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Essays on Interconnectedness and Systemic Risk

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Abstract

This dissertation contains two papers about systemic risk and interconnectedness.

Defuse the Bomb. In Part I we present a simulation model of contagion in interbank networks. We find that the frequency of contagion is non-monotonic in connectivity. We also explore the role of heterogeneity, finding how it interacts with connectivity in affecting contagion risk. In general, high levels of heterogeneity seem to widen the interval of connectivity levels in which contagion is possible. Heterogeneity has, in general, stabilizing effects under the hypothesis of random shocks, while it is detrimental when shocks are targeted to the most relevant institutions. We also find that too-connected-to-fail banks pose higher contagion risk than too-big-to-fail banks. We then put forward a complete interbank model which includes a short-term and a long-term market. Banks also engage in asset-liability management to satisfy capital requirements. We find that the objectives of a micro-prudential and a macro-prudential regulation may be misaligned when banks interact in a complex system. Balance sheet composition, fire-sale losses and capital requirements interact in complex ways in determining the probability of contagion.

TailDep. In Part II we develop a theoretical model of systemic risk defining it as the risk generated *by* and *within* the financial system. The model highlights how systemic risk is a network externality stemming from the dependence structure chosen by institutions in a decentralized equilibrium. Systemic risk can be offset by a stabilization policy which can be optimally funded by a tax based on institutions' centrality. We find that the intensity of the stabilization policy is linked to the leading eigenvalue of the financial network, which then becomes a measure of systemic risk. A t-copula model is then used to estimate the tail dependence (TailDep) network through which we are able to track the evolution of systemic risk and to quantify the systemic importance of financial institutions in the recent years.

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Contents

Introduction

Despite the frequency of economic and financial breakdowns (Reinhart and Rogoff, 2009) and the efforts of scholars and regulators, the concept of *systemic risk* still lacks a unanimous rigorous definition. As noted by eminent authors (Bisias et al., 2012; Hansen, 2013; Taylor, 2010), the term has been employed to indicate a variety of phenomena, sharing the common feature of being system-wide malfunctioning events. As a consequence, a number of indicators aiming at quantifying systemic risk has been developed, ranging from those which focus on tail dependencies across assets (Acharya et al., 2012, 2010; Adrian and Brunnermeier, 2011; Brownlees and Engle, 2012), to those based on network stress testing and contagion via direct credit exposures (Battiston, Puliga, Kaushik, Tasca and Caldarelli, 2012; Upper, 2011).

The recent crisis bursted at the end of a period characterized by high asset prices and low volatility. With the advent of derivative products, financial institutions found a way to hedge their risks, transferring them to those who were believed to be better able to manage them.

Low volatility and hedging derivatives have induced institutions to feel safer. They thus increased their leverage up to a threshold which later resulted to be critical. The so-called volatility paradox became clear, unfortunately, too late. Volatility tends indeed to be very low in periods preceding a systemic breakdown. The result is that, exactly in those phases when systemic fragility is building up, standard risk indicators depict a safe world.

Of course, part of this puzzle may be solved by a causality mechanism, running from low risk indicators to higher leverage. However, the deficiencies of such indicators clearly emerged as they failed to take into account this increased leverage as a factor of risk.

The challenge is, in first place, practical, but it is ultimately caused by theoretical lacunas. The array of indicators mentioned above was developed after the recent crisis. However, two main general criticisms may be addressed to them. First of all, they all rely on publicly available data. Since intermediaries have access to these data, it is not clear why they do not exploit them in order to have a more accurate picture of the systemic risk threatening the economy. Secondly, they all lack a theoretical foundation of systemic risk as a *market failure*.

Among the various interpretations of systemic risk, the more appropriate seems to be the view of systemic risk as an externality originating from market imperfections and direct linkages (Stiglitz, 2009, 2011), eventually resulting in excessive risk taking and risk misallocation.

However, so far, nobody has ever attempted to propose a model capturing these aspects. Instead, two streams of literature have developed in the last years, focusing on separate questions. The first one builds on the early contributions of Bernanke and Gertler (1989), Bernanke et al. (1999), and Kiyotaki and Moore (1997) and explores the dynamics of an economy characterized by financial frictions. Recent contributions (Brunnermeier and Sannikov, 2012; Cúrdia and Woodford, 2009; Gerali et al., 2010; Gertler and Karadi, 2011; Gertler and Kiyotaki, 2010; He and Krishnamurthy, 2012, 2013) have extensively explored the influence of the financial sector on asset prices and output dynamics in settings characterized by imperfect markets and asymmetric information. The other stream of research draws from the science of complex systems and, inspired by the pioneering article of Allen and Gale (2000), investigates the resilience to contagion of networks of intermediaries linked in a web of financial contracts (Acemoglu, Malekian and Ozdaglar, 2013; Acemoglu, Ozdaglar and Tahbaz-Salehi, 2013; Amini et al., 2012, 2013; Battiston, Delli Gatti, Gallegati, Greenwald and Stiglitz, 2012*a*,*b*; Caccioli et al., 2012; Gai and Kapadia, 2010; Iori et al., 2006; Nier et al., 2007).

The first approach has the clear advantage of modeling endogenous interactions among agents and of adding a dynamic dimension to the analysis. The second provides instead a realistic picture of the credit market and allows for extreme levels of heterogeneity.

This thesis is framed in the current debate about systemic risk and macro-prudential regulation and makes steps towards the understanding of systemic risk as a result of the interconnectedness of the financial system.

In Part I, *"Defuse the Bomb: Rewiring Interbank Networks"*, we present a network model of contagion in interbank markets, where we assess which features of a financial network are relevant for its stability. The diversification of traders and their heterogeneity interact in complex ways to influence the probability of a shock to become systemic. We then make a step further with respect to the current network literature by providing a richer and micro-founded model. Here, banks actively manage their short-term exposures in order to meet liquidity demands and regulatory capital requirements. The results highlight the complex interaction between network structure and banks' balance sheet composition, which results in a strong misalignment of micro-prudential and macro-prudential objectives. Indeed we find that what from a micro-prudential perspective is clearly beneficial, e.g. large cash reserves and no fire-sale losses, turns out to have the opposite effect when the financial sector is seen as a complex system of interacting agents.

Part II, *"TailDep for the Measurement of Systemic Risk"*, is instead related to the econometric analysis of financial markets aiming at quantifying systemic risk. Despite the limits of market data as input for systemic risk measurement, financial markets remain a crucial source of information regarding investors' sentiment and trading strategies. Thus, we are interested in investigating whether markets, more specifically OTC derivative markets, are able to convey an adequate level of information regarding the stability of the financial system. With this purpose, we analyze the CDS spread time series for 108 global financial institutions, quantifying their degree of interconnectedness through statistical techniques. We then put forward a definition of systemic risk, as the risk generated *by* and *within* the system. A micro-founded network model is then used to quantify it and relate it to the intensity of government intervention required to avoid amplification effects. The model highlights how both the systemic risk indicator and the systemic importance of institutions depend crucially on financial interconnections. Systemic risk is a network externality, originating from imperfect information and coordination failures. The model can be estimated using the TailDep methodology, which is based on a copula model, through which conditional probability of default can be inferred. We find that the system has become increasingly connected in the last ten years and we detect the set of the most systemically relevant institutions. The indices of systemic risk and systemic importance are identified in the model and display the expected dynamics.

Part I

Defuse the Bomb: Rewiring Interbank Networks

1 Introduction

The financial crisis of the late 2000s has forced economists, both in the academia and in regulatory bodies, to confront themselves with the role of the financial system's architecture. Facing the defaults of large financial institutions, regulators found themselves uncertain about the consequences that they would have triggered in the entire financial system. Uncertainty about the actual web of financial relationships prevented any reasonable forecast about the eventual path a crisis would follow.

The years preceding the crisis were characterized by low volatility, high asset prices and by a flourishing industry of financial innovation. Meanwhile banks were increasing their leverage and the system was reaching high levels of interconnectedness, both via credit relations and via derivative contracts. The "volatility paradox" illustrated by Adrian and Brunnermeier (2011) and Brunnermeier and Sannikov (2012) describes exactly this situation in which low volatility is associated to, or is even a cause of, higher leverage, but markets fail to take into account such increase in leverage as a factor of risk.

Low volatility, low perceived risks and, additionally, low interest rates, created an incentive for financial institutions to expand their business within the financial sector itself. Haldane and May (2011) argue that two thirds of the growth of banks' balance sheets in the years preceding the crisis were due to increasing intra-financial claims. The interconnectivity of the system reached such levels that a shock, spreading through such a complex system of claims, would have lead to unpredictable consequences (Haldane, 2009).

Moreover, the advent of hedging derivatives created new form of financial linkages, introducing two additional sources of uncertainty. The first is related to the OTC nature of most of them, which, despite providing instruments specifically designed for the needs of traders, makes them difficult to monitor at a system-wide level. The second one concerns the fact that derivative products, in particular credit derivatives, add complexity to the financial system, which has seen the emergence of a shadow banking sector and the creation of connections between parts of the system which had traditionally been separate, e.g. the banking and the insurance industries.

It is thus not surprising that regulators, after the crisis unfolded, shifted their attentions from a micro-prudential approach to financial stability to a macro-prudential one. Macro-prudential regulation should indeed assess the risks the system faces as a whole, even as a result of the endogenous amplification of apparently small shocks Borio and Drehmann (2009), and taking into consideration the general equilibrium effects of bank regulation (Hanson et al., 2011).

The regulatory framework of Basel III tries to address these issues by taking into consideration higher capital requirements for systemically important banks and by instituting a countercyclical capital buffer which would counterbalance the intrinsic pro-cyclicality of a fixed capital requirement.

Despite skepticism remains regarding the actual anti-cyclicality of countercyclical capital buffers (Repullo and Saurina Salas, 2011), any step towards a macro-prudential approach to financial regulation constitutes an advancement for the future of financial stability.

In this paper we intend to contribute to the debate on macro-prudential regulation by assessing which structure of the financial system is more resilient to exogenous shocks, and which conditions, in terms of balance sheet compositions, capital requirements and asset prices, guarantee the higher degree of stability. We use techniques drawn from the theory of complex networks, since they provide a powerful tool to match the empirical properties of real-world interbank markets.

Our approach to financial markets, seen as complex systems in which agents locally interact between themselves, highlights crucial points for its regulation.

At first, we focus on a standard model of contagion through interbank exposures which has been popularized by Gai and Kapadia (2010) and subsequent contributions, and find that the knife-edge property of diversification persists under a variety of assumptions regarding the architecture of the financial system, its connectivity, the heterogeneity of exposures and heterogeneity of size of traders. Connectivity is, indeed, both a risk sharing device and a risk amplification factor. The probability of observing systemic crises is non-monotonic in connectivity, reaching a peak for intermediate values, while the severity of contagion episodes, when they happen, worsen as connectivity increases. This leads to robust-yet-fragile systems, in which contagion is a rare event but when it happens, it involves the entire system. We also find that systems of heterogenous institutions are more stable to random shocks, confirming the conjecture of Haldane (2009). However, heterogeneity poses high risks when too-big-to-fail or too-connected-to-fail banks are distressed. Our results demonstrate how the contagion risk stemming from their default is particularly high and that connectivity matters more than size as far as contagion risk is concerned.

Then, we provide a richer model of contagion which includes both losses from direct credit exposures, as in the previous framework, and liquidity shocks deriving from defaulting banks and capital requirements. We model a short-term interbank market which clears at each time step according to a perfect information equilibrium, in which agents take into account their liquidity needs, the liquidity shocks coming from their creditors and the ability of their debtors to repay. In this complex framework a clear misalignment of micro-prudential and macro-prudential objectives emerge. The most interesting role is certainly played by liquidity reserves and fire-sale prices. Larger cash reserves always worsen the stability of the system, since they allow banks to keep lower capital buffers. As for the fire-sale price, we find that fire-sale losses induce a more prudent behavior of creditors. Indeed fire-sale losses make debtors more likely to be illiquid. When creditors seek to obtain the desired amount of liquidity, the illiquidity of a debtor will induce them to increase their demand of liquidity to other debtors until the desired amount is obtained. This process is thus likely to cause the closure of short-term exposures with other non-illiquid banks, removing channel of transmission of shocks. In this sense we cast new light on the challenges regulators face when designing an appropriate set of micro-incentives for macro-stability and contribute to the debate on the proper set-up of regulatory requirements (Hanson et al., 2011; Myerson, 2014).

After reviewing, in Section 2, the literature related to our contribution, in Section 3 we present an established model for the analysis of contagion risk and asses how the network configuration, the heterogeneity of the system and its connectivity interact in determining the stability of the market. More specifically, we use two network models for the financial system, the Erdős and Rényi (1960) random graph model, which results in a system in which banks have a very similar number of connections, and a fitness model, which instead results in very heterogenous structures, so that the distribution of the number of connections has a power-law tail decay. We also assess how contagion risk changes depending on whether banks evenly distribute their claims to their creditors or they assign heterogenous weights to various creditors. In the latter case "contagious links" (Amini et al., 2012), i.e. large exposures relative to the capital buffer, are more likely to emerge, making the system more fragile. For the Erdős and Rényi (1960) network we also assess the role of size heterogeneity when exposures are heterogenous. Larger banks are indeed both a source of instability when distressed, but they also have larger capital buffers to absorb shocks.

In Section 4 we put forward a complex model which combines the previous contagion dynamics with a sequential equilibrium market clearing of the short-term market *á la* Eisenberg and Noe (2001). Here, we assess the resilience of the system for varying degrees of connectivity and various hypothesis on balance sheet structures, capital requirements and fire-sale prices, finding evidence of misalignments between micro and macro-prudential regulation. Section 5 concludes with the policy implications. Despite the network terminology is kept at the minimum, we invite the reader which is not familiar with it to refer to Appendix A for elucidations.

2 Related Literature

Networks model have become increasingly popular to study contagion dynamics in financial markets (Allen and Babus, 2008; Chinazzi and Fagiolo, 2013).

The first generation of models of financial networks was primarily concerned with the propagation of liquidity shocks hitting banks' liabilities because of depositors' stochastic liquidity needs. Interbank markets exist because they serve as insurance devices to avoid the cost of excessive cash reserves. Their existence is pareto-superior to autarchy despite the risk of contagion.

In this framework one or more banks face excess liquidity demand that they have to meet by withdrawing interbank deposits. If debtors banks are illiquid and cannot fully pay back their liabilities, creditors may themselves become illiquid, triggering a cascade of defaults. The focus of the first generation of contagion models is on stylized and analytically tractable structures for the interbank markets, e.g. ring networks or fully connected networks, consequently assuming a *deterministic* configuration of market and ruling out any source of uncertainty other than a stochastic liquidity shock. Allen and Gale (2000) and Freixas et al. (2000) adopt the same modeling framework for the financial system of Diamond and Dybvig (1983) and highlight the trade-off that interbank connections impose: on the one hand an increasing number of linkages allows shocks to be diversified-away, but, on the other hand, they constitute channels of transmission. In their models, complete network structures provide the highest level of resilience to contagion and are pareto-superior to an autarchic market. Their results are confirmed by Acemoglu, Ozdaglar and Tahbaz-Salehi (2013) under the hypothesis that the aggregate shock is smaller than the aggregate liquidity of the system. If this is the case, a complete network is more resilient than a ring network. However, for larger aggregate shocks, the two configurations are equally more fragile than clustered structures.

In this stream of literature, a methodological contribution come from Eisenberg and Noe (2001). They introduce the concept of payment clearing vector, i.e. a vector of interbank payments which is consistent with a perfect information equilibrium when banks have to fully repay their interbank and external liabilities. This equilibrium satisfies the three criteria of (i) limited liability of firms, (ii) priority of debt over equity and (ii) proportionality in repayments. The authors then identify the condition for the existence and the uniqueness of such equilibrium vector.

We place under the first generation of models also the ones that have tried to give a micro-foundation to an interbank network. As the aforementioned contributions, their set-up builds on Diamond and Dybvig (1983) and contagion is seen an illiquidity phenomenon. Despite they provide a reacher behavioral model where the network is *endogenous*, the resulting equilibrium structure is nevertheless *determin-*

istic. Babus (2007) show that banks manage to reach a connectivity threshold above which contagion does not happen, while Leitner (2005), Castiglionesi and Navarro (2008) and Acemoglu, Ozdaglar and Tahbaz-Salehi (2013) demonstrate that, because of coordination problems and network externalities, an equilibrium network does not, in general, coincide with that chosen by a (contagion-averse) social planner.

However, their set-up is very stylized, as well as the resulting network structures, which do not resemble those described by a stream of research devoted to the empirical analysis of interbank systems (Bech and Atalay, 2010; Boss et al., 2004; Cont et al., 2013; Iazzetta and Manna, 2009; Iori et al., 2008; Soramäki et al., 2007). Financial networks are extremely sparse, since only a minor share of the possible links actually exist, they present a core-periphery structures, with large institutions concentrating a large fraction of the market, the degree distribution is fat tail, clustering is high and the structure is disassortative. Moreover, the distribution of the link weight is power-law and an extremely high correlation exist between in-degree and out-degree.

The need to better model the set of interbank relations has become particularly relevant after the crisis of the late 2000s. The collapse of large financial entities has often forced policy-makers to intervene to avoid the risks of a wide-spread contagion that may have followed such credit events. *Uncertainty* about the network structure is a key ingredient of the second generation of contagion model, whose focus has also been extended to contagion as a solvency phenomenon.

Among second generation models, contagion is often seen as a consequence of marking to market the exposures to insolvent counterparties. The writing-down of exposures causes an erosion banks' capital buffers, possibly leading to their insolvency. While in the first generation of models cash reserves are a cushion against the propagation of defaults, in the second generation capital plays this role. Despite the lack of micro-foundation (an interbank network is often given by an underlying network formation process) network theory and large scale simulations allow to test the resilience of several configurations of the interbank market.

Findings are often coherent in identifying a non-monotonic relation between connectivity and systemic risk. Gai and Kapadia (2010), Battiston, Delli Gatti, Gallegati, Greenwald and Stiglitz (2012*b*), Battiston, Delli Gatti, Gallegati, Greenwald and Stiglitz (2012*a*) and Caccioli et al. (2012) assess the trade-off between diversification and network externalities in the case of insolvency cascades, while Iori et al. (2006) and Gai et al. (2011) consider the case of contagion through illiquidity. All these studies highlight that the prevailing effect, among diversification and network externalities, depends non-linearly on connectivity. Another feature which is often identified is the robust-yet-fragile property of interbank networks, which arises in certain ranges of connectivity in which contagion is a very rare phenomenon, but, when it happens, it brings the entire system to collapse. Another factor which, together with connectivity, influences contagion risk is the presence of what Amini et al. (2012) define *contagious link*, i.e. exposure exceeding the creditor's capital buffer. The authors show indeed that the size of the sub-graph identified by a sequence of contagious links is the key determinant of contagion for (asymptotically) large networks.

While most authors restrict their attention to network formation models which yield relatively simple and homogenous structures (e.g. regular graphs or Erdős and Rényi (1960) graphs), Amini et al. (2013), Caccioli et al. (2012), Georg (2013), Montagna and Lux (2013) and Roukny et al. (2013) explore contagion dynamics in more realistic cases, in which banks vary markedly in their number of connections and the distribution of the number of counterparties follow a power-law. Amini et al. (2013) argues that heterogeneity is detrimental for systemic stability, while Georg (2013) shows instead that, in case of random defaults, very heterogenous structures are the most resilient. The other contributions show how conclusions may vary depending on connectivity, capitalization, market liquidity and the nature of the exogenous default triggering the cascade (i.e. whether it is completely random, or targeted to the largest or more connected bank).

3 The Role of Network Structure in a Typical Model of Contagion

In this section we explore the role of structural characteristics on the spreading of contagion through an interbank network. We focus on such features as the network connectivity, the degree distribution and the distribution of the link weights.

Since our purpose is to assess the impact of the architecture of the financial system on its resilience, we focus on an established network model where direct credit linkages provide the only channel for contagion.

3.1 A Typical Model of Interbank Networks

We begin by introducing a model which has been extensively studied in the literature (Amini et al., 2013; Caccioli et al., 2012; Gai and Kapadia, 2010; Nier et al., 2007), and which represents the most essential set up to analyze contagion in financial networks. The financial system is modeled as a static network of credit exposures between banks, encompassing any sort of interbank claims, independently on their maturity and liquidity. In this class of models, the crucial assumption is that the time scale of the default cascade is so quick that bank do not manage to react and modify their exposures.

Assets (A_i)	Liabilities (L_i)
	Capital (E_i)
Interbank Assets (A_i^{IB})	Interbank Liabilities (L_i^{IB})
External Assets (M_i)	Customer Deposits (D_i)

Table 1: Example of banks' balance sheet in the baseline model of contagion.

The interbank system is thus represented by a graph $G = (I, V)$. $I = \{1, \ldots, n\}$ represents the set of financial institutions (nodes of the graph) and $V \subseteq I \times I$ is the set of the edges linking the banks, i.e. the set of ordered couples $(i, j) \in I \times I$ indicating the presence of a loan made by bank i to bank j.

Every edge (i, j) is weighted by the face value of the interbank claim, $A_{i,j}^{IB}$. Clearly if $(i, j) \notin I \times I$, then $A_{i,j}^{IB} = 0$.

This set up allows to represent the system of interbank claims by a single weighted n-by-n matrix IB,

$$IB = \begin{bmatrix} 0 & A_{1,2}^{IB} & \ldots & A_{1,n}^{IB} \\ A_{2,1}^{IB} & 0 & \ldots & A_{2,n}^{IB} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n,1}^{IB} & A_{n,2}^{IB} & \ldots & 0 \end{bmatrix}$$

in which interbank assets are along the rows, while columns represent vectors of interbank liabilities. From this matrix, can derive the total exposure of bank i in the interbank market, $A_i^{IB} = \sum_j A_{i,j}^{IB}$, and its total interbank liabilities, $L_i^{IB} = \sum_j A_{j,i}^{IB}$.

The balance sheet structure of banks, in this model, is very simple and stylized, as shown in Table 1. In addition to interbank assets, other items makes up the asset-size of the balance sheet. Since they are not relevant in this modeling framework, we group them in a single asset class that we name external and indicate as M. On the liability side of the balance sheet, a capital buffer E serves as a shock absorber. The remaining sources of funding, apart from the interbank liabilities, do not serve particular purposes in this sort of model, so that we can group them under the name of customer deposits (D).

It is not uncommon, in contagion literature, to add external assets fire-sales when a bank defaults (Gai and Kapadia, 2010; Nier et al., 2007). Following Cifuentes et al. (2005) the usual assumption is that the price, P, decreases exponentially in the quantity of assets sold, X, so that $P = P_0 e^{-\alpha X}$, for a given initial price P_0. However, since the our primary focus is now on the role of the network architecture in the spreading of contagion through direct linkages, this assumption would be superfluous.

The network structure of the market is thus the only shock transmission channel. As soon as a bank is insolvent, i.e. has negative capital because of excessive losses, it is declared bankrupted and its creditors suffer losses equivalent to the face value of their exposures. This is consistent with a short-term horizon, when banks experience close-to-zero recovery rates

The approach, in this framework, is static, since banks do not put in practice any strategy to restore their capital buffer. This assumption will be abandoned in the modeling framework we present in the next section.

3.2 Methodology

The model presented above underlays five different architectures of the financial system that we will stress-test: three employ the Erdős and Rényi (1960) network model (cases *ER1*, *ER2* and *ER3*) and two use the fitness model (De Masi et al., 2006) which generates scale-free degree distributions[1] (cases *FIT1* and *FIT2*). In all these scenarios we keep a fixed ratio of capital to total assets at 4% to preserve comparability with previous contributions, such as Gai and Kapadia (2010) and Caccioli et al. (2012). For the same reasons, the ratio of interbank assets to total capital is fixed at 20%. We leave the assessment of different portfolio compositions to Section 4. The five models that we will analyze differ for the degree of heterogeneity of traders and for the variable which is heterogeneously distributed among banks. Details are provided below.

ER1: Homogeneous banks with homogeneous exposures. The Erdős and Rényi (1960) network model is used to generate interbank connections and banks are assumed to have the same asset size. Interbank claims are evenly distributed among the outgoing links, so that there is no single exposure which is more dangerous than the others. This case reflects an homogenous market, in which only the in- and out-degree vary, and in which banks seek the maximal amount of diversification for a given set of creditors.

ER2: Homogeneous banks with heterogeneous exposures. As in the previous case, all banks have the same asset size and the network is Erdős and Rényi (1960). However, we now allow banks to unevenly distribute their exposures across creditors, in such a way the the link weight is power-law distributed. For each bank we extract a number of weights equal to its out-degree from a power-law distribution, we then assign interbank claims to the links proportionally to the respective weights This represents a scenario in which overexposures may be present, implying the existence of contagious links. The assumption about the distribution of link weights has been made in accordance to empirical findings (Cont et al., 2013; Soramäki et al., 2007).

ER3: Heterogeneous banks with heterogeneous exposures. In this case we allow for heterogeneity also in the asset size. First, an Erdős and Rényi (1960) network is generated. Then, link weights are drawn from a power-law distribution and assigned

[1]Appendix A explains the differences between the two models and their main features.

the links. Total assets are assigned to banks proportionally to their interbank exposures ($A^{IB} + L^{IB}$) in such a way that, on average, interbank assets represent 20% of total assets. The result is a network in which link weights are power-law distributed and asset sizes are power-law distributed as well. The presence of heterogeneity in balance sheet sizes implies the presence of money center hubs, whose ambiguous role as shock absorbers or shock amplifiers will be assessed.

