University of Pisa

Department of Civil and Industrial Engineering

Master Degree in Chemical Engineering

Master Thesis

Analogies between Internal Model Control and Predictive Control algorithms

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" Volli, e volli sempre, e fortissimamente volli."

Vittorio Alfieri

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Abbreviations

SISO	**S**ingle **I**nput **S**ingle **O**utput
MIMO	**M**ultiple **I**nput **M**ultiple **O**utput
OL	**O**pen **L**oop
CL	**C**losed **L**oop
FB	**F**eed**B**ack
IMC	**I**nternal **M**odel **C**ontrol
PR	**P**hisical **R**ealizability
LQG	**L**inear **Q**uadratic **G**aussian
MPC	**M**odel **P**redictive **C**ontrol
s.t.	**s**ubject **t**o
DOB-IMC	**D**isturbance **O**bserver **B**ased **I**nternal **M**odel **C**ontrol
AE	**A**lgebraic **E**quivalent
iff	**i**f and **o**nly **i**f
FOPTD	**F**irst **O**rder **P**lant with **T**ime **D**elay
SOPTD	**S**econd **O**rder **P**lant with **T**ime **D**elay
RGA	**R**elative **G**ain **A**rray

Symbols

$\mathbb{R}$	Set of real numbers
$\mathbb{R}^n$	Euclidean space of dimension n
$\mathbb{R}^{n \times n}$	Matrix of dimension $n \times n$
A	Matrix of the states
B	Matrix of the inputs
C	Matrix of the outputs
C	Classic FB controller (in Chapter 2)
F	Stabilizing state feedback adopted in DOB-IMC
G	Model
H	Transformation matrix
J	Cost function of optimal control problem.
K	Solution of Discrete Algebraic Riccati Equation
K_f	Kalman filter
L	Luenberger observer
N	Horizon length
P	Plant
Q	IMC controller
Q_{obs}	Weighting matrix for the choice of the observer (for states)
R_{obs}	Weighting matrix for the choice of the observer (for inputs)
x	Vector of system states
u	Vector of system inputs
y	Vector of system outputs
d	Disturbance

r	Set point
e	Prediction error
$\hat{x}$	Vector of estimated states of system
$\hat{y}$	Vector of estimated outputs
$\hat{d}$	Vector of estimated disturbances of system
x_{ss}	Vector of steady-state values for states
u_{ss}	Vector of steady-state values for inputs
y_{ss}	Vector of steady-state values for outputs
Λ	Relative gain array
λ	Filter time constant
λ	Eigenvalues of matrix A (in Chapter 3)
ϵ	state estimation error
σ	Percentage of uncertainty

UNIVERSITY OF PISA

Abstract

Chemical Engineering
Department of Civil and Industrial Engineering

Master Thesis in Chemical Engineering

Analogies between Internal Model Control and Predictive Control Algorithms

by Marina POLIGNANO

Internal Model Control (IMC) is a well-known control strategy provided with simple tuning rules requiring a model in order to control a given single-input single-output plant; furthermore, it allows an easy and straightforward closed-loop analysis. However, it has some limitations. For instance, it cannot be applied to open-loop unstable plants, it does not cope easily with constraints, and disturbance rejection may be sluggish for disturbances other than output steps.

On the other hand, Model Predictive Control (MPC), that still requires the definition of a model, has no limitation from the point of view of the nature of the plant, but it does not give allows simple CL analysis.

IMC and MPC have many common features but, at the same time, they are also quite different control strategies: the goal we want to achieve in this work is to find a compromise between them that should have advantages of both control structures.

In this work a *Disturbance Observer Based Internal Model Control (DOB-IMC)* structure is proposed: it works with an augmented model, classical IMC controller design is left unchanged, while the block standing for the model has been replaced by an *observer block*, where predicted states are "filtered" through a Luenberger observer, known to deal better with dynamic disturbances rather than classic IMC deadbeat observer.

Afterwards, this structure has been extended to open-loop unstable plants through application of the *Q parametrization*, and to integrating plants as well.

The effectiveness of this control scheme has been validated through several simulations: first, different kind of Single-Input Single-Output linear systems have been tested; then, as a pratical application, the multivariable ”Shell oil fractionator” case study has been simulated with unmeasured disturbance.

To my family.

Chapter 1

Introduction

1.1 Motivation

Internal Model Control (IMC) and Model Predictive Control (MPC) are two different and well-known strategies in their field: indeed, literature is full of papers and books describing these control structures, with their own features and properties.

Moreover, literature is also full of papers talking about their affinities and diversities: in fact, at the same time there can be found several differences and analogies among them: it is common to think that MPC comes out from IMC, but it is important to identify what they commonly share and what they does not.

IMC is a solid control structure applicable to open-loop stable plants, since it gives good responses and, at the same time, it lends itself to transparent and straightforward closed-loop analysis, even in front of plant-model mismatches. One of the disadvantages of IMC is that it is not that much versatile, since, for instance, it cannot be applied to open-loop unstable plants, it does not deal well with saturated actuators, etc.

On the other side, MPC is much more versatile because it does not suffer from any restriction due to the nature of the plant, it copes very well with constraints and always gives an optimal response, since MPC control strategy minimizes a given *cost function*; however, closed-loop analysis of a system controller through Model Predictive Control is not easy and straightforward as it used to be in IMC.

In order to find ”analogies between Internal Model Control and Predictive Control Algorithms”, attention must be focused on shared features that can be helpful in finding a rearranged control structure which could be placed in the middle between these two.

Both of them requires a *model* in order to control a given plant: this is a very important aspect in this work, since a lot of attention will be focused on how this two models are built. Then, MPC requires the adoption of an *observer* for model dynamics: IMC structure does not need it, though it can be seen like a particular situation in which there is adopted a particular kind of observer.

Furthermore, *Q parametrization* has been here investigated, principally for two reasons: the first is that this parametrization gives instructions about stabilizing any controller provided with a stable transfer function, and second because IMC can be considered as a particular example of application of this parametrization; it has been very useful for the extension of the theory here devoloped for open-loop stable plants to open-loop unstable plants.

Here there will be explained the development and the features of a single-input single-output control structure which has most of elements belonging IMC, but with some innovations brought by MPC and Linear Quadratic Gaussian (LQG) control strategy.

All the implementations here have been done in dicrete-time domain, principally in state space, even if mostly of the analisys will be still explained using Laplace transform in continuous time, for a matter of clearness and simplicity.

1.2 Literature review

Finding an interface between these two control structures has become a quite popular task, so literature is provided with a lot of previous works trying to find similarities between them.

Among the huge amount of papers and books, there can be remembered Morari and Zafiriou [13] which developed an IMC version for open-loop unstable plants; Scali et al. [18, 19] developed an analytical design for Linear Quadratic Optimal Control provided with optimal ISE controller and filter; Amobrose and Heath, [2], worked on an IMC for Input-Constrained Multivariable Processes, following anti-windup instructions given by Zheng in [22]; Youla et al. in [21] and Kučera in [10] developed a parametrization of all stabilizing controllers, that, even if is not directly linked to IMC, has been one of the key point of this dissertation project.

All these precedent works have been here taken into account to develop a *Disturbance Observer Based Internal Model Control* structure.

1.3 Thesis overview

This disseration is made up of 7 chapters:

- In the current Chapter, the aim of this project has been discussed, together with a brief description of the principal features and problems of the control strategies here investigated.

  Moreover, there are also some examples of common features of the two structures and a literature review, where some of the principal references here adopted are listed.

- In Chapter 2 Internal Model Control has been analyzed in depth: first, it has been developed in continuous time through Laplace Transform, and then the entire discussion has been turned into a discrete-time context. Sensitivity functions have been furthermore implemented.

- In Chapter 3 there is a quick summary of notes about optimal control: in particular, we can first find a list of basic definitions, then there is the *optimal control problem* expressed in terms of minimization of a cost function J, and then some typical strategies of resolution of the optimal control problem are listed, such as *Receding Horizon* or H_∞ control. Finally LQG and MPC control structure have been described.

- Chapter 4 is the heart of the theory of this thesis, since proposed Disturbance Observer-Based IMC structure is here presented, provided with two different configurations: open-loop stable plants and open-loop unstable plants framework, namely an extension of the open-loop configuration to which *Q parametrization* has been applied; sensitivity functions for these "new" structures have been derived as well.

- In Chapter 5, DOB-IMC has been tested on several linear systems, each of them exhibits a specific feature; moreover, robustness has been here tested adopting the *Robust Stability* criterion suggested by [13], where possible. Plants analyzed are: first order with time delay both stable and unstable processes, a second order stable process provided with time delay and inverse response, and an integrating plant.

- In Chapter 6 we pass from theoretical applications to a more practical one: in fact, DOB-IMC has been here applied to a well-known case study, the *Shell Oil Fractionator*, developed following instructions given by [12]

- Chapter 7 is the chapter of Conclusions, when results are pointed out and summarized, giving furthermore new possible research directions.

Chapter 2

Internal Model Control

The starting-point of this dissertation project is *Internal Model Control* structure: in fact, the criterion adopted in this work is to begin looking at the IMC structure, modifying it little by little, in order to obtain a new control scheme that can be compared to the optimal ones, that are LQG and MPC (described in 3); so, it is really important to describe in a detailed way Internal Model Control, in order to well understand features and properties belonging to this control structure.

2.1 Structure

Figure 2.1 shows Internal Model Control structure: it is basically made up of

- a Controller, $Q(s)$, whose tuning rules will be widely delineated in Section 2.2.1 and 2.3.1;
- a Plant, $P(s)$;
- a Model, $G(s)$.

This control structure is basically based on the inversion of the model $G(s)$: this implies that the more $G(s)$ is close to real structure of the plant $P(s)$, the better results can be achieved, and the more robust is the output $y(s)$.

2.2 Continuous time

First, a continuous time version of IMC has been developed following instructions given by [13] ; then, the entire structure has been converted in discrete-time in Section 2.3 : so, a brief decription of both schemes has been provided.

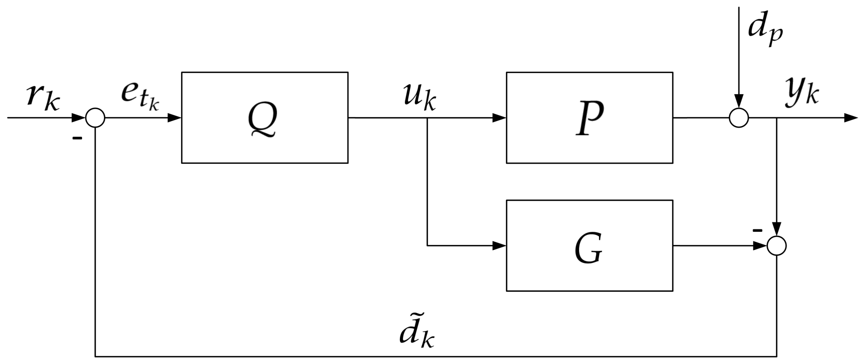


FIGURE 2.1: Internal Model Control block diagram

2.2.1 Tuning rules

IMC controller Q is based on the inversion of the model G; this implies that both plant and model need to be OL stable and, furthermore, that the model needs the be decomposed into *Minimum Phase* and *Non-Minimum Phase* part:

$$G(s) = G_{nmp}(s)G_{mp}(s) \tag{2.1}$$

The non-minimum part system of $G(s)$ includes non-invertible terms, that are terms that would make the controller $Q(s)$ unstable: these system parts are, specifically

- Time delays, $G_{td}(s) = e^{-\theta s}$;
- Zeros $\in RHP$, $G_{pz}(s) = \dfrac{-\alpha s + 1}{\alpha s + 1}$ with $\alpha > 0$;

2.2.1.1 Model decomposition

The decomposition of the model is made up of *steps*: as an example, consider a *second order orocess with time delay (SOPTD)* , namely

$$G(s) = e^{-\theta s}\frac{-\alpha s + 1}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{2.2}$$

with $\alpha > 0, \tau_1 > 0, \tau_2 > 0$

1. Factorization

$$G = \left(\frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)}\right)(-\alpha s + 1)\, e^{-\theta s}$$

2. Building of the *All-Pass filter*

$$G = \underbrace{\frac{\alpha s+1}{(\tau_1 s+1)(\tau_2 s+1)}}_{MinimumPhase} \underbrace{\frac{-\alpha s+1}{\alpha s+1}e^{-\theta s}}_{Non-MinimumPhase}$$

The decompositions for model G are

$$G_{mp} = \left(\frac{\alpha s+1}{(\tau_1 s+1)(\tau_2 s+1)}\right)$$

$$G_{nmp} = \left(\frac{-\alpha s+1}{\alpha s+1}e^{-\theta s}\right)$$

In numerical terms, let be

$$P(s) = G(s) = \frac{-3s+1}{(5s+1)\,(2s+1)}e^{-2s} \tag{2.3}$$

so, steps are

1. Factorization

$$G = \frac{1}{(5s+1)\,(2s+1)}\left((-3s+1)e^{-2s}\right)$$

2. All-Pass Filter

$$G = \underbrace{\frac{3s+1}{(5s+1)\,(2s+1)}}_{MP}\underbrace{e^{-2s}\frac{-3s+1}{3s+1}}_{NMP}$$

thus, the non-minimum and minimum part of the model are

$$G_{mp} = \frac{3s+1}{(5s+1)\,(2s+1)}$$
$$G_{nmp} = e^{-2s}\frac{-3s+1}{3s+1}$$

2.2.1.2 Controller

Let X be the Input to the control loop, that could be a Reference signal, r whether a disturbance d; the nominal controller is built as follows (please see [13] for further informations):

$$Q_{nom} = f(G, X) = (G_{mp}X_{mp})^{-1}\{G_{nmp}^{-1}X_{nmp}\}_*$$

- X_{mp} is the Minimum Phase of the input X;
- The terms in curly braces $\{\dots\}_*$ stands for the elimination of unstable terms;
- For Minimum Phase Models, i.e. $G = G_{mp}$, we have $Q_{nom} = G_{mp}^{-1} = G_m^{-1}$;
- For step input, we have $Q_{nom} = G_{mp}^{-1}$.

Remark: For the aim of this thesis, Reference Signal will always be a *step input*; thus, the algorithm adopted in this discussion is, in a simpler way

$$Q_{nom} = G_{mp}^{-1}$$

In case of inputs provided with their own dynamics, X needs to be decomposed in the same way as seen for G in Section 2.2.1.1.

Since Q_{nom} requires a model inversion, it is sometimes necessary to implement a Filter for the *Physical Realizability* of the controller , making sure that

$$n_{poles} \geq n_{zeros} \tag{2.4}$$

Moreover, together with the filter, *the only tuning parameter*, λ appears in the structure.

IMC filter structure is different, depending on the type of the plant

- type-1 filter,
$$F(s) = \frac{1}{(\lambda s + 1)^n} \tag{2.5}$$
for step-like inputs;
- type-2 filter,
$$F(s) = \frac{(n\lambda s + 1)}{(\lambda s + 1)^n} \tag{2.6}$$
for ramp-like inputs.

We deal only with step-like inputs, so filter (2.6) will be not taken into account.

λ acts as a sort of *detuning*, i.e. the higher is λ the smoother is the response, but, as a consequence, it becomes also more and more sluggish.

Final expression for the controller is

$$Q = Q_{nom}F$$

For the model (2.3), it has been assumed to be $\lambda = 10$; thus, final expression for the controller is

$$Q = \frac{(5s+1)(2s+1)}{(3s+1)(10s+1)}$$

Referring to the classical feedback control scheme (see Figure 2.2), the controller takes the form

$$C = \frac{Q}{1-PQ}$$

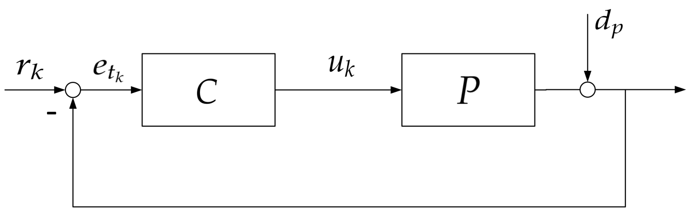


FIGURE 2.2: Classic FB control block diagram

where C is the controller of a typical FB structure.

2.2.2 Sensitivity functions

Sensitivity functions are obtained considering the two contributions to the final output y, respectively y_r for the tracking problem and y_d for the disturbance problem, according to the Superposition principle; then, expression for the output y can be so written:

$$y = \frac{PQ}{1+Q(P-G)}r + \frac{1-GQ}{1+Q(P-G)}d = y_r(s) + y_d(s) \tag{2.7}$$

So, calling S= *sensitivy* and T= *complementary sensitivity*

$$S = \frac{1-GQ}{1+Q(P-G)} \tag{2.8a}$$

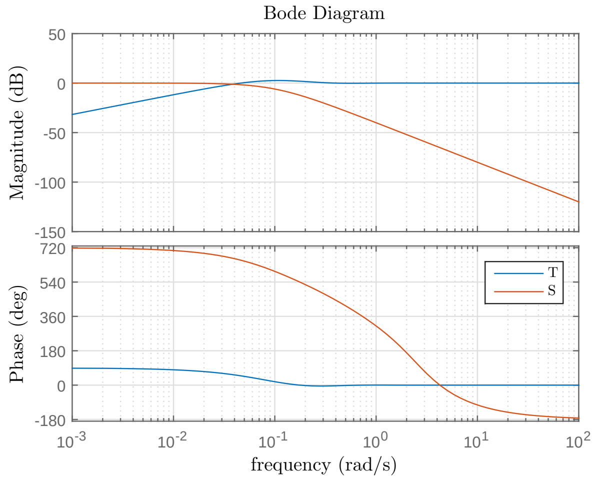


FIGURE 2.3: IMC sensitivity functions for continuous time system (2.3)

$$T = \frac{PQ}{1 + Q\,(P - G)} \tag{2.8b}$$

In the nominal case, namely $P = G$, S and T are reduced to

$$S = 1 - GQ \tag{2.9a}$$

$$T = GQ \tag{2.9b}$$

This implies linear expressions between inputs and outputs. In Figure 2.3 there are Bode diagrams for (2.3), for which T and S are described by (2.9b) and (2.9a), since we are considering the nominal case; responses for step inputs are shown Figure 2.4: we can see response for a step-like change in set point (blue line), and response for a step-like disturbance (red line).

2.3 Discrete time

IMC is commonly described in continuous time, but for the aim of this project, it needed to be turned into a *discrete-time* IMC, since MPC and LQG will be here implemented in discrete-time domain: this conversion needs some preliminary steps, such as:

1. Sample time choice, T_s: it can be taken, following an empirical rule, as

$$T_s = 0.1 \div 0.2 \min(\tau, \theta)$$

   As a rule, sampling interval *must* be multiple of constant delay; in this work, T_s has been usually taken equal to 1.

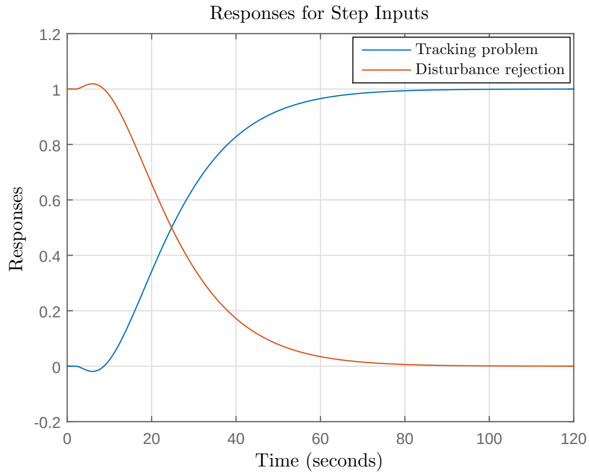


FIGURE 2.4: IMC response for a step-like input for continuous time system (2.3)

2. Conversion from Laplace transform to z-transform, through Matlab command `c2d`;
3. Passage from z-transform to state-space model, when necessary[1] (`ss` command).

   A model in state space is represented by the following equations:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k \\ y_k = Cx_k + Du_k \end{cases} \tag{2.10}$$

   with $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times p}, C \in \mathbb{R}^{p\times n}, D \in \mathbb{R}^{p\times p}$. When working with strictly proper plants (this is the case), $D = 0$; otherwise, if $n_{poles} = n_{zeros}$, $D \neq 0$. Furthermore, here a SISO system has been described, where $p = 1$.

Converting model (2.3) with a sample time of $T_s = 0.1 min\left(\tau_1, \theta\right) = 0.5s$, model in z transform becomes

$$G(z) = z^{-4}\frac{-0.072886\left(z - 1.289\right)}{\left(z - 0.9048\right)\left(z - 0.7788\right)} \tag{2.11}$$

Converting $G(z)$ in a state-space model, we obtain the following set of matrices:

[1]MPC is completely developed in state-space variables. In IMC instead, there is the first part, i.e. the tuning, in z-transform domain; once the model is decomposed as seen in Section 2.2.1.1, control loop will be solved using state-space

$$A = \begin{bmatrix} 0 & 1.135 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0.7788 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0.9048 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.5 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.1655 & -0.1458 & 0 & 0 & 0 \end{bmatrix}, \quad D = 0$$

2.3.1 Tuning

Similarly to Section 2.2.1.1, IMC controller in discrete time needs model decomposition; so, again

$$G(z) = G_{nmp}(z)G_{mp}(z)$$

where G_{nmp} contains:

- Zeros outside the unit circle, i.e. $|z| > 1$ (the corresponding for positive zeros in continuous domain);
- Zeros whose value is close to the point (-1,0); they may cause ringing, a phenomenon because of which the control output y shows ripple around the steady-state value.
- Time delays, i.e. z^{-k}.

2.3.1.1 Model decomposition

In order to explain the decomposition process steps, I assume the model to be a Second Order Process with Time Delay (SOPTD), namely

$$G(z) = z^{-k} \frac{z - a_1}{(z - p_1)(z - p_2)}$$

with $|a_1| > 1$; in other words, $G(z)$ has a zero outside the unit circle.

