Università degli Studi di Pisa

FACOLTÀ DI INGEGNERIA
Corso di Laurea Magistrale in Ingegneria Aerospaziale

TESI DI LAUREA

Variational approach to the problem of optimal propeller design

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Anno Accademico 2014–2015

Abstract

The aim of this paper is to evaluate the theoretical efficiency of propellers with non-planar blade, optimally shaped.

It is well known that non-planar wing configurations can significantly reduce the induced drag [5], hence this can be of interest also for propeller design. Furthermore the adoption of a curvilinear blade system can be justified not only for an efficiency improvement, but also for reason that concerns the structure and the noise reduction [12], [1].

A solution to the optimum rotor problem, in the context of propeller vortex theory, was given by Goldstein [7]. He considered straight blade propellers and expressed the optimum circulation function via a trigonometrical series of Bessel functions. However, such were the difficulties of computation, even after the solution was found, that Theodorsen resorted to the use of rheoelectrical analogy to evaluate the circulation function, unfortunately without great success [14]. Accurate tabulated values of the Goldstein function covering a wide range of parameters became available with an extensive mathematical effort by Tibery and Wrench [15]. Although this work is based on a completely different approach, Goldstein results are fundamental to validate the procedure for the case of straight blade.

In this dissertation, a variational formulation[1] of the optimum rotor problem is proposed in order to support the optimization of more complex blade configurations, such as the non-planar ones. The first step of the formulation consists into finding a class of functions (representing the circulation distribution along the blade) for which the thrust and the aerodynamic resisting moment functionals are well defined. Then, in this class, the functional to be minimized is proved to be strictly convex; taking into account this result, it is proved that the global minimum exists and is unique.

Some of the configurations analysed are:

- Classical straight blade
- Parabolic blade
- Elliptical blade
- Superelliptic blade

Configurations with the same value of maximum dimensions and performances required are compared in the case of single and multiple blade propellers.

[1] This formulation can be seen as the extension of the one proposed in [11].

The main difficulty in the functionals' evaluation, is the fact that an analytical expression of the velocity induced by a semi-infinite helical vortex filament do not exist[2]. For this reason the Euler-Lagrange equation associated with the variational problem is not obtained and a direct method is used. In particular the Ritz Method is adopted.

Another task to deal with, is the evaluation of the singular integral representing the induced velocity. A two-dimensional quadrature rule, based on Legendre polynomials, is used [9].

This procedure is implemented in a MATLAB program that, given the parametric expression of the curve representing the blade, allows the evaluation of the momentum in the required condition and plots the optimal circulation along the curve.

[2]Okulov proposed an expression for an infinite helical vortex [10].

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