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A jam-jar model of life expectancy and limits to life

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A jam-jar model of life expectancy and limits to life[1]

Les Mayhew and David Smith[2]

Abstract

One of great success stories in the UK is that people are living longer. Improvements in life expectancy at older ages have particularly accelerated in recent decades. However, there are three unanswered questions - namely, what will life expectancy be in the medium term (10 – 20 years, say), will it go on rising indefinitely and what will be the variation in age of death? We find there is a need for better information about life expectancy at both the population and individual level. Policies must be durable, especially anything to do with pensions, health and social care, or housing needs. In this paper we present a new method for forecasting life expectancy based on decomposition techniques. The key advantages are more certainty over which age groups are affected and more accurate information about possible limits to life. Results are presented for England and Wales and the implications discussed. A concluding section briefly compares our approach with other methods.

Key words: life expectancy - decomposition - limits to life - ageing populations

1. Introduction

One of great success stories in the United Kingdom (UK) is that people are living longer. Male life expectancy at birth is now almost 80 years, having advanced 14 years since 1950, thanks to improvements in health and wellbeing, fewer accidents, better health care and higher standards of living. This success in turn presents the country with a huge economic opportunity if extra years are spent in prosperity and good health, but economic danger if not.

Indeed, realising the full potential of older citizens of the UK will be central to the Government's response to changing economic circumstances and the drive to build a strong, fair economy for the twenty-first century. However, the challenges posed by an ageing society come at a cost in terms of pensions, the higher cost of health and social care and infrastructural change. Such changes affect the wider economy, public expenditure and taxes, which impact individuals. These effects manifest in obvious ways such as having to work for longer, increases in pension age, higher taxes, more doctor visits and so on.

It is becoming increasingly clear, through recent policy changes, that people will be required to take reasonable steps in planning for their financial needs in old age and to become less dependent on the state. This new era will mean facing up to an increasingly broader range of personal circumstances, for example, people with quite different mixes of income and assets or family circumstances on which to draw upon e.g. for care needs.

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[1] A link to a more detailed peer-reviewed companion paper extended to other developed countries may be found in Population Studies:

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These changes find increasing resonance in a range of government policies from pensions through to social care, especially in terms of cost curtailment and transfers of responsibility from government to individuals. The old assumption that the state would look after citizens from cradle to grave can no longer be relied upon, although its demise has never been officially confirmed.

These changes will mean new coping mechanisms are needed to help individuals and families, including better information both on how to navigate the system and about the real costs of ageing. One crucial dimension of this is increases in life expectancy as these will mean that available resources, for example pension savings, need to last for longer. Any increase in the certainty over how long a population or an individual will live is thus beneficial regardless of whether it is changes to state pension age, employment, estimating the demand for hospital beds or tax planning.

The problem is that life expectancy is not a deterministic process or as predictable as the grains of sand in an egg-timer. Rather it is derived from an average of the experiences of deceased individuals in a period of time, or the experiences of cohorts of individuals born in certain years. In practice, it is easier to predict life expectancies for populations than it is for individuals as some randomness is removed. Even so, we know that official forecasts of life expectancy have underestimated the rates of improvement, often spectacularly, with consequent impacts on medium to long term government planning (e.g. Booth 2006; Shaw 2007; Mayhew and Smith, 2012).

Improvements in life expectancy at older ages are occurring at a fast pace but have particularly accelerated in recent decades. Figure 1 shows the trend in life expectancy for 60 year old men and women for England and Wales since records began in 1841. It shows a steady growth for both sexes though the improvement for women has been faster, with the gap between men and women widening from under one year in the 1800s before plateauing at just over 4.5 years in the 1970s. Since then the gap has narrowed considerably and now stands at just under 3 years. This is expected to narrow further or even converge before 2030 if current trends are extrapolated; moreover this also applies to other start ages, not only aged 60 (e.g. see Mayhew and Smith 2014).

There are several reasons for this turnaround in male fortunes, among the most notable of which is the demise in male smoking habits and shifts in employment patterns away from hazardous occupations (e.g. see Preston, S. H. and H. D. Wang, 2006, and Murphy and Di Cesare, 2012, or Trovato 2005). Another factor affecting both sexes has been the reduction in year-on-year fluctuations caused mainly by wars, epidemics and cold winters. This has meant that short term trends in life expectancy have become easier to predict, but it still leaves three important unanswered questions - namely, what will life expectancy be in the medium term (10 – 20 years, say), will it go on rising indefinitely and what will be the variation in age of death (Mayhew and Smith, 2012)?