FIT1: Heterogeneous banks with homogeneous exposures. We move towards a more realistic architecture for the financial system, which shows a fat-tail degree distribution. We generate a network using a fitness model, with an exogenous distribution of the fitness parameter that follows a power-law. In order to close the model, the high level of heterogeneity in the connectivity imposes heterogeneity also in the asset size. However, in this case, exposures remain homogeneous, so that no link is more dangerous than any other. Since we first build the network and then assign assets to bank proportionally to their interbank exposures ($A^{IB} + L^{IB}$), the distribution of the asset size is power-law and the fixed ratio of interbank assets to total assets is maintained on average.

FIT2: Heterogeneous banks with heterogeneous exposures. Here, we allow also for heterogenous exposures so that, once the network is generated using the fitness model, we draw the value of the exposures from a power-law distribution with the same exponent of the distribution of the fitness parameter. Total assets are then assigned proportionally to total interbank exposures ($A^{IB}+L^{IB}$) in order to maintain the interbank ratio fixed at 20% on average.

In the simulations we shall vary the average degree of the network, which represents the average number of creditors and debtors a randomly chosen bank has. As such, the average degree is a measure of diversification at the level of single institutions, but, from a systemic point of view, it is also a measure of connectivity, which thus represents channels through which a shock may propagate. As in previous contributions (Caccioli et al., 2012; Gai and Kapadia, 2010), we will exogenously set into default a bank and analyze the steady-state effect of contagion, i.e. the number of defaulted banks when the process of contagion stops. We will focus on the *frequency of contagion*, defined as the probability that at least 10%[2] of the intermediaries default, and on the *extent of contagion*, i.e. average fraction of defaulted banks, provided that it is larger than 10%.

When heterogeneity is introduced (as in cases *ER2*, *ER3*, *FIT1* and *FIT2*) through exogenous power-law distributions, also the exponent of the distribution will be subject to analysis, since it represents the degree of heterogeneity. Indeed, smaller exponents correspond to fatter tails and, consequently, to higher levels of

[2]We do not consider the threshold of 5% as in Gai and Kapadia (2010) and caccioli **?** since it is not robust when heterogeneity is marked.

heterogeneity.

As we mentioned above, the triggering event of contagion is the exogenous default of a bank. We assume three targets of the exogenous shock: (i) *Random*, in which the exogenously defaulting bank is picked up randomly, with equal probability for each bank; (ii) *Too-connected-to-fail*, in which we set into default the banks with the largest number of creditors; (iii) *Too-big-to-fail*, in which the exogenous default hits the bank with the largest asset size, and which is feasible only in those cases in which the asset size is heterogeneous, i.e. cases *ER3*, *FIT1* and *FIT2*.

One of the crucial policy implications of this contribution comes indeed from our results on the role of "large" financial institutions, either in terms of connectivity or in term of asset size. When heterogeneity is introduced, the system presents potential "too-connected-to-fail" and "too-big-to-fail" banks. On the one hand, they may have detrimental effects because of their large exposures in the interbank market. On the other hand, their magnitude allows them to better absorb shocks, since their higher level of diversification or their larger capital buffers prevent them to go bankrupt after the default of a relatively small exposures.

We will present the computational results of 500 simulations performed for a range of average degree considered relevant and plausible. The experiment is repeated for each network structure. Our financial system comprises 1,000 banks.

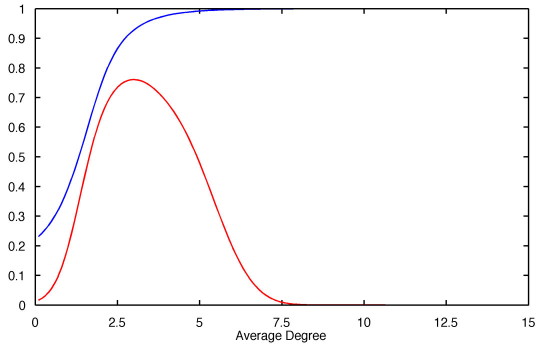
3.3 Results

In this section we will present the results of the simulations conducted on each of the five cases described in Subsection 3.2. In general, the non-monotonic effect of connectivity is confirmed. Heterogeneity in a given variable plays an ambiguous role, depending on the initial shock and on the heterogeneity of other variables, while "too-connected-to-fail" institutions poses higher contagion risk that "too-big-to-fail" banks.

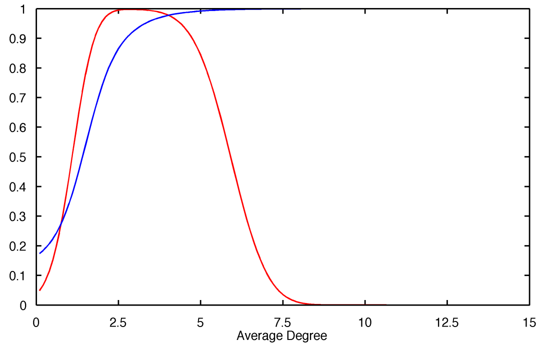
ER1: Homogeneous banks with homogeneous exposures

Figure 1 reports the results for case *ER1*, in which banks have the same size and exposures are evenly distributed across debtors.

Random Default. The non-monotonic role diversification has on financial stability is evident in 1(a). Indeed, for small values of the average degree both the extent and the frequency of contagion is increasing. Then, once the average number of counterparties becomes large enough, diversification prevails and the frequency of contagion becomes negligible. The system has therefore two phase transitions which delimit a *contagion window*: the lower phase transition is sightly below the value of 1 and the upper phase transition is around 7.5. Between these values contagion is



(a) Random Default



(b) Too-Connected-To-Fail

Figure 1: Frequency (red) and extent (blue) of contagion for case *ER1*.

a very frequent event, with probability peaking over 0.7 for average degrees around 3. However, while the frequency of contagion declines beyond a certain threshold, this is not the case for the extent of contagion. Near the upper phase transition the system exhibits the *robust-yet-fragile* property: contagion is a rare event, but, once it breaks up, it affects the entire network.

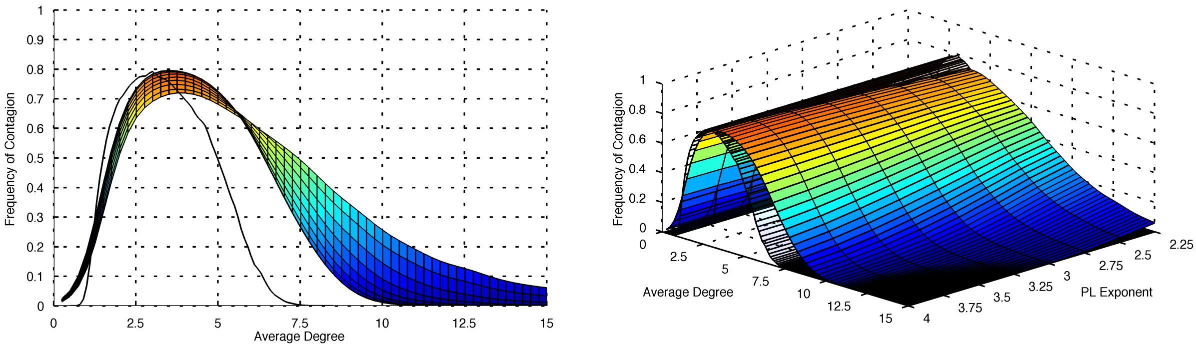
This experiment suggests that the same random shock may have completely different effects depending on the level of connectivity of the interbank market.

Too-Connected-To-Fail. 1(b) shows how the default of the most connected institution is unable to significantly affect the extent of contagion, which closely resemble that of the random attack case. Also the width of contagion window is not affected. However, contagion becomes a certain event for values of connectivity between 2.5 and 4. This highlights that the most connected institutions, even in an Erdős and Rényi (1960) interbank network, pose a relevant contagion risk.

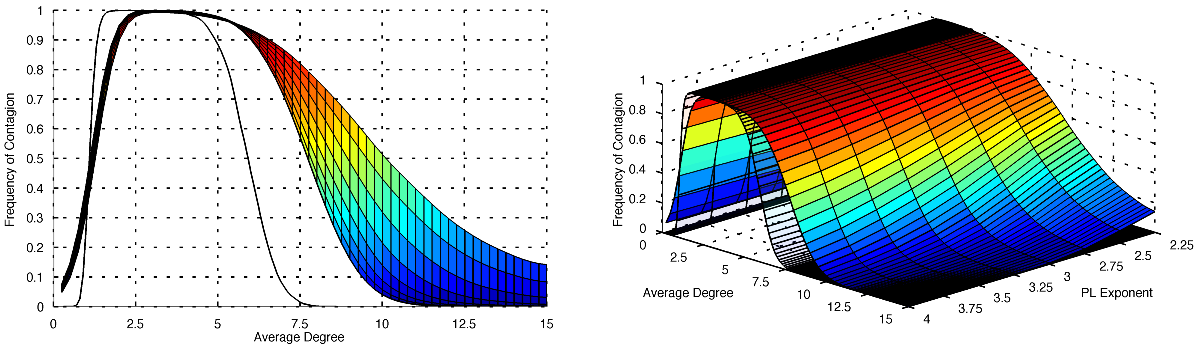
ER2: Homogeneous banks with heterogeneous exposures

Figure 2 reports the results on the frequency of contagion in case *ER2*, in which banks have the same size and link weights follow a power-law distribution. Figure 3 shows instead the results on the extent of contagion. In these figures, opaque plots are added to show and compare the results of case *ER1* in the corresponding scenario.

Random Default. In Figure 2(a) we note how the presence of an uneven distribution of claims induces a much wider contagion window. The effect is particularly evident for low exponents, i.e. for high levels of heterogeneity in the link weights distribution. For values of the power-law exponent below 2.5 the contagion windows does not close even for connectivity levels of 15. This indicates that diversifying by increasing the number of counterparties is ineffective if contagious links persist. Through these relevant links, contagion has still a way to spread. However, for



(a) Random Default

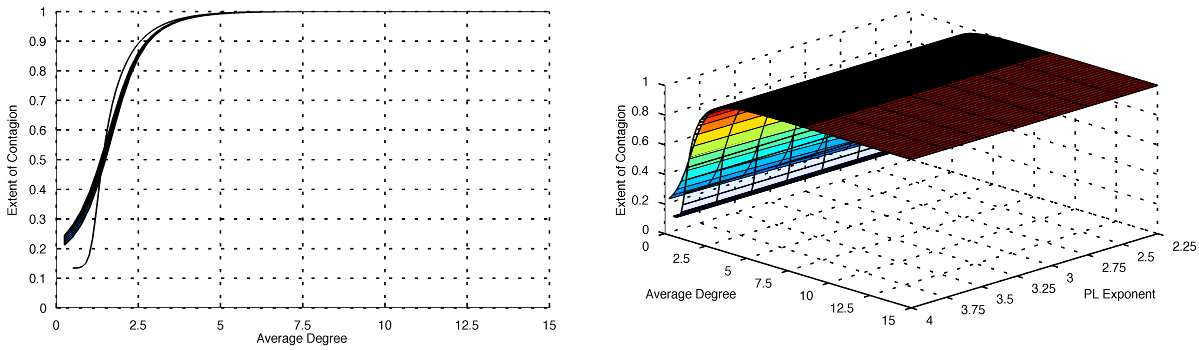


(b) Too-Connected-To-Fail

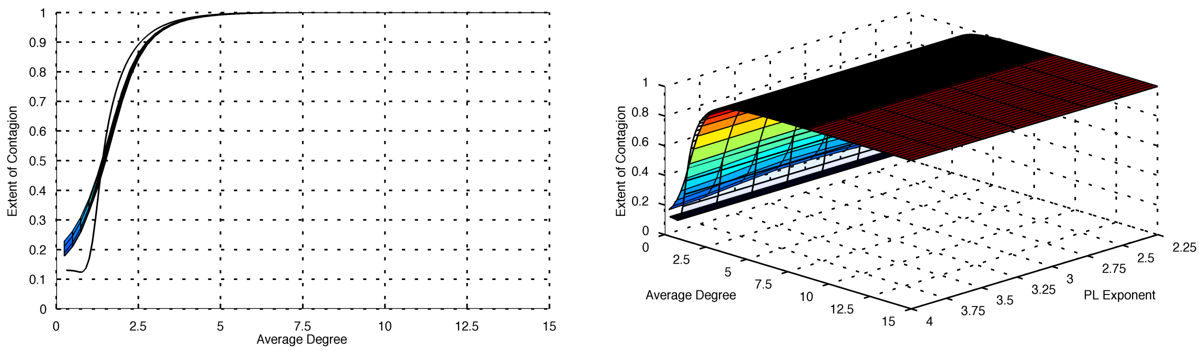
Figure 2: Frequency of contagion for case *ER2*.

small exponents the frequency of contagion is lower than in the *ER1* case when the degree of connectivity is small. This gain in stability is lost for higher degrees of interconnectedness. It seems that the heterogeneity of exposures interact in complex ways with connectivity by making the system more stable for small average degrees, between 1 and 3 or 4.5, depending on the cases, and offsetting the benefits of diversification for large average degrees. Figure 3(a) shows how the extent of contagion follow a patter that resembles that of case *ER1*, irrespective of the power-law exponent, with the exception of small average degrees, were the deviation between the results in the two cases follow the same pattern as in the frequency of contagion.

Too-Connected-To-Fail. When the most connected institution is set into default decreases in the power-law exponent do not bring any positive effect. Figure 2(b) show how the contagion window is wider and the probability of contagion is again one for connectivity levels between 2.5 and 4. The width of the contagion window, as in the previous cases, depends negatively on the exponent. However, as in the case of random default, for value of the average degree between 1 and 4, heterogeneous exposures make the system more stable. Again, with reference to Figure 3(b), the extent of contagion display the familiar pattern in connectivity, with no evident effect stemming from different power-law exponents and whose comparison with case *ER1* resembles the one made for the random default case.



(a) Random Default



(b) Too-Connected-To-Fail

Figure 3: Extent of contagion for case *ER2*.

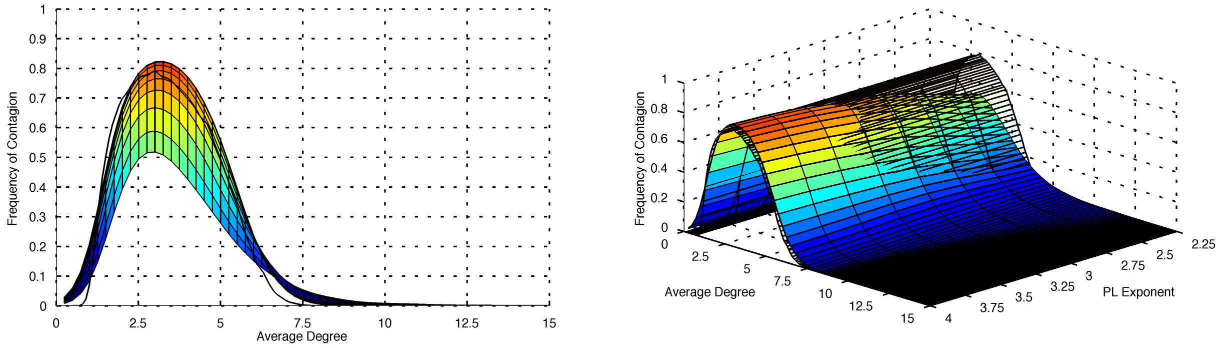
ER3: Heterogeneous banks with heterogeneous exposures

Figure 4 reports the results on the frequency of contagion in case *ER3*, in which banks have the different size and exposures are unevenly distributed across debtors. In them, opaque graphs are added to show and compare the results in the same shock scenario of case *ER1*. The power-law exponent in the plots is the one of the distribution of the exposures, which also the distribution of asset size depends.

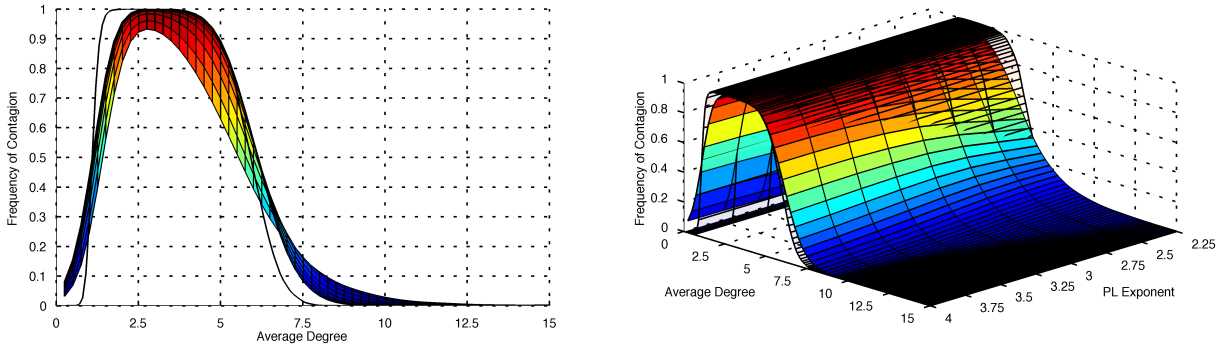
Random Default. In Figure 4(a) we see wider contagion windows than in case *ER1*. However the difference in minimal for larger exponents and it closes always below an average degree of 15. In the center of the contagion widow heterogeneity has instead a positive role. Heterogeneity seems to reduce the probability of contagion. Hence, if the system is not very connected, increasing heterogeneity both in the exposures and in the asset size brings more stability to the system. This may be due to the shock absorption potential of money-center banks and to the heterogeneity of links as well. However, we also see that for larger exponents, the speed at which the frequency of contagion decreases for increasing levels of connectivity is lower, eventually making contagion a possible event also for large average degrees and thus widening, as confirmed by 5(a), the region of average degree in which the system is robust-yet-fragile. However, there is an intermediate region of average degree in which the extent of contagion in case *ER3* is less than in case *ER1*.

Too-Connected-To-Fail. Size heterogeneity seems to have a positive effect also when the most connected bank is set into default. Indeed, we see from Figure 4(b) that for small values of the exponent, the frequency of contagion is reduced with respect to case *ER1*. Contagion widows are also much tighter than in case *ER2*, despite being wider than case *ER1*. We note that for average degrees between 1 and 6, case *ER1* is always more stable that case *ER1*, for any exponent choice, both with regard to the frequency and the extent of contagion.

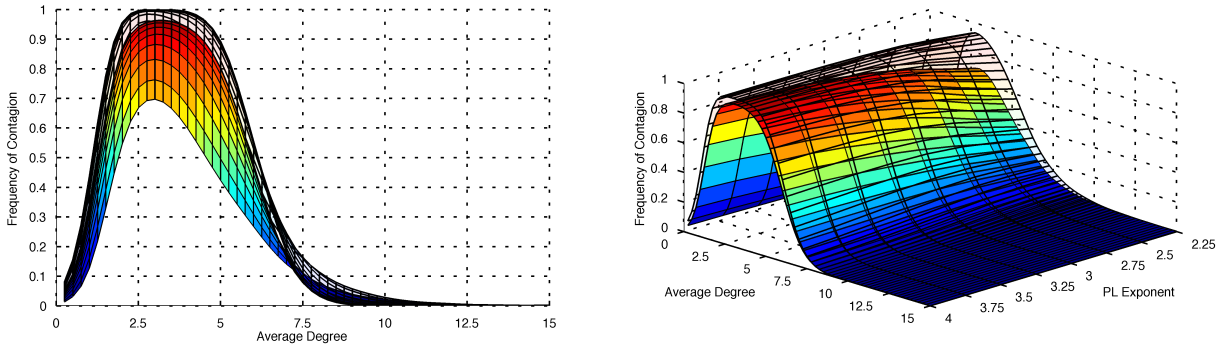
Too-Big-To-Fail. Figure 4(c) reports the contagion results when the biggest bank is set into default and compares it with the result of the "too-connected-to-fail" default in this case, which is, for easiest visualization, reported in opaque in Figure 4(c). The effects of the largest institution are severe than those stemming from the default of the most connected one. In particular, for high levels of heterogeneity in claims, and thus size, the frequency of contagion decreases, and the difference is particularly marked near the peak of the frequency of contagion. The key message is that size alone matters less than connectivity in spreading contagion. Hitting several banks with possibly small losses has higher potential for contagion than hitting fewer with larger losses. The extent of contagion shown in Figure 5(c) does not seem to be affected in a relevant way by the initial default of the largest or most connected bank.



(a) Random Default

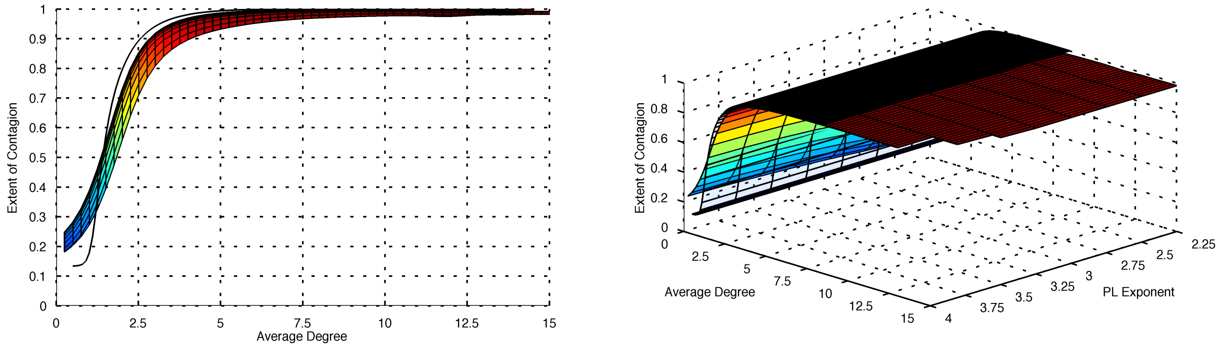


(b) Too-Connected-To-Fail

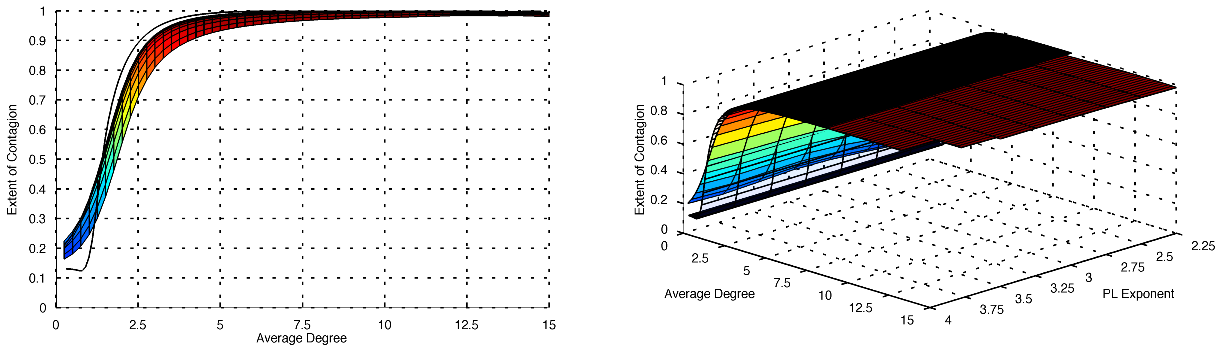


(c) Too-Big-To-Fail

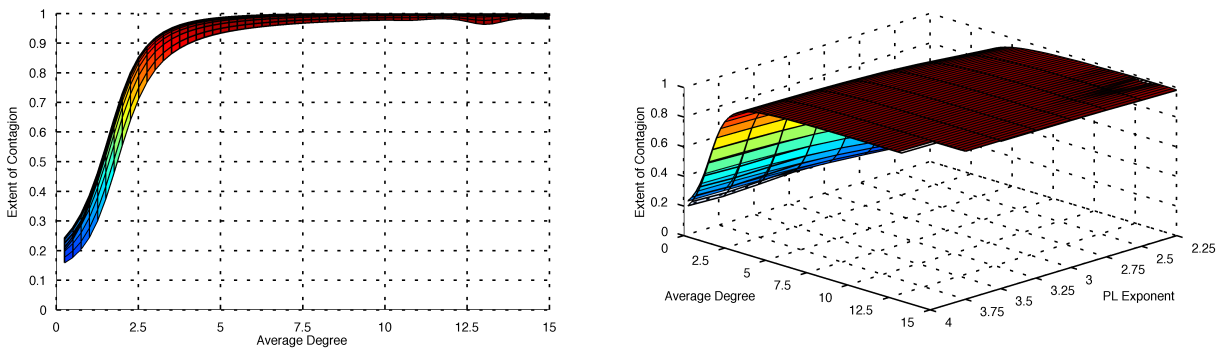
Figure 4: Frequency of contagion for case *ER3*.



(a) Random Default



(b) Too-Connected-To-Fail



(c) Too-Big-To-Fail

Figure 5: Extent of contagion for case *ER3*.

