How many alternative eggs should you put in your investment basket ?

by

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Abstract

There is some debate about how many stocks can effectively eliminate most of the unsystematic risk in an equity portfolio. Estimates range from 10 to 40. Given the growing proliferation of pooled investment vehicles aimed at the UK's pension fund industry, where these pools consist of various combinations of alternative asset classes and alternative investment strategies, in this paper we investigate the limits of diversification amongst these less conventional investments. Our results indicate that 40% of the time series risk can be eliminated by combining 8 strategies, but only a further 4% from combining 12. We also find that an investor could reduce 60% of the dispersion in terminal wealth of an alternative investment basket – which is arguably what investors should really be concerned with – by combining 6 of these less conventional asset approaches to investment, but only a further 20% by combining 15.

Keywords: Alternative asset classes, portfolio diversification, diversifiable risk.

JEL Classification: G0

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1. Introduction

"*Don't put all your eggs in one basket*" is advice that our grandmothers' might have given to us and, who in turn, might have received the same advice from their own grandmothers. This advice was formalised in an investment context by Markowitz and others in the 1950s. By constructing portfolios with assets that were imperfectly correlated with one another, Markowitz *et al* demonstrated that the risk inherent in the portfolio would decline as successive assets were added to it, until eventually the volatility of the portfolio would equate to the average covariance of the assets comprising the portfolio. This work therefore highlighted the importance of the covariance of returns between assets, and drew a distinction between undiversifiable and diversifiable risk, where the latter could be progressively eliminated by adding more and more assets to the portfolio. The work therefore explained how investors could take advantage of one of the few free lunches available in economics.

But how many stocks would produce a portfolio with only undiversifiable risk ? This question was first addressed by Evans and Archer (1968). Randomly drawing equities from a pool of assets to construct a large number of n-asset portfolios, their results indicated that most of the diversifiable risk could be eliminated by forming portfolios of eight to ten randomly selected stocks. Reference to this result can be found in all basic finance text books. Since that time many other researchers have addressed the same question[2]. In much later work Statman (1987) concluded that the number was closer to thirty or forty stocks. However, even in the event that the number is double Statman's estimate many mutual fund investors could still be said to be overdiversified, that is, holding portfolios of more assets than are required to reduce diversifiable risk to zero, effectively paying higher transactions charges to manage more assets than they need to hold.

LHabitant and Learned (2002) pose the same question but with respect to hedge funds. For investors looking to invest in hedge fund of funds, how many hedge funds should be included in this basket ? Using broadly the same approach and techniques, drawing individual hedge funds randomly from the TASS database they conclude that in terms of naive diversification, that most of the diversification benefits

[2] See for example Elton and Gruber (1977), Lorie (1975), Tole (1982), or O'Neal (1997).

are achieved by forming fund of funds comprising just five to ten individual hedge funds. In this paper we too extend the original work of Evans & Archer but in the context of a range of alternative asset classes or alternative strategies rather than to either individual stocks or hedge funds[3].

In this paper we specify a pool of possible alternative investments that include all those strategies and classes that UK pension funds have invested in already, and/or comprise the alternative pooled vehicles that are currently available to them. From this pool we randomly create portfolios consisting of 2 to 23 of these asset classes, using 100,000 draws for each set of n-asset portfolios. Our results indicate that 40% of the time series risk can be eliminated by combining 8 asset classes, but only a further 4% from combining 12. We also find that an investor could reduce 60% of the dispersion in terminal wealth – which is arguably what investors should really be concerned with – by combining 6 of these less conventional approaches to investment, but only a further 20% by combining 15.

The rest of this paper is organised as follows: in Section 2 we outline the data, our methodology and the tests we use to analyse the issues above; in Section 3 we present a discussion of main results and findings; and in Section 4 we conclude the paper.

2. Motivation

The question of how many alternative asset classes might constitute a sufficient number to diversify unsystematic risk is of more than just academic interest. Over the past few years, in response to the growing demand of UK pension funds, asset managers have launched pooled investment vehicles that consist of a range of alternative asset classes and alternative investment strategies (see Appendix 1). The *raison d'etre* for such investment vehicles is that many small and medium-sized pension funds do not have sufficient expertise to choose and monitor a range of alternative investment, nor sufficient assets to invest in the range of alternatives that say the Yale University endowment fund holds in its portfolio. Furthermore, there

[3]Throughout this paper we refer to 'alternative asset classes', by this we mean both distinct asset classes, such as infrastructure, but also to alternative investment strategies, as represented by the hedge fund indices in our work.

seems little doubt that if these products prove to be successful retail versions of the products will follow.

But the issues surrounding the construction of a such an investment vehicle, are far more significant than even those related to constructing a fund of hedge funds, and certainly more significant than those involved in constructing ordinary mutual funds. There are a whole range of difficult, practical issues involved including: a range of liquidity considerations; the tax treatment of the alternatives, which may vary considerably; and perhaps most significantly issues surrounding pricing. Given these issues it is even more crucial to establish how many alternative investments are sufficient to reduce unsystematic risk to an acceptable level. Too few could leave investors exposed to unnecessarily high levels of risk. Too many could lead to excessive management costs.

