

## Università di Pisa <br> University of Adelaide

## Doctoral Thesis

## Multi-channel Techniques for 3D ISAR

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## Declaration of Authorship

I, Federica Salvetti, declare that this thesis titled, 'Multi-channel Techniques for 3D ISAR' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed:

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"A person who never made a mistake never tried anything new."

Albert Einstein

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## Abstract

Doctor of Philosophy

## Multi-channel Techniques for 3D ISAR

by Federica Salvetti

This thesis deals with the challenge of forming 3D target reconstruction by using spatial multi-channel ISAR configurations. The standard output of an ISAR imaging system is a 2 D projection of the true three-dimensional target reflectivity onto an image plane. The orientation of the image plane cannot be predicted a priori as it strongly depends on the radar-target geometry and on the target motion, which is typically unknown. This leads to a difficult interpretation of the ISAR images. In this scenario, this thesis aim to give possible solutions to such problems by proposing three 3D processing based on interferometry, beamforming techniques and MIMO InISAR systems. The CLEAN method for scattering centres extraction is extended to multichannel ISAR systems and a multistatic 3D target reconstruction that is based on a incoherent technique is suggested.

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## Abbreviations

| 2D | $\mathbf{2}$ Dimensional |
| :--- | :--- |
| 3D | $\mathbf{3}$ Dimensional |
| DSTO | Defence Science Technology Organization |
| EM | ElectroMagnetic |
| FMCW | Frequency Modulated Continuous Wave |
| FT | Fourier Transform |
| IC | Image Contrast |
| ICBA | Image Contrast Based Autofocus |
| IFT | Inverse Fourier Transform |
| InISAR | Interferometric Inverse Synthetic Aperture Radar |
| ISAR | Inverse Synthetic Aperture Radar |
| IPP | Image Projection Plane |
| LoS | Line of Sight |
| MC-CLEAN | MultiChannel CLEAN |
| M-ICBA | Multichannel Image Contrast Based Autofocus |
| MIMO | Multiple Input Multiple Output |
| MF | Matched Filter |
| PLS | Probabilistic Least Square |
| Pol-CLEAN | Polarimetric CLEAN |
| PSF | Point Spread Function |
| PRF | Pulse Repetition Frequency |
| PRI | Pulse Repetition Interval |
| RADAR | RAdio Detection And Ranging |
| RCS | Radar Cross Section |
|  |  |


| RD | Range Doppler |
| :--- | :--- |
| RT | Radon Transform |
| RX | Reciving |
| SAR | Synthetic Aperture Radar |
| SLL | Side Lobe Level |
| SNR | Signal to Noise Ratio |
| TX | Transmitting |

## Symbols

A
received signal amplitude of the $i^{\text {th }}$ spatial channel
focusing parameter of the ICBA algorithm corresponding to the accellera B
signal bandwidth

## C

speed of light
model coordinates
coordinates of the model rotated along the trajectory
D
horizontal baseline length
vertical baseline length
size of the antenna along a given cross-range direction $\underline{E}$
energy of a time-delay section in the $i^{t h}$ spatial channel $\underline{F}$
$f_{0}$
$f_{d}$
$F \quad$ percentage of initial energy of the ISAR image

## $\underline{G}$

H
$h_{H}$
$h_{V}$
scatterer's height along the horizontal baseline scatterer's height along the vertical baseline

|  | $\underline{I}$ |
| :---: | :---: |
| $i_{\text {LoS }}$ | LoS unit vector |
| $I(\tau, \nu)$ | ISAR image in the range-Doppler domain |
|  | $\underline{J}$ |
| $J(\boldsymbol{\alpha}$ | cost function of the soft assignment |
|  | $\underline{K}$ |
| $k_{0}$ | wavenumber |
|  | $\underline{L}$ |
|  | $\underline{M}$ |
| $M_{\xi x}$ | rotation matrix |
| $m m d_{k}(n)$ | mean matching distance |
|  | $\underline{N}$ |
|  | $\underline{O}$ |
| O | centre of the reference system |
|  | $\underline{P}$ |
| $\boldsymbol{P}_{\boldsymbol{j}}$ | position at time $t=0$ for a generic scatterer in the local reference system $T_{j}$ |
| $\mathbf{p}_{T m}$ | $m^{\text {th }}$ tx coordinates in the local system of reference |
| $\mathbf{p}_{R n}$ | $n^{\text {th }}$ rx coordinates in the local system of reference |
|  | $\underline{Q}$ |
| $Q$ | number of sensor in the multistatic network |
|  | $\underline{R}$ |
| $R_{0}$ | radar-target distance |
| $\boldsymbol{R}_{\mu}$ | yaw matrix |
| $\boldsymbol{R}_{\nu}$ | pitch matrix |
| $\boldsymbol{R}_{\xi}$ | roll matrix |
|  | $\underline{S}$ |
| $S_{R}(f, t)$ | spectrum of the time-varying spatial multichannel received signal |
|  | T |
| $T_{i}$ | transmitted pulse duration |
| $T_{\xi}$ | reference system embedded in the radar |
| $T_{i}$ | transmitted pulse duration |


| $T_{\text {obs }}$ | observation time |
| :--- | :--- |
| $T_{R}$ | pulse repetition interval |
| $T_{x}$ | time-varying reference system embedded in the target |
| $T_{y}$ | reference system $T_{x}$ at $t=0$ <br>  <br> $v_{r}$ |
|  | $\underline{\boldsymbol{U}}$ |
| $w(\tau, \nu)$ | $\underline{\text { radial velocity }}$ |
| $W(f, t)$ | $\underline{\text { Point Spread Function }}$ |
|  | $\underline{\text { domain where the 2D FT of the reflectivity function is defined }}$ |
|  | $\underline{\boldsymbol{X}}$ |
| $y_{i j}$ | $\underline{i^{t h}}$ |
|  | $\underline{\text { y }}$ |

## Greek Symbols

|  | $\underline{\boldsymbol{\alpha}}$ |
| :--- | :--- |
| $\alpha$ | azimuth angle <br> angular resolution of a rectangular antenna |
| $\alpha_{c r}$ | soft assignment matrix |
| $\boldsymbol{\alpha}$ | assignment probability between the $k^{\text {th }}$ and the $i^{\text {th }}$ scatterer |
| $\alpha_{i, k}$ | $\underline{\boldsymbol{\beta}}$ |
|  | $\underline{\boldsymbol{\gamma}}$ |
| $\gamma$ | $\underline{\text { empiric parameter to adjust the threshold } \Lambda}$$\underline{\boldsymbol{\delta}}$ |
| $\delta_{c r}$ | cross-range resolution <br> $\delta_{r}$ |
| $\delta_{\tau}$ | range resolution |
| $\Delta_{\nu}$ | pulse duration at the output of the MF |
| $\Delta_{\tau}$ | Doppler resolution |
| $\Delta_{y_{1}}$ | time delay resolution |
| spatial resolution along $y_{1}$ |  |


| $\Delta_{y_{2}}$ | spatial resolution along $y_{2}$ |
| :---: | :---: |
|  | $\underline{\epsilon}$ |
| $\varepsilon_{i, k}$ | euclidean distance between $k^{\text {th }}$ and the $i^{\text {th }}$ scatterer |
| $\epsilon_{h}$ | height error |
| $\epsilon_{\phi}$ | $\phi$ error |
| $\epsilon_{\Omega}$ | effective rotation angle error |
|  | $\underline{\zeta}$ |
| $\zeta$ | compression factor |
|  | $\underline{\theta}$ |
| $\theta$ | elevation angle |
|  | $\underline{\boldsymbol{\lambda}}$ |
| $\lambda$ | wavelength |
| $\Lambda$ | threshold to identify unreliable assignments |
|  | $\mu$ |
| $\mu$ | chirp rate |
|  | $\underline{\nu}$ |
| $\nu$ | Doppler frequency |
|  | $\underline{\rho}$ |
| $\rho$ | reflectivity function |
|  | $\underline{\sigma}$ |
| $\sigma_{\epsilon_{h}}$ | standard deviation of $\epsilon_{h}$ |
|  | $\underline{\tau}$ |
| $\tau$ | time delay |
|  | $\underline{\phi}$ |
| $\phi$ | rotation angle between $T_{\xi}$ and $T_{x}$ |
|  | $\underline{\varphi}$ |
| $\varphi$ | angle between the scatterers trace and the abscissa axis |
|  | $\underline{\chi}$ |
|  | $\underline{\psi}$ |
| $\Psi(a, b)$ | function to be minimised for the estimation of $\Omega_{e f f}$ and $\phi$ |


| $\boldsymbol{\Omega}_{e f f}$ | effective rotation vector |
| :--- | :--- |
| $\boldsymbol{\Omega}_{\boldsymbol{T}}$ | total angular rotation vector |

## Math Operators

A $\}$
$E\}$
$\delta_{i, j}$
$T$
expectation
expectation
Dirac delta function
transpose operator

## Chapter 1

## Introduction

### 1.1 Background and Motivation

Radar imaging has been widely investigated in the last few years and increasingly used for a wide range of applications as radar systems, unlike optimal imaging, are capable of working day and night and in all weather conditions.

In particular, Inverse Synthetic Aperture Radar (ISAR) is a radar technique used to obtain high-resolution images of remote targets using radio waves. Developed in the in the early '80s, it has been widely investigated in the last few decades and highly exploited for a large number of tasks such as non-cooperative target identification and classification or enhancement of the cross-range resolution of rotating targets.

The basis of ISAR systems lies in the extension of the concept of Synthetic Aperture Radar (SAR), referring to a geometry in which the synthetic aperture is formed by exploiting the movement of the target and possibly the movement of the radar platform. The SAR synthetic aperture is generated by means of the movement of the platform that carries the radar only. In SAR scenarios, the antenna illuminates an area that is usually static during the synthetic aperture formation while in ISAR scenarios the target motion is not usually under the radar operator's control. The non-cooperation of the target in ISAR operating scenarios
is the main difference between these two techniques and one of the main challenges of the Inverse Synthetic Aperture Radar image formation processing.

The unknown target motion leads to the key problem of unknown geometry and dynamics of the radar-target system during the coherent integration time. Such a limitation results in a difficult interpretation of the ISAR image and consequently causes difficulties in fundamental tasks as recognition and classification of the target.

The standard output of an ISAR imaging system is, in fact, a 2D projection of the true three-dimensional target reflectivity onto an image plane.
An example of 2D ISAR image obtained by processing real data acquired while illuminating a ship is depicted in Figure 1.1.


Figure 1.1: 2D ISAR image.
The orientation of the image plane strongly depends on the radar-target geometry and on the target motion, which is typically unknown. The result is that that the target projection onto the image plane cannot be predicted and the interpretation of the ISAR images becomes complicated. Under these conditions, the projected two-dimensional image can only provide limited information and is not sufficient for identifying and recognizing the target.

To solve this problem, the idea is to form a 3D reconstruction of the target as to completely avoid the problems related to the orientation of the imaging plane and the consequent misinterpretation of the 2D ISAR image.

The aim of this work is to develop effective techniques to form 3D images of non-cooperative targets. All the techniques developed in this thesis exploit a multichannel ISAR system. The received signals are used to estimate the orientation of the image plane allowing for the target 3D shape reconstruction. The first described technique is based on the interferometric principles and gives a solution to the problem of scattering centres extraction on ISAR images obtained from a spatial multi-channel radar configuration. The second technique is based on a tomographic approach and allows to overcome the shadowing problems. The third technique exploits a MIMO InISAR system to obtain the three-dimensional target reconstruction. Moreover, a method for multistatic 3D target reconstruction that is based on a incoherent technique is proposed and analysed.

All the developed techniques are tested on simulated data whilst the interferometric technique is also tested on real data-sets.

### 1.2 Thesis Outline and Major Contributions

This thesis is organized as follows. The first part provides a review of the ISAR signal modelling for the monostatic case. Then, the multichannel signal model is introduced and the developed techniques are detailed. Results on simulated data and, where available, on real data-sets are provided for all the developed techniques.

The main contributions of this thesis are:

- Extend the CLEAN technique. The developed Multi-Channel CLEAN (MCCLEAN) technique is a useful tool for scattering centre extraction and makes use of ISAR images obtained from a spatial multi-channel radar configuration.
- Develop a procedure in order to analyse the performances of the 3D interferometric ISAR processing.
- Develop a method for 3D ISAR imaging of non-cooperative target by using beamforming techniques.
- Develop a method for multistatic 3D reconstruction based on a incoherent technique.
- Develop a technique for 3D InISAR imaging of non-cooperative target in a colocated MIMO configuration based on the monostatic-bistatic equivalence theorem. A comparison between the 3D reconstruction performances of a multichannel InISAR system and a MIMO InISAR system is provided.
- Demonstrate the effectiveness of the proposed techniques on simulated and, where available, on real data-sets.

During the research activity, the following papers have been published:

- M. Martorella, F. Salvetti, and D. Stagliano. "3D target reconstruction by means of 2d-isar imaging and interferometry." In Radar Conference (RADAR), 2013 IEEE, pages 1-6, 2013.
- M. Martorella, D. Stagliano, F. Salvetti, and N. Battisti. "3D interferometric isar imaging of non - cooperative targets." Aerospace and Electronic Systems, IEEE Transactions on, October 2014.
- Hai-Tan Tran, Elisa Giusti, Marco Martorella, Federica Salvetti, Brian W.H. Ng and An Phan. "Estimation of the total rotational velocity of a noncooperative target using a 3D InISAR system." 2015 IEEE International Radar Conference, May 2014.
- Federica Salvetti, Douglas Gray, and Marco Martorella. "Joint use of twodimensional tomography and ISAR imaging for three-dimensional image formation of non-cooperative targets." In EUSAR 2014; 10th European Conference on Synthetic Aperture Radar; Proceedings of, pages 1-4, June 2014.
- Federica Salvetti, Daniele Staglianò, Elisa Giusti, Marco Martorella. "Multistatic 3D ISAR Image Reconstruction." 2015 IEEE International Radar Conference, May 2014.


### 1.2.1 Chapters outline

## Chapter 2

## Inverse Synthetic Aperture Radar Imaging

The aim of this chapter is to recall the fundamental concepts of ISAR theory.
The concept of 'resolution', fundamental in ISAR imaging, is reminded and the main differences between SAR and ISAR concepts are highlighted. Particular attention is given to the monostatic ISAR signal modelling and the 2D image formation since these will be extended to a multistatic signal modelling and a 3D image formation processing.

## Chapter 3

## 3D Interferometric ISAR Imaging

First, the multichannel signal model is discussed in this chapter.
Then, the MultiChannel CLEAN technique for scattering centre extraction in ISAR images obtained from a spatial multi-channel radar configuration is presented.

Finally, the algorithm for 3D target reconstruction based on the interferometric principles is detailed.

## Chapter 4

## 3D Interferometric ISAR Performance Analysis

In this chapter, simulated data results of the interferometric method are presented and evaluated through the developed performance analysis procedure.

First, the performance analysis theory is detailed, then simulation results are
examined. Simulated data are obtained by generating the backscattered signal from two point-like scatterer models.

## Chapter 5

## Real Data Analysis

Two real dataset are used to test the algorithm. The data are first processed by using the Multichannel-ICBA to focus the obtained ISAR images. Then, the InISAR processing is applied.

## Chapter 6

## Multistatic 3D ISAR Imaging Reconstruction

This chapter focuses on describing a method for multistatic 3D target reconstruction that is based on a incoherent technique. This idea stems from the consideration that even though InISAR imaging has proven an effective tool to produce 3D target reconstruction, it suffers of shadowing problems.

The proposed solution to this problem is to use of spatially distributed radar network and to combine the 3D reconstructions obtained at each InISAR system.

## Chapter 7

## Joint Use of Two-dimensional Tomography and ISAR Imaging for Threedimensional Image Formation of Non-cooperative Targets

A novel tomographic approach to form three-dimensional ISAR images of noncooperative moving targets is the key idea of this chapter.

The idea behind this concept is to overcome some limitations of the interferometric algorithm. In fact, this technique does not allow for scatterers belonging to the same range-Doppler cell to be resolved in the ISAR image. Conversely, with the approach proposed in this chapter, is possible to achieve height resolution and generate a 3D reconstruction that takes into account also scatterers that are not visibile.

The beamforming processing is detailed and results of simulations are presented.

## Chapter 8

## 3D Colocated MIMO ISAR Imaging

The objective of this chapter is to propose a 3D target reconstruction method based on the InISAR processing described in Chapter 3 by using a co-located MIMO configuration. The idea is to exploit the advantages that a co-located MIMO configuration gives, related to the concept of virtual aperture. Simulation results show a comparison between the 3D reconstruction performances of a multichannel InISAR system and a MIMO InISAR system.

## Chapter 2

## Inverse Synthetic Aperture Radar Imaging

### 2.1 Introduction

Radar imaging has been increasingly used for a wide range of applications. This is because radar systems are capable of working day and night (unlike optical imaging) and in all weather conditions. They can penetrate clouds and smoke. Some radars can penetrate foliage, buildings, soil and human tissue and above all are able to detect, track and image objects with high accuracy at long range. Synthetic Aperture Radar (SAR) and Inverse Synthetic Aperture Radar (ISAR) are radar techniques to generate high resolution Electro-Magnetic (EM) images of natural and man-made objects by coherently processing returned signal from a moving target at a different aspect angle relative to the radar. The change of the aspect angle is given by the relative motion and rotation between the radar and the target[1]. The standard output of a radar imaging system is a two-dimensional (2D) image, that is a 2 D projection of the true three-dimensional (3D) target reflectivity. Radar images are often compared with photographic images as both are the result of some transformation that maps a 3D object in a 2D space. However, their distinctive mapping techniques and the considered image features lead to a
very different interpretation of these two types of image. The main differences that deserve to be pointed out are related to geometry and radiometry. From the geometrical point of view, the images are the results of a target projection onto different planes. For an optical sensor, in fact, this 2D plane coincides with the focal plane of the sensor while for an imaging radar the orientation of the Image Projecting Plane (IPP) depends on the sensor position relative to the target and on the target's motions. Furthermore, in the case of non-cooperative targets, it cannot be predicted a priori. Regarding the radiometric differences, the exploited EM waves uses drastically different carriers. This profoundly affects the observation result and the images highlight different characteristics of the target. Some quality indexes, such as geometrical resolution, radiometric resolution, signal-tonoise ratio (SNR) etc. are usually used to characterized radar images as well as other type of images. In radar imaging, the detail in which an object can be observed depends on how fine is the spatial resolution. Consequently, this feature becomes crucial in order to form high quality radar images.

This chapter is to describe the fundamental concepts of ISAR theory.
First, Section 2.2 will portray a brief history of radar and radar imaging to recall the fundamental discoveries of our forefathers, which made it possible to achieve nowadays radar technology.

Secondly, the concept of 'resolution' will be recalled in Section 2.3.

Section 2.4 will compare SAR and ISAR technologies and highlight the main differences, which lead to very different signal processing.

The monostatic ISAR signal model and the conventional 2D image formation processing will be presented in Section 2.5 and 2.6, to be extended in Chapter 3 to multi-channel ISAR and 3D image formation.

### 2.2 Brief history of radar and radar imaging

The term RADAR was coined in 1940 by the United States Navy as an acronym for RAdio Detection And Ranging [2, 3]. The history of radar can be dated back to the experiments carried out by Heinrich Hertz in the late 19th century. He experimentally validated the James Clerk Maxwell's theory on electromagnetism and discovered that radio waves are reflected by metallic objects while carrying out his experiments on radio waves. In fact, Hertz noticed that surrounding objects were interfering with his radio waves. In 1900, Nikola Tesla proposed a wireless system able to locate objects and even to measure their distance by using reflected radio waves. Tesla tried to clarify the idea behind his initiative as:
"When we raise the voice and hear an echo in reply, we know that the sound of the voice must have reached a distant wall or boundary, and must have been reflected from the same. Exactly as the sound, so an electrical wave is reflected, and the same evidence can be used to determine the relative position or course of a moving object such as a vessel at sea".

Despite his brilliant vision, Tesla's ideas were forgotten, only to be rediscovered time and time again in years to come. Four years later the German engineer Christian Hülsmeyer applied for a patent for the Telemobiloskop (Telemobiloscope), a simple ship detection device intended to help avoid collisions in fog. The Telemobiloskop and some patent drawings [4] are shown in Figure 2.1 and Figure 2.2 respectively.


Figure 2.1: The Telemobiloscope


Figure 2.2: First page and azimuth encoding of the original patent DE 165546, Hülsmeyer, 1904.

Only two weeks after his patent application, Hülsmeyer gave a public demonstration of his device in which he successfully bounced signals onto - and received reflections back from, a ship approaching the Hoenzollern Bridge over the River Rhine in Cologne. The device was only designed to detect an object, not to assess how far away the target object was but the principle and the system required had been created. The Telemobiloscope was primarily a spark-gap transmitter connected to an array of dipole antennas, and a coherer receiver with a cylindrical parabolic antenna that could rotate 360 degrees. Unfortunately, Hülsmeyer's idea had no follow on and was almost immediately forgotten, mainly because of the poor technological background of that time. G. Marconi was the first to clearly understand the possibilities of using short waves to detect targets. In 1922, in an address to a joint meeting of the Institute of Electrical Engineers and the Institute of Radio Engineers in New York, he suggested using radio waves to detect ships [5]:
"As was first shown by Hertz, electric waves can be completely reflected by conducting bodies. In some of my tests I have noticed the effects of reflection and deflection of these waves by metallic objects miles away. It seems to me that it should be possible to design apparatus by means of which a ship could radiate or project a divergent beam of these rays in any desired direction, which rays, if coming across a metallic object, such as another steamer or ship, would be reflected back to a receiver screened from the local transmitter on the sending ship, and thereby immediately reveal the presence and bearing of the other ship in fog or thick weather. One further advantage of such an arrangement would be that it would be able to give warning of the presence and bearing of ships, even should these ships be unprovided with any kind of radio."

Neither Hülsmeyer's nor Marconi's achievements were able to pique the interest of their political class. It took at least a decade before specific studies to realize his system were addressed, and at least fifteen years before such devices reached acceptable operating performances. Breit and Tuve were the first to apply the pulse technique to measure the distance, in 1925. The application was to determine the height of the ionosphere [6]. Only in the early '30s the first (accidental) objects detections were performed. The US were the first to study and build radar systems. In the early '30s the first (accidental) objects detections boosted the investments and consequently the research in this field. The organization that took on the financial burden of this first phase was the Naval Research Laboratory. Studies progressed at a slow pace and the first patented system dates back to 1934 [7]. It can be pointed out that this device, as indeed those of all the other countries, was a continuous wave system, easier to be designed with respect to the pulsed radar. The first pulsed radar are dated back to 1936. They worked at a frequency of $28,3 \mathrm{MHz}$ and pulse duration 50 sec . The maximum range was initially only 2.5 miles but later on was brought to 25 miles in a few months. A new incentive to the study of radar systems came with the development of tubes capable of providing high power. However it is only in 1941 that a series of pulsed radar were installed on the bigger units of US Navy. Other than US, also Italy, England and Germany carried on studies and researches on radar systems. It worth noticing
that France started independent studies on radar too, but these researches were almost immediately interrupted due to the instability caused by the second world war. The Germans built very good systems during the war, but these did not play as a decisive role as the American and British radars. In England, the first proposal to the government to allocate funds for research in radar is 1935. At the end of 1935 was built the first continuous wave prototype and in 1936 was fabricated the first pulsed device. This exploited a frequency of 25 MHz and, in 1939 , 200 MHz . Then, after the exchange of technical information with the Americans in 1940, the developments increased dramatically. The construction of the multi-cavity magnetron, proposed by the British Randall and Bootes, decisively contributed to this progress. In Italy, the current government didn't encourage studies on radar system, even though many researches were carried on by local scientists. In 1933 G. Marconi demonstrated the possibility of detecting obstacles by means of the reflection of electromagnetic waves during an experiment attended from the Italian military authorities. Following the experiments of Marconi, the young engineer U. Tiberio carried out a research on how to use a system that exploit e.m waves to detect aircraft and ships at a great distance. The study contained the basic radar equation and the basic schemes of the continuous wave and pulsed apparatuses. Unfortunately, Tiberio's research was basically ignored and a poor team were provided to help him dealing with this issue. Between 1935 and 1940 he realized the first pulsed radar prototype ('GUFO') at the naval academy of Livorno. The government only realized the importance of radar systems after the disaster in the naval battle of Cape Matapan and gave the necessary resources to build a first series of pulsed systems. Radar techniques dramatically improved in the following years. In 1951 Carl Wiley conceived the SAR concept [8], giving rise to the origins of radar imaging. In 1957 the Willow Run Laboratories of the University of Michigan developed the first operational system for the US Department of Defense (DoD). This was classified. Unclassified SAR systems were successfully built by NASA in the 1960s. Following these first experiments, NASA completed other significant missions such as the SIR-A (1981), SIR-Band (1984) and SIR-C (1994) missions. In 1978 was launched the first spaceborne SAR system, SEASAT-A. This
was a turning point in the radar imaging history since the results obtained with SEASAT-A demonstrated the importance of radar imaging for the observation of the earth and led to the lunch of other spaceborne SAR systems with improved resolution, wider coverage and faster revisit times. In order to overcome some spaceborne SAR systems issues such as cost, revisiting time and resolution, a number of airborne SAR system have also been developed.

Years later, in the early '80s, Walker and Ausherman gave rise to the ISAR concept by presenting the idea of radar imaging of rotating objects with fixed antennas [9, 10]. For the first time, the Doppler information generated by the rotation of an object was exploited to separate echoes returning from different parts of the object along a cross-range axis. A two-dimensional (2D) image of the target is obtained by using such a Doppler separation, together with the time-delay separation (along the radar range). The resulting 2D image is mapped onto an image plane.