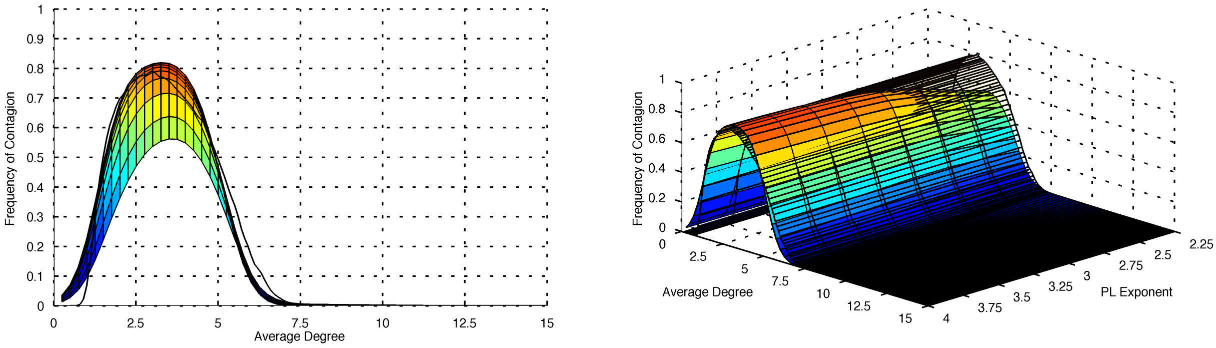
FIT1: Heterogeneous banks with homogeneous exposures

We now consider a scale-free distribution for the degree and homogeneous exposures. The distribution of the asset size is also power-law and proportional to the total interbank exposures. Figure 6 and 7 show, respectively, the frequency and the extent of contagion, and compare the results with the corresponding attack scenario of case *ER1*, with the only exception of the too-big-to-fail case which is compared with the too-connected-to-fail-one.

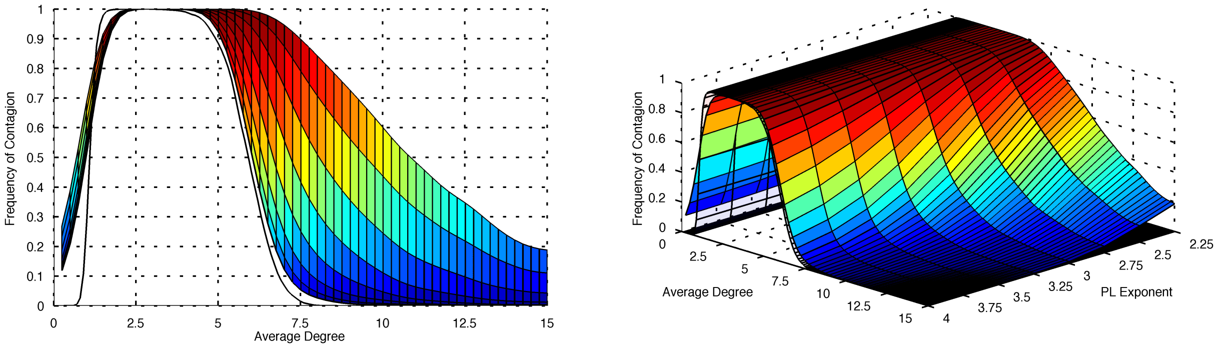
Random Default. In Figure 6(a) we clearly see the beneficial role played by an heterogeneous distribution of the number of counterparties. Indeed the frequency of contagion, for small exponents, is clearly lower than in the *ER1* case in most of the contagion window. Moreover, for values of average degree corresponding with the peaks of the frequency of contagion, smaller exponents have a marked positive effect on the stability of the system. However, the contagion window becomes larger and the structure remains robust-yet-fragile even for larger values of the average degree, no matter which degree of heterogeneity we consider. We also see from Figure 7(a) that the extent of contagion is higher than in the *ER1* for very small values of the average degree, possibly reflecting the presence of interconnected hubs which, despite the low connectivity of the network, are connected enough to trigger rounds of defaults. The extent of contagion is however lower for intermediate values of connectivity.

Too-Connected-To-Fail. All the beneficial effects of heterogeneity are lost if we assume that the most connected institution is set into default. Despite Figure 7(b) shows that the extent of contagion is not strongly affected, the frequency of contagion is now extremely high even for large average degrees and increases together with the heterogeneity of banks. For power-law exponents larger than 3, the contagion window do not close even for average degrees as large as 15.

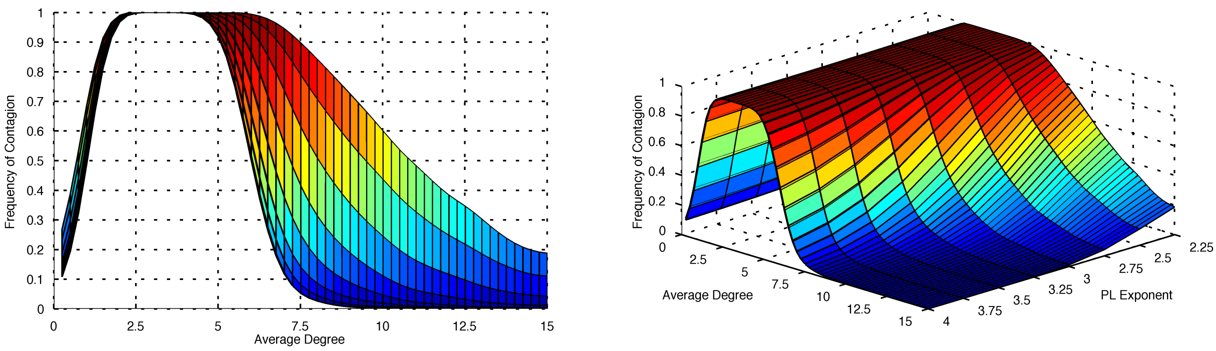
Too-Big-To-Fail. Figure 6(c) and 7(c) show the results when the largest institution is set into default and compare them with the *too-connected-to-fail* scenario. In this case the results almost perfectly coincide. This is due to the fact that exposures are homogenous and assets are proportional to total exposures. This makes the too-connected-to-fail banks almost perfectly coincide with the too-big-to-fail ones.



(a) Random Default

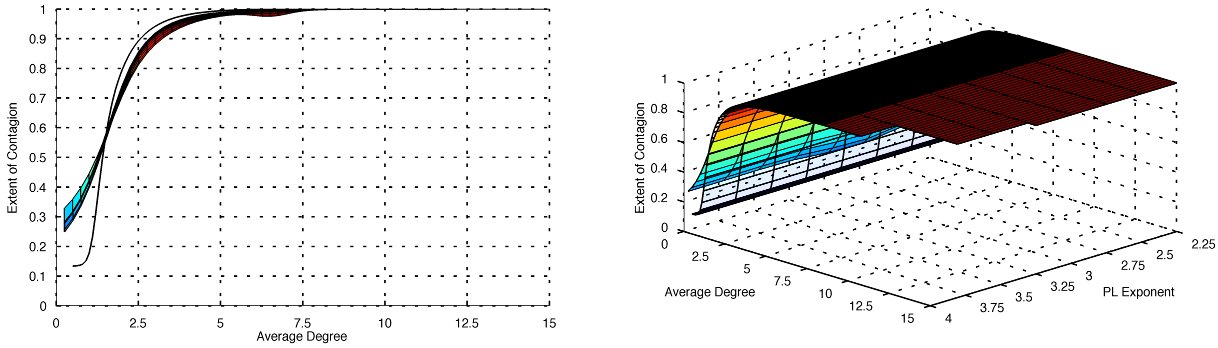


(b) Too-Connected-To-Fail

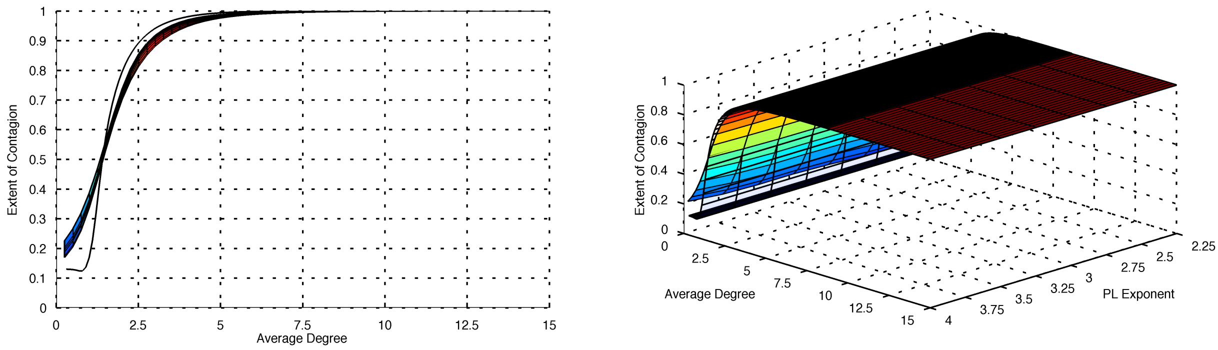


(c) Too-Big-To-Fail

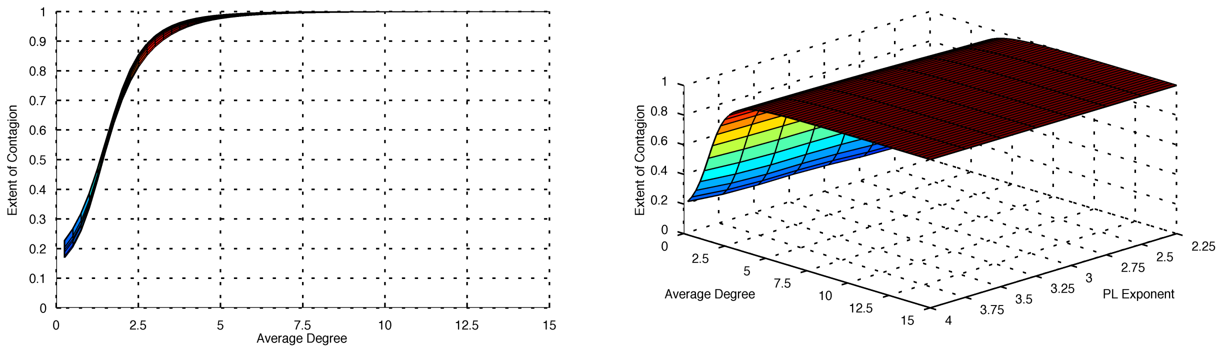
Figure 6: Frequency of contagion for case *FIT1*.



(a) Random Default



(b) Too-Connected-To-Fail



(c) Too-Big-To-Fail

Figure 7: Extent of contagion for case *FIT1*.

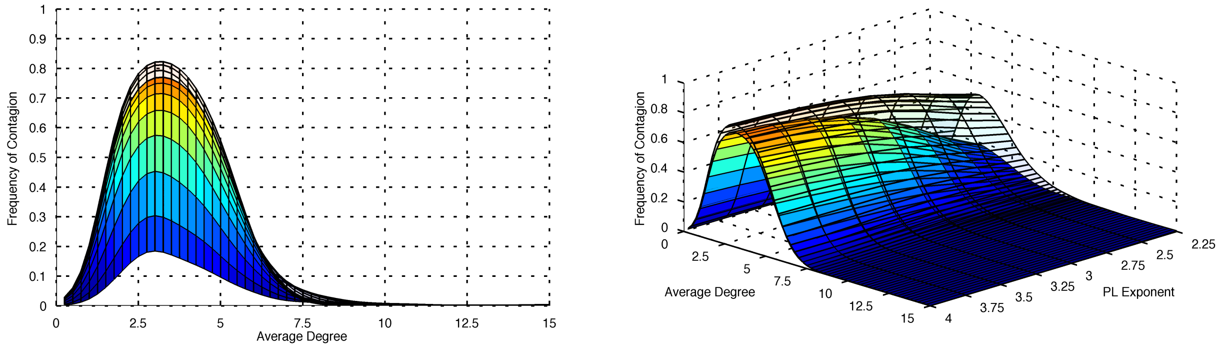
FIT2: Heterogeneous banks with heterogeneous exposures

Finally we allow for heterogeneous link weights, connectivity and asset size, analyzing the model that most resembles real interbank networks. We analyze the results on the fitness model when exposures are drawn from a power-law which has the same exponent of the fitness distribution. Figures 8 and 9 reports the results and compare them with the corresponding attack scenario of case *ER3*, with the exception of the too-big-to-fail scenario which is compared to the too-connected-to-fail one.

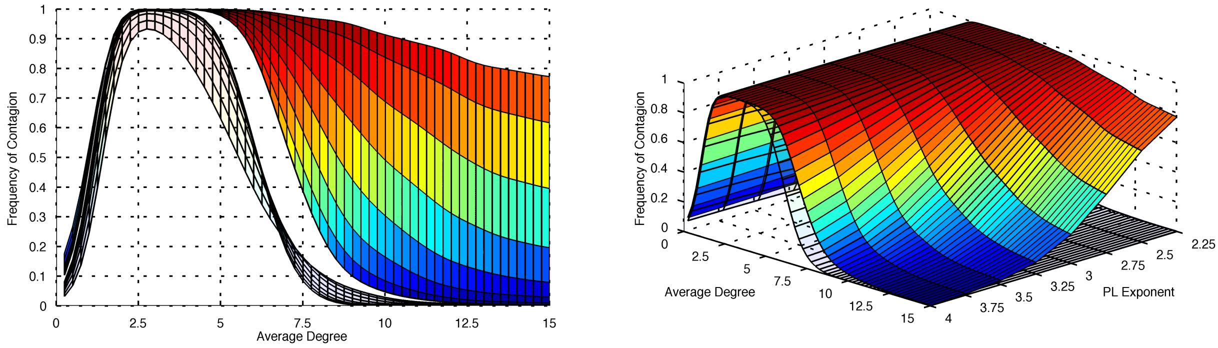
Random Default. Figures 8(a) and 9(a) reaffirm the beneficial role played by heterogeneity of market players when shocks are random. Indeed the frequency of contagion is clearly lower than in case *ER3* and, also, than in case *FIT1*. Heterogeneity plays a stabilizing role despite the presence of contagious links and highly connected hubs. Moreover heterogeneity seems always to be beneficial, since the frequency of contagion, for a given average degree, decreases with the power-law exponent. For exponents as small as 2.25 the maximum frequency of contagion is the lowest seen so far, not even reaching 20%.

Too-Connected-To-Fail. Conclusions are reversed when the most connected bank defaults (Figures 8(b) and 9(b)). In this case the frequency of contagion remains extremely high for large values of connectivity, unless heterogeneity is particularly small. Despite the extent of contagion does not seem to vary from the *ER3* case, the frequency is always higher. The frequency of contagion decreases at an extremely slow speed when heterogeneity is high, making contagion a likely event also for average degrees as large as 15. Less heterogeneity, as determined by larger power-law exponent, allows the contagion window to close more quickly, however, results on the extent of contagion show that episodes of total default still happen even for high connectivity and large exponents.

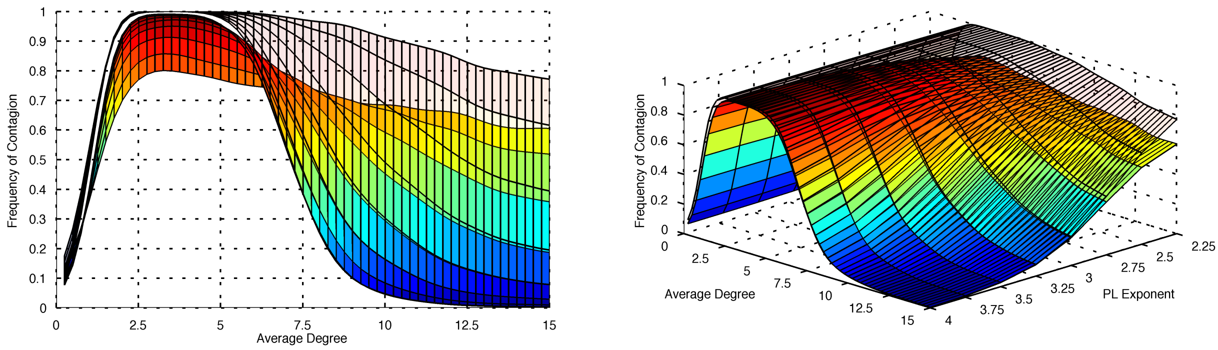
Too-Big-To-Fail. When instead the largest bank is set into default, contagion effects remain severe. However, as shown in Figure 8(c), the largest bank always poses less contagion risk than the most connected one. The heterogeneity of exposures breaks the tight connection between too-big-to-fail and to-connected-to-fail bank that characterizes case *FIT1* and allows to properly assess which of the two matter the most from a systemic point of view. In this scenario heterogeneity seems to play a beneficial role for small values of the average degree, while, for largest values, it prevents the contagion window to close. This result suggest that, in a model which closely resemble a real-world financial network, connectivity matter most than the size of the total assets and of the exposures.



(a) Random Default

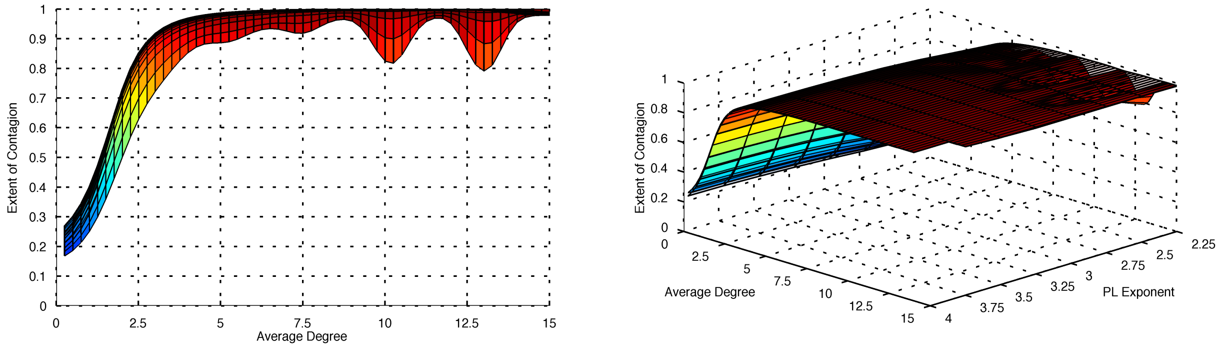


(b) Too-Connected-To-Fail

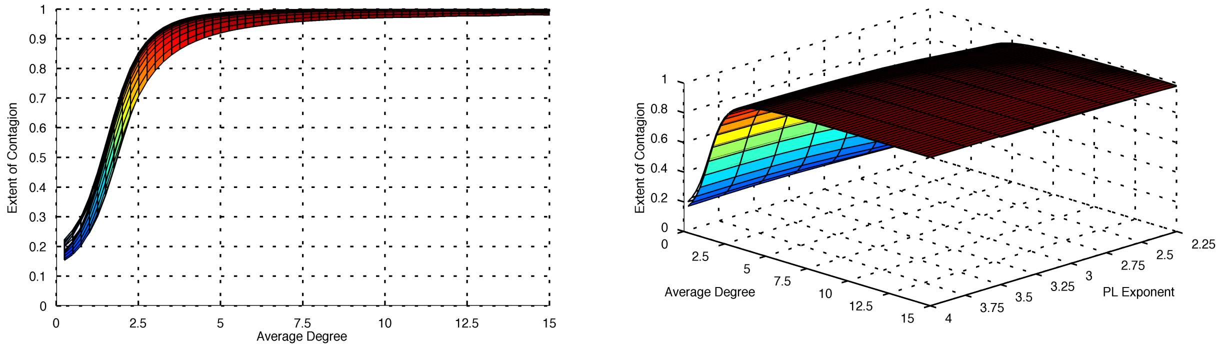


(c) Too-Big-To-Fail

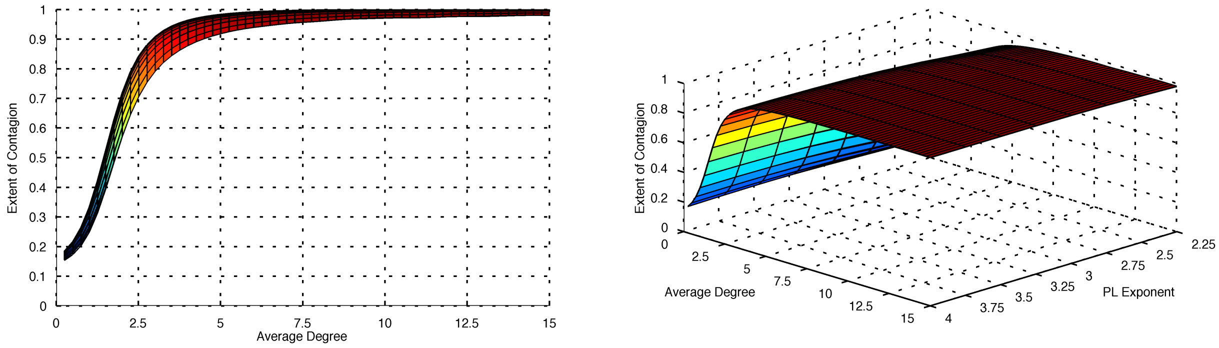
Figure 8: Frequency of contagion for case *FIT2*



(a) Random Default



(b) Too-Connected-To-Fail



(c) Too-Big-To-Fail

Figure 9: Extent of contagion for case *FIT2*

Assets (A_i)	**Liabilities (L_i)**
Liquid Assets/Cash (C_i)	Capital (E_i)
Long-term Interbank Assets (A_i^l)	Long-term Interbank Liabilities (L_i^l)
Short-term Interbank Assets (A_i^s)	Short-term Interbank Liabilities (L_i^s)
Illiquid Assets/Mortgages (M_i)	Customer Deposits (D_i)

Table 2: Example of banks' balance sheet in the richer model of contagion.

4 Capital Requirements and Asset-Liability Management

In this section we put forward a more complete model of contagion, which encompasses losses due to direct credit exposures, bank runs and assets fire-sales. The interbank market now comprises two layers, one for the long-term exposures and the other for the short-term exposures. While the former cannot be modified during the default cascade, the latter may undergone changes due to liquidity hoardings. Montagna and Kok (2013) is the contribution that is closest to ours in terms of the modeling framework, even though also Battiston, Delli Gatti, Gallegati, Greenwald and Stiglitz (2012*a*) and Roukny et al. (2013) consider the presence of different maturities in interbank lending.

This enhanced modeling framework allows to explore the interplay between the network architecture of the market and the balance sheet structure of intermediaries, highlighting their complex interaction that may lead to misalignments between micro- and macro-prudential policies.

4.1 A Richer Model of Contagion and Bank Runs

The financial system comprises two networks, one consisting of the short-term exposures and the other consisting of the long-term ones. Banks' balance sheets include now more items than in the previous section and an example is reported in Table 2. The various categories of assets are in fixed proportions to total assets according to the following rations, valid for all banks i:

$$\lambda = \frac{C_i}{A_i}$$
$$\iota = \frac{A_i^l + A_i^s}{A_i}$$
$$\tau = \frac{A_i^s}{A_i}.$$

Once the size of the total assets A_i is known, the parameters λ, ι and τ allow to uniquely identify the composition of bank i's assets.

Consistently with the previous section, we indicate with A^l_{ij} the long-term exposure of bank i to bank j, with A^s_{ij} the short-term exposure, while L^l_{ij} and L^s_{ij} indicate the long-term and short-term borrowing, respectively, of bank i from bank j.

Capital has to satisfy a constraint given by the regulators. Capital is initially assigned to banks in such a way to amount to a given fraction, e^*, of the risk-weighted assets, RWA_i, plus a capital buffer β, expressed in percentage terms. Hence

$$E_i = (e^* + \beta)RWA_i$$

in which the RWA are defined as

$$RWA_i = \gamma_C C_i + \gamma_{IB}(A^l_i + A^s_i) + \gamma_M M_i \tag{1}$$

where γ_C, $\gamma_I B$ and γ_M represents the weights assigned to liquid assets, interbank loans and illiquid assets respectively. Typically $\gamma_C = 0$, and this will the assumed henceforth.

Dynamic Capital Adjustment

While the above relations describe the initial state of the system, the model itself is truly dynamic, and a first source of time-variation of the system is the need of banks to satisfy the minimum capital requirement $E_i(t) \geq e^* RWA_i(t)$, which induces them to modify the composition of their assets, by withdrawing interbank deposits and, in some cases, by selling illiquid assets.

In particular, as soon as a bank experiences losses which erode its capital below the regulatory threshold, i.e. $E_i(t) < e^* RWA_i(t)$, the bank will try to meet the required capital ratio by, first, reducing its short-term interbank exposures and then, if this adjustment is not sufficient, by selling its illiquid assets.

For each bank i we define its regulatory hoarding of cash as:

$$\bar{d}_i(t) = \min\left(A^s_i(t), \max\left(0, A^l_i(t) + A^s_i(t) + \frac{\gamma_M}{\gamma_{IB}} M(t)_i - \frac{1}{e^* \gamma_{IB}} E_i(t)\right)\right) \tag{2}$$

which thus represents the amount of liquidity it has to withdraw from the market in order to maintain the regulatory capital.

If hoarding is not enough, the bank will try to sell its illiquid assets. However, the success of this strategy crucially depends on the fire-sale price of illiquid assets. Assume that the fair value of a unit of illiquid assets is 1, then, when the fire-sale price p is larger than $1 - e^* \gamma_M$, banks will have to engage in regulatory fire-sales for a quantity of

$$\bar{f}_i(t) = \min\left(M_i(t), \max\left(0, \frac{\gamma_{IB}(A_i^l(t) + A_i^s(t) - \bar{d}_i(t)) + \gamma_M M_i(t) - E_i(t)/e^*}{\gamma_M - (1-p)/e^*}\right)\right). \tag{3}$$

If instead $p < 1 - e^*\gamma_M$, banks will never find convenient to fire-sale illiquid assets in order to meet their capital requirements. Indeed, in this case, large price discounts will induce losses which more than offset the reduction of the RWA, and $\bar{f}_i(t)$ would hence be zero.

A bank will try to adjust its RWA as far as it is possible. Once its capital ratio irremediably falls below the minimum requirement, the bank as no other choice that to keep operating with that ratio. Indeed, the only available solution would be to raise capital, which is an action that cannot be pursued in the short time scale of a contagion scenario, which is the focus of our analysis.