3. Data and methodology

3.1. *Data*

We have created a database consisting of the total returns on 24 alternative investment categories. The definition of an alternative investment may vary depending upon the investor. However, until recently for most UK pension funds the vast majority of their asset portfolios consisted of UK equities, and UK government bonds, the "alternative" to these asset classes was generally an investment in overseas developed economy, large cap equities mainly in the USA, Europe (ex UK) and Japan. In our set of alternatives we therefore include indices representing the returns on investments in emerging market equities and on small cap stocks. Along with these two equity indices we encompass hedge funds, private equity, commodities, property, high yield and emerging market bonds, currency, managed futures and infrastructure. The full list of indices used to proxy the returns and risks on these strategies along with their sources are shown in Table 1.

In designing any experiment of this nature we could clearly have chosen other subsets of these asset classes and strategies. However, we needed indices that were available in total return formats and also that had a fairly significant time series history. All of the indices used and represented in Table 1 are available in a monthly format going back to at least January 1995. In Table 2 we present some basic descriptive statistics on these indices, where the statistics are based upon monthly data from January 1995 to December 2007.

2.2. *Methodology*

Using the in-sample returns on the indices listed in tables 1 and 2 we create equally weighted portfolios of increasing size n (n=2 to 23) by randomly selecting indices from our data set. These portfolios are rebalanced annually with the total portfolio value at the end of December each year redistributed equally among the constituent indices. For each portfolio, we build a time series of returns and use it to generate various statistics (compound annual return, volatility, skewness, kurtosis etc). For each portfolio size, this process is repeated 100,000 times to obtain 100,000 observations of each statistic. This is necessary not only to estimate the mean behaviour of a portfolio of size n, but also to examine the cross sectional variation in the results – something which is not typically reported in such exercises.

But as well as calculating the standard moments for the time series properties of the portfolios we also focus on “downside risk”. We calculate measures of downside deviation, semi-deviation and loss standard deviation. However, the results and the conclusions that one might draw from each of these sets of statistics were not materially different and so we report the results for downside deviation which is defined as follows:

$$\text{Downside deviation} = \left(\frac{\left(\sum_{i=1}^{N} (r_i - r_{MAR})^2 \right)}{n} \right)^{1/2} \qquad (1)$$

where r_i is the return for period i, n is the number of periods, r_{MAR} is the “minimum acceptable return” which can be defined by the user and where if $(r_i - r_{MAR}) \geq 0.0$ then $(r_i - r_{MAR})$ is set equal to 0.0. We set r_{MAR} to be equal to zero. For hedge fund analysis this number is often set as the target, or hurdle rate of return.

The statistics above are all designed to help understand the risk profile of a basket of alternatives. However, investors will also be interested to understand how this basket behaves relative to traditional asset classes. In other words, how does the correlation between any portfolio of n alternatives and say a portfolio of long-only equities change as the number of alternatives in the basket also changes ? We therefore calculate the correlation over the full sample of each portfolio of alternatives with the return on the S&P 500 composite index and on the MSCI World equity index.

Terminal wealth statistics

However, in considering the number of mutual funds necessary to reduce risk to its undiversifiable minimum, O'Neal (1997) argued that investors should not only consider the time series properties of their portfolios, as we do above by calculating these statistics, but also their terminal values. And more specifically the distribution of that terminal value.

The intuition behind this is as follows. One could be unfortunate enough to get in to a taxi and enjoy a very smooth ride, but ultimately not arrive at one's chosen destination. A traveller may instead be willing to put up with a bumpy cab ride that does get them to their chosen destination. In an investment context then investors should care at least as much about the dispersion of their terminal wealth as they do the volatility of that wealth over time. Since long-term investors like pension funds should be focussed on the end result, or the value of their "terminal wealth" we follow O'Neal and calculate a set of additional statistics to explore the impact of diversification among alternatives on the distribution of terminal wealth outcomes. These include: a measure of short fall probability; the mean of this shortfall; and also the semi-variance of portfolio returns.

We define shortfall probability as:

$$\text{Shortfall probability} = \frac{\text{Number of observations below } \bar{r}}{n} \qquad (2)$$

where $\bar{r}$ is the equally weighted average return over the whole sample of all 24 alternative asset classes or strategies, and n is the number of observations. However, it is one thing to know the probability of a shortfall, but the scale of the likely shortfall is also important. We calculate the mean portfolio shortfall as follows:

$$\text{Mean shortfall} = \sum_{i=1}^{n} \frac{r_i - \bar{r}}{n} \qquad (3)$$

where r_i is the return on portfolio i.