### 2.3 Image Resolution

A radar image can be defined as a spatial distribution of the EM reflectivity of an object mapped onto an imaging plane from a distribution of currents on the object's surface [11]. The EM field irradiated by the radar when incident on an object induces a set of currents on the object's surface, which in turns produces a scattered EM field. This EM field partially propagates back to the radar, which measures the backscattered electric intensity, that is the target reflectivity. The object's reflectivity may be observed in finer detail depending on the spatial resolution up to isolate the reflectivity contributions from different parts of the object. Under certain constraints, conventional ISAR images are represented in the Range and Cross-range domain. High resolution in both range and cross-range is then necessary to enable radar imaging capabilities in a radar system and achieve a high quality image. The term resolution generally refers to the ability of a system to distinguish two measured quantities. For radar systems the resolution is a measure of how far two point-like scatterers with equal magnitude have to be separated to
be recognized as two and consequently to appear separate in the radar image. to The range resolution is the ability of the system to distinguish two scatterers along the range direction, while the cross-range resolution is the ability of the system to distinguish two scatterers along the cross-range direction. The resolution is determined by the impulse response of the radar system, that is the Point Spread Function (PSF). In the case of SAR and ISAR systems, it is typically assumed that the target is composed of independent point-like scatterers and the impulse response is usually approximated by means of a two-dimensional sinc-like function. The concepts of range and cross-range resolution will be described in the following subsections.

### 2.3.1 High Range Resolution

In pulsed radar, the range resolution is commonly related to the transmitted pulse duration $T_{i}$. In fact, the persistence of an echo is usually approximated with the length of the transmitted pulse. Consequently, it is possible to distinguish a second close range scatterer only when the the first return fade away. In other words, when the delay between the two scatterers' echo is at least $T_{i}$. The range resolution can then be expressed as:

$$
\begin{equation*}
\delta r=\frac{c T_{i}}{2} \tag{2.1}
\end{equation*}
$$

Where $c$ is the speed of light. From eq.(2.1) can be derived that to improve the range resolution it is necessary to reduce the pulse duration. However, the transmitter peak power should be increased when reducing $T_{i}$ not to affect the probability of detection and false alarm and to achieve equal performances. This problem is overcame by the pulse compression theory [12-14]. The pulse compression is achieved by jointly using wide-band transmitted signals and a Matched Filter (MF). The MF ensures the maximum Signal to Noise Ratio (SNR) at its output by producing the transmitted signal autocorrelation function, as depicted in Figure 2.3. In addition, wide-band transmitted signals are necessary to have a compression gain. A compression gain is obtained when the compression factor $\varsigma$


Figure 2.3: Matched Filter's block diagram
is greater than 1. The compression factor can be defined as follows:

$$
\begin{equation*}
\varsigma=\frac{T_{i}}{\delta \tau} \tag{2.2}
\end{equation*}
$$

Where $\delta \tau$ is the pulse duration at the output of the MF, that is the minimum delay between two scatterers below which the two contributions cannot be distinguished. The range resolution can then be defined as:

$$
\begin{equation*}
\delta r=\frac{c \delta \tau}{2}=\frac{c}{2 B} \tag{2.3}
\end{equation*}
$$

Eq.(2.3) express the relation between the transmitted signal bandwidth $B$ and the range resolution. Moreover, the time duration of a pulse is reduced by the compression factor $\varsigma$ at the output of the MF. From that, the expression pulse compression. It is worth pointing out that to achieve an effective compression it is necessary to use phase and/or frequency modulations. Besides, pulse compression can produce undesirable effects in terms of SideLobe Level (SLL). In fact, sidelobes can appear in the compressed signal as a side effect and they must be attenuated or, when possible, suppressed, to avoid masking other echoes. Both analog and digital modulation are common in literature. Among analog modulations, Chirp signals are the most popular. In fact they not only are easy to generate, but are also robust against noise and Doppler effect and allow to control the SLL. In summary, pulse compression is a technique to achieve high resolution by transmitting wideband signal with transmitted peak power limited by the pulse duration and with unchanged performances at long ranges.

### 2.3.2 High Cross-range Resolution

As stated before, the cross-range resolution can be defined as the ability of the radar system to separate two independent contributions in the cross-range direction. The cross-range resolution is strongly linked to the angular separation in the same direction. As the angular resolution is inversely related to the antenna size, in the past antennas of large dimensions where built to obtain radar system with desirable azimuth and elevation resolution. As an example of this dependence, the angular resolution of a rectangular antenna can be roughly determined by the following expression:

$$
\begin{equation*}
\alpha_{c r} \simeq \frac{\lambda}{D_{c r}} \tag{2.4}
\end{equation*}
$$

where $D_{c r}$ is the size of the antenna along a given cross-range direction (usually named as azimuth and elevation), $\alpha_{c r}$ is the angular resolution (expressed in radians)along the same cross-range direction and $\lambda$ is the radar wavelength. However, higher angular resolution should be needed to enable the radar system to perform image processing. The first reason is that it should be scaled from angular to spatial coordinates. Secondly, the cross-range resolution is dependent on the target range $R$ as clearly shown in eq.(2.5)

$$
\begin{equation*}
\delta_{c r}=R \alpha_{c r}=\frac{R \lambda}{D_{c r}} \tag{2.5}
\end{equation*}
$$

Long ranges implies poor cross-range resolution as well as wider antennas produce finer resolution. However, it is obvious that is not either a practical or effective solution a to build antennas of large dimensions. Equivalent practical limitations occur when exploiting antenna arrays. In fact, the number of antennas to be used to achieve desirable cross-range resolution would be often unreasonable. The concept of synthetic aperture has been introduced in order to overcome such a problem. SAR and ISAR techniques have been developed in literature to achieve radar images with both range and cross-range resolution [15-17].

### 2.4 SAR and ISAR Concepts

As pointed out in Section 2.3.2, real aperture antennas or antenna arrays do not provide sufficient cross-range resolution for imaging systems. The concept of synthetic aperture has been developed to overcome such a limitation. Carl Wiley was the first to introduce the notion and to develop the theory of a SAR system in the early 50 's. The idea is as simple as revolutionary: a virtual array is obtained by moving a single element along a given trajectory and by transmitting and receiving from locations separated in space in a given time interval. Figure Figure 2.4 shows the comparison between a real and a synthetic aperture formation. Under


Figure 2.4: Synthetic aperture radar vs real aperture array
the hypothesis of a static illuminated scene during the synthetic aperture formation $\left(t_{1} \div t_{N}\right)$, it can be stated that the signal acquired by a synthetic aperture radar is physically identical with the signal acquired by a radar that makes use of a real array. Obviously, as the synthetic aperture formation involves the movement of the sensor, some approximations are necessary to neglect the effect of the element motion. The stop and go assumption is typically used. The radar motion is assumed to be discrete, so transmission and reception will be carried out when the SAR system is stopped at a particular position. The hypothesis of stop and
go is acceptable even for real scenario as the short round trip delay allow to consider negligible the element offset created by the distance covered by the radar, between transmission and reception of each pulse. Originally, Synthetic Aperture Radar (SAR) systems were used mostly for Earth observation applications such as ocean, land, ice, snow and vegetation monitoring. However, SAR systems were soon considered an important resource for a wider range of applications as military and homeland security purposes because of the ability to form high resolution images from remote platforms in all day/all weather conditions. Another way to look at the problem of forming a synthetic aperture to generate high resolution images is Inverse Synthetic Aperture Radar (ISAR). While in SAR the synthetic aperture can be seen as a coherent processing of echoes that comes from different view angles, the terms Inverse SAR refers to the fact that the radar is assumed to be stationary whilst the target is moving. The synthetic aperture is not formed by means of the movement of the platform that carries the radar but rather it is formed by exploiting the movement of the target. In fact, the inverse synthetic aperture is achieved when there is a variation of the target-radar aspect angle, which is crucial for the image formation process. The concept of ISAR system is illustrated in Figure Figure 2.5. The term $\vartheta\left(t_{i}\right)$ indicate the aspect angle, which changes along with the target motion. It is worth pointing out that the crossrange profile of a target is a function of the radial velocity of each scatterer with respect to the radar, whether the radar or the target or even both are moving. It then could be argued that is possible to look at synthetic aperture from a SAR or ISAR point of view by simply changing the reference system. The SAR geometry is achieved by placing the reference system on the stationary target while ISAR scenario is obtained by placing the reference system on the stationary sensor. However, the two systems, and consequently the two theoretical formulations, are very different due to a crucial detail, the target co-operation. In ISAR scenarios the target motion is usually not known a priori whereas in SAR geometry the area illuminated by the antenna is usually static during the synthetic aperture formation and the platform motion is well-known. Target cooperation can be interpreted as


Figure 2.5: ISAR system concept
system geometry knowledge, which enables straightforward image formation processing. Specifically, a radar image is obtained by coherently process the echoes received by the radar at different aspect angles. The antenna-target path difference $\Delta R\left(t_{i}\right)$ may change with the platform motion during the integration time. As a result, the received signal phase term changes over time and the phase difference must be compensated. The same concept applies to ISAR systems where the distance between the radar and the target $R_{0}\left(t_{i}\right)$ changes over time. The relative motion of the target with respect to the radar must be known a priori to apply the correct compensation. It is clear, then, that SAR and ISAR image formation techniques must differ since in ISAR systems the target is often non-cooperative and the radar-target relative motion must be estimated by applying autofocusing algorithms at time, since different targets have different motions with respect to the radar.

### 2.5 ISAR signal model

Let the ISAR system geometry to be introduced as depicted in Figure 2.6.


Figure 2.6: ISAR system geometry

The system is composed of a the transmitter/receiver antenna indicated as TX/RX.

The reference system $T_{\xi}=\left[\xi_{1}, \xi_{2}, \xi_{3}\right]$ is embedded in the radar system. The axis $\xi_{2}$ is aligned with the radar Line of Sight $(\mathrm{LoS})$ while $\xi_{1}$ and $\xi_{3}$ correspond to the horizontal and vertical axes respectively. The term $R_{0}(t)$ denotes the time varying distance between the radar and the reference point on the target.

The target is usually considered as a rigid and composed of $M$ point-like scatterers. The relative radar-target motion can then be considered as the composition of two contributions: a translational motion and a rotation motion. The translational motion is the motion of the reference point with respect to the radar. In practical conditions, manoeuvring targets or external forces on the targets produce rotation motions that are represented by the angular rotation vector $\boldsymbol{\Omega}_{\mathbf{T}}(t)$, which is applied to the center $O$ of the target. The vector $\boldsymbol{\Omega}_{\mathbf{T}}(t)$ includes the aspect angle variation due to both the translational motion and the own rotational motion of the target. Its projection onto the plane orthogonal to the LOS defines the effective rotation vector $\boldsymbol{\Omega}_{\mathrm{eff}}(t)$. As demonstrated in [18], the plane orthogonal to $\boldsymbol{\Omega}_{\mathrm{eff}}(t)$ is the imaging plane.

Then, a reference system $T_{x}=\left[x_{1}, x_{2}, x_{3}\right]$ is defined, that is embedded in the target and centred in $O$. The plane $\left(x_{1}, x_{2}\right)$, whose axes correspond to the cross-range
and range coordinates, coincides with the imaging plane. The axis $x_{2}$ is aligned with $\xi_{2}$ whereas $x_{3}$ with $\boldsymbol{\Omega}_{\mathrm{eff}}(t)$.

The reference system $T_{x}$ is time varying since the effective rotation vector changes its orientation in time. Thus, we define the reference system $T_{y}=\left[y_{1}, y_{2}, y_{3}\right]$ to be coincident with $T_{x}$ at time $t=0$. As it will be clear in the following, the plane $\left(y_{1}, y_{2}\right)$ identifies the image plane and its axes are associated with the cross-range and range coordinates respectively.

Consider a pulse denoted as $s_{T}\left(t_{f}\right)$ with temporal duration $T_{i}$, where $t_{f}$ is the fast time. The radar transmits a pulse train formed by a number of pulses separated by a time $T_{R}$, which is defined as Pulse Repetition Interval (PRI). Typically, $T_{i} \ll T_{R}$. Under the stop and go assumption the radar-target relative motion can be neglected and it can be assumed that the transmission of $s_{T}\left(t_{f}\right)$ and the reception of its echo occur instantaneously. Then, the received signal can be expressed as follows:

$$
\begin{equation*}
s_{R}\left(t_{f}, t\right)=\int_{V} f(\mathbf{y}) s_{T}\left(t_{f}-\tau(\mathbf{y}, t), t\right) d \mathbf{y} \tag{2.6}
\end{equation*}
$$

where

- $t$ is the slow time variable
- $\mathbf{y}$ is the vector that locates the position of an arbitrary point on the target w.r.t. the reference system $T_{y}$
- $f(\mathbf{y})$ is the target reflectivity function defined in the volume V of the target.
- $\tau(\mathbf{y}, t)=\frac{2 R(\mathbf{y})}{c}$ is the round trip delay time of a scattering centre located in $\mathbf{y}$ at the slow time $t$.

The transmitted signal $s_{T}\left(t_{f}\right)$ is the same at each pulse.

As described in Section 2.3.1, pulse compression is achieved by jointly using wideband transmitted signals and a Matched Filter. At the output of the matched
filter complex base-band received signal in the frequency $(f) /$ slow-time domain can be written as:

$$
\begin{equation*}
S_{R}(f, t)=W(f, t) \int_{V} f(\mathbf{y}) e^{-j 4 \pi f \tau(\mathbf{y}, t)} d \mathbf{y} \tag{2.7}
\end{equation*}
$$

The function $W(f, t)$ is the domain where the two-dimensional Fourier Transform (2D-FT) of the reflectivity function is defined. I will be shown it to determine the scattering centerPoint Spread Function(PSF) and can be expressed as:

$$
\begin{equation*}
W(f, t)=\left|S_{T}(f)\right|^{2} \operatorname{rect}\left(\frac{t}{T_{o b s}}\right) \tag{2.8}
\end{equation*}
$$

where $T_{\text {obs }}$ is the observation time. For standard active radar systems $\left|S_{T}(f)\right|^{2} \simeq$ rect $\left(\frac{f-f_{0}}{B}\right)$ and eq.(2.8) can be re-written as:

$$
\begin{equation*}
W(f, t)=\operatorname{rect}\left(\frac{f-f_{0}}{B}\right) \operatorname{rect}\left(\frac{t}{T_{\text {obs }}}\right) \tag{2.9}
\end{equation*}
$$

The function $\operatorname{rect}(x)$ is equal to 1 for $|x|<0.5$, otherwise 0 .

Under the assumption that the size of the target is much smaller tha the radartarget distance, the straight iso-range approximationcan be applied and the term $\tau(\mathbf{y}, t)$ in eq. (2.7)can be written as follows:

$$
\begin{equation*}
\tau(\mathbf{y}, t)=\frac{2 R(\mathbf{y}, t)}{c}=\frac{2}{c}\left[R_{0}(t)+\mathbf{y}^{T} \cdot \mathbf{i}_{L o S}(t)\right] \tag{2.10}
\end{equation*}
$$

where

- $R_{0}(t)$ identifies the distance between the radar and a reference point on the target at time $t$
- $\mathbf{i}_{\text {LoS }}(t)$ is the LoS unit vector at time $t$.

The principle of the straight iso-range approximation is depicted in Figure 2.7


Figure 2.7: Straight iso-range approximation

Under the assumption of a small $T_{\text {obs }}$, the total rotation vector can be considered as constant and the image plane fixed with respect to $T_{\xi}$ :

$$
\begin{equation*}
\boldsymbol{\Omega}_{\boldsymbol{T}}(t) \cong \boldsymbol{\Omega}_{\boldsymbol{T}}, \quad 0 \leq t \leq T_{\text {obs }} . \tag{2.11}
\end{equation*}
$$

Let choose the axis $x_{3}$ and the vector $\boldsymbol{\Omega}_{\text {eff }}$ to be parallel. Then, the inner product $x_{2}(\boldsymbol{y}, t)=\boldsymbol{y} \cdot \mathbf{i}_{L o S}(t)$ can be calculated by solving the following differential equation system:

$$
\left\{\begin{array}{l}
\dot{\mathbf{x}}(t)=\boldsymbol{\Omega}_{\boldsymbol{T}} \times \mathbf{x}(t)  \tag{2.12}\\
\mathbf{x}(0)=\mathbf{y}
\end{array}\right.
$$

The resulting closed form solution is shown as follows [19-21]:

$$
\begin{equation*}
\mathbf{x}(t)=\mathbf{a}+\mathbf{b} \cos (\Omega t)+\frac{\mathbf{c}}{\Omega} \sin (\Omega t) \tag{2.13}
\end{equation*}
$$

with

$$
\begin{align*}
& \mathbf{a}=\frac{\left(\Omega_{T} \cdot \mathbf{y}\right)}{\Omega^{2}} \Omega_{T} \\
& \mathbf{b}=\mathbf{y}-\frac{\left(\Omega_{T} \cdot \mathbf{y}\right)}{\Omega^{2}} \Omega_{T}  \tag{2.14}\\
& \mathbf{c}=\Omega_{\boldsymbol{T}} \times \mathbf{y} \\
& \Omega=\left|\Omega_{T}\right|
\end{align*}
$$

It should be pointed out that $\boldsymbol{\Omega}_{T}=\left(0, \Omega_{T_{2}}, \Omega_{e f f}\right)$ is in accordance with the chosen reference system.

Under the assumption of a small $T_{\text {obs }}$, the term $\mathbf{x}(t)$ in eq. (2.13) can be reasonably approximated by its first-order Taylor series around $t=0$ and the result is expressed in eq.(2.15):

$$
\begin{equation*}
\mathbf{x}(t) \cong \mathbf{a}+\mathbf{b}+\mathbf{c} t=\mathbf{y}+\mathbf{c} t \tag{2.15}
\end{equation*}
$$

resulting in:

$$
\begin{equation*}
x_{2}(\boldsymbol{y}, t)=y_{2}+\Omega_{e f f} y_{1} t \tag{2.16}
\end{equation*}
$$

with $\Omega_{e f f}=\left|\Omega_{e f f}\right|$ is the magnitude of the effective rotation vector. It is worth highlighting that the effective rotation vector $\Omega_{e} f f$ is the only term that has to be considered because it takes into account the actual target rotation. In fact, any rotation around the $y_{2}$ axis does not produce any radar-target variation and consequently it has no effect on the image formation process.

The received signal can now be obtained by substituting eq.(2.15) in eq.(2.7). The resulting expression is:

$$
\begin{equation*}
S_{R}(f, t)=W(f, t) \int_{V} f(\mathbf{y}) e^{-j \frac{4 \pi f}{c}\left(R_{0}(t)+y_{2}+\Omega_{e f f} y_{1} t\right)} d \mathbf{y} \tag{2.17}
\end{equation*}
$$

Another way to achieve this result is to consider the LoS unit vector in the reference system $T_{y}$ to be expressed as:

$$
\boldsymbol{i}_{L o S}=\left[\begin{array}{c}
\sin \left(\Omega_{e f f} t\right)  \tag{2.18}\\
\cos \left(\Omega_{e f f} t\right) \\
0
\end{array}\right]
$$

The resulting inner product in eq. (2.10) can be then written as follow:

$$
\begin{equation*}
x_{2}(\mathbf{y}, t)=\mathbf{y} \cdot \mathbf{i}_{L o S}(t)=y_{2} \cos \left(\Omega_{e f f} t\right)+y_{1} \sin \left(\Omega_{e f f} t\right) \tag{2.19}
\end{equation*}
$$

As a consequence, eq. (2.7) can be written as:

$$
\begin{equation*}
S_{R}(f, t)=W(f, t) \int_{y_{1}} \int_{y_{2}} f\left(y_{1}, y_{2}\right) e^{-j \frac{4 \pi f}{c}\left(R_{0}(t)+y_{2} \cos \left(\Omega_{e f f} t\right)+y_{1} \sin \left(\Omega_{e f f} t\right)\right)} d y_{1} d y_{2} \tag{2.20}
\end{equation*}
$$

The 2D reflectivity function is denoted by the term $f\left(y_{1}, y_{2}\right)$, which represents the projection of the 3D target reflectivity onto the IPP as follows:

$$
\begin{equation*}
f\left(y_{1}, y_{2}\right)=\int_{y_{3}} f(\mathbf{y}) d y_{3} \tag{2.21}
\end{equation*}
$$

It is clear that eq. (2.20) and eq. (2.17) are equivalent under the hypothesis of small variation of the total aspect angle in the observation time $\left(\Delta \Theta=\Omega_{e f f} T_{o b s} \simeq\right.$ 1 degree).

### 2.6 ISAR image formation

Usually, a conventional 2D ISAR image is formed by means of the Range-Doppler (RD) technique. As described in the previous sections, the first step is to obtain range compression throughout the matched flter. The following step is perform the radial motion compensation. Radial motion compensation consists of removing the phase term that depends on the radial motion of the reference point, that is the term $-\frac{4 \pi f}{c} R_{0}(t)$ in eq. (2.17). Many approaches have been developed in literature to focus the image. Such methods are called autofocusing techniques as they use only the received signal. In fact, in typical operating scenarios, no external data are available. In this thesis, only the Image Contrast Based Autofocus (ICBA) will be described in Section 2.6 .1 as it has been used to focus the ISAR images obtained with real data. Lets now assume that perfect radial motion compensation has been applied and the phase term dependant on $R_{0}(t)$ has been removed from eq. (2.17). The compensated signal is then:

$$
\begin{equation*}
S_{R}\left(Y_{1}, Y_{2}\right)=W\left(Y_{1}, Y_{2}\right) \int_{y_{1}} \int_{y_{2}} f\left(y_{1}, y_{2}\right) e^{-j 2 \pi\left(y_{1} Y_{1}+y_{2} Y_{2}\right)} d y_{1} d y_{2} \tag{2.22}
\end{equation*}
$$

The spacial frequencies $Y_{1}$ and $Y_{2}$ are defined as:

$$
\left\{\begin{array}{l}
Y_{1}(f, t)=\frac{2 f}{c} \sin \left(\Omega_{e f f} t\right)  \tag{2.23}\\
Y_{2}(f, t)=\frac{2 f}{c} \cos \left(\Omega_{e f f} t\right)
\end{array}\right.
$$

The compensated signal described in eq. (2.22) consists of two terms: the function $W\left(Y_{1}, Y_{2}\right)$ and the Fourier transform of the target reflectivity. The term $W\left(Y_{1}, Y_{2}\right)$ describes the region in the spatial frequencies domain where the signal is defined. The ISAR image can be read as the estimate of the projected reflectivity function. The ISAR image is represented in the range/cross-range domain and can be obtained by simply applying the Inverse Fourier Transform (IFT) to eq.(2.22). When considering a target as composed of $M$ point-like scatterers, the target reflectivity function can be seen as the summation of all the contributions as:

$$
\begin{equation*}
f\left(y_{1}, y_{2}\right)=\sum_{k=1}^{M} \rho_{k} \delta\left(y_{1}-y_{1 k}, y_{2}-y_{2 k}\right) \tag{2.24}
\end{equation*}
$$

where

- $y_{1 k}$ is the cross-range coordinate of the $k^{t h}$ scattering centre
- $y_{2 k}$ is the range coordinate of the $k^{t h}$ scattering centre
- $\rho_{k}$ is the reflectivity value of the $k^{\text {th }}$ scattering centre

It is worth noticing that eq. (2.24) does not takes into account the interactions between the scatterers. Under these assumptions and substituting eq. (2.24) in eq. (2.22) we obtain:

$$
\begin{equation*}
S_{R}\left(Y_{1}, Y_{2}\right)=W\left(Y_{1}, Y_{2}\right) \sum_{k=1}^{M} \rho_{k} e^{-j 2 \pi\left(y_{1 k} Y_{1}+y_{2 k} Y_{2}\right)} \tag{2.25}
\end{equation*}
$$

The IFT of eq.(2.25) is the convolution between the 2D-IFT of $W\left(Y_{1}, Y_{2}\right)$, that is $w\left(y_{1}, y_{2}\right)$ and the projection of the target reflectivity function on the image plane:

$$
\begin{equation*}
I\left(y_{1}, y_{2}\right)=w\left(y_{1}, y_{2}\right) \otimes \otimes f\left(y_{1}, y_{2}\right)=\sum_{k=1}^{M} \rho_{k} w\left(y_{1}-y_{1 k}, y_{2}-y_{2 k}\right) \tag{2.26}
\end{equation*}
$$

By making the assumption of small aspect angle variation the polar domain defined from $W\left(Y_{1}, Y_{2}\right)$ and depicted in Figure 2.8 can be approximated by a rectangular domain. As a consequence, eq. (2.23) can be approximated as follows:


Figure 2.8: Fourier domain

$$
\left\{\begin{array}{l}
Y_{1}(f, t) \approx \frac{2 \pi f_{0}}{c} \Omega_{e f f} t  \tag{2.27}\\
Y_{2}(f, t) \approx \frac{2 \pi f}{c}
\end{array}\right.
$$

Where the frequency $f$ in $Y_{1}$ has been substituted by the central frequency $f_{0}$. The rectangular window intercepts the angular sector at the coordinate $Y_{2}=\frac{2 f_{0}}{c}$ and is shown in Figure 2.9 Consequently to the rectangular domain approximation, the spatial frequencies $\left(Y_{1}, Y_{2}\right)$ can be treated as independent variables. Furthermore, a variable change can be applied from the spatial frequency to the delay time $(\tau)$ and Doppler ( $\nu$ ) coordinated. Eq. (2.25) is rewritten as:

$$
\begin{equation*}
S_{R}(f, t)=W(f, t) \sum_{k=1}^{M} \rho_{k} e^{-j 2 \pi\left(f \tau_{k}+t \nu_{k}\right)} \tag{2.28}
\end{equation*}
$$



