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# *Network Flow Techniques: An application to Integrated Energy Systems*

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*"I have no special talents. I am only*
*passionately curious".*
*Albert Einstein*

# Abstract

Energy has become one of the most crucial problem in modern society and for this reason it must be investigated, analyzed and solved.

The main goal of this thesis is to look at the economics of energy by analyzing the relationship between the availability of energy resources and economic activity in a network context. The idea is to provide an efficient distribution of energy resources.

The methodology that has been used is that of Network Flow Optimization. A particular attention has been given to a Multi period Generalized Network Flow model proposed in literature, in order to solve Integrated Energy Systems problems.

# Acknowledgements

I would like to express my sincere appreciation to Professor Riccardo Cambini for his supervision, for his positive attitude and for his confidence in me.

Furthermore, I would like to express my deep gratitude to my parents, Mauro and Maria, for being present in every moments of my life, for all the sacrifices they have made to support me and for teaching me the good values of life.

Moreover, I would like to thanks Alban, my sweetheart, for encouraging me to always have the best from life and for supporting me in every moments.

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Finally, I wish to thank all the people who have been present during these years at University.

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# Nomenclature

$i$, $j$, $k$, $s$, $u$, $v$: Nodes

$c_{ij}(l,t)$: Per unit cost of the energy flowing from node $i$ to node $j$, corresponding to the *l-th* linearization segment, during time $t$.

$b_j$: Supply (if positive) or negative of the demand (if negative) at node $j$, during time $t$.

$e_{ij.max}$: Upper bound on the energy flowing from node $i$ to node $j$.

$e_{ij.min}$: Lower bound on the energy flowing from node $i$ to node $j$.

$e_{ij}$: Energy flowing from node $i$ to node $j$, corresponding to the *l-th* linearization segment, during time $t$.

M or E: Set of arcs.

N or V: Set of nodes.

T: Set of time periods.

G: Set of arcs representing electricity generation. $G \subset M$.

$L_{ij}$: Set of linearization segments on the energy flowing from node $i$ to node $j$.

$\eta_{ij}(l)$: Efficiency parameter associated with the arc connecting node $i$ to node $j$, in the *loth* linearization segment.

$SO2_i(t)$: Sulphur dioxide emissions rate associated with the fuel consumed by power plant $i$, during time $t$.

$\alpha_i$: Removal efficiency of the pollution control equipment installed at power plant $i$. If no pollution equipment exists at power plant $i$, then $\alpha_i$=0.

NSO2: U.S. national $SO_2$ limit.

$u_{ij}$: Upper bound on the energy flowing from node $i$ to node $j$, during period $t$.

$l_{ij}$: Lower bound on the energy flowing from node $i$ to node $j$, during period $t$.

$r_{ij}(t)$: Efficient parameter associated with the arc from node $i$ to node $j$, in period $t$.

$c_f$: Residual capacity.

$w$: Weight function.

$p$: Weight.

# Introduction

*"Take the money out of the economy: an economy could continue to function via barter, albeit in an awkward, limited and inefficient way. Take the energy out of the economy: the economy would immediately contract immensely or stop".*

Energy is one of the most important element of our daily life. We use energy to work, to power our vehicles, to warm our houses, to cook, to play music, to light, to wash and dry clothes, to communicate ... in a way or in another we are strictly connected and dependent on energy. This explains why we have to give energy a lot of attention from an economic, a politic and a social points of view.

Energy can be defined as follow *"Measure of the ability of a body or system to do work or produce a change, expressed usually in joules or kilowatt hours (kWh). No activity is possible without energy and its total amount in the universe is fixed. In other words, it cannot be created or destroyed but can only be changed from one type to another. The two basic types of energy are (1) Potential: energy associated with the nature, position, or state (such as chemical energy, electrical energy, nuclear energy). (2) Kinetic: energy associated with motion (such as a moving car or a spinning wheel)".*

The main characteristic of energy is that it is not a single commodity, we can find it from different sources and forms. Energy sources, e.g. crude oil,

natural gas, coal, hydro, uranium, wind, sunlight or geothermal deposits, can be used to produce energy commodities such as gasoline, diesel fuel, natural gas, coal or electricity. This characteristic has helped the transition to new energy sources during the time according to costs and availability. The substitution of one form of energy for another has developed gradually, some of the major changes took place over millennia and have proceeded at different stages of development in different parts of the world.
Moreover the transition of different forms of energy has changed the demand side, by changing life styles, raising standards of living and helping urbanization.

The aim of this dissertation is to look at the economics of energy, which studies how economic agents -firms, households, governments- supply and demand energy resources and commodities, convert, transport them in order to satisfy their increasing needs. Moreover it includes markets and regulatory institutions which play an important role in establishing prices and allocations. Another relevant aspect is the involvement of environment that in the last years is one of the most crucial problem of the modern society.
The objective is to analyze the relationship between the availability of energy resources and the economic activities in a network context under the idea of providing an efficient distribution of energy resources. Complex relations, constraints and influences of the economic, social and political systems are at the basis of this analysis. Many intuitive expectations are invalidated, for example the possession of abundant energy resources for some countries has been no a guarantee for a good economy performance, while the "absence" has been no obstacle to achieve economic prosperity for other countries.

In particular my attention focuses on methodologies, i.e. network flow techniques, to analyze the problem of allocating energy resources in the supply chain and not only.
These methodologies can be used to study integrated realities of all dimen-

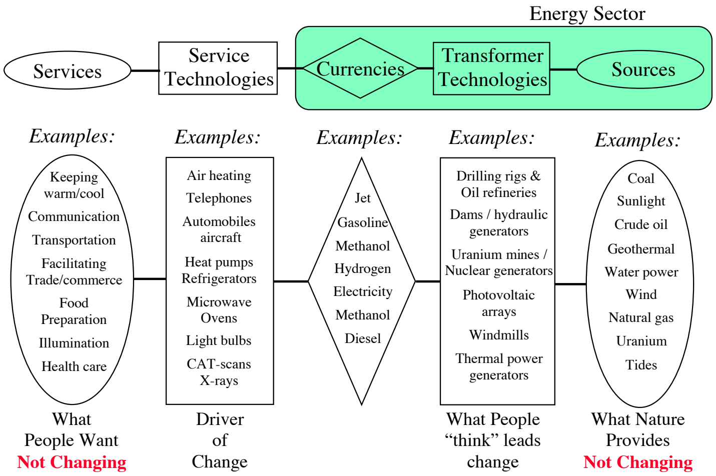


Figure 1: Complete Energy System (Scott, 1995).

sion.

Modern cities for example, can be considered as integral part of modern energy systems, so one can look at urban energy systems to analyze how they are organized in terms of delivering services such as heating, cooling, lighting, mobility, communication..., and how they can improve in an efficient way in a period in which per capita energy use is increasing, the energy system structure is becoming more complex and service provision is driven by innovations.

The questions that are at the basis of modern energy systems are: How people will behave in the future?What kind of services will they want?And how to best deliver those services in an efficient way?

All these aspects have to be taken into account by modern societies in planning energy provisions. A very simple outline is presented in Fig.**??**.

In the last years a lot attention has been given to renewable energy sources, but the big problem of these sources is that they are intermittent, that means they are not able to provide a uniform distribution of energy. The

remedy could be the integration between different sources in order to have a compensation, that is the aim of smart grids. A smart grid can be defined as an evolved electricity network that can manage electricity demand in a sustainable, reliable and economic manner, built on advanced infrastructure and tuned to facilitate integration of all involved.
Another relevant aspect that could be investigated into is that of economic dependence on energy sector in particular by European countries, whose supply comes from outside the area, from markets that are not hold in check. This is the question that nowadays is in discussion in European and Global Institutions.

This thesis is structured as follows. In chapter one, a brief review of the most significant energy transitions in human history is presented, in order to understand the growing dependence on energy of modern society. Moreover, to highlight the crucial importance of this issue, it is analyzed the relationship between energy and economy. Finally, the growing concept of Urban Metabolism is illustrated.
Chapter two is dedicated to the description of Energy Systems, with particular attention to Urban Energy Systems, Distributed Energy Systems and Integrated Energy Systems. In the final section, several measures adopted in the framework of energy policy in Europe are described, in order to see the institutional side of energy and what have been done and what could be done from this perspective to increase energy efficiency.
In chapter three, a short review of the most important problems concerning Network Flows is presented. The aim is to present different techniques to optimize Network Flows. After a review of the basic notions about networks, the attention is given to three different kind of network flow problems that is to say: the Shortest Path Problem, the Maximum Flow Problem and the Minimum Cost Flow Problem.

The last chapter, chapter four, focuses on the application of Network Flows techniques to energy system, a deterministic and a stochastic model are described.

# Chapter 1

## 1.1 Historical background

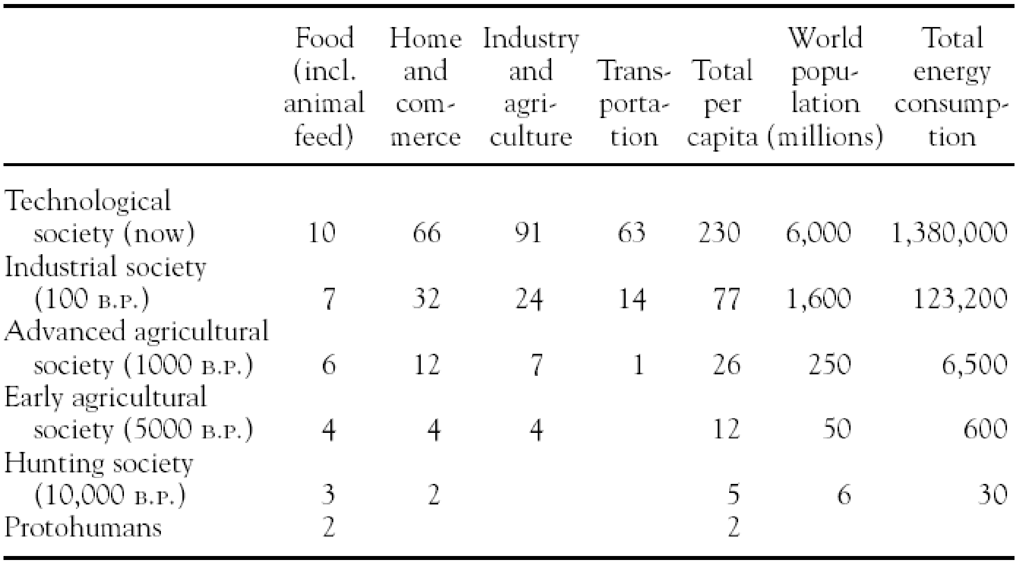
Modern society has turned back on the past for what concerns renewable energy; wind, water, sunlight are again at the center of new innovations, for this reason it is important to remark the most significant energy transitions in human history.

At the beginning of human existence, man's needs for energy were very simple: he used fire to keep warm and to cook.
However the inefficient use of fire, in this period, did not help to keep under control energy flow.
The first energy transition took place when the first permanent settlement happened. Settled communities were able to devote time domesticating draft animals and moreover to use fire in a more productive way. Fire started to be used for producing metals and other durable materials.
These new innovations improved man's life by enabling him to increase his diet, to keep warm, to provide light, to better control the environment and to protect himself.
Several millennia later, the second transition happened, but it proceeded at different stages of development in different parts of the world.
The development of mechanical equipments based on water and wind, substi-

tuted human and animal exertion with renewable energy flows, that increased in power and efficiency.

The use of sails to capture the power of the wind accelerated the trade around the world. Water-wheel was at the beginning used for irrigation and for girding cereals; moreover, later, water was used to power a variety of machines such as blacksmithing, tanning, fulling and woodturning.

It is estimated that energy consumed per capita doubled, see Fig.1.2 and Fig. 1.3.



| | Food (incl. animal feed) | Home and commerce | Industry and agriculture | Transportation | Total per capita | World population (millions) | Total energy consumption |
|---|---|---|---|---|---|---|---|
| Technological society (now) | 10 | 66 | 91 | 63 | 230 | 6,000 | 1,380,000 |
| Industrial society (100 B.P.) | 7 | 32 | 24 | 14 | 77 | 1,600 | 123,200 |
| Advanced agricultural society (1000 B.P.) | 6 | 12 | 7 | 1 | 26 | 250 | 6,500 |
| Early agricultural society (5000 B.P.) | 4 | 4 | 4 | | 12 | 50 | 600 |
| Hunting society (10,000 B.P.) | 3 | 2 | | | 5 | 6 | 30 |
| Protohumans | 2 | | | | 2 | | |

Figure 1.2: Average daily per capita energy consumption in different historical eras (Units of energy= 1000 calories per day).
Source: Simmons I. G., *Changing the Face of the Earth: Culture, Environment, History.* Second Edition. Oxford: Blackwell, p. 27, 1997.

However until the Middle Ages, with relatively small population and modest per capita consumption, it was possible to maintain a balance between renewable energy supply and energy demand. In this period wood was used

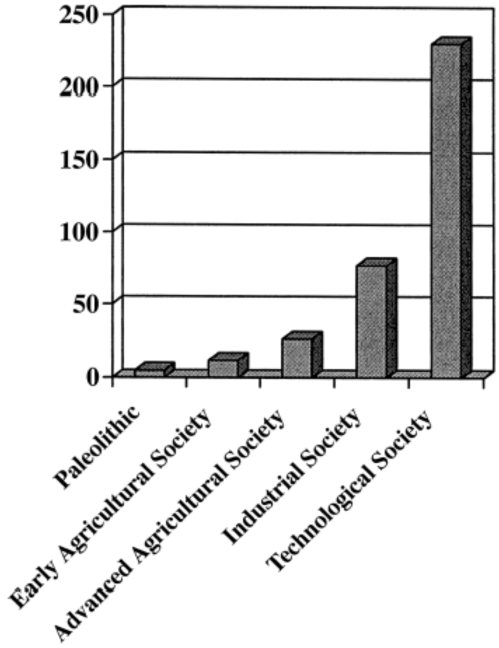


Figure 1.3: Energy consumption per capita in different eras. Data from Fig.1.2.

as a fuel, while wind and water to provide power. But since that moment local production of wood and its consumption as a fuel were not coordinated, some areas were devastated and a prosperous agriculture allowed the growth of wide urban settlements that required a lot of fuel.
With the development of metal technology high energy requirements grew up. Copper was the first metal that diffused, its reduction temperature is fairly low, but it was not abundant.
On the other hand iron was considerably more abundant, but it is required a much higher reduction temperature. During the Middle Ages iron production expanded, but forests were depleted. The scarcity of wood led to the use of coal as a fuel.

The third transition happened when after millennia of dependence on animate sources of energy and biomass fuels, gradually we passed to fossil fuels, that still today are the principal energy commodities.

Coal mines were developed in particular in England. By 1800, lots of coal was used in households, iron industry required a lot of provisions. The early mining of coal was based on outcrops above the water table, but as soon as the increasing demand had to be satisfied deeper mines were required and to mine there was an urgent need of a new form of mechanical power, other than water-wheels.

In 1692 Thomas Savery invented a coal fired steam engine for draining mines, but it had some practical disadvantages and it was not successful. Later Newcomen applied the idea of Papin of using a piston to increase the heights through which water could be lifted. Important improvements arrived between 1763 and 1782, when Watt introduced an enclosed cylinder in which steam could not only power the downward stroke but also the upward stroke. Further improvements were sufficiently to extend the use of steam engine in several sectors included transport.

The discovery of coal-based technology for smelting iron and the invention of steam engine provided the basis for the Industrial Revolution which led to a restructure of the urban energy services, through network infrastructures.

Technological innovations had a significant impact on energy demand coming from growing urban populations and firms. To improve safety of people and to lengthen the working day, street light was a new priority of that time. Oil lamps entered the houses and lightened the streets but demand for higher quality stimulated the researchers. Light produced by gas or oil improved but gas was expensive and electricity spread.

Edison designed the first electrical generating system, which provided electricity to customers through copper wires. It was the success of electric bulb that helped the commercialization of Edison's integrated power and lighting system. This is the latest energy transition, that since that time have pushed modern economies to increase their consumption of fossil fuels indirectly as electricity. New forms of electricity generation have been introduced, such as nuclear fission, wind turbine, photovoltaic cells, but hydrocarbons, crude oil and natural gas are still the dominant fuels generators.

All of these developments had enormous personal and collective consequences: improving the quality of life has been the principal individual benefit because of increased food harvest, improved health and longevity, spread of education and leisure opportunities, enhanced of personal mobility. However the great energy transitions did not decline the disparities between rich and poor societies and this is still today a crucial issue.

Other consequences have been the growth of the world population, the rise of economic power of nations thank to Industrialization, the extension of military capabilities, the expansion of trade and the globalization of human affairs. Finally the downsides have been greater environmental burdens and wars for the provisions of energy resources. These are today big problems at the basis of future choices in terms of energy that are discussed globally by Institutions.

## 1.2 Energy and Economy relationship

In the past, improvements in energy efficiency usually coincided with improvements in the nation's overall productive efficiency. Beginning with the industrial revolution, increased energy use has fueled economic development in advanced industrialized societies (Fouquet, 2008). There is agreement that the use of available energy resources, such as coal, combined with technology innovations allowed to impressive growth results. However this has not happened in the last decades. Since 1973 progress in energy efficiency has been accompanied by market slowdown in productivity; the explanation could be that those enhancements are bounded. Therefore it is crucial to understand which is the relationship between energy and economy in order to highlight the importance of this issue.

The starting point is to investigate the role of energy in production. Business and financial economists pay significant attention to the impact of oil and other energy prices on economic activity, but the mainstream theory of economic growth pays little or no attention to the role of energy or other natural resources in promoting economic growth. Moreover institutional aspects has to be considered since they influence the role of energy in a complex way.

All economic processes, in particular the production one, require energy and there are limits to the substitution of other factors of production for energy, so energy is always an essential factor of production (Stern, 1997a). It is a necessary but not sufficient input and moreover according to the law of thermodynamics, nothing can be changed without it. A simplified production function could take the form of GDP=f(capital, labor, energy, land, materials and know-how), where the contribution of each factor to output depends on the development of the economy, physical conditions and location, factor price and factor productivity. By using the production function approach we have that the optimal energy input and thus the energy demand can be determined by first order conditions where the marginal productivity is equal to the factor price.

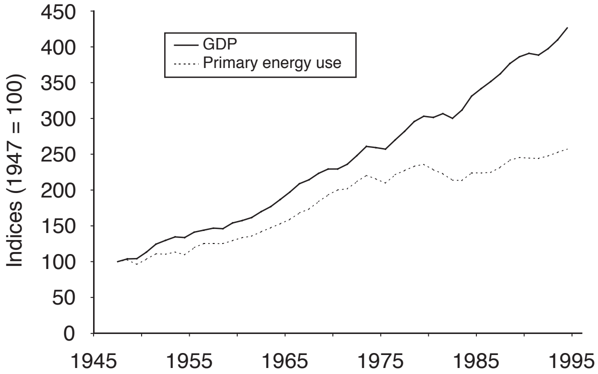


Figure 1.4: U.S. gross domestic product (GDP) and total primary energy use. GDP is in constant dollars, i.e. adjusted for inflation. Both variables are indexed to 100 in 1947 [36].

Taking the example of U.S. economy, as it can be seen in Fig.1.4, it can be observed that in the period 1973 to 1991, the trend between energy consumption and GDP hardly changed. In order to explain the reasons for this break in the trend an examination of the factors that could reduce or strengthen the relationship between energy use and economic activity, could be done through the neoclassical production function that can be represented as follows:

$$\mathrm{Q}_1, \ldots Q_{\mathrm{m}} = f(A, X_1, \ldots X_{\mathrm{n}}, E_1, \ldots E_p),$$

where $\mathrm{Q}_i$ are various outputs such as goods and services, $\mathrm{X}_j$ are various inputs such as capital and labor, $\mathrm{E}_k$ are different energy inputs, and A is the technological progress as defined by the total factor productivity. The relationship between energy and GDP can be affected by shifts in the composition of energy inputs or output. Samuel Schurr was among the first that recognize the economic importance of energy quality, that is the contrast between different forms of energy. Overtime the composition of energy use has changed, and according to Schurr the general shift toward higher quality fuels reduces the amount of energy required to produce a dollar's worth of GDP. On the other hand it can be remarked that usually, over the course of economic development, also the output mix changes. At the beginning there is a shift from agriculture toward heavy industry, and later on toward services and lighter manufacturing. Following this path the result will be an increase in energy used per unit of output in the early stages of development and then a reduction in energy used later in the process. However recent studies show that the shift toward the service sector contributed to a decoupling of energy and economic growth. Service industries still need large amount of energy and input resources, for this reason the energy-intensity ratio[1] is not decreasing.

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[1] Energy efficiency of a nation's economy can be measured by using the Energy/GDP ratio, that represents the amount of energy required to produce a dollar's worth of good and services. Energy-intensity can be written as a function of energy efficiency ($e_{it}$) and economic activity component, i.e. sectoral activities ($s_{it}$). Specifically:
$e_t = (\mathrm{E}_{t\,\overline{\mathrm{Y}}\,t}) = \sum_i (\mathrm{E}_{it\,\overline{\mathrm{Y}}\,it})(\mathrm{Y}_{it\,\overline{\mathrm{Y}}\,t})\ \sum e_{it} s_{it},$
$where E_t$ is the aggregate energy consumption in year $t$, $\mathrm{E}_{it}$ is the energy consumption in sector $i$ in year $t$, $\mathrm{Y}_t$ is the GDP in year $t$, and $\mathrm{Y}_{it}$ is a measure of economic activity in sector $i$ in year $t$.

### Causality in the energy-GDP relationship

In order to deeper analyze the relationship between economic growth and energy use, it could be useful to focus on empirical testing. Ordinary linear regression or correlation methods can not be used to establish a causal relation between these variables, this is because when two variables appear to trending over time, they result to be correlated simply because of the shared. Two methods to test causality among economic growth and energy use are the Granger causality test and the cointegration analysis.