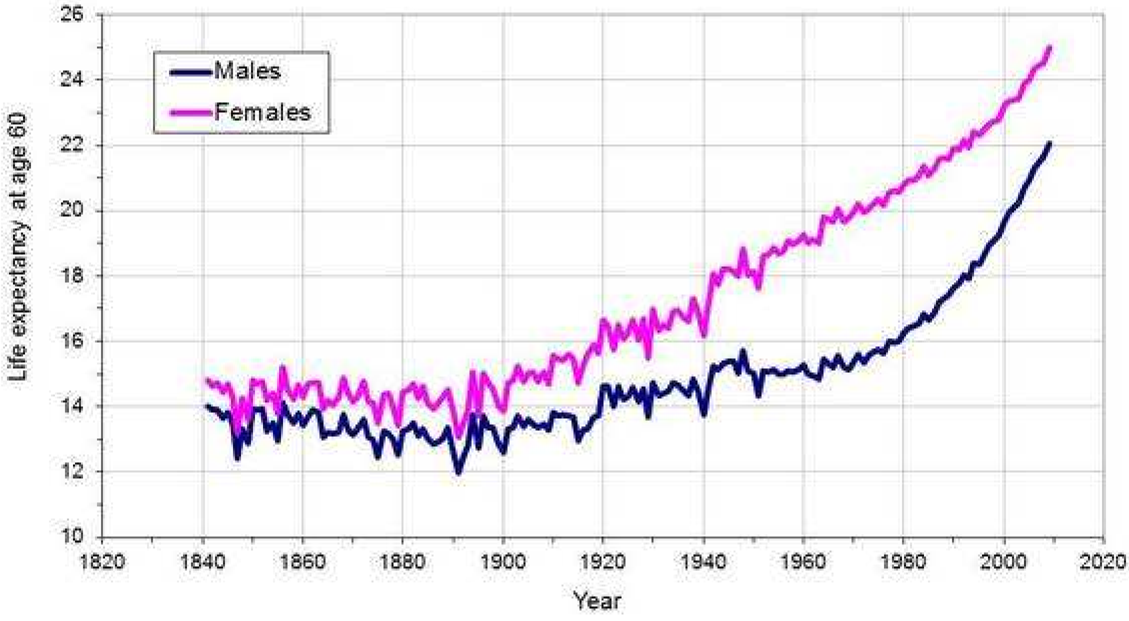

Figure 1: Changes in male and female life expectancy in England and Wales at age 60 from 1841 to 2009

The corollary is that if life expectancy does continue to grow, in which age groups will the growth come from – people in their 60s, 70s or older, or even centenarians say? In short, the need for better information about life expectancy is demonstrable at both the population and individual level - for framing government policies to planning personal finances. Policies must be durable, especially anything to do with pensions, health and social care, or housing. In this paper we present a new method for forecasting life expectancy based on decomposition techniques. The key advantages are more certainty over which age groups are affected and more accurate information about possible limits to life. We call this the 'jam-jar model' of life expectancy, for reasons which are explained later.

In what follows, Section 2 sets out the methodology while in Section 3, we provide our main results. In particular, we demonstrate how regularities in ten-year life expectancies can be used to forecast future life expectancy and discuss the assumptions and limitations of the method therein. An important feature of our approach is the exploitation of a 'natural ceiling in the data', progress towards which is statistically calibrated using a logistic function. In the concluding section, we discuss the implications of our findings in the wider context of society and in terms of other methods for forecasting life expectancy.

2. Changes in life expectancy within discrete age intervals

It is standard practice to measure life expectancy at different ages e.g. from birth, age 30 or some other age. For example, life expectancy for England and Wales (E&W) males at birth in 2009 was 78.04 years (source HMD), suggesting that at age 60 there should be 18.04 years of life remaining. However, the data show that life expectancy at age 60 was actually 22.04 years because of survival and selection effects. We therefore seek a method for calculating life expectancy within discrete age limits that addresses this anomaly. The aim is to allow us to reassemble component life expectancies to produce estimates of whole life expectancy over any desired age range.