$G(z)$ could be splitted adopting the following instructions, taken from [4]:

1. Factorization of all the zeros outside the unit circle, time-delays and ringing zeros;

$$G(z) = \frac{1}{(z-p_1)(z-p_2)} z^{-k}(z-a_1)$$

2. Building the *All-Pass Filter*

$$G(z) = \frac{z-\frac{1}{a_1}}{(z-p_1)(z-p_2)} z^{-k} \frac{(z-a_1)}{\left(z-\frac{1}{a_1}\right)}$$

3. Normalization of the function, making sure that $G_{nmp}(1) = 1$ and $G_{mp}(1) = G(1)$

$$G(z) = \underbrace{\frac{\left(z-\frac{1}{a_1}\right)(1-a_1)}{(z-p_1)(z-p_2)\left(1-\frac{1}{a_1}\right)}}_{MinimumPhase} \underbrace{z^{-k} \frac{(z-a_1)\left(1-\frac{1}{a_1}\right)}{\left(z-\frac{1}{a_1}\right)(1-a_1)}}_{Non-MinimumPhase}$$

finally,

$$G_{mp}(z) = \frac{\left(z-\frac{1}{a_1}\right)(1-a_1)}{(z-p_1)(z-p_2)\left(1-\frac{1}{a_1}\right)}$$

$$G_{nmp}(z) = z^{-k} \frac{(z-a_1)\left(1-\frac{1}{a_1}\right)}{\left(z-\frac{1}{a_1}\right)(1-a_1)}$$

As in continuous time, an example will make these concepts clearer.

Take (2.11) as model: the steps for decomposing $G(z)$ are

1. Factorization

$$G(z) = \left(\frac{-0.072866}{(z-0.9048)(z-0.7788)}\right)\left(z^{-4}(z-1.289)\right)$$

2. Building the All-Pass Filter

$$G(z) = \left(\frac{-0.072866\left(z-\frac{1}{1.289}\right)}{(z-0.9048)(z-0.7788)}\right)\left(z^{-4}\frac{(z-1.289)}{\left(z-\frac{1}{1.289}\right)}\right)$$

3. Normalization for the adjustment of gains

$$G(z) = \underbrace{\frac{-0.072866\left(z - \frac{1}{1.289}\right)(1 - 1.289)}{(z - 0.9048)(z - 0.7788)\left(1 - \frac{1}{1.289}\right)}}_{MP} \underbrace{z^{-4}\frac{(z - 1.289)\left(1 - \frac{1}{1.289}\right)}{\left(z - \frac{1}{1.289}\right)(1 - 1.289)}}_{NMP}$$

and, finally

$$G_{mp} = 0.0939\frac{(z - 0.7759)}{(z - 0.9084)(z - 0.7788)}$$
$$G_{nmp} = -0.77591z^{-4}\frac{(z - 1.289)}{(z - 0.7759)}$$

2.3.1.2 Removal of ringing zeros

Once the Minimum Phase model is built, it is necessary to eliminate all those zeros in $G_{mp}(z)$ which may give ringing in the output.

Ringing could be avoided replacing these zeros with zeros in the origin (which, in the controller, will become poles in the origin).

So, if P is the number of "ringing zeros" in the minimum phase part of $G(z)$, the expression for $G_{mp}(z)$ becomes

$$G_{mp}^{mod}(z) = G_{mp}(z) \cdot z^P \frac{\prod_{j=1}^{P}(1 - \phi_j)}{\prod_{j=1}^{P}(z - \phi_j)}$$

with ϕ_j as ringing zeros.

2.3.1.3 Controller

The design of a discrete-time IMC controller is exactly the same as the one done in continuos time (see Section 2.2.1.2; as in Laplace transform, Q still needs a filter for the PR; the discrete-time filter constant α is related to λ in (2.5) through the following expression:

$$\alpha = e^{-\frac{T_s}{\lambda}}$$

and the expression of type-1 filter in z-transform is the following:

$$F(z) = \frac{(1-\alpha)z^{-1}}{1-\alpha z^{-1}} = \frac{(1-\alpha)}{(z-\alpha)} \tag{2.12}$$

and it makes $Q(z)$ Physically Realizable; furthermore, as said in continuos time, λ is also a detuning parameters, since it manages to attenuate the effect of control input u_k. In the example with (2.11) as model, IMC phisically realizable controller is

$$Q(z) = 10\frac{(z-0.9084)\,(z-0.7788)}{z-0.7759}\frac{(1-\alpha)}{(z-\alpha)}$$

keeping $\lambda = 10$ as it was in continuous time, α is 0.9512.

2.3.2 Sensitivity functions

T and S are exactly the same as in continuous time, previously described (Sec 2.2.2): their expressions are:

$$S = \frac{1-GQ}{1+Q\,(P-G)} \tag{2.13a}$$

$$T = \frac{PQ}{1+Q\,(P-G)} \tag{2.13b}$$

In the nominal case $P = G$, S and T are reduced, simply, to

$$S = 1 - GQ \tag{2.14a}$$

$$T = GQ \tag{2.14b}$$

Trends of sensitivites for system (2.11) are shown in Figure 2.5, while response for step-like set point change (blue line) and disturbance (red line) are shown in Figure 2.6.

2.4 Open loop unstable plants

Internal Model Control is a control structure developed on purpose for OL Stable Plants: in fact, for its own features, it is not able to stabilize the closed-loop sistem in the case of unstable plants.

Several attemps to determine an IMC structure for OL Unstable system have been tried: among them, it is remarkable the example of tuning given by Morari and Zafiriou in [13], who followed instruction given by Youla et al. in [21].

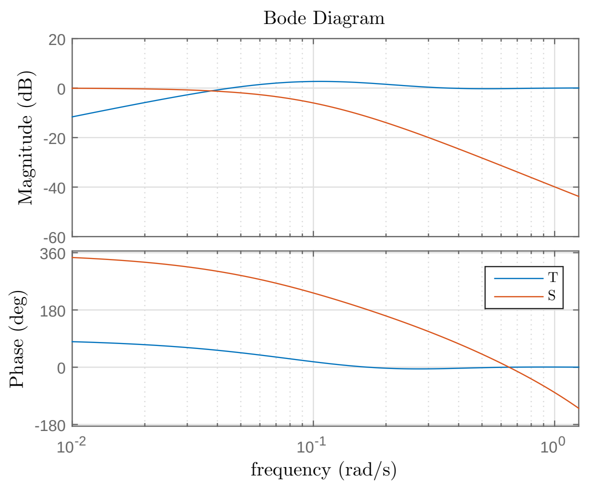


FIGURE 2.5: IMC sensitivity functions for discrete-time system (2.11)

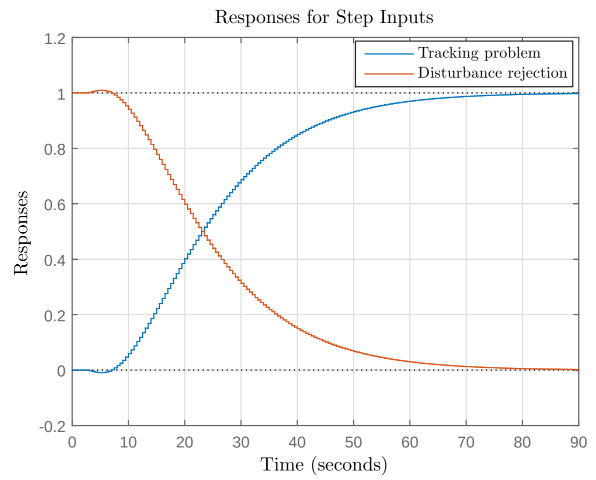


FIGURE 2.6: IMC response for a step-like input for the discrete time system (2.11)

Consider a plant with k poles $p_i \in$ RHP: define the allpass

$$b_p(s) = \prod_{i=1}^{k} \frac{-s + p_i}{s + p_i^H} \tag{2.15}$$

Theorem 1. Assume, in the nominal case, that $P = G$ has k unstable poles at $p_1, \dots p_k \in$ RHP and l poles in the origin. Assume, furthermore, that there exists Q_0 that stabilizes P. Then, all controllers which stabilize P are parametrized by

$$Q = Q_0 + b_p^2 s^{2l} Q_1 \tag{2.16}$$

with Q_1 as whatever stable transfer function.

This result uses same principles adopted in Chapter 4 for the development of a DOB-IMC for unstable plants, but it is still quite close to IMC configuration. The one we adopted, instead, is on purpose closer to optimal control structures like LQG and MPC.

Chapter 3

Optimal control and estimation

Another important part of control theory that needs to be investigated, as it is very important for the final aim of this work, is optimal control field.

3.1 Controllability, stabilizability and observability for Linear Systems

Given a linear system

$$\begin{cases} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k \end{cases} \tag{3.1}$$

Definition 1. (3.1) is controllable if and only if, given any initial x_0, it is possible to make x_k reach the origin in n steps.

From the previous sentence it can be derived that a linear system (3.1) is said to be controllable if and only if the matrix

$$R(A, B) = \begin{bmatrix} B & AB & \dots & A^{n-1}B \end{bmatrix}$$

has full rank.

$R(A, B)$ is called controllability matrix.

(Hautus Lemma) A linear system (3.1) is controllable if and only if the matrix

$$\begin{bmatrix} \lambda I - A & B \end{bmatrix}$$

has full rank for any λ eigenvalue of A.

If a system is not controllable, then there exists a transormation matrix T such that

$$\tilde{A} = T^{-1}AT = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}, \qquad \tilde{B} = T^{-1}B \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \tag{3.2}$$

Defining $\tilde{x} = \begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix}$ the vector of the states for system (3.2), states $x^{(1)} \in \mathbb{R}^r$ and $x^{(2)} \in \mathbb{R}^{n-r}$ evolve with the following dynamics:

$$\begin{cases} x_{k+1}^{(1)} & = A_{11}x_k^{(1)} + B_1 u_k + A_{12}x^{(2)} \\ x_{k+1}^{(2)} & = A_{22}x_k^{(2)} \end{cases} \tag{3.3}$$

It can be immediately noticed that state $x^{(2)}$ is not directly controlled by u_k.

Definition 2. A lynear system (3.2) is stabilizable if

1. $|\lambda\left(A_{22}\right)| < 1$
2. $rank\left(R\left(A_{11}, B_1\right)\right) = n$

Using Hautus Lemma, a linear system (3.1) is stabilizable if and only if the matrix

$$\begin{bmatrix} \lambda I - A & B \end{bmatrix}$$

has full rank for any $|\lambda\left(A\right)| \geq 1$. A matrix whose eigenvalue all stand inside the unit circle is called Hurwitz.

Definition 3. System (3.1) is observable if, for any $t_1 > 0$, the initial state x_0 can be determined from the time history of the input $u(t)$ and the output $y(t)$ in the time vector $[0, t_1]$. Observability matrix, $O(A, C)$ is defined as follows:

$$O(A, C) = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \tag{3.4}$$

and the linear system is observable iff $O(A, C)$ has full column rank.

3.2 Linear quadratic regulator

Consider linear system described by (3.1): we need to be find a control strategy providing a sequence of control inputs that can be defined *optimal*.

In linear optimal control, control strategy is obtained solving an optimization problem: when the problem is quadratic, we have the *Linear quadratic regulation (LQR) problem*, which consists of minimizing the following unconstrained cost function:

$$J\left(x, \mathbf{u}\right) = \frac{1}{2}\sum_{k=0}^{N-1}\left[x_k^T Q x_k + u_k^T R u_k\right] + \frac{1}{2}x_N^T P x_N \tag{3.5}$$

subject to

$$x_{k+1} = Ax_k + Bu_k \tag{3.6}$$

where $\mathbf{u} = \left(u_0, \ldots u_k\right)^T$ is a vector of control inputs at different time steps (with $0 < k < N-1$), R is *symmetric and positive definite*, Q and P are *symmetric and positive semidefinite*; N is a finite number called *horizon*; furthermore, in LQR problems it is always assumed that the system is stabilizable.

LQR minimization problem is solved using a dynamic programming, developed by Bellman [3] and gives an optimal control law, that is

$$u_k = -K_0 x_k \tag{3.7}$$

which minimizes cost function J.

The dynamics of the Closed Loop system evolves according to the following equation:

$$x_{k+1} = \left(A - BK_0\right) x_k \tag{3.8}$$

hance, CL system is stable if and only if

$$\left|\lambda\left(A - BK_0\right)\right| < 1 \tag{3.9}$$

It is important to notice that this approach, called *receding horizon*, solves, at each sampling time, an optimization problem over a finite horizon, but it actually applies only the *first* element of $\mathbf{u}$. It is possible to demonstrate that, following this method, there is no warranty that (3.9) is satisfied.

This aspect led to take into account turning (3.5) into (3.10), where the horizon is not anymore a finite value, but it goes to infinity, $N \to \infty$:

$$J(x, \mathbf{u}) = \frac{1}{2} \sum_{k=0}^{\infty} \left[x_k^T Q x_k + u_k^T R u_k \right] \tag{3.10}$$

subject to

$$x_{k+1} = A x_k + B u_k \tag{3.11}$$

Assuming that linear system (3.1) is both stabilizable and controllable, and that Q and R are positive definite, eq (3.10) gives as solution

$$u_k = -K x_k \tag{3.12}$$

with

$$K = \left(R + B' \Pi B \right)^{-1} B' \Pi A \tag{3.13}$$

where Π is the solution to the *Discrete Algebraic Riccati Equation (DARE):*

$$\Pi = Q + A' \Pi A - A' \Pi B \left(R + B' \Pi B \right)^{-1} B' \Pi A \tag{3.14}$$

This approach provides CL stability for any value of Q and R.

3.3 Luenberger observer

State estimation problem will be shortly introduced the before talking about observers.

For each sampling k, together with the state x_k, there is an estimation of the state, $\hat{x}_k$, which evolves as a "copy" of the Plant, with the following dynamics:

$$\hat{x}_{k+1} = A \hat{x}_k + B u_k \tag{3.15}$$

The role of the observer in this copy is to "correct" the estimation equation with a feedback based on the prediction error:

$$e_{pred} = y_k - C \hat{x}_k \tag{3.16}$$

where y_k is the measured output and $C\hat{x}_k$ the estimated one.

Dynamics evolution of the estimated states together with the observer evolve as follows:

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + L\left(y_k - C\hat{x}_k\right) \tag{3.17}$$

with $L \in \mathbb{R}^{n\times p}$ is the observer gain.

The difference between the actual state and its estimation is called estimation error, ϵ:

$$\epsilon_k = x_k - \hat{x}_k \tag{3.18}$$

whose nominal dynamics is

$$\epsilon_{k+1} = Ax_k + Bu_k - A\hat{x}_k - Bu_k + L\left(y_k - C\hat{x}_k\right) = (A - LC)\,\epsilon_k \tag{3.19}$$

from which, it is evident that, given any initial value ϵ_0 for $k \longrightarrow \infty, \epsilon_k$ goes to zero if and only if matrix $(A - LC)$ is Hurwitz.

An observer chosen such that $(A - LC)$ is Hurwitz is called *Luenberger observer*, while an observer such that the eigenvalues of $(A - LC)$ are all equal to zero is called *deadbeat observer*.

Internal Model Control can actually be considered as a control structure adopting a deadbeat observer, if we consider model G as a system that evolves as a copy of the original plant P.

Observer could be chosen in different ways, like using *pole placement technique*, with which the observer gain L could be chosen by arbitrary assignment of the poles of the dynamic matrix $A - LC$.

3.4 Kalman filter

Pole placement technique is not always the best option: indeed, its action could be too much aggressive since it places poles too far away from the original system, i.e. from the eigenvalues of matrix A, making the system ”unnatural”.

A useful way to find an observer definable optimal was suggested by Kalman in [8].

The system is in the form

$$\begin{cases} x_{k+1} &= Ax_k + Bu_k + w_k \\ y_k &= Cx_k + v_k \end{cases} \tag{3.20}$$

with w_k and v_k are white noises with Q and R as covariances;

1. choosing a large Q means large process noise (w_k) and leads to estimated states able to respond fast to changes in the measured output;
2. choosing a large R means large output noise (v_k) and leads to estimated states that respond carefully,i.e. slowly, to unexpected changes in the measured output.

Kalman filter K_f articulates the calculations of the estimation in two different steps: as first step, $\hat{x}$ is updated according to the estimation dynamics (3.15), then the estimation passes through Kalman filter.

Steps can be summarized as follows:

1. Time update

$$\begin{cases} \hat{x}_{k|k-1} & = A\hat{x}_{k-1|k-1} + Bu_{k-1} \\ P_k^- & = AP_{k-1}A^T + Q \end{cases} \tag{3.21a}$$

2. Measurement update

$$\begin{cases} K_{f_k} & = P_k^- C^T \left(CP_{k-1}C^T + R \right)^{-1} \\ \hat{x}_{k|k} & = \hat{x}_{k|k-1} + K_{f_k} \left(y_k - C\hat{x}_{k|k-1} \right) \\ P_k & = P_k^- - K_{f_k} C P_k^- \end{cases} \tag{3.21b}$$

where P_k^- is the covariance of the estimated state at time update and P_k is the covariance of the estimated state at measurement update.

After few measurements, the Kalman filter does not change anymore going ahead with the iterations; it converges to the *Steady-State Kalman filter*.

Adopting this observer, dynamics of the estimation error are

$$\epsilon_{k+1} = \left(A - AK_f C \right) \epsilon_k \tag{3.22}$$

Relation between Kalman filter and Luenberger Observer is

$$L = AK_f \tag{3.23}$$

3.5 Linear Quadratic Gaussian control

Linear Quadratic Gaussian control (LQG) gave birth to MPC, since it is its unconstrained version, and is very useful in order to better understand how these optimal stategies work.

Putting all the elements mentioned and explained in the previous sections together leads to LQG control scheme: there is a stabilizing state feedback, K, and an observer, L; an example of LQG block diagram is shown in Figure 3.1

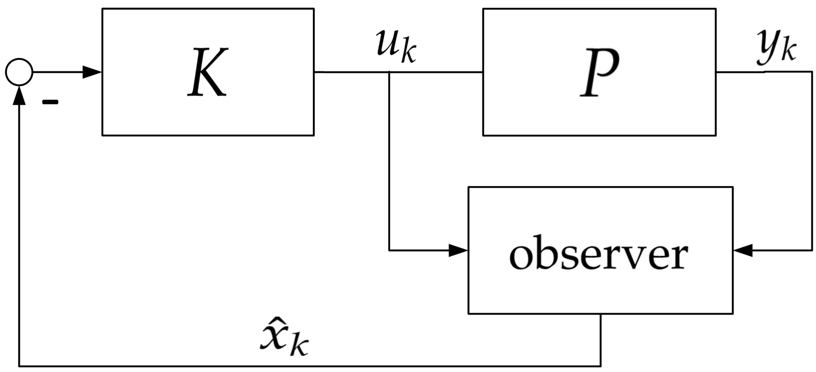


FIGURE 3.1: Linear Quadratic Gaussian Control

This structure is the basic one, it will be well explained and enlarged in following sections, to explain how it manages to follow the reference signal, r_k, and to remove the effect of disturbances[17].

3.5.1 Steady-state problem

To make LQG reach the Reference Signal and to remove offsets when disturbances occur, it is necessary to introduce a block which calculates steady-state values for each sample time, called targets; steady-state system is

$$\begin{cases} x_{ss} &= Ax_{ss} + Bu_{ss} \\ y_{ss} &= Cx_{ss} \end{cases} \tag{3.24}$$

from which

$$\begin{bmatrix} x_{ss} \\ u_{ss} \end{bmatrix} \underbrace{\begin{bmatrix} I-A & -B \\ C & 0 \end{bmatrix}^{-1}}_{M} \begin{bmatrix} 0 \\ y_{ss} \end{bmatrix} = M \begin{bmatrix} 0 \\ y_{ss} \end{bmatrix} \tag{3.25}$$

with y_{ss} equal to the reference signal r_k at T_s. Thus, in the controller block in Figure 3.1 it is included both state feedback K and target calculation matrix M .

Adopting deviation variables, that are

$$\tilde{x}_k = \hat{x}_k - x_{ss} \tag{3.26a}$$

$$\tilde{u}_k = u_k - u_{ss} \tag{3.26b}$$

we can re-define the optimal control law (3.10)

$$J\left(\tilde{x}, \tilde{\mathbf{u}}\right) = \frac{1}{2} \sum_{k=0}^{\infty} \left[\tilde{x}_k^T Q \tilde{x}_k + \tilde{u}_k^T R \tilde{u}_k\right] \tag{3.27}$$

which has as solution

$$\tilde{u}_k = K \tilde{x}_k \tag{3.28}$$

thus

$$u_k = K\left(\hat{x}_k - x_{ss}\right) + u_{ss} \tag{3.29}$$

(3.29) guarantees the output perfectly follows set point and it does not exhibit any offset left.

3.6 Model Predictive Control: offset-free design using disturbance models

Finally, what is missing in this framework is Model Predictive Control (MPC): it took birth by the end of 70s, and it is nowadays the most widespread advanced control strategy adopted in industrial processes.

Here it is described a SISO version of a Linear MPC which is the configuration that can be better compared to IMC and LQG to find analogies and differences.

MPC structure is shown in Figure 3.2: it is made up of several blocks, each of those will be briefly examined in the following pages.

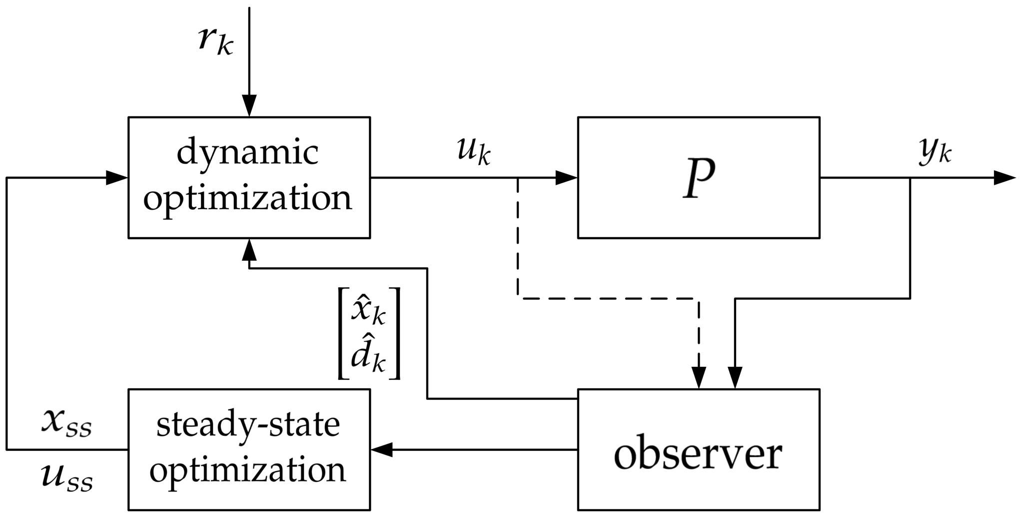


FIGURE 3.2: Model Predictive Control

3.6.1 Disturbance model and observer

In order to achieve offset-free performance, model we are working here with MPC is a *augmented*, developed as follows:

$$\begin{cases} \hat{x}_{k+1|k} & = A\hat{x}_{k|k} + Bu_k + B_d\hat{d}_{k|k} \\ \hat{d}_{k+1|k} & = \hat{d}_{k|k} \\ \hat{y}_k & = C\hat{x}_k + C_d\hat{d}_{k|k} \end{cases} \tag{3.30}$$

This system can be re-written in a more compact way, i.e.

$$\begin{gathered} \begin{bmatrix} \hat{x}_{k+1|k} \\ \hat{d}_{k+1|k} \end{bmatrix} = \begin{bmatrix} A & B_d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k \\ y_k = \begin{bmatrix} C & C_d \end{bmatrix} \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \end{bmatrix} \end{gathered} \tag{3.31}$$

Here $\hat{d}_k$ is an additional state working as a disturbance model, thanks to which it is possible to remove any offset.