Figure 2.9: Fourier domain-rectangular approximation
with

$$
\left\{\begin{array}{l}
\tau=\frac{2 y_{2}}{c}  \tag{2.29}\\
\nu=\frac{2 f_{0} \Omega_{e f f} y_{1}}{c}
\end{array}\right.
$$

By applying the 2D-IFT to eq. (2.28) we obtain the ISAR image in the delay time-Doppler frequency domain:

$$
\begin{equation*}
I(\tau, \nu)=w(\tau, \nu) \otimes \otimes \sum_{k=1}^{M} \rho_{k} \delta\left(\tau-\tau_{k}, \nu-\nu_{k}\right)=\sum_{k=1}^{M} \rho_{k} w\left(\tau-\tau_{k}, \nu-\nu_{k}\right) \tag{2.30}
\end{equation*}
$$

The term $w(\tau, \nu)$ is the two-dimensional Fourier transform of $W(f, t)$ and represents the Point Spread Function of the system. $W(f, t)$ is expressed as:

$$
\begin{equation*}
W(f, t)=\operatorname{rect}\left(\frac{f-f_{0}}{B}\right) \operatorname{rect}\left(\frac{t}{T_{o b s}}\right) \tag{2.31}
\end{equation*}
$$

while $w(\tau, \nu)$ is

$$
\begin{equation*}
w(\tau, \nu)=B T_{o b s} \operatorname{sinc}(\tau B) \operatorname{sinc}\left(t T_{o b s}\right) e^{j 2 \pi f_{0} \tau} \tag{2.32}
\end{equation*}
$$

where $\operatorname{sinc}(x)=\frac{\sin (\pi x)}{\pi x}$. The resolution in the delay time/Doppler dimensions can be derived from the system PSF, i.e. from $w(\tau, \nu)$. By picking the first null of both the sinc functions in the two directions we obtain:

$$
\left\{\begin{array}{l}
\Delta_{\tau}=\frac{1}{B}  \tag{2.33}\\
\Delta_{\nu}=\frac{1}{T_{o b s}}
\end{array}\right.
$$

The target ISAR image in the range/cross-range dimensions can be derived from eq. (2.30) by inverting eq. (2.29), which is a scaling operation:

$$
\left\{\begin{array}{l}
y_{1}=\frac{c}{2 f_{0} \Omega_{e f f}} \nu  \tag{2.34}\\
y_{2}=\frac{c}{2} \tau
\end{array}\right.
$$

The spatial resolution can then be written as:

$$
\left\{\begin{array}{l}
\Delta_{y_{1}}=\frac{c}{2 f_{0} \Omega_{\text {eff }} T_{\text {obs }}}  \tag{2.35}\\
\Delta_{y_{2}}=\frac{c}{2 B}
\end{array}\right.
$$

It is clear from eq.(2.34) that the range coordinate can directly be obtained from the delay time. Conversely, the cross-range coordinate require the knowledge of the effective rotation vector, which is typically unknown. In fact, the effective rotation vector strongly depends on the target motion. The target motion is unknown in usual operating scenarios where the target is non-cooperative. By referring at the chosen system geometry as depicted in Figure 2.6, it is worth pointing out that $\boldsymbol{\Omega}_{\text {eff }}$ is the projection on the plane orthogonal to the LoS of the total rotation vector $\boldsymbol{\Omega}_{\boldsymbol{T}}$ :

$$
\begin{equation*}
\boldsymbol{\Omega}_{\boldsymbol{e f f}}(t)=y_{2} \times\left|\boldsymbol{\Omega}_{\boldsymbol{T}}(t) \times y_{2}\right| \tag{2.36}
\end{equation*}
$$

Therefore the cross-range axis $y_{1}$ can be written as:

$$
\begin{equation*}
y_{1}=y_{2} \times \boldsymbol{\Omega}_{e f f} \tag{2.37}
\end{equation*}
$$

If the effective rotation vector $\boldsymbol{\Omega}_{\text {eff }}$ is constant during the observation time, it is possible to define the image plane as a plane that the effective rotation vector is normal to, and the LOS unit vector lies in [1]. It is clear that unknown $\boldsymbol{\Omega}_{\text {eff }}$ implies unknown image plane $\left(y_{1}, y_{2}\right)$. It should be noticed that the effective rotation vector eq. (2.37) is not dependant on $t$. This is because the assumption of constant $\Omega_{\text {eff }}$ in $T_{\text {obs }}$ holds.

To scale the image along the cross-range dimension the effective rotation vector must be estimated. This represents a great challenge in the ISAR processing. It is important to underline that the spatial resolution in the cross-range direction become finer as the aspect angle variation increase. However, the range-Doppler technique rely on the hypothesis of small angle variation. As a consequence, this assumption defines the upper bound in the cross-range resolution that can be achieved with this algorithm.

To conclude, ISAR image processing allow to obtain high resolution images of non-cooperative targets. Typically, a target ISAR image is formed in the time delay/Doppler frequency domain by means of the Range Doppler technique. Several assumptions have been made:

- Straight iso-range approximation
- Small aspect angle variation
- Constant target's rotation vector

The following issues must be taken into consideration:

- ISAR images are the 2D projection of a 3D object onto a plane, namely the Image Projection Plane.
- The effective rotation vector depends on the target motion, which is typically unknown. $\Omega_{\text {eff }}$ is not known a priori and consequently the image plane as well. As a result, the target projection shown in the image is unknown, leading to a difficult interpretation of the ISAR image.
- The Point Spread Function (PSF), which defines the imaging system response, is not known a priori.
- The cross-range resolution is not known a priori. The image cannot be scaled from the time delay/Doppler frequency domain to the range/cross-range coordinates unless some parameters are estimated. It should be mentioned that the scaling process in crucial in may applications, for example to determine the size of the target.
- In a real scenario both the variables in the signal domain $(f, t)$ and in the image domain $(\tau, \nu)$ are discrete variables. ambiguity in the ISAR image space can be generated by the sampling interval of the spatial frequencies. A non-ambiguity region can be identified. Any target bigger than the nonambiguity region will be seen as "folded" in the image. The size of the non ambiguity function along the cross-range direction depends on the unknown value of $\boldsymbol{\Omega}_{\text {eff }}$. Consequently, is very difficult to predict the target size along the cross-range dimension.

The ISAR image formation chain is summarised in Figure 2.10.


Figure 2.10: ISAR image processing block diagram

### 2.6.1 Image Contrast Based Autofocus

Autofocus algorithms can be classified as parametric and non-parametric [22]. Among the parametric techniques, the Image Contrast Based Autofocus (ICBA) is described here. Such a technique is based on the idea that ISAR images will be more focused as the value of IC increases [23].

The ICBA is essentially organized in two main steps:

1. Preliminary estimation of the focusing parameters

These are provided by an initialization technique that makes use of the radon transform (RT) and of a semi-exhaustive search.
2. Estimation refinement

This is obtained by maximizing is the Image Contrast function.[23]

By estimating the motion parameters, the term $R_{0}(t)$ responsible for image distortions is removed.

Usually verified hypothesis are relatively small observation time and relatively smooth target motions. Under this assumptions, the residual distance between the radar and the target can be expressed by mean of an $L^{t h}$ order polynomial as follows:

$$
\begin{equation*}
R_{0}(t)=\sum_{l=0}^{L} \frac{1}{l!} \omega_{l} t^{l} \tag{2.38}
\end{equation*}
$$

where $\omega_{l}$ represents the focusing parameters. It is clear from eq (2.38) that the term $R_{0}(t)$ can be estimated throughout the estimation of the focusing parameters as:

$$
\begin{equation*}
\hat{\boldsymbol{\omega}}=\arg \left(\max _{\boldsymbol{\omega}}[I C(\boldsymbol{\omega})]\right) \tag{2.39}
\end{equation*}
$$

where $\boldsymbol{\omega}=\left[\omega_{1}, \omega_{2}, \cdots, \omega_{L}\right]^{T}$.
As preaviously mentioned, the function IC gives a measure of the image focusing. It represents the normalized effective power of the image intensity and can be espressed as follows:

$$
\begin{equation*}
I C(\boldsymbol{\omega})=\frac{\sqrt{A\left\{[I(\tau, \nu ; \boldsymbol{\omega})-A\{I(\tau, \nu ; \boldsymbol{\omega})\}]^{2}\right\}}}{A\{I(\tau, \nu ; \boldsymbol{\omega})\}} \tag{2.40}
\end{equation*}
$$

In which

- $A[\cdot]$ denotes the expectation operator
- $I(\tau, \nu ; \boldsymbol{\omega})$ is the ISAR image magnitude or intensity (power) after the target's translational motion compensation by using $\boldsymbol{\omega}$ as focusing parameters
$I(\tau, \nu ; \boldsymbol{\omega})$ can be written as:

$$
\begin{equation*}
I(\tau, \nu ; \boldsymbol{\omega})=\left|2 D-F T\left[S_{R}(f, t) \exp j \frac{4 \pi f}{c} R_{0}(\boldsymbol{\omega}, t)\right]\right|^{p} \tag{2.41}
\end{equation*}
$$

where $p=1$ and $p=2$ denote the image amplitude and the image intensity respectively. The term $R_{0}(\boldsymbol{\omega}, t)$ is the radial motion model evaluated with the $\boldsymbol{\omega}$ motion parameters. When the image is focused correctly, the contrast is hight and each scatterer is represented as a pronounced peak in the ISAR image. Conversely, in the case of a defocused image, the image intensity levels are concentrated around the mean value and the contrast is low.

### 2.7 Conclusion

In this chapter the ISAR signal processing theory has been recalled. The main differences between SAR and ISAR have been analysed. The main issues related to the ISAR processing have been emphasized in order to put the attention on the gaps that will be filled in this thesis. The monostatic received signal model has been described. This will be used in the next chapters and extended for a multichannel system. Finally, the processing to form a conventional 2D ISAR image has been illustrated. This will be used to describe the novel 3D ISAR image formation techniques presented in this thesis.

## Chapter 3

## 3D Interferometric ISAR Imaging

### 3.1 Introduction

Inverse Synthetic Aperture Radar is a technique to form electromagnetic images of non-cooperative targets [24] [25]. The conventional output of the ISAR processing is a 2D image of objects, obtained by projecting the 3D target's reflectivity function onto an image plane, namely the Image Projection Plane. The image plane orientation strongly depends on the radar-target geometry and dynamics, which are typically unknown. As a consequence, such projection is unknown a priori. This often results in a difficult interpretation of an ISAR image. As an example, sensible tasks such as target classification or recognition by using ISAR images may be difficult to apply if the projection of the target cannot be predicted. It is possible to find in literature some techniques to estimate the orientation of the image projection plane [26]. However, the applicability of such techniques are to be yet proven, as well as their effectiveness. A drastic solution to this problem is to process the received signal to form 3D ISAR images. This approach has the advantage of completely overcoming the issue of the unknown projection. Several methods to form 3D ISAR images can be found in the literature that attempt to address this problem. Among this methods, a first approach is to form 3D ISAR images by using single sensor ISAR image sequences [27] [28]. The 3D position
of each target scattering center can be estimated throughout a set of view angles produced by 3D target motions. Even though the use of a single sensor is a cost-effective solution, the drawback is that this approach relies on long target observation time intervals and on a 3D structure of the target's motions [29]. Another approach exploits the interferometric principles and makes use of multiple sensors [30] [21] [31] [32]. The interferometric approach does nor require long observation time intervals and 3D target motions but is based on the assumption that each resolution cell a single scatterer.

In this chapter, the problem of 3D target reconstruction is addressed and solved by using a method that makes use of multiple sensors and interferometry. The orientation of the ISAR image plane is recovered by estimating the target's effective rotation vector (modulus and phase). The knowledge of the image plane completely eliminates the problem of image interpretation. Moreover, the knowledge of the effective rotation vector allows for cross-range scaling and consequently the target can be sized along the range/cross-range coordinates.

The developed mathematics allow to retrieve the height of each scatterer that composes the target with respect to the image plane by means of the effective rotation vector estimation. The 3D ISAR image of the target is then obtained and the object is fully sized along the three dimensions. This opens the door for a wider range of applications. Firstly, target classification and recognition can now be applied. Scondly, other applications as traffic harbour/air control, collision avoidance etc. can makes use of ISAR images.

A partial theoretical foundation of this approach has been introduced in [30] [21]. Nevertheless, some critical aspects such as the scattering center extraction and the 3D reconstruction alignment are still unclear and need to be faced.

This chapter aims at describing the 3D-Interferometric ISAR imaging processing by paying particular attention to the above mentioned problems. The scattering center extraction from an ISAR image is a crucial step as it allows to greatly compress the target information and consequently reduce the computation burden. The Interferometric processing carries out the 3D reconstruction by locating the
extracted dominant scatterers in a 3D space. The scattering center extraction is performed by applying an extended version of the CLEAN technique [33], namely the Multichannel CLEAN technique (MC-CLEAN).

Simulated and real data were used to evaluate and confirm the effectiveness of the proposed technique. Results of the simulated data will be detailed in Chapter 4 while the real data analysis will be carried out in Chapter 5.

The multichannel received signal model is discussed in Section 3.2, the Multichannel CLEAN technique is defined in Section 3.3 and the 3D target reconstruction technique is detailed in 3.4.

### 3.2 Multi-channel Received Signal Model

The goal of this section is to define the system geometry and to derive the signal model.

### 3.2.1 System geometry



Figure 3.1: ISAR system geometry

Consider a system composed of three antennas located at points $\mathbf{A V}_{\xi}, \mathbf{A C}_{\xi}$ and $\mathbf{A H}_{\xi}$. The antennas lie on a horizontal and a vertical baseline respectively, as shown in Figure 3.1. In the chosen geometry, the three antennas are both transmitting and receiving. It should be mentioned that such a configuration would
require the use of orthogonal codes in order to separate the three channels. A discussion on 3D Colocated MIMO InISAR systems will be carried out in Chapter 8. The use of one transmitter and a number of co-located receivers with equal effectiveness may represent a more cost-effective solution.

Briefly recalling the geometry described in Section 2.5, three reference systems must be defined: the reference system $T_{\xi}$ embedded in the radar system, the timevarying reference system $T_{x}$ embedded in the target and the reference system $T_{y}$ defined as $T_{x}$ at $t=0$. The reference system $T_{x}$ is time varying since the effective rotation vector changes its orientation in time. Having to deal with three reference systems, points in the 3D space will be denoted by using a subscript according to the specific reference system coordinates, e.g. $\mathbf{A C}_{\xi}$ and $\mathbf{A C}_{x}$ are the same point expressed with the reference system coordinates $T_{\xi}$ and $T_{x}$ respectively.

The origin of the reference system $T_{\xi}$ corresponds to array phase centre. The axis $\xi_{2}$ is chosen to be aligned with the radar Line Of Sight (LOS) while $\xi_{1}$ and $\xi_{3}$ correspond to the horizontal and vertical baselines respectively.

The plane ( $x_{1}, x_{2}$ ) defines the imaging plane and its axes correspond to the crossrange and range coordinates. The axis $x_{2}$ is aligned with $\xi_{2}$ while $x_{3}$ with $\boldsymbol{\Omega}_{\mathrm{eff}}(t)$.

Finally, the target is assumed a rigid body composed of $M$ point-like scatterers with complex amplitude $\rho$. The position at time $t=0$ for a generic scatterer, defined in $T_{y}$, is denoted by $\mathbf{P}_{y}=\left[y_{1}, y_{2}, y_{3}\right]$.

### 3.2.2 Received signal modeling

Let denote the analytical signal associated with a wide-band transmitted pulse as $s_{T}(t)$. As derived in Chapter 2 and according to [34], the spectrum of the timevarying spatial multichannel received signal relative to an ideal point scatterer, in free space conditions, can be written as follows:

$$
\begin{equation*}
S_{R}^{(i)}(f, t)=\rho^{(i)} \exp \left\{-j \frac{4 \pi f}{c} R_{0}^{(i)}(t)\right\} \exp \left\{-j \frac{4 \pi f}{c}\left[\mathbf{P}_{y} \cdot \mathbf{i}_{L O S_{y}}^{(i)}(t)\right]\right\} \cdot W(f, t) \tag{3.1}
\end{equation*}
$$

where

$$
\begin{equation*}
W(f, t)=\operatorname{rect}\left[\frac{t}{T_{o b s}}\right] \cdot \operatorname{rect}\left[\frac{f-f_{0}}{B}\right] \tag{3.2}
\end{equation*}
$$

is the domain where the two-dimensional Fourier Transform (2D-FT) of the reflectivity function is defined and:

- $f_{0}$ is the carrier frequency
- $B$ is the transmitted signal bandwidth
- $T_{o b s}$ is the observation time
- $R_{0}^{(i)}(t)$ is the modulus of vector $\mathbf{R}_{0}^{(i)}(t)$ which locates the position of focusing point $O$
- $\mathbf{i}_{L O S_{\xi}}^{(i)}(t)$ is the LOS unit vector of $\mathbf{R}_{0}(t)$ expressed with respect to $T_{\xi}$
- $\rho^{(i)}$ is the reflectivity function
- $i \in\{A V, A C, A H\}$

The reflectivity functions $\rho^{(i)}$ can be assumed identical when the baselines are short compared to the radar-target distance, i.e. $\rho^{(i)}=\rho$. The function rect $(x)$ is equal to 1 for $|x|<0.5$, otherwise 0 .

The radial movement of the focusing point $O$ generates the phase term $e^{( }-$ $\left.j \frac{4 \pi f}{c} R_{0}(t)\right)$. This must be removed by motion compensation techniques.

The received signal after motion compensation can be written as follows:

$$
\begin{equation*}
S_{R}^{(i)}(f, t)=\rho \exp \left\{-j \frac{4 \pi f}{c}\left[\mathbf{P}_{x}(t) \cdot \mathbf{i}_{L O S_{x}}^{(i)}(t)\right]\right\} W(f, t) \tag{3.3}
\end{equation*}
$$

where

- $\mathbf{i}_{L O S_{x}}^{(i)}(t)$ is the LOS unit vector expressed with respect to $T_{x}$
- $\mathbf{P}_{x}(t) \cdot \mathbf{i}_{L O S_{x}}^{(i)}(t)$ is a scalar product expressing the distance between the focusing point $O$ and the projection of the scatterer onto the LoS

It is important to underline that $\mathbf{P}_{x}(t) \cdot \mathbf{i}_{L O S_{x}}^{(i)}(t)$ is invariant with respect to the chosen reference system. As a consequence, the scalar product in (3.1) can be rewritten in the $T_{x}$ reference system as shown in (3.3).

Assuming that the rotation vector is constant during the overall observation time, i.e. $\boldsymbol{\Omega}_{\boldsymbol{T}}(t) \cong \boldsymbol{\Omega}_{\boldsymbol{T}}$ for $|t|<T_{\text {obs }}$, the image plane can be considered fixed with respect to $T_{\xi}$ and the position of the target's point scatterer $\mathbf{P}_{x}(t)$ can be calculated by solving the following differential equation system:

$$
\left\{\begin{array}{l}
\dot{\mathbf{P}}_{x}(t)=\boldsymbol{\Omega}_{\boldsymbol{T}} \times \mathbf{P}_{x}(t)  \tag{3.4}\\
\mathbf{P}_{x}(0)=\mathbf{P}_{y}
\end{array}\right.
$$

where the position of the scatterer is referred to as the $T_{x}$ reference system.

As derived in Chapter 2 (eq. (2.12)-(2.15)), the resulting closed form solution with the term $\mathbf{P}_{x}(t)$ approximated by its first-order Taylor series around $t=0$ is shown as follows [19-21]:

$$
\begin{equation*}
\mathbf{P}_{x}(t) \cong \mathbf{a}+\mathbf{b}+\mathbf{c} t=\mathbf{P}_{y}+\mathbf{c} t \tag{3.5}
\end{equation*}
$$

This approximation ensures constant Doppler frequency of each scatterer. The Range-Doppler technique can then be applied to form the 2D ISAR image of a target [25] [34] [35] by using of a 2D-FT.

The rotation of the reference system $T_{x}$ of an angle $\phi$ with respect to $T_{\xi}$ is described by the matrix $\boldsymbol{M}_{\xi \boldsymbol{x}}$, which is defined as follows:

$$
\boldsymbol{M}_{\xi x}=\left[\begin{array}{ccc}
\cos \phi & 0 & \sin \phi  \tag{3.6}\\
0 & 1 & 0 \\
-\sin \phi & 0 & \cos \phi
\end{array}\right]
$$

By means of the rotation matrix $\boldsymbol{M}_{\boldsymbol{\xi} \boldsymbol{x}}$, the LOS unit vectors $\mathbf{i}_{\text {LOS }_{\boldsymbol{x}}}^{(i)}(t)$ can be written as the normalized difference between the positions of each sensor and the origin
of $T_{x}$ :

$$
\begin{align*}
& \mathbf{i}_{L O S_{x}}^{A V}(t) \triangleq \frac{\mathbf{P}_{x}(t)-\mathbf{A} \mathbf{V}_{x}(t)}{\left|\mathbf{P}_{x}(t)-\mathbf{A} \mathbf{V}_{x}(t)\right|}=\left[\begin{array}{lll}
\frac{-d_{V} \sin \phi}{\sqrt{R_{0}(t)^{2}+d_{V}^{2}}} & \frac{R_{0}(t)}{\sqrt{R_{0}(t)^{2}+d_{V}^{2}}} & \frac{-d_{V} \cos \phi}{\sqrt{R_{0}(t)^{2}+d_{V}^{2}}}
\end{array}\right] \\
& \mathbf{i}_{L O S_{x}}^{A C}(t) \triangleq \frac{\mathbf{P}_{x}(t)-\mathbf{A} \mathbf{C}_{x}(t)}{\left|\mathbf{P}_{x}(t)-\mathbf{A} \mathbf{C}_{x}(t)\right|}=\left[\begin{array}{lll}
0 & 1 & 0
\end{array}\right]  \tag{3.7}\\
& \mathbf{i}_{L O S_{x}}^{A H}(t) \triangleq \frac{\mathbf{P}_{x}(t)-\mathbf{A} \mathbf{H}_{x}(t)}{\left|\mathbf{P}_{x}(t)-\mathbf{A} \mathbf{H}_{x}(t)\right|}=\left[\begin{array}{lll}
\frac{-d_{H} \cos \phi}{\sqrt{R_{0}(t)^{2}+d_{H}^{2}}} & \frac{R_{0}(t)}{\sqrt{R_{0}(t)^{2}+d_{H}^{2}}} & \frac{d_{H} \sin \phi}{\sqrt{R_{0}(t)^{2}+d_{H}^{2}}}
\end{array}\right]
\end{align*}
$$

where $\mathbf{A V}_{x}(t), \mathbf{A C}_{x}(t)$ and $\mathbf{A H}_{x}(t)$ are the positions of the antennas with respect to $T_{x}$ and $d_{V}$ and $d_{H}$ denote the vertical and horizontal baseline lengths, respectively.

Therefore, the scalar product in (3.3) can be written as [35]:

$$
\begin{gather*}
\mathbf{P}_{x}(t) \cdot \mathbf{i}_{L O S_{x}}^{A V}(t) \cong\left(\mathbf{P}_{y}+\mathbf{c} t\right) \cdot \mathbf{i}_{L O S_{x}}^{A V}(t)= \\
=\left(y_{1}+c_{1} t\right)\left(\frac{-d_{V} \sin \phi}{\sqrt{R_{0}(t)^{2}+d_{V}^{2}}}\right)+\left(y_{2}+c_{2} t\right)\left(\frac{R_{0}(t)}{\sqrt{R_{0}(t)^{2}+d_{V}^{2}}}\right)+\left(y_{3}+c_{3} t\right)\left(\frac{-d_{V} \cos \phi}{\sqrt{R_{0}(t)^{2}+d_{V}^{2}}}\right) \cong \\
\cong y_{2}+c_{2} t-\frac{d_{V}}{R_{0}}\left[\left(y_{1}+c_{1} t\right) \sin \phi+\left(y_{3}+c_{3} t\right) \cos \phi\right]=K_{0}^{A V}+K_{1}^{A V} t \tag{3.8}
\end{gather*}
$$

where

$$
\begin{array}{r}
K_{0}^{A V} \triangleq y_{2}-\frac{d_{V}}{R_{0}}\left(y_{1} \sin \phi+y_{3} \cos \phi\right) \\
K_{1}^{A V} \triangleq c_{2}-\frac{d_{V}}{R_{0}}\left(c_{1} \sin \phi+c_{3} \cos \phi\right) \tag{3.10}
\end{array}
$$

The approximation $R_{0}(t) \cong R_{0}(0)=R_{0}$ is assumed acceptable for a small observation time. The terms $\sqrt{R_{0}(t)^{2}+d_{V}^{2}}$ and $\sqrt{R_{0}(t)^{2}+d_{H}^{2}}$ are approximated as $R_{0}$. The parameters $c_{1}, c_{2}$ and $c_{3}$ are the three components of the vector $\mathbf{c}$ introduced in (2.13).