Note also that insolvent banks, i.e. those with $E_i(t) \leq 0$, will withdraw all their funds from the short-term market, i.e. $\bar{d}_i(t) = A_i^s(t)$ for insolvent banks.

Equilibrium in the Short-term Market

The dynamic adjustment of short-term exposures trigger runs in the interbank market. We assume that runs happen in a shorter time scale than contagion through counterparty losses. More specifically, we model runs as perfect information equilibria. Eisenberg and Noe (2001) provide the basic set up, which, however, we need to adapt for our framework, in which the regulatory hoarding constitute the initial demand shock.

Moreover, we give more micro-foundation to the clearing algorithm by making rationality assumption on banks' behavior instead of the simplistic one of proportional hoarding with respect to short-term debtors. Indeed, this pure proportionality assumption may artificially lead banks to illiquidity if a debtor is illiquid. Suppose that bank i hoards a quantity d_i of funds, proportionally splitting this amount among its short-term creditors in quantities d_{i1}, $d_{i,2}$, ..., d_{in}. If a debtor j is illiquid its supply of funds to i will be $s_{ji} < d_{ij}$. If the algorithm stops here, it may be the case that also i becomes illiquid, thus reducing the fund it may supply to its hoarding creditors. A more realistic assumption is that, if i is not able to meet its demand for liquidity because of j's illiquidity, it may increase the quantities hoarded from the other debtors, up to the point in which the supply of funds from liquid debtors is enough to meet i's demand for funds.

As soon as a bank j is found to be illiquid, perfect information in the short-term market makes withdrawing all the fund from it a weakly-dominating strategy for all its short-term creditors. However no payments will be made and exposures are marked down to zero, since slow and costly default procedures will be initiated by the supervisors.

It is important to note that runs to illiquid banks happen only when illiquidity is *revealed* by the failure to make a required payment. Indeed, in the short time scale we assume, updated balance sheet information are not made public, and the only source of information are market demand and supply of funds.

To make the analysis more precise we give the following definition of illiquid bank.

Definition 1 (*Illiquid bank*)**.** A bank i is illiquid if the total amount of funds it can raise is not enough to meet the demand for funds $\sum d_{ji}(t)$ of its creditors.

If we define as $\Lambda(t)$ the set of banks which become illiquid at time t, then the amount of funds bank i can raise is given by $C_i(t) + \sum_{j \notin \Lambda(t)} A^s_{ij}(t) + pM_i(t)$.

It is clear from the definition that whether a bank is illiquid crucially depends on whether its short-term debtors are liquid or not.

With this behavioral framework in mind we can define an *equilibrium in the short-term market.*

Definition 2 (*Equilibrium in the short-term market*)**.** An equilibrium in the short-term market is a matrix of liquidity demands, $D^*(t) = (d^*_{ij}(t))^n_{i,j=1}$, where d^*_{ij} is the equilibrium demand for cash of i to j, and a matrix of liquidity supply, $S^*(t) = (s^*_{ij}(t))^n_{i,j=1}$, where s^*_{ij} is the equilibrium supply of cash of i to j, such that:

(i) $\sum_{j \notin \Lambda(t)} d^*_{ij}(t) = \min\left(\sum_{j \notin \Lambda(t)} A^s_{ij}(t), \max\left(\bar{d}_i(t), \sum_j d^*_{ji}(t) - C_i(t) \right)\right) \quad \forall i$

(ii) $\sum_j s^*_{ij}(t) = \min\left(\sum_j d^*_{ji}(t), C_i(t) + \sum_{j \notin \Lambda(t)} s^*_{ji}(t) + pf^*_i(t) \right) \quad \forall i$

(iii) $f^*_i(t) = \min\left(M_i(t), \max\left(\bar{f}_i(t), \frac{\sum_j d^*_{ji}(t) - C_i(t) - \sum_{j \notin \Lambda(t)} s^*_{ji}(t)}{p} \right)\right)$

(iv) demands are proportional to exposures: $\frac{d^*_{ij}(t)}{d^*_{ik}(t)} = \frac{A^s_{ij}(t)}{A^s_{ik}(t)} \quad \forall i \in I, \forall j, k \notin \Lambda(t)$

(v) supplies are proportional to demands: $\frac{s^*_{ij}(t)}{s^*_{ik}(t)} = \frac{d^*_{ji}(t)}{d^*_{ki}(t)} \quad \forall i, j, k \in I$

(vi) $i \in \Lambda(t)$ if and only if $\sum_j s^*_{ij}(t) < \sum_j d^*_{ji}(t)$

(vii) market clears: $d^*_{ij}(t) = s^*_{ji}(t) \quad \forall i \in I, \forall j \notin \Lambda(t)$

Equilibrium demands and supply are then computed by an iterative algorithm as suggested by Eisenberg and Noe (2001).

Condition (i) states that banks' total demand for cash cannot exceed the amount of their short-term interbank assets as that it should be at least as large to include both its regulatory demand and creditor's hoarding exceeding available cash. Condition (ii) indicates that banks' total supply of cash does not exceed the total demand they have to meet and the liquidity they are able to raise via cash, supply of liquidity from short-term creditors and assets fire-sales. Condition (iii) implies that equilibrium fire-sales cannot exceed the available illiquid assets and should be enough to take into account regulatory fire-sales and the liquidity demand of creditors exceeding cash reserves and funds obtained from the short-term interbank market.

Conditions (iv) and (v) are the formalization of the behavioral assumptions we made in our micro-foundation. Condition (vi) is the definition of illiquid bank in equilibrium. Note that this condition, together with condition (v), implies that, for an illiquid bank i, it holds that $s^*_{ij}(t) < d^*_{ji}(t)$ for every creditor j.

The last condition is a market clearing condition with illiquidity, in which the market clears with equality only for liquid debtors, while there is excess demand to illiquid debtors.

The clearing of the market conveys a crucial signal to financial institutions since it revels which banks are unable to make the promised payments. In accordance with current financial regulation, as soon as a bank is found illiquid it does not make *any* payment and is then subject to regulatory supervision.

Since default procedures due to illiquidity are costly and time consuming processes, whose outcome is uncertain, and since shorter maturity does not necessarily imply higher seniority, we assume that all the creditors, both long-term and short-term, of illiquid banks mark-to-market their exposures.

Default Cascade

As in the previous section a first channel of default cascade is losses due to direct counterparty's default, to which we add illiquidity due to bank runs.

Defaulted banks, either because insolvent or because illiquid, induce losses to their direct lenders. They thus find themselves with a reduced capital buffer which, if negative, makes them insolvent.

A reduction of the capital buffer will force banks to adjust their RWA, thus triggering an additional round of runs. When the short-term market clears, new illiquid banks default and, together with the banks that became insolvent in the previous period, trigger another round of insolvencies.

The default cascade continues as long as no additional defaults happen.

We can define as $\Delta(t)$ the set of banks which are in default as of time t. Then, assuming a zero recovery rate in the short run, we may update the exposures of institutions as follows:

$$
\begin{aligned}
&A^s_{ij}(t+1) = A^s_{ij}(t) - d^*_{ij}(t) && \text{if} \quad j \notin \Delta(t) \\
&A^s_{ij}(t+1) = 0 && \text{if} \quad j \in \Delta(t)
\end{aligned}
$$

fort the short-term, and

$$
\begin{aligned}
&A^l_{ij}(t+1) = A^l_{ij}(t) && \text{if} \quad j \notin \Delta(t) \\
&A^l_{ij}(t+1) = 0 && \text{if} \quad j \in \Delta(t)
\end{aligned}
$$

for the long-term.

Certainly, defaulted banks continue to have liabilities to their creditors. However, as default procedures start, the timing and the amount of the final obligations become exogenous to our model. Hence, according to the update rules, we remove defaulted banks from the system by wiping-away all their interbank liabilities.

Illiquid assets are updated according to

$$
M_i(t+1) = M_i(t) - f^*_i(t)
$$

while cash reserves follow the rule

$$
\begin{aligned}
&C_i(t+1) = C_i(t) + \sum_{j \notin \Delta(t)} s^*_{ji} + p f^*_i(t) - \sum_j s^*_{ij} && \text{if} \quad i \notin \Delta(t) \\
&C_i(t+1) = C_i(t) + \sum_{j \notin \Delta(t)} s^*_{ji} + p f^*_i(t) && \text{if} \quad i \in \Delta(t)
\end{aligned}
$$

The dynamics of the capital buffer can thus be expressed as follows:

$$
E_i(t+1) = E_i(t) - (1-p) f^*_i(t) - \sum_{j \in \Delta(t)} \left(A^s_{ij}(t) + A^l_{ij}(t) \right) \tag{4}
$$

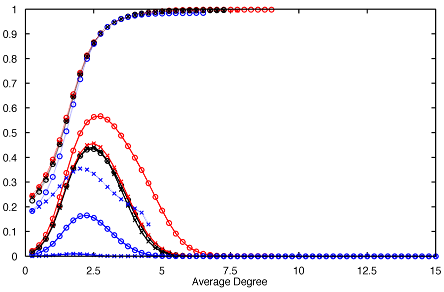
Finally, the set of defaulted banks is updated as follows:

$$
\Delta(t+1) = \Delta(t) \cup \Lambda(t+1) \cup_{\{i \mid E_i(t+1) < 0\}} i.
$$

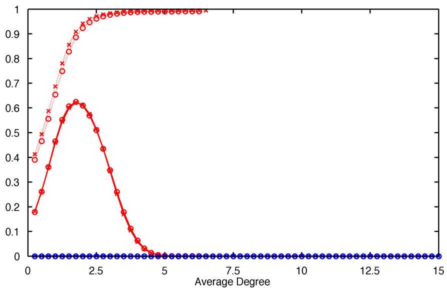
We are interested in the steady-state of the default cascade. i.e. at the state of the system at time t^* when $\Delta(t^*+1) = \Delta(t^*)$.

4.2 Results

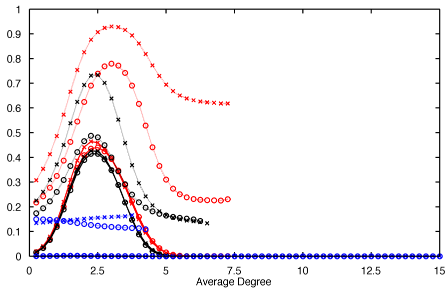
In this subsection we discuss the results obtained for the simplest configuration of the interbank market, i.e. the case of a Erdős and Rényi (1960) network with



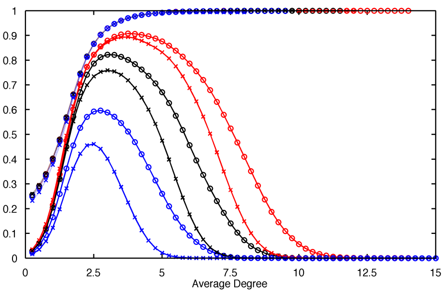
(a) $\lambda = 0.08$, $\tau = 0.02$



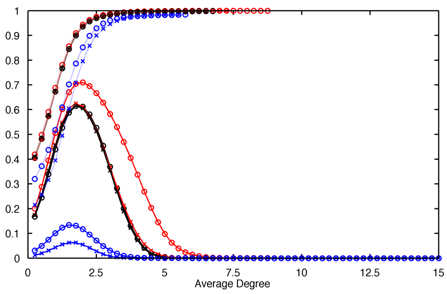
(b) $\lambda = 0.08$, $\tau = 0.10$



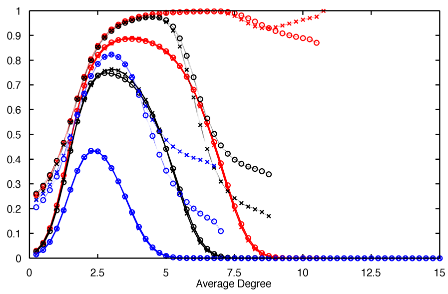
(c) $\lambda = 0.08$, $\tau = 0.18$



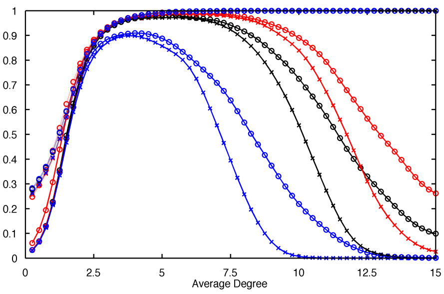
(d) $\lambda = 0.40$, $\tau = 0.02$



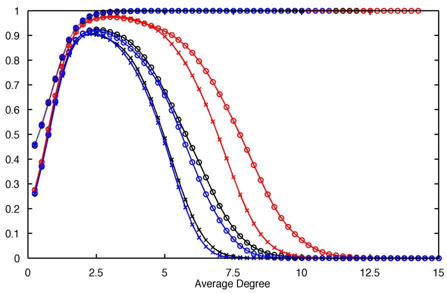
(e) $\lambda = 0.40$, $\tau = 0.10$



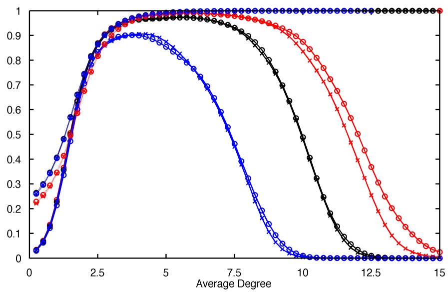
(f) $\lambda = 0.40$, $\tau = 0.18$



(g) $\lambda = 0.72$, $\tau = 0.02$



(h) $\lambda = 0.72$, $\tau = 0.10$

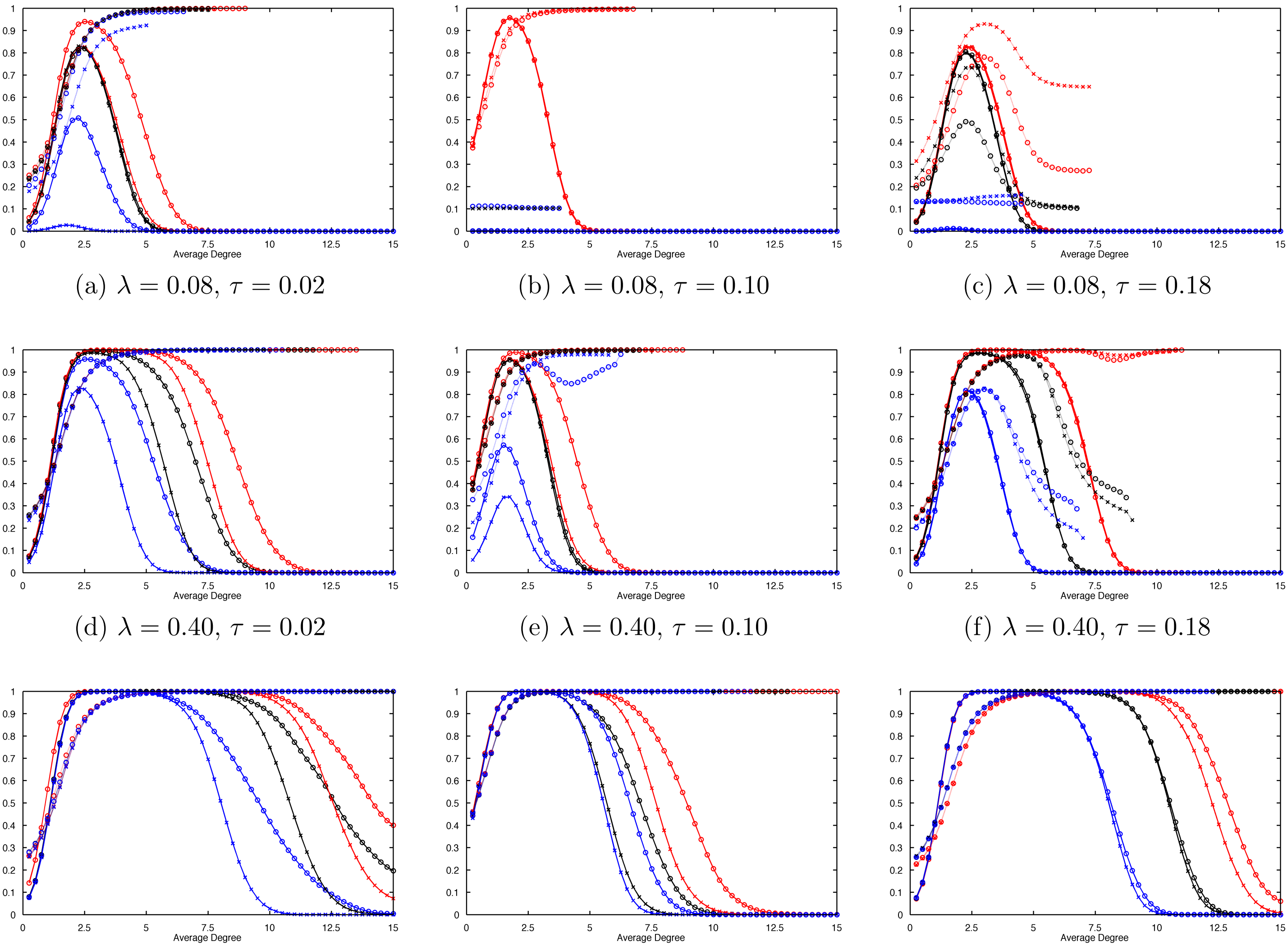


(i) $\lambda = 0.72$, $\tau = 0.18$

Figure 10: Frequency (straight lines) and extent (dotted lines) of contagion for case *ER1* under random initial default. Capital requirements: 6% (red), 8% (black), 10% (blue). Fire-sale price: 1 (∘-marks), 0.5 (x-marks).

homogenous exposures and homogenous asset sizes.

As in the previous section the network includes 1000 banks and results are shown from 500 simulations. In addition to connectivity, we vary the additional parameters of the richer model. We explore different combination of liquid assets in portfolio, λ, short-term exposures, τ, minimum capital ratio, e^* and fire-sale price, p. In all the simulations we keep ι, the share of total interbank exposures to total assets, fixed at 20% and the capital buffer at 2.5%. Risk weights are fixed at $\gamma_M = 0.5$ for illiquid assets, which is the weight assigned by regulators to residential loans, and at $\gamma_{IB} = 1$ for interbank claims, which correspond to the regulatory weight for loans to BBB- institutions, thus implying a conservative scenario. Figures 10 and 11 report, respectively, the results for a random initial default and the too-connected-to-fail



(a) $\lambda = 0.08$, $\tau = 0.02$ (b) $\lambda = 0.08$, $\tau = 0.10$ (c) $\lambda = 0.08$, $\tau = 0.18$

(d) $\lambda = 0.40$, $\tau = 0.02$ (e) $\lambda = 0.40$, $\tau = 0.10$ (f) $\lambda = 0.40$, $\tau = 0.18$

(g) $\lambda = 0.72$, $\tau = 0.02$ (h) $\lambda = 0.72$, $\tau = 0.10$ (i) $\lambda = 0.72$, $\tau = 0.18$

Figure 11: Frequency (straight lines) and extent (dotted lines) of contagion for case *ER1* under too-connected-to-fail default. Capital requirements: 6% (red), 8% (black), 10% (blue). Fire-sale price: 1 (○-marks), 0.5 (x-marks).

initial default.

The Figures immediately highlight the complex interactions of balance sheet composition and connectivity, which we disentangle by discussing the effect of each variable.

Connectivity

Connectivity has the same qualitative effect we found in the previous section. For small values, increasing connectivity increases both the frequency and the extent of contagion, since additional transmission channels more than offset the role of diversification. Its effect on the frequency of contagion is reversed for larger values of the average degree, so that contagion becomes a rarer events due to diversification.

However, with only one exception, the extent of contagion is always increasing in connectivity, making the network robust-yet-fragile for those levels of connectivity at which the frequency is low, but the extent is extremely high.

Minimum Capital Ratio

The effect of higher regulatory capital ratio is unambiguously positive. The frequency of contagion is always reduced by increasing the minimum capital, as well as the extent of contagion, despite the differences are often slight. This was not, however, a trivial result. Indeed, while higher capital ratios provide a larger buffer to absorb losses, they also raise the threshold that triggers runs in the short-term market because of the adjustments of the RWA.

Fire-sale Price

The results from two different assumptions for the fire-sale price, 1 and 0.5, are probably the most counterintuitive and interesting, since they clearly highlight how the interplay between network architecture and behavioral rules can yield unexpected results. Indeed, the frequency of contagion is always lower when banks incur losses as they sell their illiquid assets. The reason for this fact is that banks, when facing fire-sale losses, have higher probability of becoming illiquid. All short-term creditors will thus try to obtain the desired amount of liquidity from other debtor banks. This implies that a larger fraction of credit lines are reduced and, in part, closed. The closure of short-term credit lines with short-term debtors removes possible channels through which contagion may eventually spread, thus grating higher stability to the system.

The beneficial effect of fire-sale losses is more evident when the fraction of short-term interbank claims is small and when diversification is greater, i.e. in those cases in which every short-term link carries a smaller weight and thus is more likely to be removed during a run.

Initial Cash Reserves

As far as contagion through interbank exposures is concerned, a higher share of liquid items in portfolio is detrimental for the stability of the system. Indeed, despite they provide a cash buffer preventing liquidity shocks to spread, they also lower the RWA and thus the initial absolute amount of capital available to absorb losses.

The result is that the most stable configuration is reached when liquid assets are at their minimum. When $\lambda = 0.08$ and $\tau = 0.10$ we clearly have the most stable configuration, in which a capital buffer at least as large as 8% is able to prevent any systemic episode to happen.

Share of Short-term Exposures

The role of the share of short-term exposures seem to be non-linear, possibly displaying an optimal value. Indeed, for each level of λ, an intermediate value of $\tau = 0.10$ makes the system more stable than the smaller value of 0.02 and the larger value of 0.18.

Indeed while it is true that an higher share of short-term exposures induces higher risks of runs and thus of illiquidity, it is also true that runs are a mechanism through which connections are removed, thus removing channels of contagion. Hence, this trade-off at systemic level between liquidity shocks and possibility of removing channels of transmissions seems to yield an optimal intermediate value. When comparing the two extreme values, we note that a $\tau = 0.18$ makes the system more resilient than $\tau = 0.02$, showing how the positive marginal effect of disconnections may be larger than the marginal increase of the risk associated to bank runs.

5 Conclusions

In this paper we explored the interplay between heterogeneity, network structure and balance sheet composition in the spreading of contagion.

In the first part, using an established model of contagion, we have proved that the system presents phase transitions in connectivity. Indeed connectivity is both a driver of contagion, as it provides the channel for shocks to propagate, but it is also an hedge against contagion, via diversification.

Also heterogeneity has an ambiguous role. If heterogeneity regards exclusively the link weights, the main effect is a widening of the interval of connectivity levels in which contagion is possible. This is due to the fact that diversification cannot, in this case, prevent contagious links to exist, which are a necessary condition for contagion to arise.

When size heterogeneity is introduced, also some positive effects are seen. Indeed big banks seems to act as shock absorber, making contagion a less likely phenomenon. Heterogeneity in connectivity provides additional stabilization when the initial default is random. However, this comes with the cost of an extremely high contagion risk when the most connected or the largest institution is initially distressed.

We showed how "too-interconnected-to-fail" banks are more dangerous that the "too-big-to-fail" ones. In our model, despite being very correlate, the two set of institutions do not necessarily overlap. We then proved that the total amount of distressed loans matters less than the number of creditors being initially hit by the default.

In our richer model of contagion, which includes default cascades, endogenous bank runs and asset-liability management, we highlighted the complex interactions between network structure and balance sheet composition. We proved that larger capital requirements are effectively able to stabilize the system, while larger liquid reserves, despite providing a buffer in case of liquidity run, induce banks to keep a smaller amount of capital, thus making them vulnerable to contagion.

The relative weight of short-term and long-term exposures also matters in this framework and an intermediate balance between the two seems optimal. Short-term exposures are indeed both a channel for liquidity shocks, but they can also be easily removed, preventing shock to propagate.

Finally, the role of fire-sales highlight the complexity of this kind of models in which several channels of contagion operate. Indeed, fire-sale losses imply higher risk of illiquidity. Hoarding banks will then seek funds from other non-illiquid banks, reducing their exposures to them and, eventually, leading to a more likely closure of the credit lines. This effectively removes channel for the propagation of contagion. In this sense, fire-sale losses induce a more prudent behavior.

This paper also provides policy suggestions for the regulation of the financial system. The role of "too-interconnected-to-fail" and "too-big-to-fail" institutions in financial markets is ambiguous, since they act as shock absorbers in case of random attack, but pose relevant systemic risk if distressed. Nevertheless, we proved that "too-interconnected-to-fail" banks should be the primary concern for a contagion-averse regulator, since their distress is more likely to trigger systemic breakdowns.

Capital requirements should also be rethought in the light of the trade-offs highlighted by our complex system approach, together with the incentives micro-prudential regulation should set. Indeed, such incentives may be strongly mis-aligned with macro-prudential objectives, if not designed in a systemic perspective. Our paper has indeed clearly highlighted how those conditions that from a micro-prudential point of view are extremely desirable, e.g. larger liquid reserves and no fire-sale losses, may induce, at a systemic level, wrong incentives that translate into systemic fragility.

Appendix

A Network Theory

A network is simply a collection of points connected by links, which we may formalize as a set $G = (I, V)$, where I is the set of vertices (nodes), while V is the set of couples $(i, j) \in I^2$ representing the edges, which may be ordered or unordered, and we shall then speak of directed or undirected graphs respectively.

Any network can be unambiguously represented by an adjacency matrix $A(G)$, whose elements a_{ij} take the value of zero or one depending on whether $(i, j) \notin V$ or $(i, j) \in V$. If the network is undirected the adjacency matrix is symmetric. Moreover, whenever links have different weights, representing different intensities in the connections one may define a weighted matrix $W(G)$ whose elements w_{ij} represent the weight of the link from i to j if a link between them exist, while they are zero if no link is present between them.