In the framework of this augmented model , state estimation dynamics with observer, seen in (3.17), becomes

$$\begin{bmatrix} \hat{x}_{k|k} \\ \hat{d}_{k|k} \end{bmatrix} = \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \end{bmatrix} + \begin{bmatrix} L_x \\ L_d \end{bmatrix} e_k \tag{3.32}$$

Where L_x and L_d make an augmented observer [1].

$$L = \begin{bmatrix} L_x \\ L_d \end{bmatrix} \tag{3.33}$$

Even e_k is an augmented version for the prediction error:

$$e_k = y_k - \begin{bmatrix} C & C_d \end{bmatrix} \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \end{bmatrix} \tag{3.34}$$

Putting together (3.31) and (3.34), expression obtained is the one for the so-called *Kalman predictor step*:

$$\begin{gathered} \begin{bmatrix} \hat{x}_{k+1|k} \\ \hat{d}_{k+1|k} \end{bmatrix} = \begin{bmatrix} A & B_d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} e_k \\ y_k = \begin{bmatrix} C & C_d \end{bmatrix} \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \end{bmatrix} \end{gathered} \tag{3.35}$$

Matrices B_d and C_d could have different values; a common choice for them and for the observer is ([15])[2]

$$B_d = 0, \qquad C_d = 1$$
$$L_x = 0, \qquad L_d = 1$$

This model allows to choose how the disturbance model works by choice of matrices B_d and C_d under the only condition that (3.31) has to be detectable; detectability can be checked as follows (See [17])

Lemma 1. Augmented system (3.31) is detectable if and only if the pair $[A, C]$is detectable, and if

$$\text{rank} \begin{bmatrix} I - A & -B_d \\ C & C_d \end{bmatrix} = n + n_d \tag{3.36}$$

with n_d as dimension of the disturbance.

[1]observers are optimal, chosen as taught by Kalman, and shown in Section 3.4
[2]For unstable systems, such as control of levels, these matrices requires other values to work

Corollary 1. The dimension of the disturbance has to be such that the augmented system is detectable is equal or less than the dimension of the measured ouput: calling $n_y = dim(y)$,

$$n_d \leq n_y$$

Remark: Since the system investigated is a single input-single output, dimension n_y is necessarily equal to 1, and so is n_d.

3.6.1.1 Analogies between LQG and MPC

In Section 3.5 Linear Quadratic Gaussian Control has been investigated, but once analyzed this augmented model, it can be observed that model adopted for LQG can be considered as a particular sub-category of this augmented model, with

$$B_d = 0, \qquad C_d = 1$$

Furthermore, LQG could be implemented even choosing different values for B_d and C_d: in other words, an augmented model can be adopted in LQG, but the expression of the targets calculation matrix changes: in fact, steady-state dynamics with an augmented model is the following:

$$\begin{cases} x_{ss} &= Ax_{ss} + Bu_{ss} + B_d\hat{d}_k \\ d_{ss} &= \hat{d}_k \\ y_{ss} &= Cx_{ss} + C_d\hat{d}_k \end{cases} \tag{3.37}$$

from which

$$\begin{bmatrix} x_{ss} \\ u_{ss} \end{bmatrix} \underbrace{\begin{bmatrix} I - A & -B \\ C & 0 \end{bmatrix}^{-1}}_{M} \begin{bmatrix} B_d\hat{d}_k \\ y_{ss} - C_d\hat{d}_k \end{bmatrix} = M \begin{bmatrix} B_d\hat{d}_k \\ y_{ss} - C_d\hat{d}_k \end{bmatrix} \tag{3.38}$$

3.6.2 Controller

Figure 3.2 shows that MPC strategy is made up of two separate blocks:

1. *Steady-State Optimization*, which calculates optimal values of states and inputs;
2. *Dynamics Optimization*, which generates a sequence of optimal inputs for a time window made up of N elements (*N= Prediction Horizon*).

According to the Receding Horizon Control (RHC) strategy, only the first element of this sequence of inputs is sent to the Plant: hence, for each sampling time, both blocks repeat their calculations.

3.6.2.1 Steady-state optimization

This block calculates optimal steady-state values,i.e. the targets, for states, x_{ss}, input u_{ss} and output, y_{ss}.

The optimal steady-state control problem is so defined:

$$\min_{x_{ss},u_{ss},y_{ss}} (y_{ss} - r_k)^2 = \min_{x_{ss},u_{ss},y_{ss}} \left(y_{ss}^2 + r_k^2 - 2y_{ss}r_k\right) \tag{3.39}$$

subject to:

$$u_{min} \leq u_{ss} \leq u_{max} \tag{3.40a}$$

$$x_{ss} = Ax_{ss} + Bu_{ss} + B_d\hat{d}_{k|k} \tag{3.40b}$$

$$y_{ss} = Cx_{ss} + C_d\hat{d}_{k|k} \tag{3.40c}$$

The problem can be written in a compact form:

$$\min_{z}(\mathbf{z}^{\mathbf{T}}H_s\mathbf{z} + \mathbf{z}^{\mathbf{T}}f) \tag{3.41}$$

subject to:

- Equality Constraints

$$A_{eq}\mathbf{z} = b_{eq} \tag{3.42}$$

- Inequality Constraints

$$A_{in}\mathbf{z} \leq b_{in}\hat{d}_{k|k} \tag{3.43}$$

with the above matrices and vectors so defined:

$$\mathbf{z} = \begin{bmatrix} \mathrm{x_s} \\ \mathrm{u_s} \\ \mathrm{y_s} \end{bmatrix}, H_s = \begin{bmatrix} 0_{n\times n} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, f = \begin{bmatrix} 0_{n\times n} \\ 0 \\ -\mathrm{r_k} \end{bmatrix}$$

$$A_{eq} = \begin{bmatrix} A-I & B & 0 \\ C & 0 & -I \end{bmatrix}, b_{eq} = \begin{bmatrix} -B_d \\ -C_d \end{bmatrix}$$

$$A_{in} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, b_{in} = \begin{bmatrix} u_{max} \\ -u_{min} \end{bmatrix}$$

Remark: r_k and $\hat{d}_{k|k}$ could change for each sampling, so vector f (and so the solution of the optimal stady-state problem) must be computed *online*.

3.6.2.2 Dynamic optimization

As said before, the role of this block is to calculate, for each sample time, a sequence of optimal control inputs, even if only the first of these inputs is to the plant P.

Given the following Deviation Variables:

$$\begin{cases} \tilde{x}_j & = \hat{x}_{k+j|k} - x_{ss} \\ \tilde{u}_j & = u_{k+j|k} - u_{ss} \end{cases} \tag{3.44}$$

and the Prediction Horizon N, dynamics optimal control problem is defined as:

$$\min_{\tilde{x}_j,\tilde{u}_j} \sum_{j=0}^{N-1} \left(\tilde{x}_j^T Q \tilde{x}_j + \tilde{u}_j^T R \tilde{u}_j \right) + \tilde{x}_N^T P \tilde{x}_N =$$

$$\min_{\tilde{x}_j,\tilde{u}_j} \sum_{j=0}^{N-1} \left(\tilde{x}_j^T Q \tilde{x}_j + R \tilde{u}_j^2 \right) + \tilde{x}_N^T P \tilde{x}_N$$

s.t.

$$\tilde{x}_{j+1} = A\tilde{x}_j + B\tilde{u}_j \tag{3.45}$$

$$u_{min} - u_{ss} \leq \tilde{u}_j \leq u_{max} - u_{ss} \tag{3.46}$$

With $Q = C^T C$, R as a scalar value, and P obtained as solution of the *Discrete Algebraic Riccati Equation (DARE)*.

We can write all this in a compact way as

$$\min_{z_d} (\mathbf{z_d}^T H_d \mathbf{z_d} + \mathbf{z_d}^T q) \tag{3.47}$$

s.t.

$$D\mathbf{z_d} = e \tag{3.48}$$

$$G\mathbf{z_d} \leq h \tag{3.49}$$

with

$$\mathbf{z_d} = \begin{bmatrix} \tilde{x}_0 \\ \tilde{u}_0 \\ \tilde{x}_1 \\ \tilde{u}_1 \\ \vdots \\ \vdots \\ \tilde{x}_N \end{bmatrix}, H_d = \begin{bmatrix} Q & 0 & \dots & \dots & \dots & 0 \\ 0 & R & 0 & & & 0 \\ \vdots & 0 & Q & \ddots & & \vdots \\ \vdots & & \ddots & R & \ddots & \vdots \\ \vdots & & & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & \dots & P \end{bmatrix}, q = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{bmatrix}$$

$$D = \begin{bmatrix} I & 0 & 0 & \dots & \dots & 0 \\ -A & -B & I & 0 & & 0 \\ \vdots & 0 & -A & -B & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & -A & -B & I \end{bmatrix}, e = \begin{bmatrix} \hat{x}_{k|k} - x_s \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 1 & 0 & \dots & \dots & \dots & \dots & \dots & 0 \\ 0 & -1 & 0 & \ddots & \ddots & & & & \vdots \\ \vdots & \ddots & \ddots & 0 & 1 & \ddots & & & \vdots \\ \vdots & & \ddots & 0 & -1 & \ddots & \ddots & & \vdots \\ \vdots & & & \ddots & \ddots & \ddots & 0 & 1 & 0 \\ 0 & \dots & \dots & \dots & \dots & \dots & 0 & -1 & 0 \end{bmatrix}, h = \begin{bmatrix} (u_{max} - u_{ss}) \\ -(u_{min} - u_{ss}) \\ \vdots \\ \vdots \\ (u_{max} - u_{ss}) \\ -(u_{min} - u_{ss}) \end{bmatrix}$$

The solution to the quadratic problem calculates at each sampling T_s the vector $\mathbf{z_d}$, thus the control input a time 0 as deviation variable, $\tilde{u}_0$, from which the final expression of desired input u_k

$$u_k = \tilde{u}_0 + u_{ss} \tag{3.50}$$

And u_k so obtained is optimal.

Chapter 4

Proposed Observer Based IMC structure

In this chapter a Disturbance Observer Based Internal Model Control (DOB-IMC) is proposed, and its features and properties are delineated.

As first step, there is the presentation of the proposed structure for OL stable plants; afterwards, the structure has been extended to OL unstable plants, according to instructions given by [7, 10, 21].

4.1 Model

The model is represented by the following equations, taken from the MPC control scheme (the so-called *Kalman Predictor Step*):

$$\begin{aligned} \begin{bmatrix} \hat{x}_{k+1|k} \\ \hat{d}_{k+1|k} \end{bmatrix} &= \begin{bmatrix} A & B_d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \left(y_k - \begin{bmatrix} C & C_d \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{d}_k \end{bmatrix} \right) \\ \hat{y}_k &= \begin{bmatrix} C & C_d \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{d}_k \end{bmatrix} \end{aligned} \tag{4.1}$$

Hence, instead of the usual model, an augmented one is taken into account, where $\hat{d}_k$ is introduced as additional state.

Furthermore, conventional IMC deadbeat observer is replaced by a Luenberger observer, in order to overcome the poorness of performance that Internal Model Control normally achieves with the occurrence of a disturbance on control input u_k.

4.2 Open loop stable plants

The modified structure initially proposed for open loop stable plants is shown in Figure 4.1:

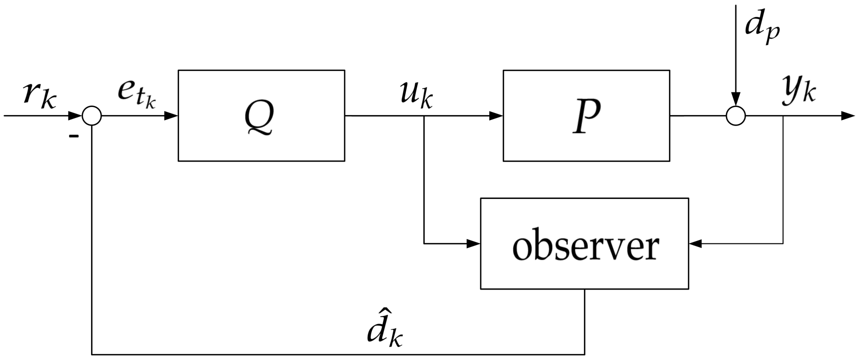


FIGURE 4.1: DOB-IMC block diagram for open loop stable plants

As it can be easily seen from the picture, the block standing for the model in the original IMC scheme is replaced by an observer block; moreover, the feedback branch in this structure is represented by $\hat{d}_k$, which is supposed to replace the conventional IMC return difference between measured output and estimated one, namely $\tilde{d}_k = y_k - C\hat{x}_k$.

In this framework controller design is left unchanged and its tuning perfectly follows the tuning istructions given in Chapter 2.

Remark: In order to make this structure work, it is necessary that *what is fed back to the controller is exactly the same quantity as it used to be in classical IMC*, that is

$$\hat{d}_k = \tilde{d}_k$$

This is true only if the equations of the model shown in (4.1) collapse to the ones describing (2.10), that is the conventional model adopted in IMC; this happens only under certain specific conditions, i.e.

$$B_d = 0, \qquad C_d = 1 \tag{4.2a}$$

$$L_x = 0, \qquad L_d = 1 \tag{4.2b}$$

However, we want to obtain stability independently from the choices of both the observer and the matrices, so the next step consists of working on a system capable to turn the observer output into the classical return difference $\tilde{d}_k$.

4.2.1 Algebraic equivalence of two different systems

To make structure in Figure 4.1 work, system (4.1) needs to be turned into an *algebraically equivalent* one, having $\tilde{B}_d$ and $\tilde{C}_d$ described by (4.2a) with different observers than those in (4.2b).

$\tilde{L}_x$ and $\tilde{L}_d$ are evaluated used the following corollary, taken from [16]

Corollary 1. Given two systems in the form (4.1), defined by matrices

i. $(A, B, C, B_d, C_d, L_x, L_d)$

ii. $(A, B, C, \tilde{B}_d, \tilde{C}_d, \tilde{L}_x, \tilde{L}_d)$;

assuming that

- The pair (A, B) is stabilizable, the pair (A, C) is detectable, and

$$rank \begin{bmatrix} A - I & B_d \\ C & C_d \end{bmatrix} = n + n_d$$

  where $n = dim(A)$ and $n_d = dim(\hat{d}_k)$.

- The matrix

$$\begin{bmatrix} A & B_d \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} L_x \\ L_d \end{bmatrix} \begin{bmatrix} C & C_d \end{bmatrix}$$

  is strictly Hurwitz.

these two systems are Algebrically Equivalent (AE) if there exists $H_{12} \in \mathbb{R}^{n_d \times n}$ and $H_{22} \in \mathbb{R}^{n_d \times n_d}$ (invertible) satisfying:

$$\begin{bmatrix} A - I & \tilde{B}_d \\ C & \tilde{C}_d \end{bmatrix} \begin{bmatrix} H_{12} \\ H_{22} \end{bmatrix} = \begin{bmatrix} B_d \\ C_d \end{bmatrix} \tag{4.3}$$

and such that $\tilde{L}_x = L_x + H_{12} L_d$ and $\tilde{L}_d = H_{22} L_d$.

Once these new values are known, it is sufficient to replace system (i) with (ii) to make the control structure work.

In this way, in this way, augmented system can be obtained with the matrices and the observers chosen arbitarily; for instance, it is possible to choose (see [14, 20] for further details):

- L_x set through pole placement technique or, better, defined as a *Kalman filter*, $L_d = 1$;

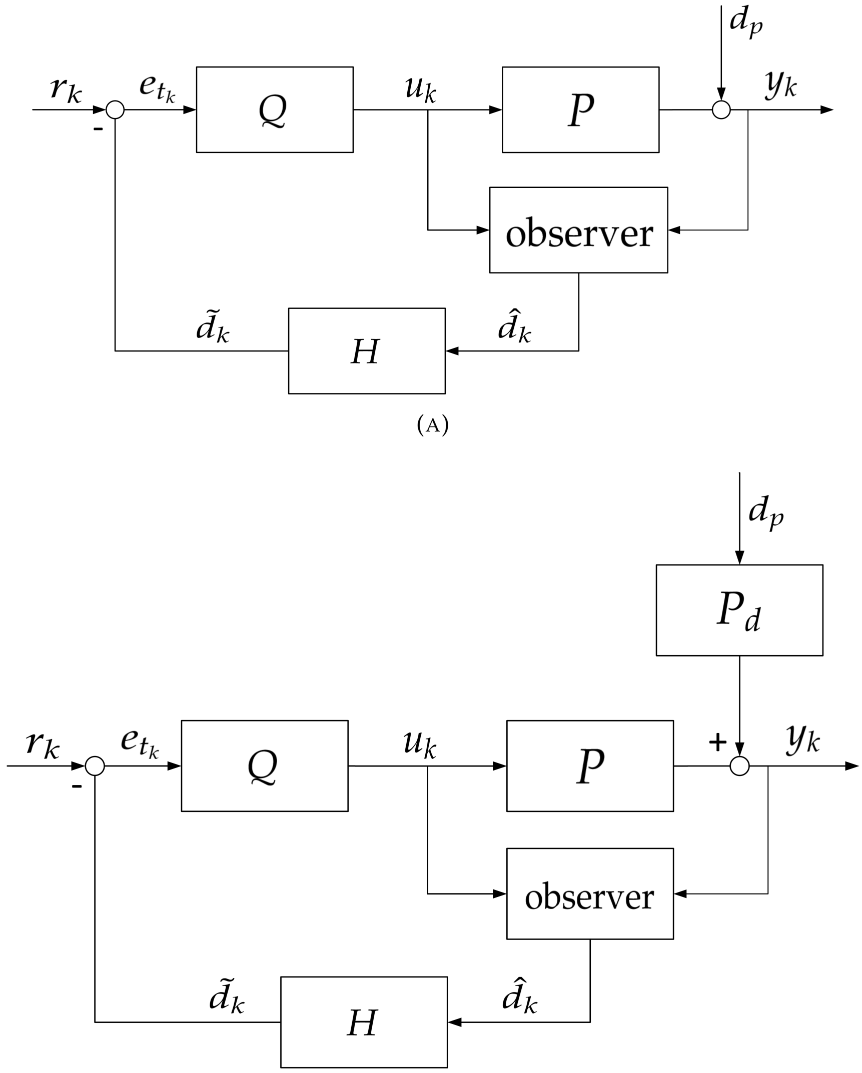
- $B_d = L_x$, $C_d = 1 - CL_x$

4.2.2 Introduction of a transformation block

Since we do not want to change B_d, C_d into $\tilde{B}_d$ and $\tilde{C}_d$, it is necessary to modify structure of Figure 4.1 into something new, capable to turn the observer output $\hat{d}_k$ into the desired feedback $\tilde{d}_k$; so, an additional block H is added to the control scheme, thanks to which I have

$$\tilde{d}_k = H\hat{d}_k$$

where , in the OL stable case, H is equal to H_{22} calculated in (4.3). The adjustment is shown in Figure 4.2a, which can be more generally represented by Figure 4.2b, where the disturbance enters with its own dynamics, P_d.



(B)

FIGURE 4.2: DOB-IMC block diagram for open loop stable plants, with the transformation block H
4.2a: step-like disturbance
4.2b: disturbance with a certain dynamics P_d

4.2.3 Sensitivity functions

Sensitivity functions are useful and powerful instruments thanks to which it is possible to verify if response is well-shaped and to analyze robustness of control schemes; sometimes finding these functions is easy, such as with classical FB o IMC structure. On the contrary, finding CL transfer functions for this block diagram is not immediate and straightforward, since it needs some preliminary information, such as the knowledge of the observer transfer function.

4.2.3.1 Observer block trasfer functions

To define CL transfer functions T and S for the Observer-Based IMC, it is necessary to derive, first, the *transfer function for the observer block*: this can be obtained considering that che observer block has, as it is easy to see in Figure 4.3:

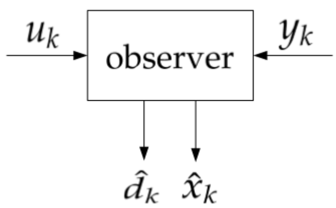


FIGURE 4.3: DOB-IMC, observer block

- two inputs, u_k and y_k;
- two outputs, $\hat{x}_k$ and $\hat{d}_k$

so, basically, there is the need to build several transfer functions.

Equations for the observer block are:

$$\begin{aligned} \begin{bmatrix} \hat{x}_{k+1|k} \\ \hat{d}_{k+1|k} \end{bmatrix} &= \begin{bmatrix} A & B_d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \left(y_k - \begin{bmatrix} C & C_d \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{d}_k \end{bmatrix} \right) \\ \hat{y}_k &= \begin{bmatrix} C & C_d \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{d}_k \end{bmatrix} \end{aligned} \tag{4.4}$$

with

$$L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} A & B_d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} Ł_x \\ Ł_d \end{bmatrix}$$

For a matter of simplicty and shortness, some terms are grouped:

$$\hat{x}^a_k = \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} A & B_d \\ 0 & 1 \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \bar{C} = \begin{bmatrix} C & C_d \end{bmatrix}, \bar{L} = \bar{A}L$$

so it can be written (4.4) in a more compact way

$$\hat{x}^a_{k+1|k} = (\bar{A} - \bar{L}\bar{C})\,\hat{x}^a_{k|k-1} + \bar{B}u_k + \bar{L}y_k \tag{4.5}$$

while the equations for estimated states and disturbance kept separated are

$$\hat{x}_{k+1|k} = \left(\begin{bmatrix} A & B_d \end{bmatrix} - L_1\bar{C}\right)\hat{x}^a_{k|k-1} + Bu_k + L_1 y_k \tag{4.6a}$$

$$\hat{d}_{k+1|k} = \left(\begin{bmatrix} 0 & I \end{bmatrix} - L_2\bar{C}\right)\hat{x}^a_{k|k-1} + L_2 y_k \tag{4.6b}$$

Considering a generic system G described through state-space variables with (A, B, C, D), its transfer function is

$$G(z) = C\,(zI - A)^{-1}\,B + D \tag{4.7}$$

(4.7) can be also written as

$$G(z) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \tag{4.8}$$

in the rest of this section, we will adopt this notation for the description of transfer functions.