Hickman and Estell (1969) proposed a similar idea which they termed ‘partial life expectancies’. In their context they argued that partial life expectancies are relevant to discussions on the economic costs of illness, since ‘partial life expectancies may be related to the ages at which the economic contributions ...are usually greater’ (p2244). Pollard (1982), posing a different question, showed that the change in expectation of life can be expressed as a weighted function of mortality changes at individual ages plus interaction effects. Arriaga (1984) develops this further, setting out the basic equations for measuring temporary expectancies using a discrete life table approach rather than the continuous methods of Pollard.

In this paper, we proceed similarly but our aims are different. Our definition of life expectancy is similar to Arriaga's temporary life expectancy but our focus is on trends in life expectancy within specified age intervals. In particular, a technique is presented that gives the expectation of a person aged 60, 70, 80, 90 etc. reaching age 70, 80, 90, 100, etc. Under our approach, a ten-year life expectancy at age 60, for example, means that everyone survives to age 70; a two-year expectancy at 60 means that a person can only expect to live two of a possible ten years and so on. It should thus be easier to pinpoint in which age intervals future increases in life expectancy are more likely to occur by fitting bounded functions to trends in the data from any start age whether at birth or some later age, in our case 60.

The results strongly indicate that contributions to life expectancy have transferred in a predictable wave-like fashion to older ages as opposed to a process in which each age interval has contributed equally. For example, in 1950 when male life expectancy at 60 in E&W was 15 years, the age range 80-90 only contributed 9.1% to this figure, but by 2009, when life expectancy was 22 years, they contributed 18.5%. In fact, we can imagine each decade of life as a ‘jam-jar’ which fills to the brim with life years, with extra life years being added to the each decade’s jam-jars at different rates, filling the early ones first, until all are full.

Segmenting life expectancy by age

We can derive the future expectation of life for a life currently aged x_1 by calculating the area under the population curve and dividing by the starting population (i.e. so that we turn the population curve into an individual survival curve for a standard member of the population).

Figure 2 shows a survival curve divided into segments based on age. Each value of l_x is joined by a straight line on the assumption that population decreases linearly between two ages. The standard equation for expectation of future life at age x_1 is

$$e_{x_1} = \frac{1}{l_{x_1}} \sum_{y=x_1}^{x_5} l_y - \frac{1}{2} = \frac{1}{l_{x_1}} \sum_{y=x_2}^{x_5} l_y + \frac{1}{2}$$