Equivalently, we can calculate the scalar products for the other two elements, as follows:

$$
\begin{align*}
\mathbf{P}_{x}(t) \cdot \mathbf{i}_{L O S_{x}}^{A C}(t) & \cong K_{0}^{A C}+K_{1}^{A C} t  \tag{3.11}\\
\mathbf{P}_{x}(t) \cdot \mathbf{i}_{L O S_{x}}^{A H}(t) & \cong K_{0}^{A H}+K_{1}^{A H} t \tag{3.12}
\end{align*}
$$

with

$$
\begin{align*}
& K_{0}^{A C} \triangleq y_{2}  \tag{3.13}\\
& K_{1}^{A C} \triangleq c_{2}  \tag{3.14}\\
& K_{0}^{A H} \triangleq y_{2}+\frac{d_{H}}{R_{0}}\left(y_{3} \sin \phi-y_{1} \cos \phi\right)  \tag{3.15}\\
& K_{1}^{A H} \triangleq c_{2}+\frac{d_{H}}{R_{0}}\left(c_{3} \sin \phi-c_{1} \cos \phi\right) \tag{3.16}
\end{align*}
$$

By substituting the scalar products of (3.8), (3.11) and (3.12) into (3.3), the received signal model can be written as follows:

$$
\begin{gather*}
S_{R}^{(i)}(f, t)=\rho \exp \left\{-j \frac{4 \pi f}{c}\left[K_{0}^{(i)}+K_{1}^{(i)} t\right]\right\} W(f, t)= \\
=\rho \exp \left\{-j \frac{4 \pi f K_{1}^{(i)}}{c} t\right\} \operatorname{rect}\left(\frac{t}{T_{o b s}}\right) \exp \left\{-j \frac{4 \pi K_{0}^{(i)}}{c} f\right\} \operatorname{rect}\left(\frac{f-f_{0}}{B}\right) \tag{3.17}
\end{gather*}
$$

According to the RD technique, the polar Fourier domain in which the signal is defined can be approximated by a rectangular window. As a result, the frequency $f$ in the term $\exp \left\{-j \frac{4 \pi f K_{1}^{(i)}}{c} t\right\}$ in (3.17) can be substituted by the central frequency $f_{0}$. After the application of a 2D-FT, the analytical form of the complex ISAR image in the delay-time $(\tau)$ and Doppler $(\nu)$ domain can be obtained. The result is shown in (3.18).

$$
\begin{align*}
I^{(i)}(\tau, \nu) & =R D_{\substack{\rightarrow \rightarrow \tau \\
t \rightarrow \nu}}\left\{S_{R}^{(i)}(f, t)\right\} \\
& =B T_{\text {obs }} \rho e^{j 2 \pi f_{0}\left(\tau-\frac{2}{c} K_{0}^{(i)}\right)} \operatorname{sinc}\left[T_{\text {obs }}\left(\nu+\frac{2 f_{0}}{c} K_{1}^{(i)}\right)\right] \operatorname{sinc}\left[B\left(\tau-\frac{2}{c} K_{0}^{(i)}\right)\right] \tag{3.18}
\end{align*}
$$

where $\operatorname{sinc}(x)=\frac{\sin (\pi x)}{\pi x}$ and $R D_{\substack{f \rightarrow \tau \\ t \rightarrow \nu}}\{\cdot\}$ indicates the operation of image formation by means of the Range-Doppler approach.

### 3.3 Multi-Channel CLEAN technique

The CLEAN technique is a method for scattering center extraction. It performs the extraction of target features such as position and complex amplitude of dominant scatterers. Having to deal with a multichannel system composed by three antennas, the single channel CLEAN technique were modified [33]. A first modification of the single channel CLEAN was proposed by [36] where the Polarimetric CLEAN (Pol-CLEAN) technique was presented. The Multi-Channel CLEAN (MC-CLEAN) presented in this thesis adopt a similar concept as the Pol-CLEAN, but it is extended to ISAR images obtained from a spatial multi-channel radar configuration. This represents the major innovation of the proposed MC-CLEAN algorithm.

### 3.3.1 Signal separation

As previously mentioned, the Range-Doppler technique can be applied if the Fourier domain in which the reflectivity function is defined can be approximated with a rectangular window. As a consequence, the spatial frequencies can be assumed independent variables and the received signal model in (3.17) can be written as the product of two terms, one depends on the slow time $(t)$ and one depends on the frequency $(f)$, as follows:

$$
\begin{align*}
S_{R}^{(i)}(f, t) & \cong S_{1}^{(i)}(t) S_{2}^{(i)}(f)= \\
& =\rho \exp \left\{-j \frac{4 \pi f_{0} K_{1}^{(i)}}{c} t\right\} \operatorname{rect}\left(\frac{t}{T_{o b s}}\right) \exp \left\{-j \frac{4 \pi K_{0}^{(i)}}{c} f\right\} \operatorname{rect}\left(\frac{f-f_{0}}{B}\right) \tag{3.19}
\end{align*}
$$

The two components $S_{1}^{(i)}(t)$ and $S_{2}^{(i)}(f)$ can be expressed as follows:

$$
\begin{align*}
& S_{1}^{(i)}(t)=A_{1}^{(i)} \exp \left(j 2 \pi\left(\eta+f_{d} t+\frac{\mu}{2} t^{2}\right)\right) \text { rect }\left(\frac{t}{T_{\text {obs }}}\right) \\
& S_{2}^{(i)}(f)=A_{2}^{(i)} \exp \left(j 2 \pi f \tau_{0}\right) \operatorname{rect}\left(\frac{f-f_{0}}{B}\right)  \tag{3.20}\\
& \rho^{(i)}=A_{1}^{(i)} A_{2}^{(i)} \cong \rho
\end{align*}
$$

where $f_{d}$ is the Doppler frequency, $\mu$ is the chirp rate, and $\tau_{0}$ is the time delay associated with the scattering center. It is worth noting that the chirp rate $\mu$ is related to the signal model, which accounts for a quadratic radial motion.

### 3.3.2 Feature extraction

The MC-CLEAN is an iterative technique and mainly performs four main steps:

- locating the brightest scattering center, namely the dominant scatterer, in one of the multichannel ISAR images;
- recovering its coordinates in the time delay-Doppler image plane $\left(\tau^{*}, \nu^{*}\right)$;
- removing it from all ISAR images;
- extracting the next dominant scatterer;

It is worth pointing out that the CLEAN technique estimates the selected scatterers PSF and subtract it from the ISAR image in order to remove the scattering centres.

Denote $I^{(i)}(\tau, \nu)$ as the ISAR image reconstructed by the $i^{\text {th }}$ receiver.

The dominant scatterer is found within the three images. The parameters $\tau^{*}, \nu^{*}$ and $i^{*}$ that correspond to the range index, the Doppler index and the index of the image that contains the dominant scatterer, are extracted by means of:

$$
\begin{equation*}
\left(\tau^{*}, \nu^{*}, i^{*}\right)=\arg \max _{(\tau, \nu, i)}\left\{\left|I^{(i)}(\tau, \nu)\right|\right\} \tag{3.21}
\end{equation*}
$$

with $\tau \in\{1,2, \ldots, P\}, \nu \in\{1,2, \ldots, N\}$ and where $P$ and $N$ are the number of range and Doppler bins.

Once $i^{*}$ has been found, its contribution for the other two spatial channels of $I^{(i)}(\tau, \nu)$ must be estimated to avoid reselecting the same scatterer at the next iteration and to delete it from each channel.

By referring to $S_{1}^{(i)}(t)$ in (3.20), the parameters $A_{1}^{(i)}, f_{d}$ and $\mu$ are firstly estimated. The constant $\eta$ does not affect the shape of the PSF, then can be neglected.

Assuming that the PSF is the same for all the ISAR images, the scattering center deletion can be performed in a single channel and then be applied to the remaining channels by adjusting the corresponding $A_{1}^{(i)}$ parameter. Such a deletion is performed by making use of a cost function that depends only on the absolute value of the range profile, consequently ensuring to handle the optimization problem in the real domain. It is important to highlight that only the magnitude $\left|A_{1}^{(i)}\right|$ of $A_{1}^{(i)}$ must be estimated at this stage. Conversely, the phase component is estimated separately and directly in the image domain.

According to [36], the following optimization problem can be stated:

$$
\begin{equation*}
\left\{\hat{f}_{d}, \hat{\mu},\left|\hat{A}_{1}^{i^{*}}\right|\right\}=\arg \min _{\left(f_{\mathrm{d}}, \mu,\left|A_{1}^{i^{*}}\right|\right)}\left\{E_{d_{i^{*}}}\left(f_{d}, \mu,\left|A_{1}^{i^{*}}\right|\right)\right\} \tag{3.22}
\end{equation*}
$$

where $E_{d_{i^{*}}}=\int\left|d_{i^{*}}(\nu)\right|^{2} d \nu$ is the energy of a Doppler section in the $i^{*} t h$ spatial channel, with $d_{i^{*}}(\nu)=\left|I^{\left(i^{*}\right)}\left(\tau^{*}, \nu\right)\right|-\left|S_{1}^{\left(i^{*}\right)}(\nu)\right|$ and $S_{1}^{\left(i^{*}\right)}(\nu)=F T_{t \rightarrow \nu}\left\{S_{1}^{\left(i^{*}\right)}(t)\right\}$ where the $F T_{t \rightarrow \nu}\{\cdot\}$ operator implements the Fourier Transform.

The procedure to estimate the frequency component of the PSF follows similar steps. In fact, the parameters $\tau_{0}$ and $A_{2}^{i}$ are estimated as:

$$
\begin{equation*}
\left\{\hat{\tau}_{0},\left|\hat{A}_{2}^{i^{*}}\right|\right\}=\arg \min _{\left(\tau_{0},\left|A_{2}^{i^{*}}\right|\right)}\left\{E_{g_{i^{*}}}\left(\tau_{0},\left|A_{2}^{i^{*}}\right|\right)\right\}, \tag{3.23}
\end{equation*}
$$

where $E_{g_{i^{*}}}=\int\left|d_{g_{i^{*}}}(\tau)\right|^{2} d \tau$ is the energy of a time-delay section in the $i^{*}$ th spatial channel, with $g_{i^{*}}(\tau)=\left|I^{\left(i^{*}\right)}\left(\tau, \nu^{*}\right)\right|-\left|S_{2}^{\left(i^{*}\right)}(\tau)\right|$ and $S_{2}^{\left(i^{*}\right)}(\tau)=I F T_{f \rightarrow \tau}\left\{S_{2}^{\left(i^{*}\right)}(f)\right\}$ where the $I F T_{f \rightarrow \tau}\{\cdot\}$ operator implements the Inverse Fourier Transform.

The solution of the optimization problem in (3.22) and (3.23) is achieved by using genetic algorithms [37].

The scattering center PSF in the $i^{t h}$ spatial channel is computed as follows:

$$
\begin{equation*}
I_{P S F}^{(i)}(\tau, \nu)=\left|2 D-F T_{\substack{f \rightarrow \tau \\ t \rightarrow \nu}}\left\{\hat{S}_{1}^{(i)}(t) \hat{S}_{2}^{(i)}(f)\right\}\right| \angle\left(I^{(i)}(\tau, \nu)\right) \tag{3.24}
\end{equation*}
$$

where the phase extracted from the ISAR image is multiplied by the 2D-FT of the product of the estimates of the time and frequency components.

Then, at each iteration, the scattering center must be deleted from the ISAR image by using eq. (3.25) in order to extract the following dominant scattering centre:

$$
\begin{equation*}
I_{k+1}^{(i)}(\tau, \nu)=I_{k}^{(i)}(\tau, \nu)-I_{P S F_{k}}^{(i)}(\tau, \nu) . \tag{3.25}
\end{equation*}
$$

where $k$ denotes a generic iteration.

When the scatter have been removed, the estimation of the PSF is achieved by minimizing the image energy.

The MC-CLEAN stops when the residual energy in the ISAR image at the $k^{t h}$ iteration is lower than a pre-set threshold $\Gamma$. The threshold $\Gamma$ is usually chosen as a percentage $F$ of the initial energy and depends on the energy content and on the SNR of the initial ISAR image as follows: (3.26):

$$
\begin{equation*}
\Gamma=F E^{(\mathbf{I}(\tau, \nu))} \frac{S N R}{S N R+1} \tag{3.26}
\end{equation*}
$$

where $E^{(\mathbf{I}(\tau, \nu))}=\sum_{i=1}^{3} E^{\left(I^{(i)}(\tau, \nu)\right)}$, with $E^{\left(I^{(i)}(\tau, \nu)\right)}=\iint\left|I^{(i)}(\tau, \nu)\right|^{2} d \tau d \nu$.
The iterations stop when $E_{k}^{(\mathbf{I}(\tau, \nu))} \frac{S N R}{S N R+1}<\Gamma$.
The block diagram of the MC-CLEAN technique is shown in Figure 3.2 .

### 3.4 Three-dimensional target reconstruction

The height of each scatterer with respect to the image plane depends on the angle $\phi$ and on the interferometric phases measured from two orthogonal baselines, as described in [21]. Due to the short baseline lengths compared to the radar-target distance, it is acceptable to assume that the ISAR images obtained from the three sensors show the same image projection plane. However, the length of the baselines have to be chosen carefully in order to maximise the system performances


Figure 3.2: MC-CLEAN block diagram.
and, above all, in order to obtain an unambiguous altitude measurement. It can be demonstrated [38][39] that the phase difference between the return echoes received by the orthogonal antennas along either the vertical or the horizontal baselines is a periodic function with period $2 \pi$. Thus, the height estimation is unambiguous if the baseline lengths satisfy the following upper bounds:

$$
\begin{equation*}
d_{H} \leq \frac{\lambda R_{0}}{2 h_{H}} \quad, \quad d_{V} \leq \frac{\lambda R_{0}}{2 h_{V}} \tag{3.27}
\end{equation*}
$$

where $d_{H}$ and $d_{V}$ are the the baseline length along the vertical and the horizontal direction. $h_{H}$ and $h_{V}$ are the projections of the scatterer with the maximum height onto the imaging plane and $\lambda=\frac{c}{f_{0}}$ is the wavelength.

It is worth mentioning that non ambiguous phase measurements are needed to estimate scatterer's heights. In fact, the phase drift loose its continuity after the scattering extraction process and consequently the phase unwrapping is not possible.

As discussed in [38], it would be important to have a long baseline in order to minimize phase error measurement. Equivalently, short baselines would be desirable in order to minimize the view angle. Then, it is fundamental to dimension the system in order to achieve a trade-off between these conflicting requirements.

### 3.4.1 Joint estimation of the angle $\phi$ and $\Omega_{e f f}$

The received signals can be used to compute the phase differences at the peak of the sinc functions in (3.18) for the horizontal and vertical configurations:

$$
\begin{align*}
& \triangle \theta_{V}=\frac{4 \pi}{c} f_{0}\left(K_{0}^{A C}-K_{0}^{A V}\right)=\frac{4 \pi}{c} f_{0} \frac{d_{V}\left(y_{1} \sin \phi+y_{3} \cos \phi\right)}{R_{0}}  \tag{3.28}\\
& \triangle \theta_{H}=\frac{4 \pi}{c} f_{0}\left(K_{0}^{A C}-K_{0}^{A H}\right)=\frac{4 \pi}{c} f_{0} \frac{d_{H}\left(y_{1} \cos \phi-y_{3} \sin \phi\right)}{R_{0}} \tag{3.29}
\end{align*}
$$

The point $\mathbf{P}_{y}\left(\mathbf{P}_{x}(0)\right)$ is firstly mapped onto the point $\mathbf{P}_{\xi}=\left[y_{1 \xi}, y_{2 \xi}, y_{3 \xi}\right]$ from the reference system $T_{x}$ to the reference system $T_{\xi}$ at $t=0$ :

$$
\begin{equation*}
\mathbf{P}_{\xi}=\mathbf{M}_{\xi \mathbf{x}}^{-1} \cdot \mathbf{P}_{x}(0)^{T}=\mathbf{M}_{\xi \mathbf{x}}^{-1} \cdot \mathbf{P}_{y}^{T} \tag{3.30}
\end{equation*}
$$

where the symbol $(\cdot)^{T}$ is the transpose operator.
At this stage we have obtained the analytical expression of its coordinates with respect to $T_{\xi}$.

By substituting (3.30) in (3.28)-(3.29) the phase differences $\triangle \theta_{V}$ and $\triangle \theta_{H}$ can be rewritten as a function of the coordinates $y_{3 \xi}$ and $y_{1 \xi}$ respectively. After inverting the obtained equations, these coordinates can be expressed as a function of the phase differences:

$$
\begin{align*}
& \triangle \theta_{V}=\frac{4 \pi}{c} f_{0} \frac{d_{V} y_{3 \xi}}{R_{0}} \Rightarrow y_{3 \xi}=\frac{c}{4 \pi f_{0}} \frac{R_{0}}{d_{V}} \Delta \theta_{V}  \tag{3.31}\\
& \triangle \theta_{H}=\frac{4 \pi}{c} f_{0} \frac{d_{H} y_{1 \xi}}{R_{0}} \Rightarrow y_{1 \xi}=\frac{c}{4 \pi f_{0}} \frac{R_{0}}{d_{H}} \Delta \theta_{H} \tag{3.32}
\end{align*}
$$

Finally, the coordinates of $\mathbf{P}_{y}$ can be expressed in the $T_{x}$ reference system by remapping the coordinates from $T_{\xi}$ to $T_{x}$ as follows:

$$
\begin{equation*}
\mathbf{P}_{y}=\mathbf{M}_{\xi \mathbf{x}} \cdot \mathbf{P}_{\xi}^{T} \tag{3.33}
\end{equation*}
$$

Among the three components, $y_{3}$ corresponds to the height of the scatterer with respect to the image plane. The component $y_{3}$ can be expressed as a function of the phase differences and the angle $\phi$ as follows:

$$
\begin{equation*}
y_{3}=y_{3 \xi} \cos \phi-y_{1 \xi} \sin \phi=\frac{c}{4 \pi f_{0}} R_{0}\left(\frac{\triangle \theta_{V}}{d_{V}} \cos \phi-\frac{\triangle \theta_{H}}{d_{H}} \sin \phi\right) \tag{3.34}
\end{equation*}
$$

Furthermore, $\boldsymbol{\Omega}_{\text {eff }}$ and $\phi$ can be jointly estimated by expressing the term $\mathbf{c}$ in equation (2.13) with respect to the reference system $T_{x}$ :

$$
\begin{equation*}
\mathbf{c}=\boldsymbol{\Omega}_{\boldsymbol{T}} \times \mathbf{P}_{y} \Rightarrow c_{2}=y_{1} \Omega_{e f f} \tag{3.35}
\end{equation*}
$$

where $\boldsymbol{\Omega}_{\boldsymbol{T}}=\left(0, \Omega_{T 2}, \Omega_{e f f}\right), \Omega_{e f f}$ is the modulus of $\boldsymbol{\Omega}_{e f f}$ and $\Omega_{T 2}$ is the coordinate of $\boldsymbol{\Omega}_{\boldsymbol{T}}$ along the $x_{2}$ axis. It should be pointed out that this result originate from the selection of $T_{x}$. In fact, this reference system is chosen in order to have the $x_{3}$ and $x_{2}$ axes aligned with $\boldsymbol{\Omega}_{\text {eff }}$ and the LoS respectively. Furthermore, it is clear that $\Omega_{\text {eff }}$ is the component that takes into account the actual target rotation. In fact, $\Omega_{T 2}$ is alinged with the LoS and consequently does not produce any aspect angle changes. Thus, the first component $\Omega_{T 1}$ must be zero.

Another way to express the term $c_{2}$ by taking into account the Doppler component as follows:

$$
\begin{equation*}
\nu_{A C} \triangleq-\frac{2 f_{0}}{c} K_{1}^{A C} \cong-\frac{2 f_{0}}{c} c_{2} \quad \Rightarrow \quad c_{2}=-\nu_{A C} \frac{c}{2 f_{0}} \tag{3.36}
\end{equation*}
$$

After some algebra we obtain:

$$
\begin{equation*}
\nu_{A C}=-\frac{R_{0} \Omega_{e f f}}{2 \pi}\left(\frac{\triangle \theta_{H}}{d_{H}} \cos \phi+\frac{\triangle \theta_{V}}{d_{V}} \sin \phi\right) \tag{3.37}
\end{equation*}
$$

As a result, equation (3.37) can be rewritten by considering only the contribution due to the $k^{\text {th }}$ scatterer as follows:

$$
\begin{equation*}
Z_{k}=a Y_{k}+b X_{k} \tag{3.38}
\end{equation*}
$$

where $Z \triangleq \nu_{A C}, Y \triangleq-\frac{R_{0}}{2 \pi d_{H}} \triangle \theta_{H}, X \triangleq-\frac{R_{0}}{2 \pi d_{V}} \triangle \theta_{V}, a \triangleq \Omega_{e f f} \cos \phi$ and $b \triangleq$ $\Omega_{e f f} \sin \phi$.

The term $Z_{k}$ corresponds to the Doppler value of the $k^{\text {th }}$ scatterer relative to the central receiver. Therefore, the interferometric phases are calculated from the matrices $\triangle \theta_{H}$ and $\triangle \theta_{V}$ only in the scatterers location as identified by the CLEAN processing, i.e their range/Doppler bins. In this way, the terms $X_{k}, Y_{k}$ and $Z_{k}$ are real values and equation (3.38) represents the equation of a plane.

Then, the estimates of $\Omega_{\text {eff }}$ and $\phi$ can be calculated throughout the estimation of $a$ and $b[40]$. This can be done by evaluating the regression plane, that is the plane that minimizes the sum of the square distances of the points $P_{k}=\left(X_{k}, Y_{k}, Z_{k}\right)$ from the plane itself.

The problem can be mathematically solved by minimizing the function:

$$
\begin{equation*}
\Psi(a, b)=\sum_{k=1}^{M}\left[Z_{k}-\left(a Y_{k}+b X_{k}\right)\right]^{2} \tag{3.39}
\end{equation*}
$$

Finally, the estimation of $\Omega_{e f f}$ and $\phi$ can be derived from the estimates $\tilde{a}$ and $\tilde{b}$ as described below:

$$
\begin{align*}
& \hat{\Omega}_{e f f}=\sqrt{\tilde{a}^{2}+\tilde{b}^{2}} \\
& \hat{\phi}=\arctan \left(\frac{\tilde{b}}{\tilde{a}}\right) \tag{3.40}
\end{align*}
$$

Figure 3.3 summarises the step to be performed in order to obtain a 3D image of the target, The signal received from the three antennas is compensated by using the Multichannel Image Contrast Based Algorithm (M-ICBA). Then, the scattering centres are extracted from the ISAR images by using the Multichannel CLEAN (MC-CLEAN) technique. The inter-element phase differences are used to jointly estimate the effective rotation vector and the altitude of the scatterers with respect to the image plane. Finally, the 3D spatial coordinates of the scattering centres are used to form a 3D image of the moving target.

It worth noticing that the M-ICBA must be applied to focus the ISAR images from different spatial channels with respect to the same focusing point on the


Figure 3.3: 3D reconstruction processing flowchart
target. This step is necessary to achieve the correct 3D reconstruction. In the considered scenario, the receiving antennas lies on a plane orthogonal to the LoS and the radar-target distance is much grater that the baseline lengths. For these reasons, it is acceptable to assume that the distortions in all the three ISAR images are caused by the same phase term. Under these hypotheses, it can be applied the standard ICBA described in Section 2.6.1 to a reference channel. Then, the same focusing parameters can be applied to the ISAR images formed by the other receivers.

In this case, the function $I C$ for the AC receiver can be expressed as follows:

$$
\begin{equation*}
I C\left(v_{r}, a_{r}\right)=\frac{\sqrt{A\left\{\left[I^{A C}\left(\tau, \nu ; v_{r}, a_{r}\right)-A\left\{I^{A C}\left(\tau, \nu ; v_{r}, a_{r}\right)\right\}\right]^{2}\right\}}}{A\left\{I^{A C}\left(\tau, \nu ; v_{r}, a_{r}\right)\right\}} \tag{3.41}
\end{equation*}
$$

in which $I^{A C}$ is the ISAR image formed at the AC receiver, $v_{r}$ and $a_{r}$ represent the focusing parameters corresponding to the radial speed and acceleration. Thus, the optimization problem to be solved becomes:

$$
\begin{equation*}
\left(\hat{v}_{r}, \hat{a}_{r}\right)=\arg \left(\max _{v_{r} a_{r}}\left[I C\left(v_{r}, a_{r}\right)\right]\right) \tag{3.42}
\end{equation*}
$$

As previously stated, the estimates of the motion parameters are supposed to be the same for all the spatial channels. Then, focusing parameters of the AC receiver can be used to focus the ISAR images formed at the other receivers.

It is worth pointing out that in operating scenarios where the above mentioned conditions are not met, other multistatic autofocusing algorithm must be applied as the one described in [41].

### 3.5 Conclusion

In this chapter a 3D target reconstruction method that makes used of a novel scattering centre extraction (MC-CLEAN) has been detailed. This method makes use of a multi-channel interferometric ISAR system which is composed of three TX/RX antennas lying on two orthogonal baselines with an L-shape.

The dominant scatterers, within the three ISAR images, are extracted by means of the MC-CLEAN technique. The Multichannel CLEAN technique provides the scattering centres range-Doppler position.

The analytical expressions for calculating the height of the extracted point scatterers (w.r.t the IPP) is derived by means of a joint estimation of the effective rotation vector magnitude and the angle $\phi$. This allows for the reconstruction of the 3 D shape of the target.

## Chapter 4

## 3D Interferometric ISAR Performance Analysis

### 4.1 Introduction

Simulated and real data were used to evaluate the effectiveness of the 3D reconstruction technique described in Chapter 3.

In this chapter, simulated data results are presented and evaluated through the developed performance analysis procedure, which is detailed in 4.2.

Simulated data are obtained by generating the backscattered signal from two pointlike scatterer models. Several scenarios have been produced to test the proposed algorithm by varying the target's motion and the system geometry. The target 3D reconstruction is accomplished by applying perfect motion compensation. For the purpose of verifying the efficiency of the algorithm, it is important to align the 3D cloud of points with respect to a pre-set reference system. This operation is fundamental as it allows reconstructed targets to be compared directly with the target's models.