Using superposition principle

$$\hat{x}(z) = f_{y,x}(z)y(z) + f_{u,x}u(z) \tag{4.9a}$$

$$\hat{d}(z) = f_{y,d}(z)y(z) + f_{u,d}u(z) \tag{4.9b}$$

with

- $f_{y,x}$ as transfer function between $y(z)$ and $\hat{x}(z)$

$$f_{y,x} = \left[\begin{array}{c|c} \bar{A} - \bar{L}\bar{C} & \bar{L} \\ \hline \begin{bmatrix} A & B_d \end{bmatrix} - L_1 C & L_1 \end{array}\right]$$

- $f_{u,x}$ as transfer function between $u(z)$ and $\hat{x}(z)$

$$f_{u,x} = \left[\begin{array}{c|c} \bar{A} - \bar{L}\bar{C} & \bar{B} \\ \hline \begin{bmatrix} A & B_d \end{bmatrix} - L_1 C & B \end{array}\right]$$

- $f_{y,d}$ as transfer function between $y(z)$ and $\hat{d}(z)$

$$f_{y,d} = \left[\begin{array}{c|c} \bar{A} - \bar{L}\bar{C} & \bar{L} \\ \hline \begin{bmatrix} 0 & I \end{bmatrix} - L_2 C & L_2 \end{array}\right]$$

- $f_{u,d}$ as transfer function between $u(z)$ and $\hat{d}(z)$

$$f_{u,d} = \left[\begin{array}{c|c} \bar{A} - \bar{L}\bar{C} & \bar{B} \\ \hline \begin{bmatrix} 0 & I \end{bmatrix} - L_2 C & 0 \end{array}\right]$$

4.2.3.2 CL transfer functions

As known, the output of a system y can be considered as a sum of two different contributions, that are the reference tracking problem, represented by the the complementary sensitivity function T, and the disturbance rejection problem, represented by the sensitivity function S.

So, thanks to the Superposition principle, we can write

$$y(z) = T(z)r(z) + S(z)d(z)$$

and $T(z)$ and $S(z)$ are calculated separately.

The algebraic expression for complementary sensitivity T is

$$T = \frac{PQ}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d}} \tag{4.10}$$

while the equation for sensitivity, S, in the general case of disturbances occurring provided with a certain dynamics process P_d:

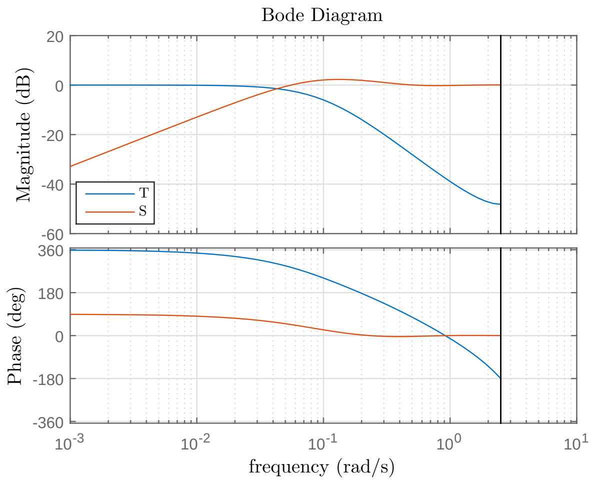


FIGURE 4.4: DOB-IMC Sensitivity functions for open loop stable plants

$$S = \frac{P_d\left(1 + QH_{22}f_{u,d}\right)}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d}} \tag{4.11}$$

Furthermore, there are particular situations:

- case $P_d = 1$, disturbance on the output

$$S = \frac{\left(1 + QH_{22}f_{u,d}\right)}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d}}$$

- case $P_d = P$, input disturbance entering just before the plant

$$S = \frac{P\left(1 + QH_{22}f_{u,d}\right)}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d}}$$

As an example, CL transfer functions for a nominal case $P(s) = G(s)$ with model defined as (2.3) are displayed in Figure 4.4.

These functions appear to be well-designed, since they clearly show that this control structure works as it is supposed to, i.e. the complementary sensitivity rolls off to zero at high frequency and is equal to 1 at low frequencies (it means perfect tracking of the Reference Signal), while the sensitivity rolls off to zero at low frequency.

Further information about how to calculate CL transfer function for a stable configuration of DOB-IMC can be found in Appendix B

4.3 Open loop unstable plants

When the Plant $P(z)$ is OL unstable, i.e. it has one or some unstable poles [1], control scheme provided in Figure 4.2 is not able anymore to stabilize the process output: the structure needs to be modified.

In order to do this, notes given by *Q parametrization* [7], have been very useful, which states that provided that both plant $P(z)$ and controller $Q_{IMC}(z)$ are stable, *the Parametrization of All Stabilizing Controllers* needs:

- a Luenberger observer, whose gain is L;
- a *stabilizing state feedback* F, chosen making sure that

$$\bar{A} = A + BF$$

  is Hurwitz.

The observer already existed in structure in Figure 4.2, so there is only the need to add the state feedback.

Control scheme adopted for Unstable Plants is shown in Figure 4.5

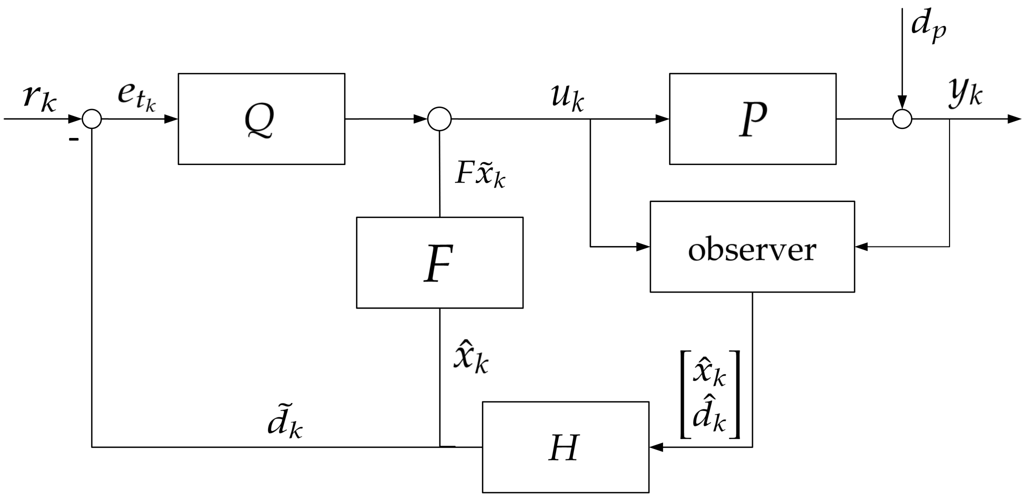


FIGURE 4.5: DOB-IMC block diagram for open loop unstable plants

In the OL stable systems structure, even though there are two outputs exiting from the observer block, i.e. $\hat{x}_k$ and $\hat{d}_k$, only the disturbance estimation $\hat{d}_k$ is fed back to the controller, provided the transformation discussed in Section 4.2.2; here, instead, vector of states $\hat{x}_k$ is a key point for the construction of the control input, since $\hat{x}_k$ first passes

[1] An unstable pole in discrete time is a pole p_1 such that $|p_i| > 1$

through H and then the resulting $\tilde{x}_k$ is sent to the stabilizing state feedback F, and the exiting product $F\tilde{x}_k$ is a part of the final control input expression, that is

$$u_k = v(k) + F\tilde{x}_k$$

.

In this case, the structure of matrix H is $H = \begin{bmatrix} H_{11} & H_{12} \\ 0 & H_{22} \end{bmatrix}$, so the tranformation equation becomes:

$$\begin{bmatrix} \tilde{x}_k \\ \tilde{d}_k \end{bmatrix} = H \begin{bmatrix} \hat{x}_k \\ \hat{d}_k \end{bmatrix} \tag{4.12}$$

Control input exiting from the controller $Q(z)$ is called v_k : definining $e_{fb} = r_k - \tilde{d}_k$,

$$v_k = Qe_{fb}$$

Remark: OL Stable Plant control structure could be considered as a particular case for the general structure shown in Figure 4.5, where F is chosen to be zero.

4.3.1 IMC controller tuning

Let the unstable model be

$$G(z) = z^{-n} \frac{z - a_1}{(z - b_1)(z - b_2)}$$

where b_1 is an unstable pole.

The state-space description for $G(z)$ is

$$\begin{cases} x_{k+1} = Ax_k + Bu_k \\ y_k = Cx_k \end{cases} \tag{4.13}$$

The idea is that the IMC controller should keep working as it already used to do in OL stable configuration, taking care of both tracking system and disturbance problem, while F has, as only task, to make the output y_p stable.

In order to do this, IMC controller has been tuned using a different model from (4.13), since matrix A has an eigenvalue λ_1 standing outside the unit circle: so , Q is built using *the same tuning Rules* descripted in Section 2.3.1, but the model to be inverted during the decomposition is the pre-stabilized one, i.e.

$$\begin{cases} x_{k+1}^p = \hat{A}x_k^p + Bu_k \\ y_k^p = Cx_k^p \end{cases} \tag{4.14}$$

where $\hat{A} = A + BF$ is guaranteed stable as a requirement for the choice of F.

4.3.2 CL transfer functions

Transfer functions for the observer block are those defined in Section 4.2.3.1: expression for the sensitivity functions is

$$T_{unst} = \frac{PQ}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d} - PFH_{11}f_{y,x} - FH_{11}f_{u,x} - PFH_{12}f_{y,d} - FH_{12}f_{u,d}} \tag{4.15}$$

while the sensitivity is, considering a disturbance with a certain dynamics P_d

$$S_{unst} = \frac{P_d\left(1 + QH_{22}f_{u,d} - FH_{11}f_{u,x} - FH_{12}f_{u,d}\right)}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d} - PFH_{11}f_{y,x} - FH_{11}f_{u,x} - PFH_{12}f_{y,d} - FH_{12}f_{u,d}} \tag{4.16}$$

which becomes

- for disturbances on the output, i.e. $P_d = 1$

$$S_{unst} = \frac{\left(1 + QH_{22}f_{u,d} - FH_{11}f_{u,x} - FH_{12}f_{u,d}\right)}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d} - PFH_{11}f_{y,x} - FH_{11}f_{u,x} - PFH_{12}f_{y,d} - FH_{12}f_{u,d}}$$

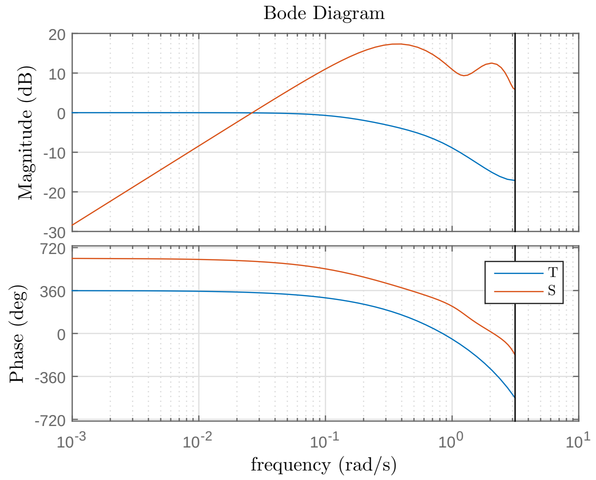
- for input disturbances, $P_d = P$

$$S_{unst} = \frac{P\left(1 + QH_{22}f_{u,d} - FH_{11}f_{u,x} - FH_{12}f_{u,d}\right)}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d} - PFH_{11}f_{y,x} - FH_{11}f_{u,x} - PFH_{12}f_{y,d} - FH_{12}f_{u,d}}$$

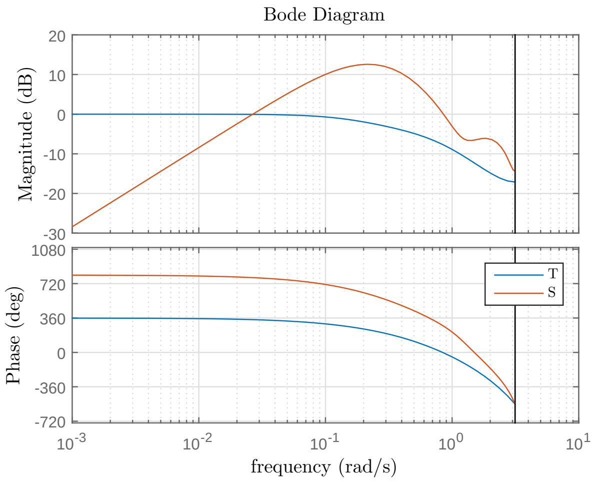
For instance, considering a nominal case with model

$$G(s) = e^{-2s}\frac{-2s+1}{(2s-1)(5s+1)} \tag{4.17}$$

Bode diagrams for T_{unst} and S_{unst} are shown in Figure 4.6: in Figure 4.6a there is an output disturbance and in Figure 4.6b the disturbance occurs immediately before the plant.



(A)



(B)

FIGURE 4.6: DOB-IMC Sensitivity functions for open loop unstable plants,
4.6a: for a step-like disturbance occurring on the output
4.6b: for an input disturbance

From these pictures we can see that disturbances are amplified for high frequencies and rejected at low frequencies, but in an industrial framework high frequencies are not relevant.

For more information about how to obtain the expressions for CL transfer functions in the Unstable configuration, go to Appendix B.

Remark: It is imporant to notice that

$$T + S \neq 1$$

both in stable and unstable plants.

In Internal Model Control, sum between S and T is always equal to 1: this because IMC has a *one degree of freedom controller*; this proposed Disturbance Observer Based IMC, instead, has a *two degree of freedom controller*, since there is more than one tuning parameter, that are:

- in the OL unstable configuration , λ, R_{obs} and R_{fb};
- in the OL stable configuration, λ and R_{obs}

where R_{obs} and R_{fb} are weighting matrices for the choice respectively of the observer L and the stabilizing state feedback F.

Chapter 5

Applications: linear systems

In this chapter DOB-IMC is applied to several kinds of Linear systems; these examples have been chosen to show how this control structure works if applied to different kind of plants and to compare results obtained with well-known control structures, such as Internal Model Control and Model Predictive Control.

Furthermore, these systems have been tested from the robustness point of view as well, in order to verify if this structure could afford some mismatches between Plant and Model, which thing occurs very often in real systems; this is an important aspect, since it is really hard to have a model which is the exact ″copy″ of the Plant; some mismatches might always been taken into account.

Remark: In the whole work, only stricly proper plants are simulated: this is consistent with reality since most of the physical processes are strictly proper; furthermore, it has been done in order to avoid alegraic loops, which would make both DOB-IMC and IMC resolutions in dicrete time not able to give a stable response.

5.1 Procedure

This section briefly summarizes the operations needed to obtain all the elements required for the analysis.

Assuming $P(s)$ to be the plant and $G(s)$ the model, and once chosen a certain sample time T_s, $P(s)$ and $G(s)$ are converted from Laplace transform into z-transform; then, $G(z)$, discrete-time expression for the model, is decomposed into Minimum and Non-Minimum phase, respectively P_{mp} and P_{nmp}, according to Section 2.3.1.1.

Afterwards, there is the inversion of the model, in order to build the ideal controller $Q_{id} = P_{mp}^{-1}$ and then the insertion of the filter, as a sort of "detuning" for the control input: final expression for the controller is

$$Q(s) = \frac{Q_i d(z)}{F(s)} = Q_{id}(s) \frac{1}{(\lambda s + 1)^n} \tag{5.1}$$

in continuous time, and

$$Q(z) = \frac{Q_{id}(z)}{F(z)} = Q_{id}(z) \frac{1-\alpha}{(z-\alpha)^n} \tag{5.2}$$

in discrete time, where $0 \leq \alpha \leq 1$: the greater are λ (or, equivalently, α) and n values, the stronger is the detuning effect: as a result, high values of these parameters give a sluggish output, but with stronger robustness property. It is important to achieve a reasonable trade-off between fast response and robust performance.

Another element required is the observer, chosen accordingly to methods discussed in Chapter 3.

When the system is open loop unstable, a stabilizing state feedback F is needed: this has been here built as optimal, i.e. F is a steady-state Kalman filter[1].

Once done all the preliminary steps, we can see and analyze responses.

Remark: Reading this section, it emerges immediately that every reasoning part refers to continous time plants, in order to make analysis more immediate and easy to undestand; this does not affect the results and the remarks done in the whole chapter.

5.2 First order plants

5.2.1 Open loop stable plant

Let the Plant be

$$P(s) = \frac{1}{5s+1} e^{-\theta s} \tag{5.3}$$

namely a first-order lag plant with delay, and whose pole $p = -\frac{1}{5} \in LHP$.

Time delay belongs to the non-minimum phase part of the plant, so in the decomposition it is separated from the "good" stuff, that is the minimum phase part, selected for the building of the controller.[2].

[1] This is the choice adopted for this work; nevertheless, F could be chosen in other ways, such as using *pole placement technique*, here avoided because considered too much aggressive, since CL poles could be placed too far away from A original eigenvalues

[2] This remark about time delays is explained only here, but it will be still valide for *every* plant provided with delay here analyzed

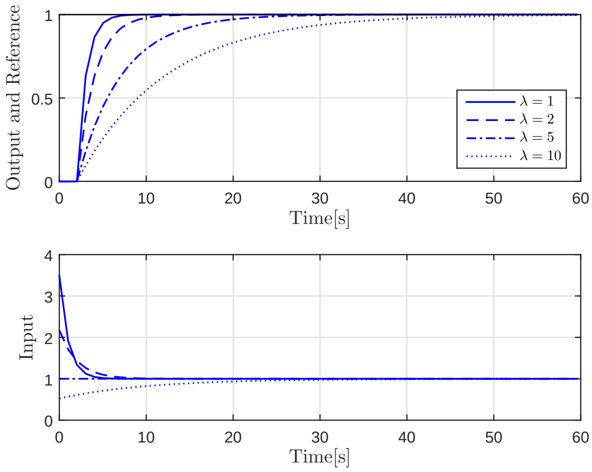


FIGURE 5.1: First order OL stable system, nominal responses for different values of λ

5.2.1.1 Nominal case

For the moment it is assumed not to have mismatches between plant and model, $G(s) = P(s)$.

Responses to the nominal case for a constant step input $r_k = 1$ can be seen in Figure 5.8, for different values of λ: as expected, response becomes sluggish for increasing values of the filter time constant.

As further instrument for analysis, it is possible to study frequency response of CL transfer functions, calculated as explained in Section 4.3.2; Bode diagram of sensitivity functions is represented in Figure 5.2, that shows that complementary sensitivity T rolls off faster for bigger values of the filter constant; on the opposite side, a growing filter constant values determines a slower rolling off of the sensitivy; a fair trade-off between these two opposite trends needs to be found.

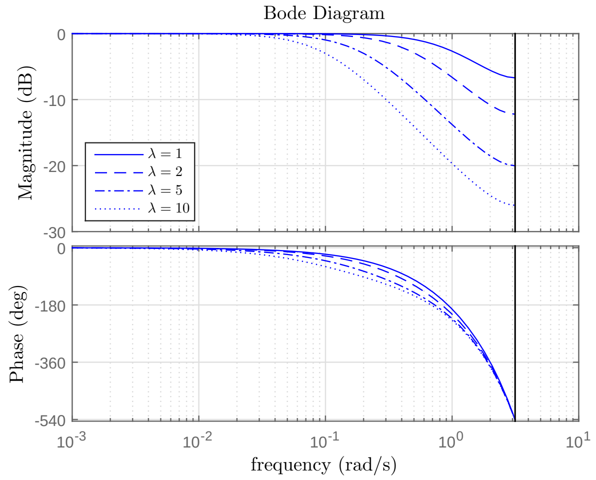
In order to find a good value for λ, robustness analysis could be very helpful.

5.2.1.2 Non-nominal case: robustness analysis

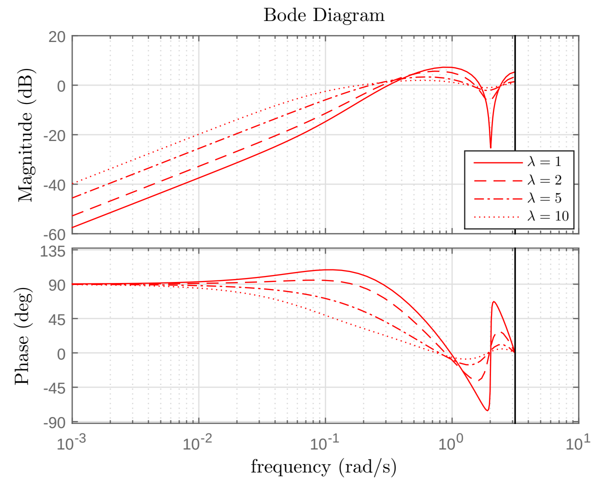
When the model does not exactly represent the actual plant, i.e. $P \neq G$, it is possible to express the mismatch between P and G in terms of *multiplicative uncertainty* l_m, using the following expression:

$$P = G\,(1 + l_m) \tag{5.4}$$

thus



(A)



(B)

FIGURE 5.2: First order OL stable system, closed loop transfer functions:
5.2a: complementary sensitivity
5.2b: sensitivity

$$|l_m| = \left|\frac{P-G}{G}\right| \leq \bar{l}_m \tag{5.5}$$

where $\bar{l}_m$ is the upper bound for l_m.