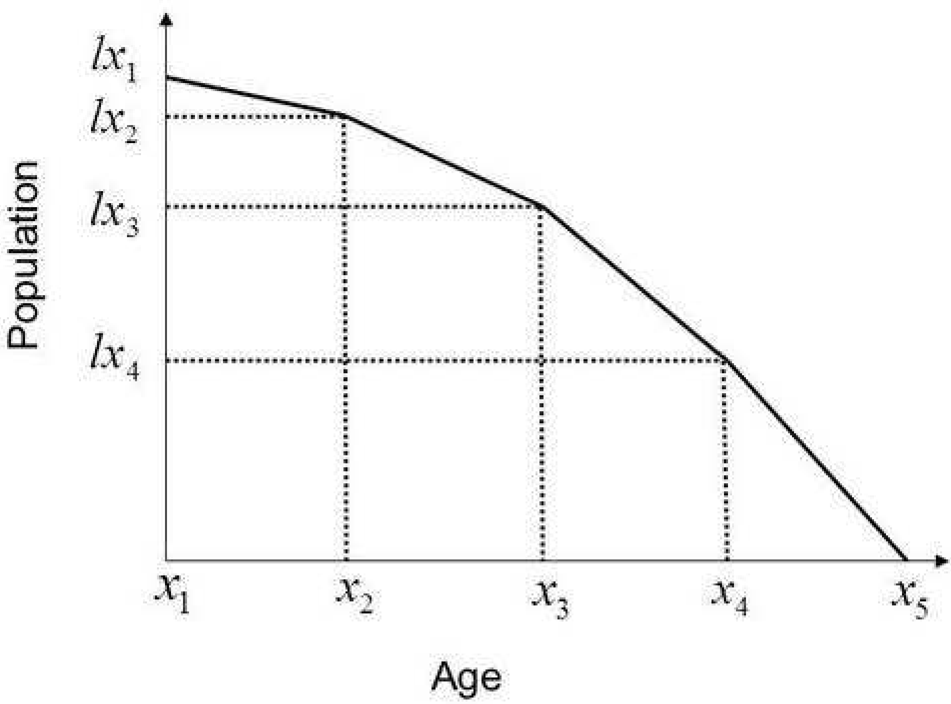

Figure 2: Survival curve S(x) divided into segments

In effect, what we are calculating is a series of rectangles and triangles. Assuming that we are increasing age by 1 each time i.e. $x_{n+1} - x_n = 1$ then the area of each rectangle is simply the height or number of people alive at age x_{n+1}, and the area of each triangle is $\frac{1}{2}(l_x - l_{x+1})$. Therefore

$$
\begin{aligned}
e_{x_1} &= \frac{1}{l_{x_1}}\left\{\left(l_{x_2} + \frac{1}{2}\left(l_{x_1} - l_{x_2}\right)\right) + \left(l_{x_3} + \frac{1}{2}\left(l_{x_2} - l_{x_3}\right)\right) + \left(l_{x_4} + \frac{1}{2}\left(l_{x_3} - l_{x_4}\right)\right) + \left(l_{x_5} + \frac{1}{2}\left(l_{x_4} - l_{x_5}\right)\right)\right\} \\
&= \frac{1}{l_{x_1}}\left\{\sum_{y=x_2}^{x_5} l_y + \frac{1}{2}\left(l_{x_1} - l_{x_5}\right)\right\} \\
&= \frac{1}{l_{x_1}}\sum_{y=x_2}^{x_5} l_y + \frac{1}{2}
\end{aligned}
$$

(because $l_{x_5} = 0$)

Contribution to expected Life

We can now break this future lifetime into two parts – from ages x_1 to x_3 and from x_3 to x_5 and see how much of the total expected future lifetime each section gives.

Define:

$e_{x_n(x_i:x_j)}$ as the future expected life of someone currently aged x_n between the ages of x_i and x_j

Hence

$$e_{x_1(x_1:x_3)} = \frac{1}{l_{x_1}}\left\{\left(l_{x_2} + \frac{1}{2}\left(l_{x_1} - l_{x_2}\right)\right) + \left(l_{x_3} + \frac{1}{2}\left(l_{x_2} - l_{x_3}\right)\right)\right\}$$
$$= \frac{1}{l_{x_1}}\left\{\sum_{y=x_2}^{x_3} l_y + \frac{1}{2}\left(l_{x_1} - l_{x_3}\right)\right\}$$

And

$$e_{x_1(x_3:x_5)} = \frac{1}{l_{x_1}}\left\{\left(l_{x_4} + \frac{1}{2}\left(l_{x_3} - l_{x_4}\right)\right) + \left(l_{x_5} + \frac{1}{2}\left(l_{x_4} - l_{x_5}\right)\right)\right\}$$
$$= \frac{1}{l_{x_1}}\left\{\sum_{y=x_4}^{x_5} l_y + \frac{1}{2}\left(l_{x_3} - l_{x_5}\right)\right\}$$

And we can see that

$$e_{x_1} = e_{x_1(x_1:x_3)} + e_{x_1(x_3:x_5)}$$

We can extend this idea to as many ages as we want. For example, if we have a population that we are studying from age 60, where the population dies out at age 110 then:

$$e_{60} = \frac{1}{l_{60}}\sum_{y=61}^{110} l_y + \frac{1}{2}$$

And we can break this population down into contributions to expected life from each 10 years of age i.e. ages 60-70, 70-80, …, 100-110.