For the measure of downside risk we follow L'Habitant and Learned (2002) we calculate a measure of the semi-deviation of the dispersion of terminal wealth. This measure is also consistent with the measures of shortfall probability and mean shortfall. The statistic is defined as follows:

$$\text{Semi-deviation} = \left(\frac{\left(\sum_{i=1}^{N} \left(r_i - \bar{r}\right)^2 \right)}{n_l - 1} \right)^{1/2} \qquad (4)$$

where n_i is the number of periods where r_i is below $\bar{r}$ and where if $(r_i - \bar{r}) \geq 0.0$ then $(r_i - \bar{r})$ is set equal to 0.0.

4. Results

4.1. *Time-series portfolio properties*

In Chart 1 we present the average (mean and median) values for the nominal annual return on the n-asset portfolios. The average (as one might expect from random sampling) is fairly stable and converges quickly on the equally-weighted average of the nominal return of all 24 indices over the full sample period, approximately 12.5%pa.

In Chart 2 we present an analogous chart, but using the time series standard deviation of the same set of portfolios as the metric. The first point to note is that the average standard deviation line traces the familiar "L" shaped pattern, falling quite rapidly at the start and levelling off after the once n reaches between eight to ten. In Chart 3 we plot the marginal change in average portfolio standard deviation as we move progressively into portfolios with higher numbers of asset classes. By the time we reach 8 asset classes the average standard deviation has been reduced by 40%, but only by 45% by the time there are 14 asset classes in the portfolios. The marginal benefit in terms of risk reduction of investing in more than 8 strategies is therefore relatively small. And these benefits may be outweighed by the marginal costs of managing more strategies.

Taken together one might also conclude that say five asset classes are sufficient to get most of the benefits of diversification since once n equals 5, 35% of the volatility has been removed from the average portfolio. However, the maximum and minimum lines in Chart 2 show how this would be a riskier approach since the spread of possible standard deviation outcomes is much wider than for say an 8 asset class approach, at least with a random choice of these alternatives.

In Chart 4 we present the average, maximum and minimum skews of these portfolios. The skew for an equally weighted portfolio of these alternatives is negative; but the average skew of the portfolios becomes progressively worse, rather than improving as the numbers of asset classes increases. This means that in terms of mean skew investors would be better off with fewer rather than more alternatives asset classes in their investment basket. This is consistent with the results of Amin and Kat (2002) who found that for portfolios of hedge funds, increasing the number of funds not only lowered the standard deviation but also increased the negative skewness of the fund of funds. They found that this increase in negative skewness was the result of the co-skewness between the returns. However, the dispersion of skew experience narrows significantly as the minimum and maximum skew lines show in the Chart.

In Chart 5 we present the downside deviation of the portfolios. Arguably, it is this element of risk that investors usually wish to avoid. This chart also shows how the marginal benefit of adding more alternatives declines as the number of alternatives in the portfolio rises. The majority o the downside deviation is reduced with a portfolio of six to eight alternatives; a result that suggests that a smaller number of alternatives might be suitable if investors wish to reduce the downside risk over time of their basket of alternatives.

As well as understanding the time series properties of these alternative portfolios, it is also important to understand how they might behave relative to an asset class that they may be designed to replace or to which they may be combined. To understand this aspect of investing in these alternatives, we calculated the full sample correlation of the portfolios with both the S&P500 composite and MSCI World equity indices. The average results of these calculations are shown in Chart 6. The average correlation rises quite sharply as we move from one or two alternatives to eight to ten. For our set of alternatives the mean correlation is always below 60% for the S&P500 and reaches just over 65% when the MSCI index is used. Given the very high correlation between developed economy equity markets these much lower levels of correlation may be attractive to investors. Chart 7 shows the variation of the correlation coefficient, by plotting the mean correlation for the S&P500 along with the associated minimum and maximum correlation statistics. The range is very large. Between portfolios consisting of 0ne to eight alternatives, the minimum correlation is negative. But arguably it is the range above the mean that is more relevant, since lower levels of correlation with any equity investment are probably to be welcomed. This maximum correlation (from 100,000 draws) is fairly stable at around 70 to 75% for most of the n-asset portfolios. This may be sufficiently low for many investors, depending upon their investment objectives.

3.2. *Terminal wealth statistics*

Investors are naturally concerned about how bumpy the journey to their end point proves to be, but ultimately they would like to know that they will at least make it to their final destination. In this exercise we choose the last observation of our data sample, December 2007, to be our chosen end point. In Chart 8 we plot the standard deviation of terminal wealth at this end point, and in Chart 9 the marginal change in this measure of risk.

These results contrast sharply with charts 2 and 3 for the time series standard deviation of returns. Eight asset classes were enough to reduce the time series standard deviation by 40% whereas for the terminal wealth standard deviation this reduction is almost 65%. Beyond 8 assets the reduction in time series standard deviation is marginal, but for the terminal wealth measure adding another 10 alternatives leads to a further 20% reduction in volatility.

These results have important implications for an investor seeking to invest in alternative asset classes. Although diversifying beyond 8 assets offers extremely marginal benefits in terms of time series standard deviation, if the investor wants more certainty that the risk level they expect is the one that they actually experience (as measured by the standard deviation of terminal wealth) then they should diversify much more.