The case to be considered now is to have (5.3) as model, with equal uncertainty $\sigma = 25\%$ on each parameter, namely τ, K and θ: l_m is obtained through Equation 5.5, and its Bode diagram is shown in Figure 5.3:

So, *generally*, l_m is low at low frequencies and it grows at higher frequencies. It can be defined a set of plants Π such that

$$\Pi = \left\{P : \left|\left(P - \tilde{P}\right)\tilde{P}^{-1}\right| \leq \bar{l}_m\left(\omega\right)\right\} \tag{5.6}$$

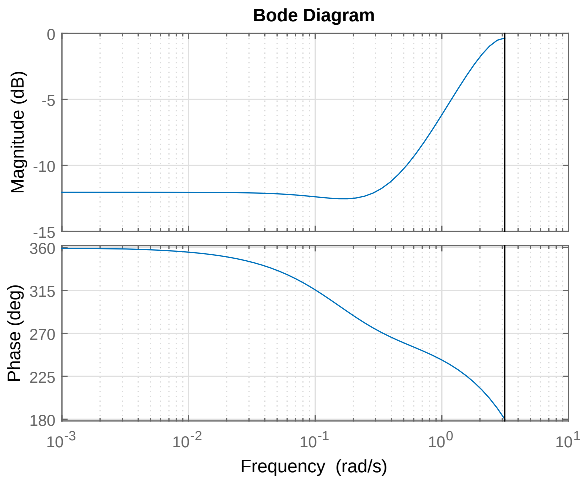


FIGURE 5.3: First order OL stable system, Bode diagram of multiplicative uncertainty l_m

The necessary and sufficient condition for *Robust Stability* is (see [13, 18, 19]

$$|T(i\omega)| \leq \bar{l}_m^{-1}(\omega), \forall\omega \tag{5.7}$$

So, assuming a certain percentage of uncertainty, the trade-off could consist in finding the lowest value of λ able to satisfy (5.7): translated into a Bode diagram framework, this means that complementary sensitivity T plot needs to be at most tangent to $l_m(\omega)^{-1}$ Bode diagram.

For model 5.3, calling K, τ, θ plant parameters and $\tilde{K}, \tilde{\tau}, \tilde{\theta}$ model parameters, and fixing $\sigma = 25\%$ as mismatch on every parameter, namely

$$K = \tilde{K}(1+\sigma) \tag{5.8a}$$

$$\tau = \tilde{\tau}(1+\sigma) \tag{5.8b}$$

$$\theta = \tilde{\theta}(1+\sigma) \tag{5.8c}$$

the filter constant value satisfying (5.7) is $\lambda_{SR} = 0.98$, which corresponds to the red line in Figure 5.4, with the tangency detail zoomed Figure 5.5; thus, it is possible to take any $\lambda \geq \lambda_{SR}$ being sure to satisfy stability robustness condition.

As an example, Figure 5.4 shows nominal and uncertain output response for a filter time constant equal to 3.

Figure 5.6 shows the difference between the nominal output (solid line) and the one with uncertainty on all parameters (dashed line) for a given $\lambda = 1$: in this case, the

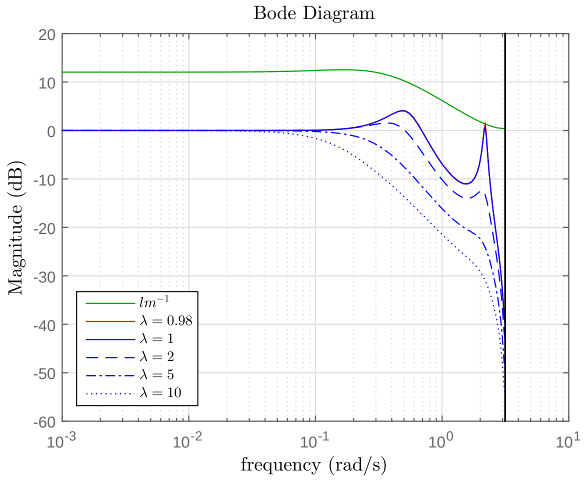


FIGURE 5.4: First order OL stable system, Bode diagram for robust stability

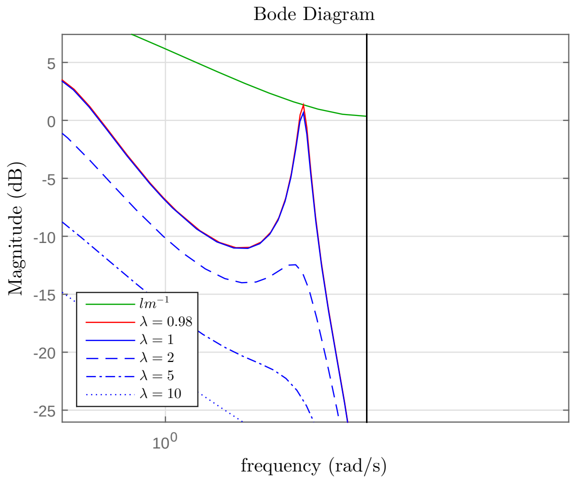


FIGURE 5.5: First order OL stable system, Bode diagram for robust stability; detail of the tangency point between l_m^{-1} and T

situation is quite close to the robust stability limit (it has been made on purpose, in order to show how the control structure works in the worst case scenario), but the response could be made more robust by increasing λ value, keeping in mind that a growing value for the filter constant makes the resulting output slower.

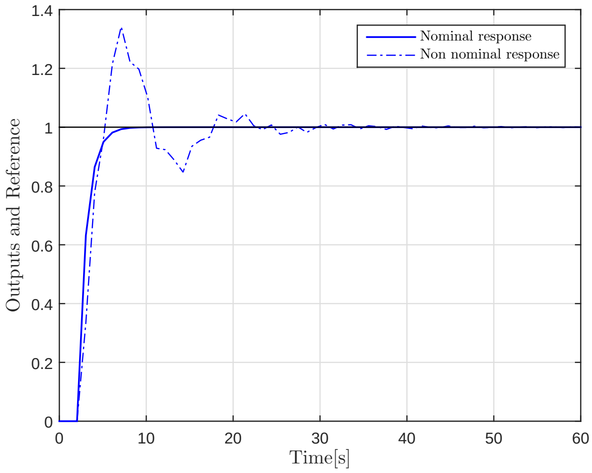
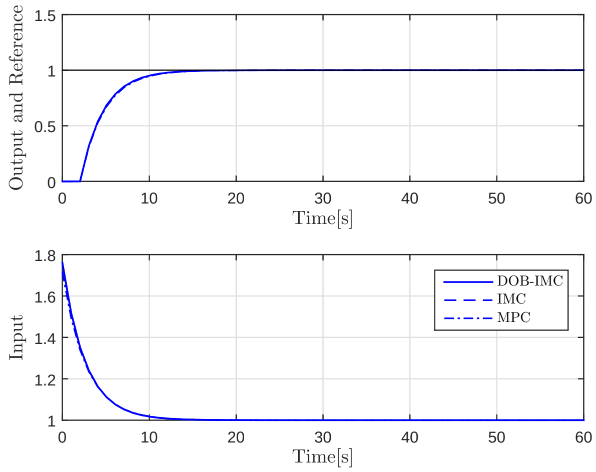


FIGURE 5.6: First order OL stable system, comparison between nominal response, and uncertain response ($\sigma = 25\%$)

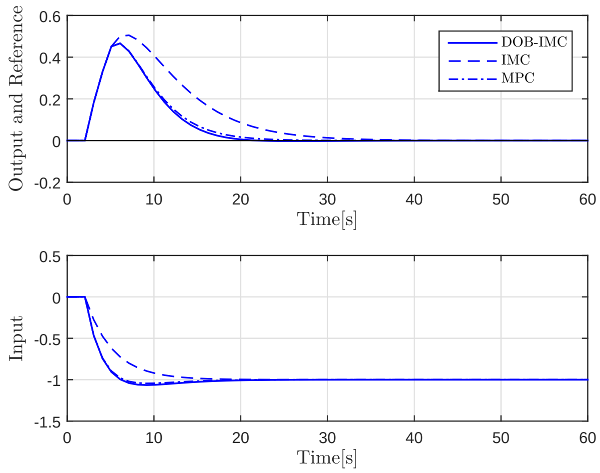
For *each* case here analyzed, tuning parameters have been set in the following way:

- observer parameters are equal in MPC and DOB-IMC;
- λ, tuning parameter for IMC controller, is the same for both IMC and DOB-IMC;
- in order to find some tuning parameters to work with, controllers have been designed in such a way that the output response for every control strategy for a change in set point is the same. Afterwards, disturbance rejection problem is analyzed.

Coming back to the nominal case, once chosen a suitable value for the filter constant, comparison between responses to already existing control schemes IMC and MPC in time domain, are shown in Figure 5.7



(A)



(B)

FIGURE 5.7: First order OL stable system, comparison (in nominal conditions) between IMC, MPC and DOB-IMC

with the following tuning parameters:

1. IMC: $\lambda = 3$, $n = 1$;
2. MPC:
   - for the observer choice, $Q_{obs} = I$ and $R_{obs} = 1$;
   - for the quadratic optimization problem, $Q_{opt} = C^T C$ and $R_{opt} = 0.5$;
   - for the matrices of the augmented model 3.30, $Bd = L_x$ and $C_d = 1 - L_x C$;
3. DOB-IMC:
   - for the IMC standard controller, $\lambda = 3$, $n = 1$;

- for the observer choice, $Q_{obs} = I$ and $R_{obs} = 0.5$;
- for the matrices of the augmented model 3.30, $B_d = L_x$ and $C_d = 1 - L_x C$;

Figure 5.7a shows that, without any disturbance, there is actually no difference between IMC and DOB-IMC, as desired; furthermore, as expected, DOB-IMC works exactly as a classical IMC, since feedback branch $\tilde{d}_k$ is 0.

Figure 5.7b, instead, shows disturbance rejection problem: here we see that, differently from Figure 5.7a, DOB-IMC and IMC exhibit quite different trends, since DOB-IMC response set with these tuning parameters (solid line) is closer to MPC rather than IMC.

This is a first validation of DOB-IMC, since it respects the expectation of faster responses in the case of input disturbances.

5.2.2 Open loop unstable plants

The Plant taken under exam now is

$$P(s) = \frac{1}{5s - 1} e^{-\theta s} \tag{5.9}$$

that has a pole $p = \frac{1}{5} \in RHP$

5.2.2.1 Nominal case

Nominal responses for a step input with different values λ are shown in Figure 5.8, and their corresponding Bode diagrams represented in Figure 5.9: even in this case, results perfectly mirror what was expected, the higher is λ, the slower y_k settles down.

As for the OL stable case, a trade-off value between speed in the response and robust performance, thus a good value for the tuning parameter λ needs to be found.

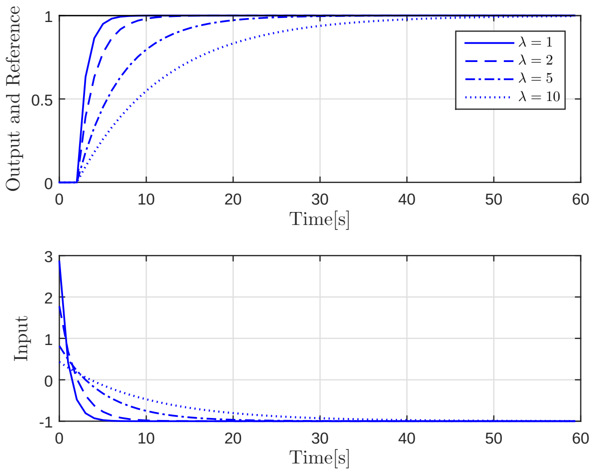


FIGURE 5.8: First order OL unstable system, nominal responses for different values of λ

5.2.2.2 Non-nominal case: robustness analysis

In this case robustness analysis modalities are different because for the OL Unstable Plant control structure defining a multiplicative uncertainty appears quite difficult, since feedback F acts stabilizing the state belonging to system $G = (A, B, C, D)$, while the IMC controller is based on the inversion of the pre-stabilized system, described by (4.14).

So, once fixed an uncertainty value $\sigma = 25\%$ on each plant parameter, different values for tuning parameters have been tested in order to decide which one is the best.

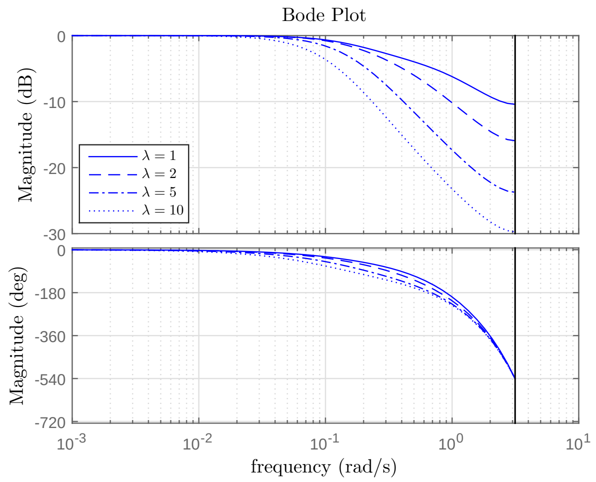
In this case, there is another factor to take into account, since there are not only IMC controller and observer as control instruments, but there is also the stabilizing state feedback F.

Keeping $Q_{fb} = C^T C$ fixed, it is interesting to see how the system reacts to different values of R_{fb}: only for the moment, λ is kept equal to 1.

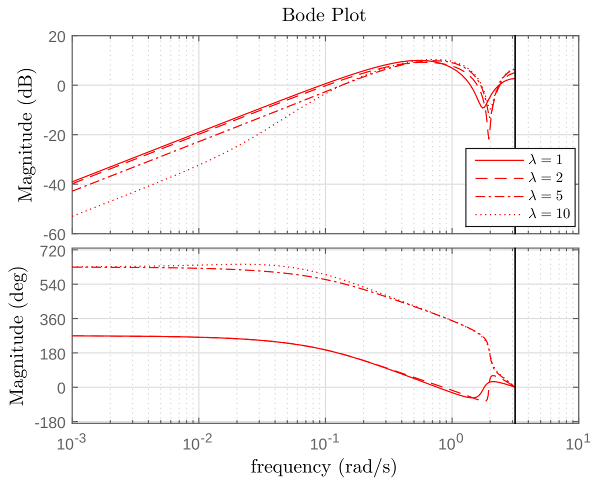
From Figure 5.10 emerges that, apart from $R_{fb} = 0.1$ (case shown separately in Figure 5.10a), for which the corresponding output response is unstable, the choice of R_{fb} does not affect so much the performance; so, it is possible to choose the desired R_{fb} without loss of stability or robustness.

Consider now IMC controller tuning parameters: given a fixed value for R_{fb}, namely $R_{fb} = 0.5$[3], responses for different values of λ need to be investigated: results of the parametric Analysis are shown in Figure 5.11: since for $1 \leq \lambda \leq 5$ time response does

[3]to keep continuity with the previous sections



(A)

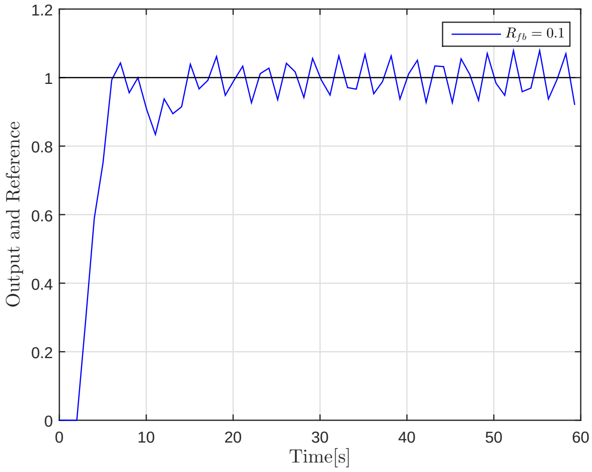


(B)

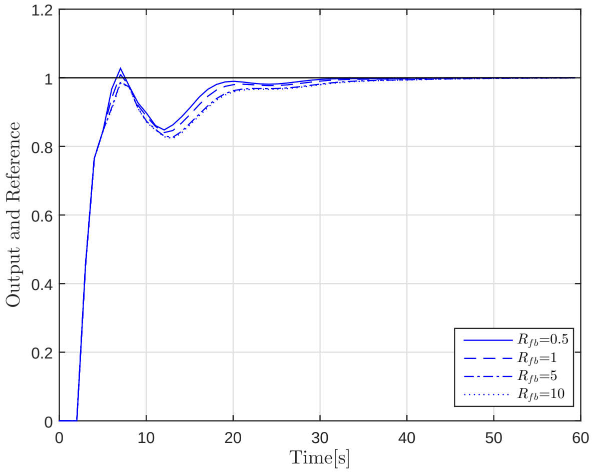
FIGURE 5.9: First order OL unstable system, closed loop transfer functions:
5.9a: complementary sensitivity
5.9b: sensitivity

not change that much, it is possible to assume , as previosly done, $\lambda = 2.5$: this choice has been done in order to keep the same value used for an OL stable plant, and also for having a λ that could be a good compromise between robustness and velocity ($\lambda = 5$ already starts to exhibit a too much sluggish response).

A comparison between nominal and uncertain responses in a situation very close to instability can be looked at in Figure 5.12: since, unfortunately, there is not a specific limit value to test, the situation analyzed is the closest as possible to unstability, i.e. $R_{fb} = 0.11$ and $\lambda = 1$.



(A)



(B)

FIGURE 5.10: First order OL unstable system, robustness test: parametric analysis for different values of the state feedback parameter R_{fb}
5.10a shows response for $R_{fb} = 0.1$
5.10b shows resposes for the other values of R_{fb}*tested*

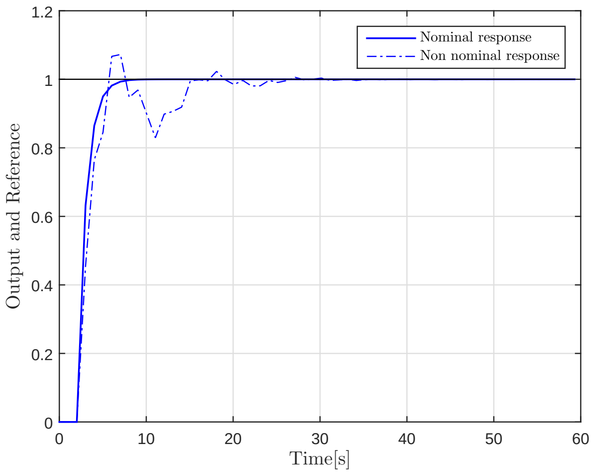


FIGURE 5.12: First order OL unstable system, comparison between nominal an uncertain response ($\sigma = 25\%$)

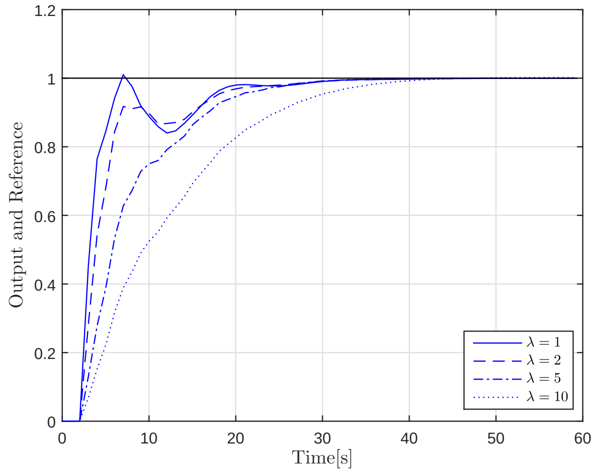


FIGURE 5.11: First order OL unstable system, robustness test: parametric analysis for different values of the filter constant

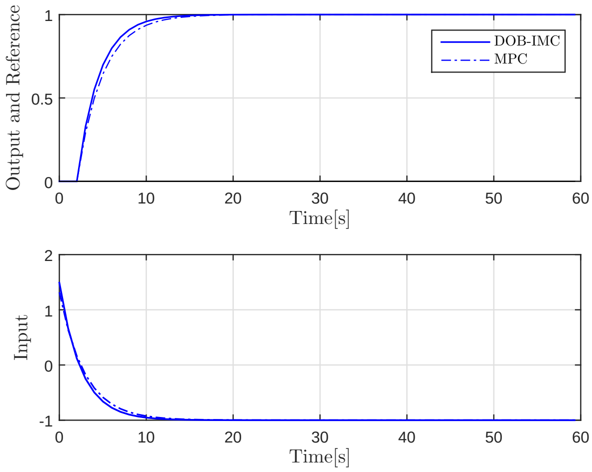
In this case, time response obtained will be compared only to MPC, since classical Internal Model Control does not manage to control OL unstable plants[4].

Tuning parameters are:

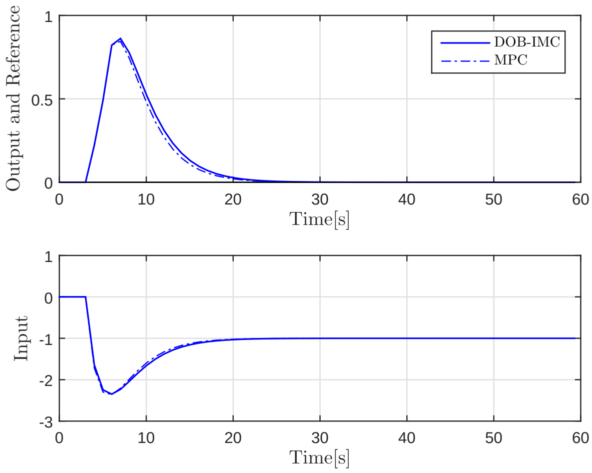
1. DOB-IMC:
   - for the IMC standard controller, $\lambda = 2.5$, $n = 1$;
   - for the stabilizing state feedback F, $Q_{fb} = C^T C$, $R_{fb} = 0.5$;
   - for the observer choice, $Q_{obs} = I$ and $R_{obs} = 1$;
   - for the matrices of the augmented model 3.30, $Bd = L_x$ and $C_d = 1 - L_x C$.
2. MPC:
   - for the observer choice, $Q_{obs} = I$ and $R_{obs} = 1$;
   - for the quadratic optimization problem, $Q_{opt} = C^T C$ and $R_{opt} = 0.5$;
   - for the matrices of the augmented model 3.30, $Bd = L_x$ and $C_d = 1 - L_x C$.

From the comparison of the two different control strategies so tuned, it is easy to observe that DOB-IMC in an OL Unstable configuration is able to achieve results very close to those given by MPC.