First, Section 4.2 will introduce the performance analysis mathematics. Then, Section 4.3 will detail the results of three significant simulations.

### 4.2 Performance analysis

Several numerical simulations have been run in order to evaluate the performances of the interferometric 3D reconstruction method. Gaussian noise has been added to the raw data to introduce a given SNR in the data domain.

Firstly, it is essential a realignment process between the model, which is expressed with respect to $T_{y}$, and the reconstructed target, which is expressed with respect to the image plane. This step allows to compare each scatterer of the reconstructed target with each scatterer of the reference model. For this reason, the model is rotated to align its heading to the assigned trajectory. Then the reconstructed target is rotated to align the image plane to the horizontal plane where the reference model is expressed.

Secondly, each scatterer of the 3D reconstructed target is assigned to the correspondent scatterer of the model via a soft assignment technique.

Finally, the algorithm performances are evaluated throughout chosen performance indicators.

The performance analysis block diagram is shown in Figure 4.1.


Figure 4.1: Performance analaysis flowchart

### 4.2.1 Scatterers realignment

## 1. Rotation of the model along the trajectory

The model is rotated along the trajectory by using the rotation matrix $\boldsymbol{R}_{t}$
as follows:

$$
\begin{gather*}
\boldsymbol{C}_{r t}=\boldsymbol{R}_{t} \cdot \boldsymbol{C}_{m}  \tag{4.1}\\
\boldsymbol{R}_{t}=\boldsymbol{R}_{\mu} \cdot \boldsymbol{R}_{\nu} \cdot \boldsymbol{R}_{\phi}
\end{gather*}
$$

where $\boldsymbol{C}_{m}$ is a $3 \times M$ matrix expressing the coordinates of the model; $\boldsymbol{R}_{\mu}, \boldsymbol{R}_{\nu}$ and $\boldsymbol{R}_{\xi}$ describe the yaw, pitch and roll rotation matrices respectively. Finally, the matrix $\boldsymbol{C}_{r t}$ describes the three-dimensional coordinates of the model rotated along the trajectory.

## 2. Image plane rotation

The reconstructed target is rotated to align the image plane to the horizontal plane by means of the rotation matrix $\boldsymbol{M}_{\xi x}^{-1}$ and the estimated rotation angle $\hat{\phi}$.

### 4.2.2 Soft assignment

The proposed soft assignment method is based on a Probabilistic Least Squares (PLS) approach [42][43]. Each scatterer of the model is associated to each reconstructed scatterer by means of a soft assignment procedure. This procedure assigns a probability to these associations by using weighting coefficients. The sum of all possible assignments for a given scatterer adds to the unity. The optimization problem is given by:

$$
\begin{equation*}
\hat{\boldsymbol{\alpha}}=\arg \{\min \{J(\boldsymbol{\alpha})\}\} \tag{4.2}
\end{equation*}
$$

where

$$
\begin{equation*}
J(\boldsymbol{\alpha})=\sum_{i=1}^{M} \sum_{k=1}^{K} \alpha_{i, k} \varepsilon_{i, k}^{T} \varepsilon_{i, k} \alpha_{i, k} \tag{4.3}
\end{equation*}
$$

is the cost function; $K$ is the number of the extracted scattering centers and $\varepsilon_{i, k}$ is the euclidean distance between the $i^{\text {th }}$ scatterer of the model and the $k^{\text {th }}$ scatterer of the reconstructed target; The $M \times K$ matrix $\boldsymbol{\alpha}$ containing the soft assignments
can be expressed as:

$$
\boldsymbol{\alpha}=\left[\begin{array}{ccc}
\alpha_{1,1} & \cdots & \alpha_{1, K}  \tag{4.4}\\
\vdots & \vdots & \vdots \\
\vdots & \alpha_{i, k} & \vdots \\
\vdots & \vdots & \vdots \\
\alpha_{M, 1} & \cdots & \alpha_{M, K}
\end{array}\right]
$$

Each element of the matrix $\alpha_{i, k}$ denotes the probability that the $k^{t h}$ extracted scatterer belongs to the $i^{\text {th }}$ model's scatterer and is written as follows:

$$
\left\{\begin{array}{l}
\alpha_{i, k}=\frac{\left(\varepsilon_{i, k}^{T} \varepsilon_{i, k}\right)^{-1}}{\sum_{p=1}^{M}\left(\varepsilon_{p, k}^{\varepsilon_{p}} \varepsilon_{p, k}\right)^{-1}}  \tag{4.5}\\
\sum_{i=1}^{M} \alpha_{i, k}=1 \quad \forall k=1, \ldots, K
\end{array}\right.
$$

Finally, each scatterer of the reconstructed target is assigned to the model's scatterer with the highest $\alpha_{i, k}$.

### 4.2.3 Performance indicators

Two different types of errors must be taken into into account that could affect the results: the assignment error and the scatterers height estimation error.

The assignment error is related to the identification of unreliable assignments. An unreliable assignment is declared when a scatterer's height error $\epsilon_{h}$ is greater than a fixed threshold $\Lambda$, which is expressed as follows:

$$
\begin{equation*}
\Lambda=E\left[\bar{\epsilon}_{h}(s)\right]+\gamma \cdot \sigma_{\varepsilon_{h}} \tag{4.6}
\end{equation*}
$$

where $E[\cdot]$ denotes the expectation operator considering all the $S$ Monte Carlo runs of the simulation. The term $\bar{\epsilon}_{h}(s)$ is defined as follows:

$$
\begin{equation*}
\bar{\epsilon}_{h}(s)=\frac{1}{K} \sum_{k=1}^{K} \epsilon_{h}(k, s) \tag{4.7}
\end{equation*}
$$

The height error of the $k^{\text {th }}$ scatterer at the $s^{t h}$ Monte Carlo step is defined as:

$$
\begin{equation*}
\epsilon_{h}(k, s)=\left|h_{m}(k, s)-h_{r}(k, s)\right| \tag{4.8}
\end{equation*}
$$

where $h_{m}(k, s)$ and $h_{r}(k, s)$ are the heights related to the model and to the reconstructed target respectively. The variable $\sigma_{\varepsilon_{h}}$ is the standard deviation of $\epsilon_{h}$ whereas $\gamma \in \mathbb{R}$ is a parameter that can vary to empirically adjust the threshold $\Lambda$.

When all the unreliable assignments are discarded, the scatterers height estimation error is performed using the same procedure.

The mean error on the estimated values of the rotation angle $\hat{\phi}$ and the effective rotation vector $\hat{\Omega}_{\text {eff }}$ are computed as follows:

$$
\begin{equation*}
E\left[\epsilon_{\Omega}(s)\right]=\frac{1}{S} \sum_{s=1}^{S} \epsilon_{\Omega}(s) \tag{4.9}
\end{equation*}
$$

and

$$
\begin{equation*}
E\left[\epsilon_{\phi}(s)\right]=\frac{1}{S} \sum_{s=1}^{S} \epsilon_{\phi}(s) \tag{4.10}
\end{equation*}
$$

where $E[\cdot]$ is the expectation operator. The parameters $\epsilon_{\Omega}(s)$ and $\epsilon_{\phi}(s)$ are defined as follows:

$$
\begin{gather*}
\epsilon_{\Omega}(s)=\left|\hat{\Omega}_{e f f}(s)-\Omega_{e f f}\right|  \tag{4.11}\\
\epsilon_{\phi}(s)=|\hat{\phi}(s)-\phi| \tag{4.12}
\end{gather*}
$$

where $\Omega_{e f f}$ and $\phi$ are a priori known.

### 4.3 Simulation results

Several simulations have been run. The main significant simulations are reported here to prove the effectiveness of the proposed method. The simulations consist on a target moving along a given trajectory defined by the roll, pitch and yaw directions. The target motions are characterized by their own motion components and the targets are assumed to be rigid bodies.

The first simulation sees a target composed of $M=35$ ideal point-like scatterers and resembles the shape of an airplane. In the second simulation, the target resembles the shape of a boat and it is composed of $M=36$ ideal point-like scatterers. The third simulation considers a cross-shaped antenna to evaluate possible performance differences due to the addition of the fourth sensor.

### 4.3.1 Simulation 1: Airplane target model

The target is shown in Figure 4.2 and Figure 4.3.


Figure 4.2: Airplane model target.


Figure 4.3: Airplane model target.

For the purpose of simplicity and clarity of visualization, the model is depicted with its principal axes to be coincident with the X-Y-Z Cartesian coordinates.

The parameters used in the simulation are shown in Table 4.1.

| Number of freq. | 256 | $\boldsymbol{\gamma}$ | 1.2 |
| :---: | :---: | :---: | :---: |
| Radar sweeps | 128 | Radar-target dist. | 10 Km |
| Bandwidth | 300 MHz | Target velocity | $80 \mathrm{~m} / \mathrm{s}$ |
| Carrier freq. | 10 GHz | Roll/Pitch/Yaw | $0^{\circ} / 20^{\circ} / 75^{\circ}$ |
| $\boldsymbol{T}_{\text {obs }}$ | 0.6 s | Monte Carlo runs | 50 |

Table 4.1: Simulation Parameters - Airplane

The setup geometry is depicted in Figure 4.4.


Figure 4.4: Airplane set-up geometry.
Examples of simulated raw data and the resulting ISAR images for two different SNR values are illustrated in Figure 4.5 and Figure 4.6


Figure 4.5: Simulated raw data. (a) $\mathrm{SNR}=0 \mathrm{~dB}$; (b) $\mathrm{SNR}=-15 \mathrm{~dB}$.


Figure 4.6: Amplitude ISAR images. (a) $\mathrm{SNR}=0 \mathrm{~dB}$; (b) $\mathrm{SNR}=-15 \mathrm{~dB}$.
The Multi-Channel CLEAN technique is used to extract the scattering centres and gives the range/Doppler positions of the extracted scatterers. Results are shown in Figure 4.7 and Figure 4.8.


Figure 4.7: Scattering centres extracted by the MC-CLEAN. (a) SNR=0 dB;
(b) $\mathrm{SNR}=-15 \mathrm{~dB}$.


Figure 4.8: MC-CLEAN residual image. (a) $\mathrm{SNR}=0 \mathrm{~dB}$; (b) $\mathrm{SNR}=-15 \mathrm{~dB}$.
By looking at Figure 4.7 it can be noticed that the MC-CLEAN technique performs a good scatterers extraction for both the considered SNR.

After the scattering centres extraction, the algorithm uses the inter-element phase differences and the $(\tau, \nu)$ position of the extracted scatterers to estimate the height of each scatterer w.r.t. the image plane. The final 3D reconstruction result is shown in Figure 4.9 and Figure 4.10, where the reconstructed targets are superimposed onto the models.

$X[m]$
Figure 4.9: Results of the 3D reconstruction - airplane.


Figure 4.10: Results of the 3D reconstruction - airplane.
The height error and the standard deviation of the height error curves are depicted in Figure 4.11 and Figure 4.12. The curves are expressed in meters and are shown as the SNR and the baseline length change.


Figure 4.11: Height error - Airplane. (a) Height error with unreliable assignments; (b) Height error without unreliable assignments.


Figure 4.12: Standard deviation of the height error - Airplane. (a) With unreliable assignments; (b) without unreliable assignments.

The baseline lengths are chosen in order to satisfy the constraint expressed in (3.27). Short baselines produce larger phase error measurement and therefore larger height estimation errors occur. On the other hand, the height error tends to increase when the baseline length exceeds the upper bounds. This is because the target is not contained in the unambiguous window.

It is worth noticing that the values of both the height error and the standard deviation decrease significantly when the unreliable assignments are discarded. In addiction, the curves follow a decreasing trend when the SNR increases and when the baseline length increases, accordingly with the theory. It can be inferred that the correct identification of the unreliable assignments, and consequently the choice of an appropriate threshold, is crucial in order to obtain reliable results.

The robustness of the selected threshold $\Lambda$ has been tested by varying the parameter $\gamma$. Results do not show significant variations. As expected, this parameter have no impact on the performances of the proposed method.

Figure 4.13 shows the height error $E\left[\bar{\epsilon}_{h}(k)\right]$ computed by preserving the positive or negative sign of $\epsilon_{h}$.


Figure 4.13: Signed error without unreliable assignments - Airplane.
The estimator results unbiased.

The estimates of $\Omega_{e f f}$ and $\phi$ are depicted in Figure 4.14-4.17. As already noticed for the other values, it can be observed a descendant pattern as the SNR and the baseline lengths increase.


Figure 4.14: Mean error of the estimate of $\Omega_{e f f}$ - Airplane.


Figure 4.15: Standard deviation of the estimate of $\Omega_{e f f}$ - Airplane.


Figure 4.16: Mean error of the estimate of $\phi$ - Airplane.


Figure 4.17: Standard deviation of the estimate of $\phi$ - Airplane.

### 4.3.2 Simulation 2: boat target model

The target is shown in Figure 4.18. Again, the model is depicted with its principal axes to be coincident with the X-Y-Z Cartesian coordinates to ease the visualization.


Figure 4.18: Boat model target
Table 4.2 shows the simulation parameters.

| Number of freq. | 256 | $\gamma$ | 1.2 |
| :---: | :---: | :---: | :---: |
| Radar sweeps | 128 | Radar-target dist. | 10 Km |
| Bandwidth | 300 MHz | Target velocity | $8 \mathrm{~m} / \mathrm{s}$ |
| Carrier freq. | 10 GHz | Roll/Pitch/Yaw | $0^{\circ} / 0^{\circ} / 80^{\circ}$ |
| $\boldsymbol{T}_{\text {obs }}$ | 0.8 s | Monte Carlo runs | 50 |

Table 4.2: Simulation Parameters - Boat

The setup geometry is shown in Figure 4.19.


Figure 4.19: Boat set-up geometry.
An example of simulated raw data and the resulting ISAR image for different SNR values are illustrated in Figure 4.20 and Figure 4.21

compensated frequency-slow time signal


Figure 4.20: Simulated raw data. (a) $\mathrm{SNR}=0 \mathrm{~dB}$; (b) $\mathrm{SNR}=-15 \mathrm{~dB}$.


Figure 4.21: Amplitude ISAR images. (a) $\mathrm{SNR}=0 \mathrm{~dB}$; (b) $\mathrm{SNR}=-15 \mathrm{~dB}$.
Results of the Multi-Channel CLEAN technique are shown in Figure 4.22 and Figure 4.23.


Figure 4.22: Scattering centres extracted by the MC-CLEAN. (a) $\mathrm{SNR}=0 \mathrm{~dB}$;
(b) $\mathrm{SNR}=-15 \mathrm{~dB}$.


Figure 4.23: MC-CLEAN residual image. (a) $\mathrm{SNR}=0 \mathrm{~dB}$; (b) $\mathrm{SNR}=-15 \mathrm{~dB}$.
It is worth noticing that the MC-CLEAN technique performs in both cases a good scatterers extraction despite the ISAR image with lower SNR is obviously degraded.


Figure 4.24: Results of the 3D reconstruction - boat.
The behaviour of the height error and the standard deviation of the height error expressed in meters are depicted in Figure 4.25 and Figure 4.26. As in the first simulation, the figures are shown as the SNR and the baseline length change. Obviously, the baseline lengths meet the constraint expressed in (3.27).


Figure 4.25: Height error - Boat. (a) Height error with unreliable assignments; (b) Height error without unreliable assignments.



Figure 4.26: Standard deviation of the height error - Boat. (a) With unreliable assignments; (b) without unreliable assignments.

As in the first simulation, the values of both the height error and the standard deviation decrease significantly when the unreliable assignments are discarded. Furthermore, both the height error and the standard deviation values decrease as the SNR increases and as the baseline length increases.

Also in this second simulation the estimator is verified unbiased, as shown inFigure 4.27 .


Figure 4.27: Signed error without unreliable assignments - Boat.
The same descendant pattern can be observed in Figure 4.28-4.31, which show the mean error and the standard deviation of the estimates of $\Omega_{e f f}$ and $\phi$.


Figure 4.28: Mean error of the estimate of $\Omega_{e f f}$ - Boat.


Figure 4.29: Standard deviation of the estimate of $\Omega_{e f f}$ - Boat.


Figure 4.30: Mean error of the estimate of $\phi$ - Boat.


Figure 4.31: Standard deviation of the estimate of $\phi$ - Boat.
It should be pointed out that this pattern changes for a baseline length equal to 5 .
This is because the baseline length exceeds the upper bound expressed in (3.27).

### 4.3.3 Cross-shaped antenna array

Consider a cross-shaped antenna as shown in Figure 4.32


Figure 4.32: System geometry - cross-shaped array.

Simulation parameters are detailed in Table 4.3 and results are shown in Figure 4.33-4.37.

| Number of freq. | 256 | $\boldsymbol{\gamma}$ | 1.2 |
| :---: | :---: | :---: | :---: |
| Radar sweeps | 128 | Radar-target dist. | 10 Km |
| Bandwidth | 300 MHz | Target velocity | $150 \mathrm{~m} / \mathrm{s}$ |
| Carrier freq. | 10 GHz | Roll/Pitch/Yaw | $-60^{\circ} / 50^{\circ} / 90^{\circ}$ |
| $\boldsymbol{T}_{\text {obs }}$ | 0.7 s | Monte Carlo runs | 50 |

Table 4.3: Simulation Parameters.


Figure 4.33: MC-CLEAN processing, $\operatorname{SNR}=-20 \mathrm{~dB}$. (a)Amplitude ISAR image; (b) Extracted scatterers; (c)Residual image.

Also in this case, a good 3D reconstruction is achieved. According with the theory, the curves of the performance indicators follow a descendant trend as the SNR and the baseline lengths increase. Finally, the algorithm performances increase when unreliable assignments are discarded.

However, results with equal effectiveness can be achieved with an L-shaped array, which represents a more cost effective and easier to realize solution.

SUPERIMPOSITION OF TARGET AND MODEL


Figure 4.34: 3D reconstruction.


Figure 4.35: Height error. (a)Height error with unreliable assignments; (b) Height error without unreliable assignments.


Figure 4.36: Standard deviation of the height error. (a)Standard deviation of the height error with unreliable assignments; (b) Standard deviation of the height error without unreliable assignments.


Figure 4.37: Signed error without unreliable assignments.

### 4.4 Conclusion

In this chapter, the main simulation results of the 3D interferometric algorithm described in Chapter 3 have been detailed.

Three scenarios with two different targets have been used to test the algorithm. Raw data have been simulated and conventional two-dimensional ISAR images have been obtained. In all the simulations, the MC-CLEAN effectively extracts the scattering centres and the algorithm is able to reconstruct the 3D shape of the target.

A performance analysis has been carried out in order to compare the 3D reconstructed targets with the models.

First, the model is rotated as to be aligned to its trajectory. Then, the reconstructed target is rotated by means of the rotation matrix $\boldsymbol{M}_{\xi x}$. A PLS approach is used to assign each reconstructed scatterer to the correspondent scatterer of the model. A threshold $\Lambda$ is given to identify the assignment errors and consequently to discard the unreliable assignments.

Simulation results have shown that unreliable assignments significantly deteriorate the performances of the proposed method. It is then crucial to chose the appropriate threshold.

Performance indicators exhibit a descendant pattern as the SNR and the baseline lengths increase, accordingly with the theory.

From the study conducted on the described simulated data sets, it can be concluded that this method performs good 3D reconstructions for all the considered SNR, for all the analysed scenarios.

## Chapter 5

## Real Data Analysis

### 5.1 Introduction

The 3D interferometric reconstruction method was tested on two real datasets.

Conversely from simulated data, the Multichannel Image Contrast Based Autofocus algorithm is applied to real data as no knowledge a priori is given.

Section 5.2 and Section 5.2 will show the real data analysis results. Conclusion will be discussed in Section 5.4

### 5.2 Case study 1

In this section, the 3D interferometric reconstruction method previously described is tested on a first set of real data. A measure campaign took place on the $21_{\text {th }}$ March 2014 at the 'Giancarlo Vallauri' institute, which is located at the Naval Academy of Livorno (Italy). This specific scenario was chosen to test the algorithm because of the continuous traffic of ships in front of the coastline.

The sensor used for the measurements is a dual interferometric low-powered FMCW radar provided by Metasensing. The radar is called Habra 2 and exploits a four
elements antenna array with one transmitter and three receivers in a L-shape configuration. The radar and the antenna are shown in Figure 5.1 and Figure 5.2.


Figure 5.1: HABRA 2.


Figure 5.2: HABRA 2 - Antenna.
It is worth pointing out that in Figure 5.2 the transmitter is located in the upper right corner.

The radar parameters are listed in Table 5.1.

| Central frequency | 9.6 GHz |
| :---: | :---: |
| Bandwidth | 300 MHz |
| PRF | 0.75 KHz |
| Tx Power | 23 dBm |
| $\boldsymbol{d}_{H}$ | 0.175 m |
| $\boldsymbol{d}_{V}$ | 0.43 m |

Table 5.1: Radar Parameters

The measure campaign scenario is depicted in Figure 5.3. The radar was placed on top of an about four-storey high building that overlooks the sea.


Figure 5.3: Measure campaign scenario - 'G. Valauri' institute@the Naval Academy of Livorno.

The data were acquired and post processed as follows.

First, data in the observation time were processed to produce Range-Doppler maps. The chosen Range-Doppler map in Figure 5.4 clearly shows a target located at roughly 2300 m in range. Then, the fragment of the Range-Doppler map containing the target was cropped and the successive processing was applied to the cropped image.


Figure 5.4: Range-Doppler map.
Subsequently, the ISAR image was formed as shown in Figure 5.5.


Figure 5.5: ISAR image before motion compensation.

The obtained ISAR image shows distortions due to the defocusing. Then, the image was refocused by means of the Multichannel ICBA, described in Chapter 3. The M-ICBA makes use of an initialisation method to provide an accurate initial guess for the iterative search of the image contrast maximisation. This initialization technique is based on the fact that the range profile time history consists of several stripes due to the main scatterers' range migration [23]. This 'bands' are almost linear. Then, a rough estimation of the radial velocity $v_{r}$ can be made by using the Radon Transform (RT) under the following assumptions:

- The distance between the radar and the generic $i^{\text {th }}$ scatterer varies linearly with slope equal to $v_{r}$.
- The distance between the radar and the focusing point varies approximately in a linear way as the scatterers.

The radial velocity can then be expressed as:

$$
\begin{equation*}
v_{r}=\tan (\hat{\varphi}) \tag{5.1}
\end{equation*}
$$

where $\varphi$ is the the angle between the scatterers trace and the abscissa axis. The parameter $\hat{\varphi}$ is the estimates of $\varphi$, obtained by selecting the $\varphi$ that maximise the RT [44]. In fact, the Radon transform is an integral along a path, and in its simplest form this path is a straight line.

For the data under analysis, the RT is shown in Figure 5.6.


Figure 5.6: Radon transform.

In Figure 5.7 is depicted the Image Contrast (precisely, $-I C$ ) section along the radial acceleration in correspondence of the down peak.


Figure 5.7: Image Contrast maximization.
The range profiles after motion compensation are illustrated in Figure 5.8 while Figure 5.9 shows the ISAR images on the Three receiving channels after applying the M-ICBA.


Figure 5.8: Range profiles after motion compensation.


Figure 5.9: ISAR image after motion compensation.
After motion compensation, the MC-CLEAN is applied. The result is shown in Figure 5.10


Figure 5.10: Scattering centres extracted by using the MC-CLEAN technique.
Finally, the proposed interferometric approach was applied. The resulting 3D reconstruction is shown in Figure 5.11-5.14.

Figure 5.11 shows the 3D reconstruction from three different prospectives whilst Figure 5.12-5.14 show the 3D reconstruction along the Range/Cross-range, Height/Crossrange and Height/Range planes respectively.

3D-ISAR Target from Reference Sensor


Figure 5.11: 3D reconstruction.

It is worth pointing out that is possible to size the target along the cross-range coordinate due to the estimation of the effective rotation vector $\boldsymbol{\Omega}_{\text {eff }}$. In fact, the interferometric algorithm jointly estimates the effective rotation vector and the rotation angle $\phi$. For this data set, these values have been estimated as follows:

$$
\begin{aligned}
\hat{\Omega}_{e f f} & =0,0032[\mathrm{rad} / \mathrm{s}] \\
\hat{\phi} & =72,5264^{\circ}
\end{aligned}
$$

This is consistent with a side-view of the vessel. From the reconstruction the target can be sized as about 90 m in length and about 18 m in height. Unfortunately, during this campaign, we couldn't recover the AIS data and consequently it was not possible to precisely match the reconstructed target with a particular ship.


Figure 5.12: 3D reconstruction - Range/Cross-range.


Figure 5.13: 3D reconstruction - Height/Cross-range.

However, a possible match with a ship as in Figure 5.15 can be assumed from the measured size of the target.


Figure 5.15: Gulf Express cargo ship

| Built | 1999 |
| :---: | :---: |
| Size | $85 \times 17 \times \cdot \mathrm{m}$ |
| Draught | 6.255 m |

Table 5.2: Gulf Express master data


Figure 5.14: 3D reconstruction - Height/Range.

### 5.3 Case study 2

A second measure campaign was conducted within the activities of the NATO SET 196 (Multichannel/Multistatic Radar Imaging of Non-cooperative Targets Joint Trials), which ran a five-day joint trial in Livorno from the $29_{t h}$ September 2014 to the $3_{t h}$ October 2014. The trials were hosted by the Istituto Vallauri of the Italian Navy located in Livorno (Italy). Representatives from eight nations participated in the trials. The radar systems of each work group simultaneously acquired data of a number of maritime and air targets.