The dispersion in wealth at the end of our sample period, as measured by the standard deviation of terminal wealth takes into account both positive and negative deviations from the mean. However investors will naturally be less concerned about outcomes above then mean than those below it. For this reason we examine the shortfall probability in Chart 10. The benchmark we use is the mean return on the 24 investment strategies which is 11.01%. Since 10 of the alternatives have a compound annual return of less than this, if only one asset class is chosen there is a 42% chance that the return will be lower than the mean. Thereafter the shortfall probability declines in an almost linear fashion as more indices are added to the portfolio from 42% to zero. If for example an investor chose to invest in only 5 strategies, based on the reduction in time series standard deviation, they would still have a 33% chance of achieving a return lower than the mean. Once again this illustrates that a more diverse approach might be prudent.

The major shortcoming of the shortfall probability is that it does not account of the magnitude by which returns fall short of the mean, for this reason we calculate the mean shortfall shown in Chart 11. If only one alternative is chosen then the mean shortfall is 5.0% per annum which equates to an 88.6% total return over our 13 year sample period. Combining the mean shortfall with the shortfall probability we can see that a portfolio of 5 strategies has a 33% chance of returning 1.6%pa (23% total return) below the mean which is clearly substantial.

Finally, in Chart 12 we presents the semi-deviation statistics for the terminal wealth o the portfolio, since investors will probably be more concerned about not achieving their target return than beating the target at their chosen terminal date. The results are broadly consistent with those presented above, with the vast majority of benefit achieved in terms of the reduction of portfolio semi-deviation by the time n equals six; and certainly by the time n equals eight.

4. Conclusions

Our results show that increased diversification significantly decreases the time series standard deviation of the portfolios, but that the marginal decrease from adding more alternatives decreases rapidly. This implies that if an investor is only concerned with this element of risk they should hold a portfolio of between eight and ten alternatives. However for investors with long-term investment horizons the dispersion of terminal wealth outcomes might be a more appropriate measure. Here we find that holding more than the number of alternatives suggested by considering the time series standard deviation might be preferable. The downside to this reduction in the dispersion in terminal wealth is that it may come at the cost of increased negative skew in portfolio returns over time.

These results represent a first step to understanding the limits to the benefits of naive diversification that might exist with regard to building a portfolio of alternative asset classes and or alternative investment strategies. And given the interest amongst institutional investors in this investment alternative investment universe, we believe our results will be of interest to such investors, to their advisors and to the product providers seeking to satisfy this investment need.

However, unlike the related studies that have sought to establish the limits to diversification with regard to portfolios of individual equities or hedge fund of funds, our results are hampered to some extent by the limited number of alternative asset classes and strategies and in particular by the availability of suitable time series proxies for the investment experience in each. We constructed a database of 24 such proxies. Over time however it will be possible to revisit these results to see whether our basic conclusions hold as the alternative universe continues to expand encompassing new strategies as it does so.

So how many how many alternative eggs should you put in your investment basket ? There is no definitive answer since it will ultimately depend on investor attitudes to risk, and to their objectives, but overall it is probably around eight !

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Chart 1: Mean portfolio return