Granger causality tests whether one variable in a relation can be meaningfully described as a dependent variable and the other variable as an independent variable; whether the relation is bidirectional; and/or whether meaningfully relation exists. Many analysts have used this test to test if energy use causes economic growth or vice versa in the context of a bivariate vector autoregression. Generally the results were inconclusive but when significant results where obtained, the majority showed that the causality goes from output to energy use. For example Stern (1993) tested US data (1947-1990) for Granger causality in a multivariate setting using a vector autoregression (VAR) model of GDP, energy use, capital and labor inputs. The multivariate methodology is important because changes in energy uses are countered by the substitution of other factors of production. The measure of energy use was made by its thermal equivalents and by the Divisia aggregation method[2]. Moreover energy use was weighted to take into account the composition of energy inputs, because growth effects of energy are due to

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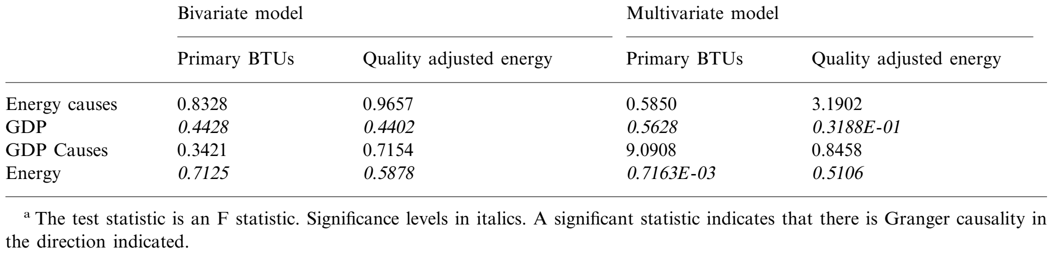
[2] Divisia is an aggregation method used in economics that permits variable substitution among material types without imposing a priori restrictions on the degree of substitution. Divisia aggregation in this context, is an appropriate way to aggregate energy use for investigating its role in the economy.

the substitution with higher quality energy resources (Jorgenson, 1984, Hall et al., 1986). The results of Stern study are shown in Fig.1.5, and as can been seen in both the Bivariate and the Multivariate model, energy is found to "Granger-cause" GDP. These results are consistent with the price-based studies of Hamilton (1983) and Burbridge and Harrison (1984), that will not be analyzed here.



| | Bivariate model | | Multivariate model | |
|---|---|---|---|---|
| | Primary BTUs | Quality adjusted energy | Primary BTUs | Quality adjusted energy |
| Energy causes | 0.8328 | 0.9657 | 0.5850 | 3.1902 |
| GDP | *0.4428* | *0.4402* | *0.5628* | *0.3188E-01* |
| GDP Causes | 0.3421 | 0.7154 | 9.0908 | 0.8458 |
| Energy | *0.7125* | *0.5878* | *0.7163E-03* | *0.5106* |

[a] The test statistic is an F statistic. Significance levels in italics. A significant statistic indicates that there is Granger causality in the direction indicated.

Figure 1.5: Energy GDP causality tests USA 1947-1990. [37].

Another kind of test is the Cointegration Test which can be applied only to linear models of integrated time series. The irregular trend in time series is known as a stochastic trend as opposed to a simple linear deterministic time trend. Usually GDP and energy use are integrated. Cointegration analysis aims to uncover causal relations among variables by determining if the stochastic trends in a group of variables are shared by the series so that the total number of unique trends is less than the number of variables. The presence of cointegration can be interpreted as the presence of a long-run equilibrium relationship between the variables considered. Stern (1998) tested the cointegration between energy use and economic activity by using a multivariate model with US data from 1948 to 1994. To test cointegration

Johansen methodology[3] (Johansen and Joselius, 1990) was used in the Multivariate Vector Error Correction Model (VECM). The VECM is given by:

$$\Delta y_{\mathrm{t}} = \gamma + \alpha\beta[1, t, \gamma_{\text{t-1}}] + \Gamma_{\mathrm{i}}\Delta y_{\text{t-1}} + \epsilon_{\mathrm{t}},$$

Where $y$ is a vector of variables (in logarithms), $\epsilon_{\mathrm{t}}$ is a vector of random disturbances, $\Delta$ is the first difference operator, $t$ is a deterministic time trend, $\gamma$ is a vector of coefficient to be estimated, $\alpha$ is a matrix of adjustment coefficients to be estimated, $\beta$ is the matrix of cointegrating vectors to be estimated, and finally $\Gamma_{\mathrm{i}}$ are matrices of short-run dynamics coefficients to be estimated. The aim of the test is to estimate $\alpha$ and $\beta$. The cointegrating vectors indicate that energy and GDP are present in both cointegrating relations but the elements of $\alpha$ indicate that these cointegrating relations affect the equation for energy use only.

The result of the test indicates that there is a statistically significant relation between energy use and GDP, but the direction of causality was from economic activity to energy use, and this is consistent with the results obtained by Stern (1993). Other analysts have found that energy, GDP and energy prices cointegrate and that when all three variables are included there is a mutual causation between energy and GDP.

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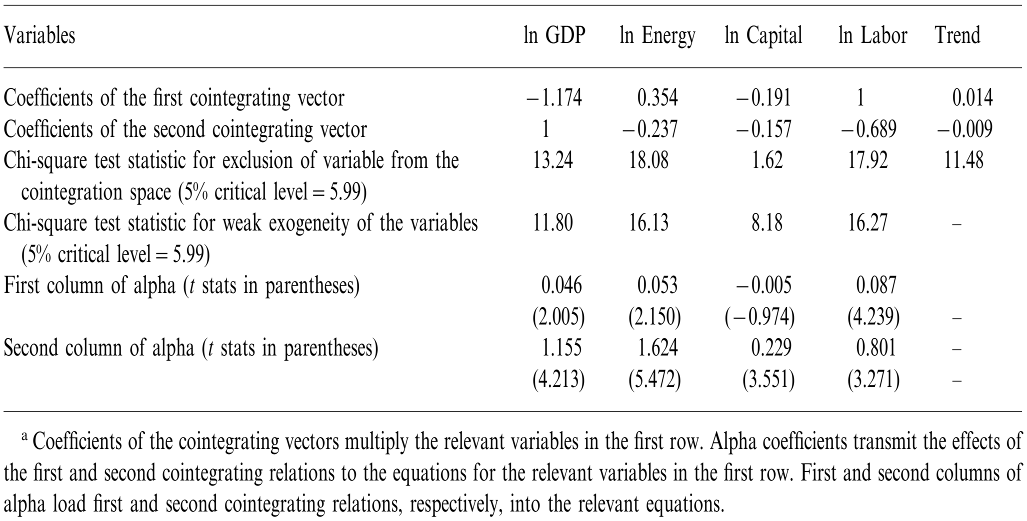
[3] Johansen methodology, is a procedure for testing cointegration of several time-series, that allows for more than one cointegrating relationship, unlike Granger method, but it is subject t8 asymptotic properties, i.e. large sampler.



| Variables | ln GDP | ln Energy | ln Capital | ln Labor | Trend |
|---|---|---|---|---|---|
| Coefficients of the first cointegrating vector | −1.174 | 0.354 | −0.191 | 1 | 0.014 |
| Coefficients of the second cointegrating vector | 1 | −0.237 | −0.157 | −0.689 | −0.009 |
| Chi-square test statistic for exclusion of variable from the cointegration space (5% critical level = 5.99) | 13.24 | 18.08 | 1.62 | 17.92 | 11.48 |
| Chi-square test statistic for weak exogeneity of the variables (5% critical level = 5.99) | 11.80 | 16.13 | 8.18 | 16.27 | – |
| First column of alpha (*t* stats in parentheses) | 0.046 | 0.053 | −0.005 | 0.087 | |
| | (2.005) | (2.150) | (−0.974) | (4.239) | – |
| Second column of alpha (*t* stats in parentheses) | 1.155 | 1.624 | 0.229 | 0.801 | – |
| | (4.213) | (5.472) | (3.551) | (3.271) | – |

[a] Coefficients of the cointegrating vectors multiply the relevant variables in the first row. Alpha coefficients transmit the effects of the first and second cointegrating relations to the equations for the relevant variables in the first row. First and second columns of alpha load first and second cointegrating relations, respectively, into the relevant equations.

Figure 1.6: Cointegration model[a].[37].

## 1.3 Urban metabolism

*"No other century -no millennium- in human history can compare with the twentieth for its growth in energy use. We have probably deployed more energy since 1900 than in all of human history before 1900".*
*(McNeill, 2000)*

In the recent years cities have grown in a complex way due to interlinked geographical forces and institutional frameworks. In particular cities are now dependent on access to resources and ecosystems outside their boundaries and this is a result of globalization.

The rapid expansion in size, density and complexity, has been accompanied by increasing energy flows of inputs and outputs, that enter, exit and/or accumulate within and external of the cities' boundaries. Understanding this system made by interactions and interdependencies can help to shape energy provisions and uses in a more efficient way. Measuring and understanding cities' resources and energy inputs, outputs and storage could be done through Urban metabolism, that offers a platform for expanded urban system analysis.

In the context of energy, Urban metabolism has emerged from a growing understanding of the limited availability of fossil fuels and their impact on the environment as well efficiency use. Urban metabolism was firstly discussed by Karl Marx in 1883. He used the concept of metabolism to describe material and energy interactions between nature and society, through human activity. Marx wrote that man lives from nature -and is a natural being himself- but in addition, he also transforms nature to produce his material needs. (Marx in Foster, 2000) In his analysis, metabolism took on both a specific ecological meaning and a wider social meaning, that we can call socio-ecological metabolism.

The concept was later on applied by Abel Wolman that in 1965 wrote a pioneering article "The Metabolism of Cities". In the article he proposed a model of a hypothetical American city of one million people, to actually calculate the inputs of materials and outputs of waste for such an urban system, taking UM to a quantitative proof of concept. While Marx used the concept of UM putting the emphasis on the social organization of harvesting of Earth's materials, Wolman developed UM as a method to analyze cities through the quantification of inputs-water, food, fuels-outputs and waste, in order to develop sustainable cities. Since Wolman first study a lot of researches have been undertaken, a chronological review is described in Fig.1.7 .

Urban metabolism may be defined as *"the sum total of the technical and socio-economic processes that occur in cities, resulting in growth, production of energy, and elimination of waste"* (Kennedy et al., 2007).
According to this methodology UM is similar to the metabolism of an organism that consumes resources and excretes wastes.
*"Cities transform raw materials, fuel, and water into the built environment, human biomass and waste"* (Decker et al., 2000).
However a city is more complex than a single organism, because it includes lots of different organisms. It can be defined an artificial organism whose objective is to follow the model of natural ecosystems, that are usually energy

| Author (year) | City or region of study | Notes/contribution |
|---|---|---|
| Wolman (1965) | Hypothetical US city of 1 million people | Seminal study |
| Zucchetto (1975) | Miami | Emergy approach |
| Stanhill (1977); Odum (1983) | 1850s Paris | Emergy approach |
| Hanya and Ambe (1976). | Toyko | |
| Duvigneaud and Denayeyer-De Smet (1977) | Brussels | Includes natural energy balance |
| Newcombe et al. (1978); Boyden et al. (1981) | Hong Kong | Particularly comprehensive metabolism study |
| Girardet (1992) | | Recognized link to sustainable development of cities |
| Bohle (1994) | | Critiqued metabolism perspective for studying food in developing cities |
| European Environment Agency (1995) | Prague (comprehensive metabolism study) | Energy use data for Barcelona and seven other European cities given in the report. |
| Nilson (1995) | Gävle, Sweden | Phosphorus budget |
| Baccini (1997). | Swiss Lowlands | |
| Huang (1998). | Taipei | Emergy approach |
| Newman (1999); Newman et al. (1996) | Sydney | Adds liveability measures |
| Stimson et al. (1999) | Brisbane & Southeast Queensland | Framework relating urban metabolism to quality of life. |
| Hermanowicz and Asano (1999) | | Water |
| Hendriks et al. (2000). | Vienna & Swiss Lowlands | |
| Warren-Rhodes and Koenig (2001). | Hong Kong | |
| Baker et al. (2001) | Phoenix & Central Arizona | Nitrogen balance |
| Sörme et al. (2001) | Stockholm | Heavy metals |
| Svidén and Jonsson (2001) | Stockholm | Mercury |
| Obernosterer and Brunner (2001) | Vienna | Lead |
| Færge et al. (2001) | Bangkok | Nitrogen & Phosphorus |
| Chartered Institute of Wastes Management (2002) | London | |
| Gasson (2002) | Cape Town | |
| Barrett et al. (2002) | York, UK | Materials |
| Obernosterer (2002) | | Metals |
| Sahely et al. (2003). | Toronto | |
| Emmenegger et al. (2003) | Geneva | |
| Burstrom et al. (2003) | Stockholm | Nitrogen & Phosphorus |
| Gandy (2004) | | Water |
| Lennox and Turner (2004) | | State of the Environment report |
| Hammer and Giljum (2006) | Hamburg, Vienna and Leipzig | Materials |
| Kennedy et al. (2007) | | Review of changing metabolism |
| Schulz (2007) | Singapore | Materials |
| Barles (2007a) | Paris | Historical study of nitrogen in food metabolism |
| Forkes (2007) | Toronto | Nitrogen in food metabolism |
| Zhang and Yang (2007) | Shenzhen, China | Develops eco-efficiency measure |
| Ngo and Pataki (2008) | Los Angeles | |
| Chrysoulakis (2008) | | New project under EU 7th framework |
| Schremmer and Stead (2009) | | New project under EU 7th framework |
| Barles (2009, 2007b) | Paris | Analysis of central city, suburbs and region. |
| Zhang et al. (2009) | Beijing | Emergy approach |
| Niza et al. (2009) | Lisbon | Materials |
| Deilmann (2009) | | Studies relationship between metabolism and city surface |
| Baker et al. (2001) | | Water |
| Thériault and Laroche (2009) | Greater Moncton, New Brunswick | Water |
| Browne et al. (2009) | Limerick, Ireland | Develops measure of metabolic efficiency |

Figure 1.7: Chronological review of Urban Metabolism studies [19].

self-sufficient. UM is a quantitative framework that enables policy-makers to identify early trends, set priorities, develop indicators and establish policy directives. It provides information about energy efficiency, material cycling, waste management and infrastructure and finally is an important tool to understand energy use in communities.

## Methodologies

The basic rationale behind the urban metabolism concept is that the relationship between the environment and an urban system can be described by systematically recording all flows to and from the environment in physical terms.

The behavior of energy as a flow, follows two basic principles of thermodynamics: First Law -energy transforms to another form and is neither created nor destroyed; Second Law -in all processes of energy, some energy will be degraded in quality and transformed into waste heat.

UM has evolved into two distinct approaches: Odum's Emergy method and mass balance accounting.

Emergy is one of the most important concept for studying energy flows. It is defined as the available energy used directly or indirectly to make product or deliver a service. It measures the work of nature and humans in generating products and services as a common metric of environmental and economic values (Odum, 1996: Odum 2006).

The Emergy method incorporate environmental, social and economic aspects into a common unit of non-monetary measure and objectively assesses the sustainability of systems and processes. It can be used to compare the sustainability of different supply chains. It is measured in emjoules, a unit that refers to the available energy of one kind consumed in transformations. However it is practically difficult to express all urban processes in common units. Emergy accounting faces challenges of inadequate data as well as difficulties of integrating and/or comparing materials and energy represented in different units. The complexity of this approach and its resulting limited application is due to converting flows to the seJ metric (Huang, 1998; Huang, Chen, 2009; Huang, Hsu, 2003; Odum, 1996).

Thus, there are other methodologies that deal with energy-material flux such as: material flow analysis (MFA), mass balance, life cycle assessment (LCA), Economic Input-Output Life Cycle Assessment (EIO-LCA).

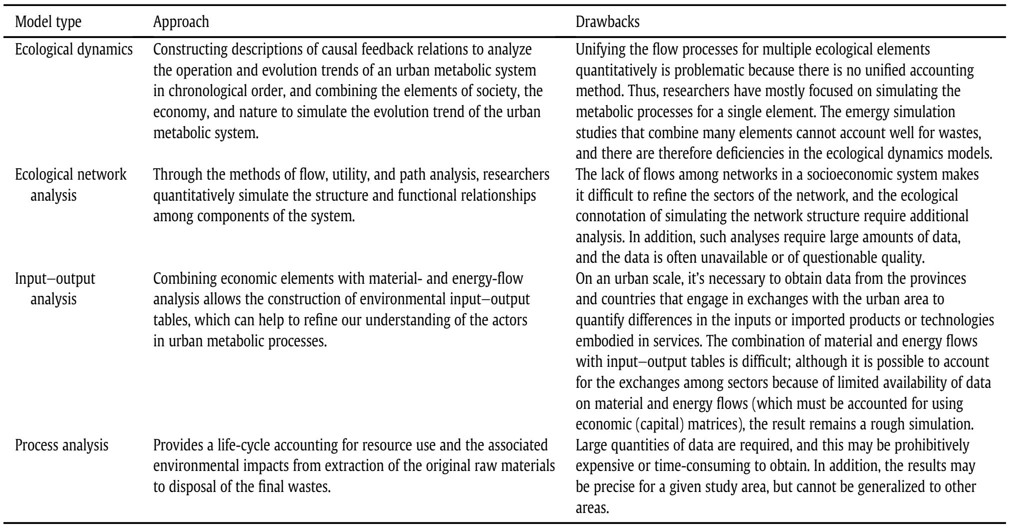
MFA provides a framework for analyzing the ways urban areas transform natural resources. It measures the material flowing into the system, the stocks and flows within it, and the resulting outputs from the system to other systems in the form of pollution, waste or exports. (Sahely, Dudding, Kennedy, 2003) MFA is based on the principle of mass conversion where mass in=mass out+stock changes.

Mass balance is an application of the physical principle of conservation of mass, therefore the mass of inputs into a process has to balance the mass of outputs as products, emissions or wastes, plus any exchange in stocks.

International Standard Organization defines Life cycle assessment as the complication and evaluation of the inputs, outputs and the potential environmental impacts of a product system throughout its life cycle.

Finally the Economic Input-Output Life Cycle Assessment method estimates the materials and energy resources required for, and the environmental emissions resulting from activities in the economy.

All these methodologies can be used jointly or separately according to Researchers needs. Fig.1.8 shows merits and drawbacks of each UM method.



| Model type | Approach | Drawbacks |
|---|---|---|
| Ecological dynamics | Constructing descriptions of causal feedback relations to analyze the operation and evolution trends of an urban metabolic system in chronological order, and combining the elements of society, the economy, and nature to simulate the evolution trend of the urban metabolic system. | Unifying the flow processes for multiple ecological elements quantitatively is problematic because there is no unified accounting method. Thus, researchers have mostly focused on simulating the metabolic processes for a single element. The emergy simulation studies that combine many elements cannot account well for wastes, and there are therefore deficiencies in the ecological dynamics models. |
| Ecological network analysis | Through the methods of flow, utility, and path analysis, researchers quantitatively simulate the structure and functional relationships among components of the system. | The lack of flows among networks in a socioeconomic system makes it difficult to refine the sectors of the network, and the ecological connotation of simulating the network structure require additional analysis. In addition, such analyses require large amounts of data, and the data is often unavailable or of questionable quality. |
| Input–output analysis | Combining economic elements with material- and energy-flow analysis allows the construction of environmental input–output tables, which can help to refine our understanding of the actors in urban metabolic processes. | On an urban scale, it's necessary to obtain data from the provinces and countries that engage in exchanges with the urban area to quantify differences in the inputs or imported products or technologies embodied in services. The combination of material and energy flows with input–output tables is difficult; although it is possible to account for the exchanges among sectors because of limited availability of data on material and energy flows (which must be accounted for using economic (capital) matrices), the result remains a rough simulation. |
| Process analysis | Provides a life-cycle accounting for resource use and the associated environmental impacts from extraction of the original raw materials to disposal of the final wastes. | Large quantities of data are required, and this may be prohibitively expensive or time-consuming to obtain. In addition, the results may be precise for a given study area, but cannot be generalized to other areas. |

Figure 1.8: Comparison of the main urban metabolism simulation methods [41].

Urban metabolism is much more than an accounting exercise since urban metabolism can also influence sustainable urban design and inform policy analysis (Kennedy et al, 2011). Therefore UM focuses its attention not only on environmental impacts, but also on economic and social dimensions of sustainable cites. We can conclude that the main goal of UM studies is to make citizens and companies aware of these impacts and as a result to promote society collaboration and smarter decision-making processes.

# Chapter 2

## 2.1 Urban Energy Systems

Cities are now the dominant form of spacial organization in which people live, economies operate, technologies are generated and used. According to United Nations, by the year 2030 nearly two thirds of the global population will be located in cities. This trend can be explained in large part by economic and social forces, as cities offer their citizens new opportunities for business, education, security, and community, therefore it will cause city planners and key infrastructure stakeholders to face the big problem of providing good quality services, meeting environmental targets and providing energy services to an increasing number of customers. Moreover it can be observed a strong trend of convergence of developing countries toward level of urbanization of developed countries, that will require equitable access to clean-energy services, energy securities as well as environmental capabilities, that for this reason have to be considered in the study of Urban Energy System.

We now turn the attention on the definition of Energy System.

Jaccard's (2005) defines an Energy System as *"the combined processes of acquiring and using energy in a given society or economy".* This definition is relevant to identify at least three important features.

The first one deals with *"combined processes"*: Delivering energy services requires many different steps including resource extraction, refining, transportation, storage, and conversion to end service. While the urban environment may be physically separate from many of these processes, they should be considered in an overall analysis if they are ultimately being used to service urban demands[4].

The second one deals with *"acquiring and using"*: Energy systems represent a balance between supply and demand. Historically cities might be seen as centers of passive demand which must be supplied from an ex-urban source, but recent work suggests that there are now significant opportunities for in-city energy generation [5].

Finally the issue of *"given society and economy"* highlights that: Energy System is a socio-technical system, comprised of more than just pipelines, fuels, and engineering equipments. Markets, institutions, consumer behaviors and other factors affect the way technical infrastructures are constructed and operated.

A Urban System can be defined as a network of towns, cities and their

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[4] See: Kennedy C, Steinberger J, Gasson B, Hansen Y, Hillman T, Havránek M, et al. Methodology for inventorying greenhouse gas emissions from global cities. *Energy Policy* 38(9):4828-4837, 2009

[5] See: "Foresight Sustainable Energy Management and the Built Environment Project" *Final project report.* Tech. Rep. London: Government Office for Science; 2008. Retrieved from http://www.bis.gov.uk/foresight/our-work/projects/published-projects/sustainable-energy-management-and-the-built-environment.

hinterlands, which can be seen as a system in the sense that it depends on the movements of labor, goods and services, and capital through the network. It is a phenomenon that has to be analyzed from a functional perspective, in addition to territorial and administrative perspectives. A Urban Energy System is therefore a network that relates the use and the provision of energy services with the functional Urban System.

The development of Urban Energy Systems arises some big problems that are today faced by several administrative and political institutions from all over the world and in particular by Europe. The high density of the population and the concentration of economic activities arise the problem of energy supply capability. In many larger cities it is required a large-scale imports of renewable energy generated elsewhere. To stop this reliance on foreign provisions, it could be useful to plan Urban Energy System in a more integrated way. Increasing Energy Networks synergies could be an opportunity to improve energy efficiency delivery. Therefore the question that is presented here is to investigate what could be the benefits if cities are organized to integrate their energy use. There are several issues to be taken into account, the first one is to understand how people will behave in the future and what kind of services they will ask for. The related aspect is to understand how to deliver those services in the most efficient way. Another important issue is to understand where technology moves in order to invest in the right places. Finally, a crucial issue is that of energy saving, that actually seems to be the best way to solve the problem of energy provision capabilities.

## 2.2 Distributed Energy System (DES)

In the 1900's, energy has been commonly generated in large power plants operating in a central location and transmitted to consumers via transmission and distribution networks. The typical structure was that of a Centralized Energy System in which a large number of consumers are located within a large area. A Distributed Energy System can be regarded as the opposite of a Centralized Energy System. Usually, both electric power and district heat are produced in large scale units. However, If we take into consideration consumers that are scattered in a region it could be a good question to ask whether the suppliers should be smaller and closer to consumers rather than far away from large units.