$$e_{60(60:70)} = \frac{1}{l_{60}}\left\{\sum_{y=61}^{70} l_y + \frac{1}{2}\left(l_{60} - l_{70}\right)\right\}$$
$$e_{60(70:80)} = \frac{1}{l_{60}}\left\{\sum_{y=71}^{80} l_y + \frac{1}{2}\left(l_{70} - l_{80}\right)\right\}$$
$$e_{60(80:90)} = \frac{1}{l_{60}}\left\{\sum_{y=81}^{90} l_y + \frac{1}{2}\left(l_{80} - l_{90}\right)\right\}$$
$$e_{60(90:100)} = \frac{1}{l_{60}}\left\{\sum_{y=91}^{100} l_y + \frac{1}{2}\left(l_{90} - l_{100}\right)\right\}$$
$$e_{60(100:110)} = \frac{1}{l_{60}}\left\{\sum_{y=101}^{110} l_y + \frac{1}{2}\left(l_{100} - l_{110}\right)\right\}$$

And:

$$e_{60} = e_{60(60:70)} + e_{60(70:80)} + \ldots. + e_{60(100:110)}$$

Another way at looking at contributions

When we calculate a term such as $e_{60(70:80)}$ we are calculating a value based on a life currently aged 60 surviving to age 70 and then contributing these expected years. We can therefore express this in a different way.

Instead of looking at $e_{60(70:80)}$ we can look at $e_{70(70:80)}$ i.e. the amount of expected life that a person who is now aged 70 can expect to live over the next 10 years.

$$e_{70(70:80)} = \frac{1}{l_{70}} \left\{ \sum_{y=71}^{80} l_y + \frac{1}{2}(l_{70} - l_{80}) \right\}$$

We can see that:

$$e_{60(70:80)} = \frac{1}{l_{60}} \left\{ \sum_{y=71}^{80} l_y + \frac{1}{2}(l_{70} - l_{80}) \right\} = \frac{l_{70}}{l_{60}} \frac{1}{l_{70}} \left\{ \sum_{y=71}^{80} l_y + \frac{1}{2}(l_{70} - l_{80}) \right\} = \frac{l_{70}}{l_{60}} e_{70(70:80)}$$

This of course is intuitive as the term on the left is the expected life between the ages of 70 and 80 of someone currently aged 60, while the term on the right is the expected life between the ages of 70 and 80 of someone currently aged 70 multiplied by the probability that someone currently aged 60 reaches the age 70 i.e. a conditional probability.

Hence:

$$e_{60} = e_{60(60:70)} + \frac{l_{70}}{l_{60}} e_{70(70:80)} + \ldots + \frac{l_{100}}{l_{60}} e_{100(100:110)}$$

3. Results

Table 1(a) and 1(b) show the results of applying decomposition to male and female life expectancy at age 60 in England and Wales. The contribution from each decade of life is given in the rows for each given calendar year starting in 1950. Column totals give overall life expectancy at age 60 calculated by adding together the ten-year expectancies above. A final column shows the gain in years over a sixty year time span from 1950 to 2010.

age band	1950	1960	1970	1980	1990	2000	2010	gain 1950 to 2010 (years)
60-70	8.6	8.6	8.7	8.8	9.0	9.3	9.5	0.9
70-80	5.0	5.0	5.0	5.5	6.0	6.8	7.6	2.6
80-90	1.4	1.5	1.5	1.8	2.3	3.1	4.1	2.7
90-100	0.1	0.1	0.1	0.2	0.3	0.5	0.9	0.8
100+	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
total	15.1	15.2	15.3	16.3	17.6	19.7	22.1	7.0

(a) Men

age band	1950	1960	1970	1980	1990	2000	2010	gain 1950 to 2010 (years)
60-70	9.2	9.3	9.3	9.4	9.4	9.6	9.7	0.5
70-80	6.4	6.8	7	7.2	7.5	7.9	8.4	2.0
80-90	2.3	2.8	3.2	3.6	4.1	4.6	5.4	3.1
90-100	0.2	0.4	0.5	0.6	0.9	1.1	1.5	1.3
100+	0.0	0.0	0.0	0	0	0	0.1	0.1
total	18.1	19.3	20.0	20.8	21.9	23.2	25.1	7.0

(b) Women

Table 1: Male and female ten-year life expectancies from age 60 for England and Wales: 1950 to 2010

Although in earlier decades improvements in life expectancy for women were greater than for men, the results show an overall 7-year gain for both men and women in the period under review as men improve. For men, gains between ages 60 and 70 have been less than one year for the key reason that the likelihood of reaching age 70 is already high and close to the ten-year limit. Most of the gains have been made between ages 70 to 80 and 80 to 90 at 2.6 and 2.7 years respectively where there has been more scope for improvements. A similar story applies to women, except that more of the gains have occurred in the 80 to 90 age bracket than between 70 to 80.