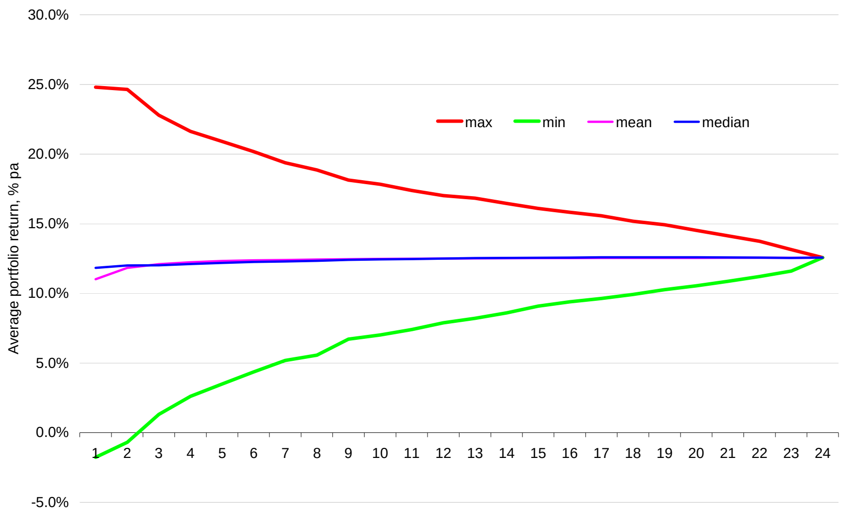

Chart 2: Portfolio standard deviation

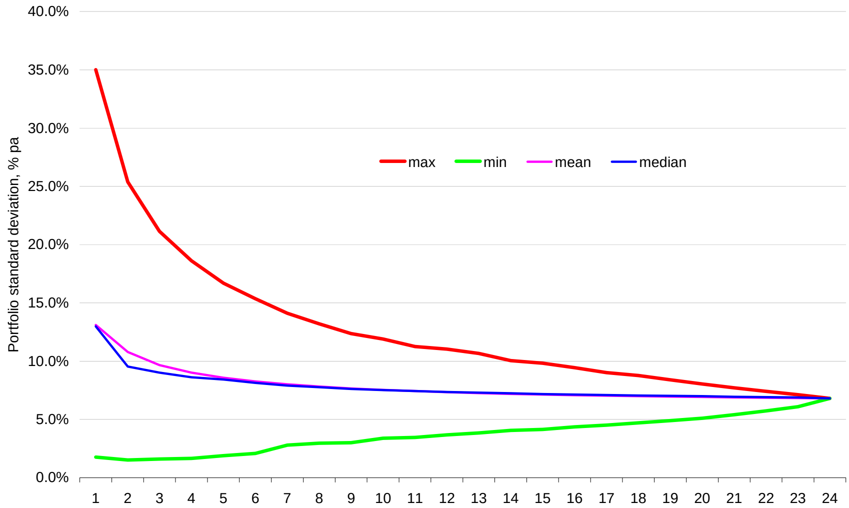

Chart 3: Marginal change in portfolio standard deviation

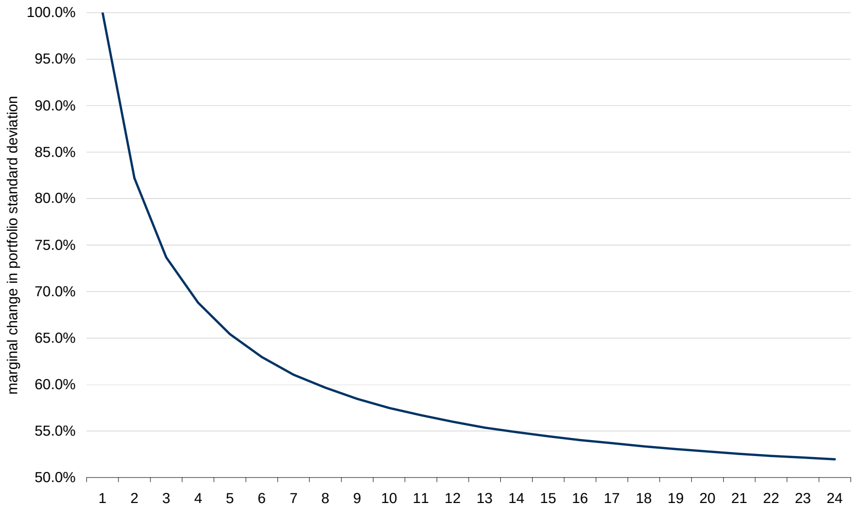

Chart 4: Portfolio skewness

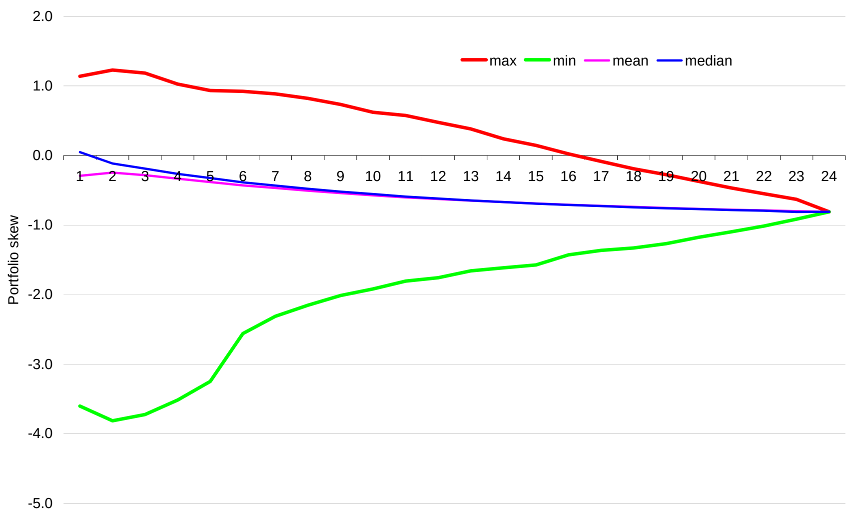

Chart 5: Downside deviation

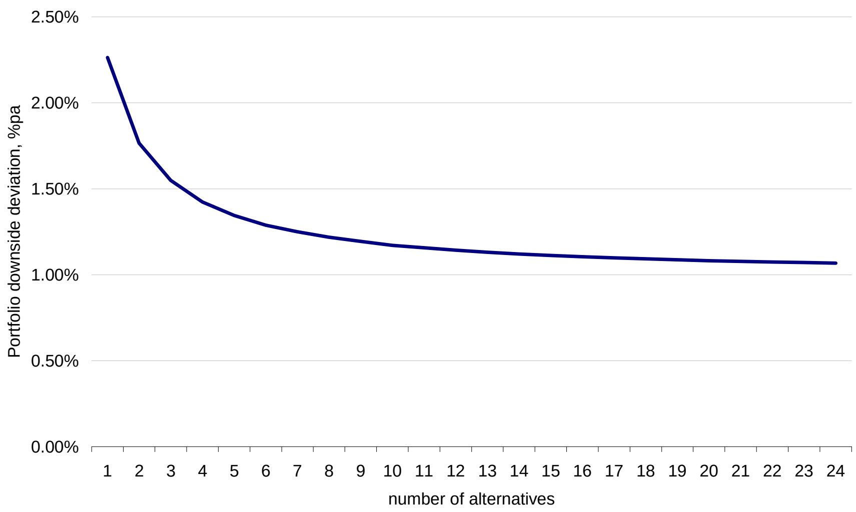

Chart 6: Alternatives and equity market correlation

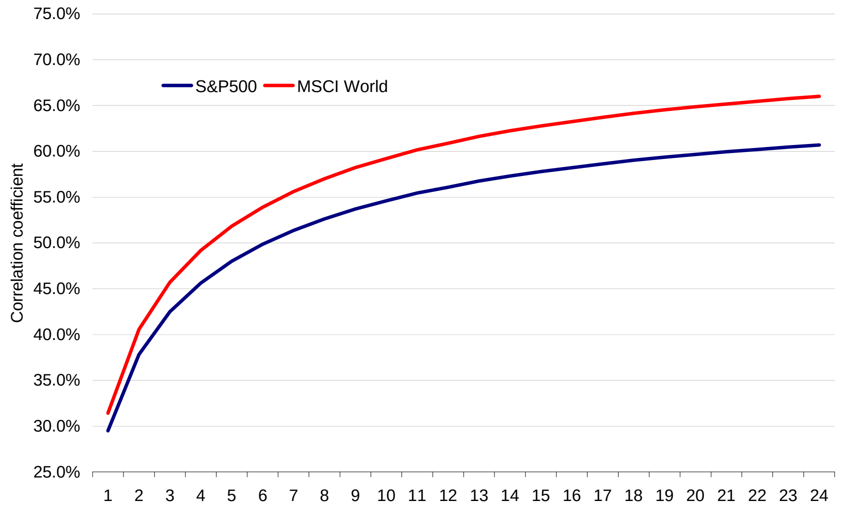

Chart 7: Alternatives and S&P500 correlation, max and min

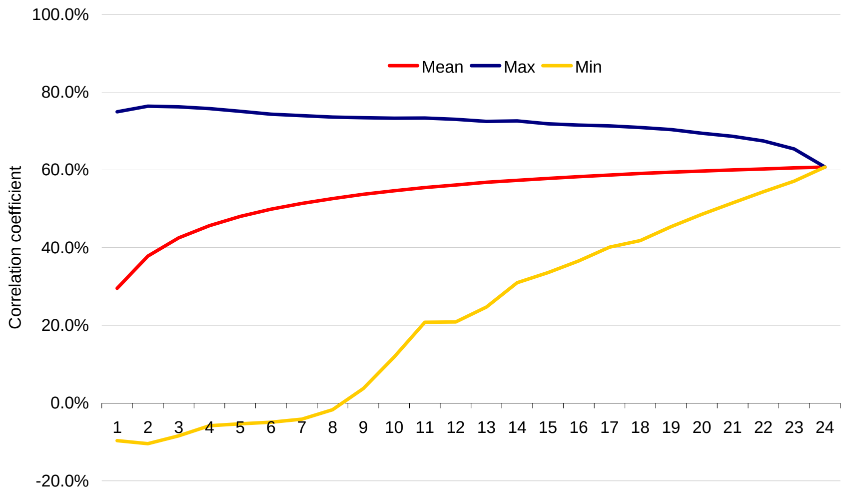

Chart 8: Standard deviation of terminal wealth

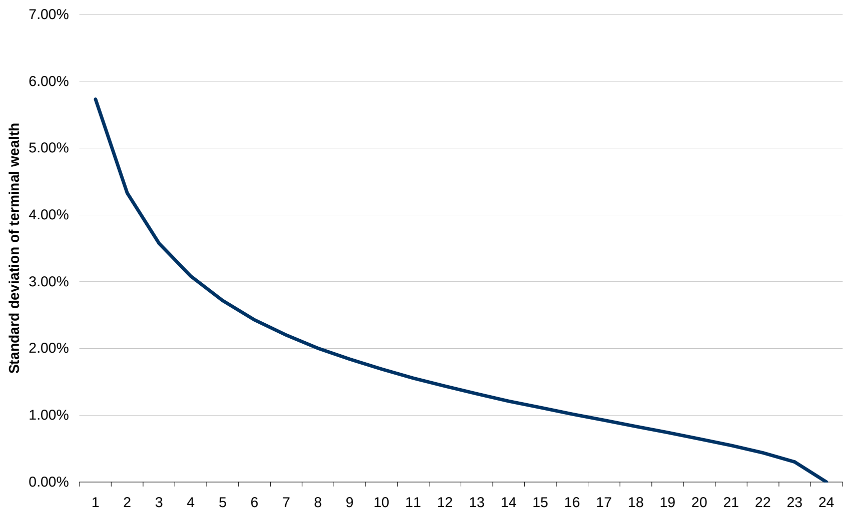

Chart 9: Marginal change in standard deviation of terminal wealth

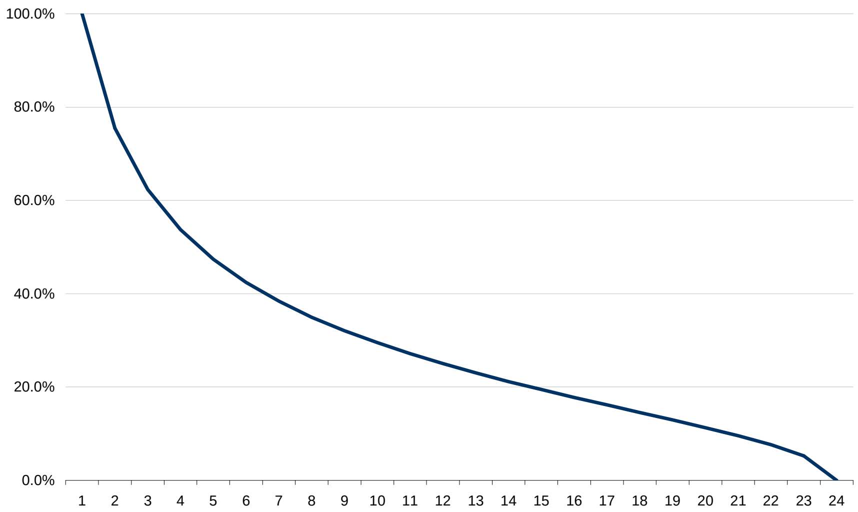

Chart 10: Shortfall probability of terminal wealth

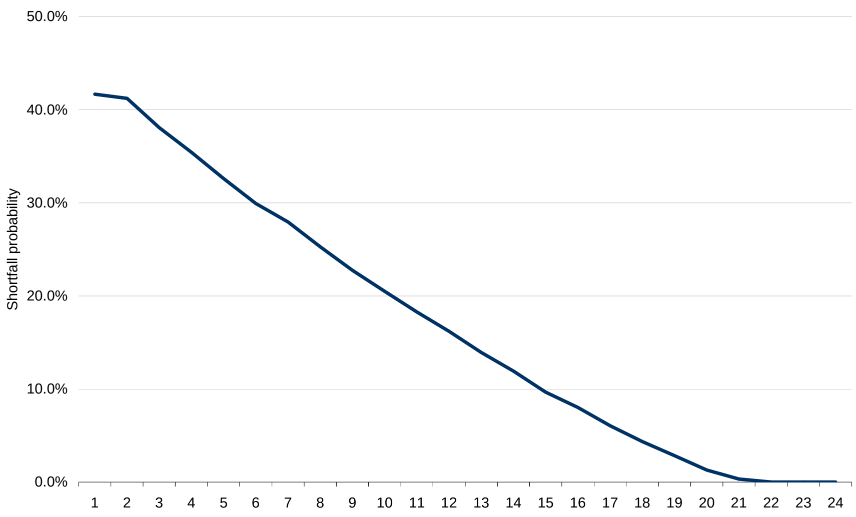

Chart 11: Mean shortfall of terminal wealth