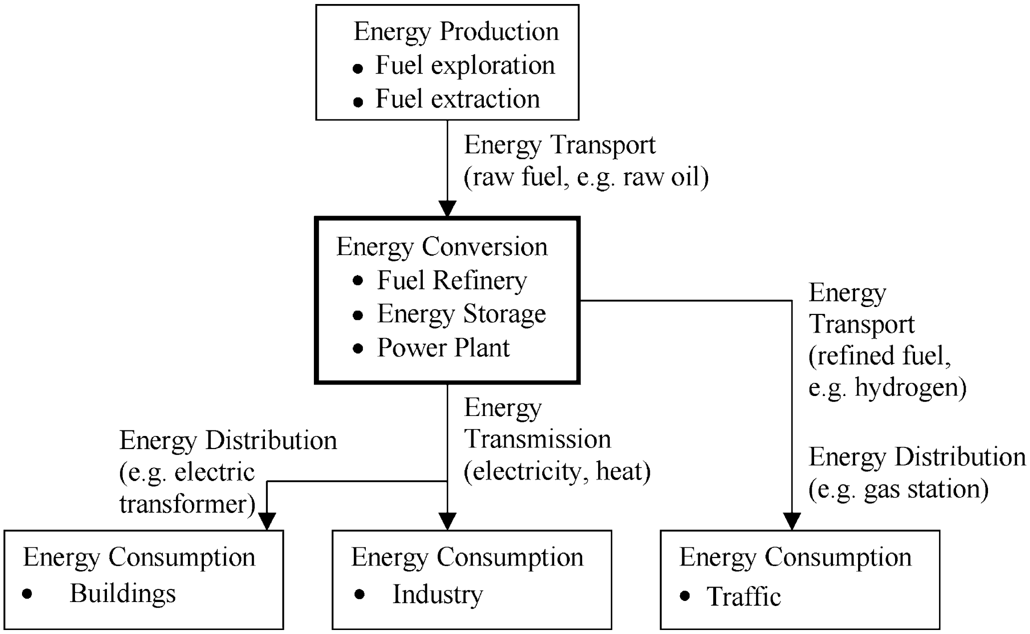

Figure 2.9: Centralized Energy System [2].

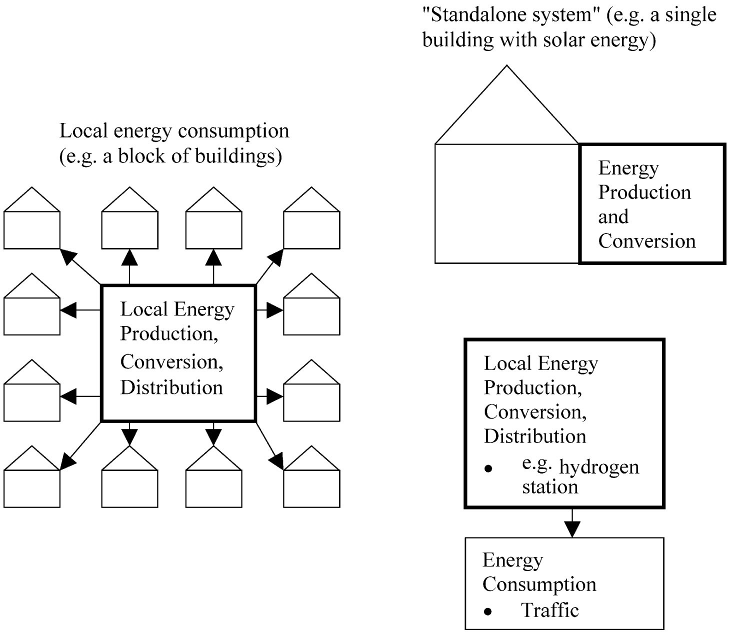

Figure 2.10: Decentralized Energy System [2].

In today's open energy market, Distributed Energy Systems[6] (DES) have obtained an increasing important role. An example can be seen in Fig. 2.11.

A Distributed Energy System is a complex system comprising a number of energy suppliers and consumers, district heating pipelines, heat storage facilities and power transmission lines in a region. The main characteristic is that energy conversion units are situated close to energy consumers, and large units are substituted by smaller ones. Another definition presents DES as a network of energy suppliers and consumers connected by e.g. electric lines or water pipes for transporting the energy flows between the network

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[6] The term ”distributed” seems to be most common, however the term ”decentralized” is also used, especially in European literature.

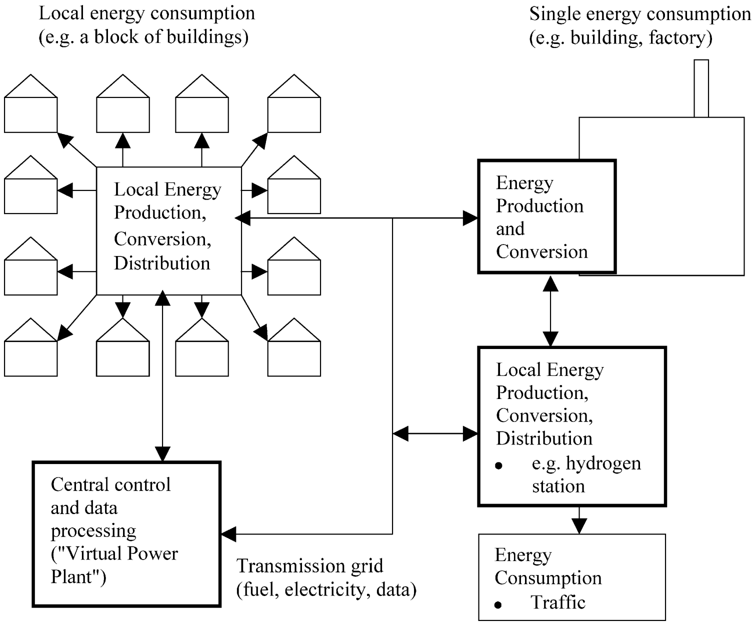

Figure 2.11: Distributed Energy System [2].

components. After having presented some definitions of DES, the attention now turns on understanding the link between Centralized and Decentralized Energy Systems. The basic question deals with what can be decentralized in terms of Energy System and how this decentralization could affect the system. The Online Source for Public Economics[7] defines decentralization as "*transferring power and resources*". However, in the energy context it means more than simply situating energy conversion units closer to consumers and from substituting large units providers with smaller ones. According to the World Bank[8], decentralization is wide concept that includes political, admin-

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[7] Decentralization.org -The Online Source for Public Economics. http://www.decentralization.org/Active_Pages/Faqs.asp.

[8] The World Bank Group: Decentralized NET. http://www.worldbank.org/publicsector/decentralization/Different.htm.

istrative, fiscal and market aspects.

Starting from the political aspect, democratization emerged. In the economic context democratization means that the decision-making power of corporate managers shifts to a larger group of public stakeholders that includes workers, customers, suppliers and a broader public. As a consequence energy consumers are given the task to make decisions and hence local responsibilities increase with respect to political definitions, laws and rules. A problem that may occur because of more units providing energy is that of bureaucracy. According to Vartiainen et Al., licenses procedures of implementation of energy distributed technology can be slow and complicated. However this problem could be solved by simplifying procedures included in every State Organization, from the central body to local municipalities.

From the economic point of view, the question that arises in a decentralized context could be that of ownership. The most common opinion seems that energy conversion technology should be owned by energy utilities. As a consequence the entrance of numerous small operators into the liberalized energy market would increase competition.

In terms of social factors, special knowledge and experience would be required to better perform in such energy system. The required number of staff increased when energy system are decentralized, and this is a relevant aspects because new employment opportunities could be created at local level. This in turn, causes a need for high quality of education of employers and employees that will end with a positive attitude toward new energy technologies.

Current energy systems are going probably to be something in between centralized and decentralized systems. In order to analyze if the energy system of a region is centralized or decentralized, we have to look at some parameters easily measurable and comparable.

First of all, we assume that an energy system consists of "units" and "consumption nodes", where the units are referred to power plants or fuel extraction sites, while a "consumption node" is represented by a single building or grid interface. The main indicator to assess decentralization is the number of consumption nodes per unit, however other indicators such as the number of units per region, the unit size, or the distance between a unit and a consumption node could be evaluated. Therefore the more decentralized is an energy system in a region, the smaller is the number of consumption nodes per unit, larger is the number of units in that region, smaller is the size of each unit, smaller is the average distance between a unit and a consumption node and greater is the number of delivers in the market.

### Benefits and drawbacks of DES

Distributed Energy system has become an object of interest recently, thus the benefits and drawbacks of this system have not largely discussed, however some of the most remarkable aspects are presented in Fig.2.12.

The first aspect that has been analyzed is flexibility. Flexibility in the energy context can be referred first of all to scalability, that is the ability of a network, in this case represented by a DES, to handle a growing amount of energy demand in a capable way in order to face the respective growth. This is seen as a good property of DES. Moreover the flexibility of a DES can be seen as the ability of this kind of network to adapt the system to a wide range of fuels, as well as to employ a variety of energy conversion technologies. When technology advances, obsolete units can be replaced by new on easily, or they can be converted in order to avoid building new power plants. Flexibility has also some negative aspects such as for e.g. the continuous need of new components compatible with the changes produced by new technology discoveries, or by some problems affecting the supply chain. It may also happen that a solution do not last for a long time, or new laws and rules are required in order to better organize new plants, but all this increases the uncertainty of the system.

Another important aspect is reliability. It can be defined as the ability of an energy system to secure energy supply at reasonable prices.

A DES improves local and global well-being of humans because it avoids wide electricity blackouts since there are several units that are autonomous

| Sector of sustainability | Benefits | Drawbacks |
|---|---|---|
| Flexibility | • scalability to changes in heat and electricity demand | •compatibility of the components required |
| | • open to new technologies | • life-cycle of single solutions is not necessarily long |
| | • flexibility for different fuels because of versatile technologies | • new laws and rules needed |
| | • adaptable to the “future of networks” | • unsure if common standards will be found |
| | • takes into account the changing individual needs via decentralized responsibility in decision-making | |
| Reliability | • not vulnerable to external risks | • may increase risk of hazards in consumption point due to extra devices |
| | • no wide electricity blackouts because of independency on electricity distribution | |
| Local and global well-being of humans | • improved employment possible | • some people may find increased responsibility as difficult and new technology as bizarre |
| | • new local market opportunities and competition | • “someone’s bread can be another one’s death” |
| | • gives a feeling of independence and self-control | |
| | • can “teach” private energy consumers | |
| Environment | • no deteriorated landscape due to large power plants and lines | • local distribution of emissions |
| | • decrease in emissions due to elimination of transmission losses | • effects of possible new fuel infrastructure (e.g. natural gas network) |
| Utilization of local resources and networks | • utilization of existing infrastructure | • may require changes in existing infrastructure at the beginning |
| | • more effective utilization of building sites | • increased need for education and training |
| | • utilization of local fuels | |
| | • utilization of information networks | |

Figure 2.12: Summary of benefits and drawbacks of DES [2].

in local production. The only drawback for people of small context could be the increased responsibility, but on the other hand employment would be higher.

For what concern the environment aspect, we have that more units providing energy would offer new local opportunities and hence market competition, moreover from the consumer side we have more awareness of the energy use efficiency. Smaller power plants and lines do not deface the environment, and the emission of dangerous waste would be reduced thank to a more efficient way of production and to a strict system of control.

Lastly, a DES would improve the use of local fuels and of existing infrastructures even if it can be required to change, adapt or build them as well as building sites.

International Energy Agency in a recent World Energy Output, foresees a rapid growth for the distributed electricity generation. It is expected that the annual distributed electricity output will grow by 4.2% between 2000 and 2030.

Up to now I have presented what is a Distributed Energy System and what are its key aspects, however the aim of my dissertation is to show methodologies that can be used for a distribution of energy network in order to minimize the overall costs and the environmental damages, while delivering the hourly energy services required by customers.

## 2.3 Integrated Energy System

In the last years improvements in deregulation policies and competition have increased decentralized energy-related decision making.
This process has led each subsystem to support specific procedures and strategies according to its own values system and interests from an economical, political and environmental point of view.
Recently, the rapid depletion of fossil fuels coupled with the increasing demand for energy, from both developed and developing countries has raised the question of developing an economical and efficient energy system that depends on the performance of the electric power system as well as the associated fossil fuel network.

An Integrated Energy System can be defined as a set of relations, interactions and interdependencies in the energy sector. Considering Integrated Energy Systems implies dealing with complex systems in which synergy between the various components is best exploited.
In a globalized world, every Country's decision produces not only positive or negative effects within the Country's borders, instead it produces a lot of consequences in the rest of the world. For this reason when one decide to analyze an important issue as energy system, it is necessary to enlarge the setting to take into account all the relevant aspects and thus provide possible new solutions to some basic problems.
The first one that is in discussion, in particular in European countries, is that concerning electric power industry that is becoming more competitive and scrutinized from the performance point of view, and has some problem in the price metric. The problem is to provide an optimal allocation of en-

ergy resources to meet electricity demand at minimum cost and subject to physical constraints.

Another question is that of fuel provision contracts. The fuel purchased in the spot market has increased with respect to long term contracts, which have become less and shorter in duration. This creates more uncertainty on the supply side that makes the system more vulnerable.

Moreover we have to take into account externalities on the environmental side. In the last periods, in particular in developed countries, the awareness of the environmental problem caused by pollution, seems to dominate the political policy of the Bigs in the world, and has increased pressure to internalize these externalities associated with the energy production. Plants have to implement specific programs and are subject to a rigorous normative.

Another important consideration includes the interconnected and interdependent nature of fuel infrastructure, that has made the system more integrated, but also dependent on international questions as well as wars and problems between countries.

## 2.4 Measures to reduce energy inefficiency in Europe

European integration development is currently one of the most discussed matter within European Union countries.
Since it involves the crucial issue of energy policy, it is important to understand the role energy has played throughout the more than fifty years' history of this process. Energy appears to be a driver of the European integration, indeed it is not possible to explain the origins of the European Union without considering what happened in Europe after the World War II (Lucas, 1977).

In 1946 was established the European Coal Organization (ECO) and then the Organization for European Economic Cooperation (OEEC) in 1948, where energy was considered the cornerstone of the European integration.
In 1951, at the basis of the creation of the European Coal and Steel Community (ECSC) there were energy-related challenges that Europe had to face during this period. Further more the establishment of the European Atomic Energy Community (EURATOM) in 1957, with the previous mentioned communities, were the pillars of the European Economic Community (EEC).

However, paradoxically, the European energy policy has been very weak during this period, indeed for e.g. in 1980 it was considered a "spectacular failure" of integration (Anderson, 2000).
Recently, after the Lisbon Treaty and the Treaty on the Functioning of the European Union (TFEU), the Eu's involvement on energy policies seemed to take off to face climate changes and the exploding demand of energy consumption.

Today, Europe's energy networks, that is the infrastructure to transport electricity, gas, oil and other fuels from producers to consumers, need to be reorganized in a more efficient way.

The key strategies and technologies identified in literature as the foundation for sustainable energy systems include: enhancing energy efficiency, expanding renewable energy, improving fossil fuels technologies and advancing novel technologies.

The *European energy policy* has three main objectives:

- competitiveness
- sustainability
- security supply

The Eu's goal of renewable energy can be reached by expanding the use of bioenergy, that would contribute to improve energy security and sustainable development, compacting therefore the climate change. In order to implement this goal, the first step that was done in 2005 by the Commission of the European Union was the *Biomass Action Plan*.

The aim of this plan is to accelerate the development of bioenergy by creating market-based incentives. In this way Europe can cut its dependence on fossil rules, cut greenhouse gas emission and stimulate economic activity in rural areas.

In 2006, the Commission approved the *Green Paper*, a European strategy for sustainable, competitive and secure energy system, which was revised in 2010. This *Green Paper* seeks views on how the European Union can better promote the new energy network using all the instruments at its disposal such as TEN-E [9]. Further more it suggests some projects that the EU could promote to reinforce the solidarity and security of supply in European energy network. The link between all member States of EU, could therefore enabling them to benefit of an *internal energy market.*

In 2006, the Commission adopted a comprehensive *Energy Efficiency Action Plan* to create a common framework of legislation, policies and measures to reduce by 20 % the Union's primary energy consumption by 2020.

In 2007, the Commission issued the first *EU Energy Action Plan* whose aims were to improve the yield of energy production and distribution, to decrease the impact of transport on energy consumption, to facilitate the investments in this sector, to encourage rational energy consumption and to promote energy market liberalization.

For what concerns the incentives to the development of new energy technologies, the *Strategic Energy Technology Plan* was approved in 2008.

In the same year the Commission proposed the climate package "*20-20-20 by 2020*", whose targets are: a 20 % reduction in EU greenhouse gas emissions from 1990 levels, raising the share of EU energy consumption produced from renewable resources to 20 % and a 20% improvement in the EU's energy

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[9] TEN-E states for Trans-European Energy Networks. The TEN were created by the European Union by Articles 154-156 of the Treaty of Rome, with the goals of the creation of an internal market and the reinforcement of economic and social cohesion. TEN-E has the specific objective of creating a more competitive internal energy market in electricity and natural gas as well as to promote the security of energy supply and the use of renewable energy sources as a contribution to further sustainable development policy. The way to achieve this objective is through projects of common interest. Call for proposal are published annually and then they are evaluated by the internal Evaluation Committee, and successful proposals thereafter pass to the Committee on Financial Assistance to TEN-E for financial support of the European Union budget.

efficiency.

Lately, the EU energy and climate goals has been incorporated in the *Energy 2020 Strategy*, for a smart and sustainable growth, that was approved by the European Council in 2010.
A flagship initiative, "*Resource efficient Europe*", has been developed to support the sift toward a resource efficient, low-carbon economy.

Future goals to be reached in the next decade are:

- *Energy efficiency*: The Commission has proposed several measures to increase efficiency at all stages of the energy chain: generation, transformation, distribution and final consumption. To implement this policy it is required the mobilization of the public opinion as well as of decision makers and market operators.
- *Variety of input fuels*: The aim is to reduce the heavy dependence on one type of fuel and/or on one type of technology.
- *Reduction of import dependence*: Energy savings and diversification of input fuels will help the EU to be less vulnerable to volatile import prices of resources.
- *Cleanliness of new energy generation*: Future perspectives move toward small-size and decentralized units in order to reduce the environmental impacts.
- *Flexibility in infrastructures*: It helps diversification, in particular for what concerns the availability of different suppliers from different places.

- *Smart energy network*: Forward looking EU energy policy, now supports the integration of Distributed Energy Systems, promoting a more sustainable energy network through Research  Development efforts.
- *Simplification*: The installation of new plants should become simple, less time consuming, less costing and more transparent.

In the following sections my attention will focus above all on the optimization phase, providing different optimization algorithms and some case studies that I have chosen from the literature review about the optimization of Energy Systems.

# Chapter 3

## 3.1 Introduction

This chapter is dedicated to a short overview of the most important elements concerning Network Flows. Before discussing what a network is in the mathematical context and which are the techniques to optimize flows within a network, I would like to highlight the importance and the motivation for studying networks.

Taha (2002) reported that as much as 70 % of real-world mathematical programming problems can be represented by network-related models. For example, in the energy context the electricity industry depends on electricity grids to power homes and factories; Internet is the largest network ever created that has changed the frequency, quantity and quality of information shared; road, rail, airline services and sea cargo networks are the lifeblood of the global economy as they allow for the distribution of food, raw materials, health supplies and consumer products. So, everywhere one looks, networks are perceived as crucial elements for a globalized economy and for this reason I think that a lot of Research studies have to be addressed to this topic in order to offer ways to make them functioning in the best way.

Network optimization is crucial in many different areas, including Operational Research, Applied Mathematics, Computer Science, Engineering, Management and Economics.

Going back from the origin the first network flow optimization problem was applied by Charles Babbage for England' postal system transportation optimization during the middle of the 19th century. Afterwards Gustav Kirchhoff and other engineers applied this tool to analyze electrical circuits. Starting from the World War II, significant improvements were done in particular concerning formal methods for making intelligent logistical decisions, and today it finds a wide range of applications especially in solving industrial problems.

There are three fundamental problems in the study of network optimization. The first one is called the *Shortest Path Problem (SPP)*, which deals with the problem of finding the path of shortest length from a starting node called *source* to an ending node called *sink*.
The second one is the *Maximum Flow Problem.* The goal of this problem is to find the maximum possible flow that can be routed from a *source* node to the *sink*.
Finally, but only in this short presentation, the fundamental problem I present, applied in an Energy system context, is the *Minimum Cost Flow Problem.* This problem can be stated as follows: given a network where each arc has an associated cost per unit of flow, a lower bound, an upper bound, and where there are *demand* nodes, *supply* nodes and *transhipment* nodes,

we have to find the minimum cost flow in the network such that all demands are satisfied.

Network optimization is a special type of linear programming model. Network models have three main advantages over linear programming:
The first one is that they can be solved very quickly and for this reason they are used in many applications such as real-time decision making for which linear programming would be inappropriate. The second advantage is that they have naturally integer solutions. By recognizing that a problem can be formulated as a network program, it is possible to solve special types of integer programs without resorting to the ineffective and time consuming integer programming algorithms. Furthermore they are intuitive. Network models provide a language for talking about problems that is much more intuitive than the "variables, objective and constraints" language of linear and integer programming.

In the following sections my goal is not to focus on the details of Network Flow Theory, but instead to describe the general problems and the most common algorithms to solve them.

## 3.2 Preliminaries

In this section I introduce some key elements of Graph theory such as the basic definitions of graphs, paths, flows and other related notions. Graph concepts are important tools in the Mathematical branch of Operational Research since they are very intuitive through their representative figures even if they involve hidden complex structure. Further more they can be employed to represent a wide range of different problems by using a simple and unified language.

### 3.2.1 Graph and flows

Graph Theory's origin can be placed in 1936 when Euler, the mathematician and physicist, considered the *Königsberg bridge problem*, in which he considered the problem of walk crossing each of the seven bridges of Königsberg city only once. [10] It took 200 years before the first book on Graph Theory was written. Thus the first one that was published in 1936, was *"Theorie der endlichen und unendlichen Graphen"* ( Teubner, Leipzig, 1936) by KOYNIG. Since then Graph Theory has developed into an extensive and popular branch of mathematics, which has been applied to many problems in mathematics, computer science, economics and other scientific and not-so-scientific areas.

**Definition 3.1** A *directed graph* or *digraph* G=(V, E) consists of a finite set V(G)={$v_1$,..., $v_n$} of elements defined *nodes* and a set $E(G) = \{e_1,\ldots,$ $e_m\} \subseteq V \times V$ of pairs of distinct nodes called *arcs.*

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[10] For a more detailed analysis of Euler's study see Danesi, M. (2006). Labirinti, quadrati magici e paradossi logici. *"Il problema dei ponti di Königsberg di Eulero"*(pp.89-110). Bari: Dedalo.

**Definition 3.2** An *arc* $e$=($v$,$u$), is viewed as an ordered pair, and is to be distinguished from the pair $e$=($u$,$v$).
We can say that an arc such $e$=($v$,$u$) is outgoing from node $v$ and incoming to node $u$; in other words we can say that $u$ is an outward neighbor of $v$ and that $v$ is an inward neighbor of $u$. Moreover we define the arc $e$=($v$,$u$) incident to $v$ and to $u$ and we call $v$ the *start node* and $u$ the *end node* of a the arc. The degree of a node is the number of arcs that are incident to $v$.

A graph G can be represented as a plane figure by drawing a line (or a curve) between the points $v$ and $u$ (vertices) of the *arc* $e$=($v$,$u$). A representation of a directed graph can be seen in Fig. 3.13.