A network is a natural representation of an interbank market. Banks represent the nodes of the graph, edges are given by lending relations and their weight is the value of the exposures. By taking this point of view on the financial system, one is able to analyze its properties and its architecture, in order to identify the relevant features for its stability. Indeed we are interested in the network structure of an interbank market for its consequences in the transmission of liquidity shocks and default cascades.

Network Statistics

A first step towards the understanding of the stability of financial systems passes through the analysis of their structure itself. Despite not exhaustive of the entire set of topological feature one my compute in a network, the following list provides an overview of the statistics which are both economically meaningful and relevant for financial stability. In the following definitions we consider a network of n nodes, whose adjacency matrix is A and whose weighted matrix is W.

Definition 3 (Node Degree)**.** The in-degree k_i^{in} and out-degree k_i^{out} of a node in a directed networks are the number of incoming and outgoing links respectively:

$$k_i^{in} = \sum_{j=1}^{n} a_{ij} \quad \text{and} \quad k_i^{out} = \sum_{j=1}^{n} a_{ji}.$$

In the context of an interbank network the in-degree represent the number of creditors and the out-degree the number of debtors.

Definition 4 (Node Strength)**.** The in-strength s_i^{in} and out-strength k_i^{out} of a node in a directed networks are the total amount of weight carried by its incoming and outgoing links respectively:

$$s_i^{in} = \sum_{j=1}^{n} w_{ij} \quad \text{and} \quad s_i^{out} = \sum_{j=1}^{n} w_{ji}.$$

The definition parallels that of node in- and out-degree and, in our framework, can be interpreted as the total amount of interbank assets and liabilities.

Definition 5 (Connectivity)**.** Connectivity is the fraction of possible links that the network actually displays. Calling l the number of existing edges, in a directed graph, connectivity is given by

$$c = \frac{l}{n(n-1)}.$$

Connectivity of thus a measure of the fraction of possible interbank relations which actually exist. It thus provide a measure of diversification and also of the channels of transmissions through which a shock may flow. A closely related concept is that of average degree

Definition 6 (Average Degree)**.** The average degree is the average in-degree, or, equivalently, the average out-degree, of the n nodes in the network

$$\bar{k} = \sum_{i=1}^{n} k_i^{in} = \sum_{i=1}^{n} k_i^{out}$$

Which thus represents the average number of counterparties a bank has, and thus even more clearly represents both the average level of diversification and the average number of possible sources of shock.

Definition 7 (Average Path Length)**.** A path is a sequence of vertices such that each pair of consecutive vertices in the sequence are connected by an edge. The number path of length r from i to j is the element i, j of A^r. The average path length is the average shortest path between any two nodes.

Despite most studies on interbank contagion have, so far, focused on connectivity, also the average path length should be taken into consideration when exploring the resilience of an interbank markets. Indeed it represents the average number of connections separating two banks and may thus be relevant for the timing and the severity of a default cascade.

Definition 8 (Reciprocity)**.** In directed networks, reciprocity is the fraction of links for which a link in the opposite direction exists. An expression for reciprocity is

$$r = \frac{TrA^2}{l}.$$

Reciprocity represents the frequency of reverse lending relationships. Certainly, from an empirical point of view, it is interesting to note how several contributions have found high levels of reciprocity in real interbank networks (Bech and Atalay, 2010; Soramäki et al., 2007). This possibly reflects the role of what Cocco et al. (2009) define preferential lending, i.e. the importance of non-economic foundations for interbank lending.

Definition 9 (Clustering)**.** In an undirected network, the clustering coefficient is defined as the probability that two nodes, which are connected with another node, are connected between themselves:

$$C = \frac{\text{number of triangles} \times 3}{\text{number of connected triples}} = \frac{1}{n}\sum_{i=1}^{n} \frac{(A^3)_{ii}}{k_i(k_i-1)}.$$

In the definition of the clustering coefficient, we consider, for simplicity, the case of an undirected network, which can be derived from a directed one if the directionality of a link is neglected. k_i indicates the (undirected) degree of node i, i.e. the number of connections i has, and $(A^3)_{ii}$ represents the i-th element in the diagonal of A^3. The clustering coefficient is a measure of how tight interbank relations are at local level. An high clustering coefficient indicates that the counterparties of a given bank are very likely to make transactions also between themselves.

Definition 10 (Assortativity and Disassortativity)**.** A network is said to be assortative if nodes with a certain degree are more likely to be connected with nodes with similar degree. It is said to be disassortative if the opposite holds. A simple measure of assortativity in undirected networks is

$$m = \frac{cov(k_i, ANND_i)}{\sigma(k_i)\sigma(ANND_i)} \in [-1, 1]$$

where $ANND_i$ is the average nearest neighbor degree, i.e. the average degree of node i's neighbors.

As for the clustering coefficient, we presented only the undirected version of the assortativity coefficient. Interbank markets tend to be disassortative, in the sense that small banks tend to trade with large banks and viceversa. This may be symptomatic of the presence of banking groups, in which small subsidiaries preferentially trade with the parent company.

The previous statistics are either referred to single nodes, or are synthetic network-level measures that summarize, in a single number, a series of local features. Other relevant information may instead come from the statistical *distribution* of certain local characteristics. First and foremost, the distribution of the degree, i.e. the number of counterparties, provides important information regarding the structure of an interbank network, since it is able to quantify the level of heterogeneity of its nodes.

Definition 11 (Degree Distribution)**.** Given a network, construct a sequence of possible degrees $\{1, 2, \dots\}$ and a sequence of probabilities $\{p_1, p_2, \dots\}$, where p_k is the frequency of nodes with degree k. The quantities $\{p_1, p_2, \dots\}$ thus define a probability distribution over degrees $\{1, 2, \dots\}$, which is defined as *degree distribution.*

More appropriately, in the context of directed graphs, we would dealt with a *joint degree distribution* $\{p_{k^{in}k^{out}}\}$ representing the probability that a node have in-degree k^{in} and out-degree k^{out}.

Models of Network Formation

The previous overview of definition can be applied to any arbitrary network. However, when one seeks to build a network displaying some desired features, he has to confront with the theory of network formation, which provides a set of models that, because of different assumptions on the mechanism of link formation, are able to generate a corresponding set of networks with specific statistical peculiarities. Here, we intend to provide a brief description of the two network formation models employed in our analysis, namely the random graph model by Erdős and Rényi (1960) and the fitness model[3].

Random Graphs

The random graph model due to Erdős and Rényi (1960) is a model in which, given a set of N nodes a link from a node i to a node j exist with a probability p which is constant for each pair of nodes. In the network there are $N(N-1)$ possible directed links to be created, resulting in an expected number of edges in the network equal to $pN(N-1)$, so that the (expected) average degree is $p(N-1)$. Indeed each node has $(N-1)$ nodes which it can connect to. It follows that both the distribution of the in- and out-degree follow a binomial distribution:

$$p^{in}(k) = p^{out}(k) = \binom{N-1}{k} p^k (1-p)^{N-k-1}$$

[3]For a more complete overview of network formation models one may refer to standard textbooks as Newman (2010) or to the reviews by Albert and Barabási (2002) and Chakrabarti and Faloutsos (2006).

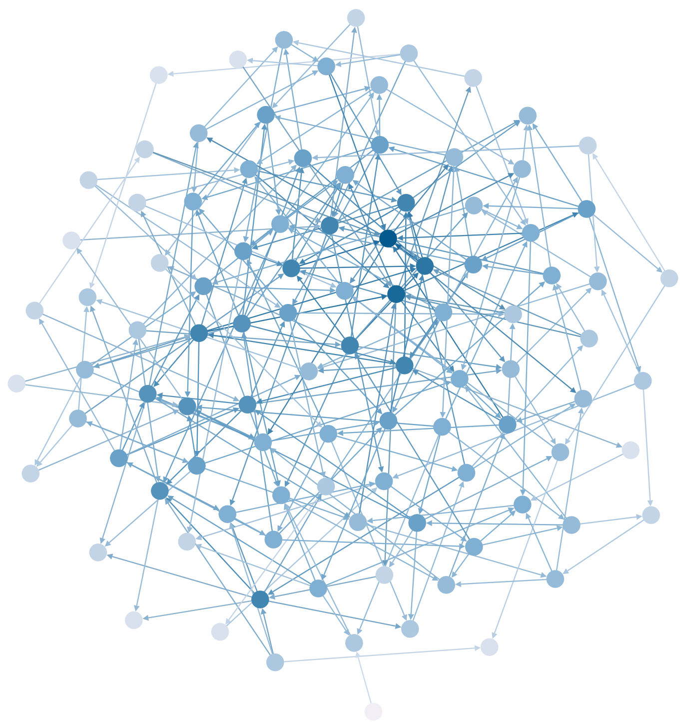


Figure 12: Example of Erdős and Rényi (1960) random graph with 100 nodes and average degree of 3.

If c denotes average degree of a random graph, asymptotically, as $N \to \infty$, the degree distribution converges to a Poisson(c)

$$p(k) = \frac{e^{-c}c^k}{k!}$$

which is the reason why the model is sometimes referred to as Poisson random graph.

Since the probability of forming a link is homogenous, the resulting network structure does not present marked heterogeneity. In a Poisson distribution the dispersion around the mean is limited and deviations from it are exponentially rare. An interbank network generated using this model will thus provide of an homogenous market, in which banks tend to have similar levels of connectivity, i.e. their specific number of counterparties does not significantly vary from the average.

The Erdős and Rényi (1960) graph is also said to be small-world, since it presents a short average path length and its diameter, i.e. the longest of the shortest paths linking two nodes, grows at a much lower rate than N, precisely as $\log(N)$. The clustering coefficient is equal to the probability of a link's existence, p.

This kind of models has been extensively applied for the study of contagion in financial networks, e.g. in the contributions from Nier et al. (2007), Gai and Kapadia (2010), Iori et al. (2006) and Montagna and Kok (2013).

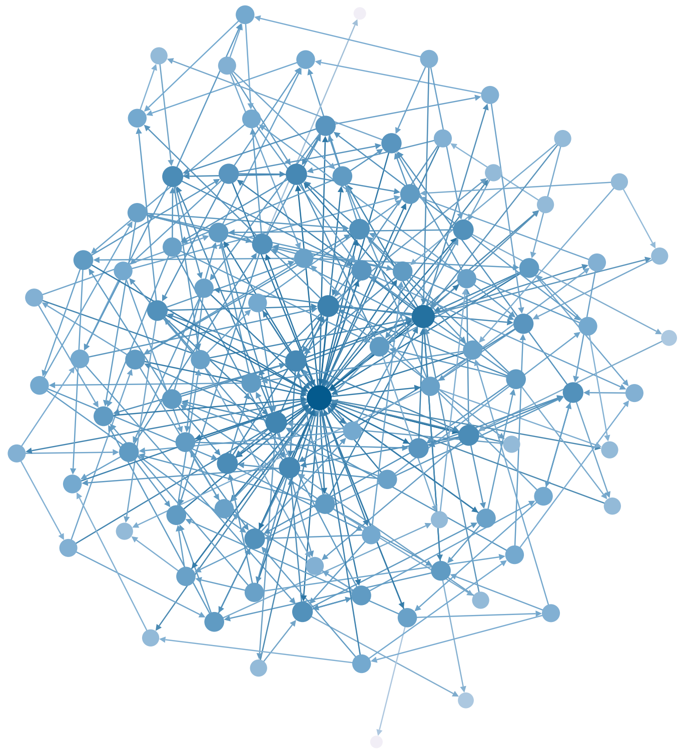


Figure 13: Example of graph generated with a fitness model with 100 nodes and average degree of 3. The distribution of the fitness is power-law with exponent 2.5.

Fitness Model

The fitness model is a very flexible model of network formation which is able to provide a wide range of structural features. Every node i is endowed with a fitness parameter, x_i, which is a measure of its "attractiveness", and links are formed between nodes with a probability which is a function of the fitness of the nodes. More formally, if we define p_{ij} as the probability that a link exists from i to j, this probability is given by

$$p_{ij} = f(x_i, x_j)$$

for a generic function f.

Depending on the shape of the function f and on the probability distribution ρ of the fitness, various properties may emerge.

In general the expected in-degree for a node with fitness x is

$$k^{in}(x) = n \int_{-\infty}^{+\infty} f(t, x)\rho(t)dt \equiv nF_{in}(x)$$

while the expected out-degree is

$$k^{out}(x) = n \int_{-\infty}^{+\infty} f(x, t)\rho(t)dt \equiv nF_{out}(x).$$

Clearly, the two expressions coincide if f is symmetric, i.e. $f(x_i, x_j) = f(x_j, x_i)$, meaning that the fitness parameter represents the attractiveness of the node irrespective of the direction of the relation to be established.

Under the assumption of invertibility of F^{in} and F^{out} and of differentiability of their inverse, one can derive an analytical expression for the probability of observing nodes with in-/out-degree equal to a generic k:

$$P^{in}(k) = \rho\left[F_{in}^{-1}\left(\frac{k}{n}\right)\right] \cdot \frac{d}{dk}F_{in}^{-1}\left(\frac{k}{n}\right)$$
$$P^{out}(k) = \rho\left[F_{out}^{-1}\left(\frac{k}{n}\right)\right] \cdot \frac{d}{dk}F_{out}^{-1}\left(\frac{k}{n}\right)$$

One may also compute higher order properties of networks built via fitness models. Focusing on undirected models, where $p_{ij} = f(x_i, x_j) = f(x_j, x_i)$ represents the probability of an undirected link between i and j, closed form solutions are available for the clustering coefficient of nodes with fitness x

$$C(x) = n^2 \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,t)f(t,s)f(s,x)\rho(t)\rho(s)dtds}{k(x)^2}$$

and for the average degree of their neighbors

$$ANND(x) = \frac{n}{k(x)}\int_{-\infty}^{+\infty} f(x,t)k(t)\rho(t)dt.$$

Despite this is just a brief summary of the properties of a fitness model[4], it should be clear enough that its flexibility has the potential to take into account a number of target properties. This is the reason why authors as De Masi et al. (2006) and Montagna and Lux (2013) suggest the fitness model in order to match the empirical features of real interbank networks.

In our (directed) interbank network we use a fitness model with an additive linking function

$$p_{ij} = f(x_i, x_j) = c(x_i + x_j) \tag{5}$$

where c is a constant that we tune in order to obtain the desired average degree.

We then draw a series of n fitness parameters $\{x_1, x_2, \ldots, x_n\}$, one for each bank from a power-law distribution with exponent $\beta > 2$ and minimum value x_0

$$P(x) = ax^{-\beta} \quad x > x_0 \tag{6}$$

[4]For a more detailed description of its properties we invite the reader to refer to Caldarelli et al. (2002), Caldarelli (2007) and Servedio et al. (2004).

Solving the integration for the expected in- and out-degree we find that

$$F_{in}(x) = F_{out}(x) = \frac{cx_0^{2-\beta}}{\beta - 2} + \frac{acx_0^{1-\beta}}{\beta - 1}x$$

Inverting these functions and using the formulas for the degree distribution we see that

$$P^{in}(k) = P^{out}(k) \propto (k - \xi)^{-\beta} \tag{7}$$

where ξ is a positive constant that depends on the parameters of the fitness model. This means that our model is able to replicate a power-law tail decay of the degree distribution, which is a feature often observed in real networks (Caldarelli, 2007; Newman, 2010), including in interbank markets (Bech and Atalay, 2010; Boss et al., 2004; Cont et al., 2013; Iazzetta and Manna, 2009; Iori et al., 2008; Soramäki et al., 2007) and which is thus symptomatic of high levels of heterogeneity in the connectivity of financial institutions. The model is flexible enough to allow us to tune the exponent of the tail decay.

Part II

TailDep for the Measurement of Systemic Risk

6 Introduction

The risk of wide-spread contagion became a concern for academics and policy-makers when the recent crisis unfolded. Uncertainty regarding the actual degree of interconnectedness of the financial system prevented any knowledgeable assessment of the consequences exogenous shocks would have triggered.

The dominating approach to financial regulation, before the crisis, was primarily micro-prudential (Hanson et al., 2011), i.e. based on a partial equilibrium view of the financial system, in which entities were seen as stand-alone risk takers.

Standard measures of riskiness were the Value-at-Risk (VaR) and the Expected-Shortfall (ES). VaR is defined as the largest loss a bank may experience with probability $(1-\alpha)$, i.e. $P(L > VaR_\alpha) = \alpha$, while the ES is the expected value of the loss, conditional on being larger than VaR, i.e. $ES_\alpha = \mathrm{E}(L|L \geq VaR_\alpha)$.

These measures, despite being flexible enough to take into account macroeconomic shocks, are specific to single institutions and fail to give any measure of the risks generated by the system itself.

Our view of systemic risk is that of a risk generated *by* and *within* a financial system. As such, idiosyncratic risk indicators, e.g. leverage, liquidity mismatch, VaR and ES, should not enter in an appropriate measure of systemic risk. They remain, however, useful instruments to identify idiosyncratic risks, which may be amplified by systemic factors.

In order to generate systemic risk, financial connections are crucial, since they identify the dependence structure which interlinks the financial institutions in a system. Their existence is able to generate amplification effects[5], which are a source of risk on their own, and which we regard as systemic risk. In this paper we propose a quantification procedure for systemic risk, which entirely rely on the interconnectedness of the financial system.

We model the financial system as a network of institutions which chose their degree of dependence on other institutions. The systemic risk of the system is seen

[5]Recent contributions on financial networks (Amini et al., 2012; Battiston, Delli Gatti, Gallegati, Greenwald and Stiglitz, 2012*a*,*b*; Gai and Kapadia, 2010; Lenzu and Tedeschi, 2012; Montagna and Kok, 2013; Nier et al., 2007) and industrial networks (Acemoglu et al., 2012; Carvalho and Gabaix, 2013; Gabaix, 2011) have addressed the issue of the amplification effects generated by local interlinkages.

the probability of its entire collapse, conditional on a shock hitting some part of the system. The magnitude and the location of the shock is asymptotically irrelevant in determining the steady-state risk of default of financial entities, which is exclusively determined by the equilibrium network.

A policy-maker which is averse to systemic risk will have then an incentive to design a stabilization policy whose intensity depends on the leading eigenvalue of the network. If a tax is imposed on banks' assets and banks are risk neutral, they will be willing to pay a tax rate which is proportional to their eigenvector centrality, which we show to be the key determinant of their susceptibility to systemic risk.

Such tax accomplishes four roles: (i) it entirely finances the stabilization policy, (ii) it provides banks with an information on the centrality of their counterparties, and thus on their systemic importance, (iii) it reduces the degree of interconnectedness of the system in equilibrium, (iv) it aligns banks' incentives to those of the policy-maker.

We view our concept of systemic risk a theoretically strong one. Several measures have taken into account the probability of default, or joint default, of entities or the tail correlation of single entities to the market (Acharya et al., 2010; Adrian and Brunnermeier, 2011; Huang et al., 2012). However, most of them lack a theoretical motivation and they all suffer of an idiosyncratic bias: it is not clear why, if excessive correlation is a problem, investors do not take it into account and the regulator's intervention is needed.

In our model, systemic risk emerges because the linking decisions of banks affect also the stability of banks linking to them, because of imperfect information and network externalities. Imperfect information is indeed a necessary ingredient when dealing with financial systems, especially after the rapid expansion of OTC derivatives and of the shadow banking system, which have made the financial network extremely complex and opaque (Haldane, 2009).

We estimate systemic risk with the TailDep methodology, which is based on the tail dependence of CDS spread, which is a proxy of the probability of conditional default. Tail dependence is indeed a mathematical concept expressing the probability of a random variable to experience large (asymptotic) movements, conditional of another variable being at its asymptotic value. In the context of CDS spread, this reflect the probability of conditional default and this methodology allows to build a network where each link represents the probability of default of an institution conditional on the default of another institution.

Importantly one has to notice that the network of tail dependencies does not tell anything about the risk of default of individual entities since, as mentioned above, links are defined as conditional probabilities. The fundamental contribution of TailDep is that of assessing the probability of distress of a reference entity conditional on another one being in distress. For this reason it allows to assess the systemic risk generated by the network of interconnections.

The concerns of scholars and regulators have recently begun to be channeled towards the potential sources of risk stemming from OTC derivatives, in particular Credit Default Swaps (ESRB, 2012; IMF, 2013; Noyer, 2010). A number of critics have also been addressed to CDS products, among which we may recall Fostel and Geanakoplos (2012), who argue that the introduction of CDSs of mortgage products in 2005 and 2006 led mortgage bonds and house prices to increase, thus fueling the housing bubble which eventually bursted. Others highlight how, by creating a false sense of security, CDS incentivize leverage and excessive risk taking, in such a way that, despite idiosyncratic risk is reduced (or at least it appears to be so), systemic risk is actually increased. Lastly, another point of view is that of Heise and Kühn (2012) and Stulz (2010) who see the dangers of CDSs in generating new forms of financial dependencies and interconnections across financial institutions.

However, we view such instruments as important signaling devices of the perception markets have of the interconnectedness of the financial system. Nevertheless, we claim that regulators should engage in extensive data collection in order to obtain their own estimates of the interconnectedness of the financial system. The data to be gathered should encompass also OTC, shadow banking and off-balance sheet exposures, information which are not available to investors and not easily priceable into CDS contracts. Confidential collection of such data would allow regulators to monitor the building up of systemic risk *before* a crisis, allowing them to intervene in due time, without imposing banks the public disclosure of sensitive information.

After discussing the related literature in Section 7, in Section 8 we present our dataset. Section 9 provides a first overview of the evolution of the interconnectedness of the financial system using a dynamic conditional correlation model and principal component analysis. Section 10 put forward our model and discusses the copula methodology used to estimate it. Section 11 presents the results of the estimation and Section 12 concludes.

7 Related Literature

The spirit of our work is closer to the ones by Acharya et al. (2010), Adrian and Brunnermeier (2011), Billio et al. (2012) and Huang et al. (2012) who tried to quantify systemic risk from market data. Acharya et al. (2010) provides a theoretical model in which a planner wants to maximize a welfare function taking into consideration the stability of the overall system. An optimal tax that depends on the expected shortfall[6] and the systemic expected shortfall[7] of banks. Such tax allows to align the incentives of banks to those of the planner. They estimate these measures using high frequency equity and CDS data and assess their predictive power.

Adrian and Brunnermeier (2011) propose the CoVar and forward CoVar measures as indicators of systemic importance and as predictor of future distress respectively. CoVar is estimated via quantile regression of the VAR of the system conditional on a given institution being at its VAR level. The forward CoVar is instead estimated by regressing the VAR on lagged values of market and balance sheet data. However no theoretical motivation is provided for these measures and it is not clear how a forward CoVar may be superior in predicting future distress than simply using the market and balance sheet data used to estimate it.

Billio et al. (2012) provide an extensive analysis of the correlations of the returns of 100 financial institutions in the two decades preceding the 2008 crisis. They find that co-movements became increasingly marked, despite it is not clear why only 36 components result from a sample of 100 variables. They then construct a network of Granger-casual relations, which became particularly dense in the crisis years. Measures of centrality in the networks have good out-of-sample predictive power on future losses of market value.

Finally, Huang et al. (2012) propose a systemic risk index, the distressed insurance premium (DIP), which is based on the risk-neutral probabilities of default implied by CDS spreads, which measures the premium an insurer would require to cover losses in the entire banking sector. They then rank banks according to their marginal contribution to the DIP, which is strongly driven by the asset size, and relate it to losses in market value and capital shortfall under regulatory assessment. However, no theoretical model is provided.

From the theoretical side, our contribution relates to the concept of negative spillovers and network externalities which have been recently subject to a flourishing academic debate (Acemoglu, Ozdaglar and Tahbaz-Salehi, 2013; Acharya, 2009; Allen et al., 2010; Allen and Gale, 2000; Battiston, Delli Gatti, Gallegati, Greenwald and Stiglitz, 2012*b*; Krishnamurthy, 2010; Lorenzoni, 2008) and which have boosted increasing attention to the role of the complex structure of the financial system, which is seen as a network of contractual linkages (Amini et al., 2012; Battiston, Delli Gatti, Gallegati, Greenwald and Stiglitz, 2012*a*; Gai and Kapadia, 2010; Montagna and Kok, 2013; Nier et al., 2007).

From the empirical side, our work relates also to the growing literature that uses

[6]The expected shortfall is the expected loss, conditional on losses larger than the VAR of a bank, i.e. $ES = \mathrm{E}[L|L \geq VAR_\alpha]$, where L indicate the loss and VAR_α is the institution's VAR relative to quantile α

[7]The systemic expected shortfall is defined as the expected capital shortfall of an institution conditional on the overall system being undercapitalized

copula models to study the dependence structure of CDS spreads (Christoffersen et al., 2013; Lucas et al., 2014; Oh and Patton, 2013). Their focus is however more on the application of an econometric methodology than on the estimation of a theoretical model, while, in our contribution, the use of a copula model will be ancillary to our theoretical framework.