The particular chosen data-set was acquired the $2_{t h}$ October with the same lowpowered FMCW dual interferometric radar system $H A B R A 2$ used for the fist real data-set, with the parameters listed in Table 5.3.

| Central frequency | 9.6 GHz |
| :---: | :---: |
| Bandwidth | 300 MHz |
| PRF | 611.5 Hz |
| Tx Power | 23 dBm |
| $\boldsymbol{d}_{H}$ | 0.35 m |
| $\boldsymbol{d}_{V}$ | 0.6 m |

Table 5.3: Radar Parameters

Figure 5.16 shows the radar positioned on a terrace on top of the Instituto Vallauri, which is about three-storey high that overlooks the sea.

The antenna array is again composed of a transmitter and three receivers in a L-shape configuration.


Figure 5.16: HABRA 2.
The measure campaign scenario and the radar location are depicted in Figure 5.17. The figure shows three radar systems $H A B R A 1, H A B R A 2$ and $H A B R A 3$ illuminating the sea. The radar coordinates (latitude and longitude) and their pointing direction with respect to the north are summarised in Table 5.4.

|  | Radar position | Pointing |
| :---: | :---: | :---: |
| HABRA 1 | $43.524860^{\circ}, 10.309020^{\circ}$ | $254^{\circ}$ |
| HABRA 2 | $43.526163^{\circ}, 10.309288^{\circ}$ | $252^{\circ}$ |
| HABRA 3 | $43.526367^{\circ}, 10.306558^{\circ}$ | $176^{\circ}$ |

Table 5.4: Radars coordinates (latitude and longitude) and pointing direction


Figure 5.17: Measure campaign scenario - 'G. Valauri' institute@the Naval Academy of Livorno.

Several cooperative maritime and air targets were used to test the systems (Astice A 5379, MBN 1132, Rubber duck, CESSNA-182, Ultralight aircraft - p92, Ultralight aircraft - sinus. In the considered acquisition scenario the target is the training ship Astice (A 5379) shown in Figure 5.18 along with its main characteristics.


Figure 5.18: Astice A 5379

| Weight | 180 tons |
| :---: | :---: |
| Size | $33.25 \times 6.47 \times \sim 12 / 13 \mathrm{~m}$ |
| Draught | 2.1 m |

Table 5.5: Gulf Express master data

The Astice's trajectory tracked with the GPS data is shown in Figure 5.19. It can be noticed that the distance between the radar and the target at the GPS time 14.04 is about 1550 m, consistently with the RD map shown in Figure 5.21.


Figure 5.19: Trajectory of the ship Astice.
Five corner reflectors were placed on Astice as shown in Figure 5.20


Figure 5.20: Corner reflector locations on Astice.
The data were acquired and post processed as follows.

First, data in the observation time were processed to produce Range-Doppler maps. The chosen Range-Doppler map in Figure 5.21 shows a target located at roughly 1550 m in range, consistently with the position of Astice. Another target is visible at zero-Doppler, which is another ship used in the campaign, at that time anchored at $\backsim 650 \mathrm{~m}$.

The fragment of the Range-Doppler map containing the target was cropped and the processing was applied on the cropped image.


Figure 5.21: Range-Doppler map.
The ISAR image of the target is shown in Figure 5.22


Figure 5.22: ISAR image before motion compensation.
As explained in Section 5.2, the M-ICBA inizialization process makes use of the Radon transform, which is shown in Figure 5.23.


Figure 5.23: Radon transform.
In Figure 5.24 is depicted the Image Contrast (precisely, $-I C$ ) section along the radial acceleration in correspondence of the down peak.


Figure 5.24: Image Contrast maximisation.
The range profiles after motion compensation are illustrated in Figure 5.25 while Figure 5.26 shows the ISAR image on the central channel after applying the MICBA.


Figure 5.25: Range profiles after motion compensation.


Figure 5.26: ISAR image after motion compensation.
Figure 5.27 illustrates a sketch of the corner reflector positions, top view. By comparing the ISAR image in Figure 5.26 and Figure 5.27 it is possible to recognize the two scatterers on the back of Astice and one scatterer located on the deckhouse. The radar is able to see the stern and part of the side of the ship.


Figure 5.27: Corner reflectors position on Astice.
The result of the MC-CLEAN scattering center extraction is shown in Figure 5.28.


Figure 5.28: MC-CLEAN- extracted scatterers.
Finally, the effective rotation vector $\boldsymbol{\Omega}_{\text {eff }}$ and the rotation angle $\phi$ were estimated in order to obtain the 3D shape of the target. For this data set, these values have been estimated as follows:

$$
\begin{aligned}
\hat{\Omega}_{e f f} & =0,0461[\mathrm{rad} / \mathrm{s}] \\
\hat{\phi} & =163,5041^{\circ}
\end{aligned}
$$

These estimations are consistent with a view angle close to the top view.
The resulting 3D reconstruction is shown in Figure 5.29-5.30.


Figure 5.29: 3D reconstruction.



Figure 5.30: 3D reconstruction - Height/Cross-range.

Figure 5.29 shows the 3D reconstruction whilst Figure 5.30 shows the 3D reconstruction along the XY, XZ and YZ planes respectively.

It is worth pointing out that the estimated heights of the two scatterers at the back of Astice are almost identical, consistently with the corner reflector positions.

Furthermore, the dimensions of the reconstructed target are consistent with the size of the real target, Astice.

### 5.4 Conclusion

This chapter details the results of the 3D interferometric algorithm applied to real data.

Two data set acquired with the Metasensing radar $H A B R A 2$ in two different scenarios were used to test the algorithm. From the Range-Doppler map, the
location of interest was cropped. Then, the data were processed by applying the MICBA technique to focus the ISAR image and the MC-CLEAN technique to extract the scattering centres. Results show that in both cases the 3D reconstruction is consistent with the real targets.

Concluding, this method is effective and allows for a satisfactory 3D target reconstruction in both simulated and real data.

However, some issues need to be taken into account:

1. Baseline length constraints:

The altitude measurement is unambiguous if the baseline lengths satisfy the upper bounds expressed in eq. (3.27). Phase unwrapping is not possible as there is no continuity of phase drift after the bright scatterers extraction. As a consequence, non ambiguous phase measurements are needed to estimate the scatterer's heights.
2. Radar-target geometry:

If the target is squinted with respect to the radar system's LoS, the InISAR method can be applied with minor modifications. The radar reference system must be projected onto the plane perpendicular to the target reference system and the phase error must be compensated.
3. Shadowing effects:

The interferometric technique is not able to separate several scatterers located in the same range-Doppler cell in the ISAR image. When this occurs, the scatterers located at a particular $(\tau, \nu)$ position are wrongly treated as a single scatterer, to which is associated an incorrect phase estimation. This leads to a wrong estimation of this single scatterer's altitude w.r.t. the image plane and the 3D reconstruction results incorrect. This problem is addressed in Chapter 7 where is proposed a novel 3D target reconstruction method that makes use of a tomographic approach. Moreover, scatterers that are not visible to the radar cannot be identified and placed in a 3D coordinate system. This leads to a partial reconstruction of the target. This problem
is addressed in Chapter 6, where a incoherent technique for multistatic 3D reconstruction is proposed.

## Chapter 6

## Multistatic 3D ISAR Imaging Reconstruction

### 6.1 Introduction

This chapter focuses on describing a method for multistatic 3D target reconstruction that is based on a incoherent technique. Although the multichannel InISAR processing described in Chapter 3 has demonstrated to be an efficient method to generate 3D images of the target, issues related to shadowing are a limitation for this technology. In fact, one of the key problem of a multichannel InISAR system is that scatterers that are not visible to the radar cannot be identified. Consequently, they cannot contribute to the 3D reconstruction and the produced 3D image may result incomplete. Moreover, monostatic ISAR image processing can fail in some specific radar-target geometries, e. g. when a target is moving along the LoS.

The idea is to overcome such limitations by using a net of spatially distributed ISAR systems as depicted in Figure 6.1.

Multistatic configurations are proven to overcome a number of limitations related to monostatic systems. Among several advantages, from the prospective of


## Sensor \#1

Figure 6.1: Example of multistatic configuration

3 D reconstruction, multistatic configurations allow to overcome geometrical and shadowing issues.

However, a critical aspect is how to combine the data from different radar systems [45].

A first approach is a coherent data fusion. The data are combined at the rowdata level and although this could be considered the optimal solution, practical difficulties can preclude its implementation. Some crucial issues are:

- Time and phase synchronization of all the radars
- Target's aspect angle and rotation vector knowledge - otherwise the signal could not be mapped in the full 3D Fourier space
- Target's coherence over the multistatic angles

The second solution is to incoherently combine the output of each radar processing in the image domain. This method is more robust against synchronization and target motion estimation errors. Thus, it is a more practical approach.

However, the 2D ISAR images obtained at each monostatic system cannot be easily combined. In fact, the 2D image obtained at the output of each ISAR processing is the representation of the target reflectivity function along a particular image projection plane, which is different for each radar system.

The proposed solution to this problem is to join together the monostatic 3D reconstructions of each InISAR system, obtained with the algorithm described in Chapter 3 [40]. Multiple look angles will produce 3D reconstructions of different parts of the target. Then, the incoherent data combination in the image domain requires the alignment of the 3 D reconstructions.

### 6.2 Multistatic 3D Interferometric ISAR

### 6.2.1 Pre-alignment processing

InISAR processing exploits the phase differences measured from two orthogonal baselines in order to estimate the scattering centres heights with respect to the image plane.

Each InISAR system produces a three-dimensional target reconstruction from a particular aspect angle, expressed with respect to the specific acquisition reference system. To join together the different 3D images it is then necessary a prealignment step to express the estimated coordinates of the scatterers with respect to the same reference system. First, each 3D reconstruction is rotated of the rotation angle $\phi$ (w.r.t. the reference system embedded in the radar) estimated during the interferometric processing. Then, under the assumption of a completely known multistatic acquisition geometry, all the reconstructions are rotated with respect to a fixed reference system. However, inaccurate $\phi$ estimations and shadowing effects at each InISAR system can cause misalignments. After the pre-alignment step, a further alignment algorithm is necessary to perform the complete multistatic data fusion.

### 6.2.2 Multistatic 3D reconstruction fusion

At a high level, the flowchart of the proposed algorithm is shown in Figure 6.2


Figure 6.2: Algorithm flowchart

The reconstruction fusion algorithm aims at combining the reconstructions obtained at each InISAR system in order to achieve the 3D target image as complete as possible.

First, the alignment process is performed between each pair of the partial reconstructions.

Within each pair, one 3D image will be identified as the reference. Lets suppose that the reference is composed by $N$ scatterers while the other reconstruction is composed by $M$ scatterers. Then, the main steps are as follows:

## 1. Height error calculation

The height error $\varepsilon_{n m}$ between the $n^{t h}$ scatterer of the reference and all the scatterers of the other reconstruction is defined as follows:

$$
\begin{equation*}
\epsilon_{n m}=\left|h_{n}-h_{m}\right| \tag{6.1}
\end{equation*}
$$

It should be pointed out that is possible to use this approach only if the data from the three sensors are coregistered at least in the focusing point. A potential multistatic ISAR autofocusing method is described in [41]. Under this assumption, only the height error can be used for a reliable comparison. In fact, the $x-y$ - coordinate cannot be used as the shadowing effect shifts
the centroids of the reconstructed targets. The only coordinate consistent for all the 3D reconstructions is along the $z-$ axis.
2. Comparison of the height error to a threshold $\Upsilon$

When the height error is lower than the threshold $\Upsilon$, a possible match $m_{k}(n)$ for the $n^{\text {th }}$ scatterer is declared, with $k=1 \ldots K_{n}$. The parameter $K_{n}$ indicates the total number of matches for the particular $n^{\text {th }}$ scatterer.

## 3. Alignment

For each $m_{k}(n)$, the reference is aligned to the other 3D reconstruction by overlapping the $n^{\text {th }}$ to the possible match.
4. Number of correspondences and mean matching distance calculation

After the alignment process, all the $x-, y-, z-$ coordinates are consistent for both the 3D reconstructions. The total number of correspondence $t c_{k}(n)$ can now be obtained by calculating the euclidean distance between the scatterers of the two reconstructions and comparing to the threshold. If this distance is lower than the threshold, a correspondence is declared. The mean matching distance $m m d_{k}(n)$ is also calculated as reliability indicator:

$$
\begin{equation*}
m m d_{k}(n)=\frac{1}{C} \sum_{i=1}^{C} d_{i} \tag{6.2}
\end{equation*}
$$

where $C$ is the number of total correspondences and $d_{i}$ is the $i^{\text {th }}$ euclidean distance between the scatterers that are declared as a correspondence for the particular alignment.
5. Choice of the best alignment

For the particular $n^{\text {th }}$ scatterer of the reference reconstruction, the best alignment will be considered the one with the maximum number of correspondences. If more then one couple $(n, m)$ gives the same number of correspondence, the one with the minimum mean match distance will be chosen.

This process is repeated for all the $N$ scatterers of the reference reconstruction.

For each particular pair of 3D images, the final alignment is then chosen according to the position that gives the maximum number of correspondences. If several configurations generate the same number of correspondences, the most reliable alignment is chosen, that is the one with the minimum mean match distance. The block scheme of the algorithm for a single pair of 3D reconstructions is shown in Figure 6.3.


Figure 6.3: Algorithm block scheme for a single pair of 3D reconstructions The complete reconstruction is achieved by iteratively joining together the partial reconstructions, repeating the same process. The number of iteration depends on the number $Q$ of InISAR systems. The algorithm stops after $Q-1$ iterations. Again, the final reconstruction is chosen according with the maximum number of matches and the minimum mean match distance.

### 6.2.3 Simulation results

A simulated multistatic scenario with three 3D InISAR systems has been considered to test the algorithm. The target is assumed to be a rigid body composed of $N=36$ ideal point-like scatterers as shown in Figure 6.4.


Figure 6.4: Target model
The simulation parameters roll, pitch and yaw describe the target's motion. The multistatic scenario is simulated by varying these parameters to obtain the desired view angle for each sensor. The shadowing effect is generated by making use of blockage planes to identify the visible scatterers. The blockage planes used in this simulation are shown in Figure 6.5.




Figure 6.5: Visible scatterers identification
For two InISAR systems of the multistatic network the visible scatterers are found by using a single blockage plane perpendicular to the LoS in the target's centroid. For the third sensor, two blockage planes are used to simulate a further shadowing. The view angles are set so that the object is not completely illuminated by the multistatic sensor's network in order to test the robustness of the algorithm in the case of missing scatterers in the overall 3D target's shape reconstruction.

As described in Section 6.2.1, the 3D reconstruction fusion algorithm can be applied only after having expressed the coordinates of the reconstructed scatterers with respect to a fixed reference system. The result of this pre-alignment process is shown in Figure 6.6.

(a) 3D Pre-alignment result

(b) Top view

Figure 6.6: Pre-alignment result

It is worth pointing out that the 3D images obtained at each sensor are affected by misalignments as the centroids are shifted. As clear from Figure 6.6, after the pre-alignment process it is only possible to have a rough estimation of the target occupancy and the scatterers' fusion cannot be performed as it is.

The final alignment of each pair of 3D reconstructions is depicted in figurename 6.7.

Finally, the complete 3D reconstruction fusion is shown in Figure 6.8. The top views of the final 3D fusion and the model are depicted in Figure 6.9 to highlights that the shape of the final reconstruction is incomplete. As stated before, several scatterers are missing due to the choice of the views angles. In fact, some scattering centres of the target are not visible to all the three monostatic systems. The algorithm results robust against missing scatterers and its performances are not


Figure 6.7: Partial 3D reconstructions
compromised. In conclusion, the partial fusions are correctly aligned and the overall target shape is properly reconstructed.

Results show that the algorithm successfully reconstruct the complete 3D shape of the target. However, the method is based on the fact that the 3D reconstructions obtained at each monostatic InISAR system must have in common a number of scatterers. Then, the monostatic InISAR systems should be positioned in order to obtain enough difference in aspect angle to have a view of the target as complete as possible. On the other hand, the monostatic InISAR systems should be positioned not too far apart to avoid illuminating completely different parts of the target.


Figure 6.8: Complete 3D reconstruction

(a) 3D reconstruction fusion

(b) Model

Figure 6.9: Complete 3D reconstruction and model top views

### 6.2.4 Conclusion

In this chapter, a multistatic 3D ISAR imaging reconstruction method have been proposed. Each monostatic InISAR system produces a 3D image of the target. The 3D reconstruction fusion is performed in the image domain by consistently joining together the monostatic 3D reconstructions. A pre-alignment process is necessary since the Image Projection Plane might be different for all the radars. The coordinates of the reconstructed scatterers are expressed with respect to a fixed reference
system and the image fusion is performed. To find the best alignment, the algorithm works iteratively and uses the height error, the euclidean distance and the mean matching distance between the scatterers as distinctive parameters for a reliable association. Simulation results have shown that the proposed method is able to provide a correct 3D reconstruction fusion of the object. Since the method rely on the fact that the initial 3D reconstructions have in common a number of scatterers, the positioning of the InISAR systems in the multistatic configuration has to be carefully considered.

## Chapter 7

## Joint Use of Two-dimensional Tomography and ISAR Imaging for Three-dimensional Image Formation of Non-cooperative Targets

### 7.1 Introduction

The objective of this chapter is to introduce a novel tomographic approach to form three-dimensional ISAR images of non-cooperative moving targets. A 2D ISAR image is obtained at each element of two orthogonal arrays. Beamforming techniques are used to coherently combine the inter-element phase differences. The 3D shape of the target is obtained by estimating the effective rotation vector modulus, the image projection plane orientation angle and finally the height of each target's scatterer (w.r.t. the image plane). Three-dimensional ISAR image formation allows to overcome several limitations of the conventional 2D ISAR processing. One of the main challenges in ISAR imaging is the interpretation of
the 2D image when the image plane is unknown. In fact, the orientation of the image plane depends on the relative motion between the radar and the target and in the case of non-cooperative targets cannot be predicted a priori. A related problem is the cross-range scaling. After the Range-Doppler processing, the ISAR image is represented in the time delay/Doppler frequency domain. It would be desirable to scale the image in order to size the target both in range and in crossrange. On the other hand the cross-range resolution depends on the effective rotation vector. This parameter takes into account the target angular motion and cannot be predicted a priori.

In literature quite a few methos that deal with the formation of 3D ISAR images can be found. Some methods exploit a sequence of ISAR images obtained from a single sensor [46], [47], [27] or from multistatic configurations [48]. However, these methods often require either a priori knowledge of the target's motions or the cross-range scaling problem to be solved [49]. Other techniques makes uses of interferometry [30], [32], [50]. Nevertheless, issues as phase unwrapping or baseline constraints have to be faced.

The 3D reconstruction method proposed in Chapter 3 is based on the interferometric principles. A 3D image of the target is obtained by the estimation of the orientation of the image plane ( $\left.\Omega_{e f f}, \phi\right)$. By following some baseline constraints, there is no need of phase unwrapping and the method shows to achieve good target reconstructions for each of the considered SNR.

However, this technique and most of the above mentioned 3D ISAR imaging approaches does not allow for scatterers belonging to the same range-Doppler cell to be resolved in the ISAR image (layover effect).

To fill this gap, the idea is to use a tomographic approach to obtain threedimensional ISAR images of non-cooperative moving targets. In fact, it is possible to achieve height resolution by means of beamforming techniques.

Tomographic techniques are widely used in Synthetic Aperture Radar (SAR), where a three-dimensional SAR image can be generated by means of multiple
pass processing [51], [52]. However, one of the main differences between SAR and ISAR processing lies on the knowledge of the radar-target geometry. In fact, while in SAR scenarios the radar-target geometry is fixed and consequently perfectly known, in the ISAR environment the orientation of the imaging plane is unknown and has to be taken into account during the image processing.

This Chapter is organized as follows: the 2D ISAR image processing is briefly recalled in Section 7.2. Section 7.3 describes the tomographic method to obtain a 3D ISAR image of non-cooperative targets and simulation results are presented in Section 7.4.

### 7.2 ISAR System Model

### 7.2.1 System Geometry

Consider the system geometry in Figure 7.1 showing a cross-shaped antenna array configuration. In this configuration, two uniform linear arrays composed by $K_{V}$ and $K_{H}$ transmit/receive elements are located along the $\xi_{3^{-}}$and $\xi_{1}$-axis of the reference system $T_{\xi}=\left[\xi_{1}, \xi_{2}, \xi_{3}\right]$ embedded in the radar. The array phase center is in $k_{H}=k_{V}=0$. The axis $\xi_{2}$ is chosen to be aligned with the radar LoS.


Figure 7.1: Imaging system geometry.

The reference system $T_{x}=\left[x_{1}, x_{2}, x_{3}\right]$ is time-varying as is embedded on the target and rotates along with the LOS in the rotation centre $O$. The radartarget distance is $R_{0}$. Axes $x_{1}, x_{2}$ define the image plane and correspond to the cross-range and range coordinates respectively. The axis $x_{3}$ is aligned to the effective rotation vector $\boldsymbol{\Omega}_{e f f}(t)$. As the reference system $T_{x}$ is time varying, is defined a reference system $T_{y}=\left[y_{1}, y_{2}, y_{3}\right]$ which corresponds to $T_{x}$ at $t=0$. The parameter $\phi$ is the rotation angle between $T_{x}$ and $T_{\xi}$ whilst $\theta_{m}$ and $\alpha_{m}$ denote the elevation and azimuth angles of the $m_{t h}$ scatterer, with $1 \leq m \leq M$ in the $T_{\xi}$ coordinate system. The target is assumed to be a rigid body composed of $M$ ideal independent point-like scattering centres. It is important to mention that the described radar-target geometry can be applied to any real scenario by projecting the radar reference system onto the plane perpendicular to the target reference system and by subsequently compensating the phase error.

### 7.2.2 2D ISAR Imaging

Let briefly recall the main steps to obtain a conventional 2D ISAR image as described in Chapter 3. As described in to [40], the spectrum of the time-varying spatial multichannel signal received from the $k^{\text {th }}$ antenna, relative to an ideal point scatterer, after motion compensation can be expressed as:

$$
\begin{equation*}
S_{R}^{(k)}(f, t)=\rho^{(k)} \exp \left\{-j \frac{4 \pi f}{c}\left[\mathbf{P}_{y} \cdot \mathbf{i}_{L O S_{\xi}}^{(k)}(t)\right]\right\} \cdot W(f, t) \tag{7.1}
\end{equation*}
$$

where

$$
\begin{equation*}
W(f, t)=\operatorname{rect}\left[\frac{t}{T_{o b s}}\right] \cdot \operatorname{rect}\left[\frac{f-f_{0}}{B}\right] \tag{7.2}
\end{equation*}
$$

is the Point Spread Function (PSF) and $\mathbf{P}_{y}$ is the position at time $t=0$ for a generic scatterer, defined in the reference system $T_{y}$ and $\mathbf{i}_{L O S_{\xi}}^{(k)}$ is the LOS unit vector relative to the $i^{t h}$ antenna. Furthermore, it is acceptable to assume that $\rho^{(k)}=\rho$. In fact, the reflectivity functions $\rho^{(k)}$ can be considered identical due to the small dimension of the array compared with the radar-target distance $R_{0}$.

At each element, the received signal is used to form a 2D ISAR image of the target by means of the RD technique. As described in [40], the analytical expression of the ISAR image point spread function from the $k^{t h}$ antenna for a single scatterer located at $\mathbf{P}_{y}$ can be obtained by using a 2D Fourier Transform (2D-FT) after radial motion compensation:

$$
\begin{align*}
I^{(k)}(\tau, \nu)=B & \cdot T_{\text {obs }} \cdot \rho \cdot e^{j 2 \pi f_{0}\left(\tau-\frac{2}{c} \Gamma_{0}^{(k)}\right)} \\
& \cdot \operatorname{sinc}\left[T_{\text {obs }}\left(\nu+\frac{2 f_{0}}{c} \Gamma_{1}^{(k)}\right)\right] \cdot \operatorname{sinc}\left[B\left(\tau-\frac{2}{c} \Gamma_{0}^{(k)}\right)\right] \tag{7.3}
\end{align*}
$$

where $\operatorname{sinc}(x)=\frac{\sin (\pi x)}{\pi x}$ and $d_{V}=d_{H}=d$.
The parameters $\Gamma_{0}^{(k)}$ and $\Gamma_{1}^{(k)}$ are defined for the vertical array as follows:

$$
\begin{align*}
\Gamma_{0}^{(k)} & =\Gamma_{0 V}^{(k)}=y_{2}+\frac{k \cdot d}{R_{0}}\left(y_{1} \sin \phi+y_{3} \cos \phi\right)  \tag{7.4}\\
\Gamma_{1}^{(k)} & =\Gamma_{1 V}^{(k)}=c_{2}+\frac{k \cdot d}{R_{0}}\left(c_{1} \sin \phi+c_{3} \cos \phi\right)
\end{align*}
$$

and for the horizontal array as:

$$
\begin{align*}
\Gamma_{0}^{(k)} & =\Gamma_{0 H}^{(k)}=y_{2}-\frac{k \cdot d}{R_{0}}\left(y_{1} \sin \phi-y_{3} \cos \phi\right) \\
\Gamma_{1}^{(k)} & =\Gamma_{1 H}^{(k)}=c_{2}-\frac{k \cdot d}{R_{0}}\left(c_{1} \sin \phi-c_{3} \cos \phi\right) \tag{7.5}
\end{align*}
$$

The MC-CLEAN technique $[40,53]$ described in Chapter 3 is then used to locate and extract the brightest scattering centres in the 2D ISAR images.

### 7.3 Three-dimensional ISAR Beamforming

The tomographic method to obtain a three-dimensional image of moving target is described here. The height of each scattering centre is recovered throughout the joint estimation of the rotation angle $\phi$ and the modulus of effective rotation vector $\Omega_{e f f}$.