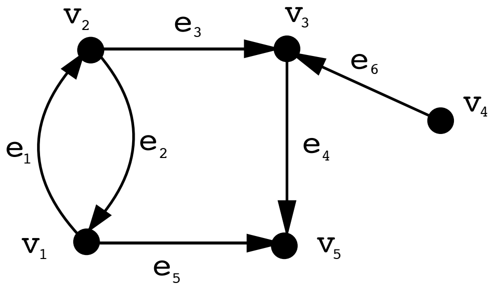


Figure 3.13: Example of directed graph

**Definition 3.3** An *undirected graph* G=(V, E) consists of a finite set V(G)= {v$_1$,. . . , v$_n$} of elements defined nodes and a set E(G)={e$_1$,. . . , e$_m$} $\subseteq$V $\times V$ of pairs of not ordered nodes called *arcs*.

An example of undirected graph is provided in Fig. 3.14 :

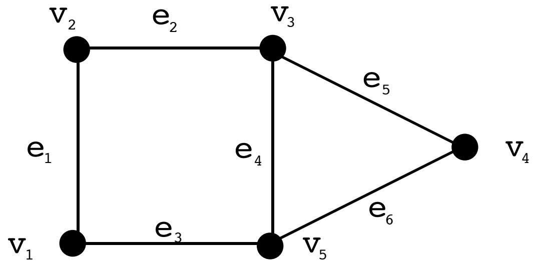


Figure 3.14: Example of undirected graph

### 3.2.2 Paths and Cycles

**Definition 3.4** Given a graph G=(V, E) (*directed* or *undirected*), where V=$\{v_1,\ldots,v_n\}$ and E=$\{e_1,\ldots,\ e_m\}$, we call *pathP* of G, a sequence of nodes $(n_1,\ \ldots,\ n_k)$ with $k \geq 2$ and a corresponding sequence of $k-1$ arcs such that the $i-th$ arc in the sequence is either $(n_i,\ n_{i+1})$ in which case is called a *forward arc* of the *path* or ( $n_{i+1},n_i$) in which case is called a *backward arc* of the *path*.

Nodes $n_1$ and $n_k$ are usually called respectively the *start node* or *origin* and the *end node* or *destination* of P. Further more a *path* is said to be *forward* (or *backward*) if all its arcs are *forward* (or *backward*) arcs. Finally we can indicate with $P^+$ and $P^-$ the sets of forward and backward arcs of P.

**Definition 3.5** Given a graph G=(V, E) (*directed* or *undirected*), where

V=$v_1$,..., $v_n$ and E=$e_1$,..., $e_m$, we call *cycle*, a path for which the *start node* and the *end node* are the same.

A path is called *simple cycle* if it contains no repeated arcs and no repeated nodes, with the exemption of the start and the end nodes.

We call *Hamiltonian cycle* a simple forward cycle that contains all the nodes of the graph.

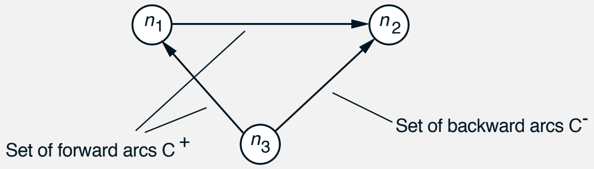
Moreover we said that a graph is *acyclic* if it contains no *simple cycle.* On the other hand we can speak of *connected* graph if for each pair of nodes $v$ and $u$, there is a path starting at $v$ and ending at $u$. If for each pair of nodes $v$ and $u$, there is a forward path starting at $v$ and ending at $u$, the graph is *strongly connected.* In Fig. 3.15 are reported some examples concerning *simple forward path*, *simple cycle* and in figure (c) is presented a situation in which it is shown that the sequence of nodes ($n_1$, ..., $n_k$) is not sufficient to identify a path, so it is necessary to look at the sequence of arcs.

I conclude this brief and no exhaustive presentation of the basic notions of network flows with the definition of a *subgraph*, a *tree* and a *spanning tree.* Given a graph G=(V, E) we call *subgraph* of G, $G^{'} = (V^{'}, E^{'})$ if $V^{'} \subset V$ and $E^{'} \subset E$.

A *tree* is defined as a connected acyclic graph, while a a *spanning tree* is a subgraph of G, which is a *tree* and includes all nodes of G. In addition it can be shown that a subgraph is a *spanning tree* if and only if it is connected and it contains N-1 arcs.

Figure 3.15: Examples of cycles and paths.

(a) A simple forward path $P = (n_1, n_2, n_3, n_4)$.



(b) A simple cycle $C = (n_1, n_2, n_3, n_1)$ which is neither forward nor backward.

(c) Path $P = (n_1, n_2, n_3, n_4, n_5)$ with corresponding sequence of arcs $\{(n_1, n_2), (n_3, n_2), (n_3, n_4), (n_5, n_4)\}$.

Fig. 3 Illustration of various types of paths and cycles. The cycle in (b) is not a *Hamiltonian cycle*; it is *simple* and contains all the nodes of the graph, but it is not forward. Note that for the path (c), in order to resolve ambiguities, it is necessary to specify the sequence of arcs of the path (rather than just the sequence of nodes) because both ($n_3$, $n_4$) and ($n_4$,$n_3$) are arcs.

### 3.2.3 Path Flows and Conformal Decomposition.

I now turn the attention to another important notion to describe network flows which is the *path*.

A *simple path flow* is a flow vector $x$ with components of the form:

$$x_{\mathrm{u,\,v}} = \begin{cases} a \quad if(u,v) \in P^{+} \\ -a \quad if(u,v) \in P^{-} \\ 0 \quad otherwise, \end{cases}$$

where $a$ is a positive scalar and $\mathrm{P}^{+}$ and $\mathrm{P}^{-}$ are the sets of forward and backward arcs, of some simple path P. If P is a cycle, $x$ is called *simple cycle flow.*

A relevant aspect of any flow vector is that it can be decomposed into a set of *conforming simple path flows.*

A path P conforms a flow vector $x$ if $x_{u,v} > 0$ for all forward arcs $(u,\ v)$ of P and $x_{u,v} < 0$ for all backward arcs $(u,\ v)$ of P. In other words a path conforms to a flow vector if it it carries flow from the *start node* to the *end node.*

Proposition 3.6 states the conditions for a *conformal decomposition.*

***Proposition 3.6 Conformal Realization Theorem***

*A nonzero flow vector x can be decomposed into the sum of t simple path flows vectors* $x^{1}$, $x^{2}$, …, $x^{t}$ *that conform to x, with t being at most equal to the sum of the numbers of arcs and nodes V+E. if x is a circulation then* $x^{1}$, $x^{2}$, …, $x^{t}$ *can be chosen to be simple cycle flows, and* $t \leq V$.

## 3.3 Network flow techniques

In this section I present a short review of the most used methodologies, discussing theoretical and practical relevant aspects.

### 3.3.1 The Shortest Path Problem. SPP

*"It is desired to find a set of nodes. What is the shortest path between two specified set of nodes in G?"*

The Shortest Path Problem is an important combinatorial problem that occupies a big area of Research in Network Flow Optimization. It involves a general network structure in which the only relevant parameter is the cost. The goal is to determine the shortest, cheapest, or most reliable path between one or many pairs of nodes in a network.

Shortest path problems arise in a variety of practical settings. Some typical applications concern the *transportation planning* (i.e. how to determine the route road such that the driver can reach the destination in the shortest time), *salesperson routing*, *Investment planning*, *plant and facility layout*, *message routing in communication systems*, etc..

The shortest path problem can be posed in a number of ways; for example, finding a shortest path from a single origin to a single destination, or finding a shortest path from each of several origins to each of several destinations, or finding shortest paths from one origin to several destinations, or finding various types of constrained shortest paths between node (e.g. the shortest path visiting specified nodes), or finding the shortest cycle through a node, etc..

Formally we can define the SPP in general as follows:

***Definition 3.7 The Shortest Path Problem*** Given a directed graph G=(V,E), we associate to every arc $e = (v, u) \in E$ a weight $p(u, v) \in \Re$.
For every directed path $P = \{v_1$, e$_1$, …, e$_{p-1}$, v$_p\}$ we define weight p(P) of the path P the sum of the arcs' weights belonging to P, thus:

$$\text{p(P)}=\sum_{(u,v)\in P} p(u,v).$$

Given two nodes $s \in V$ and $t \in V$, we have to find an oriented path P* in G, from s to t with the minimum weight.

In particular we can consider the SPP a minimum cost flow problem with the goal of sending one unit of flow from the source node $s$ to very other node in the network.
Let $x_{ij}$ be the amount of flow on arc $(i,j)$, and let $c_{ij}$ be the cost of traversing the arc $(i,j)$, the shortest path problem can be formulated as follows:

Minimize: $\sum_{(i,j)\in E} \mathrm{c}_{ij}\mathrm{x}_{ij}$

Subject to: $\sum_{\{j:(i,j)\in E\}} \mathrm{x}_{ij} - \sum_{\{j:(j,i)\in E\}} \mathrm{x}_{ji} = (n-1), \quad i=s \quad (2)$

$$\sum_{\{j:(i,j)\in E\}} x_{ij} - \sum_{\{j:(j,i)\in E\}} \mathrm{x}_{ji} = \text{-}1, \quad i = V - \{s\} \quad (3)$$

$$x_{ij} \geq 0 \qquad\qquad (i,j) \in E$$

In the above formulation, the objective function minimizes the total cost of sending ($n$-1) units of flow node $s$ to all other nodes of the network.
The constraint (2) is the mass balance constraint (node balance, conservation of flow constraint) for the origin, while the constraint (3) is the mass balance constraint for all other nodes. The SSP can be solved using several, very efficient algorithms.
The most important are:

- Dijkstra's algorithm, that is discussed below;
- Bellman-Ford algorithm, which solves the single source shortest path problems if weights are negative;
- Floyd-Warshall algorithm, which solves pairs shortest path;
- Johnson's algorithm, which solves all pairs shortest path and it may be faster than Floyd-Warshall algorithm on sparse graphs;

Among them, in this context, I chose to describe the famous Dijkstra's algorithm.

### Dijkstra's algorithm

Dijkstra's algorithm is a wide used and simple to implement algorithm, to solve SPP.

The basic assumption that we have to do is that all arcs length must be non-negative.In addition we have to introduce a specially designated node $s$, and assume without any loss of generality that the network G contains a directed path from $s$ to every other node. We can ensure this condition by adding an artificial arc $(s, j)$, with a suitably large arc length, for each node $j$.
The basic idea of the algorithm is to fan out from node $s$ and label nodes in order of their distances from $s$.
It is described as a *label-setting algorithm* because at every iteration $k$ is updated the value of the minimum distance of every node from the origin node. The aim of the iteration is to determine the $k$-th node nearest the *start node* (for $k$=1,2,...) until the $k$-th node is the *end node*.
At the beginning all nodes are assigned tentative distance labels (temporary shortest path distances), and then iteratively, the shortest path distance to a node or set of nodes at each step is determined.
The (temporary) distance label of node j (shortest path distance or minimum cost directed path from the source node s to node j) is denoted by $d(j)$. The label is *permanent* once we know it represents the shortest distance from s to j, otherwise it is temporary. All nodes that are assigned temporary shortest distances are stored in a data structure called LIST.

Initially, it is assigned to the source node $s$ a permanent label of zero, and to each other node $j$ a temporary label equal to $d_{sj}$ if $(s, j) \in E, and \infty$ otherwise, which means that the length from the starting point $s$ to other nodes is unknown.
At each iteration, the label of a node $i$ is its shortest distance from the

source node along a path whose internal nodes are permanently labeled. The algorithm selects a node $i$ with the minimum temporary label, makes it permanent, and scans arc E($i$) to update the distance labels of adjacent nodes. The algorithm associates predecessor index, denoted by *pred(i)*, to each node in order to show the latest node prior to $i$ in the research of the shortest path from $s$ to $j$.
The algorithm terminates when it has assigned all nodes as permanently labeled.

Dijkstra's algorithm can be represented as follows:

---

```
begin
    P:={s}; T:=V-{s};
    d(s):=0 and pred(s):=0;
    d(j):=$c_{sj}$ and pred(j):=s if (s,j) $\in E, and\ d(j)$ :=$\infty$ otherwise;
    while P$\neq E$ do
    begin
        (node selection) Let $i \in T$ be a node for which d(i)=min{d(j):j $\in T$}
        $P := P \cup \{i\}; T := T - \{i\}$
        (distance update) for each $(i,j) \in E(i)$ do
            if d(j)>d(i)+$c_{ij}$ then d(j):=d(i)+$c_{ij}$ and pred(j):=i
    end;
end.
```

---

As shown by the algorithm, the process finishes when all nodes are visited. The nodes that are not connected with the starting point remain assigned $\infty$.

Unfortunately, the Dijkstra's Algorithm can not be applied to every situation to solve SPP, because it fails to find the shortest path with the minimum weight when some weights are negative. At first glance, this situation seems impossible to happen because for e.g. there will never be a negative path that reduces the time for a journey, however in some other situations it may happen. To solve this kind of problems we may apply another method-The Bellman Ford Algorithm. The Bellman Ford Algorithm is similar to Dijkstra's Algorithm but it repeats the updating process of Dijkstra's Algorithm for each checkpoint instead of the neighboring checkpoints only, and repeat this for every checkpoint, in this way it is updated every time.

### 3.3.2 The Maximum Flow Problem

*" What, given capacities on the arcs, is the maximum flow that can be sent between any two nodes?" (Ahuja)*

A crucial characteristic of a network is its capacity to carry flow. For this reason the Maximum Flow Problem (MXF) has been studied for over thirty years. The Maximum Flow Problem is to find a feasible flow through a single-source node $s$ and a single-sink node $t$, such that the flow is maximum without exceeding the capacity of the network
The Maximum Flow Problem and the Shortest Path Problem are complementary. They are different because they capture different aspects: in the SPP all arcs are costs, while in the MXP all arcs are capacities. Together provide the foundation upon which much of the algorithm methodology of the minimum cost flow is built. To understand the importance of the MXP I recall some real world applications that arise in diverse settings such as manufacturing, communication systems, distribution planning, matrix rounding and scheduling. *Baseball elimination, airline scheduling, circulation-demand problem, fairness in car sharing*, etc., are some of them.
The classical methods for solving this class of problems are:

- the Ford-Fulkerson *augmenting path* method;
- the Dinitz *blocking flow method*;
- a variant of the network simplex method;
- the Goldberg *push-relabel method.*

Central to the Maximum Flow Problem is the *Max-flow/Min-cut Theorem*, which is the most celebrated theorem in network optimization, but before to describe it, it is necessary to introduce some basic notion.

### Basic tools

In order to better understand the MXP it is necessary to introduce further basic notions. First of all we have to consider a directed graph G, with a weight at every edge.

**Definition 3.8** A network N is a directed graph G=(V, E) with a mapping $w$: E $\rightarrow\Re$ that assigns a weight to each edge. The function $w$ is called *weight function* of N.

To define a flow on a network N=(G,$w$), it is necessary to introduce additional features: We extend $w$ to a function $c$: V $\times V \rightarrow \Re$ as follows:

$$\mathrm{c}(u,\ v) = \begin{cases} w(\overline{uv}) & if \quad \overline{uv} \in E \\ 0 & otherwise. \end{cases}$$

$c$ is called the *capacity function* of $e$, and represents how much data can flow along that edge.

**Definition 3.9** A *flow* on a network N with source $s$, sink $t$, and capacity function $c$ is a function $f$: V $\times V \rightarrow \Re$ such that:

- $f$(u, $v$)<$c(u,\ v)$ $\forall u,\ v \in V$.
- For every vertex $v$ not equal to s or $t$, $\sum_{u\in V} f(u,\ v)$=$\sum_{w\in V} f(v,\ w)$.
- $\sum_{u\in V} f(s,\ u) \geq 0$ *and* $\sum_{u\in V} f(u,\ t) \geq 0$.

The value of a flow is simply $\sum_{u\in V} f(s,\ u) \geq 0$, thus the total data leaving the source. Generally speaking it means that the flow does not exceed the capacity of any edge, and at every vertex (other than the source and the sink) the quantity of data entering equals the quantity of data leaving. In addition, the source has non-negative amount of data leaving it, and the sink has non-negative data entering it.

Another useful concept is that of *residual capacity*, that can be seen as the difference between an edge's maximum allowed data flow and the flow actually passing through it. More formally:

**Definition 3.10** The *residual capacity* of an edge $\overline{uv}$ is $c_f(u,\ v)$=$c(u,\ v)$-$f(u,\ v)$.

**Definition 3.11** Let $p$ be a simple path from $u$ to v. The *residual capacity* of $p$ is $c_f(p) = min\{c_f($u, v: $\overline{uv} \in p\}$. If $c_f(p) > 0$, then $p$ is called an *augmenting path.*

An *augmenting path* is therefore a path along which more data could flow, and the flow is not maximal.

***Lemma 3.12*** *Let f be a flow on a network N=(G,c). Then f is maximal only if f has no augmenting paths.*

*Proof.* Let $f$ be a flow and suppose that an augmenting path $p$ exists with residual capacity $c_f\,(p)$. Then we can define a new flow *f'* by adding $c_f\,(p)$ along each edge in *f*. By the definition of residual capacity, *f'* satisfies the capacity restraints for each edge in *p*, and has $c_f\,(p)$, a positive number, more net flow than *f*. Therefore $f$ is not maximal. ■

The last notion I introduce is that of *vertex cut.*

***Definition 3.13*** A *vertex cut* of a flow $f$ on a network N=(G,$w$) with graph G=( V, E) is a partition of V into two disjoint sets S and T such that $s \in S, t \in T$, and $S \cup T = V$. The *net flow* of a cut (S,T) is $f$(S, T)=$\sum_u \in S, v \in T\ f(u,\ v)$, and the *capacity* is $c(S,T) = \sum_{u \in S, v \in T}\ c(u,v)$.

Since for all vertices $u$ and $v$, the flow $f(u,v)$ is non-negative, we can conclude that for any vertex cut , $c$(S,T) is bounded from below by zero. Furthermore, since the set of all cuts for a graph is finite, there exists a vertex cut of minimal capacity.

### Max-flow/Min-cut Theorem

The Max-flow/Min-cut Theorem states that for any network having a single origin node and a single destination node, the maximum possible flow from the origin to destination equals the minimum cut value for all cuts in the network. To prove this, we start by the following Lemma.

***Lemma 3.14*** *Let f be a flow on a network (G,c) with net flow v and let C be a vertex cut (S,T) with capacity k. Then v$\leq$ k.*

*Proof.* Define P=$\{\overline{xy} : x \in S, y \in T\}$, the set of edges from S to T. As the flow along each edge can not exceed the capacity along that edge for all edges within the network, it is also true for each edge in P. It therefore follows that the net flow over C $f$(S,T)$\leq$ $k$. ■

***Theorem 3.15*** *For any network (G,c), the value of the maximal flow is equal to the minimum-capacity cut.*

*Proof.* Define any flow $f$ and the digraph D$_f$ on the vertex set V with edge set E’=$\{\overline{uv}c_f(u,v)$>0$\}$. Then suppose there is a path $p$ from $s$ to t within D$_f$. In G, $p$ is a path along which every edge could carry more flow, and therefore $p$ is an augmenting path. Let m be the minimum of $c_f(p)$, the residual capacity of the forward-oriented edge from $s$ to $t$, and the set $f(u,v)$ is a backwards-oriented edge from $s$ to $t$, $m$>0 by the construction of D$_f$. Then increasing the flow of each forward-oriented edge by $m$ and decreasing the flow of each backwards-oriented edge by $m$, preserves the non-negativity of flows. Denote the augmented flow by $f$’. Since $p$ is not an augmenting path in $f$’, as augmenting the flow by $m$ either made one of the forward-oriented edges have the maximum capacity flow or one of the backwards-oriented edges have zero flow. Repeat the above process until no augmenting paths remain, remembering that since the number of total paths from $s$ to $t$ is finite in any graph, the process will end in a finite number of steps. In the resulting digraph D$_f$, denote the set of vertices reachable from $s$ as R and the set of unreachable vertices from $s$ as U, where of course $s$ is in R while $t$ is in U. For each edge from a vertex in R to a vertex in U, each forward-oriented edge must be at full capacity and each backwards-oriented edge must have zero flow. Thus the augmenting flow *f'* is a maximum flow, the cut (R,U) is a minimum cut, and moreover the flow of *f'* equals the capacity of (R,U); and now the proof is completed. ■

Let $f$ represent the amount of flow in the network from the source node $s$ to sink node $t$, then the maximum flow problem can be stated as follows:

$$\text{Maximize} \qquad f$$

$$
\begin{array}{lll}
\text{subject to} & \sum_{\{j:(i,j)\in A\}} x_{ij} = f & i = s \\
& \sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = 0 & i = N \backslash \{s,t\} \\
& -\sum_{\{j:(j,i)\in A\}} x_{ji} = -f & i = t \\
& x_{ij} \leq u_{ij} & (i,j) \in A \\
& x_{ij} \geq 0 & (i,j) \in A
\end{array}
$$

The objective of the problem is to maximize the total flow sent from the source node $s$ to the sink node $t$. The first and the third constraints are

conservation of flow (mass balance) constraints. At the origin and at the destination, the inflow and outflow are *f* and -*f*, respectively.
This kind of problem can be solved through the algorithms listed above. Below I present the Ford-Fulkerson *augmenting path* method, which is an effective approach developed by L. R. Ford and D. R. Fulkerson in 1956.

### Ford-Fulkerson *augmenting path* Algorithm

The Ford-Fulkerson algorithm is a generic method for increasing flows incrementally along paths from the source to the sink.
It is a bit different from the Dijkstra algorithm in the details, however the goal is more or less the same and it can be also used to find all the shortest path from a root node to each other.
The basic idea is that, given a feasible flow vector $x$ (i.e., one that is capacity-feasible and has zero divergence out of every node other than $s$ and $t$, and a path P from $s$ to $t$, which is unblocked with respect to $x$, we can increase the flow of all forward arcs $(i,j) \in P^+$ and decrease the flow of all backward arcs $(i,j) \in P^-$ by the positive amount

$$\delta = min\{\{c_{ij}\text{-}x_{ij} \mid (i,j) \in P_+\}, x_{ij}\text{-}b_{ij} \mid (i,j) \in P\text{-}\}\}.$$

The resulting flow vector $\overline{x}$, given by:

$$\overline{x}_{ij}= \begin{cases} x_{ij}+\delta & if(i,j) \in P^+ \\ x_{ij}\text{-}\delta & if(i,j) \in P^- \\ x_{ij} & \text{otherwise} \end{cases}$$

is feasible, and it has divergence out of $s$ that is larger by $\delta$ than the divergence out $s$ corresponding to $x$. We can refer to P as an *augmenting path*, and we refer to the operation of replacing $x$ by $\overline{x}$ as a *flow augmenting* along P.

The algorithm starts with the feasible flow vector $x$. If the lower flow bound is zero for all arcs, the zero flow vector can be used as a starting vector; otherwise, a feasible starting flow vector can be obtained by solving an auxiliary max-flow problem with zero lower flow bounds.

At each iteration the algorithm has a feasible flow vector $x$ and uses the unblocked path search method, to generate a new feasible flow vector with larger divergence out of $s$ or terminate with a maximum flow and a minimum cut capacity.

Going on the discussion we can see that with each augmentation the Ford-Fulkerson algorithm will improve the divergence of $s$ by the augmentation increment $\delta$. Thus, if $\delta$ is bounded below by some positive number, the algorithm can execute only a finite number of iterations and must terminate with an optimal solution.