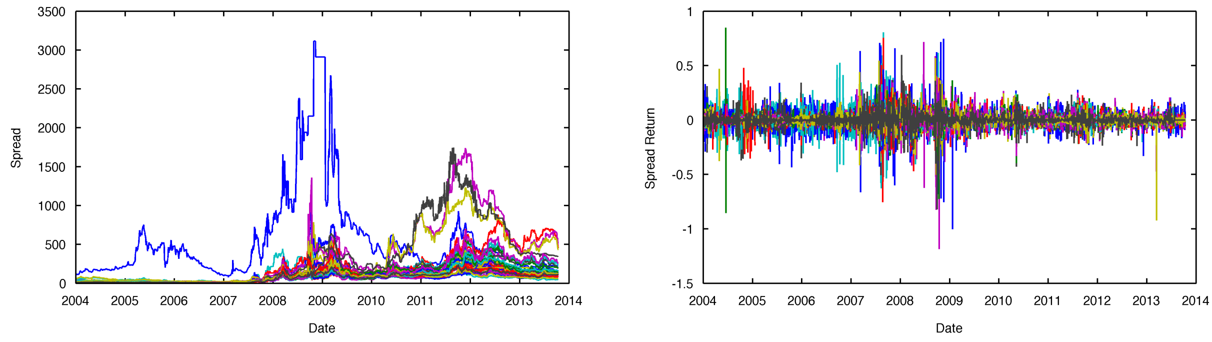
Not many papers have been written on the statistical analysis of CDS spreads. To our knowledge Cont and Kan (2011) represents the first and most extensive contribution in this field. The authors show that CDS spreads follow non stationary processes, while spread returns are stationary with positive autocorrelation. They find positive serial correlations in extreme values, conditional heteroschedasticity and two-sided heavy tails. They show how large co-movements may be observed, often unrelated to credit events, thus suggesting the presence of common risk factors. Furthermore, they showed that correlations of spread returns increase significantly after 2007 and that credit events do not necessarily lead to large upward moves in the CDS spreads. They also estimate a heavy-tailed multivariate AR-GARCH model for CDS spread returns and simulated statistical properties of CDS spread returns as well as their dependence structures.

Gündüz and Kaya (2014) perform an econometric analysis of the CDS spread of 10 European countries, finding mixed evidence of long memories in the series of CDS log-differences. They also run a Granger causality test and find evidence of causality going from CDS log-differences volatility and CDS spread. Their estimates of dynamic conditional correlations show an upward jump coinciding with the collapse of Lehman

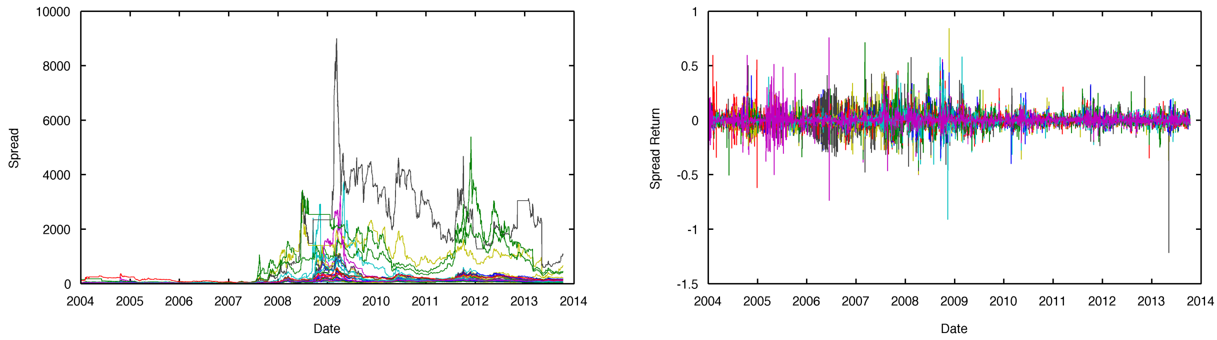
Kaushik and Battiston (2012) is another contribution in the field of the statistical analysis of CDS spreads. Here the authors introduce the tool of the ϵ-drawups to build a network of CDS securities. Two entities are linked if they are likely to experience jumps together. This allows the authors to build a topology of the risk structure in the CDS market.

8 Data

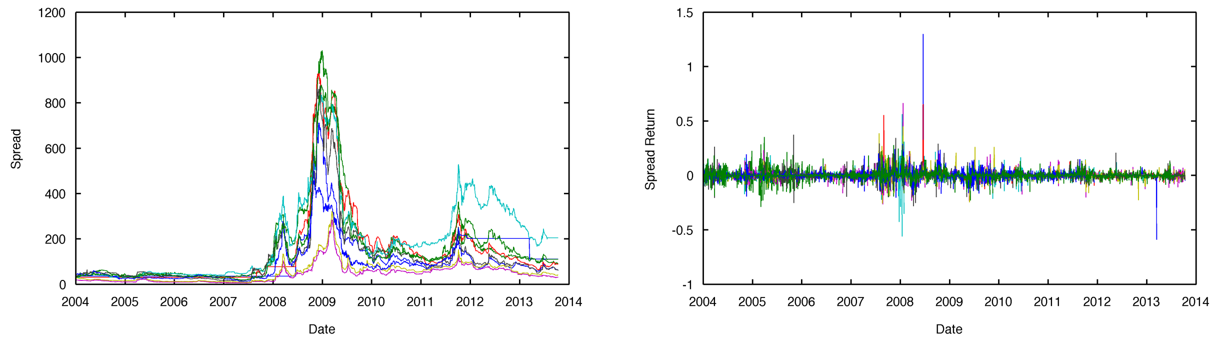
Credit Default Swaps are credit derivatives which grant protection against the default of a debt issuer. Using the terminology of CDS contracts, a protection seller is the short side, a protection buyer is the long side, while the debt issuers against whose default the contract is written is called reference entity. Under the terms of a CDS contract, the protection buyers pays an annual premium to the seller. This premium is often expressed as a percentage, named spread, of the notional amount insured. In exchange of the premium, the protection seller will refund the protection buyer of any loss incurred due to the default of the reference entity, within limits of



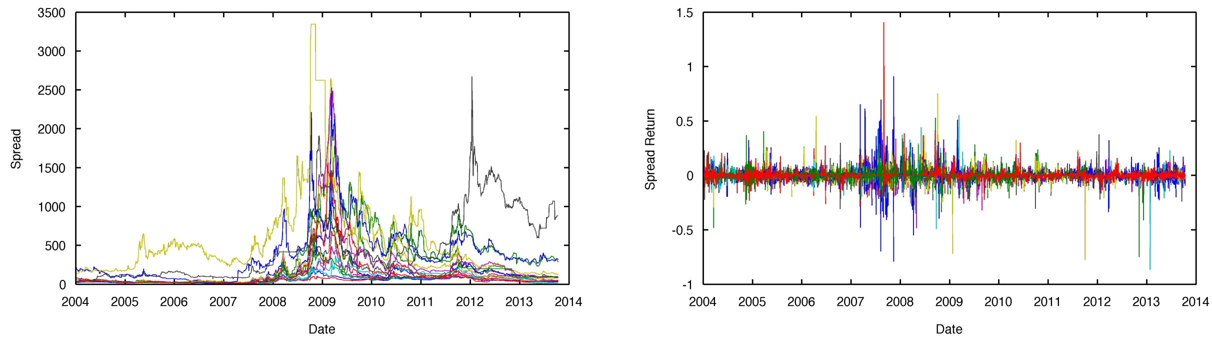
(a) Banks



(b) Insurers



(c) Real Estate



(d) Others

Figure 14: CDS spread and spread returns.

the notional amount insured.

CDS spread data are downloaded from Bloomberg. We have data available for 108 financial institutions for the time period between the 1st January 2004 and 30th September 2013.

Entities are grouped into four different categories, indeed the dataset comprises 49 banks, 33 insurers and 9 real estate companies. The remaining 17 entities, which we group under the name of 'others', mainly include the financial branches of industrial companies, asset management firms, providers of financial services and credit institutions other than banks.

Figure 14 plots the CDS spread and the CDS spread return, defined as the first difference of the logarithm of the spreads. The 2007 clearly represents clear transition year from a relatively tranquil period, characterized by low CDS spreads, to a period where investors became increasingly concerned with stability of financial sector, concerns which are indicated by the quoted premiums.

CDS spread returns provide complementary information. Relatively small daily spread changes characterize the period from 2010 onwards. Before, the years from 2007 to 2009 included present a market volatility cluster, which somehow anticipate the spikes in the CDS spread series.

Overall, three phases can be identified: the first one, from January 2004 to December 2006, correspond with the build-up of the crises, spreads are low and volatility limited; the second one, from January 2007 to December 2009, is the acute phase of the crisis, when at first, uncertainty increase, as reflected in the volatility of CDS returns, and then the risks perceived by investors rise together with the spreads; the third one, from January 2009 onwards seems to represent the mature phase of the crises, when uncertainty is reduced but perceived risk remains relevant.

In our analysis we are interested in the co-movements of CDSs net of any autocorrelation and volatility clusters. The Augmented Dickey-Fuller test conducted under different specification of the autoregressive model confirms the visual intuition and fails to reject the null hypothesis of a unit root at a 10% confidence level for all the 108 CDS spread series. This confirms the finding on the nonstationariety of (Cont and Kan, 2011).

Spread returns seem instead to follow a stationary behavior according to an ADF test, which reject the null hypothesis of non-stationariety for each of the series at a 1% confidence level. We thus filter out residuals of the spread return series by using an AR(1)-GARCH(1,1) model

$$r_{l,t} = \rho r_{i,t} + \epsilon_{l,t} \tag{8}$$

$$\epsilon_{l,t} \sim \sigma_{l,t} t_{\nu} \tag{9}$$

$$\sigma^2_{l,t} = k_l + \alpha_l \sigma^2_{l,t-1} + \beta_l \epsilon^2_{l,t-1} \tag{10}$$

and we define the standardized residuals as $v_{l,t} = \epsilon_{l,t}/\sigma_{l,t}$.

We estimate this model via maximum likelihood under the hypothesis of t-distributed errors with stochastic variance. A student-t distribution is better able to account for fat tails in the distribution of residuals, while the stochastic variance estimated via GARCH model is used to normalize each innovation and thus to offset sources of heteroschedasticity due to volatility clusters.

Two types of estimation are conducted: one for the entire sample period and the other using a 24 month rolling window. The entire sample estimation is used to estimate the dynamic conditional correlations, while 24 month rolling window are instead used to understand the dynamics of the other measures of interconnectedness. All these measures are explained in details in the following sections.

Why CDS?

We have already mentioned in the Introduction that a CDS is, essentially, an insurance contract against the default of a reference entity. Their premium is therefore a clear market information about the financial soundness of the entity.

To clarify the meaning of the CDS spreads consider a very simple model in which the default of a reference entity is a Poisson process with (time-varying) intensity μ_t, the premium is paid continuously and the recovery rate is a fixed constant ρ. We can hence define $Q(\tau)$ as the probability of the reference entity to survive at least up to time τ, which is $Q(\tau) = e^{-\int_0^\tau \mu_s ds}$. Then, $-dQ(\tau)$ represent the probability of default at time τ since it is the absolute value of the variation (which is negative) of the probability of surviving up to time $\tau + d\tau$ and τ. Finally, we indicate the with r_τ the instantaneous interest rate paid by a risk-free bond with maturity τ.

In this case the value of the protection given by the CDS is

$$V_b = (1-\rho) \int_0^T e^{r_\tau}(-dQ(\tau))$$

while the value of the short position is

$$V_s = s \int_0^T e^{r_\tau} Q(\tau) d\tau.$$

Then, by equating the two expressions, it can be shown that the fair spread for a CDS with maturity at T is given by

$$s = -(1-\rho)\frac{\int_0^T e^{-r_\tau \tau} dQ(\tau)}{\int_0^T e^{-r_\tau \tau} Q(\tau) d\tau}.$$

If, to simplify further, we assume a flat credit curve, i.e. a constant $\mu_t = \mu$, the formula simplifies to

$$s = (1-\rho)\mu$$

Keeping the recovery rate fixed, movements of the CDS spread correspond to movements of intensity of the default process driving the probability of default. In particular as $\mu \to +\infty$, also $s \to +\infty$ since the probability of surviving up to an a certain time t, no matter how small, is $Q(t) = e^{-t\mu} \to 0$, reflecting therefore the default of the reference entity. Innovations to the spread are then driven by innovations to the intensity of default.

CDS spreads are therefore the ideal measure of distress of a financial institution, and the co-movements of the CDS prices of financial entities uncover important information about the co-movements of the probability of distress of the underlying entities. Marked co-movements of the CDS spreads of two reference entities signal that markets regard the two entities as being very interlinked, so that their probabilities of default increase or decrease together in the same direction.

9 A First Look to Correlations

Since correlation of the probability of default of entities are reflected in the correlation of their spread and, consequently, in the spread returns, we begin our exploration of the interconnectedness in the financial sector by having a first look to the co-movements of CDS returns.

9.1 Dynamic Conditional Correlation

The Dynamic Conditional Correlation (DCC) estimator was put forward by Engle (2002) to estimate the correlation of returns conditional on past information. It allows to obtain daily estimates of the correlation matrix of various assets.

Consider the Constant Conditional Correlation (Bollerslev, 1990) estimator

$$\bar{R} = \frac{1}{T}\sum_{t=1}^{T} v_t v_t'$$

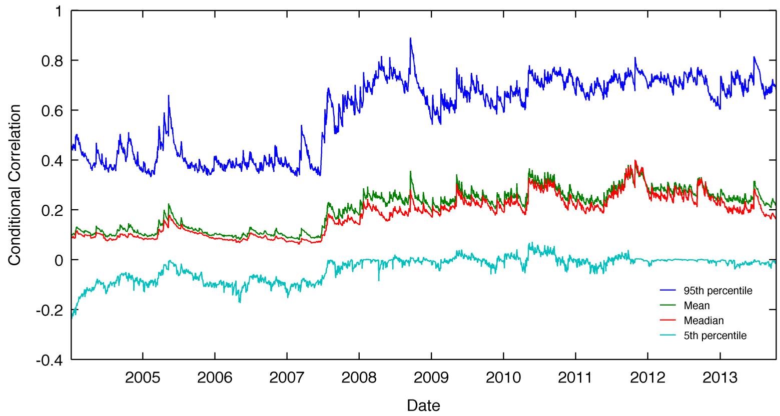


Figure 15: Dynamic Conditional Correlation of the normalized residuals of spread returns. The Figure report mean, median correlations, together with the 95th and 5th percentile using a single estimate of the model for the entire sample period.

The the one-lag DCC model is

$$H_t = \bar{R} + \alpha(v_{t-1}v'_{t-1} - \bar{R}) + \beta(H_{t-1} - \bar{R})$$

The matrix H_t contains the correlation coefficients of the normalized residuals condition on the past realizations. Thus it can be used to a first assessment of the interconnectedness of default risks in financial markets. Increasing dynamic correlations imply stronger co-movements of CDS spreads, and thus of the perceived financial soundness of financial entities.

In our analysis we perform an estimate of the DCC model for each pair of reference entities, allowing the parameters of the model to be different for each pair of reference entity. Estimations are conducted using a single model for the entire sample period.

As we seen in Figure 15, the median and the mean DCC are positive for the entire sample period, however they experienced a jump around mid 2007: while before they were fluctuating around 10%, in the second half of 2007 they started a transition to higher values, fluctuating around 20% from 2008 onwards. They change of regime is even more marked in the tails of the distribution of the correlations. The 95th percentile started from values around 40% and reached levels fluctuating around 70% from 2008 onwards. As for the 5th percentile, it started from negative values and, form 2012, it has been frequently hitting the value of zero 0, with some slight fluctuations below it.

It is clear, even with these summary data, that the co-movements of CDS returns became more marked in the second half of 2007, following the first signs of uncertainty in the American housing market.

9.2 Principal Component Analysis

The principal component analysis (PCA) is a statistical technique based on a linear transformation of the dataset. In simple words, given N variables and T observations, the PCA finds N linear combinations of the original variables, called principal components, which are: (i) uncorrelated, (ii) and sorted in decreasing order of explanatory power.

In our context, consider the zero-mean, unit-variance series of normalized residuals v_i for institution i. Define as V the matrix whose i-th column is v_i.

Then we know that

$$V^T V = \Sigma$$

where Σ is the covariance matrix of our data, which, since it is symmetric, has N distinct orthonormal eigenvectors. Unless some observations are constant Σ is positive definite, so it has strictly positive eigenvalues. It can be thus decomposed as

$$V^T V = W \Lambda W^T$$

where Λ is the matrix whose diagonal elements are the eigenvalues sorted in decreasing order and where W is the matrix whose columns are the (orthonormal) eigenvectors of Σ, ordered according to the eigenvalues in Λ.

Then

$$(WV)^T (WV) = \Lambda$$

and the matrix WV is the matrix whose columns are the principal components, whose covariance matrix is Λ. Since Λ is diagonal, this implies that the principal components are uncorrelated.

Statistical theory (Jolliffe, 2005) shows that the eigenvalues in Λ, which are the variances of the principal components, are proportional to the fraction of the total variation of the dataset which is explained by the corresponding principal components. This is the reason why, without loss of generality, the matrix Λ was built by placing the eigenvalues along the first diagonal in decreasing order and arranging the eigenvectors in matrix W accordingly. The first column of WV will thus be the component that explains a fraction $\frac{\lambda_1}{\sum_l \lambda_i}$ of the total variation of the observations.

The first principal component may then be interpreted as the leading factor driving market movements. Hence, the variance of the market explained by the first components is a clear indicator of the interconnectedness of the system. If few common factors are able to account for most of the variation of the market, these few common factors gain systematic importance and the institutions which are more strongly correlated to them represent themselves critical entities, since they are the

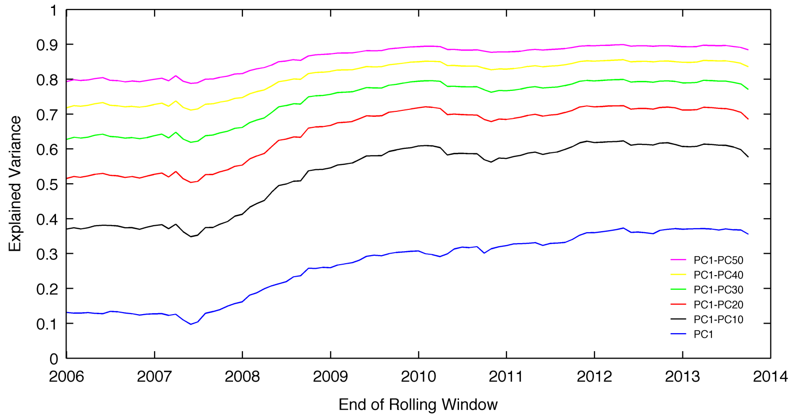


Figure 16: Variance explained by the first principal component and by principal components 1 to 10, 1 to 20, 1 to 30, 1 to 40, 1 to 50, estimated using a 24-month rolling window.

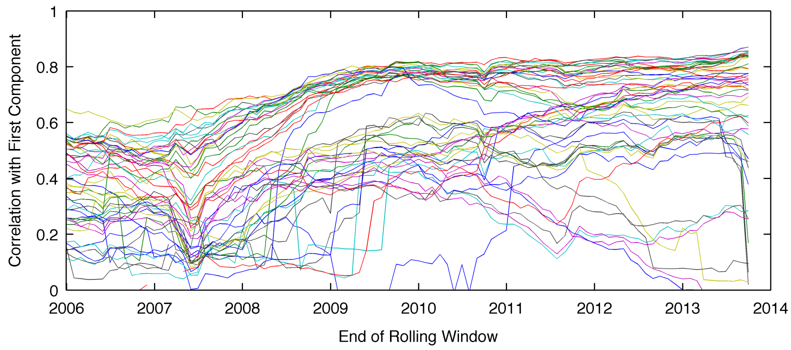
ones with the strongest connections to the systematic risk factors.

We then estimate the principal components and the explained variances in our dataset using a 24-month rolling window estimations of the normalized residuals of an AR-GARCH model. Results are reported in the figures at the date when they would have been available, i.e. the end of the rolling window.

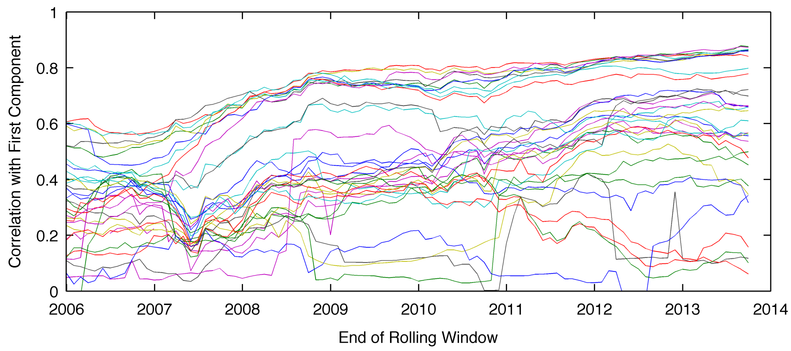
Figure 16 shows how, in the time span covered by our data, the systematic rick factors have become increasingly important in explaining market movements. The variance explained by the first component experienced a small drop at the end of the first half of 2007, starting a sustained increase afterwards. Starting from a value around 15% in 2006, the variance explained by the first component reached a level of almost 40% at the end of 2013. Similar observations hold for the variance explained by additional components. The first ten components started from values below 40% and reached levels of 60% at the end of the sample period. The increase in explained variance is progressively less marked as more components are considered. However, at the end of the sample period only 50 components are sufficient to explain 90% of the market variations.

Figure 17 reports the correlation coefficient between the normalized residuals of single reference entities and the first principal component. High correlation is a symptom that the entity tends to strongly co-move with the entire market, due to a strong connection with systematic risk factors. We separate the results according to the sector of activity of the financial institutions. Coherently with our findings on an increasing fraction of variance explained by the first component, the correlation coefficients with it are generally increasing. However, the results highlight different correlation patterns of financial entities.

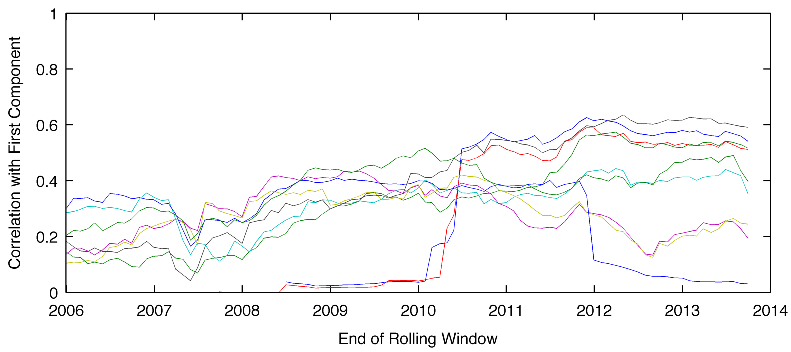
The top of the plots is dominated by European banks and insurers which, at



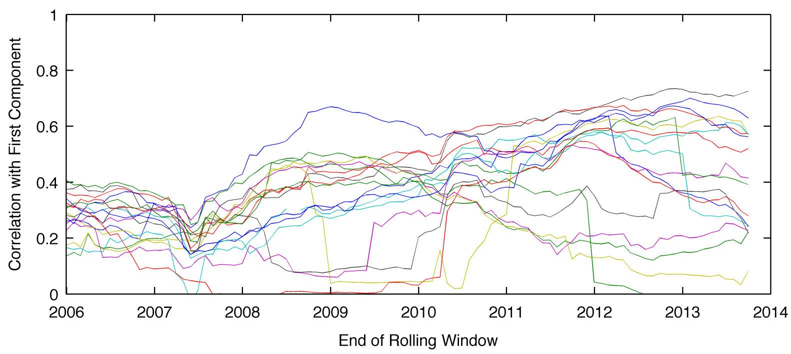
(a) Banks



(b) Insurers



(c) Real Estate



(d) Others

Figure 17: Correlation with the first principal component. The principal component and the correlations are estimated using a 24-month rolling window.

least in the last period, exhibit the highest correlation with the first component. While before the first half of 2007 correlations with the first component were all less than 60%, with only one exception, and no clear cluster of highly correlated entities could be detected, starting from the second half of 2007, European banks and insurers started converging towards high value of correlation and their departure from US entities was particularly clear in 2009 and 2010. US entities are usually ranked among the medium-highly correlated, while entities from other Countries usually show low correlation.

In the first half of 2007 the correlation with the first component experienced a sharp drop for a number of entities, reflected also in Figure 16 in a small drop of the explained variance of the various components. This could be due to a structural break in the components of systematic risk: the first half of 2007 was the time in which markets started realizing the dangers of the housing bubbles and securitized mortgages. This may have shifted investors concerns towards different risk factors., thus changing the correlation structure which had existed up to that moment.

10 TailDep

Correlations, however, do not tell the full story of a crisis. While it is certainly true that they convey an idea of how interconnected markets are and how default risks co-move, their explanatory power about the systemic risk generated by and within the financial sector is limited.

Our purpose is to derive a measure of systemic risk which is: (i) endogenous, i.e. defined entirely and exclusively by the interconnectedness of the financial system and not by exogenous factors, such as asset size, (ii) meaningful, in the sense that it can be related to physical quantities, (iii) theoretically motivated.

To deduce such a measure we need to first discuss a model of systemic risk in which the network structure of interconnections is able to generate endogenous probabilities of default.

10.1 Model

Consider an economy with N financial institutions, where we assume that N is large. Institutions may be in one of two states: distressed or not distressed. An institution is distressed if it is experiencing a relevant credit event which leads almost surely to default. We denote with $p_{i,t}$ the probability that institution i is distressed at time t. To be more precise, $p_{i,t}$ is an endogenous probability of being in distress, i.e. a probability that takes into account only the possibility of receiving shocks from the rest of the financial system, and not the possibility that i may be a source of exogenous shocks. We then define a matrix D whose element d_{ij} indicates the probability of i being in distress conditional on j being in distress. Using the network terminology, we say that if $d_{ij} > 0$, then i and j are neighbors[8].