A rotation matrix $\boldsymbol{M}_{\xi x}$ is used to express the coordinates of the vector $\mathbf{P}_{y}$ by remapping its coordinates from $T_{\xi}$ to $T_{x}$.

$$
\begin{equation*}
\mathbf{P}_{y}=\mathbf{M}_{\xi \mathbf{x}} \cdot \mathbf{P}_{\mathbf{y} \xi} \tag{7.6}
\end{equation*}
$$

In particular, the coordinate $y_{3}$ denotes the height of the scatterer with respect to the image plane and is given by:

$$
\begin{equation*}
y_{3}=y_{3 \xi} \cos \phi-y_{1 \xi} \sin \phi \tag{7.7}
\end{equation*}
$$

By means of the beamforming technique, the components $y_{3 \xi}$ and $y_{1 \xi}$ can be evaluated. A final tomographic image is formed by using all the $K$ antennas of each array. The tomographic images are focused at a particular focus elevation angle $\theta$ for the vertical array, and at a particular focus azimuth angle $\alpha$ for the horizontal array as follows [54]:

$$
\begin{align*}
\hat{I}_{V}\left(\tau_{i}, \nu_{j}\right) & = \\
& =\sum_{k=-\frac{\left(K_{V}-1\right)}{2}}^{\frac{\left(K_{V}-1\right)}{2}} I_{V}^{(k)}\left(\tau_{i}, \nu_{j}\right) \exp \left\{-j \frac{4 \pi}{\lambda} k d \sin \theta\right\}  \tag{7.8}\\
\hat{I}_{H}\left(\tau_{i}, \nu_{j}\right) & = \\
& =\sum_{k=-\frac{\left(K_{H}-1\right)}{2}}^{\frac{\left(K_{H}-1\right)}{2}} I_{H}^{(k)}\left(\tau_{i}, \nu_{j}\right) \exp \left\{-j \frac{4 \pi}{\lambda} k d \cos \theta \sin \alpha\right\} \tag{7.9}
\end{align*}
$$

For both the vertical and the horizontal arrays, the term $\hat{I}\left(\tau_{i}, \nu_{j}\right)$ in eq. (7.8) and (7.9) denotes an estimate of the $i^{\text {th }} j^{\text {th }}$ pixel value of the final tomographic ISAR image $\hat{I}(\tau, \nu)$ and obtained by weighting the contribution of all the $K$ ISAR images $I^{(k)}(\tau, \nu)$.

The tomographic processing is carried out for a contiguous range of focus planes relative to a set of elevation angles $\{\theta\}$ and a set of azimuth angles $\{\alpha\}$. With this process, the scatterers point-spread function is shown both in the elevation and in the azimuth directions.

The elevation and azimuth angles of each $m_{t h}$ scattering centre, are estimated in a cascade as follows:

$$
\begin{align*}
& \hat{\theta}_{m}=\max _{\theta}\left\{10 \cdot \log _{10}\left[\frac{\left|\boldsymbol{I}\left(\tau_{m}, \nu_{m},\{\theta\}\right)\right|^{2}}{K^{2}}\right]\right\} \\
& \hat{\alpha}_{m}=\max _{\alpha}\left\{10 \cdot \log _{10}\left[\frac{\left|\boldsymbol{I}\left(\tau_{m}, \nu_{m}, \hat{\theta}_{m},\{\alpha\}\right)\right|^{2}}{K^{2}}\right]\right\} \tag{7.10}
\end{align*}
$$

Where

- The vector $\boldsymbol{I}\left(\tau_{m}, \nu_{m},\{\theta\}\right)$ contains the complex values of the tomographic ISAR images obtained with eq. (7.8), for the particular index $\tau_{m}, \nu_{m}$;
- $\tau_{m}, \nu_{m}$ denote the $m^{t h}$ scatterer location in range and Doppler;
- the vector $\boldsymbol{I}\left(\tau_{m}, \nu_{m}, \hat{\theta}_{m},\{\alpha\}\right)$ contains the complex values of the tomographic ISAR images obtained with eq. (7.9) at the horizontal array, by using the estimated $\hat{\theta}_{m}$ obtained from the beamforming at the vertical array.

It can be noticed that the terms $k d_{V} \sin \theta$ and $k d_{H} \cos \theta \sin \alpha$ in equations (7.8) and (7.9) express the phase differences between the element located in the phase center and each $k_{t h}$ element of the vertical and horizontal arrays, respectively. According to equations (7.3)-(7.5), these phase differences can also be written as $\left(y_{1} \sin \phi+y_{3} \cos \phi\right) / R_{0}$ for the vertical configuration and as $\left(y_{1} \cos \phi-y_{3} \sin \phi\right) / R_{0}$ for the horizontal configuration. Then, the components $y_{3 \xi}$ and $y_{1 \xi}$ can be written as a function of the estimates $\hat{\theta}$ and $\hat{\alpha}$ :

$$
\begin{align*}
& \hat{y}_{3 \xi}=y_{1} \sin \phi+y_{3} \cos \phi=R_{0} \sin \hat{\theta}  \tag{7.11}\\
& \hat{y}_{1 \xi}=y_{1} \cos \phi-y_{3} \sin \phi=R_{0} \cos \hat{\theta} \sin \hat{\alpha}
\end{align*}
$$

At this stage, the magnitude of the effective rotation vector $\Omega_{e f f}$ and the rotation angle $\phi$ can be jointly estimated by expressing the term $\mathbf{c}$ in eq. (2.13) with respect to the reference system $T_{x}$ as:

$$
\begin{equation*}
\mathbf{c}=\boldsymbol{\Omega}_{\boldsymbol{T}} \times \mathbf{y} \Rightarrow c_{2}=y_{1} \cdot \Omega_{e f f} \tag{7.12}
\end{equation*}
$$

with $\boldsymbol{\Omega}_{\boldsymbol{T}}=\left(0, \Omega_{T 2}, \Omega_{e f f}\right)$.
The Doppler component at the central antenna can be used to express the term $c_{2}$ as follows:

$$
\begin{equation*}
\nu^{(0)} \triangleq-\frac{2 f_{0}}{c} \cdot \Gamma_{1}^{(0)} \cong-\frac{2 f_{0}}{c} c_{2} \Rightarrow c_{2}=-\nu^{(0)} \cdot \frac{c}{2 f_{0}} \tag{7.13}
\end{equation*}
$$

After some algebra we obtain:

$$
\begin{equation*}
\nu^{(0)}=-\frac{2 f_{0}}{c} R_{0} \Omega_{e f f} \cdot(\cos \hat{\theta} \sin \hat{\alpha} \cos \phi+\sin \hat{\theta} \sin \phi) \tag{7.14}
\end{equation*}
$$

Refferring to the single $m^{t h}$ scatterer contribution, eq. (7.14) can be rewritten as follows:

$$
\begin{equation*}
Z_{m}=a Y_{m}+b X_{m} \tag{7.15}
\end{equation*}
$$

where

- $Z \triangleq \nu^{(0)}$
- $Y \triangleq-\frac{2 f_{0}}{c} R_{0} \cdot \cos \hat{\theta} \sin \hat{\alpha}$
- $X \triangleq-\frac{2 f_{0}}{c} R_{0} \cdot \sin \hat{\theta}$
- $a \triangleq \Omega_{e f f} \cdot \cos \phi$
- $b \triangleq \Omega_{e f f} \cdot \sin \phi$

Eq. (7.15) represents the equation of a regression plane. As detailed in Chapter 3, the estimates $\tilde{a}$ and $\tilde{b}$ can be obtained through the evaluation of this regression plane. The parameters $\Omega_{e f f}$ and $\phi$ can then be estimated throughout $\tilde{a}$ and $\tilde{b}$. The final result is:

$$
\begin{equation*}
\hat{\Omega}_{e f f}=\sqrt{\tilde{a}^{2}+\tilde{b}^{2}} \quad \hat{\phi}=\arctan \left(\frac{\tilde{b}}{\tilde{a}}\right) \tag{7.16}
\end{equation*}
$$

### 7.4 Simulation Results

In this section, results from two simulations are presented to test the effectiveness of the described method.

### 7.4.1 Simulation 1

The target is assumed to be a rigid body composed of $M=35$ ideal point-like scatterers and is depicted in Figure 7.2. The simulation parameters are detailed in Table 7.1 and the resulting target reconstruction for $\mathrm{SNR}=-10 \mathrm{~dB}$ is shown in Figure 7.3-7.4.

| Number of freq. | 256 | Radar sweeps | 128 |
| :---: | :---: | :---: | :---: |
| Bandwidth | 300 MHz | Radar-target dist. | 10 Km |
| Carrier freq. | 10 GHz | Target velocity | $80 \mathrm{~m} / \mathrm{s}$ |
| $\boldsymbol{T}_{\text {obs }}$ | 0.6 s | $\boldsymbol{d}_{\boldsymbol{V}}=\boldsymbol{d}_{\boldsymbol{H}}$ | 10 m |
| Roll/Pitch/Yaw | $10^{\circ} / 20^{\circ} / 75^{\circ}$ | $\boldsymbol{K}_{\boldsymbol{V}}=\boldsymbol{K}_{\boldsymbol{H}}$ | 5 |

Table 7.1: Simulation Parameters.


Figure 7.2: Model - Airplane.


Figure 7.3: 3D target reconstruction.


Figure 7.4: 3D target reconstruction.
The algorithm successfully reconstructs the target. The reconstructed target in the imaging plane reference system is shown in Figure 7.5 while Figure 7.6 shows the tomographic image focused at the estimated height of the scatterer $M 1$ in Figure 7.5.


Figure 7.5: 3D target reconstruction with respect to the imaging plane for $\mathrm{SNR}=-10 \mathrm{~dB}$.


Figure 7.6: Amplitude ISAR image and Tomographic ISAR image relative to scatterer $M 1$ for $\operatorname{SNR}=-10 \mathrm{~dB}$.

According to eq. (7.10), the highest value of the tomographic image correspond to the scatterer estimated position in elevation and azimuth $\left(\hat{\theta}_{M_{1}}, \hat{\alpha}_{M_{1}}\right)$. The other scatterers vanish in the tomographic image according to their relative distance from the considered scattering centre. This results show that the proposed 3D ISAR tomographic processing allows non-cooperative targets at height to be focused to their correct position.

Indicators such as the mean height error, its standard deviation and the signed error have been chosen to evaluate the algorithm performances. On the top left
of Figure 7.7 is shown mean height error whilst its standard deviation is depicted on the top right corner. Both the curves are represented as the SNR increases. It is worth noticing that the behaviour of both the parameters follows a decreasing trend as the SNR increase. This is consistent with the theory.


Figure 7.7: Mean height error, Standard deviation of the height error, Signed error and Height resolution.

The mean height error computed by maintaining the positive or negative sign of each scatterer's height error is shown on the bottom left of Figure 7.7. This parameter shows that the proposed estimator is unbiased. It is worth highlighting that most of the 3D ISAR imaging methods do not allow multiple scatterers belonging to the same Range-Doppler cell in the ISAR image to be distinguished. Conversely, with the detailed ISAR processing based on the beamforming technique, is possible to overcome layover problems. More detailed 3D ISAR images can be
formed. However, the height resolution depends both on the number of antennas and the distance between the array element, as illustrated on the lower right side of Figure 7.7. As a consequence, it is important to find a trade-off between the decrement of the array elements and the inter-element spacing increment, on which depends also the spatial aliasing. The array has to be dimensioned depending on the height resolution that we want to achieve.

### 7.4.2 Simulation 2

The target consists of $M=36$ ideal point-like scatterers and resemble the shape of a boat. The simulation parameters are detailed in Table 7.2. The resulting 3D reconstructions for $\mathrm{SNR}=-20 \mathrm{~dB}$ and $\mathrm{SNR}=0 \mathrm{~dB}$ are shown in Figure 7.8-7.11.

| Number of freq. | 256 | Radar sweeps | 128 |
| :---: | :---: | :---: | :---: |
| Bandwidth | 300 MHz | Radar-target dist. | 10 Km |
| Carrier freq. | 10 GHz | Target velocity | $8 \mathrm{~m} / \mathrm{s}$ |
| $\boldsymbol{T}_{\text {obs }}$ | 0.8 s | $\boldsymbol{d}_{\boldsymbol{V}}=\boldsymbol{d}_{\boldsymbol{H}}$ | 10 m |
| Roll/Pitch/Yaw | $0^{\circ} / 0^{\circ} / 80^{\circ}$ | $\boldsymbol{K}_{\boldsymbol{V}}=\boldsymbol{K}_{\boldsymbol{H}}$ | 5 |

Table 7.2: Simulation Parameters.


Figure 7.8: 3D reconstruction $-\mathrm{SNR}=0 \mathrm{~dB}$.


Figure 7.9: 3D reconstruction $-\mathrm{SNR}=0 \mathrm{~dB}$.


Figure 7.10: 3D reconstruction $-\mathrm{SNR}=-20 \mathrm{~dB}$.

Results show that the algorithm is able to reconstruct the 3D shape of the target even with low SNR.

### 7.5 Conclusion

The proposed method exploits two perpendicular antenna arrays and allows multiple scatterers belonging to the same range-Doppler cell to be resolved. The 3 D shape of non-cooperative targets is obtained by combining 2D beamforming


Figure 7.11: 3D reconstruction $-\mathrm{SNR}=-20 \mathrm{~dB}$.
techniques and 2D ISAR imaging. The described technique consists of two main steps:

1. 2D ISAR imaging

A 2D ISAR image is obtained from each antenna by means of a 2D Fourier transform of the received signal after motion compensation.
2. 3D ISAR Beamforming and 3D shape Reconstruction

First, the $K_{V}$ ISAR images from the vertical array are used to form a tomographic image and for each scatterer (identified by using a MC-CLEAN technique) the elevation angle $\theta_{m}$ corresponding to the maximum power is found. For each $\theta_{m}$, the $K_{H}$ ISAR images for the horizontal array are used to form a tomographic image and for each scatterer the azimuth angle $\alpha_{m}$ corresponding to the maximum power is found. Then, the effective rotation vector modulus $\Omega_{\text {eff }}$ and the rotation angle $\phi$ are estimated. Finally, the height of each scatterer is calculated, allowing the 3D shape of the target to be reconstructed.

As a conclusion, simulation results show that 3D target reconstructions can be satisfactory achieved by using this method for all the considered SNR. Accordingly to the theory, performances improve as the SNR increase. Finally, the use of beamforming techniques allows to achieve height resolution and consequently to overcome layover problems.

## Chapter 8

## 3D Colocated MIMO ISAR

## Imaging

### 8.1 Introduction

MIMO (Multiple Input Multiple Output) radar technique evolved from MIMO wireless communication. Different from traditional radar systems, the MIMO radar adopts orthogonal waveforms transmitted from different antenna elements, which can avoid transmitting waveform interference and ensure the signal channels to be independent from each other. A first attempt to select different system input to enhance parameter estimation in a radar system can be dated back to the early 1970s with Raman K. Mehra 1. However, almost 10 years had to pass by before to see the first MIMO working system consisting of a circular array transmitting orthogonal waveforms at each sensor. This was conceived by the French research agency ONERA and used in the experimental program RIAS (Radar a Impulsion et Antenne Synthetique) for air surveillance purposes.

Each transmitting antenna of the MIMO radar systems can emit multiple probing signals. This is one of the main difference between MIMO radar and phased-array radar, which transmits a scaled version of the adopted waveform, and one of its main advantages. Many discussions can be found in literature about the potential
benefits that a MIMO configuration can provide. Among them, for given system design choices, can be underlined the following characteristics [55]

- Improve the performance of target detection and localisation;
- Enhance the angle estimation accuracy;
- Improve the parameter identifiability;
- Increasethe sensitivity to detect slow moving targets.

Recently, MIMO ISAR techniques have been developed to generate 3D images of the target to exploit the advantages provided by using waveform diversity. MIMO distributed ISAR imaging techniques have been used to increase the cross-range resolution, which depends on the intrinsic motion characteristics of the target (and, specifically, on its overall change of aspect angle). The use of MIMO ISAR configuration can also reduce transmitting power [56], [57]. Whilst methods to obtain three-dimensional images of the target using MIMO ISAR systems have been proposed in the last few years [58], [59], [60], the theoretical framework of a MIMO ISAR system is still in an embryonic stage. Several research problems still need to be addressed and a number of challenges have to be faced.

This objective of this chapter is to provide a useful tool to produce 3D target reconstruction based on the InISAR processing described in Chapter 3 in a colocated MIMO configurations. The idea is to exploit the advantages that a colocated MIMO configuration gives, related to the concept of virtual aperture [61]. Section 8.2 describes the basic principles of MIMO radar technology while 8.3 details the proposed method.

### 8.2 Principles of MIMO radar: the virtual array

MIMO radar systems can be categorized in two main configurations: statistical MIMO radar and coherent MIMO radar.

The statistical MIMO radar configuration consists of a number of transmitting/receiving elements largely spaced. As a consequence, the target RCS (Radar Cross Section) seen from each transmitter-receiver pairs is decorrelated [62].

In the coherent MIMO configuration, the antenna elements are co-located, i.e. closely spaced. Each sensor transmit different linearly independent signals and, due to the particular TX/RX geometry, it is assumed that each antenna sees the same (delayed) target's scattering response.

By transmitting independent waveforms, the transmit-receive arrays geometries can be exploited to form a much larger so called virtual array. Let suppose the transmitting array to be composed of $N_{T X}$ elements and the receiver array to be composed of $N_{R X}$ elements. The virtual array allows to process a number $N_{T X} \cdot N_{R X}$ of signals at the receivers, with obvious advantages in parameter identifiability, target detection and localization etc.

Let suppose that a MIMO systems transmits a set of $N_{T X}$ orthogonal waveforms $\left\{s_{m}(t)\right\}$ where $m=1 \ldots, N_{T X}$ denotes the particular transmitter

$$
\begin{equation*}
\int s_{i}(t) s_{i}^{*}(t) d t=\delta_{i j} \tag{8.1}
\end{equation*}
$$

where $\delta_{i j}$ is the Kronecker delta function defined as follows:

$$
\left\{\begin{array}{l}
\delta_{i j}=1 \text { if } i=j  \tag{8.2}\\
\delta_{i j}=0 \text { if } i \neq j
\end{array}\right.
$$

Signals coming from different transmitters are separated at the receiver by a bank of matched filters. The received signal at the output of the $k^{t h}$ matched filter of $n^{\text {th }}$ receiver can be expressed as:

$$
\begin{equation*}
s_{R_{n, k}}=\rho e^{j k_{0} T_{L O S}^{T}\left(\mathbf{p}_{T_{m}}+\mathbf{p}_{R_{n}}\right)} \tag{8.3}
\end{equation*}
$$

where $k_{0}$ is the wave number, the coefficient $\rho$ takes into account the target reflectivity, $\mathbf{i}_{L O S}^{T}$ is the LoS unit vector, the terms $\mathbf{p}_{T_{m}}$ and $\mathbf{p}_{R_{n}}$ denotes the location of the $m_{t h}$ transmitter and the $n_{t h}$ receiver elements respectively. As evident in (8.3), the phase term $\mathbf{p}_{T_{m}}+\mathbf{p}_{R_{n}}$ is the spatial convolution of the transmitter and the receiver elements locations. The received signals can then be interpreted as coming from an array which elemets are located as fiven by (8.3).

As a conclusion, we can state that a MIMO system composed of $N_{T X}+N_{R X}$ real antennas produces at the receiver the same result as an $N_{T X} \cdot N_{R X}$ elements array.

### 8.3 3D target reconstruction

The InISAR system geometry described in Chapter 3 is shown in Figure 3.1.


Figure 8.1: InISAR system geometry.
Let suppose that the three antennas transmit orthogonal waveforms and are both transmitting and receiving, i.e. the system is in a co-located MIMO configuration.

At the receiver, signal coming from different transmitters are separated by using a bank of matched filters. At the output of the matched filters, the MIMO received
signal at each antenna can be written as follows:

$$
\begin{align*}
& S_{R_{V}}(f, t)=\left[\begin{array}{c}
S_{T_{V} R_{V}}(f, t) \\
S_{T_{C} R_{V}}(f, t) \\
S_{T_{H} R_{V}}(f, t)
\end{array}\right] ; \quad S_{R_{C}}(f, t)=\left[\begin{array}{c}
S_{T_{V} R_{C}}(f, t) \\
S_{T_{C} R_{C}}(f, t) \\
S_{T_{H} R_{C}}(f, t)
\end{array}\right] ;  \tag{8.4}\\
& S_{R_{H}}(f, t)=\left[\begin{array}{c}
S_{T_{V} R_{H}}(f, t) \\
S_{T_{C} R_{H}}(f, t) \\
S_{T_{H} R_{H}}(f, t)
\end{array}\right]
\end{align*}
$$

where $S_{T_{i} R_{j}}(f, t)$ is the spectrum of the MIMO time-varying spatial multichannel received signal with transmit array element $T X_{i}$ and receiver $R X_{j}$, relative to an ideal point scatterer, after motion compensation and in free space conditions.

According to eq. (8.3) the phase of the received signal depends on the spatial convolution of the transmitter and the receiver elements locations.

Figure 8.2 depicts the corresponding equivalent virtual array obtained according to the MIMO radar theory [55].


Figure 8.2: MIMO ISAR equivalent virtual antenna array.
The idea is to treat the received signal at each antenna as a sub-L-shape configuration as depicted in Figure 8.3 and to apply the InISAR processing detailed in Chapter 3 to each receiver.


Figure 8.3: L-shaped configurations associated to the signal received at antenna V, C and H

The signal received from each sensor is processed independently in order to obtain an estimation of the height of the scatterers with respect to the image plane. It is worth to pointing out that, due to the small distance between the antenna array elements with respect to the distance to the target, the image plane does not change for the three configuration. All the 2D ISAR images are used by the MCCLEAN technique to perform the scatterers extraction. After the three height estimations have been estimated, a soft assignment for each result is performed in order to finally average the heights consistently. The final scatterers height estimation is then obtained by averaging the height estimations resulting from the three configurations. The block diagram of this procedure is shown in Figure 8.4


Figure 8.4: Block diagram of the proposed method
However, the array phase center changes for each of the sub-L-shape configurations. Consequently, the the estimators must be modified.

Let consider the signal received in $\boldsymbol{V}$. Equation (3.8) can be rewritten as follows:

$$
\begin{aligned}
\mathbf{i}_{L O S_{x}}^{V}(t) & \triangleq \frac{\mathbf{P}_{x}(t)-\mathbf{V}_{x}(t)}{\left|\mathbf{P}_{x}(t)-\mathbf{V}_{x}(t)\right|}= \\
& =\left[\begin{array}{lll}
\frac{-d_{V} \sin \phi}{\sqrt{R_{0}(t)^{2}+d_{V}^{2}}} \frac{R_{0}(t)}{\sqrt{R_{0}(t)^{2}+d_{V}^{2}}} & \frac{-d_{V} \cos \phi}{\sqrt{R_{0}(t)^{2}+d_{V}^{2}}}
\end{array}\right] \\
\mathbf{i}_{L O S_{x}}^{C V}(t) & \triangleq \frac{\mathbf{P}_{x}(t)-\mathbf{C V}_{x}(t)}{\left|\mathbf{P}_{x}(t)-\mathbf{C V}_{x}(t)\right|}= \\
& =\left[\begin{array}{lll}
\frac{-\left(d_{V} / 2\right) \sin \phi}{\sqrt{R_{0}(t)^{2}+\left(d_{V} / 2\right)^{2}}} & \frac{R_{0}(t)}{\sqrt{R_{0}(t)^{2}+\left(d_{V} / 2\right)^{2}}} & \frac{-\left(d_{V} / 2\right) \cos \phi}{\sqrt{R_{0}(t)^{2}+\left(d_{V} / 2\right)^{2}}}
\end{array}\right] \\
\mathbf{i}_{L O S_{x}}^{H V}(t) & \triangleq \frac{\mathbf{P}_{x}(t)-\mathbf{H V}_{x}(t)}{\left|\mathbf{P}_{x}(t)-\mathbf{H V} V_{x}(t)\right|}= \\
& =\left[\begin{array}{lll}
\frac{-\left(d_{H} / 2\right)(\cos \phi+\sin \phi)}{\sqrt{R_{0}(t)^{2}+\left(d_{H} / 2\right)^{2}}} & \frac{R_{0}(t)}{\sqrt{R_{0}(t)^{2}+\left(d_{H} / 2\right)^{2}}} & \frac{\left(d_{H} / 2\right)(\sin \phi-\cos \phi)}{\sqrt{R_{0}(t)^{2}+\left(d_{H} / 2\right)^{2}}}
\end{array}\right]
\end{aligned}
$$

where $\mathbf{V}_{x}(t), \mathbf{C V}_{x}(t)$ and $\mathbf{H} \mathbf{V}_{x}(t)$ are the positions of the antennas with respect to $T_{x}$.