More in detail, if the arc flow bounds are integer and the initial flow vector is also an integer, $\delta$ will be a positive integer at each iteration, and the algorithm will terminate. The same is true also if arc flow bounds and the initial flow are rational, instead if the problem data are irrational, proving termination of the Ford-Fulkerson algorithm is nontrivial.

**Inputs** Given graph G with flow capacity $c$, a source node $s$, and a sink node $t$

**Output** A flow $f$ from $s$ to $t$ which is maximum

```
1. $f(u,v) \longrightarrow 0$ for all edges $(u,v)$
2. While there is a path $p$ from $s$ to $t$ in $G_f$, such that $c_f(u,v)>0$ for all edges $(u,v)\in p$
1. Find $c_p(p)=\min\{c_p(u,v): (u,v)\in p\}$
2. For each edge $(u,v)\in p$
1. $f(u,v)\longrightarrow f(u,v) + c_f(p)$(Send flow along the path)
2. $f(u,v)\longrightarrow f(u,v) - c_f(p)$(The flow might be "returned" later)
```

### 3.3.3 The Minimum Cost Flow Problem

The Minimum Cost Flow Problem (MCFP) is the most fundamental among the flow problems, as all such problems may be easily transformed in the MCFP, and as this problem can be solved efficiently using the *network simplex algorithm.*

The Minimum Cost Flow Model consists in determining the most economic way to transport a certain amount of a good (e.g. oil, cars, ...) from one or more production facilities to one or more consumption facilities, through a transportation network. In other world, the goal of MCFP is to send flow from supply nodes to demand nodes using arcs with capacities and involve the minimum total cost of transportation given availability of supply and demand in a direct network (if the network is undirected, an undirected arc between nodes $i$ and $j$ is replaced with two directed arcs, $(i,j)$ and $(j,i)$, with the same cost and capacity as the undirected arc to obtain a directed network).

A Minimum Cost Flow Problem has several applications: distribution of a product from manufacturing plants to warehouses, or from warehouses to retailers; flow of raw materials and intermediate goods through stations in a production line; routing of automobiles through a street network; routing of calls through a telephone system; etc..

The nodes of the network may be associated with physical places (cities, warehouses, industrial facilities, stations, ... ), and the arcs to one-way communication links (e.g. road sections, railways, ...) among these places.

***Definition 3.16*** Let $x_{ij}$ denote the amount of flow on arc $(i,j)$, the Minimum Cost Flow Problem can be formulated as follows:

Minimize: $\sum_{(i,j)\in A} c_{ij}\, x_{ij}$

Subject to: $\sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b_i \quad i \in N$

$l_{ij} \leq x_{ij} \leq u_{ij} \qquad (i,j) \in A$

$x_{ij} \geq 0 \qquad (i,j) \in A$

where $c_{ij}$ is the cost of arc$(i,j) \in A$ and $b_i$ is the supply/demand at node $i$. If node $i$ is a supply node, then $b_i > 0$, whereas $b_i < 0$ for demand node, and $b_i = 0$ for a transshipment node. To have a feasible solution, $\sum_{i\in N} b_i = 0$. The aim of this problem is to minimize the cost of transporting the commodity from supply nodes to demand nodes subject to constraints. The first constraint is the mass balance constraint (flow balance or conservation of flow). It states that the difference between the total flow emanating from node $i$ (*outflow*) and entering node $j$ (*inflow*) is equal to the demand/supply at that node. The next one constraint states that flow on any arc $(i,j)$ should be between the allowable range, that is, between the lower ($l_{ij}$) and upper bounds ($u_{ij}$) of flow on arc $(i,j)$, where $l_{ij} \leq 0$ *or* $l_{ij} > 0$, *and* $u_{ij} < \infty$. *When* $u_{ij} = \infty$ *for all arcs (i.e. there is no upper bound on the arc capacity), the problem becomes an incapacitated network flow problem.*

The above problem can be solved using linear programming techniques such as the Simplex algorithm, but for network problems we have an easier and more specialized Simplex algorithm that is called the *Network Simplex Method*.

### Network Simplex Method

The Network simplex algorithm for solving the minimum cost flow problem is an adaptation of the well-known simplex method for general linear programming problems.

Because the minimum cost flow problem is a highly structured linear programming problem, when applied to it, the computations of the simplex method become considerably streamlined. We need not explicitly maintain the matrix representation (known as the simplex tableau) of the linear program and can perform all of the computation directly in the network.

In this subsection I describe the network simplex algorithm starting by the definition of the concepts of *basis structure* and describing a data structure to store and to manipulate the basis, which is a *spanning tree*. Then I show how to compute arc flows and node potentials for any basis structure [11].

The network simplex algorithm maintains a basic feasible solution at each stage. A basic solution of the minimum cost flow problem is defined by a triple (B, L, U), where B, L and U partition the arc set E. The set B denotes the set of *basic arcs*, i.e., arcs of a spanning tree[12], and L and U denote

---

[11] For a complete description see [1]

[12] A tree is a connected undirected graph with no cycles. It is a *spanning tree* of a graph G if it spans G, i.e. it includes every node of G and is a subgraph of G since every edge

the sets of *nonbasic arcs* at their lower and upper bound [1]. Therefore we can call (B, L, U) *basis structure.*

The *basis structure* is *feasible* if by setting $x_{ij}$=0 for each $(i,\ j) \in L$, and setting $x_{ij}=u_{ij}$ for each $(i,\ j) \in U$, the minimum cost flow problem has a feasible solution satisfying the constraints.

The *basis structure* is *optimum* if it is possible to obtain a set of potentials $\pi$ so that the reduced cost is $\overline{c}_{ij}$ =$c_{ij}$-$\pi(i)$+$\pi(j)$ and it satisfies the following optimality conditions:

1. $\overline{c}_{ij}$ =0, for each $(i,\ j) \in B$,
2. $\overline{c}_{ij} \geq 0$, for each $(i,\ j) \in L$,
3. $\overline{c}_{ij} \leq 0$, for each $(i,\ j) \in U$

These optimality conditions could have an economic interpretation, for e.g. if $\pi(1)$=0, then optimization condition (1.) implies that -$\pi(j)$ is the length of the tree path B from node 1 to node j. More over, $\overline{c}_{ij}$ =$c_{ij}$-$\pi(i)$+$\pi(j)$ for a nonbasic arc $(i,\ j)$ in L, is the change in cost of flow achieved by sending one unit of flow through the tree path from node 1 to node *i*, through the arc $(i,\ j)$, and then returning the flow back along the tree path from node j to node 1. So the condition (2.) implies that this kind of circulation flow is

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in the tree belongs to G. A spanning tree of a directed graph G can also be defined as a maximal set of edges of G that contains no cycle, or as a minimal set of edges that connect all nodes.

not profitable for any nonbasic arc in L [1].

Network simplex method maintain a feasible basic structure during the procedure by improving it until it becomes an optimum basic structure. The network simplex algorithm can be presented as follows:

```
begin
    determine an initial basic feasible flow $x$ and the corresponding basic structure (B, L, U);
    compute node potentials for this basis structure;
    while some arc violate the optimality condition do
    begin
        select an entering arc $(k, l)$ violating the optimality conditions;
        add arc $(k, l)$ to the spanning tree corresponding to the basis forming a cycle and augment the maximum possible flow in this cycle;
        determine the leaving arc $(p, q)$;
        perform a basis exchange and update node potentials;
    end
end
```

The network simplex method has two basic steps: the first one deals with the determination of the node potentials $\pi$ of a given basis structure (B, L, U); and the second one of computing the arc flows for a given basis structure.

The determination of node potentials can be described as follows. The first assumption that as to be made is that $\pi(1)=0$. Then, the basic idea is to start at node 1 and fan out along the tree arcs using the thread indices to compute other node potentials. Whenever this fanning out procedure visits node $j$, it has already evaluated the potential of its predecessor, for e.g. node $i$. All this imply that the procedure can compute $\pi(j)$ by using an alternative condition:

$\pi(j)=\pi(i)\text{-}c_{ij}$, *for every arc* $(i, j) \in B$.

A similar procedure can be used to compute flows on basic arcs for a given basis structure (B, L, U), but the order is reverse: it means that the staring point is the leaf node and then it moves toward the root node using the predecessor indices [1].

```
procedure Compute potentials
begin
    $\pi(1)=0$;
    $j$:=thread(1);
    while $j \neq 1$ do
    begin
        $i$:=pred($j$);
        if $(i, j) \in E$ then $\pi(j) := \pi(i) - c_{ij}$;
        if $(j, i) \in E$ then $\pi(j) := \pi(i) - c_{ji}$;
        $j$:=thread (j);
    end
end
```

```
procedure Compute flows
begin
e(i):= b(i) for all $i \in N$;
let T be the basis tree;
for each $(i, j) \in U$ do
set $x_{ij} := u_{ij}$, subtract $u_{ij}$ from e(i) and add $u_{ij}$ to e(j);
while $T \neq [1]$ do
begin
select a lead node $j$ in the subtree T;
$i$:=pred($j$);
if $(i, j) \in T$ then $x_{ij}$:= -e(j);
else $x_{ji}$:= e(j);
add e(j) to e(i);
delete node $j$ and the arc incident to it from T;
end
end
```

### 3.3.4 Variation and extension of network flow problems

The standard network flow problem has been the basis of more elaborated models during the years, in order to use it to solve several optimization problems. Variation and extensions include problems having nonlinear costs, multi commodity networks, networks with side constraints and/or side columns, generalized networks, mixed-integer networks, and fixed-charge network.

In all of the network problems considered up to now, no distinction among units flowing in the network has been done; all the models presented are *single-commodity flow problems*. However as I have anticipated before, there is also a class of network flow problems, called *multi commodity flow problem* in which a distinction among units is considered.

In a multi commodity generalization, several commodities share arcs in capacitated network. Usually, there are multiple independent copies of the network, one for each commodity, and mutual arc capacity constraints having the generalized upper bound structure (Dantzig and Van Slyke, 1964).

It is well known that in a pure network the sum of flows entering an arc is equal to the sum of flows leaving it. However, in generalized network models there may be a gain or a loss as flow traverses an arc. These models are characterized by a multiplier associated to each arc. This multiplier should be greater (less) than one and indicates that a gain (loss) of the flow through an arc. To see what is meant, we can consider the network in Fig.3.16.[13]

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[13] For a more in-depth analysis see Glover, F., Hultz, J., Klingam, D., Stutz, J. (1978). Generalized networks: A fundamental computer-based planning tool. *Management Science*, 24(12), 1209-1220.

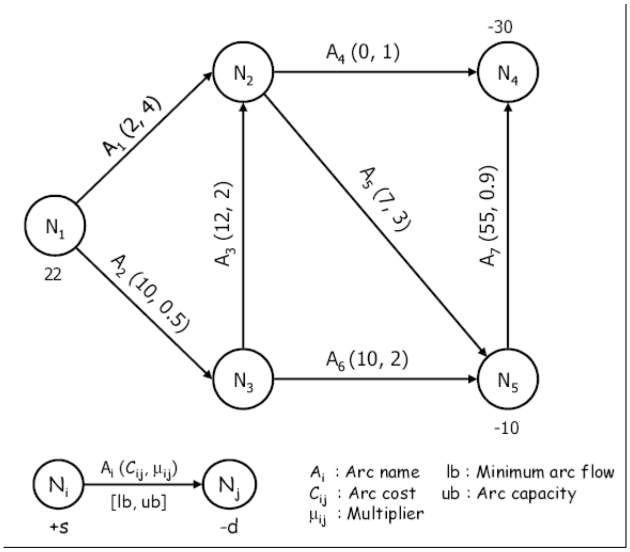

Figure 3.16: Generalized Network example

Mixed-integer networks on the other hand are characterized by some flows which are restricted to be integer. In fixed-charge problems, indeed there is a fixed cost of allowing positive flow through an arc.
In addition there are multi period network flow models in which there is an additional parameter to be considered, the *transit time* $\tau_e$, that is a positive time associated to each arc $e$ determining the amount of time it takes for flow to go from the source node to the sink node.

### Dynamic Network Flow models

More network optimization problems that have been studied in literature are *static* in nature, in the sense that they assume that it takes zero time to traverse any arc in a network and all the attributes of the network (e.g. cost to send flow on an arc) are constant overtime.[14]

Some problems with *static* network flow problems arise in real world application, where time-caring is relevant. Therefore it is required a more complex model such as the dynamic network flow model, whose *static* network flow models are simply an approximation.

Dynamic network flow models describe network structured, decision-making problems overtime. The terms *multi period*, *multistage*, *time-phased*, *staircase*, and sometimes *acyclic* are synonymous with dynamic context.

In a dynamic network, any flow must take a period of time to traverse an arc and the parameters (e.g. arc capacities) may change overtime. In these circumstances we speak of *non-static* or dynamic network flow optimization problems.

In such a problem how to control a flow is very important, since for example waiting at a node, may allow one to catch the best timing along this path.

These problems can be used for solving various problems in the power industry for example, concerning power generation, transmission and distribution.

A lot of characteristics and parameters have to be taken into account such as equipment capabilities, time factors, optimal cost and many others.

Many dynamic network flow problems are considered as extensions of static network flow problems. These includes maximum dynamic flow and minimum dynamic flow problems.

The maximum dynamic flow problem seeks a dynamic flow which sends as many as possible a commodity from a single source to a single sink of the

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[14] Static Network Flow models have been studied a lot in literature, for a more in-depth analysis see: Ahuja, Magnanti and Orlin(1991,1993); Adel'son, Dinic and Karzana(1975); Bazaraa, Iarvis and Sherali (1990); Ford and Fulkerson (1962); Gupta (1985); Iri (1969); Jensen and Barnes (1980); Lawler (1976); and Minieka (1978).

network within the time horizon T.

The minimum cost dynamic flow seeks a dynamic flow that minimizes the total shipment cost of a commodity in order to satisfy demands at certain nodes within T.

# Chapter 4

## 4.1 Application of Network Flow techniques to the Energy System

Over the past decades, real-world production and distribution networks have become increasingly complex. For this reason a strategic network design is helpful for long-term decisions regarding the configuration of the supply chain network. Typically, it involves selecting sites for the location of new facilities, deciding their number and size, choosing the distribution channels, all this in order to meet customer demands.

Of course these decisions have major impacts on long-term profitability and competitive advantage of companies, however for what concern the main topic of this thesis, that is to say, energy distribution, these are not only decisions to help companies but also to help the entire society. Important decisions have to be taken at supranational level and these impacts all the world, for these reasons it is important to develop such a tools to help the decision-making process to adopt reasonable decisions according to the main objective of energy efficiency.

According to Harrison[15], up to 80% of the total cost of a product is driven by decisions made during the design phase of the supply chain network.
Due to the globalization of the economy, redesign processes have become more frequent and they have to be structured in a more efficient way.
Since 1974, for e.g. the U.S. Department of Energy Information Administration (EIA) and its predecessor, the Federal Energy Administration (FEA), have developed several computer-based energy modeling systems, in order to analyze domestic energy-economy markets and the relationships between electric energy and all types of fuels. The first model that was employed by the FEA was the Project Independence Evaluation System (PIES) [16]. It provides a framework for developing a national energy policy through a quantitative analysis and projections of the energy systems. The main objective is to increase the economic efficiency through a better use of low cost generators and electric power trade.

To face problems of Energy System, in particular of distribution, It could be useful to use the methodology of Network Flow rather than a more general linear programming approach because of the more efficient solution procedure that can be used. Networks are an important subclass of linear programs that are intuitive, easy to solve, and have nice integrality properties.
Another important point is that problems that might not look like networks might be networks. Since we deal with network which are complex in their

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[15] See: Harrison, T. P. (2004). Principles for the strategic design of supply chains. In T.P. Harrison, H.L. Lee, and J.J. Neale (Eds.), *The practice of Supply Chain Management: Where Theory and Application Converge* (pp. 3-12). New York: Springer.

[16] For a detailed description see: Hogan W.W.. Energy policy models for project independence. *Computers and Operations Research*. 2:251-271, 1975.

structure, computational advantages in problem solving could be provided for e.g. by the Network Simplex Method, that is able to solve larger problems than the regular simplex method.
Further more the CPLEX software, a powerful optimization solver able to solve large problem, for example, has a first-rate implementation of the Network Simplex Method[17]

Since the early 1970s, a wide number of energy models[18] have been developed for policy analysis, forecasting, and to support local and global energy planning. They have different purposes, but most of them deal with a better energy supply system design given a level of demand forecast, a better understanding of the present and the future demand-supply interactions, energy-environment and energy-economy relations, and energy system planning.
An important remark of all this variety of models is that they usually tend to be highly resource intensive, in the sense that they require a lot data to be performed; and moreover it seems that the time of execution is larger, together with the computational resource requirements which are complex.

In the following sections I will present a survey, whose aim is to present some studies that have been done for energy system with emphasis on the network flow applications. The Integrated Energy Systems models that will be presented are motivated by the hypothesis that the current fragmented

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[17] See: CPLEX Division, ILOG Inc., CPLEX optimization package, USA, 1998. [online]. Available: www.cplex.com

[18] For description of the most relevant energy models see: Van Beck, N. (1999). Classification of energy models. In *Tilburg University and Eindhoven University of Technology*

decision making environment in which coal, natural gas, and electricity firms operate leads to inefficiencies.

First, I provide the definition of some approaches, the Top-Down approach and the Bottom-Up approach, that to my mind could be useful to better understand the directions of different studies. Then, I review the state of the art in the field by presenting the models and their results.

## 4.2 Top-Down and Bottom-Up Approaches

Policy-makers are interested in a better understanding of the effectiveness and the cost of policies whose purpose is to shift energy systems toward more environmentally desirable technology paths (Hourcade, 2010).
Two different approaches have been developed: Top-Down and Bottom-Up. Top-Down and Bottom-Up are the two modeling paradigms to represent interactions between the energy system and the economy (International Panel on Climate Change (IPCC) [1996]). The terms "top-down" and "bottom-up" are shorthand for aggregate and disaggregated models. Models in the first category emphasize economy-wide, while those in the second category feature sectoral and technological details. The dichotomy of energy-economy models into top-down and bottom-up approaches is sometimes traced back to competing paradigms (B*ö*bringer, Rutherford)

According to Hourcade et. al., these models produce opposite outcomes for the same problem; the reason seems to be the distinct approach in which these models consider the adoption of technologies, the decision-making behavior of economic agents and how markets and economic institutions operate over a given period.

Grubb et. Al. (1993, 433-437) stated that the Top-Down approach is associated with -but not exclusively restricted to- the "pessimistic" economic paradigm, while the Bottom-Up approach is associated with the "optimistic" engineering paradigm.

Economics considers technology as a set of techniques by which inputs such as capital, labor and energy can be transferred into outputs. The optimal technique can be obtained by constructing the production frontier which has at the basis the observed actual behavior. On the other hand Engineering studies are more independent of market behavior.

### Bottom-Up models

Bottom-Up models describe the current and the prospective competition of energy technologies, both on the supply side (substitution between primary forms of energy) and on the demand side (energy efficiency use).

These models are useful to illustrate future evolution of technologies but on the other hand they have been criticized because they do not provide a realistic scenario of micro-economic decision-making by firms and consumers, or they do not take into account from a macro-economic perspective the impacts on economic structure, on productivity, on trade and on economic growth.

According to Hourcade et. al. (1996), Bottom-Up models can be further subdivided into descriptive and prescriptive models.

Descriptive models try to provide a practical estimate of the technology mix that would result from actual decisions, based on factors such as complex preferences, intangible costs, capital constraints, attitude to risk, uncertainty, market barriers. These kind of models are considered more predictive.

Prescriptive models, on the contrary, provide an estimate for technological potential by examining the effects of acquiring only the most efficient existing technologies. These kind of models, instead are considered more explorative.

### Top-Down models

Top-Down models use to examine interactions between the energy sector and other sectors of the economy, by analyzing the impacts at micro and macro levels.

Since the late 1980's energy-economy modeling has been dominated by General Equilibrium Models, which are assumed to represent real-world microeconomic responsiveness to policies. However, the lack of General Equilibrium models is the technological flexibility beyond current practices.

In Top-Down models, energy sectors are represented in an aggregate way by means of smooth production function, and this implies that they do not incorporate different assumptions about discrete energy technologies and costs.

Table 4.17 compares the most relevant aspects of both models highlighting the differences between the two approach.

Many researchers have elaborated various hybrid models, that seeks to compensate for the limitations of one approach or the other. The aim is to combine the ethnological explicitness of the Bottom-Up models with the economic richness of the Top-Down model.

In this context we can identify two approaches. The first one attempts to couple existing large-scale Bottom-Up and Top-Down models, however heterogeneity in complexity and accounting methods can create some problems in achieving significant results.[19]

The second approach emphasizes the overall economic consistency and there-

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[19] For a more detailed analysis see Hudson, E. A. & Jorgenson, D. W. (1974). US energy policy and economic growth, 1975-2000. *Bell Journal of Economics*, 5(2), 461-514 .

fore proposes a single integrated model.[20]

| Top-Down Models | Bottom-Up Models |
|---|---|
| use an “economic approach” | use an “engineering approach” |
| give pessimistic estimates on “best” performance | give optimistic estimates on “best” performance |
| can not explicitly represent technologies | allow for detailed description of technologies |
| reflect available technologies adopted by the market | reflect technical potential |
| the “most efficient” technologies are given by the production frontier (which is set by market behavior) | efficient technologies can lie beyond the economic production frontier suggested by market behavior |
| use aggregated data for predicting purposes | use disaggregated data for exploring purposes |
| are based on observed market behavior | are independent of observed market behavior |
| disregard the technically most efficient technologies available, thus underestimate potential for efficiency improvements | disregard market thresholds (hidden costs and other constraints), thus overestimate the potential for efficiency improvements |
| determine energy demand through aggregate economic indices (GNP, price elasticities), but vary in addressing energy supply | represent supply technologies in detail using disaggregated data, but vary in addressing energy consumption |
| endogenize behavioral relationships | assess costs of technological options directly |
| assumes there are no discontinuities in historical trends | assumes interactions between energy sector and other sectors is negligible |

Figure 4.17: Characteristics of Top-Down and Bottom-Up Models

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[20] Some studies of integrated models provide specifications of market equilibrium models as *mixed complementarity problems*, see Rutherford, T. F. (1995). Extension of GAMS for complementarity problems arising in applied economic analysis. *Journal of Economic Dynamics and Control*, 19(8), 1299-1324.

## 4.3 Models description

### 4.3.1 Multi period generalized network flow model of an Integrated Energy System

The model by Quelhas, Gil, McCalley and Ryan (2007) [28] is a *generalized minimum cost flow problem* that can be solved by applying the simplex network algorithm.

The objective of the *generalized minimum cost flow problem* is to satisfy electric energy demands with available fossil fuel supplies at the minimum total cost, without violating the bound constraints. The costs that are considered are the fossil fuel production, transportation, and storage costs, the operation and maintenance cost associated with electricity generating units and the electric power transmission costs. This model is solved for the most efficient allocation of quantities and corresponding prices.

