which we can compute the discount curve on any time grid.

[88] Notice that time fractions in parenthesis are computed as ACT/ACT days, which is (ACT/360)/(ACT/360)=ACT/ACT, coherently with the first branch implied with spot rates.

[89] The formula has to be rearranged when time lags between swap dates are greater than 1 year. However, piecewise constant forward rate always implies a single unknown in (C.3.3).

C.3.2 The Protection Leg

In order to ease notation, since in this section time distances are computed using ACT/365 daily count convention, we simply write the distance between time t and time $u \geq t$ as $u - t$ implicitly assuming that this distance is computed as an actual day fraction of a non-leap year. Again, the index t stands for trade date and will be dropped. Rewrite (C.2.8) as:

$$Prot01(\{f_{jk}\}, \lambda) = \int_t^{\bar{T}_H} \bar{L}(u)\lambda \exp(-\lambda(u - t_0))du.$$

The form (C.3.5) chosen for the discount curve, induces a closed form for the integral; if $\{f_{j_k}\}$ is the set of implied forward rates relevant for selected maturity T_H, the *risky discount factor* $\Lambda_{j_k} = f_{j_k} + \lambda$ can be defined so as to set:

$$\mathcal{T}_1^* = \{t_{j_k}\}_{k=0}^{H^*}, \quad H^* = H(T_H) = \text{argmin}\{j_k : t_{j_k} \geq T_H\} \tag{C.3.6}$$

Using the set of correspondent $\{\Lambda_{j_k}\}$, we compute:

$$Prot01(\{f_{j_k}\}, \lambda) = \lambda \sum_{i=1}^{N^*} \exp(-\lambda(t_{j_{k-1}} - t_0))\bar{L}(t_{j_{k-1}}) \frac{1 - \exp(-\Lambda_{j_k}(t_{j_k} - t_{j_{k-1}}))}{\Lambda_{j_k}}.$$

C.3.3 The Premium Leg

In order to compute the Premium Leg, we set

$$\bar{\mathcal{T}}_2 = \{\bar{T}_i\}_{i=0}^{H} \tag{C.3.7}$$

where $\{\bar{T}_i\}$ is the set of CDS payment dates, all business adjusted. Again, the assumption of a flat hazard curve together with a piecewise constant forward curve allows to explicitly solve the integral regardless of any approximation scheme.
In this case, we have:

$$Prem01(\{L_{j_k}\}, \lambda) = \sum_{i=1}^{h} (\bar{T}_i - \bar{T}_{i-1})\bar{L}(\bar{T}_i) \exp(-\lambda(\bar{T}_i - t_0))$$

$$\begin{aligned} ACC01(\{f_{j_k}\}(\cdot), \mathbb{Q})) = & \int_{t_0}^{\bar{T}_1} \bar{L}(u)(u - t_0) \exp(-\lambda(u - t_0))du \\ & + \sum_{i=2}^{H} \int_{\bar{T}_{i-1}}^{\bar{T}_i} P(u)(u - \bar{T}_{i-1}) \exp(-\lambda(u - t_0))du. \end{aligned}$$

In order to explicitly compute $ACC01$, we define

$$\begin{aligned}
\{t_h^1\}_{h=1}^{H_1} &= \Big(\mathcal{T}_1 \cup \bar{\mathcal{T}}_2\Big) \cap [t_0, \bar{T}_1] \\
\{t_h^i\}_{h=1}^{H_i} &= \Big(\mathcal{T}_1 \cup \bar{\mathcal{T}}_2\Big) \cap [\bar{T}_{i-1}, \bar{T}_i] \quad i \geq 2
\end{aligned}$$

so that each of the intervals between standard coupon maturities is divided into subintervals on which the forward rate is constant and equal to $f_h^i \in \{f_{j_k}\}$.
Observe that $t_1^0 = t$ while, for $i \geq 2$, $t_0^i = T_{i-1}$ and $t_{H_i}^i = T_i$; this results in:

$$\begin{aligned}
ACC01(\{f_{j_k}\}, \lambda) =& \lambda \sum_{h=1}^{H_1} \int_{t_{h-1}^i}^{t_h^1} (u - t_0) \exp(-L_h^1(u - t_{h-1}^1) - \lambda(u - t_0)) du \\
&+ \lambda \sum_{i=2}^{H} \sum_{h=1}^{H_i} \int_{t_{h-1}^i}^{t_h^i} (u - \bar{T}_{i-1}) \exp(-L_h^i(u - t_{h-1}^i) - \lambda(u - t_0)) du
\end{aligned}$$

Define the risky discount factors $\{\Lambda_h^i\} = \{f_h^i + \lambda\}$ and solve the integral, obtaining:

$$ACC01(\{f_{j_k}\}, \lambda) = \lambda \sum_{h=1}^{H_1} \frac{\bar{L}(t_{h-1}^1) \exp(-\lambda(t_{h-1}^1 - t_0))}{\Lambda_h^1} \cdot \Big(\frac{1}{\Lambda_h^1} + t_{h-1}^1 - t_0 -$$

$$\exp(-\Lambda_h^1(t_h^1 - t_{h-1}^1)) \cdot (\frac{1}{\Lambda_h^1} + t_h^1 - t_0)\Big) + \lambda \sum_{i=2}^{H} \sum_{h=1}^{H_i} \frac{\bar{L}(t_{h-1}^i) \exp(-\lambda(t_{h-1}^i - t_0))}{\Lambda_h} \times$$

$$\left(\frac{1}{\Lambda_h^i} + t_{h-1}^i - \bar{T}_{i-1} - \exp(-\Lambda_h^i(t_h^i - t_{h-1}^i)) \cdot \Big(\frac{1}{\Lambda_h^i} + t_h^i - \bar{T}_{i-1}\Big)\right)$$

where $\bar{L}(\cdot)$ is computed as in (C.3.5).
Notice that also the coupon spread refers to a year of protection computed in ACT/360 day count convention; hence, the Premium Leg is

$$PremLeg(S, \{f_{j_k}\}, \lambda) = \frac{365}{360} \cdot S \cdot \big(Prem01(\{f_{j_k}\}, \lambda) + ACC01(\{f_{j_k}\}, \lambda)\big).$$

where S could be either the standard or the conventional spread, depending on the contract.

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