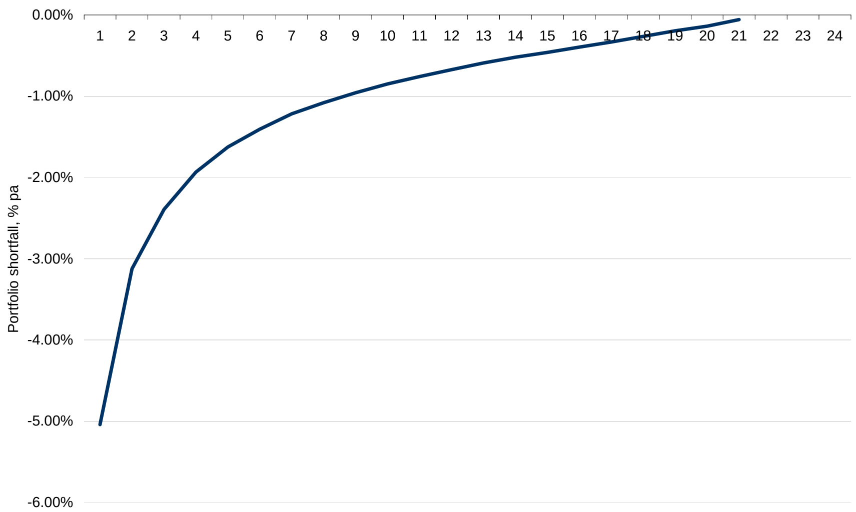

Chart 12: Semi-deviation of terminal wealth

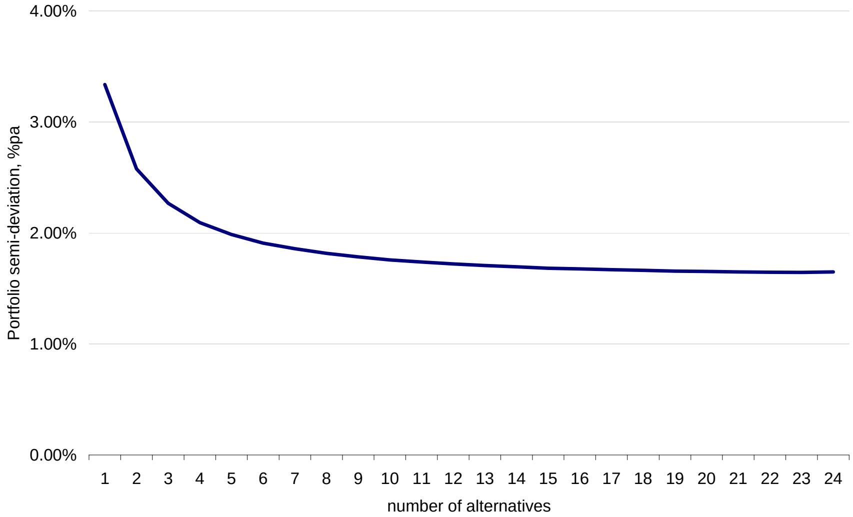

Table 1: Set of alternative indices

	Alternative	Sub category	Index constructed by
1	Equities	Emerging equities	Thomson Financial
2		Small Cap	Dow Jones Wilshire
3	Hedge funds	FoF	CSFB Tremont
4		Equity market neutral	CSFB Tremont
5		Event driven	CSFB Tremont
6		Fixed income arbitrage	CSFB Tremont
7		Global macro	CSFB Tremont
8		Long/short equity	CSFB Tremont
9	Commodities	Energy	Dow Jones AIG
10		Agriculture	Dow Jones AIG
11		Industrial metals	Dow Jones AIG
12		Gold	S&P GSCI
13		Timber	NBS
14	Commercial property	US	FTSE EPRA
15		Europe	FTSE EPRA
16		UK	FTSE EPRA
17		Asia	FTSE EPRA
18	UK residential property	UK Residential	Fathom Financial Consulting
19	Bonds	High Yield	Lehman Brothers
20		Emerging Market Debt	Merrill Lynch