We take a first order approximation of the contagion model by assuming that, for any institution i, the distress of any two neighboring institutions are independent events[9].

With a fixed probability δ an institution is guaranteed by the policy-maker, thus effectively removing it from the network. Finally, we denote by $\phi_{i,t}$ the probability that institution i does not receive shocks from its neighbors at time t, that is:

$$\phi_{i,t} = \prod_{j \neq i} (1 - d_{ij} p_{j,t-1}) \tag{11}$$

Hence we can write

$$1 - p_{i,t} = \delta + (1 - \delta)[(1 - p_{i,t-1})\phi_{i,t}] \tag{12}$$

If we assume that $p_{i,t-1}$ and $d_{ij} p_{j,t-1}$ are all very small, we can approximate this expression as

$$1 - p_{i,t} = \delta + (1 - \delta)\left[1 - p_{i,t-1} - \sum_{j \neq i} d_{ij} p_{j,t-1}\right]$$

and obtain the simple relation

$$p_{i,t} = (1 - \delta) \sum_{j \neq i} d_{ij} p_{j,t}$$

Hence, we can write the following linear dynamical system for $P_t = (p_{1,t}, \ldots, p_{N,t})^T$:

$$P_t = (1 - \delta) D P_{t-1}. \tag{13}$$

D is a nonnegative matrix, hence the Perron-Frobenius theorem guarantees us that its leading eigenvalue $\bar{\lambda}$ is positive. Moreover if λ_j is an eigenvalue of D, then $(1 - \delta)\lambda_j$ is an eigenvalue of $(1 - \delta)D$.

Standard results in the theory of linear dynamical system ensure that, for P_t not to diverge, a necessary and sufficient condition is that $(1 - \delta)\bar{\lambda} < 1$. In particular, if this is the case, the steady state distress probabilities will be zero. For this condition

[8] Our model has indeed some affinities with the mathematical modeling of epidemics on networks, e.g. Dezső and Barabási (2002), Newman (2002), Pastor-Satorras and Vespignani (2001) and Wang et al. (2003).

[9] This assumption is not made only for mathematical tractability, but also for computational feasibility. Estimating the probability of distress of each entity i conditional on the distress of any possible set of other entities would require an amount of estimations growing as 2^N, which is unfeasible for large systems.

to hold, the policy-maker has to enforce a bail-out policy such that

$$\delta > 1 - \frac{1}{\bar{\lambda}}. \tag{14}$$

Note that if $\bar{\lambda} < 1$ no bail-out is needed, since shocks will be dampened by the dependency structure of the financial system. This makes clear that $\bar{\lambda}$ is a measure of the systemic risk generated by the financial systems, and this measure is related to the efforts a policy-makers has to make in order to prevent the spreading of shocks across the system.

Suppose that no stabilization policy is enforced, then the probabilities of distress evolve according to:

$$P_t = DP_{t-1}. \tag{15}$$

If the leading eigenvalue of D, $\bar{\lambda}$, is less than one, then system is safe and the system does not generate systemic risk, otherwise P_t diverges. In this case we can find a basis of eigenvectors $\{v_1, \ldots, v_n\}$ corresponding to the eigenvalues $\{\lambda_1, \ldots, \lambda_n\}$. Without loss of generality we assume $\lambda_1 = \bar{\lambda}$ and denote v_1 as $\bar{v}$. Equation 15 can be written as $P_t = D^t P_0$ and P_0 may be expressed as a linear combination of the set of the eigenvectors: $P_0 = \sum_k \alpha_k v_k$. Therefore, substituting in equation 15, yields:

$$P_t = D^t P_0 = \sum_k \lambda_k^t \alpha_k v_k = \lambda_1^t \sum_k \left(\frac{\lambda_k}{\lambda_1}\right)^t \alpha_k v_k. \tag{16}$$

Since $|\lambda_1| > |\lambda_k|$ for any $k \neq 1$, as t increases the dominating direction of increase will be given by the leading eigenvector, $v_1 = \bar{v}$, and the rate of increase will depend on $\lambda_1 = \bar{\lambda}$, i.e.

$$P_t \sim \alpha_1 \bar{\lambda}^t \bar{v}. \tag{17}$$

Hence, bank i's probability of being shocked through interconnections is proportional to the i-th element of the leading eigenvalue. This is what in network terminology is referred to as eigenvector centrality or eigenvector score.

If the leading eigenvalue is less than one, then the system has a stable fixed point at zero and banks would not face any probability of contagious shocks in steady state. Thus, the value of their assets remains the original one, that we denote as a_i. If instead the leading eigenvalue is larger than 1, there exists a positive probability of contagious default, which is $p_{i,t}$ and increases in time. Hence the expected value of their assets as shocks flow through the system is $(1-p_{i,t})a_i$. If a stabilization policy $\bar{\delta} > \frac{\bar{\lambda}-1}{\bar{\lambda}}$ is enforced, the equilibrium asset value of bank i is certain and equal to a_n.

If banks are risk neutral and the leading eigenvalue is larger than 1, they will be willing to pay a tax equal to $t_i a_i = p_{i,t} a_i$ to finance a stabilization policy. It follows that the ratio of the optimal tax rates two banks, i and j, are willing to pay is

$$\frac{t_i}{t_j} = \frac{p_{i,t}}{p_{j,t}} = \frac{\bar{v}_i}{\bar{v}_j}. \tag{18}$$

If the policy-maker has to raise total resources of $\bar{T} = \bar{\delta} \sum_n a_n = \frac{\bar{\lambda}-1}{\bar{\lambda}} \sum_n a_n$, a tax rate

$$\bar{t}_i = \frac{\bar{v}_i}{\sum_j \bar{v}_j a_j} \bar{T} \tag{19}$$

is able to: (i) cover the financial needs of the policy-makers, (ii) satisfy the equilibrium condition for risk neutral institutions.

Moreover, we can show that such a tax is able to align the incentives of banks with those of the policy-maker, reducing the connectivity of the network and, thus, the implied network externalities.

Micro-foundation of the Network

We focus only on the linking decisions of banks, which have to chose their degree of dependence on other entities, given their net worth and liabilities, which sum up to total assets a_i. The maximization problem of banks thus becomes:

$$\max_{d_{i1},\ldots,d_{in}} \Pi_i = \pi_i(d_{i1}, \ldots, d_{i,n}) + (1 - t_i)a_i \tag{20}$$

$$\text{s.t.} \quad 0 \leq d_{ij} \leq 1 \quad \forall j \tag{21}$$

$$\text{s.t.} \quad d_{ii} = 0 \tag{22}$$

where π denotes the expected payoff of a bank as a function of the linking decisions. This function captures the gains a bank expects to obtain from connections, as well as the risks connections involve, since it includes the believes of banks about the riskiness of exposures. Function π_i, then, represents, in a sense, the set of micro-prudential incentives of banks. As such, this expected payoff does not depend on the level of systemic risk, since it captures the *a priory* believes a bank has on the default risk of other banks.

As for its shape, we assume that (i) it is twice continuously differentiable in $(0, 1)^n$, (ii) it is strictly quasiconcave, (iii) $\pi_i(0, \ldots, 0) = 0$ and (iv) for every $j \neq i$, $\lim_{d_{ij}\to 1} \frac{\partial \pi_i}{\partial d_{ij}} < 0$. Strict quasiconcavity reflects the fact that a convex combination of two connectivity levels is always better than the worst one, condition (iii) states that no connections do not yield gains nor losses and the assumptions at the extreme tells that full dependence is not desired.

Suppose that taxation does not depend on the choice of connections, then, under the above assumptions, there exists a unique set of connections $(\bar{d}_{i1}, \ldots, \bar{d}_{in})$ that maximizes 20 and it satisfies:

$$\left.\frac{\partial \pi_i}{\partial d_{ij}}\right|_{d_{ij}=\bar{d}_{ij}} = -\mu_j \tag{23}$$

where μ_j is the non-negative Lagrange multiplier associated with constraint $d_{ij} \geq 0$. If $\bar{d}_{ij} > 0$ then $\mu_j = 0$.

Consider now the policy-maker's problem:

$$\max_{d_{i1},\ldots,d_{in},t_1,\ldots,t_n,\delta} W = \left[1 - P(\xi|\delta)\right] \sum_{i=1}^{n} \left[\pi_i(d_{i1},\ldots,d_{i,n}) + (1-t_i)a_i\right] \tag{24}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} a_i t_i = T = \delta \sum_{i=1}^{N} a_i \tag{25}$$

$$\text{s.t.} \quad 0 \leq d_{ij} \leq 1 \quad \forall i, j \tag{26}$$

$$\text{s.t.} \quad d_{ii} = 0 \quad \forall i \tag{27}$$

where $P(\xi|\delta)$ is the probability of a systemic breakdown given the bailout policy δ.

As for the discussion above, if $\delta < \frac{\bar{\lambda}-1}{\bar{\lambda}}$ then $P(\xi|\delta) = 1$, if instead $\delta > \frac{\bar{\lambda}-1}{\bar{\lambda}}$ then $P(\xi|\delta) = 0$. This condition, together with the first constraint, ensures that the welfare function is strictly positive as long as $\delta < 1$ which is the case for a finite $\bar{\lambda}$.

Now consider a bailout policy $\bar{\delta} > \frac{\bar{\lambda}-1}{\bar{\lambda}}$, so that the total amount of resources to be collected is $\bar{T} = \bar{\delta} \sum_i a_i$, and consider a tax rate $\bar{t}_n = \frac{\bar{v}_n}{\sum_k \bar{v}_k a_k} \bar{T}$. Then the planner objective function becomes:

$$W = \sum_{i=1}^{n} \left[\pi_i(d_{i1},\ldots,d_{i,n}) + (1-\bar{t}_i)a_i\right], \tag{28}$$

and the bank's objective function becomes:

$$\Pi_i = \pi_i(d_{i1},\ldots,d_{i,n}) + (1-\bar{t}_i)a_i. \tag{29}$$

Hence the set of choices maximizing 29 also maximizes 28. It is clear that a tax designed in that way depends on the connection choices of the bank since $\bar{v}_n = \sum_k d_{ik}\bar{v}_k$.

Hence in equilibrium every bank will satisfy

$$\left.\frac{\partial \pi_i}{\partial d_{ij}}\right|_{d_{ij}=d^*_{ij}} = -\mu_j + \frac{1}{\lambda^*}\frac{a_i T^*}{\sum_k a_k v^*_k} v^*_j, \tag{30}$$

where the stars denote the socially optimal equilibrium values of the variables, in order to distinguish them from the spontaneous equilibrium values, indicated with bars, when no regulatory intervention was enforced. Note that $\mu_j \geq 0$, with equality for interior solutions, and $\frac{1}{\lambda^*} \frac{a_i T^*}{\sum_k a_k v_k^*} v_j^* \geq 0$, with equality if $v_j^* = 0$ or if $T^* = 0$, which is the case if $\lambda^* < 1$.

The following theorems highlight how the tax does not distort equilibrium structures which do not generate any systemic risk, while it provides an incentive for banks to reduce their degree of interconnectedness if the system exhibits systemic fragility.

Theorem 1. *If the spontaneous equilibrium yields $\bar{d}_{ij} = 0$, then also $d^*_{ij} = 0$.*

Proof. If $\bar{d}_{ij} = 0$ then $\frac{\partial \pi_i}{\partial d_{ij}}\Big|_{d_{ij}=0} < 0$. Strict quasiconcavity implies that $\frac{\partial \pi_i}{\partial d_{ij}} < 0$ for any d_{ij}. If the solution were interior, relation 30 would imply $\frac{\partial \pi_i}{\partial d_{ij}} \geq 0$, which is impossible. □

Theorem 2. *If the spontaneous equilibrium of the system is an acyclic network, then this is also a socially optimal equilibrium.*

Proof. If the acyclic network is an equilibrium when no tax is implemented, then it maximizes π_i for all i, thus any deviation cannot strictly increase the payoff of banks. Moreover, in an acyclic network the centrality of every institution is zero, thus any deviation cannot reduce the tax, which is nonnegative. Hence, the acyclic network remains an equilibrium. □

Theorem 3. *If the spontaneous equilibrium of the system yields a network with $\bar{\lambda} < 1$, then this network remains a socially optimal equilibrium.*

Proof. In this case no tax would be imposed and problem 29 would remain equal to problem 20. □

Theorem 4. *If the spontaneous equilibrium of the system is a network with $\bar{\lambda} > 1$ and $\bar{d}_{ij} > 0$, then, when the stabilization policy is enforced, $d^*_{ij} \leq \bar{d}_{ij}$, with strict inequality if $v^*_j > 0$.*

Proof. If $d^*_{ij} = 0$ then the statement is trivial. If $d^*_{ij} \neq 0$, then equation 30 implies $\frac{\partial \pi_i}{\partial d_{ij}}\Big|_{d_{ij}=d^*_{ij}} \geq 0$, with strict inequality if $v^*_j > 0$. Under the assumption of strict quasiconcavity this implies that $d^*_{ij} \leq \bar{d}_{ij}$, with strict inequality if $v^*_j > 0$. □

A tax policy as the one we have suggested has thus important implications: (i) it provides an incentive to reduce the degree of connectedness, (ii) it conveys an information on the centrality (and thus the susceptibility to contagion) of other institutions, (iii) it forces banks to internalize the network externalities they impose.

Our result is consistent with a strand of economic literature on network externalities (Brock and Durlauf, 2001; Fagiolo, 2005; Katz and Shapiro, 1985; Liebowitz and Margolis, 1994) and on asymmetric information in financial markets (Stiglitz and Greenwald, 1986). Uncertainty regarding the network structure of the economy leads agent to over-connect with respect to the socially desirable level. Linking decisions of a bank i affect its centrality, which thus in turn affects the centrality of the banks linking to it. If the centrality of other banks is ignored, institutions may only rely on their *a priori* believes, captured by their expected payoff function π_i. The presence of a tax proportional to a trader's centrality allows, in equilibrium, to establish the optimal connections as if the network structure were known.

10.2 Estimating Interconnections from Market Data

We have seen in the previous subsection a model of interconnectedness and government stabilization policy. The model have two major advantages: (i) we can find closed-form solution for the systemic importance of institution and for the level of systemic risk generated by the system, (ii) it is entirely focused on systemic-risk and exogenous probability of defaults are not needed.

While it may seem very difficult to estimate such a model, recent advancements in multivariate statistics allow to construct matrix of conditional probabilities of default. This constitutes the empirical contribution of this paper, i.e. the TailDep methodology, which aims at estimating from real data the relevant quantities identified in the model. In order to do so we need to estimate the tail dependencies of CDS returns using a copula model.

In its simplest definition an n-copula may be defined as a multivariate distribution function whose support is the unit hypercube $[0,1]^n$ and whose marginals are uniformly distributed (Embrechts et al., 2003). More specifically a copula function $C\colon [0,1]^n \to [0,1]$ is such that: (i) $\forall U = (u_1, \dots, u_n) \in [0,1]^n$, $C(U) = 0$ if at least one element of U is zero, (ii) $\forall U \in [0,1]^n$, $C(U) = u_i$ if $u_j = 1, \forall j \neq i$, (iii) $\forall U, W \in [0,1]^n$ s.t. $U \leq W$ $\mu_C([U,W]) \geq 0$, where $\mu_C([U,W])$ is the probability measure induced by the copula on the hypercube defined as $[u_1, w_1] \times \cdots \times [u_n, w_n]$.

The great advantage of modeling random variables with copulas is the possibility to disentangle the univariate margins from their dependence structure, which is what the copula formalization captures.

Several measures of correlation and dependence may be computed given a copula model (Embrechts et al., 2003, 2002), but for our purposes the concept of tail dependence it the most relevant one. The tail dependence between two random variables expresses the probability of the two variables being contemporaneously in the tail of their distribution. In the context of CDS spread returns, it amounts to the probability of two spread to experience sudden and large co-movements.

In general terms, given two spread returns, r_i and r_j and their CDFs, F_i and F_j, their coefficient of upper tail dependence is

$$\tau_{ij}^U = \lim_{u \to 1} \mathbb{P}[r_i > F_i^{-1}(u) | r_j > F_j^{-1}(u)]$$

and indicates the probability of r_i lying in the upper tail of its distribution (thus possibly tending to infinity) conditional on r_j taking an extreme value. Joe (1997) provides an alternative formulation of the tail dependence in the case of a bidimensional copula, showing how the tail dependence is an property driven by the copula structure of the data generating process:

$$\tau^U = \lim_{u \to 1} \frac{1 - 2u + C(u,u)}{1-u}.$$

Hence, fitting a copula to our data would prove to be an important source of information regarding the dependence structure between financial institutions. However, since data to be fitted must be i.i.d. and spread returns exhibit some autocorrelation and volatility clusters, we fit the copula model on the normalized residuals z_i.

Various contributions in the financial literature (Longin, 2000; McNeil and Frey, 2000; Nystrom and Skoglund, 2002) show how empirical CDFs perform poorly when the tails of the distribution are the main concern of the analysis. Thus, before estimating the copula model, we fit the top and lower 10% of the observations with a Generalized Pareto distribution, in order to account for the presence of fat tails, while we allow the interior of the distribution to be represented by the empirical one.

As a model, we chose the t-copula since, as argued by Breymann et al. (2003) and Mashal and Zeevi (2002), it provides a better model for the analysis of extreme events, and, more specifically, of extreme co-movements of the variables. A t-copula is parametrized by its degrees of freedom ν_{ij} and the Pearson correlation coefficients ρ_{ij}, which are estimated via maximum likelihood.

The t-copula, in addition to being a good fit for financial time series, has, in the bivariate case, interesting properties which allow to compute the tail dependencies analytically.

Indeed the dependence in the upper-right tail of the distribution of the residuals is defined in the standard way as

$$\tau_{ij}^U = \lim_{u \to 1} \mathbb{P}[z_i > F_i^{-1}(u) | z_j > F_j^{-1}(u)]$$

which, for a bivariate t-copula, takes a convenient closed form solution (Embrechts et al., 2003):

$$\begin{aligned}\tau_{ij}^U &= 2 \lim_{z^* \to +\infty} \mathbb{P}[z_i > z^* | z_j = z^*] \\ &= 2\left[1 - t_{\nu_{ij}+1}\left(\sqrt{(\nu_{ij}+1)\frac{(1-\rho_{ij})}{(1+\rho_{ij})}}\right)\right]\end{aligned} \tag{31}$$

where ρ_{ij} is the Pearson correlation coefficient, ν_{ij} the degrees of freedom of the bivariate t-copula and $t_{\nu_{ij}+1}$ is the CDF of a t-distributed random variable with $\nu_{ij}+1$ degrees of freedom.

As we have seen in Section 8, large jumps in the CDS spread of an entity reflect large jumps in the intensity of the Poisson process underlying the arrival of default. An infinite spread correspond to an infinite intensity of default, i.e. immediate default. The tail dependence of CDS spreads is thus a measure of conditional default and the estimated tail dependence parameter between the spread returns of institution i and institution j, τ_{ij}^U, is thus an estimate of the element d_{ij} of the matrix D used in the in the model to describe the dependence structure of the financial system. The estimated matrix of dependencies is what we define as TailDep network.

11 Results of TailDep

Once the tail dependence (TailDep) network has been estimated we can perform the estimation of the relevant measures of the model.

A graphical representation of three realizations of the TailDep network is provided in Figures 18, 19 and 20. They represent the maximum spanning tree of the TailDep network estimated in the periods 2005-2006, 2008-2009 and 2011-2012, respectively. A network can be effectively displayed by showing its maximum spanning tree, which is the graph obtained from the original one by retaining the most relevant connections and deleting the redundant ones due to closed cycles. The resulting structure is thus tree-like and connects all the entities belonging to connected components using the links with the largest weights.

A look at the Figures reveals clusters of entities grouped according to geographical location and sector of activity. Groups of European, US, Japanese and Australian companies can easily be detected and, within each of them, banks, insurers and other entities tend to have the tighter connections with entities of the same sector.

In order to identify the most connected entities we compute the eigenvector centrality of institutions, which we have proved to be a measure of systemic importance in the previous section[10]. Since this centrality measure has clear systemic risk im-

[10]We recall that this estimated network is symmetric, therefore avoiding the problems, pointed out by Bonacich and Lloyd (2001), of connected nodes receiving no centrality in directed networks.

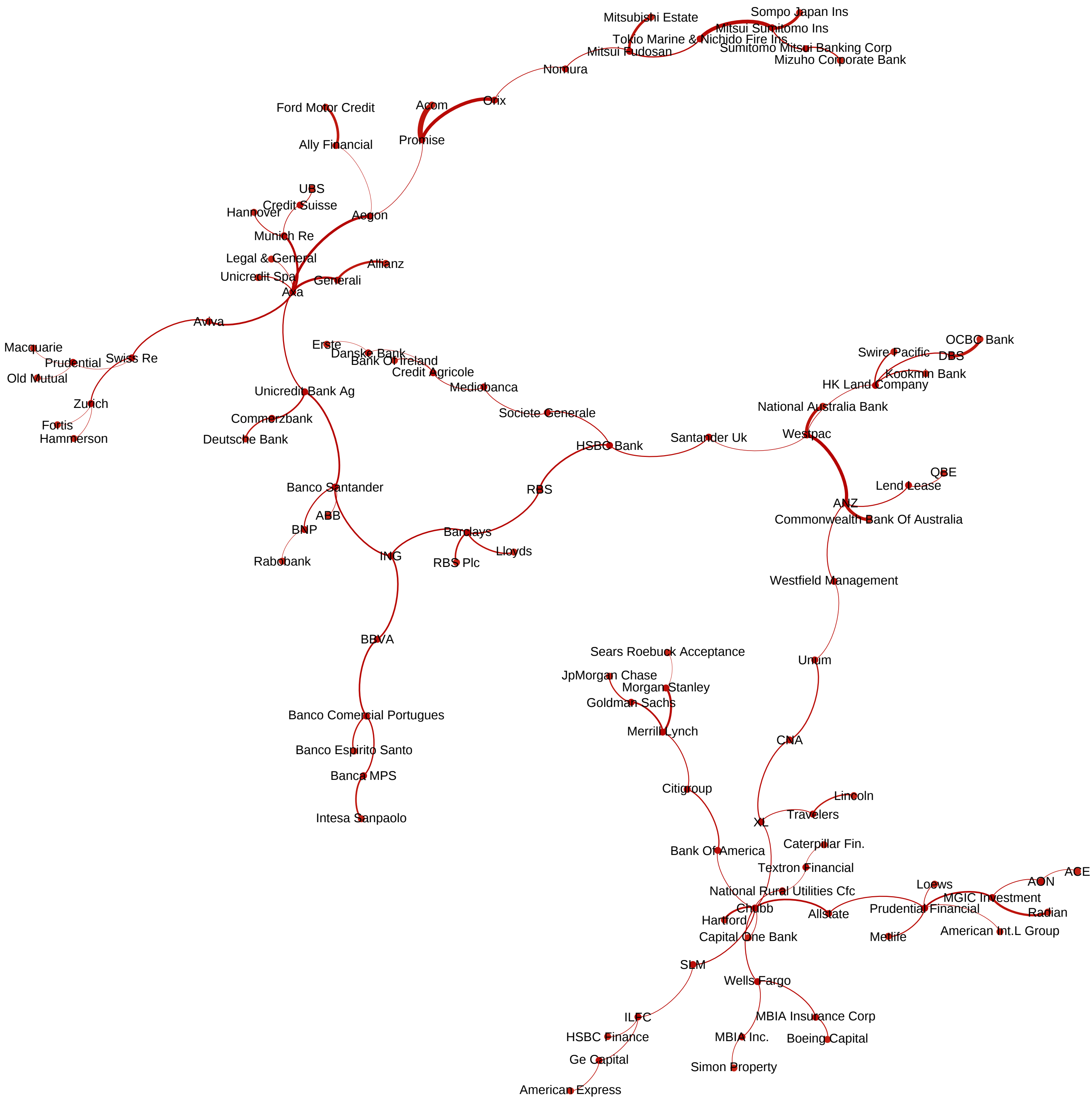


Figure 18: Maximum spanning tree of TailDep network for the period 2005-2006. The maximum spanning tree representation of a network provides a visualization of the most relevant links, deleting the redundant information contained in closed triangles.