Therefore, the scalar product in (3.3) can be written as:

$$
\begin{align*}
& \mathbf{P}_{x}(t) \cdot \mathbf{i}_{L O S_{x}}^{V}(t) \cong\left(\mathbf{P}_{y}+\mathbf{c} t\right) \cdot \mathbf{i}_{L O S_{x}}^{V}(t)=K_{0}^{V}+K_{1}^{V} t \\
& \mathbf{P}_{x}(t) \cdot \mathbf{i}_{L O S_{x}}^{C V}(t) \cong K_{0}^{C V}+K_{1}^{C V} t  \tag{8.5}\\
& \mathbf{P}_{x}(t) \cdot \mathbf{i}_{L O S_{x}}^{H V}(t) \cong K_{0}^{H V}+K_{1}^{H V} t
\end{align*}
$$

where

$$
\begin{align*}
& K_{0}^{V} \triangleq y_{2}-\frac{d_{V}}{R_{0}}\left(y_{1} \sin \phi+y_{3} \cos \phi\right) \\
& K_{1}^{V} \triangleq c_{2}-\frac{d_{V}}{R_{0}}\left(c_{1} \sin \phi+c_{3} \cos \phi\right) \\
& K_{0}^{C V} \triangleq y_{2}-\frac{d_{V}}{2 R_{0}}\left(y_{1} \sin \phi+y_{3} \cos \phi\right) \\
& K_{1}^{C V} \triangleq c_{2}-\frac{d_{V}}{2 R_{0}}\left(c_{1} \sin \phi+c_{3} \cos \phi\right)  \tag{8.6}\\
& K_{0}^{H V} \triangleq y_{2}+\frac{d_{H}}{2 R_{0}}\left[y_{3}(\sin \phi-\cos \phi)-y_{1}(\cos \phi+\sin \phi)\right] \\
& K_{1}^{H V} \triangleq c_{2}+\frac{d_{H}}{2 R_{0}}\left[y_{3}(\sin \phi-\cos \phi)-y_{1}(\cos \phi+\sin \phi)\right]
\end{align*}
$$

After some algebra we obtain the $y_{3}$ coordinate as follows:

$$
\begin{equation*}
y_{3}=y_{3 \xi} \cos \phi-y_{1 \xi} \sin \phi=\frac{c}{2 \pi f_{0}} R_{0}\left(\frac{\triangle \theta_{V}}{d_{V}} \cos \phi-\frac{\triangle \theta_{H}}{d_{H}} \sin \phi\right) \tag{8.7}
\end{equation*}
$$

The term $c_{2}$ can then be expressed by taking into account the Doppler components as follows:

$$
\begin{equation*}
\nu_{c_{2}}=2 \nu_{C V}-\nu_{V} \triangleq-\frac{2 f_{0}}{c}\left(2 K_{1}^{C V}-K_{1}^{V}\right] \cong-\frac{2 f_{0}}{c} c_{2} \tag{8.8}
\end{equation*}
$$

The Doppler component $\nu_{c_{2}}$ can be written as:

$$
\begin{equation*}
\nu_{c_{2}}=-\frac{R_{0} \Omega_{e f f}}{\pi}\left(\frac{\triangle \theta_{H}}{d_{H}} \cos \phi+\frac{\triangle \theta_{V}}{d_{V}} \sin \phi\right) \tag{8.9}
\end{equation*}
$$

As a result, equation (8.9) can be rewritten by considering only the contribution due to the $k^{\text {th }}$ scatterer as follows:

$$
\begin{equation*}
Z_{k}=a Y_{k}+b X_{k} \tag{8.10}
\end{equation*}
$$

where $Z \triangleq \nu_{c_{2}}, Y \triangleq-\frac{R_{0}}{\pi d_{H}} \triangle \theta_{H}, X \triangleq-\frac{R_{0}}{\pi d_{V}} \triangle \theta_{V}, a \triangleq \Omega_{e f f} \cos \phi$ and $b \triangleq \Omega_{e f f} \sin \phi$.

As described in Chapter 3, the problem can be mathematically solved by minimizing the function:

$$
\begin{equation*}
\Psi(a, b)=\sum_{k=1}^{M}\left[Z_{k}-\left(a Y_{k}+b X_{k}\right)\right]^{2} \tag{8.11}
\end{equation*}
$$

and the estimation of $\Omega_{e f f}$ and $\phi$ can be derived from the estimates $\tilde{a}$ and $\tilde{b}$ :

$$
\begin{equation*}
\hat{\Omega}_{e f f}=\sqrt{\tilde{a}^{2}+\tilde{b}^{2}} \quad \hat{\phi}=\arctan \left(\frac{\tilde{b}}{\tilde{a}}\right) \tag{8.12}
\end{equation*}
$$

Equivalent procedure can be followed to calculate the estimator for the sub-Lshape relative to the signal received in $\boldsymbol{C}$ and $\boldsymbol{H}$.

### 8.4 Simulation results

The same parameters of Simulation 1 in Chapter 4 have been used for the described MIMO configuration. The ISAR images at each receiver have been obtained with the Bistatically Equivalent Monostatic Theorem and the baseline lengths have been adjusted to meet the constraints expressed in (3.27). Results are shown in comparison with the multichannel InISAR system.

Figure 8.5 shows the 3D reconstructions. Figure 8.6-8.8 show the height error, the standard deviation of the height error and the cross-range error, for the multichannel and the MIMO configuration, both the cases with and without unreliable assignments.

In Figure 8.9-8.13 are shown the signed error (to show that the estimator is unbiased), the effective rotation vector estimation error and its standard deviation as well as the rotation angle error and its standard deviation.

According with the theory, the curves follow a decreasing trend as the SNR and the baselines length increase. Moreover, it is worth pointing out that the performances of all the considered parameters greatly improve when using the MIMO
system. However, the computational burden is higher and no iterations between the antennas have been considered.

### 8.5 Conclusion

In this chapter, a method to obtain a 3D image of the target by using a MIMO InISAR configuration has been described. The received signal at each TX/RX antenna is treated as a sub-L-shape array to which is applied the InISAR processing detailed in Chapter 3. Results shows that very good 3D reconstructions of the target are obtained for all the considered SNR. Moreover, a comparison with the monostatic case has been carried out. This comparison shows that the 3D reconstruction performances highly increase when adopting the MIMO configuration.


Figure 8.5: Results of the target reconstruction along three different planes. (a) superimposition of the model and the reconstructed target - Multichannel; (b) superimposition of the model and the reconstructed target - Colocated


Figure 8.6: Height error. (a) Height error with unreliable assignments - Multichannel; (b) Height error with unreliable assignments - Colocated MIMO; (c) Height error without unreliable assignments - Multichannel; (d) Height error without unreliable assignments - Colocated MIMO


Figure 8.7: Standard deviation of the height error. (a) Standard deviation of the height error with unreliable assignments - Monistatic; (b) Standard deviation of the height error with unreliable assignments - Colocated MIMO; (c) Standard deviation of the height error without unreliable assignments - Multichannel; (d) Standard deviation of the height error without unreliable assignments - Colocated MIMO


Figure 8.8: Cross-range error. (a) Cross-rang error with unreliable assignments - Multichannel; (b) Cross-rang error with unreliable assignments - Colocated MIMO; (c) Cross-rang error without unreliable assignments - Multichannel; (d) Cross-rang error without unreliable assignments - Colocated MIMO


Figure 8.9: Signed error without unreliable assignments. (a) Signed error Multichannel; (b) Signed error - Colocated MIMO


Figure 8.10: (a) Mean error of the estimate of $\Omega_{e f f}$ - Multichannel; (b) Mean error of the estimate of $\Omega_{\text {eff }}$ - Colocated MIMO


Figure 8.11: (a) Standard deviation of the estimate of $\Omega_{\text {eff }}$ - Multichannel;
(b) Standard deviation of the estimate of $\Omega_{\text {eff }}$ - Colocated MIMO


Figure 8.12: (a) Mean error of the estimate of $\phi$ - Multichannel; (b) Mean error of the estimate of $\phi$ - Colocated MIMO


Figure 8.13: (a) Standard deviation of the estimate of $\phi$ - Multichannel; (b) Standard deviation of the estimate of $\phi$ - Colocated MIMO

## Chapter 9

## Conclusion

In this thesis, the problem of forming 3D target images of non- cooperative targets is addressed.

3D target reconstruction solves the problems related to the ISAR image interpretation. In fact, the orientation of the image plane depends on the sensor position relative to the target and on the target's motions. In typical operating scenarios, targets of interest are non-cooperative. Then, the orientation of the image plane cannot be predicted a priori and the interpretation of the conventional 2D ISAR image becomes ambiguous.

A review of the basic principles of ISAR imaging and of the monostatic ISAR signal modelling has been firstly provided.

The monostatic ISAR signal model has been extended to a multichannel processing. Three multichannel ISAR processing have been developed to form 3D images of non-cooperative targets.

The first method is based on interferometry and has been tested on simulated and real datasets.

For the purpose of scattering centre extraction, the MC-CLEAN has been developed by extending the CLEAN technique to be effective when using spatial multi-channel radar configurations.

Although InISAR imaging represents an effective tool to produce 3D target reconstruction, problems related to shadowing still remain unsolved.

In order to overcome such limitations, a method for multistatic 3D target reconstruction has been introduced.

Another solution is to reconstruct the 3D shape of a target by using a tomografic approach. Such a technique is detailed in this thesis.

Furthermore, a comparison between the 3D reconstruction performances of a multichannel InISAR system and a MIMO InISAR system is carried out.

All the developed techniques have been tested on simulated data and, for the interferometric approach, on real data. Results demonstrate their effectiveness for all the considered scenarios.

Further investigations in this research area will include the study of more effective techniques for scattering centres extraction for multichannel systems in the case of distributed targets. A second possible follow on to this thesis could be also the application of other beamforming techniques to the tomographic approach in order to reduce the number of antennas and the inter-element spacing without loosing height resolution. Furthermore, based on the results obtained with the MIMO InISAR system, another important future direction could be the study of the most desirable waveforms for 3D target reconstruction and the impact on the system performances when inter-element interferences occur.

## Bibliography

[1] V.C. Chen and W.J. Miceli. Simulation of isar imaging of moving targets. Radar, Sonar and Navigation, IEE Proceedings -, 148(3):160-166, Jun 2001. ISSN 1350-2395. doi: 10.1049/ip-rsn:20010384.
[2] Translational Bureau. Radar definition. Public Works and Government Services Canada, Retrieved November 82013.
[3] Daniel N. Lapedes. McGraw-Hill dictionary of scientific and technical terms, Daniel N. Lapedes, editor in chief. McGraw-Hill New York, 1974. ISBN 0070452571.
[4] Gaspare Galati and Piet van Genderen. History of radar: The need for further analysis and disclosure. In European Radar Conference (EuRAD), 2014 11th, pages 25-28, Oct 2014. doi: 10.1109/EuRAD.2014.6991198.
[5] G. Marconi. Radio telegraphy. Proceedings of the Institute of Radio Engineers, 10:237, 1922.
[6] G. Breit and M. A. Tuve. A test of the existence of the conducting layer. Phys. Rev., 28:554-575, Sep 1926. doi: 10.1103/PhysRev.28.554.
[7] A. H. Taylor, L. C. Young, and L. A. Hyland. System for detecting objects by radio. US Patent 1981884, November 1934.
[8] Giorgio Franceschetti and Riccardo Lanari. Synthetic aperture radar processing. CRC press, 1999.
[9] Jack L. Walker. Range-doppler imaging of rotating objects. Aerospace and Electronic Systems, IEEE Transactions on, AES-16(1):23-52, Jan 1980. ISSN 0018-9251. doi: 10.1109/TAES.1980.308875.
[10] Dale A. Ausherman, Adam Kozma, Jack L. Walker, Harrison M. Jones, and Enrico C. Poggio. Developments in radar imaging. Aerospace and Electronic Systems, IEEE Transactions on, AES-20(4):363-400, July 1984. ISSN 00189251. doi: 10.1109/TAES.1984.4502060.
[11] M. Martorella. Chapter 19 - introduction to inverse synthetic aperture radar. In Rama Chellappa Nicholas D. Sidiropoulos, Fulvio Gini and Sergios Theodoridis, editors, Academic Press Library in Signal Processing: Volume 2 Communications and Radar Signal Processing, volume 2 of Academic Press Library in Signal Processing, pages 987 - 1042. Elsevier, 2014. doi: http://dx.doi.org/10.1016/B978-0-12-396500-4.00019-3.
[12] W.M. Boerner and H. Überall. Radar target imaging. Springer series on wave phenomena. Springer-Verlag, 1994. ISBN 9783540577911.
[13] Merrill Ivan Skolnik. Radar Handbook. Electronic engineering series. McGrawHill, 1990. ISBN 9780070579132.
[14] D.R. Wehner. High-resolution Radar. Radar Library. Artech House, 1995. ISBN 9780890067277.
[15] C. Oliver and S. Quegan. Understanding Synthetic Aperture Radar Images. SciTech radar and defense series. SciTech Publ., 2004. ISBN 9781891121319.
[16] J.A. Richards. Remote Sensing with Imaging Radar. Signals and Communication Technology. Springer Berlin Heidelberg, 2012. ISBN 9783642261138.
[17] V.C. Chen and M. Martorella. Inverse Synthetic Aperture Radar Imaging; Principles, Algorithms and Applications. Institution of Engineering and Technology, 2014. ISBN 9781613530139.
[18] J.L. Walker. Range-Doppler Imaging of Rotating Objects. Aerospace and Electronic Systems, IEEE Transactions on, AES-16(1):23 -52, Jan. 1980. ISSN 0018-9251. doi: 10.1109/TAES.1980.308875.
[19] F. Berizzi, E. Dalle Mese, M. Diani, and M. Martorella. High-resolution ISAR imaging of maneuvering targets by means of the range instantaneous Doppler technique: modeling and performance analysis. Image Processing, IEEE Transactions on, 10(12):1880-1890, Dec 2001. ISSN 1057-7149. doi: 10.1109/83.974573.
[20] A. Scaglione and S. Barbarossa. Estimating motion parameters using parametric modeling based on time-frequency representations. In Radar 97 (Conf. Publ. No. 449), pages $280-284$, Oct 1997. doi: $10.1049 / \mathrm{cp}: 19971679$.
[21] N. Battisti and M. Martorella. Intereferometric phase and target motion estimation for accurate 3D reflectivity reconstruction in ISAR systems. In Radar Conference, 2010 IEEE, pages 108 -112, May 2010. doi: 10.1109/ RADAR.2010.5494644.
[22] F. Berizzi, M. Martorella, B. Haywood, E. Dalle Mese, and S. Bruscoli. A survey on isar autofocusing techniques. In Image Processing, 2004. ICIP '04. 2004 International Conference on, volume 1, pages 9-12 Vol. 1, Oct 2004. doi: 10.1109/ICIP.2004.1418676.
[23] M. Martorella, F. Berizzi, and B. Haywood. Contrast maximisation based technique for 2-d isar autofocusing. Radar, Sonar and Navigation, IEE Proceedings -, 152(4):253-262, Aug 2005. ISSN 1350-2395. doi: 10.1049/ip-rsn: 20045123.
[24] Dale A. Ausherman, Adam Kozma, Jack L. Walker, Harrison M. Jones, and Enrico C. Poggio. Developments in Radar Imaging. Aerospace and Electronic Systems, IEEE Transactions on, AES-20(4):363 -400, July 1984. ISSN 00189251. doi: 10.1109/TAES.1984.4502060.
[25] D.R. Wehner. High-Resolution Radar. Artech House Radar Library. Artech House, 1995. ISBN 9780890067277. URL http://books.google.it/books? $i d=S a F j Q g A A C A A J$.
[26] D. Pastina and C. Spina. Slope-based frame selection and scaling technique for ship ISAR imaging. Signal Processing, IET, 2(3):265 - 276, September 2008. ISSN 1751-9675. doi: 10.1049/iet-spr:20070122.
[27] T. Cooke. Ship 3D model estimation from an ISAR image sequence. In Radar Conference, 2003. Proceedings of the International, pages 36 - 41, Sept. 2003. doi: 10.1109/RADAR.2003.1278706.
[28] M. Stuff, M. Biancalana, G. Arnold, and J. Garbarino. Imaging moving objects in 3D from single aperture Synthetic Aperture Radar. In Radar Conference, 2004. Proceedings of the IEEE, pages 94 - 98, April 2004. doi: 10.1109/NRC.2004.1316402.
[29] W. Nel, D. Stanton, and M.Y.A. Gaffar. Detecting 3-D rotational motion and extracting target information from the principal component analysis of scatterer range histories. In Radar Conference - Surveillance for a Safer World, 2009. RADAR. International, pages 1 -6, Oct. 2009.
[30] Genyuan Wang, Xiang-Gen Xia, and V.C. Chen. Three-dimensional ISAR imaging of maneuvering targets using three receivers. Image Processing, IEEE Transactions on, 10(3):436-447, Mar 2001. ISSN 1057-7149. doi: 10.1109/ 83.908519 .
[31] J.A. Given and W.R. Schmidt. Generalized ISAR-part ii: interferometric techniques for three-dimensional location of scatterers. Image Processing, IEEE Transactions on, 14(11):1792-1797, Nov. 2005. ISSN 1057-7149. doi: 10.1109/TIP.2005.857285.
[32] Xiaojian Xu and R.M. Narayanan. Three-dimensional interferometric ISAR imaging for target scattering diagnosis and modeling. Image Processing, IEEE Transactions on, 10(7):1094-1102, Jul 2001. ISSN 1057-7149. doi: 10.1109/ 83.931103.
[33] Y. Sun and P. Lin. An improved method of ISAR image processing. In Circuits and Systems, 1992., Proceedings of the 35th Midwest Symposium on, pages 983 -986 vol.2, aug 1992. doi: 10.1109/MWSCAS.1992.271131.
[34] M. Martorella, F. Berizzi, and B. Haywood. Contrast maximisation based technique for 2-D ISAR autofocusing. Radar, Sonar and Navigation, IEE Proceedings -, 152(4):253 - 262, Aug. 2005. ISSN 1350-2395. doi: 10.1049/ ip-rsn:20045123.
[35] F. Berizzi and M. Diani. Target angular motion effects on ISAR imaging. Radar, Sonar and Navigation, IEE Proceedings -, 144(2):87-95, 1997. ISSN 1350-2395. doi: 10.1049/ip-rsn:19970965.
[36] Marco Martorella, Andrea Cacciamano, Elisa Giusti, Fabrizio Berizzi, Brett Haywood, and Bevan Bates. CLEAN Technique for Polarimetric ISAR. International Journal of Navigation and Observation, 2008:1-13, 2008. ISSN 1687-5990. doi: 10.1155/2008/325279. URL http://dx.doi.org/10.1155/ 2008/325279.
[37] Marco Martorella, Fabrizio Berizzi, and Silvia Bruscoli. Use of Genetic Algorithms for Contrast and Entropy Optimization in ISAR Autofocusing. EURASIP Journal on Advances in Signal Processing, 2006(1):087298, 2006. ISSN 1687-6180. doi: 10.1155/ASP/2006/87298. URL http://asp. eurasipjournals.com/content/2006/1/087298.
[38] Qun Zhang, Tat Soon Yeo, Gan Du, and Shouhong Zhang. Estimation of three-dimensional motion parameters in interferometric ISAR imaging. Geoscience and Remote Sensing, IEEE Transactions on, 42(2):292 - 300, Feb. 2004. ISSN 0196-2892. doi: 10.1109/TGRS.2003.815669.
[39] Qun Zhang and Tat Soon Yeo. Three-dimensional SAR imaging of a ground moving target using the InISAR technique. Geoscience and Remote Sensing, IEEE Transactions on, 42(9):1818-1828, Sept. 2004. ISSN 0196-2892. doi: 10.1109/TGRS.2004.831863.
[40] M. Martorella, D. Stagliano, F. Salvetti, and N. Battisti. 3d interferometric isar imaging of noncooperative targets. Aerospace and Electronic Systems, IEEE Transactions on, 50(4):3102-3114, October 2014. ISSN 0018-9251. doi: 10.1109/TAES.2014.130210.
[41] Stefan Brisken, Marco Martorella, Torsten Mathy, Christoph Wasserzier, and Elisa Giusti. Multistatic isar autofocussing using image contrast optimization. In Radar Systems (Radar 2012), IET International Conference on, pages 1-4, Oct 2012. doi: 10.1049/cp.2012.1623.
[42] M.L. Krieg and D.A. Gray. Comparison of probabilistic least squares and probabilistic multi-hypothesis tracking algorithms for multi-sensor tracking. In Acoustics, Speech, and Signal Processing, 1997. ICASSP-97., 1997 IEEE International Conference on, volume 1, pages 515-518 vol.1, Apr 1997. doi: 10.1109/ICASSP.1997.599688.
[43] E. Giusti, M. Martorella, and A. Capria. Polarimetrically-Persistent-Scatterer-Based Automatic Target Recognition. Geoscience and Remote Sensing, IEEE Transactions on, 49(11):4588-4599, Nov. 2011. ISSN 01962892. doi: 10.1109/TGRS.2011.2164804.
[44] Avinash C. Kak, Malcolm Slaney, IEEE Engineering in Medicine, and Biology Society. Principles of computerized tomographic imaging. IEEE Press, New York, 1988. ISBN 0-87942-198-3. Published under the sponsorship of the IEEE Engineering in Medicine and Biology Society.
[45] P. van Dorp, M.P.G. Otten, and J.M.M. Verzeilberg. Coherent multistatic isar imaging. In Radar Systems (Radar 2012), IET International Conference on, pages 1-6, Oct 2012. doi: 10.1049/cp.2012.1624.
[46] M. Iwamoto and T. Kirimoto. A novel algorithm for reconstructing threedimensional target shapes using sequential radar images. In Geoscience and Remote Sensing Symposium, 2001. IGARSS '01. IEEE 2001 International, volume 4, pages 1607-1609 vol.4, 2001. doi: 10.1109/IGARSS.2001.977008.
[47] Kei Suwa, Kazuhiko Yamamoto, Masafumi Iwamoto, and Tetsuo Kirimoto. Reconstruction of 3-d target geometry using radar movie. In Synthetic Aperture Radar (EUSAR), 2008 7th European Conference on, pages 1-4, 2008.
[48] K. Suwa, T. Wakayama, and M. Iwamoto. Estimation of target motion and 3d target geometry using multistatic isar movies. In Geoscience and Remote Sensing Symposium, 2009 IEEE International,IGARSS 2009, volume 5, pages V-429-V-432, 2009. doi: 10.1109/IGARSS.2009.5417640.
[49] M. Martorella. Novel approach for ISAR image cross-range scaling. IEEE Transactions on Aerospace and Electronic Systems, 44(1):281-294, 2008.
[50] Changzheng Ma, Tat Soon Yeo, Qun Zhang, Hwee Siang Tan, and Jun Wang. Three-dimensional isar imaging based on antenna array. Geoscience and Remote Sensing, IEEE Transactions on, 46(2):504-515, 2008. ISSN 0196-2892. doi: 10.1109/TGRS.2007.909946.
[51] A. Reigber and A. Moreira. First demonstration of airborne sar tomography using multibaseline l-band data. Geoscience and Remote Sensing, IEEE Transactions on, 38(5):2142-2152, 2000. ISSN 0196-2892. doi: 10.1109/36.868873.
[52] F. Lombardini and A. Reigber. Adaptive spectral estimation for multibaseline sar tomography with airborne l-band data. In Geoscience and Remote Sensing Symposium, 2003. IGARSS '03. Proceedings. 2003 IEEE International, volume 3, pages 2014-2016, 2003. doi: 10.1109/IGARSS.2003.1294324.
[53] M. Martorella, F. Salvetti, and D. Stagliano. 3d target reconstruction by means of 2d-isar imaging and interferometry. In Radar Conference (RADAR), 2013 IEEE, pages 1-6, 2013. doi: 10.1109/RADAR.2013.6586064.
[54] Federica Salvetti, Douglas Gray, and Marco Martorella. Joint use of twodimensional tomography and isar imaging for three-dimensional image formation of non-cooperative targets. In EUSAR 2014; 10th European Conference on Synthetic Aperture Radar; Proceedings of, pages 1-4, June 2014.
[55] J. Li and P. Stoica. MIMO Radar Signal Processing. Wiley, 2008. ISBN 9780470391433. URL http://books.google.com.au/books?id= g6uLLWb-TqYC.
[56] Yutao Zhu, Yi Su, and Wenxian Yu. An isar imaging method based on mimo technique. Geoscience and Remote Sensing, IEEE Transactions on, 48(8): 3290-3299, Aug 2010. ISSN 0196-2892. doi: 10.1109/TGRS.2010.2045230.
[57] D. Pastina, M. Bucciarelli, and P. Lombardo. Multistatic and mimo distributed isar for enhanced cross-range resolution of rotating targets. Geoscience and Remote Sensing, IEEE Transactions on, 48(8):3300-3317, Aug 2010. ISSN 0196-2892. doi: 10.1109/TGRS.2010.2043740.
[58] Changzheng Ma, Tat-Soon Yeo, Chee Seng Tan, Jian-Ying Li, and Yong Shang. Three-dimensional imaging using colocated mimo radar and isar technique. Geoscience and Remote Sensing, IEEE Transactions on, 50(8): 3189-3201, Aug 2012. ISSN 0196-2892. doi: 10.1109/TGRS.2011.2178607.
[59] Guang Qing Duan, Dang Wei Wang, Xiao Yan Ma, and Yi Su. Threedimensional imaging via wideband mimo radar system. Geoscience and Remote Sensing Letters, IEEE, 7(3):445-449, July 2010. ISSN 1545-598X. doi: 10.1109/LGRS.2009.2038728.
[60] Xie Xiaochun and Zhang Yunhua. 3d isar imaging based on mimo radar array. In Synthetic Aperture Radar, 2009. APSAR 2009. 2nd Asian-Pacific Conference on, pages 1018-1021, Oct 2009. doi: 10.1109/APSAR.2009.5374254.
[61] B.J. Donnet and I.D. Longstaff. Mimo radar, techniques and opportunities. In Radar Conference, 2006. EuRAD 2006. 3rd European, pages 112-115, Sept 2006. doi: 10.1109/EURAD.2006.280286.
[62] A.M. Haimovich, R.S. Blum, and L.J. Cimini. Mimo radar with widely separated antennas. Signal Processing Magazine, IEEE, 25(1):116-129, 2008. ISSN 1053-5888. doi: 10.1109/MSP.2008.4408448.