#### Mathematical formulation

The multi period generalized minimum cost flow model is a Network Flow optimization model that can be formalized as follows:

Minimize $z=\sum_{t\in T}\sum_{(i,j)\in M}\sum_{l\in Lij}c_{ij}(l)\,e_{ij}(l,t)$

subject to: $\sum_{\forall k}\sum_{l\in Lik}e_{jk}(l,t)-\sum_{\forall i}\sum_{l\in Lij}\eta_{ij}(l)\,e_{ij}=b_j(t)$

$$\forall j \in N,\ \forall t \in T$$

$$e_{ij.min} \leq e_{ij}(t) \leq e_{ij.max}$$

$$\forall (i,j) \in M,\ \forall t \in T$$

The objective function $z$ represents the total cost of energy flow from the fossil fuel production sites to the electricity end users and nonelectric natural gas consumers. In the total cost are included: fuel production costs, fuel transportation costs, fuel storage costs, electricity generation costs, and electricity transmission costs.

The first constraint is the energy balance constraints for all nodes, while the second one is the flow bound constraint for all arcs.

In the matrix form the problem can be represented as follows:

Minimize $z=\underline{c}'\ \underline{e}$

subject to: $A\underline{e}=\underline{b}$

$$\underline{e}_{min} \leq \underline{e} \leq \underline{e}_{max},$$

where A is an $n \times m$ matrix, composed by $n$ nodes and $m$ arcs, and it is called the *node arc incidence matrix.* Every column of A is associated with a decision variable, while each row is associated with a node. The column $A_{ij}$ has $a$+1 in the $i$-th row, $a-1$ or $a$-$\eta_{ij}$ in the $j$-th row, and the rest of entries are made of zeros. Here, we can highlight that every column of matrix A has no more than two non-zero elements, which allows to use the structure of a network and to solve it by using Network Simplex Method, which is able to solve networks having a number of arcs in a range of several millions.

An example of the *node arc incidence matrix* is provided in the figure below for a simple Integrated Energy System.

$$
\begin{array}{cc|ccccccccccc|ccccccccccc|cccccccccc|}
 & & \multicolumn{11}{c|}{\text{Time step 1}} & \multicolumn{11}{c|}{\text{Time step 2}} & \multicolumn{10}{c|}{\text{Time step 3}} \\
 & & E_{11,1} & E_{22,1} & E_{14,1} & E_{33,1} & E_{23,1} & E_{25,1} & E_{35,1} & E_{44,1} & E_{55,1} & E_{46,1} & E_{56,1} & E_{11,2} & E_{22,2} & E_{14,2} & E_{33,2} & E_{23,2} & E_{25,2} & E_{35,2} & E_{44,2} & E_{55,2} & E_{46,2} & E_{56,2} & E_{11,3} & E_{22,3} & E_{14,3} & E_{23,3} & E_{25,3} & E_{35,3} & E_{44,3} & E_{55,3} & E_{46,3} & E_{56,3} \\
\hline
\text{Time step 1} & 1' & 1&0&-1&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0 \\
 & 2' & 0&1&0&0&-1&-1&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0 \\
 & 3 & 0&0&0&-1&1&0&-1&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0 \\
 & 4 & 0&0&1&0&0&0&0&-1&0&0&0 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0 \\
 & 4' & 0&0&0&0&0&0&0&\eta_4&0&-1&0 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0 \\
 & 5 & 0&0&0&0&0&1&1&0&-1&0&0 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0 \\
 & 5' & 0&0&0&0&0&0&0&0&\eta_5&0&-1 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0 \\
 & 6 & 0&0&0&0&0&0&0&0&0&\eta_{46}&\eta_{56} & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0 \\
\hline
\text{Time step 2} & 1' & 0&0&0&0&0&0&0&0&0&0&0 & 1&0&-1&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0 \\
 & 2' & 0&0&0&0&0&0&0&0&0&0&0 & 0&1&0&0&-1&-1&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0 \\
 & 3 & 0&0&0&1&0&0&0&0&0&0&0 & 0&0&0&-1&1&0&-1&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0 \\
 & 4 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&1&0&0&0&0&-1&0&0&0 & 0&0&0&0&0&0&0&0&0&0 \\
 & 4' & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&\eta_4&0&-1&0 & 0&0&0&0&0&0&0&0&0&0 \\
 & 5 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&1&1&0&-1&0&0 & 0&0&0&0&0&0&0&0&0&0 \\
 & 5' & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&\eta_5&0&-1 & 0&0&0&0&0&0&0&0&0&0 \\
 & 6 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&\eta_{46}&\eta_{56} & 0&0&0&0&0&0&0&0&0&0 \\
\hline
\text{Time step 3} & 1' & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0&0 & 1&0&-1&0&0&0&0&0&0&0 \\
 & 2' & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0&0 & 0&1&0&-1&-1&0&0&0&0&0 \\
 & 3 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&1&0&0&0&0&0&0&0 & 0&0&0&1&0&-1&0&0&0&0 \\
 & 4 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&1&0&0&0&-1&0&0&0 \\
 & 4' & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&\eta_4&0&-1&0 \\
 & 5 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&1&1&0&-1&0&0 \\
 & 5' & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&\eta_5&0&-1 \\
 & 6 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&0&0&0 & 0&0&0&0&0&0&0&0&\eta_{46}&\eta_{56} \\
\end{array}
\times
\begin{bmatrix}
E_{11,1} \\ E_{22,1} \\ E_{14,1} \\ E_{33,1} \\ E_{23,1} \\ E_{25,1} \\ E_{35,1} \\ E_{44,1} \\ E_{55,1} \\ E_{46,1} \\ E_{56,1} \\ \hline
E_{11,2} \\ E_{22,2} \\ E_{14,2} \\ E_{33,2} \\ E_{23,2} \\ E_{25,2} \\ E_{35,2} \\ E_{44,2} \\ E_{55,2} \\ E_{46,2} \\ E_{56,2} \\ \hline
E_{11,3} \\ E_{22,3} \\ E_{14,3} \\ E_{23,3} \\ E_{25,3} \\ E_{35,3} \\ E_{44,3} \\ E_{55,3} \\ E_{46,3} \\ E_{56,3}
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ -E_{33,0} \\ 0 \\ 0 \\ 0 \\ 0 \\ E_{6,1} \\ \hline
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E_{6,2} \\ \hline
0 \\ 0 \\ E_{33,3} \\ 0 \\ 0 \\ 0 \\ 0 \\ E_{6,3}
\end{bmatrix}
$$

Figure 4.18: node arc incidence matrix example

The objective of this optimization problem is to determine the energy flows that satisfy the demand for electricity at the minimum operating costs, subject to physical and environmental constraints.
In the mathematical formulation presented above only physical constraints are considered, however in order to include also environmental constraints that derive from the regulatory treatment of environmental externalities at national or local level, we have to incorporate restrictions on emissions.

The amount of emissions produced depends on several issues such as the fuel used, the pollution control devices installed, and the amount of electricity produced.

This model has been applied to the United States Integrated Energy System where the *Clean Air Act*[21] (CAA) imposes a strict restriction on $SO_2$ emissions. Thus the additional constraint for this specific case can be represented by:

$$\sum_{t\in T} \sum_{(i,j)\in G} \mathrm{SO2}_i\ (t)\ \times (1\text{-}\alpha_i) \times \sum_{l\in L ij} e_{ij}\ (l,t) \leq NSO2$$

The above constraint states that the sum of Sulphur dioxide emissions associated with the fuel consumed by power plants and corresponding to the energy flowing from node $i$ to node $j$ must be within national $SO_2$ limit. In linear programming it is called bundle or side constraint, which specifies a flow relationship between several of the arcs in the network flow model.

However this inequality constraint can be transformed into an equality constraint in order to incorporate it in the matrix equation.

The procedure implies the introduction of a slack nonnegative variable in the left-hand side of the equation $A\underline{e}=\underline{b}$. As a consequence some of the columns of the matrix A have more than two nonzero entries, which makes it no longer a *node-arc incidence matrix*, but instead a more general *coefficient matrix.*

---

[21] The*Clear Air Act*(CAA) is a United States federal law designed to control air pollution on national level. It directs the Environmental Protection Agency (EPA) to establish national ambient air quality standards (NAAQS) for pollutants at levels that will protect public health. For more details see: http://epa.gov/air/caa/text.html. (Last accessed 20th September 2014)

### Nodal Prices

Nodal Pricing represents the most sophisticated and efficient expression of locational energy prices. It was developed by Schweppe et al. (1988 ). This transmission pricing attempts to base prices on real-time marginal costs. Nodal prices are the prices that allow the decentralization of the optimal dispatch of power through a network.

Nodal prices are also explained as the "shadow values" related with each active constraint at optimal solution of the choice variables, and they represent the marginal costs of enforcing the constraints. In economics this definition is associated to the Lagrangian multipliers.

According to the nodal price theory, when the network is optimally dispatched, at each node the marginal utility of power is equal to the marginal cost. Moreover, from one node to another this marginal valuation can vary, depending on the capacity of the connecting lines as compared with the flow of energy. If for example no line is congested, so there are no power losses, there is a unique energy price at all nodes, on the other hand if some lines are congested, differences in the marginal nodal valuations reflect the "shadow value" of the lines.

In the following I present the Lagrangian Function. Without loss of generality it is assumed that the cost and efficiency parameters associated with each arc are constant functions, and this allows to exclude parameter $l$ for simplicity.

$$L=\sum_{t\in T}\sum_{(i,j)\in M} c_{ij}(t)\ e_{ij}(t)+$$

$$\sum_{t\in T}\sum_{j\in N} \lambda_j(t)\ [\sum_{\forall k} e_{ik}(t)\text{-}\sum_{\forall i}\eta_{ij}\ e_{ij}(t)\text{-}b_j(t)]+$$

$$\sum_{t\in T}\sum_{(i,j)\in M} \delta_{ij}(t)[\ e_{ij.min}\text{-}\ e_{ij}(t)]+$$

$$\sum_{t\in T}\sum_{(i,j)\in M} \mu_{ij}(t)[e_{ij}(t)\text{-}e_{ij.max}]+$$

$$\gamma[\sum_{t\in T}\ \sum_{(i,j)\in G} \mathrm{SO2}_i\ (t)\ \times(1\text{-}\alpha_i)\times\sum_{l\in L ij}\ e_{ij}(t)\text{-}\mathrm{NSO2}].$$

$\lambda_j$ is the Lagrangian multiplier associated with the energy balance constraint at node j, $\eta_{ij}$ and $\delta_{ij}$ are the Lagrangian multipliers associated with the lower and upper bound constraints, respectively, on the energy flowing from node $i$ to node $j$.

For optimality, the relationship between the nodal prices of two linked nodes $i$ and $j$ is:

If $(i,j) \notin G$, that means $(i,j)$ does not represent for e.g. electricity generation, then

$$\frac{\partial L}{\partial e}{}_{ij}(t) = c_{ij}(t)+\lambda_i(t)-\lambda_j(t)\eta_{ij}\text{-}\delta_{ij}(t)+\mu_{ij}(t)=0$$

From the above equation we can derive that if the flow bound constraints are not binding, the cost is zero, so $c_{ij}$=0 and there are no losses $\eta_{ij}$, therefore the nodal prices of the two linked nodes are the same $\lambda_i=\lambda_j$.

If $(i,j) \in G$, that means $(i,j)$ is an arc representing for e.g.electricity generation, then

$$\frac{L}{\partial e}{}_{ij}{=}(t) = c_{ij}(t) + \lambda_i(t) - \lambda_j(t)\eta_{ij}\text{-}\delta_{ij}(t) + \mu_{ij}(t) + \gamma SO2_i\ (t)\ \times (1\text{-}\alpha_i) = 0$$

From the above equation we can derive that the nodal price at plant node $i$ is the same as the nodal price at the the demand node $j$, if and only if the flow bound constraints are not binding, the arc cost is zero, there are no losses, and the emissions limit constraint is not binding [28].
On the other hand it has to be remarked that flow bound constraints that are binding means congestion in the associated arc. For this reason we can conclude that prices vary from node to node because of transmission line congestion and losses.
At each node, the price represents the locational marginal price, that includes the cost of energy, the cost of delivering it, losses and congestion. Nodal prices provide important economic signals by the identification of interdependencies between the fuel subsystems and the electric subsystem. It could be helpful for improvement in investments.

### An example

We can imagine an energy system composed of two utilities, one in a northern region and the other in a southern region, interconnected. The northern region operates two generating units: one oil-fired and the other gas-fired. The southern region operates three units: two coal-fired and one natural gas-fired. There are two possible suppliers of coal, one supplier of natural gas, and one supplier of oil.

The energy system can be represented in a network context as follows:

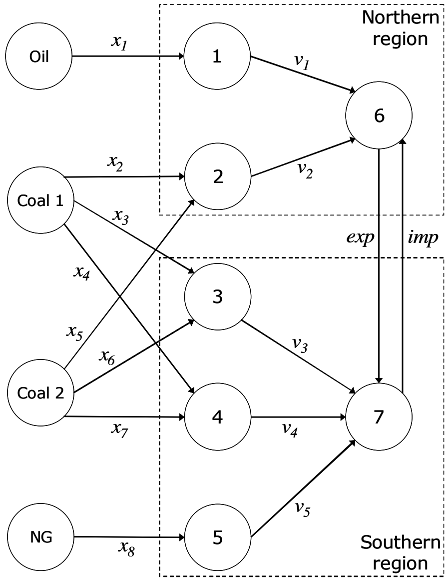


Figure 4.19: Two regions system example

The variables $\{x_1, \ldots, x_8\}$ are some fuel energy flows to the generating units, while the variables $\{v_1, \ldots, v_8\}$ are the electric energy output of the generating units.

Variables *imp* and *exp* are the interchanged energy between the two regions. It is important to remark that all these variables ($x_1, \ldots, x_8$,$v_1, \ldots, v_8$, *imp, exp)* correspond to the energy flow variable $e_{ij}$ described in the mathematical formulation above.

The objective of this problem is to satisfy the demand at the minimum operating costs, subject to some constraints such as the conservation of energy at all nodes and the units' operating ranges.

The main characteristics of the utilities and the fuel are represented in the tables 4.20 and 4.21 :

| **Region** | **Unit** | **Fuel** | **Min (MW)** | **Max (MW)** | **Incremental Heat Rate (MBtu/MWh)** | **Average Load (MW)** |
|---|---|---|---|---|---|---|
| North | 1 | oil | 150 | 600 | 9.95 | 1200 |
| | 2 | coal | 400 | 1000 | 8.93 | |
| South | 3 | coal | 70 | 500 | 10.05 | 800 |
| | 4 | coal | 70 | 500 | 10.05 | |
| | 5 | gas | 0 | 300 | 9.55 | |

Figure 4.20: Unit Characteristics

| **Supplier** | **Cost** | **Heat Value** |
|---|---|---|
| Coal 1 | 30 $/ton | 11,500 Btu/lb |
| Coal 2 | 25 $/ton | 10,200 Btu/lb |
| Natural Gas | 3.7 $/Mcf | 1,000 Btu/cf |
| Oil | 21 $/barrel | 143,500 Btu/gallon |

Figure 4.21: Fuel Characteristics

In [27] different tests have been constructed in order to analyze the impacts of different situations on nodal prices. I have decided to present only the Base Case, the Test Case 1 and the Test Case 5 because I think they are meaningful in illustrating how nodal prices are in an Integrated Energy System.

The Base Case states that there are no limitations on the fuel transportation links and on the electric power transferred between the two regions. On the other hand the situation presented in Test Case 1 supposes that there is an increase in the southern region load from 800 MW to 1100 MW. Test Case 5 imposes a limit to the coal delivered to units 3 and 4 from coal supplier 2, of 2400 tons.

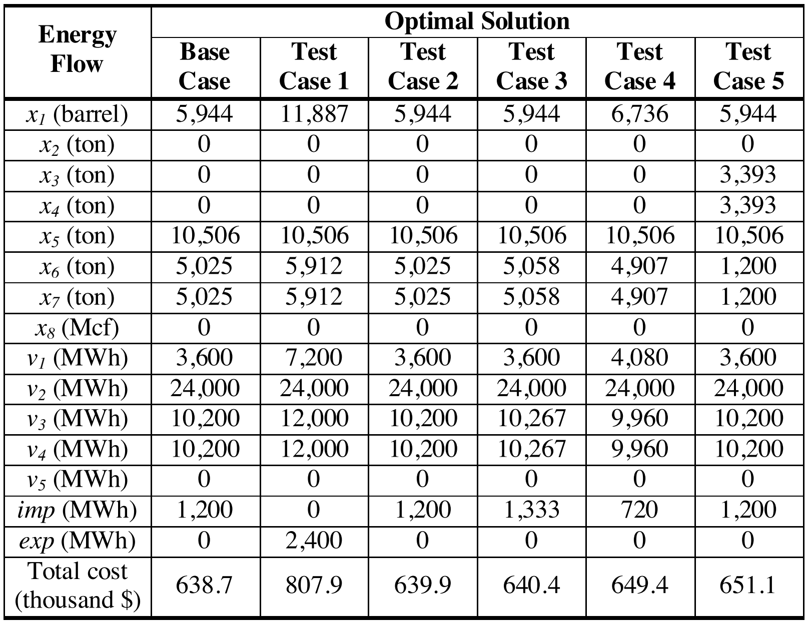
As one could imagine from the previous discussion, in the Base Case since there are no limitations on the electric power transferred between the two regions, transmission costs are zero, and so the nodal price in the northern area is equal to the nodal price in the southern area.

In the Test Case 1 since there are no congestions the transmission costs are zero, thus the nodal prices in both regions are equal. However the nodal prices are higher than the Base Case, this is because to supply the higher demand the incremental cost is higher. The nodal prices are now 34.7$ /MWh.

In the Test Case 5 there is congestion in the coal delivered in units 3 and 4 from coal supplier 2, however since there is no congestion in the tie line, the transmission cost is zero and the nodal prices at units 3 and 4 are equal. However the difference from the other two cases is that due to congestion in the coal delivery system nodal prices are higher than the Base Case.

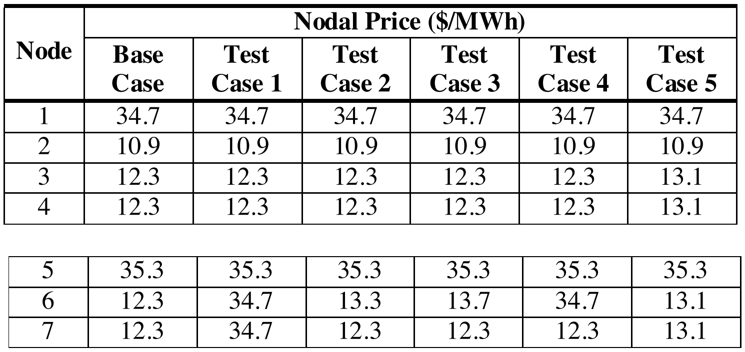
From the Tests' results (Fig.4.22, Fig.4.23) we can conclude that the nodal prices among interconnected control areas are the same, but when demand increases as in Test Case 1, the nodal prices may increase; the same happens when a constraint in the fuel production or transportation systems is imposed.

From this example and from the previous discussion we can conclude that nodal prices of an Integrated Energy System represent the marginal cost or more precisely the opportunity cost of energy at each node of the system. For this reason, nodal prices can be used as a measure of the use of all re-



| Energy Flow | Optimal Solution | | | | | |
|---|---|---|---|---|---|---|
| | Base Case | Test Case 1 | Test Case 2 | Test Case 3 | Test Case 4 | Test Case 5 |
| $x_1$ (barrel) | 5,944 | 11,887 | 5,944 | 5,944 | 6,736 | 5,944 |
| $x_2$ (ton) | 0 | 0 | 0 | 0 | 0 | 0 |
| $x_3$ (ton) | 0 | 0 | 0 | 0 | 0 | 3,393 |
| $x_4$ (ton) | 0 | 0 | 0 | 0 | 0 | 3,393 |
| $x_5$ (ton) | 10,506 | 10,506 | 10,506 | 10,506 | 10,506 | 10,506 |
| $x_6$ (ton) | 5,025 | 5,912 | 5,025 | 5,058 | 4,907 | 1,200 |
| $x_7$ (ton) | 5,025 | 5,912 | 5,025 | 5,058 | 4,907 | 1,200 |
| $x_8$ (Mcf) | 0 | 0 | 0 | 0 | 0 | 0 |
| $v_1$ (MWh) | 3,600 | 7,200 | 3,600 | 3,600 | 4,080 | 3,600 |
| $v_2$ (MWh) | 24,000 | 24,000 | 24,000 | 24,000 | 24,000 | 24,000 |
| $v_3$ (MWh) | 10,200 | 12,000 | 10,200 | 10,267 | 9,960 | 10,200 |
| $v_4$ (MWh) | 10,200 | 12,000 | 10,200 | 10,267 | 9,960 | 10,200 |
| $v_5$ (MWh) | 0 | 0 | 0 | 0 | 0 | 0 |
| *imp* (MWh) | 1,200 | 0 | 1,200 | 1,333 | 720 | 1,200 |
| *exp* (MWh) | 0 | 2,400 | 0 | 0 | 0 | 0 |
| Total cost (thousand $) | 638.7 | 807.9 | 639.9 | 640.4 | 649.4 | 651.1 |

Figure 4.22: Results of the optimization problem [27].



| Node | Nodal Price ($/MWh) | | | | | |
|---|---|---|---|---|---|---|
| | Base Case | Test Case 1 | Test Case 2 | Test Case 3 | Test Case 4 | Test Case 5 |
| 1 | 34.7 | 34.7 | 34.7 | 34.7 | 34.7 | 34.7 |
| 2 | 10.9 | 10.9 | 10.9 | 10.9 | 10.9 | 10.9 |
| 3 | 12.3 | 12.3 | 12.3 | 12.3 | 12.3 | 13.1 |
| 4 | 12.3 | 12.3 | 12.3 | 12.3 | 12.3 | 13.1 |
| 5 | 35.3 | 35.3 | 35.3 | 35.3 | 35.3 | 35.3 |
| 6 | 12.3 | 34.7 | 13.3 | 13.7 | 34.7 | 13.1 |
| 7 | 12.3 | 34.7 | 12.3 | 12.3 | 12.3 | 13.1 |

Figure 4.23: Nodal prices results [27].

sources in a system and can contribute to identify inefficiencies in the crucial interdependencies between for e.g.the electric energy subsystem and the fuel production and delivery subsystems, in order to correct them. Therefore nodal prices can be used by decision makers to support their decisions.

### Procedure

The complete procedure for obtaining the solution of the optimization problem can be divided into four steps as shown in table 4.24.

The first task that can be called *Data gathering*, is an initial phase concerning the identification of sources of information, thereafter data are collected and data gaps are solved. For example for the U.S. Integrated Energy System investigated in [28] data are gathered from the Energy Information Administration [43], the Mine Safety and Health Administration of the Department of Labor [51], the FERC [49] and the EPA [45].
The data are organized in two text files: nodes.txt and arcs.txt. The first file collects a list of nodes and relative supply/demand, while the second one is a list of all arcs involved with their relative information about the origin node, the destination node, the lower bound of the flow, etc..
Some other files with time-variant parameters are created.

The second step concerns the creation of a *node-arc incidence matrix* (or a constrain coefficient matrix) as that of Fig.4.18, in MPS format[22].
If the simulation is a multi period one, the input files nodes.txt and arcs.txt are expanded. Multi period network flow models can be seen as multiple copies of a network, so the size of the network is proportional to the number of periods.