[4] In literature there are some works presenting an IMC designed and tuned to control unstable plants: see [13]. Nevertherteless, in this work this configurations has not been taken into account.



(A)



(B)

FIGURE 5.13: First order OL unstable system, comparison (nominal case) between MPC and DOB-IMC

Furthermore, we want to investigate how much the response is affected by different values of λ: from Figure 5.14 it can be seen that, given a fixed value of R, even decreasing λ the output does not become faster than MPC.

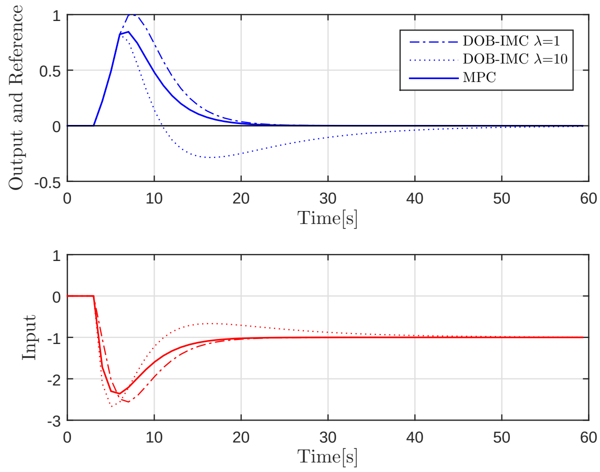


FIGURE 5.14: First order OL unstable system, comparison between MPC and DOB-IMC for different value of λ

5.3 Second order plants

The transfer function for the plant here analyzed is

$$P(s) = e^{-2s} \frac{-2s+1}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{5.10}$$

This is the Laplace function representing a Second Order Plant with Time Delay (SOPTD) and inverse response as an effect of the positive zero for $z_i = 0.5$.

First, classical IMC controller needs to be tuned. The positive zero belongs of the Non-Minimum part of the plant, so P needs to be decomposed in order to separate this term from the part of the plant adopted for the tuning of Q_{IMC}, i.e. the Minimum one.

5.3.0.3 Nominal case

Nominal response for plant (5.10) in time domain with the usual values adopted for λ can be seen in Figure 5.15:

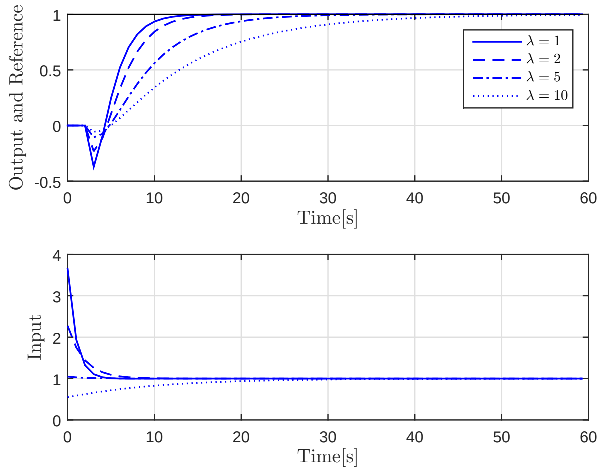
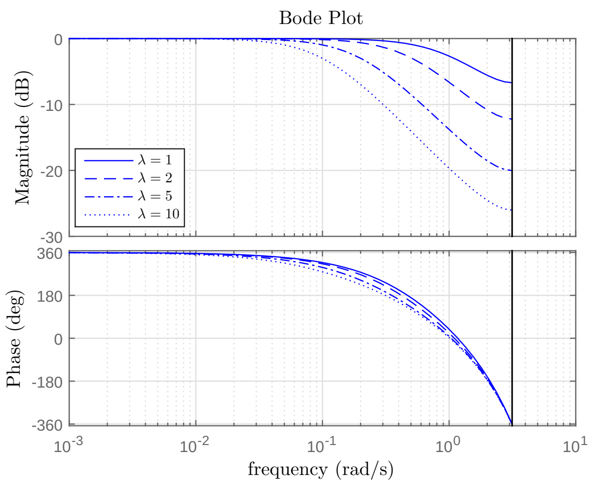


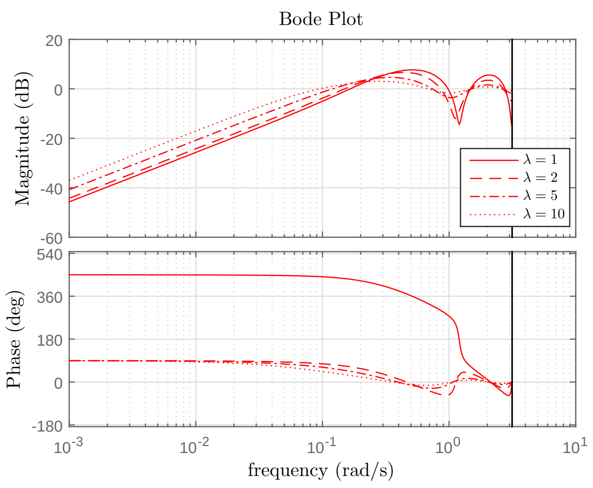
FIGURE 5.15: Second order OL stable system, nominal response for different values of λ

and Bode diagrams of CL transfer functions are represented in Figure 5.16.

In time responses plot we can notice that an high λ gives a slow response but copes better with the inverse response; Bode diagram for transfer functions show that controller has been well designed, and that the rolling off frequency of S and T changes with filter constant values.



(A)



(B)

FIGURE 5.16: Second order OL stable system with inverse response, closed loop transfer functions
5.16a: complementary sensitivity
5.16b: sensitivity

As usual, an optimal value of λ needs to be sought: robustness analysis will be helpful in this field.

5.3.0.4 Robustness analysis

Consider an percentage of uncertainty $\sigma = 25\%$ on each parameter: in other words if the model is

$$G = \tilde{K}e^{-\tilde{\theta}s}\frac{\tilde{\alpha}s+1}{(\tilde{\tau}_1 s+1)(\tilde{\tau}_2 s+1)} \tag{5.11}$$

And expressing plant parameters in terms of σ

$$K = \tilde{K}\,(1+\sigma) \tag{5.12a}$$

$$\alpha = \tilde{\alpha}\,(1+\sigma) \tag{5.12b}$$

$$\tau_1 = \tilde{\tau}_1\,(1+\sigma) \tag{5.12c}$$

$$\tau_2 = \tilde{\tau}_2\,(1+\sigma) \tag{5.12d}$$

$$\theta = \tilde{\theta}\,(1+\sigma) \tag{5.12e}$$

expression for the actual plant P in terms of σ and model parameters is

$$P(s) = Ke^{-\theta s}\frac{(\alpha s+1)}{(\tau_1 s+1)\,(\tau_2 s+1)} \tag{5.13}$$

As done for first order plants, a value of λ able to satisfy robust stability condition needs to be found: Figure 5.17 reveals that (5.7) is satisfied by $\lambda = 1.5$.

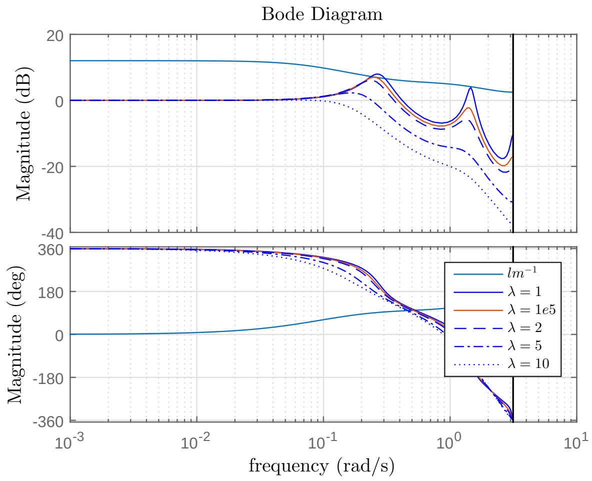


FIGURE 5.17: Second order OL stable system with inverse response, Bode diagram for robust stability

Comparison between nominal response and the one provided with uncertainty is presented in Figure 5.18

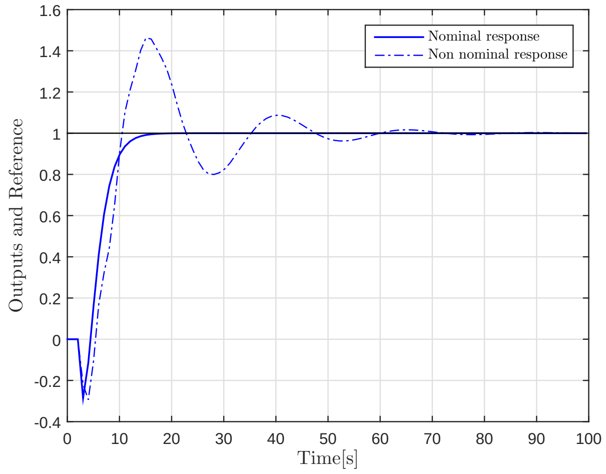
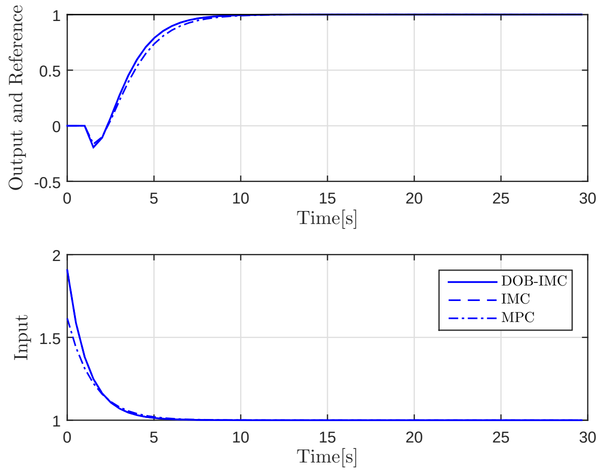
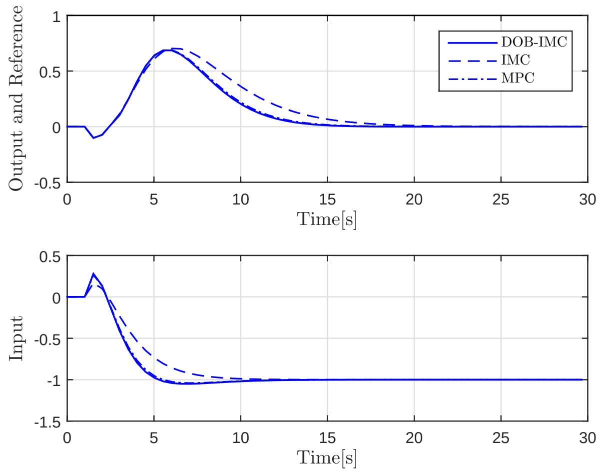


FIGURE 5.18: Second order OL stable system with inverse response, comparison between nominal response and uncertain response ($\sigma = 25\%$)

Once fixed a certain value for the filter time constant, it is possible to look at and subsequently analyze DOB-IMC time responses, compared to classic IMC and MPC, represented in Figure 5.19



(A)



(B)

FIGURE 5.19: Second order OL stable system with inverse response, comparison (nominal case) between IMC, MPC and DOB-IMC

Tuning parameters for the three control scheme are the same as those for first order plants, namely

1. IMC: $\lambda = 3$, $n = 1$;
2. MPC:
   - for the observer, $Q_{obs} = I$ and $R_{obs} = 1$;
   - for the quadratic optimization problem, $Q_{opt} = C^T C$ and $R_{opt} = 0.5$
   - for the matrices of the augmented model 3.30, $Bd = L_x$ and $C_d = 1 - L_x C$
3. DOB-IMC:

- for the IMC standard controller, $\lambda = 3$ and $n = 1$;
- for the observer choice, $Q_{obs} = I$ and $R_{obs} = 0.5$;
- for the matrices of the augmented model 3.30, $Bd = L_x$ and $C_d = 1 - L_x C$

In Figure 5.19a, responses are very close one to each other; in Figure 5.19b instead, trends of DOB-IMC and MPC are still equite close, while IMC, nevertheless has the same λ as DOB-IMC,this time gives a slower response, for the reasons previously explained.

DOB-IMC response can be further made faster or even slower by respectively decreasing or increasing the tuning parameter value.

5.4 Integrating plants

In this section there will be a short description of how this kind of plants could be controlled adopting DOB-IMC strategy, which thing should not taken for granted, since integrating dynamics are provided with some features that make it difficul to control them with this control structure.

5.4.1 Definitions and problems of an integrating plant

In continuous time, an integrating Plant has a pole in the origin, namely $p_i = 0 :$ an example of integrator in Laplace transform is shown in the following equation:

$$P(s) = \frac{K}{s} \tag{5.14}$$

This type of plants cannot be implemented in DOB-IMC: in order to understand why this implementation faces difficulties, it could be helpful converting $P(s)$ from continuous time to dicrete time.

In a discrete time context, integrator is a process with a pole $p_i = 1$, at the *border of the stability region*: an example of a discrete-time integrator is given by the following equation:

$$P(z) = \frac{K}{z-1} \tag{5.15}$$

When using a state-state description for (5.15), with its relative set of matrices (A, B, C, D) we see that matrix A has en eigenvalue in 1: this makes things difficult from a point of view of implementation, since transformation matrix described by (4.12) cannot be computed because of its very high conditiong number, which makes its inversion impossible.

The solution adopted to overcome this problem is to represent the integrating plant with a model slightly different from (5.14): namely, G should be modeled as a first order model with an high lag:

$$G(s) = \frac{K\tau}{\tau s \pm 1} \tag{5.16}$$

with $\tau \gg 1$.

It is possible to adopt this approximation because the limit of G for high values of time constant τ goes to the system gain K:

$$\lim_{\tau\to\infty} G(t) = \lim_{\tau\to\infty} K \frac{\tau}{\tau\left(s \pm \frac{1}{\tau}\right)} = \frac{K}{s} \tag{5.17}$$

This way of acting naturally involves *robustness problems*, since representing the plant with (5.16) clearly implies a plant/model mismatch.

The model can be chosen to be whether OL stable or unstable, and could be implemented in both configurations analized in Chapter 4: after several trials (here omitted), it seems that the configuration giving the best behaviour is *stable or unstable model applied to the OL unstable scheme.*

In order to analyze scheme behaviour in presence of integrating plants, several analysis needs to be done.

5.4.2 Parametric analysis

Let the plant be despcripted by a pure integrator as defined in (5.14): it is assumed the model to be

$$G(s) = \frac{K\tau}{\tau s + 1} \tag{5.18}$$

The attempt is to control it through the OL unstable Configuration scheme of DOB-IMC.

5.4.2.1 Parametric analysis for τ

First thing to do consists of searching for a time constant value τ which good represents the plant:

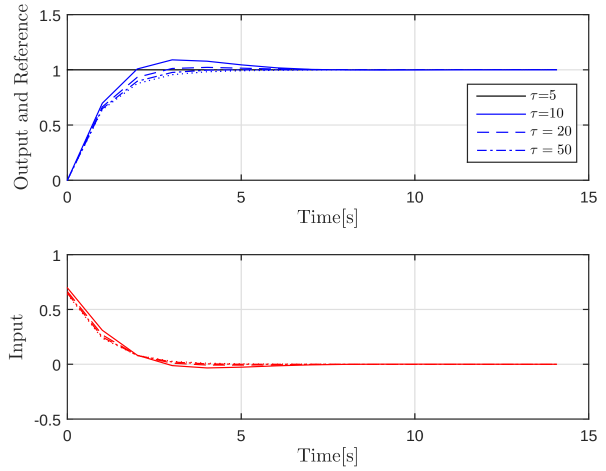


FIGURE 5.20: Integrating plant, time response for different choices of τ

In Figure 5.20 there are time responses for different values of time constant (assuming, for the moment, λ to be 1): it is evident that, for $\tau \geq 10$, there is no such a big difference in the behaviour of time responses ; thus, it is assumed to be $\tau = 20$, since zooming the figure, whose zoom is shown in Figure 5.21, this is the lowest value for which y does not exhibit underdamped response.

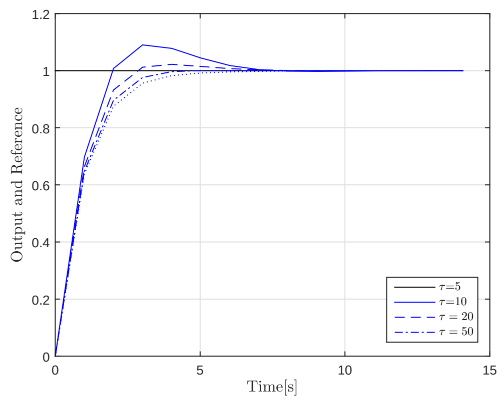
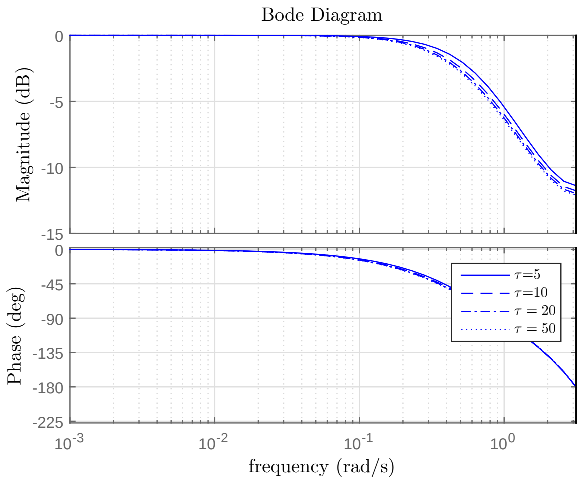
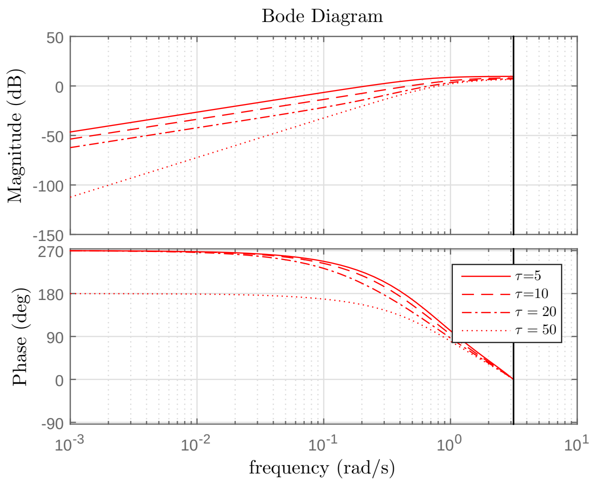


FIGURE 5.21: Integrating plant, zoom on time response for different choices of τ

Transfer functions for different values of τ can be found in Figure 5.22: the higher is τ, the faster both sensitivities roll off to zero, but difference between different values of time constant is not very relevant.



(A)



(B)

FIGURE 5.22: Integrating plant, closed loop transfer functions for different values of τ
5.22a: complementary sensitivity
5.22b: sensitivity

5.4.2.2 Parametric analysis for λ

Afterwards, a good value for λ needs to be sought: so, different filter constants have been tested, and results are shown in Figure 5.23:

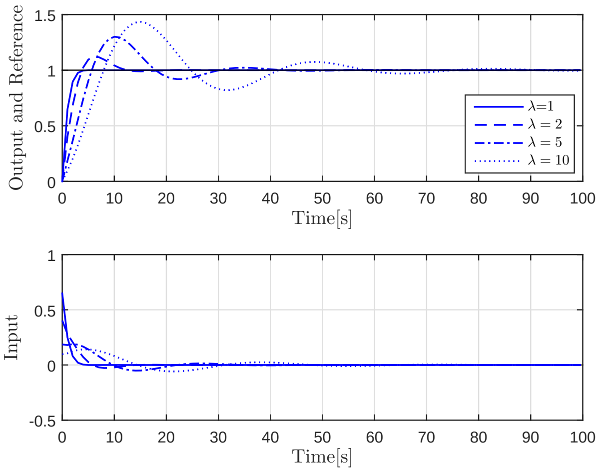
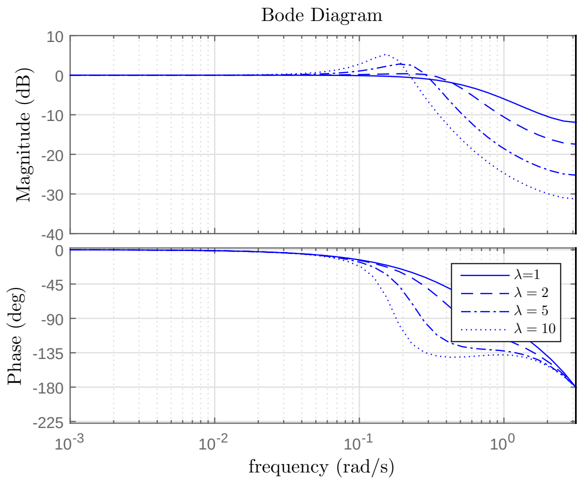


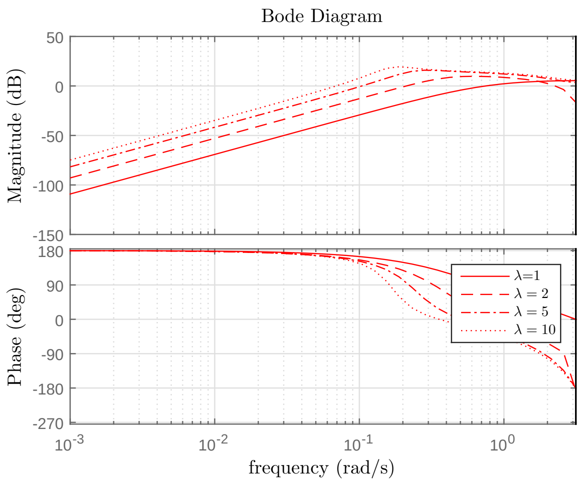
FIGURE 5.23: Integrating plant, time response for different choices of λ

Looking at this pictures, it clearly emerges that the smaller is λ, the better is the resulting performance, since an high value for the filter constant determines a response more *fluctuating*: this even because an integrating system needs to be kept under control, so filter detuning action does not need to be too much excessive).

Figure 5.24 shows CL transfer functions for values of λ analyzed.



(A)



(B)

FIGURE 5.24: Integrating plant, closed loop transfer functions for different values of λ
5.22a: complementary sensitivity
5.22b: sensitivity

Growing values of filter time constant determine a faster rolling off for T, but, on the other hand, S reels off slower; furthermore, values different from 1 generate a peak at mid frequencies.