Looking along the rows of each table we observe a gradually increasing contribution across the decades, but once we reach ages above 90 the rate of progress slows considerably. For the age bracket 100+ there are no significant gains at all, with the exception of women in 2010. This raises the important question of whether we can ever expect future increases in that age group; in the meantime it is evident that most future gains will occur in younger age bands which will, using the jam-jar analogy, ‘fill up’ to their maximum extent more rapidly.

Fitting trend lines to 10-year life expectancies

The picture presented above suggests a systematic shift in survival at higher ages. Having calculated 10-year life expectancy for each decade of life, it is important to know how longevity will behave in the future and when it will reach its natural limit in each decade of life. As we saw in Figure 1, simply projecting existing trends does not have the right mathematical properties to perform this role because it is unbounded and hence could reach uncharted levels within a few years.

Any fitted trend needs to be a non-decreasing function (if we are modelling Western economies where we expect life expectancy to continue to increase with time). We also need to ensure there is a natural limit to life expectancy meaning that we do not project infinite life spans. By working with ten-year age segments we know there is a definite limit within each age interval and so we can concentrate on age intervals still showing scope for growth. If growth trends are well established it should be possible to test functions that are good fits to the data but also trend towards this natural upper limit.

We therefore proceed as follows. Life expectancy in ten-year age bands is calculated using observed data for each calendar year. The results are presented in Figure 3 in which each set of data points represents one of five ten-year age bands starting at age 60 for each year since 1950.

The dotted extensions are trend lines based on the procedure which is outlined and justified below. For each band, we have assumed that the maximum life expectancy of ten will be reached over time, although clearly this would imply, for reasons already given, the elimination of all deaths if projected over a long enough period of time.

For the youngest age band (ages 60 to 70) the impact of the limit is minor as the life expectancy is already at or close to ten, and has been for many years. For older age bands, the true limit may, in reality, be less than ten and hence an element of uncertainty will always remain as we project forward; however, this is expected to have only a minor impact on short to medium term projections which are likely to be of most use in practice. At older ages there are signs of more rapid progress which supports the argument for retaining the current limit rather than reducing it from ten.