21	Currency		Barclays Capital
22	Managed futures		BTOP
23	Infrastructure		MSCI
24	Private Equity		Thomson Financial

Source: Thomson Financial

Table 2: Descriptive statistics

		Return	St.dev.	Skewness	Kurtosis
1	Emerging equities	24.81%	35.0%	-0.17	7.07
2	Small Cap	11.99%	18.5%	-0.53	4.01
3	FoF	12.33%	7.4%	0.12	5.83
4	Equity market neutral	10.94%	2.7%	0.46	3.66
5	Event driven	12.77%	5.5%	-3.64	29.20
6	Fixed income arbitrage	6.74%	3.6%	-3.17	20.79
7	Global macro	15.46%	10.4%	0.03	6.75
8	Long/short equity	13.93%	9.9%	0.15	7.34
9	Energy	10.30%	32.0%	0.31	3.53
10	Agriculture	-1.77%	15.5%	0.20	3.06
11	Industrial metals	4.50%	18.9%	0.65	4.14
12	Gold	6.43%	13.7%	0.81	4.45
13	Timber	-0.25%	4.1%	-0.32	6.60
14	US Property	14.87%	14.5%	-0.58	3.87
15	Europe Property	15.67%	13.0%	-0.39	3.61
16	UK Property	9.48%	16.5%	-0.11	2.65
17	Asia Property	11.70%	23.8%	0.51	9.75
18	UK Residential	18.10%	1.8%	0.69	3.15
19	High Yield	11.16%	8.7%	0.07	8.99
20	Emerging Market Debt	13.67%	13.8%	-2.88	23.79
21	Currency	5.05%	6.4%	1.15	4.83
22	Managed futures	8.29%	9.4%	0.28	3.49
23	Infrastructure	14.24%	16.5%	-0.32	4.26
24	Private Equity	13.87%	13.0%	-0.43	5.00

Source: Thomson Financial

Appendix 1

Asset manager	Pooled Alternative Investment Product
Credit Suisse	Credit Suisse Nova Diversified Growth
Schroders	Schroder Diversified Growth
JP Morgan	JP Morgan Life Diversified Growth
Morley Fund Management	Morley Diversified Strategy
Newton Phoenix	Newton Phoenix Multi-Asset
Merrill Lynch	Merrill Lynch Target Return
Fidelity International	Fidelity Diversified Growth Fund
Goldman Sachs	Goldman Sachs Balanced Strategy
UBS	UBS Targeted Return
Insight Investments	Insight Diversified Target
Baring Asset Management	Baring MM (Reduced, Optimum & Extended Risk)