The third task is the optimization phase, called *Optimization routine.*

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[22] The procedure is described in http://Ipsolve.sourceforge.net/5.5/mps-format.htm

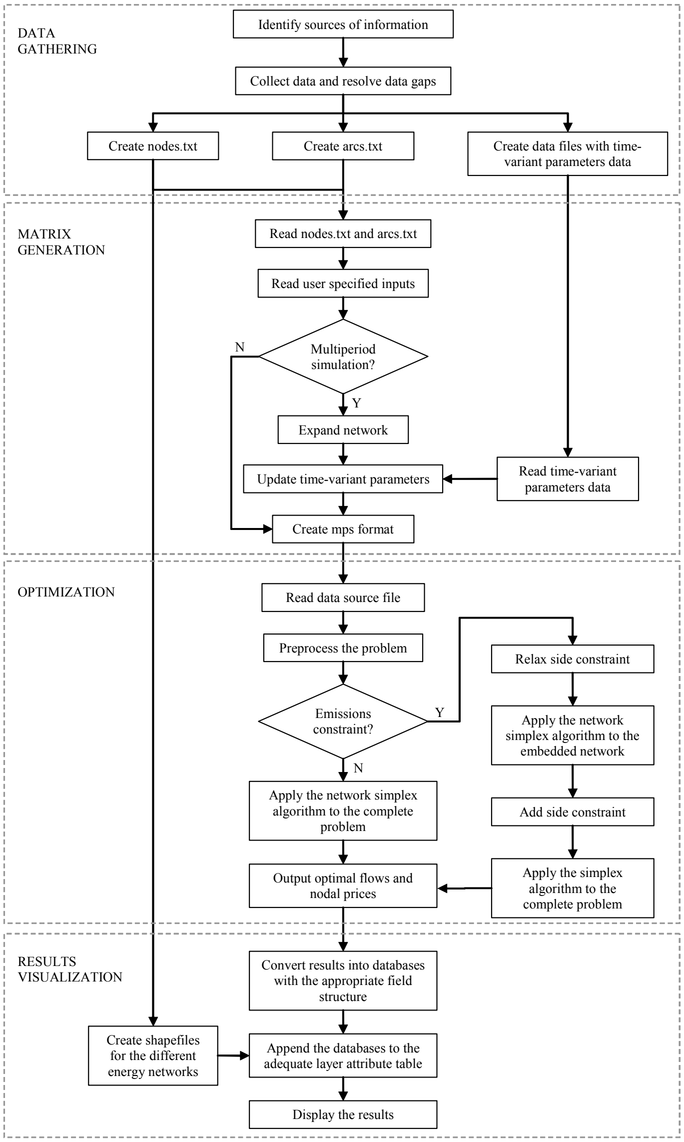

Figure 4.24: Complete procedure [29].

The mathematical model is executed in CPLEX[23], where after reading the

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[23] CPLEX is an optimization solver characterized by exceptional reliability. It is fast and robust even for poorly scaled or numerically difficult problems. Furthermore, CPLEX automatically recognizes the embedded network structure with millions of variables and constraints and solve the problem quickly.

data source files, it preprocesses the problem in order to reduce its size and to make it simpler to work with. For example some constraints which are redundant may be eliminated, e.g. nodes with only one incoming and one outcoming arc, and the overall problem adjusted.
Thereafter CPLEX solves the entire problem using the network simplex algorithm. The solution is written in a solution file text format, containing the optimal flows and the nodal prices associated to the constraints.

The last step is the *Results visualization*, maybe the most important task, in which the results are displayed in ArcView 9.1, which is a geographic information system software that shows the geographic context of the results.

### U.S. Case Studies

In the analysis provided by [29], three models are proposed: the coal network, the natural gas network and the electricity network.
The coal network model concerns the supply side of coal and it considers coal production nodes, coal-fired plants and arcs connecting them. The arcs are characterized by a lower bound that represents the existing contractual agreements and transportation costs.
The natural gas network model involves natural gas production nodes, transshipment nodes[24], storage nodes and gas-fired power plants nodes. Arcs are established between the production nodes and the transshipment nodes, between the transshipment nodes and the storage nodes, and between the

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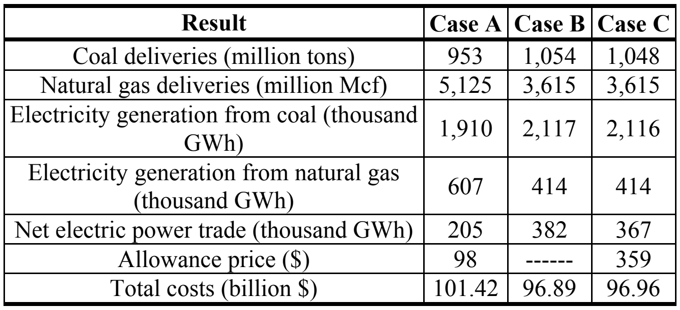
[24] Transshipment nodes are nodes that send to and receive from other nodes in the network. In this specific case they represent a junction point for flows coming into and out of a specific region.

transshipment nodes and the gas-fired plant nodes. Even this model concern the supply side, indeed the natural gas consumption of end users is an exogenously given demand in the transshipment nodes.

Finally the electricity network model which has been considered is based on the topology of the electrical grid of U.S., but in a simplify way. Three case studies are proposed: the first one (Case A) which has been called reference case, optimizes coal and natural gas flow given actual generation, demand and emissions data from 2002; the second case (Case B) is still based on the reference case but without emission constraints; the third one (Case C) considers emission constraints.

The costs considered in all these studies are the coal production and transportation costs, the natural gas production, transportation and storage costs and the electric power transmission costs. All cases have been simulated with yearly data for coal network and monthly data for natural gas and electric networks.

The results have been obtained using the network optimizer routine of CPLEX and a summary of the results is presented in table 4.25.



| **Result** | **Case A** | **Case B** | **Case C** |
|---|---|---|---|
| Coal deliveries (million tons) | 953 | 1,054 | 1,048 |
| Natural gas deliveries (million Mcf) | 5,125 | 3,615 | 3,615 |
| Electricity generation from coal (thousand GWh) | 1,910 | 2,117 | 2,116 |
| Electricity generation from natural gas (thousand GWh) | 607 | 414 | 414 |
| Net electric power trade (thousand GWh) | 205 | 382 | 367 |
| Allowance price ($) | 98 | ------ | 359 |
| Total costs (billion $) | 101.42 | 96.89 | 96.96 |

Figure 4.25: Summary of the results [29].

### 4.3.2 Analysis of an Integrated Thermal Power System

The second study by Kumar & Chebiyam (2012) [20] uses the *generalized network flow model* to solve the Integrated Thermal Power System problem. This system involves several energy systems such as the electric power system and the fossil fuel networks (coal, diesel and natural gas) which are strictly interconnected.

The idea is to find the optimal allocation of energy resources to find the electricity demand subject to physical constraints. The model includes energy source nodes, energy transformation nodes, energy storage nodes, energy demand nodes and their interconnections through arcs.

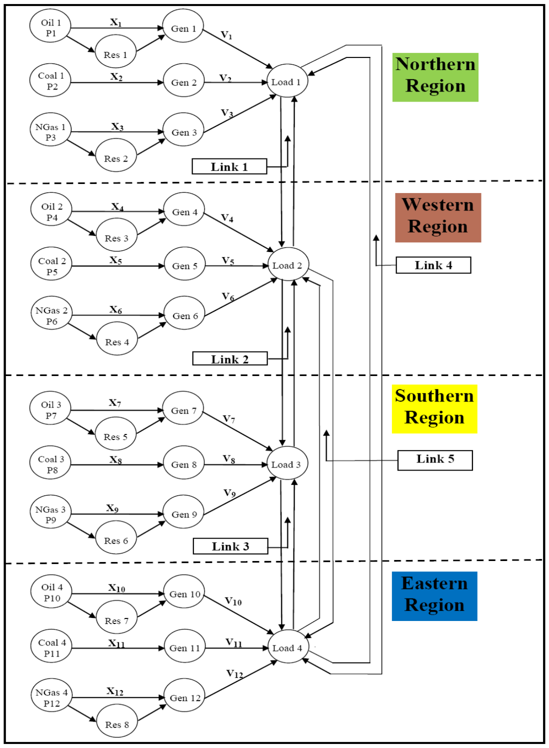
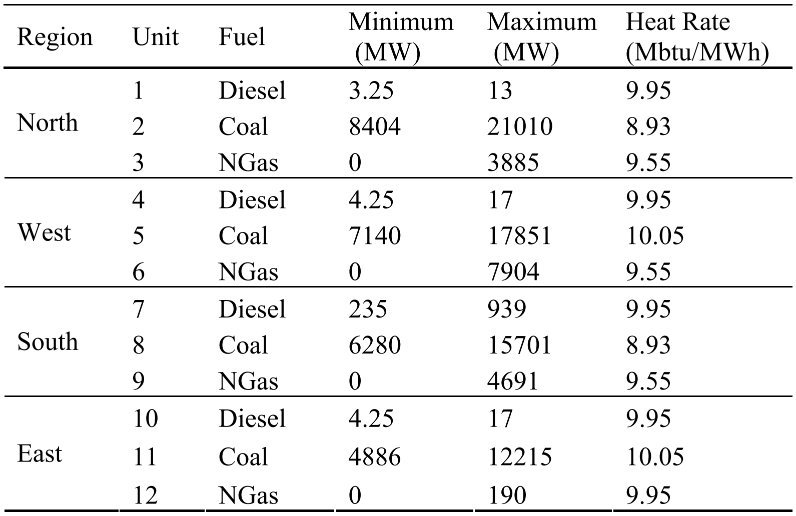


Figure 4.26: India Thermal Power System Model [20].

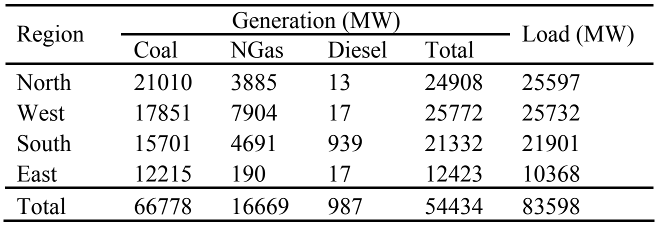
### India case study

Figure 4.26 illustrates a thermal power system model for India. Fossil fuel resources, which include coal, diesel and natural gas, are identified by nodes $\{P_1, P_2, \ldots, P_{12}\}$; storage facilities for fossil fuels are represented by $\{Res_1, Res_2, \ldots, Res_8\}$; electricity generation which is given by generators $\{Gen_1, \ldots, Gen_{12}\}$ and the loading of four regions is represent by $\{Load_1, \ldots, Load_4\}$.



| Region | Unit | Fuel | Minimum (MW) | Maximum (MW) | Heat Rate (Mbtu/MWh) |
|---|---|---|---|---|---|
| North | 1 | Diesel | 3.25 | 13 | 9.95 |
|  | 2 | Coal | 8404 | 21010 | 8.93 |
|  | 3 | NGas | 0 | 3885 | 9.55 |
| West | 4 | Diesel | 4.25 | 17 | 9.95 |
|  | 5 | Coal | 7140 | 17851 | 10.05 |
|  | 6 | NGas | 0 | 7904 | 9.55 |
| South | 7 | Diesel | 235 | 939 | 9.95 |
|  | 8 | Coal | 6280 | 15701 | 8.93 |
|  | 9 | NGas | 0 | 4691 | 9.55 |
| East | 10 | Diesel | 4.25 | 17 | 9.95 |
|  | 11 | Coal | 4886 | 12215 | 10.05 |
|  | 12 | NGas | 0 | 190 | 9.95 |

Figure 4.27: Unit characteristics [20].



| Region | Generation (MW) |  |  |  | Load (MW) |
|---|---|---|---|---|---|
|  | Coal | NGas | Diesel | Total |  |
| North | 21010 | 3885 | 13 | 24908 | 25597 |
| West | 17851 | 7904 | 17 | 25772 | 25732 |
| South | 15701 | 4691 | 939 | 21332 | 21901 |
| East | 12215 | 190 | 17 | 12423 | 10368 |
| Total | 66778 | 16669 | 987 | 54434 | 83598 |

Figure 4.28: India thermal generation and load for 2010 [20].

| Region | Unit | Fuel | Fuel cost | Fuel storage cost | Heat value |
|---|---|---|---|---|---|
| North | 1 | Diesel | * | 1$/barrel | 143500 Btu/gallon |
|  | 2 | Coal | $40/ton | ** | 11500/Btu/lb |
|  | 3 | NGas | * | 0.1$/Mcf | 1000 Btu/cf |
| West | 4 | Diesel | * | 1$/barrel | 143500 Btu/gallon |
|  | 5 | Coal | $40/ton | ** | 10200/Btu/lb |
|  | 6 | NGas | * | 0.1$/Mcf | 1000 Btu/cf |
| South | 7 | Diesel | * | 1$/barrel | 143500 Btu/gallon |
|  | 8 | Coal | $40/ton | ** | 11500/Btu/lb |
|  | 9 | NGas | * | 0.1$/Mcf | 1000 Btu/cf |
| East | 10 | Diesel | * | 1$/barrel | 143500 Btu/gallon |
|  | 11 | Coal | $35/ton | ** | 10200/Btu/lb |
|  | 12 | NGas | * | 0.1$/Mcf | 1000 Btu/cf |

Figure 4.29: Fuel characteristics [20].

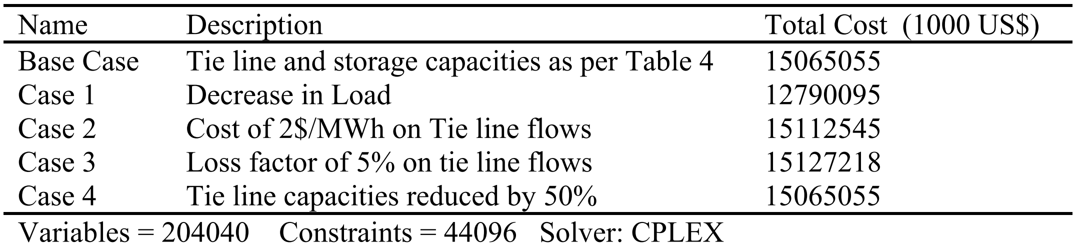
| Name | Description | Capacity |
|---|---|---|
| Tie Line 1 | West to North Link 1 | 5000 MW |
| Tie Line 2 | South to West Link 2 | 3800 MW |
| Tie Line 3 | East to South Link 3 | 3650 MW |
| Tie Line 4 | East to North Link 4 | 11650 MW |
| Tie Line 5 | East to West Link 5 | 6950 MW |
| Res1 | Diesel storage for Gen1 | 3000 barrels |
| Res2 | Gas storage for Gen3 | 13000 Mcf |
| Res3 | Diesel storage for Gen4 | 3000 barrels |
| Res4 | Gas storage for Gen6 | 26000 Mcf |
| Res5 | Diesel storage for Gen7 | 170000 barrels |
| Res6 | Gas storage for Gen9 | 16000 Mcf |
| Res7 | Diesel storage for Gen10 | 3000 barrels |
| Res8 | Gas storage for Gen12 | 650Mcf |

Res1 through Res8 are storage facilities for Diesel and Natural Gas. Gen stands for Generator Unit.

Figure 4.30: Tie line and storage capacities [20].

The mathematical formulation and the procedure to solve the network flow model of the India Integrated Thermal Energy System is the same of the above section. The input data file has been generated using MATLAB and it includes node and arc data, bounds on flows, capacity, per unit costs and time-variant parameter related to fuel costs. On the other hand the solution file contains optimal energy flows and nodal prices.

In table 4.31, India Energy System optimization study is proposed for five different Cases.



| Name | Description | Total Cost (1000 US$) |
|---|---|---|
| Base Case | Tie line and storage capacities as per Table 4 | 15065055 |
| Case 1 | Decrease in Load | 12790095 |
| Case 2 | Cost of 2$/MWh on Tie line flows | 15112545 |
| Case 3 | Loss factor of 5% on tie line flows | 15127218 |
| Case 4 | Tie line capacities reduced by 50% | 15065055 |

Variables = 204040 Constraints = 44096 Solver: CPLEX

Figure 4.31: Results [20].

### 4.3.3 Stochastic fuel costs in a generalized network flow model of Integrated Energy Systems

The models presented above assume that all information are known with certainty in advance. However, in the energy system there are several uncertainties due to different factors such as weather, equipment failures, international political events, wars, transportation problems as well as electricity generation and demands. As a result the uncertainty may cause higher costs to satisfy energy demands and also large-scale disruption of energy supply. To provide an accurate forecasting energy flow system that can be used by decision makers, it could be useful to include uncertainty in the model parameters and thus to study their effects by using stochastic programming [25]. The crucial concept behind stochastic programming that could be useful in this specific context is that of *recourse*. Recourse is the ability to take corrective actions after a random event has taken place. The most studied stochastic programming models with recourse are the two-stage linear programs. The basic idea of the two-stage stochastic programming is that optimal decisions should be based on data available at the time the decisions are made and should not depend on future observations. In particular decision maker takes some action in the first stage, after which a random event occurs, affecting the outcome of the first-stage decision. Then, a recourse decision can be made at the second-stage in order to compensate for the random event effects experienced after the first-stage decision.

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[25] See: Kall, Peter, Wallace, Stein W.(1994). Stochastic programming. Wiley-Interscience Series in Systems and Optimization.

Researchers Mulvey and Vladimirou [26] have developed a specified stochastic programming to networks by dividing nodes and arcs into separate sets corresponding to the stage to which they belong. More over they illustrated a scenario algorithm to maintain the network structure when decomposing a large-scale problem into small sub-problems.

### Solving the stochastic problem via the deterministic equivalent

In order to investigate the impacts of fuel cost and demand, it could be useful to represent these quantities as discrete random variables. This assumption is common in stochastic programming models solved with the two-stage approach.

The cost per unit flow on a fuel acquisition arc is now a random variable that can be represented as $\Pr\{c_{ij}=c_{ij}(k)\}=\mathrm{p}_{cij}(k)$, $k=1, \ldots,$ K. The electricity load (supply) modeled as a demand node can be represented by $\Pr\{b_j=b_j(l)\}=\mathrm{p}_{bj}(\mathrm{l})$, $l=1, \ldots,$ L. Given $m$ random cost variables and $n$ random demand variables in the model, we ca define a scenario $s \in S$ as an $(m+n)$ vector of values that occur jointly with probability $\pi_s$.

The application of the two-stage approach to a generalized network flow problem imply a division of all the arcs into two sets: the first-stage arcs and the second-stage arcs. In the first-stage is decided the flows on the set of first-stage arcs, then the value of uncertain variables is revealed and finally the flows on the second-stage arcs are optimized. We can represents the sets

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[26] See: Mulvey J. M., Vladimirou H.. Solving multistage stochastic networks: an application of scenario aggregation. *Networks*. 21:619-643, 1991.

as follows:

- *$M_1$: set of arcs at first-stage decision, when all parameters are deterministic;*
- *$M_1$': set of arcs at first-stage decision, when some parameters are stochastic;*
- *$M_2$: set of arcs at second-stage decision.*

In addition we can distinguish between first-stage flows $x_{ij}=e_{ij}$, $(i,j)\in M_1\cup M_1$' *and the second-stage flows* $y_{ij}=e_{ij}$, $(i,j)\in M_2$. *We can define* $\Delta^{+}{}_{i}=\{(i,j\in M\}$ *and* $\Delta^{-}{}_{i}=\{(j,i)\in M\}$.

- $N_1=\{i\text{: }\Delta^{-}{}_{i}\}$:
- $N_2=N\backslash N_1$.

## Mathematical formulation

The subproblem for scenario $s \in S$ can be represented as follows:

$$\min f_s\ (x(s),y(s))=\sum_{(i,j)\in M1}\mathrm{c}_{ij}\mathrm{x}_{ij}(\mathrm{s})+\sum_{(i,j)\in M'1} c_{ij}x_{ij}(s)+\sum_{(i,j)\in M2} c_{ij}y_{ij}(s)$$

subject to $$\sum_{(i,j)\in\Delta^{+}i}\mathrm{x}_{ij}(\mathrm{s})-\sum_{(j,i)\in\Delta^{-}i}\mathrm{r}_{ji}\mathrm{x}_{ji}(\mathrm{s})=\mathrm{b}_i \qquad \forall i\in N_1$$

$$\sum_{(i,j)\in\{\Delta_i^+\cap M_1\}} x_{ij}(s) - \sum_{(j,i)\in\{\Delta_i^-\cap M_1\}} r_{ji}x_{ji}(s)+$$

$$\sum_{(i,j)\in\{\Delta_i^+\cap M_1'\}} x_{ij}(s) - \sum_{(j,i)\in\{\Delta_i^-\cap M_1'\}} r_{ji}x_{ji}(s)-$$

$$\sum_{(i,j)\in\{\Delta_i^+\cap M_2\}} y_{ij}(s) - \sum_{(j,i)\in\{\Delta_i^-\cap M_2\}} r_{ji}y_{ji}(s) \quad \forall i \in N_2$$

$$l_{ij} \leq x_{ij}(s) \leq u_{ij} \qquad \forall (i,j) \in \{M_1 \cup M'_1\}$$

$$l_{ij} \leq y_{ij}(s) \leq u_{ij} \qquad \forall (i,j) \in M_2$$

We have to remark that the solution procedure all scenario must be considered together and the values of the first-stage decisions must be the same for all scenarios, thus $x$(s)=$x$(s')=$z$, $\forall s$, s'$\in S$; $s \neq s'$. The overall problem to minimize expected cost can be represented by the following deterministic equivalent:

$$\min \qquad \sum_{s\in S} \pi_s\ f_s(z,\ y(s)) = \sum_{(i,j)\in M1} \mathrm{c}_{ij}\mathrm{z}_{ij}(\mathrm{s}) +$$

$$\sum_{s\in S} \pi_s\ [\sum_{(i,j)\in M'1} \mathrm{c}_{ij}(\mathrm{s})\ \mathrm{z}_{ij}(\mathrm{s}) + \sum_{(i,j)\in M2} \mathrm{c}_{ij}(\mathrm{s})\ \mathrm{y}_{ij}(\mathrm{s})]$$

$$\text{subject to:} \quad \sum_{(i,j)\in\Delta+i} \mathrm{z}_{ij} - \sum_{(j,i)\in\Delta-i} r_{ji}z_{ji}(s)=b_i \qquad \forall i \in N_1$$

$$\sum_{(i,j)\in\Delta+i}\cap\{M_1\cup M'_1\}z_{ij} - \sum_{(j,i)\in\Delta-i}\cap\{M_1\cup M'_1\}r_{ij}\ z_{ij} +$$

$$\sum_{(i,j)\in\{\Delta+i}\ \cap M_2\}y_{ij}(\mathrm{s}) - \sum_{(j,i)\in\{\Delta-i}\ \cap M_2\}r_{ji}y_{ji}(s)=b_i(s)$$

$$\forall i \in N_2;\ \forall s \in S$$

$$l_{ij} \leq z_{ij}(s) \leq u_{ij} \qquad \forall (i,j) \in \{M_1 \cup M'_1\}$$

$$l_{ij} \leq y_{ij}(s) \leq u_{ij} \qquad \forall (i,j) \in M_2;\ \forall s \in S$$

By solving this problem we obtain a feasible solution $(z,\ y(S))$ for each scenario *s*. It is important to underline that since the objective is the expected value of objective functions for each scenario, all the scenarios are considered together.