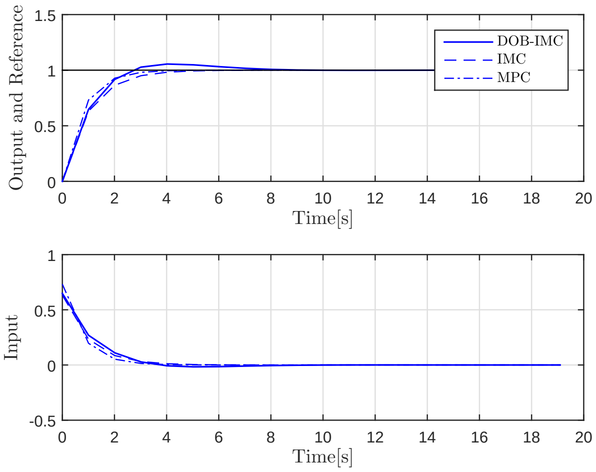
For all there reasons, *λ is assumed to be 1*.

Once fixed model and controller tuning parameters, it could be interesting to look at the comparison of performances between IMC, MPC and DOB-IMC; tuning parameters, this time, are the following (R_{fb} has been kept, for the moment, equal to 0.5 for a matter of continuity with other sections):

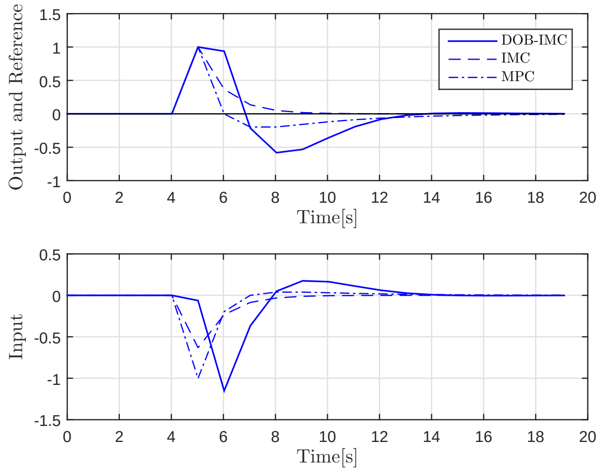
1. IMC: $\lambda = 1, n = 1$;

2. MPC:
    - for the observer, $Q_{obs} = I$ and $R_{obs} = 1$;
    - for the quadratic optimization problem, $Q_{opt} = C^T C$ and $R_{opt} = 0.5$
    - for the matrices of the augmented model (3.30), $B_d = L_x$ and $C_d = 1 - L_x C$
3. DOB-IMC:
    - for the IMC standard controller, $\lambda = 1$;
    - for the observer choice, $Q_{obs} = I$ and $R_{obs} = 1$;
    - for the stabilizing state feedback, $Q_{fb} = C^T C$ and $R_{fb} = 0.5$;
    - for the matrices of the augmented model 3.30, $B_d = L_x$ and $C_d = 1 - L_x C$

Figure 5.25 shows responses for, respectively, reference tracking problem and disturbance rejection: for the case of a change in the set point, represented in Figure 5.25a, we see that DOB-IMC, designed with tuning parameters previously specified, leads to underdamping in the output; this behaviour can be better noticed in Figure 5.26. In Figure 5.25b, instead, a step disturbance $d_p = \frac{1}{s}$ occurs at the output: it is immediate to notice that DOB-IMC response is worse than IMC and MPC, but we have to consider that model (5.18), implemented in DOB-IMC, is different from the plant itself, while both IMC and MPC have been implemented in nominal conditions, in which $G = P$.



(A)



(B)

FIGURE 5.25: Integrating plant, comparison (nominal case) between IMC, MPC and DOB-IMC

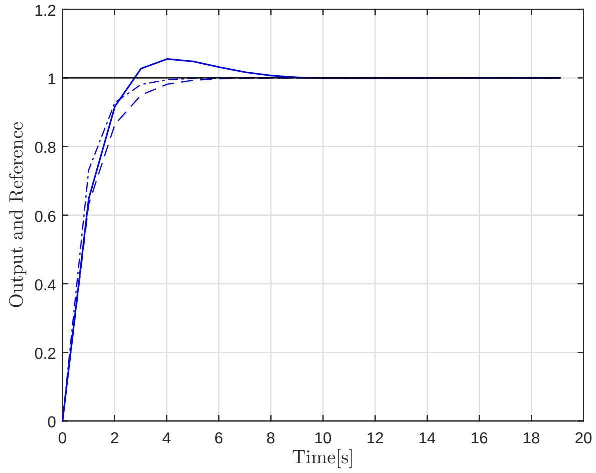
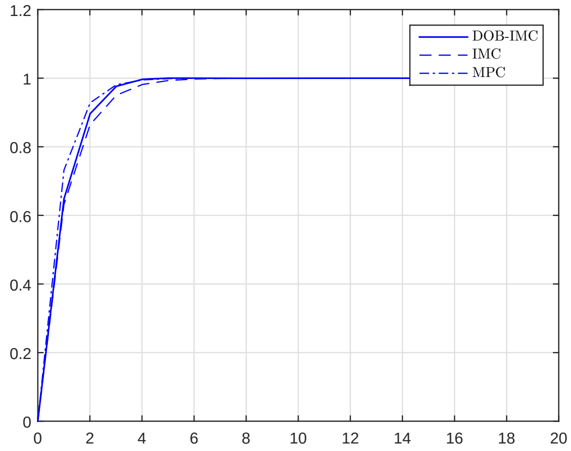
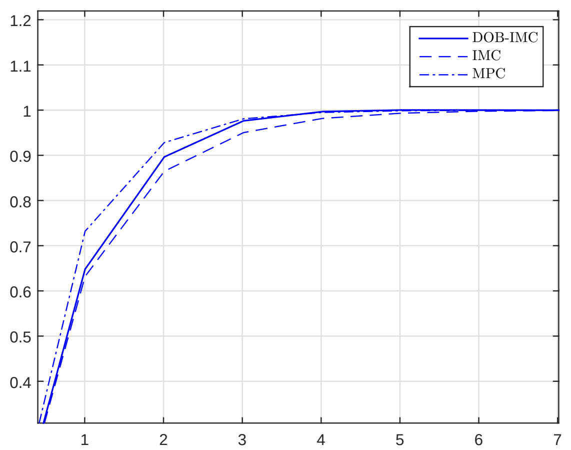


FIGURE 5.26: Integrating plant, detail of comparison between IMC, MPC and DOB-IMC for the case of a change in set point

In order to obtain a better response, different actions could be done: for instance, increasing R_{opt} to 1 (keeping λ fixed); improvement in the result can be seen in Figure 5.27



(A)



(B)

FIGURE 5.27: Integrating plant, comparison (nominal case) between IMC, MPC and DOB-IMC, with an increased value for R_{fb}, with a particular zoomed on Figure 5.27b

and the response is not underdamped anymore. The behaviour of this structure when faced to a pure integrating plants is considered as a big result achieved, since DOB-IMC initially structure is not thought for this kind of systems; however, since it is still slower than IMC and MPC, development of DOB-IMC for integrating plant should be further improved.

Chapter 6

Applications: simulated industrial process

In this chapter DOB-IMC has been applied to a well-known case study, useful to see how DOB-IMC copes with processes closer to reality, and not only with hypotetical systems.

Case analyzed is the ”Shell heavy oil fractionator”, whose definition is taken from [12]: it does not represent a real situation, but it is implemented to show and analyze how DOB-IMC deals with multivariable systems.

6.1 Process summary

The distillation column works as a fractionator, as shown in Figure 6.1:

the feed enters the column at the bottom, in a gaseous form. There are three products exiting the column, drawn off respectively at the top, side and bottom of the fractionator: they can be seen on the right-hand side of the fractionator; furthermore, on the left-hand side of the column there are three circulating reflux, again at the top, middle and bottom of the column: they have the task to remove heat carried into the column by the feed, giving it to other processes demanding for thermal duties, that could be, for instance, other fractionator columns. Heat removal is made through heat exchangers: the amount of heat removed by each reflux is defined as *heat duty*. Gains from heat duties to temperatures are defined positive. Heat removed in the two top reflux depends on the demand of the other processes: an higher duty corresponds to more heat recirculated back into the fractionator (then, it also corresponds to a smaller amount of heat given to other processes). Intermediate reflux duty is assumed to be a *measured*

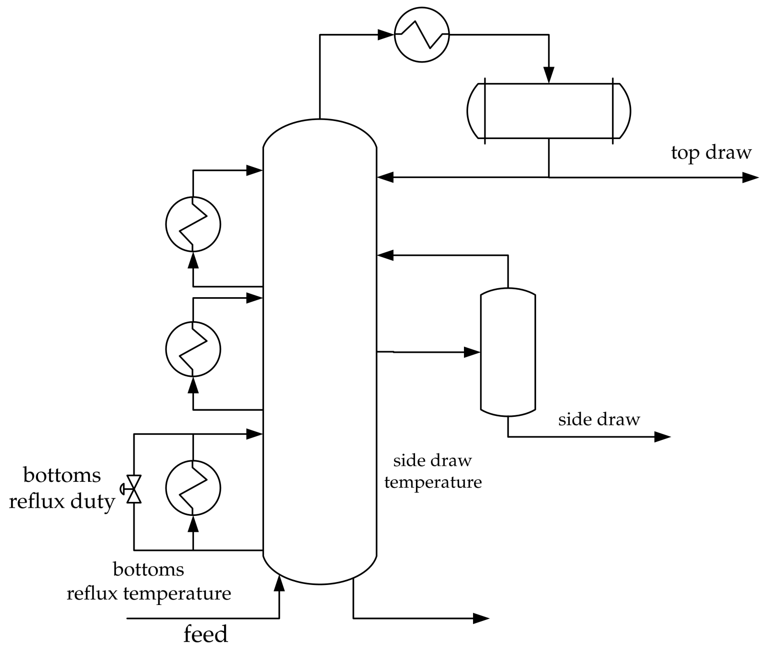


FIGURE 6.1: Shell oil fractionator

disturbance, so it can be controlled by means of a feedforward controller, while upper reflux duty acts as an *unmeasured disturbance*, being thus object of interest in our implementation.

On the other hand, bottoms reflux duty is used as *manipulated variable* , so it can be used to control the process: however, since it is used to generate steam sent other units, even if bottoms reflux duty is a manipulated variable, our interest is to keep it the lowest possible, for an evident economic advantage.

Top Draw and Side Draw are *manipulated variables* as well.

Controlled variables are the following:

- Composition of top end point flow;
- Composition of side end point flow;
- Bottoms reflux temperature.

Table 6.1 delineates and summarizes role and symbol of each variable taken into account.

TABLE 6.1: Classification of each variable with its role and symbol

Variable	Role	Symbol
Top Draw	Control Input	u_1
Side Draw	Control Input	u_2
Bottoms Reflux Duty	Control Input	u_3
Intermediate Reflux Duty	Measured Disturbance	d_m
Upper Reflux Duty	Unmeasured Disturbance	d_u
Top End Point	Controlled Output	y_1
Side End Point	Controlled Output	y_2
Bottoms Reflux Temperature	Controlled Output	y_3

Now, transfer functions of the process need to be defined. Tables 6.2 and 6.3 contains information about transfer functions between inputs and outputs of the MIMO system and of disturbances as well[1].

TABLE 6.2: Transfer functions for the MIMO process

Variables	u_1	u_2	u_3
y_1	$4.05e^{-27s}\frac{1}{50s+1}$	$1.77e^{-28s}\frac{1}{60s+1}$	$5.88e^{-27s}\frac{1}{50s+1}$
y_2	$5.39e^{-18s}\frac{1}{50s+1}$	$5.72e^{-28s}\frac{1}{60s+1}$	$6.90e^{-15s}\frac{1}{40s+1}$
y_3	$4.38e^{-20s}\frac{1}{33s+1}$	$4.42e^{-22s}\frac{1}{44s+1}$	$7.20\frac{1}{40s+1}$

TABLE 6.3: Transfer functions of unmeasured disturbances

Variables	d_m
y_1	$1.20e^{-27s}\frac{1}{45s+1}$
y_2	$1.52e^{-15s}\frac{1}{25s+1}$
y_3	$1.14\frac{1}{27s+1}$

So, the MIMO transfer function of this multivariable system is

$$G = \begin{bmatrix} 4.05e^{-27s}\frac{1}{50s+1} & 1.77e^{-28s}\frac{1}{60s+1} & 5.88e^{-27s}\frac{1}{50s+1} \\ 5.39e^{-18s}\frac{1}{50s+1} & 5.72e^{-28s}\frac{1}{60s+1} & 6.90e^{-15s}\frac{1}{40s+1} \\ 4.38e^{-20s}\frac{1}{33s+1} & 4.42e^{-22s}\frac{1}{44s+1} & 7.20\frac{1}{40s+1} \end{bmatrix} \tag{6.1}$$

[1]We will *not* deal with measured disturbance, since they are kept under control through a feedforward controller. Building this kind of additional controller goes beyond the aim of this work.

All transfer functions are linear systems of first order provided with time delay.

A sample time $T_s = 1$ has been here chosen.

In the following pages there will be results of implementation of 3×3 "Shell" case controlled through DOB-IMC; results will be furthermore compared to those obtained with classical IMC, in order to see if improvements in the resulting performance could be achieved or not.

6.2 Simulation

The implementation of this case with three manipulated variables (MV) and three controlled variables (CV) practically consists of building three different decentralized controllers, having so three different responses, taking into account the fact the output y_i given by each system "suffers" for interactions between itself and the others.

6.2.1 Interactions and relative gain array

First thing to do is choosing the best pairing between inputs with outputs, hence *Relative Gain Array(RGA)* , Λ has been evalued.

Elements of Λ are so defined:

$$\lambda_{ij} = \frac{\left(y_i/u_j\right)_{u_{l \neq j}=0}}{\left(y_i/u_j\right)_{y_{l \neq j}=0}} \tag{6.2}$$

with

$$\sum_{i=1}^{n} \lambda_{ij} = 1 \tag{6.3a}$$

$$\sum_{j=1}^{n} \lambda_{ij} = 1 \tag{6.3b}$$

Referring to MIMO transfer function matrix (6.1), Λ can be obtained as

$$\Lambda = G \circ \left(G^{-1}\right)^T \tag{6.4}$$

The resulting RGA for the process here analyzed is

$$\Lambda = \begin{bmatrix} 2.0757 & -0.7289 & -0.3468 \\ 3.4242 & 0.9343 & -3.3585 \\ -4.4999 & 0.7946 & 4.7056 \end{bmatrix} \tag{6.5}$$

which suggests the following couplings:

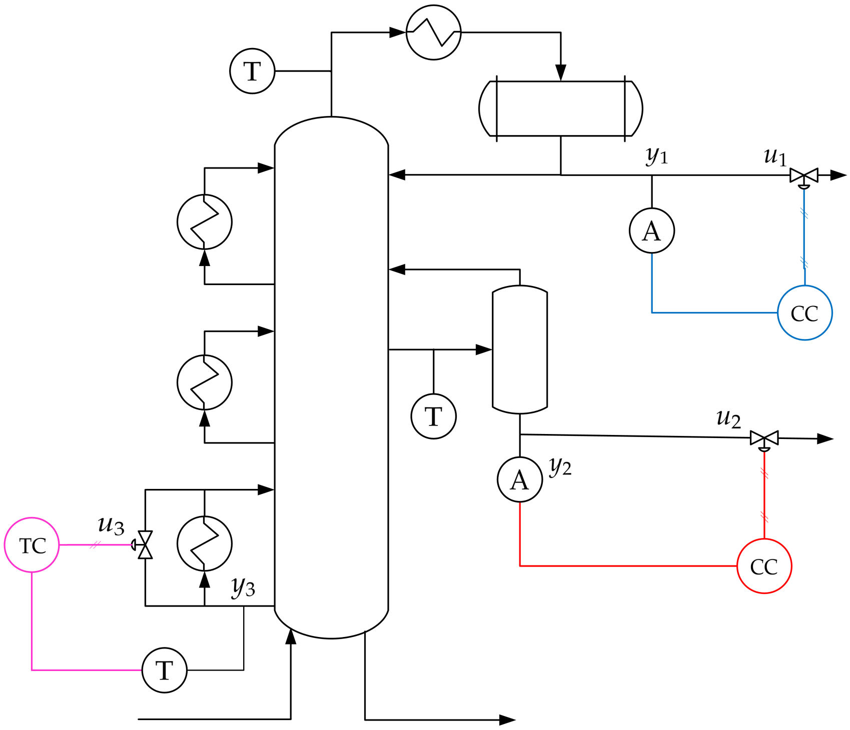


FIGURE 6.2: Shell oil fractionator with decentralized controllers

1. $u_1 \rightarrow y_1$
2. $u_2 \rightarrow y_2$
3. $u_3 \rightarrow y_3$

So, the expression of outputs y_i is

$$y_1 = g_{11}u_1 + g_{12}u_2 + g_{13}u_3 \tag{6.6a}$$

$$y_2 = g_{21}u_1 + g_{22}u_2 + g_{23}u_3 \tag{6.6b}$$

$$y_3 = g_{31}u_1 + g_{32}u_2 + g_{33}u_3 \tag{6.6c}$$

Figure 6.2 gives a complete framework of the case study here analyzed, provided with decentralized controllers: blue controller stands for system 1, red controller stands for system 2 and magenta line represents system 3.

Remark: Please note that $\Lambda(1,1) = 2.0757$ and $\Lambda(3,3) = 4.7056$, both > 1: they are signals of strong interactions between considered system and the others; $\Lambda(2,2)$, instead, is quite close to 1, so interactions between system 2 and the others is weak.

TABLE 6.4: Observers Parameters

System	1	2	3
Q_{obs_i}	$C_{11}^T C_{11}$	$C_{22}^T C_{22}$	$C_{33}^T C_{33}$
R_{obs_i}	0.5	0.5	0.5

6.2.2 Tuning

Three differents DOB-IMC controllers needs to be designed, inteverting, respectively

$$g_{11} = G(1,1) = 4.05e^{-27s}\frac{1}{50s+1} \tag{6.7a}$$

$$g_{22} = G(2,2) = 5.72e^{-28s}\frac{1}{60s+1} \tag{6.7b}$$

$$g_{33} = G(3,3) = 7.20\frac{1}{40s+1} \tag{6.7c}$$

this means that each control scheme has its own parameters to tune; furthermore, since there are strong interactions between systems, the effect of the tuning of a single controller does influence not only the behaviour of its system, but has also impact on the other two.

Furthermore, IMC controllers have also been designed, in order to compare the two control schemes.

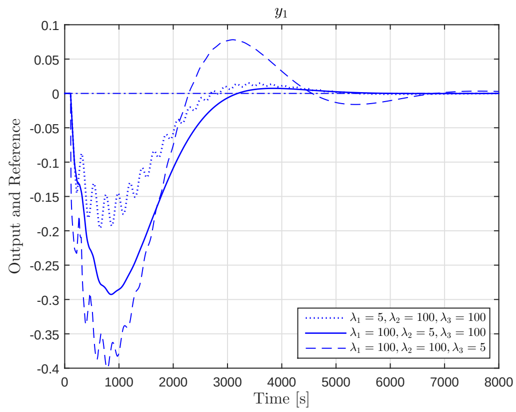
Several tests for a correct tuning have been done in order to find a configuration able to give good performances as results: as first choice, observer parameters Q_{obs_i} and R_{obs_i} have been kept fixed, and λ_i has been changed: this decision is due to several reasons, among which the most important is that changing λ_i seems to be more effective than working on the observer parameters.

First thing done has been fixing relative values of λ_1, λ_2, λ_3, tuning parameters of the three IMC controllers: considering that $\Lambda(1,1), \Lambda(3,3) >> \Lambda(2,2)$, we have to think that systems 1 and 3 needs to be detuned more than system 2, that does not interact that much in the MIMO, so the first choice adopted is

$$\lambda_1, \lambda_3 >> \lambda_2 \tag{6.8}$$

In order to prove the goodness of this choice, once fixed observer values as shown in Table 6.4, DOB-IMC has been tried with different combinations of λ_i, with results shown in Figure 6.3, subdivided into three different pictures, one for each output

- $\lambda_1 = 5, \lambda_2 = 100, \lambda_3 = 100$, represented by dotted line in Figure 6.3;



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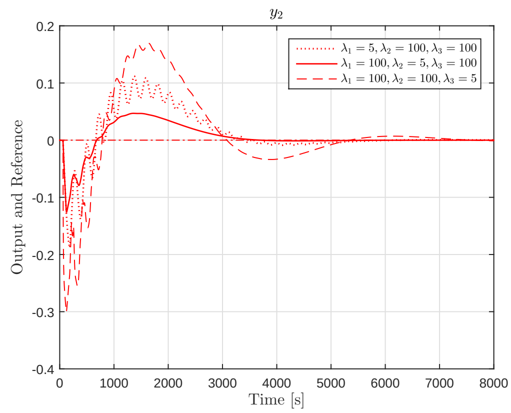

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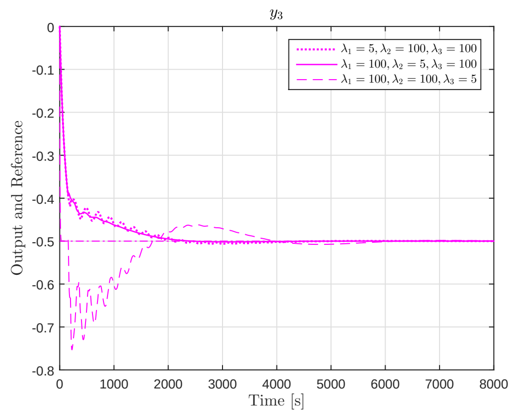


(C)

FIGURE 6.3: Time responses for different choices of $\lambda_1, \lambda_2, \lambda_3$:
6.3a: y_1, composition of top draw
6.3b: y_2, composition of side draw
6.3c: y_3, bottoms reflux temperature

- $\lambda_1 = 100, \lambda_2 = 5, \lambda_3 = 100$ represented by solid line in Figure 6.3;
- $\lambda_1 = 100, \lambda_2 = 100, \lambda_3 = 5$ represented by dashed line in Figure 6.3;

Solid line appears always to be the best situation among the three implemented. So, 6.8 appears now a justified choice; a similar analysis revealed that the relative value between λ_1 and λ_3 has not much influence on responses.