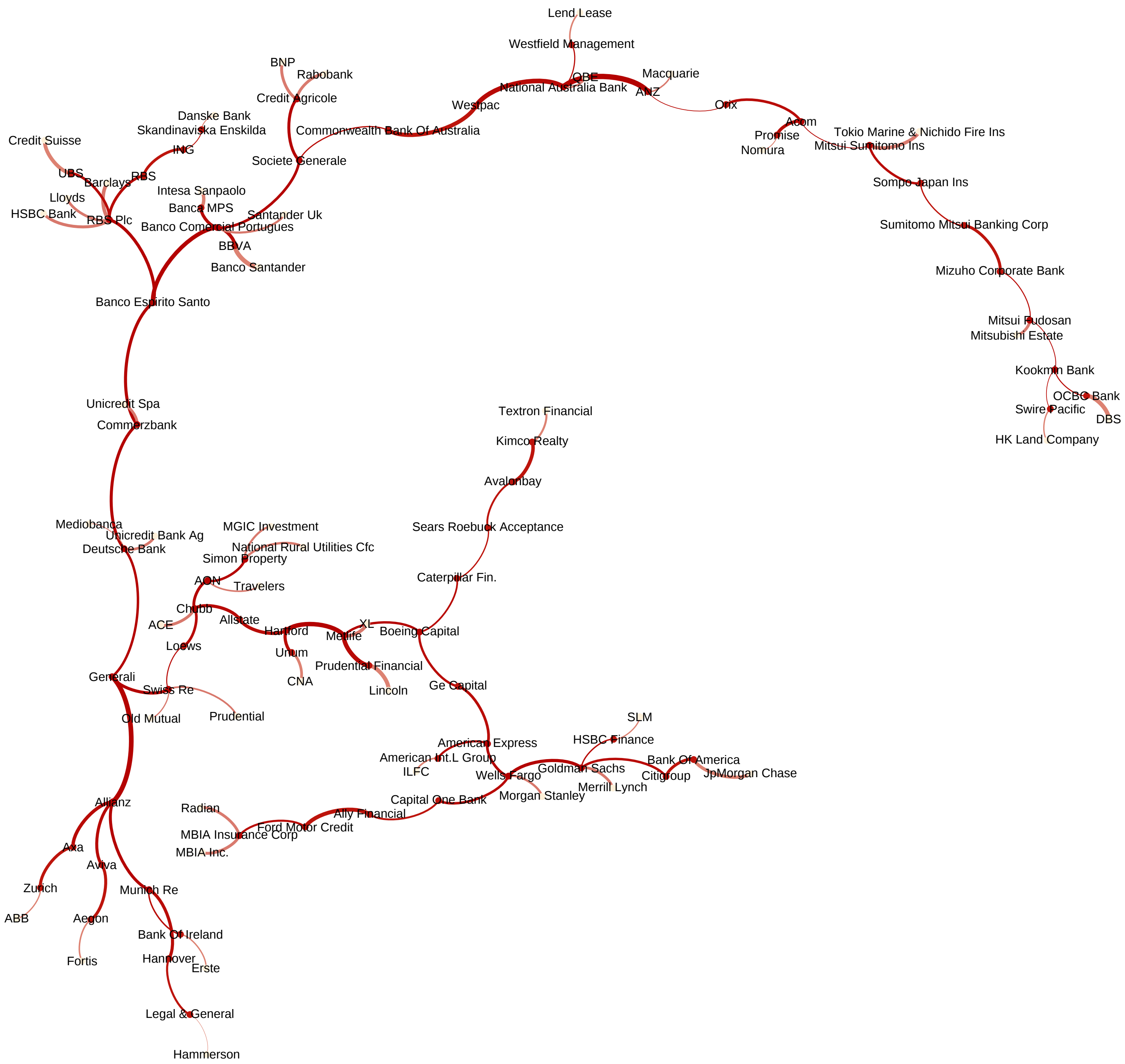


Figure 19: Maximum spanning tree of TailDep network for the period 2007-2009. The maximum spanning tree representation of a network provides a visualization of the most relevant links, deleting redundant information contained in closed triangles.

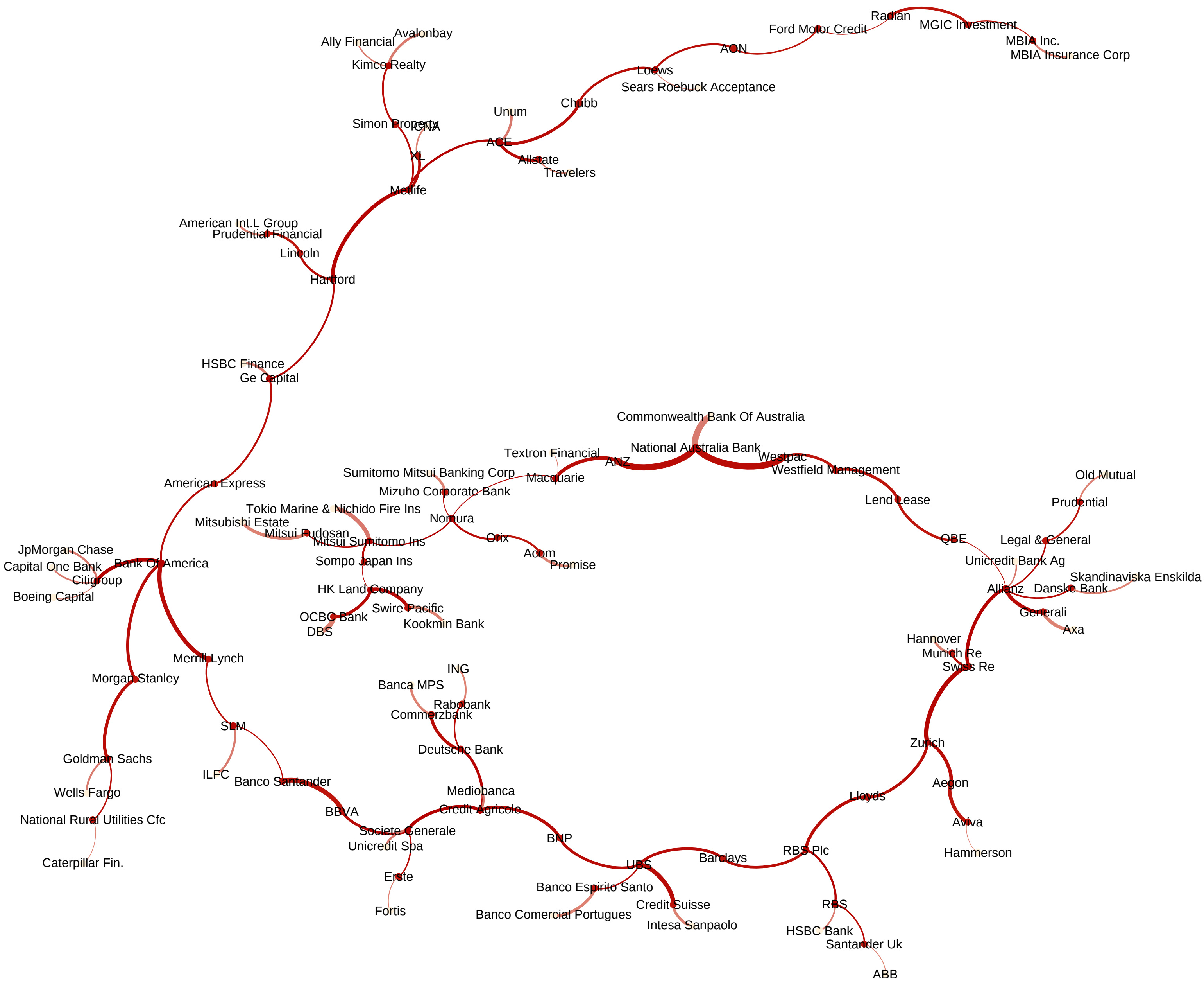


Figure 20: Maximum spanning tree of TailDep network for the period 2010-2012. The maximum spanning tree representation of a network provides a visualization of the most relevant links, deleting redundant information contained in closed triangles.

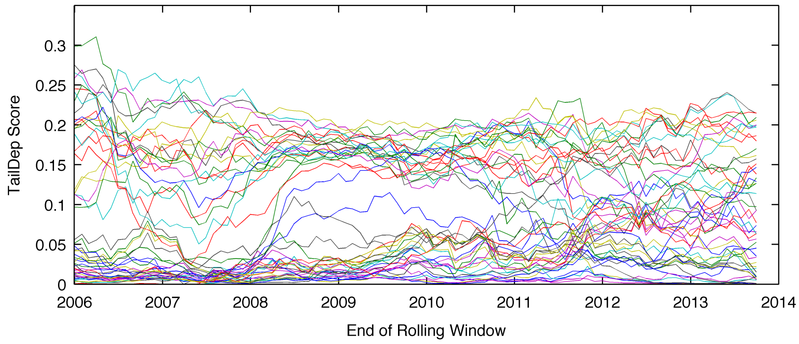
Table 3: TailDep score estimated in the periods 2005-2006, 2008-2009 and 2011-2012. The score is in percentage terms and is normalized in such a way that the norm of the corresponding eigenvector is unitary.

2005-2006		2008-2009		2011-2012	
Institution	Score	Institution	Score	Institution	Score
Axa	26.34	Banco Espirito Santo	20.26	Aegon	23.82
Generali	26.16	Banco Comercial Português	19.27	Credit Suisse	22.70
Unicredit Bank Ag	25.60	RBS Plc	19.12	Banco Santander	22.32
Allianz	24.44	Deutsche Bank	18.39	BBVA	21.97
Munich Re	23.71	Swiss Re	18.24	Zurich	21.07
Banco Comercial Português	23.09	Credit Agricole	18.19	UBS	20.25
BBVA	22.73	Allianz	18.05	Deutsche Bank	20.11
Barclays	22.72	Munich Re	17.26	Commerzbank	19.62
Banco Santander	22.36	Rabobank	17.09	Swiss Re	19.40
Aegon	20.87	Société Génerale	17.07	Credit Agricole	19.33
Commerzbank	20.37	Zurich	16.97	Société Génerale	18.74
ING	18.73	RBS	16.93	Lloyds	18.10
Deutsche Bank	17.62	Banco Santander	16.76	Generali	17.08
Lloyds	17.34	BBVA	16.66	BNP	16.85
Banca MPS	16.73	Aegon	16.42	Allianz	16.64
Banco Espirito Santo	15.89	Generali	16.37	Aviva	16.26
HSBC Bank	15.84	Commerzbank	16.27	RBS Plc	15.34
Aviva	15.66	ING	16.02	Axa	15.31
RBS Plc	15.32	Barclays	15.70	Banca MPS	14.88
BNP	14.79	Unicredit Bank Ag	15.43	Unicredit Spa	14.80

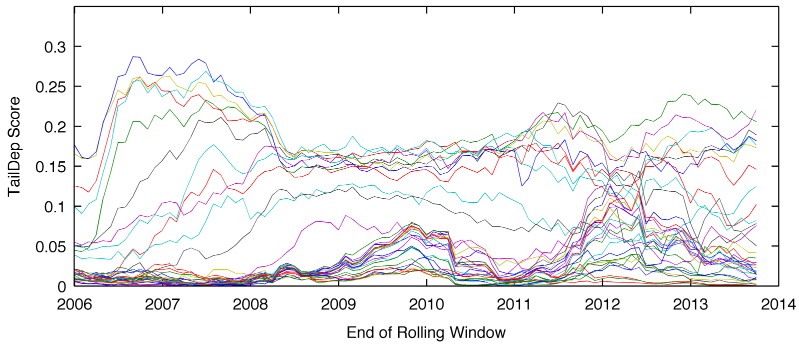
plications and since it has been estimated with the TailDep methodology, we call it TailDep score. Table 3 reports the scores of the most connected 20 entities in the sub-periods corresponding to 2005-2006, 2008-2009 and 2011-2012. We see how European entities lead the table of the most systemic institutions, with large banks and insurers progressively gaining importance.

According to our model, this means that European entities are those who would benefit the most from a stabilization program. Interestingly, a number of insurers are represented among the most connected institutions, especially in the first and last period, highlighting the increasing importance insurance companies are gaining at a systemic level. In the middle period, which corresponds to the acute phase of the crisis, the top three entities in terms of centrality are distressed European banks, probably reflecting their susceptibility to contagion risk.

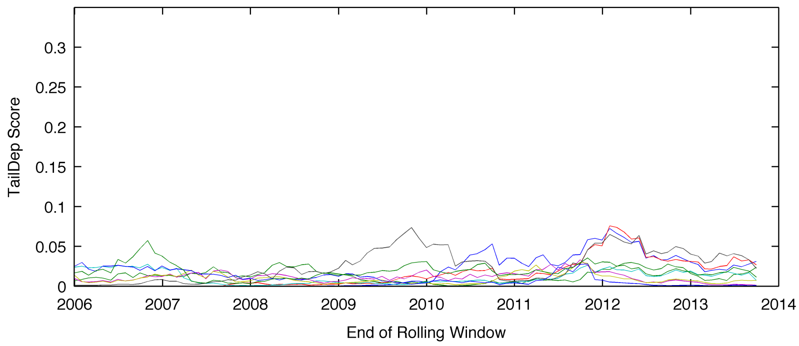
By comparing the centrality scores in the three periods, one may not how the largest European institutions gradually gained importance. The width of the edges in the three Figures, which is proportional to the tail dependence, also highlights a generalized increase in the tail dependencies of financial firms. Dependencies increased, in particular, between European institutions, possibly reflecting the weak-



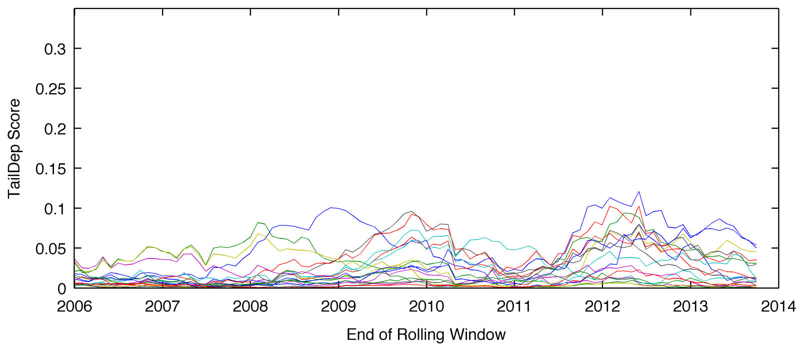
(a) Banks



(b) Insurers

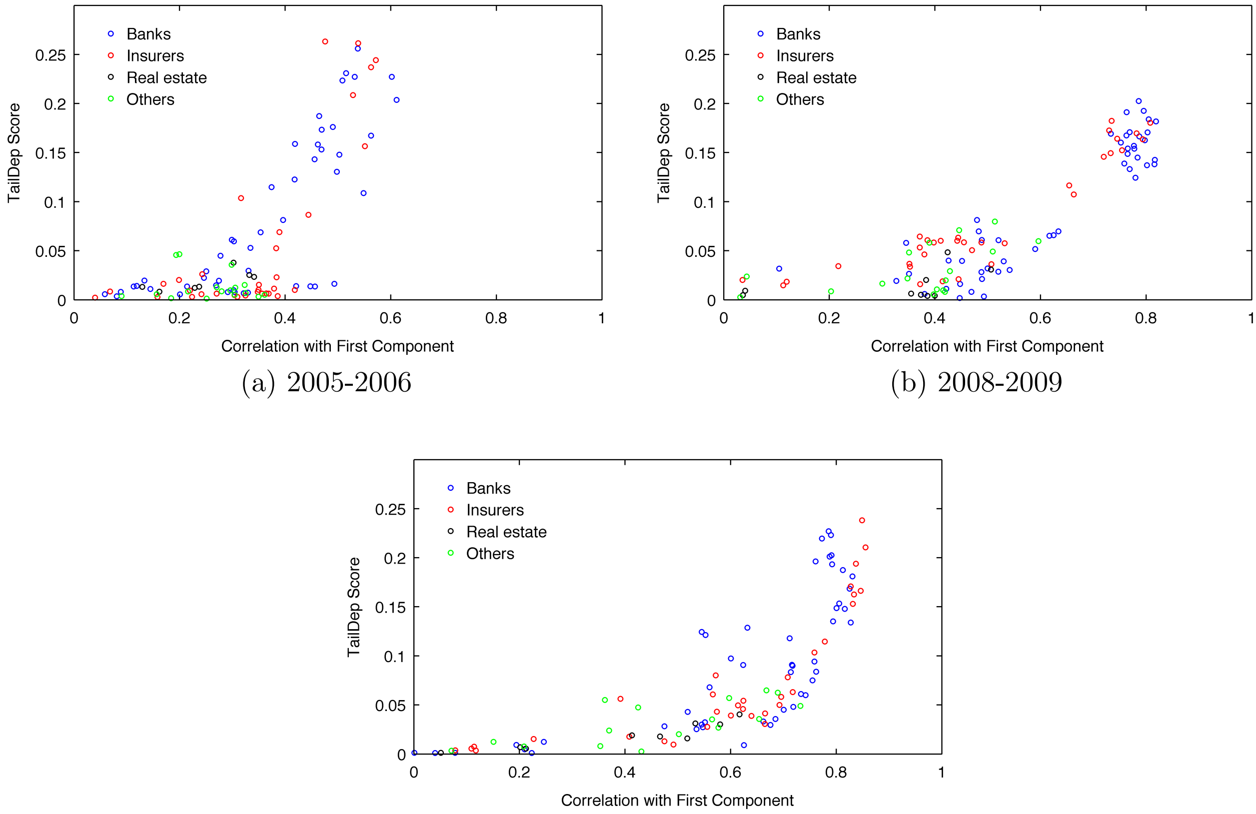


(c) Real Estate



(d) Others

Figure 21: Evolution of TailDep score computed using the eigenvector centrality in the tail dependence network estimated with a 24-month rolling window. The score is normalized in such a way that the norm of the corresponding eigenvector is unitary.



(a) 2005-2006

(b) 2008-2009

(c) 2011-2012

Figure 22: Relation between correlation with the first principal component and TaiDep score in selected sample periods.

ening of the macro-economic conditions, which may have increased the dependencies of financial institutions between themselves. The raise, in the ranks, of large central European entities may indeed reflect the increasing concerns of investors regarding the riskiness of uncertain intra-financial linkages.

Figure 21 provides the time evolution of the centrality score of each institutions. The TailDep score has been normalized so that the corresponding eigenvector has norm one. One can clearly note how banks and insurers have always been the most connected entities along the whole sample period. As for banks and insurers one can observe dynamics which are consistent with those found in Figure 17 for the correlation with the first principal component. Indeed in the period from 2008 to 2011 included, the system seem to be more polarized, with European entities being clearly more central than other institutions, reflecting the presence of different sets of institutions with different levels of exposures to systemic risk. In the end of the sample period, however, this clear grouping disappears, large banks and insurers started occupying the most central positions and, probably as a consequence of this fact, the distribution of the TailDep score became less polarized around two regimes.

This observations are corroborated also by Figure 22 which plots the relation

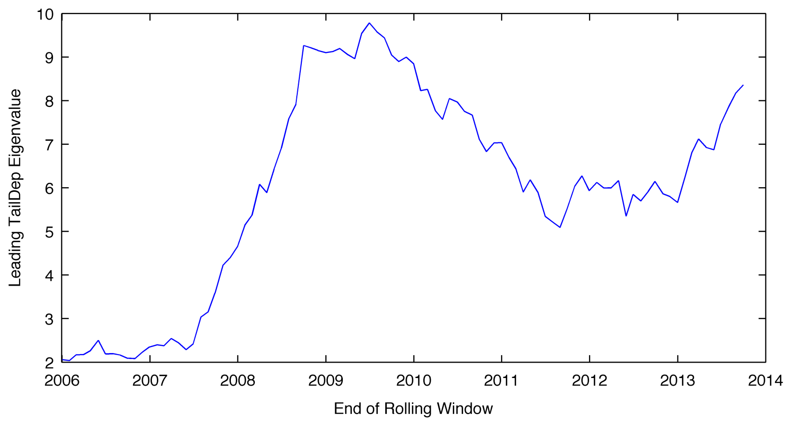


Figure 23: Leading eigenvalue of TailDep network, which is a measure of systemic risk and interconnectedness of the system.

between the correlation with the first component and the TailDep score. The relation is clearly non-linear and, in 2008-2009, a group of entities was forming a cluster displaying both high correlation with systematic factors and high TailDep score.

Figure 23 shows the leading eigenvalue of the TailDep network estimated using a 24-month rolling window. As we have shown in Section 10.1, the leading eigenvalue of the dependence network is a measure of systemic risk since it is related to the intensity of government's intervention in mitigating the contagion risk. We see that the eigenvalue markedly increased from the end of the first half of 2007, it remained at high levels until the second half of 2009, and started decreasing afterwards. It then fluctuated at intermediate values from mid 2011 to the end of 2012, when it again started increasing.

The first marked increase is a clear effect of the financial crisis, however, most of the systemic risk had already been priced by the TailDep network before the Lehman episode. The decrease seems instead to be due to the European Sovereign debt crisis. In that period financial institutions have become increasingly connected to their government and a segmentation of the European market happened. Indeed, the lower level of systemic risk represented by the TailDep eigenvalue coincides with the peak of the sovereign crisis. In the second half of 2012 sovereign concerns were alleviated, thus weakening the sovereign-bank nexus. The dependence of financial institutions among themselves subsequently began increasing again, together with the systemic risk.

12 Conclusions

In this paper we put forward a view of systemic risk as the risk generated *by* and *within* the financial system. We provided a model where its quantification is achieved by solving for an optimal stabilization policy undertaken by a policy-maker. The policy-maker can fund this policy by taxing banks according to their centrality in the financial system. If such a tax is implemented, banks' incentives coincides with the regulators' and a lower degree of interconnectedness is chosen by banks in equilibrium.

Our contribution, from an operational point of view, allows to measure the risk generated by a financial system, with no need to bring into its quantification exogenous risk measures, which are instead sources of idiosyncratic risk. Idiosyncratic risk regards indeed a triggering event, whose impact on the overall system depends on its inherent fragility.

From a theoretical point of view, we put forward a model with a large number of heterogenous banks, showing how, in an economy viewed as a complex interconnected system, the quantification of systemic risk is feasible. Moreover, we highlighted that regulators should be concerned about the risk generated by the system itself, which stems from network externalities, worsened by imperfect information.

Finally, from an empirical point of view, we proposed the TailDep methodology, which is based on the estimation of the conditional probabilities of default through a copula model. Our theoretical framework allowed us to identify the relevant measure of systemic risk and systemic importance of institutions, and to observe their evolution in the last two decades.

While the use of CDS spreads for the estimation of financial dependencies necessarily implies the use of public information, regulators, which have access to confidential data, may effectively adopt the TailDep methodology as an early warning signal of the building up of systemic fragility.

In conclusion, we would like to remark the need to proper data collection by regulators. Indeed, confidentiality issues and the increasing complexity of markets often prevent full disclosures of the financial linkages institutions have. Regulators, by collecting and analyzing exposure data in a confidential manner, may put in place a set of incentives for financial institutions in order to align, in due time, banks' objectives with those dictated by a macro-prudential regulation.

Concluding Remarks

In this thesis we made two contributions to the current debate on systemic risk and macro-prudential regulation. Our approach was to consider the economy as a complex system of interacting agents, in which the probability of a systemic event crucially depends on its interconnectedness.

In Part I, *"Defuse the Bomb: Rewiring Interbank Networks"*, we investigated the resilience to contagion of interbank networks under a variety of assumptions regarding their architecture and banks' portfolio. We showed how diversification in the interbank market works both as a risk sharing strategy and as a device to increase the channel of shock transmission. Hence, contagion risk is non-monotonic in diversification and there exists an interval of connectivity in which the system is robust-yet-fragile, i.e. the probability of contagion is low but, when it happens, it affects the entire system. More heterogenous structures in terms of connectivity, due to a power-law distribution of the number of connections among traders, are able to sensibly stabilize the system when shocks are random. However, too-connected-to-fail and too-big-to-fail banks may exist, posing high contagion risk if distressed. Moreover, the former are more systemically relevant than the latter, calling from a more holistic view on the systemic importance of institutions, which does not solely rely on banks' size.

We also explored a more complete model of interbank market, in which a short-term and a long-term segment are present. Banks have to meet regulatory capital requirements and this may induce runs, which are cleared as a perfect information equilibria. This dynamic and micro-founded model highlights the challenges regulators face when the market is seen as a complex system of interacting agents. We explored the effects of connectivity, capital requirements, portfolio composition and fire-sale losses, and found how micro-prudential objectives may strongly diverge from macro-prudential targets.

In Part II, *"TailDep for the Measurement of Systemic Risk'*, we, first, developed a statistical analysis of the CDS spreads in order to identify the correlation patterns that emerged during and after the crisis. European banks and insurers are the entities which are more strongly connected with the systematic factors driving the largest share of the overall market movements. We then proposed a definition of systemic risk as the risk generated *by* and *within* the financial system. We proposed a model that is able to identify it and relate it to the dependence structure linking financial institutions in a network. Our measure of systemic risk is the only determinant of the intensity of the stabilization policy a regulator should pursue in order to avoid systemic breakdowns.

We then estimated the dependence structure of the global financial system using the TailDep methodology: a t-copula model is fitted to the data using a 24-month rolling window, tail dependencies are estimated and a time series of networks linking reference entities according to their tail dependencies is obtained. The model provided us with the relevant measure of systemic risk, i.e the leading eigenvalue of the dependence matrix, and of systemic importance of entities, i.e. their eigenvector centrality. We found that systemic risk rapidly increased from the end of the first half of 2007, well before the Lehman episode. Its decrease corresponded to the phase of the European sovereign distress, and its post-crisis minimum corresponded with the peak of the sovereign crisis, reflecting the segmentation that global and European financial markets experienced in that phase. Afterwards, the sovereign-financial nexus was weakened by central banks' intervention, and our measure of systemic risk began rising again. Throughout the years, the most central institutions are European entities, and, recently, the largest European banks, together with a number of insures, gained systemic importance.

Our contributions to the understanding of systemic risk crucially relied on the view of the financial system as an interconnected network of institutions. Eminent authors (Brunnermeier and Oehmke, 2012; Caballero, 2010; Haldane, 2009; Hansen, 2013; Schweitzer et al., 2009; Stiglitz, 2010) have welcomed network theory as a promising field of research. However, many of them blame its lack of microfoundation. In both our contribution, we tried to meet this theoretical request, highlighting how the behavior of agents in a complex system may yield, in general equilibrium, results which do not coincide with conclusions reached in a partial equilibrium perspective.

We hope our models will serve as a basis to build theoretically robust early warning signals of incipient crisis and to design an appropriate set of micro-incentives in order to grant stability to the entire system. The need for macro-prudential regulation today is felt more strongly than ever in history, and the consciousness of the importance of an efficient and stable financial system has rooted in the academic and regulatory environments. Theoretical and empirical investigations on systemic risk are thus increasingly demanded, and we hope to have met, at least in part, this demand.

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