### Rolling two-stage procedure

When we apply such a model in reality it's impossible to obtain a good solution by assuming simply that all data are known in advance. Decision makers have to found their decisions on forecasts future costs for example. For this reason a stochastic model has been proposed in [40].
The stochastic problem as advanced above has to be solved using the two-stage approach, see Appendix A. The problem that arises in the two-stage approach is that all uncertain elements are revealed at the beginning of the *second-stage*, so to overcome it we can apply the two-stage approach repeatedly in a rolling procedure, which can be seen in the following figures.

The procedure illustrated in the figures above is of a 4-period problem. Figure 4.32 is the first-stage, and all the remaining periods are the second-

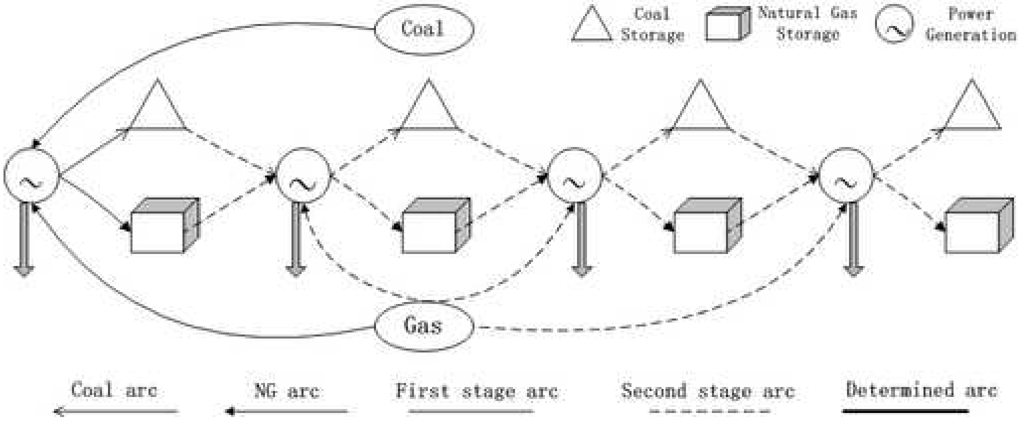

Figure 4.32: Rolling two-stage approach: first period [40].

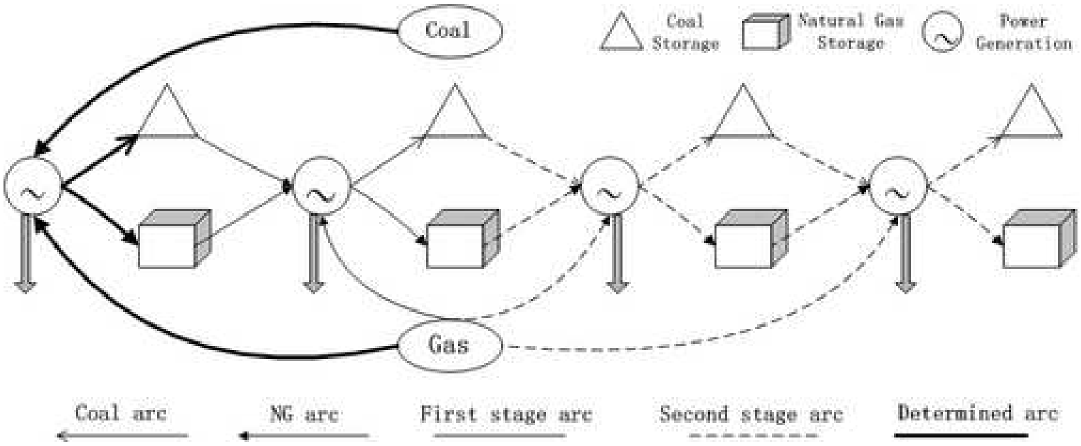

Figure 4.33: Rolling two-stage approach: second period [40].

stages at the beginning. In this phase one set of fuel cost forecasts is used to generate scenarios. In the second period, as shown in Fig.4.33, the decision variables of the first period are removed and shifted to period 2, that now becomes the first-stage, while periods 3 and 4 are on the second-stage. When new information comes, the fuel cost forecast is adjusted. The procedure

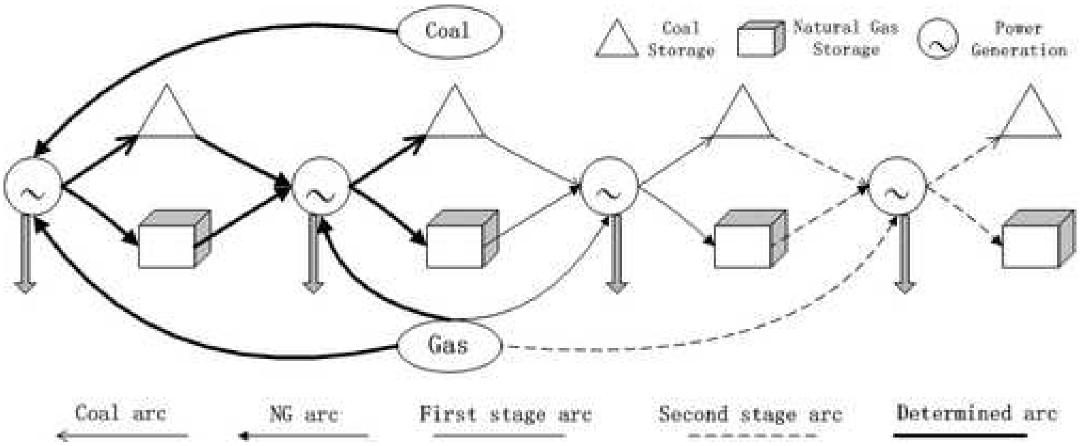

Figure 4.34: Rolling two-stage approach: third period [40].

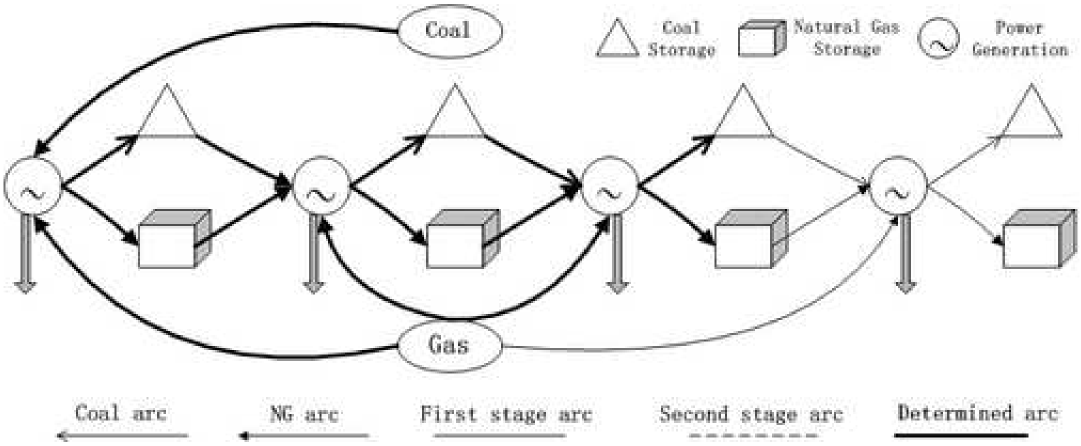

Figure 4.35: Rolling two-stage approach: fourth period [40].

continues until the last period Fig.4.35, until no uncertainties remain, and the new problem that has to be solved is a deterministic one as shown in Fig.4.36.

To use the rolling procedure, it is required to generate scenarios with predicted fuel costs. For example for U.S., EIA provides a monthly updated

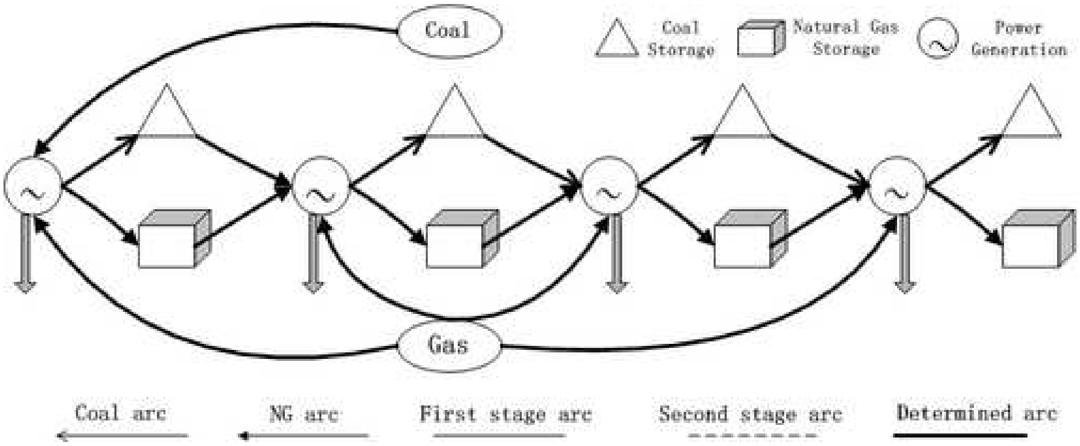

Figure 4.36: Rolling two-stage approach: Complete procedure [40].

short term energy estimate[27], which can be used with high reliability by analysts.

**Solutions**

For the purpose of presenting the solutions to the stochastic problem and comparing them with those of the deterministic model I illustrate the numerical study of [40] for the problem of Fig.4.37.

This is a single period network model of an Integrated Energy System with two coal suppliers, two natural gas suppliers, five generation plants and two electricity demand centers. Here the uncertainty is above all on natural gas prices, indeed the coal contracts are assumed to have long durations, while natural gas contracts tend to be much shorter due to floating in prices. For this reason in order to apply the two-stage approach we have to divide

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[27] http://www.eia.gov/forecasts/steo.

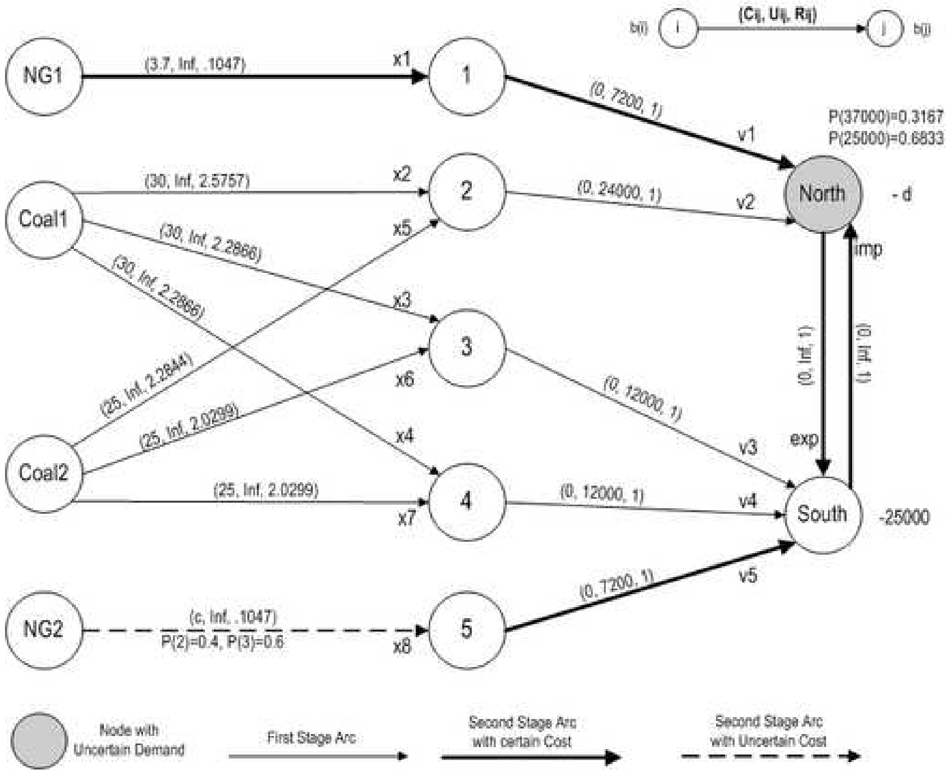

Figure 4.37: Numerical example: Energy system [40].

coal arcs from natural gas and electricity arcs, the first are put in the first-stage, while the seconds are put in the second-stage, thus:

- $M_1$={*(coal1, 2), (coal1, 3), (coal1, 4), (coal2, 2), (coal2, 3), (coal2, 4), (3, South), (4, South)*}
- $M'_1$={*(2, North)*}
- $M_2$={*(natural gas1, 1), (natural gas2, 5), (1, North), (5, South), (North, South), (South, North)*}

The mathematical model presented above can be solved by three different approaches which lead to three different solutions as illustrated in Fig.**??**. The first solution approach is called *Recourse Problem Solution* (RP) since it operates in two steps: in the first step a solution is given before the uncertainty is realized, and then other solutions, the recourse solutions are found after the definition of the random variables. It is obtained by solving the stochastic problem with the rolling two-stage procedure.
The second solution approach is the *Wait and See Solution* (WS) that is obtained by solving simply the subproblem for each scenario $s \in S$. Following this approach we have that the decision maker know which scenario will occur before making the first-stage decision. The difference between the optimal values of RP and WS is called *expected value of perfect information* (EVPI)[28].
The last solution approach, according to [40], is the *Expected Value Solution* (EV), that operates by substituting to each random variable its expected value and then by simply solving the deterministic problem of section 4.3.1.. In other words it replaces the actual price of the deterministic model with the mean value of its forecast over a year.

As we can see from Fig.4.38 the weighted average flows, that is to say the average of set of flows weighted for their probabilities, for each scenario of the RP solution is closer to the WS solution. On the contrary the EV solution differs from RP in particular in the use of South natural gas.

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[28] In decision theory, the expected value of perfect information (EVPI) is the price that one would be willing to pay in order to get access to perfect information. In this specific context it represents how much one would expect to gain if one were told what would happen before making the decision

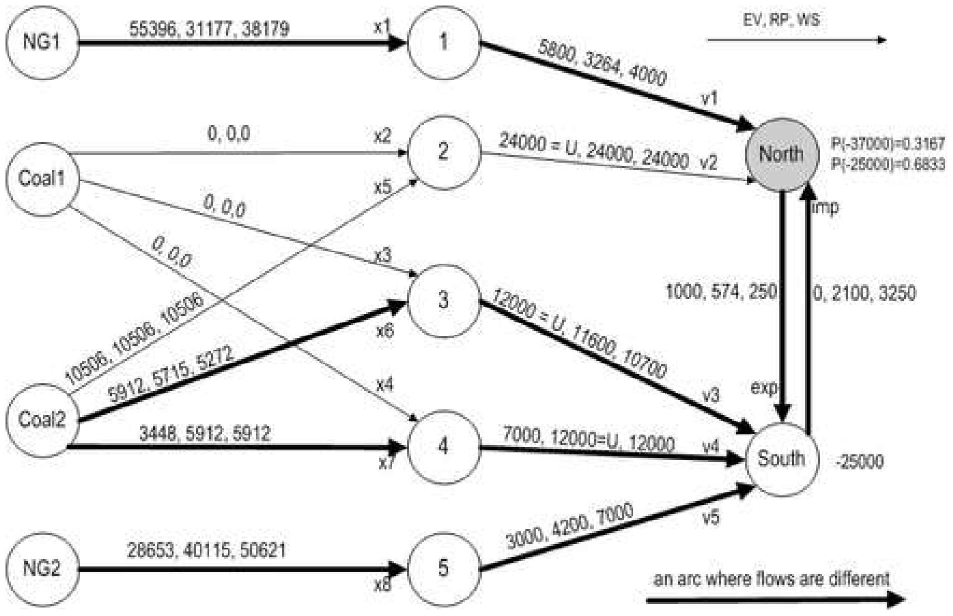
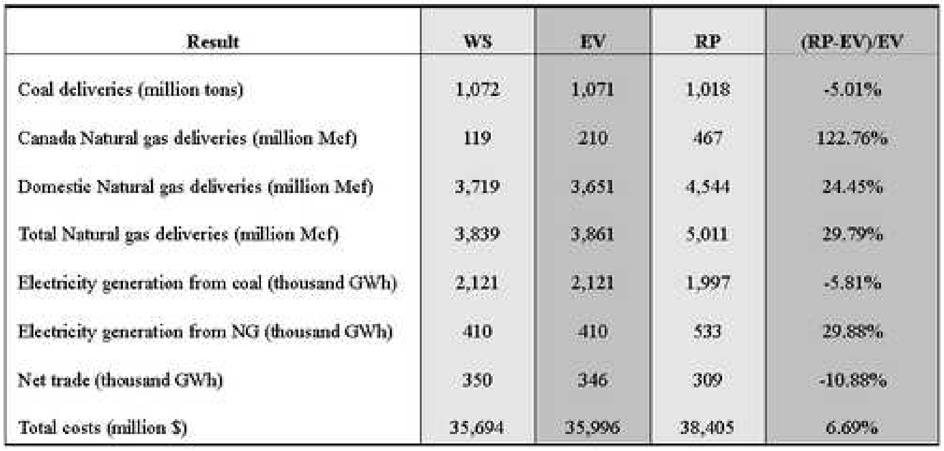


Figure 4.38: Comparison solution approaches [40].



| Result | WS | EV | RP | (RP-EV)/EV |
|---|---|---|---|---|
| Coal deliveries (million tons) | 1,072 | 1,071 | 1,018 | -5.01% |
| Canada Natural gas deliveries (million Mcf) | 119 | 210 | 467 | 122.76% |
| Domestic Natural gas deliveries (million Mcf) | 3,719 | 3,651 | 4,544 | 24.45% |
| Total Natural gas deliveries (million Mcf) | 3,839 | 3,861 | 5,011 | 29.79% |
| Electricity generation from coal (thousand GWh) | 2,121 | 2,121 | 1,997 | -5.81% |
| Electricity generation from NG (thousand GWh) | 410 | 410 | 533 | 29.88% |
| Net trade (thousand GWh) | 350 | 346 | 309 | -10.88% |
| Total costs (million $) | 35,694 | 35,996 | 38,405 | 6.69% |

Figure 4.39: Total flow comparison [40].

As shown in Table 4.39, in the RP solution approach which contains uncertainty, we have that coal deliveries are lower than the other two approaches, while for e.g. natural gas deliveries are higher, especially those imported from Canada. As a result one could conclude that the electricity generated from coal-fired plants would be reduced and more electricity would be generated from natural gas. More over in this way the electricity trade among regions would be decreased by more or less 10% and this represent a saving in the transmission costs.

On the contrary the solutions proposed by WS and EV are more in favor of coal. However it seems that the RP approach is closer to the 2002 data, for this reason it is better appreciated.

## 4.4 Stochastic model and deterministic model

Since most energy investments and operations in the field involve decisions that could be irreversible, a stochastic approach model can be used. Uncertainty indeed, dominates the energy systems all over the world, in particular for what concerns fuel availability and prices, electricity prices and also demand from end consumers.

As we have seen in the previous section the results indicate that RP solution approach, which includes uncertainties, is closer to real data, so the stochastic model could be a better tool to investigate and predict how the whole system would react in front of real world changes. The stochastic model has the capability of forecasting.

On the other hand we have that the deterministic model that has been illustrated in the previous section is unlike to the real world events. Imperfect information over data is always a crucial drawback in analyzing Energy Systems. However the stochastic model seems to be more complicated in the solution procedure since several scenarios have to be generated with for e.i. predicted fuel costs.

Another drawback of using a stochastic model instead of a deterministic one could be that in the first one the solution tends to be more diversified according to which solution approach is applied. Moreover predictions about for e.g. fuel prices may be inaccurate, for this reason it is necessary to study the impact of degree of uncertainty.

We can conclude that the stochastic model is simply an extension of the deterministic model which not only take into account uncertainties, but it is closer to the real world energy systems events.

# Conclusion

In this thesis I have presented how a crucial issue such as energy can be analyzed in a comprehensive way. As a part of national planning effort, most countries need to decide on energy policy, i.e. how to use resources to satisfy their increasing energy needs. Unfortunately, primary energy forms are limited and the demand for energy is very high, so the only solution to the energy problem could be a more efficient use.

Since 1970s a wide variety of models have been developed for investigating energy systems or sub-systems, for number of different purposes, but in particular with the aim of providing a better supply system, given a level of forecast demand. The common denominator of them has been that they view, for example, the fuel system only in terms of contracts, without taking into account all the operations concerning the fuel production, storage, transportation costs and capabilities.

An integrated, interdependent energy system that involves the coal, the gas, the petroleum and the electric sub-systems, could be more significant to study global characteristics. For this reason recently the attention focuses on Integrated Energy System that looks at the overall system efficiency.

In my opinion, a network description could be very powerful, since it captures all the activities involved in the supply chain, avoiding fragmentation that could affect the decision making process leading to potential inefficiency at a global level. Although a representation through a network could be complex, the fundamental advantage of this approach is the ability to apply optimization techniques to analyze different configurations. By using network flow programming model, one can take the advantage of a much faster procedure to obtain some important results from an high dimension problem. The generalized network flow model presented here, allows to look at the economic side of this complex network, because it employs the notion of nodal prices, that are obtained as a by-product of the optimization procedure and identifies the interdependences between the fuel subsystems and the electric subsystem. They represent an economic signal for efficient operations in the energy system.

I think that several improvements have to be applied to best settle an Integrated Energy System, for example we have to take into account the important factor of economic dependence from primary energy suppliers, that some countries are not able to control, maybe uncertainty must be much more investigated. A stochastic generalized network flow model has been presented, however it addresses only the problem of uncertain price and demand, so It could be interesting to include also the uncertainty on the primary energy suppliers.

# Appendix A

# Two-Stage Stochastic Programming

When we speak of *Two-Stage Stochastic Programming*, one should think of the decision process taking place in two stages, namely respectively: the *first-stage* and the *second-stage*. At the beginning, values for the *first-stage* variables $x$ are chosen, thereafter in the *second-stage*, upon a realization of the random parameters, a recourse action is taken in case of infeasibilities. To every recourse action is attached a particular cost. The expected cost of the optimal recourse action is then added to the objective function.
A standard formulation for this problem is:

$$\mathrm{Min}_x c^T x + E[Q(x, \xi(\omega))]$$

$$\text{Subject to } \mathrm{A}x{=}b,\ x \geq 0$$

where $Q(x, \xi(\omega))$ is the optimal value of the second stage problem:

$\mathrm{Min}_x q^T y$

Subject to $Tx+Wy=h$, $y\geq 0$

$x$ and $y$ are vectors of the first and the second stage decision variables. The *second-stage* problem depends on the data $\xi = (q, h, T, W)$, which can be random. The expectation E depends on the probability distribution of $\xi(\omega)$. T is called the technological matrix, while W is the recursive matrix.
At the *first-stage* one should take a "here-and-now" decision $x$ before the realization of the uncertain data $\xi$, while at the second stage after the realization of $\xi$, one should optimize the behavior according to the random parameters. The *second-stage* can be considered as a penalty for the violation of the constraint $Tx=h$.
As presented in above standard formulation problem, at the *first-stage* one should optimize (minimize) the cost $c^T$ of the *first-stage* decision plus the expected cost of the optimal *second-stage* decision. $q^T y$ is the cost of the recourse action, while $Wy$ is the compensation for a possible inconsistency of the system $Tx\geq h$[1].

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[1] For more details see: [26], chapter 12.

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