So, tuning parameters here adopted are

TABLE 6.5: Tuning Parameters DOB-IMC for the multivariable system "Shell oil fractionator"

System	1	2	3
λ	100	8	100
R_{obs}	1	0.1	0.1

6.2.3 Performance and comparison with IMC

Responses obtained with tuning parameters listed in Table 6.5 are now compared to classic Internal Model Control; in order to have results the more comparable as possible, each IMC controller has the same values of λ_i that it used to have in DOB-IMC.

6.2.3.1 Reference tracking problem

Figure 6.4 is referred to the case in which there is a change in set point of system 3, $r_3 = -0.5$, which is desidered to be kept the lowest possible, while the others are constant (here we fixed $r_1 = 0, r_2 = 0$).

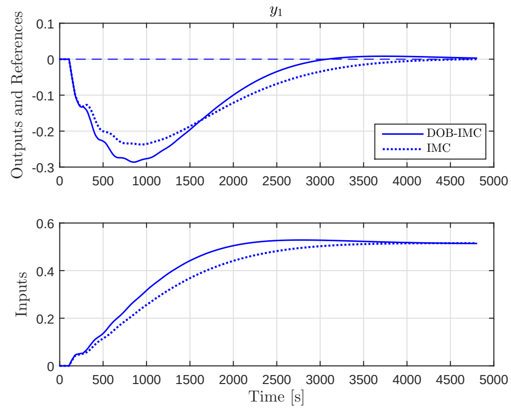
Differently from the SISO case, here responses given by DOB-IMC are quite different from those of IMC, even without disturbances occurring: this happens because interactions between systems act themselves as disturbances for each control stucture. Here we see that DOB-IMC responses are still quite fluctuating before settling down; nevertheless, some better performance, for instance that given by system 3, can be noticed.

There have been done simulations also in cases in which there are changes in Set Point of system 1 and 2: obtained results have been here omitted because they does not change that much compared to the case shown in Figure 6.4.

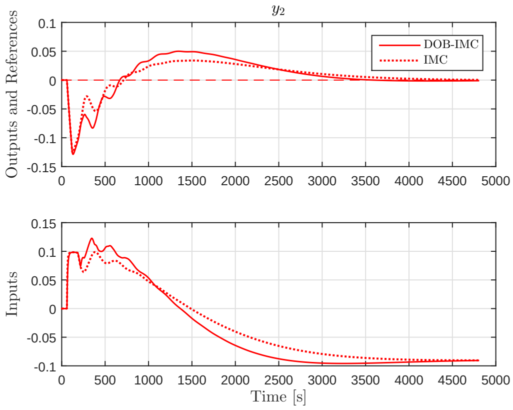
6.2.3.2 Disturbance rejection problem

Figure 6.5 instead, shows responses when an unmeasured disturbance occurs (disturbance enters in Upper Reflux Duty, and it affects each system with a different dynamics. See Table 6.3 for the list of dynamic disturbance occurring on each system).

Results show that DOB-IMC copes dyanamic disturbance better than IMC: response is still fluctuating, but we can see here that there are less peaks and a faster disturbance rejection.



(A)



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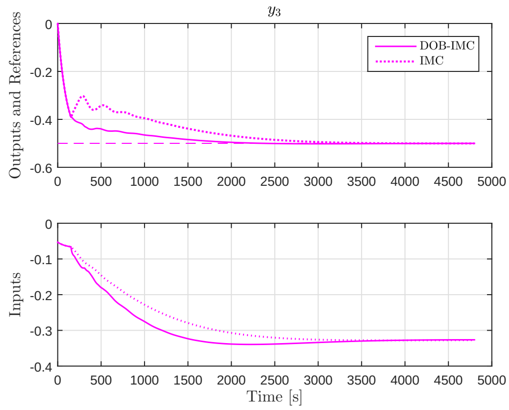
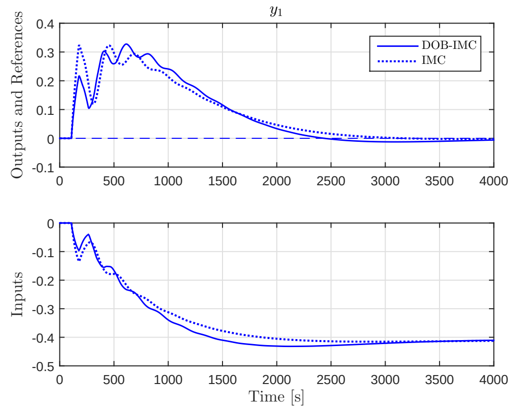
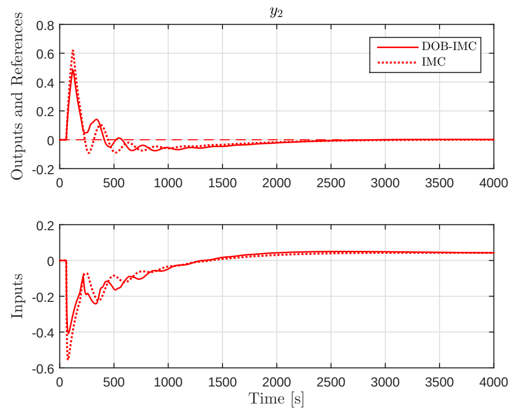


(C)

FIGURE 6.4: Time response, comparison between DOB-IMC and IMC for a change in set point of system 3
6.3a: y_1, composition of top draw
6.3b: y_2, composition of side draw
6.3c: y_3, bottoms reflux temperature



(A)



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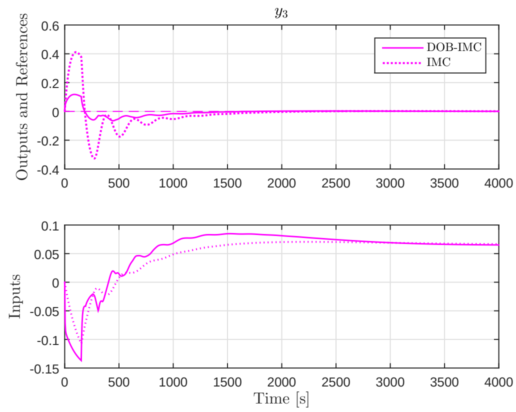

(C)

FIGURE 6.5: Time response, comparison between DOB-IMC and IMC for an input disturbance
6.5a: y_1, composition of top draw
6.5b: y_2, composition of side draw
6.5c: y_3, bottoms reflux temperature

Chapter 7

Summary and conclusions

Starting from conventional Internal Model Control (IMC) and looking at features of Linear Quadratic Gaussian (LQG) control and Model Predictive Control (MPC), we have here implemented a Disturbance Observer Based Internal Model Control (DOB-IMC) in a discrete-time context, whose block diagram is clearly based on IMC one, but has some features taken from optimal control.

For open loop stable plant, this control structure keeps design of classical IMC controller Q unchanged, making sure that closed-loop analysis are still transparent and easy to do; as additional control instrument, it uses a Luenberger observer instead of default IMC deadbeat observer, with resulting improved performance when an input disturbance occurs. Since observer needs the definition of some preliminary parameters, its introduction implies an increased number of tuning parameters to fix.

For open-loop unstable plants, DOB-IMC adopts a stabilizing state feedback F[1] which makes resulting output y stable. Contrary to the state feedback of LQG, F does not need calculation of steady-state variables to converge to the set point value and to reject disturbances, as Q is not designed inverting the unstable plant, which would give an unstable controller, but a pre-stabilized system, thanks to which DOB-IMC manages to give an output that follows reference signal and rejects disturbances. It is also important to underline that this structure introduces other tuning parameters, that are those requested by F calculation.

Furthermore, these additional parameters to regulate turned conventional one degree-of-freedom IMC regulator into a *two degree-of-freedom controller*, since λ here is not anymore the only one tuning parameter. As a verify, we checked that the sum of sensitivity and complementary sensitivity is not anymore equal to 1.

[1] from Q parametrization [10]

A variant for integrating plant has been furthermore proposed, in which model does not correspond to the plant, but it is a first order system with an high lag.

Finally, DOB-IMC has been tested to several types of linear systems in Chapter 5 and to a multivariable system in Chapter 6, exhibiting stability and robustness, and even performances closer to those given by MPC rather than IMC.

7.1 Other research possibilities

DOB-IMC has not been tested with saturations on actuators, so a constrained analysis can be done. Furthermore, in this work the advantages of Model Predictive Control have been ”imported” to Internal Model Control, but the final structure is still an IMC, even if with improved performance and more tuning parameters; it could probably be interesting ooking at the opposite direction, namely importing the features that make IMC a very good instrument in control field to MPC.

Appendix A

The Q parametrization

A.1 Introduction

In Chapter 4, DOB-IMC structure, initially developed for open-loop stable systems, has been subsequently extended to OL unstable plants: in order to do this, as mentioned before, concepts about *Q parametrization* (or, equally, *Youla parametrization*) have been adopted, following instructions given by [7, 10, 21]. The name Q parametrization derived from the fact that it can stabilize all feedback controller capable to stabilize a given system.

It is important to underline that only basic notions about Q Parametrization have been here used to make the output y converge; in fact, the whole theory could not be applied here, since we are working with an augmented model in order to reject any offset related to disturbances, inserting thus an integrator to the normal system dynamics.

The insertion of this integrator means that augmented dynamics matrix

$$\begin{bmatrix} A & B_d \\ 0 & I \end{bmatrix} \tag{A.1}$$

has an eigenvalue in 1, making it not Hurwitz.

In this section, theory about Q parametrization will be briefly explained. For further and detailed information, please see [7, 10, 21].

A.2 Augmented controller

Consider linear system

$$\begin{cases} x_{k+1} & = Ax_k + Bu_k \\ y_k & = Cx_k \end{cases} \tag{A.2}$$

A standard LQG controller is in the form

$$\hat{x}_{k+1} = (A - ALC)\,\hat{x}_k + Bu_k + Ly_k \tag{A.3a}$$

$$u_k = -K\hat{x}_k \tag{A.3b}$$

with $(A - ALC)$ and $(A - BK)$ are both Hurwitz.

Suppose, now, that control input u_k is

$$u_k = -K\hat{x}_k + v_k \tag{A.4}$$

where $v_k = We_k$: W is a stable transfer function and e_k is the *output estimation error*,

$$e_k = y_k - C\hat{x}_k \tag{A.5}$$

Putting W equal to the standard IMC controller, we can rearrange (A.3) as follows

$$\hat{x}_{k+1} = (A - ALC - BK)\,\hat{x}_k + Bu_k + Ly_k \tag{A.6a}$$

$$u_k = -K\hat{x}_k + v_k = -K\hat{x}_k + We_k \tag{A.6b}$$

Where W is a stable transfer matrix chosen arbitrarily; if we choose $W = Q$, where Q is IMC controller, control input (A.6b) is a sum of two contributions: LQG-like and IMC-like control input; in other words, a stabilizing feedback on the states manages to stabilize the output y_k, keeping original Q_{IMC} design unchanged.

Appendix B

Calculation of Sensitivity functions for DOB-IMC

This appendix illustrates how to derive expressions for sensitivity functions for DOB-IMC, already shown in Chapter 4, here investigated more in depth and explained step by step.

B.1 Reference tracking problem

Note: z dependency will be dropped for a matter of simplicity. Assuming $r = 1$ and $d_p = 0$, $y_r = Pu$, consequently $u = P^{-1}y_r$; replacing u value in (4.9), we have

$$\begin{cases} \hat{x} &= \left(f_{y,x} + f_{u,x}P^{-1}\right) y \\ \hat{d} &= \left(f_{y,d} + f_{u,d}P^{-1}\right) y \end{cases} \tag{B.1}$$

in the most general case (OL unstable plants)

$$y_r = Pu = P\left(Q\left(r - \tilde{d}\right) + F\tilde{x}\right) = P\left(Q\left(r - \tilde{d}\right) + F\tilde{x}\right)$$

where $\tilde{x}$ and $\tilde{d}$ are obtained using the transformation matrix T; replacing in Equation (4.12) x and d values shown in (B.1), I have

$$\begin{bmatrix} \tilde{x} \\ \tilde{d} \end{bmatrix} = H \begin{bmatrix} \left(f_{y,x} + f_{u,x}P^{-1}\right) \\ \left(f_{y,d} + f_{u,d}P^{-1}\right) \end{bmatrix} y = \begin{bmatrix} H_{11} & H_{12} \\ 0 & H_{22} \end{bmatrix} \begin{bmatrix} \left(f_{y,x} + f_{u,x}P^{-1}\right) \\ \left(f_{y,d} + f_{u,d}P^{-1}\right) \end{bmatrix} y \tag{B.2}$$

after few algebraic steps, we obtain

$$y = \frac{QP}{1 + QPH_{22}f_{y,d} + QH_{22}f_{u,d} - PFH_{11}f_{y,x} - FH_{11}f_{u,x} - PFH_{12}f_{y,d} - FH_{12}f_{u,d}} r$$

- for OL unstable plants,

$$T_{unst} = \frac{PQ}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d} - PFH_{11}f_{y,x} - FH_{11}f_{u,x} - PFH_{12}f_{y,d} - FH_{12}f_{u,d}}$$

- for OL stable plants, $F = 0$, so the complementary sensitivity function is

$$T_{st} = \frac{PQ}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d}}$$

B.2 Disturbance rejection problem

In this case, $r = 0$, $d_p = 1$; thus,

$$y = Pu + P_d d_p \Rightarrow u = P^{-1}\left(y - P_d d_p\right)$$

from which

$$\begin{cases} \hat{x} & = \left(f_{y,x} + f_{u,x}P^{-1}\right)\left(y - P_d d_p\right) \\ \hat{d} & = \left(f_{y,d} + f_{u,d}P^{-1}\right)\left(y - P_d d_p\right) \end{cases} \tag{B.3}$$

so, $\tilde{x}$ and $\tilde{d}$ are given by the following expression:

$$\begin{bmatrix} \tilde{x} \\ \tilde{d} \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ 0 & H_{22} \end{bmatrix} \begin{bmatrix} \left(f_{y,x} + f_{u,x}P^{-1}\right) \\ \left(f_{y,d} + f_{u,d}P^{-1}\right) \end{bmatrix} y - \begin{bmatrix} H_{11} & H_{12} \\ 0 & H_{22} \end{bmatrix} \begin{bmatrix} f_{u,x} \\ f_{u,d} \end{bmatrix} P^{-1}P_d d_p \tag{B.4}$$

Then, the output has the following expression (derived after some algebraic steps here omitted)

$$y_d = \frac{P_d\left(1 + QH_{22}f_{u,d} - FH_{11}f_{u,x} - FH_{12}f_{u,d}\right)}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d} - PFH_{11}f_{y,x} - FH_{11}f_{u,x} - PFH_{12}f_{y,d} - FH_{12}f_{u,d}} d_p$$

- for OL unstable plants,

$$S_{unst} = \frac{P_d\left(1 + QH_{22}f_{u,d} - FH_{11}f_{u,x} - FH_{12}f_{u,d}\right)}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d} - PFH_{11}f_{y,x} - FH_{11}f_{u,x} - PFH_{12}f_{y,d} - FH_{12}f_{u,d}}$$

  which

  - for disturbances on the output, i.e. $P_d = 1$, becomes

$$S_{unst} = \frac{\left(1 + QH_{22}f_{u,d} - FH_{11}f_{u,x} - FH_{12}f_{u,d}\right)}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d} - PFH_{11}f_{y,x} - FH_{11}f_{u,x} - PFH_{12}f_{y,d} - FH_{12}f_{u,d}}$$

  - for internal disturbances, $P_d = P$

$$S_{unst} = \frac{P\left(1 + QH_{22}f_{u,d} - FH_{11}f_{u,x} - FH_{12}f_{u,d}\right)}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d} - PFH_{11}f_{y,x} - FH_{11}f_{u,x} - PFH_{12}f_{y,d} - FH_{12}f_{u,d}}$$

- , for OL stable plants, as for H, some terms are missing:

$$S_{st} = \frac{P_d\left(1 + QH_{22}f_{u,d}\right)}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d}}$$

  - case $P_d = 1$

$$S_{st} = \frac{\left(1 + QH_{22}f_{u,d}\right)}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d}}$$

  - case $P_d = P$

$$S_{st} = \frac{P\left(1 + QH_{22}f_{u,d}\right)}{1 + PQH_{22}f_{y,d} + QH_{22}f_{u,d}}$$

Appendix C

Matlab codes

C.1 Open loop stable plants

Once defined plant and model in continous time, they are both converted in dicrete time domain, both in z transform and state space; state space description is made up of a set of matrices (A, B, C, D). Then, model has been decomposed in minimum phase part and non-minimum part; then, controller is built.

This part of code has been here omitted for its simplicity.

```
%------------------------------------------------------
% Augmented Model & Observer
%------------------------------------------------------

    n_states=size(A,1);         % Number of states

%%% Standard IMC Matrices for the augumented model

    Bdt=zeros(n_states,1);
    Cdt=1;

%%% Actual matrices and observer implemented

    Qf=eye(n_states);       % Weighting matrix
    Rf=1;                   % Weighting matrix
    Lx=dlqr(A',C',Qf,Rf)';  % Kalman filter
    Ld=1;

    Bd=Lx;
    Cd=1-C*Lx;

```

```
%%% Function for conversion
    % H22 is the transformation block needed for the calculation
    % of the Alebgraic Equivalent system

    [H22]=Convert_Obs(A,C,n_states,Bdt,Cdt,Lx,Ld,Bd,Cd);

%--------------------------------------------------------
%Generate Reference Signal
%--------------------------------------------------------
    r=linspace(1,1,W);

%--------------------------------------------------------
%Solving Control Loop
%--------------------------------------------------------

for k=1:W

    % Disturbance dynamics evolution
        dpd=Cdd*xd+Ddd*dp;

    % Measured outuput
        yp(k)=Cp*xp+dpd;

    % Estimated output
        y=C*x+Cd*d;

    % Prediction error
        e_pred=(yp(k)-y);

    % Observer block
        x=x+Lx*e_pred;
        d=d+Ld*e_pred;

    % Algebraic equivalent system calculation
        dt=H22*d;

    % Return difference
        e_fb=r(k)-dt;

    % Control input
        u(k)=Cq*xq+Dq*e_fb;


    % States Update
        x=A*x+B*u(k)+Bd*d;   % Model
        xp=Ap*xp+Bp*u(k);    % Plant
        xq=Aq*xq+Bq*e_fb;    % Controller
        xd=Add*xd+Bdd*dp;    % Disturbance
```

```
end
```

C.2 Open loop unstable plants

Here the building of the controller has been included, because it is based on the pre-stabilized process.

```
n_states=size(A,1);
%-----------------------------------------------------
% Augmented Model & Observer
%-----------------------------------------------------
%%% Standard IMC Matrices for the augmented model
    Bdt=zeros(n_states,1);
    Cdt=1;

%%% Actual matrices and observer implemented
    Qf=eye(n_states);        % Weighting matrix
    Rf=1;                    % Weighting matrix
    Lx=dlqr(A',C',Qf,Rf)';   % Kalman Filter
    Ld=1;

    Bd=Lx;
    Cd=1-C*Lx;

%%% Matrix for the Algebraic Equivalent system
    [H12,H22]=Convert_Obs(A,C,n_states,Bdt,Cdt,Lx,Ld,Bd,Cd);
    H11=eye(n_states);
    H=[H11, H12; zeros(1,n_states), H22];

%%% Stabilizing state feedback (from Q parametrization)
    epsilon=10^(-5);
    Q=(C'*C)+epsilon*eye(n_states);
    R=1;
    F=dlqr(A,-B,Q,R);        % Kalman Filter

%%% Pre-stabilized model to be inverted in the controller

    Ahat=A+B*F; Bhat=B; Chat=C; Dhat=D;
    Ghat=ss(Ahat,Bhat,Chat,Dhat,Ts);
    [numhat,denhat]=ss2tf(Ahat,Bhat,Chat,Dhat);
    Ghatz=tf(numhat,denhat,Ts); Ghatz=zpk(Ghatz);
%---------------------------------------------------------------
% IMC controller design
%---------------------------------------------------------------
```

```
%Decomposition into minimal phase and non-minimal phase
    [Pnmp,Pmp,Ring]=decompose(Ghatz,Ts);

%Nominal Controller
    [qid]=(Pmp)^-1;

%Tuning Parameters
    lambda=1;

%Controller Filtered
    [Q,filter,alphan,n]=IMCfilterz(qid,lambda,Ts);
    [numq,denq]=tfdata(Q,'v');
    [Aq,Bq,Cq,Dq]=tf2ss(numq,denq);

    W=200; % Number of samplings

%-------------------------------------------------------
%Ranks
%-------------------------------------------------------
    n_states=size(A,1);

%--------------------------------------------------
%Generate Reference Signal
%--------------------------------------------------
r=linspace(1,1,W);

%--------------------------------------------
%Solving Control Loop
%--------------------------------------------
for k=1:W

    % Disturbance dynamics evolution
        dpd=Cdd*xd+Ddd*dp;

    % Measured output
        yp(k)=Cp*xp+dpd;

    % Estimated output
        y=C*x+Cd*d;

    % Prediction error
        e_pred=(yp(k)-y);

    % Observer block
        x=x+Lx*e_pred;
        d=d+Ld*e_pred;

    % Algebraic equivalent system
```

```
        x_aug=H*[x;d];
        xt=x_aug(1:n_states);
        dt=x_aug(n_states+1);

    % Return difference
        e_fb=r(k)-dt;

    % Control input
        v(k)=Cq*xq+Dq*e_fb; % IMC controller
        u_lqg(k)=F*xt;      % Optimal controller
        u(k)=u_lqg(k)+v(k); % Global control input

    % States Update
        x=A*x+B*u(k)+Bd*d;  % Model
        xp=Ap*xp+Bp*u(k);   % Plant
        xq=Aq*xq+Bq*e_fb;   % Controller
        xd=Add*xd+Bdd*dp;   % Disturbance

end
```

C.3 Functions

C.3.1 Algebraic Equivalent System calculation

```
function [H12,H22]=Convert_Obs(A,C,n_states,Bdt,Cdt,Lx,Ld,Bd,Cd)

I=eye(n_states);
S=[A-I, Bdt; C, Cdt];
H=S^(-1)*[Bd;Cd];
H12=H(1:n_states);
H22=H(n_states+1);

```

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