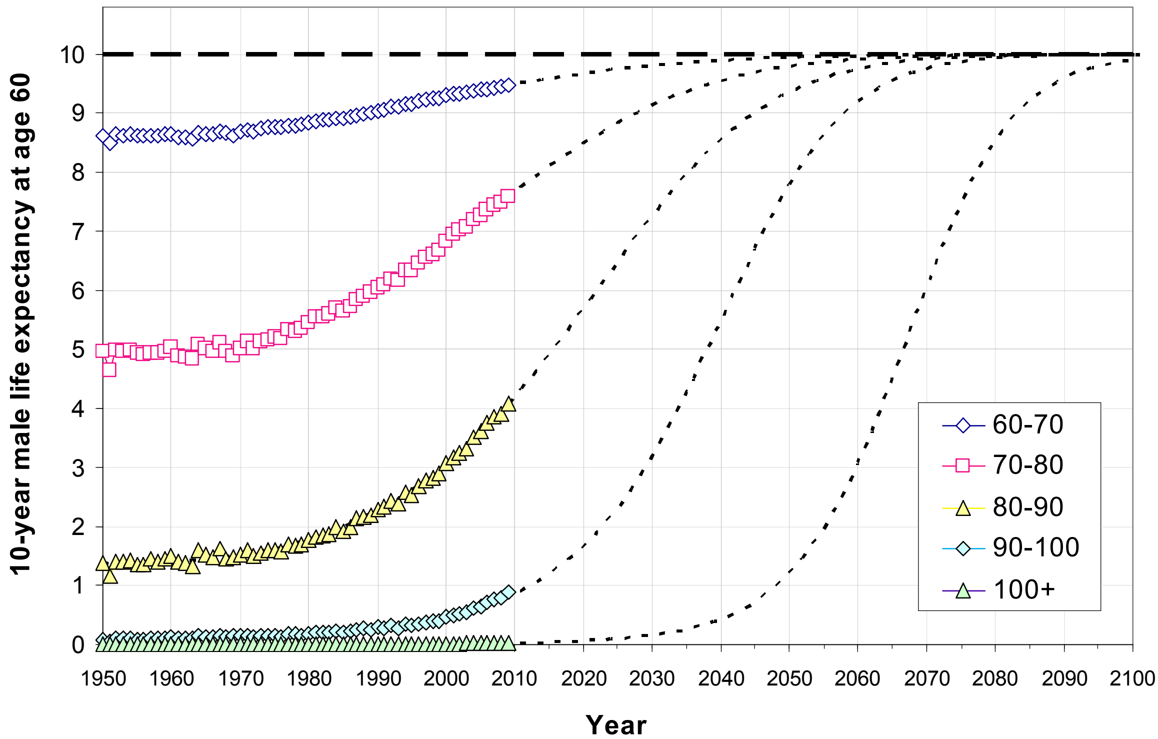

Figure 3: Trends in England and Wales male life expectancy by ten-year segments from age 60. Dotted line shows extrapolated trends based on function described in text

Basic projection model

Long range projections of life expectancy are obviously sensitive to both the projection methodology and assumptions. As already noted, we sought a function for which we could impose a limit which could not be breached, rather than a function which finds its own limit. Mayhew and Smith (2012) discuss the problem of demographic agencies imposing arbitrary limits to life expectancy which subsequently proved to be too low. By allowing the data to impose the trend, we showed that forecasts could have been better. However, we could also see our predictions starting to become unlikely post 2030 as the trend of life expectancy increased at an increasing rate.

By seeing life as a set of ten-year blocks, we are not forcing the data to fit this limit through our judgement, but rather it is a natural limit based on the design of our model. We are then able to chain-link expectations of life together to get an aggregate life expectancy projection built on the component parts. Because of its convenient properties, and the ease with which the parameters can be estimated, the most useful function we tested is a form of the logistic function which can be written as follows:

$$y_i = \frac{Ae^{f_i(t,n)}}{1+e^{f_i(t,n)}}$$

Where $f_i\ (t,n)$ is a polynomial equation of order *n* which generally takes the value of one or two. That is:

$$f_i(t) = a_i + b_i t$$

or

$$f_i(t) = a_i + b_i t + c_i t^2$$

where a_i, b_i and c_i are parameters to be estimated, *t* is calendar year, and *y* is life expectancy in age interval *i.* A is a constant defined by the user taking all age intervals to be equal and in this case is set to 10 years.

Re-arranging yields:

$$\ln\left[\frac{y_i}{A-y_i}\right] = f_i(t) = a_i + b_i t$$

or

$$\ln\left[\frac{y_i}{A-y_i}\right] = f_i(t) = a_i + b_i t + c_i t^2$$

In these equations, a_i, b_i and c_i are estimated using multiple linear regression. Note that higher order polynomials (*n*>2) could also be considered depending on the patterns observed in practice. By fitting this model to data in each age band from 1950 to 2009, we analysed how future life expectancy may be expected to develop, assuming the underlying trend is unchanged. In low mortality countries, such as those considered here, we found that first or second order polynomial equations generally performed best. In the case of England and Wales we used second order polynomials as these gave the best fit to the data.

By fitting the model to data in each age band, we analysed how future life expectancy may be expected to develop assuming the underlying trend is unchanged. Table 2 shows the regression parameters for both sexes. All coefficients are statistically significant at the 99% level of probability. As can be seen, values of R^2 are mostly above 0.95 indicating very good statistical fits to the data.

Returning to the jam-jar analogy, on combining the life expectancies in each 'jam-jar' we get total life expectancy at age 60 which is represented in the chart in Figure 4 and Table 3. This shows how each 'jam-jar' is in the process of filling up, starting in 1950 and with extra years (i.e. more 'jam') being added between 1950 and 2010. It is noteworthy that the projected added years between 2010 and 2030 will be greater than the added years from 1950 to 2010 emphasizing the rapidity of the process. The unshaded tips of the columns show the available years for improvement in each decade of life post-60. Once 10 years is reached, there are no available years left and the 'jam-jar' is full, but the chart also shows there is still many available years left in the 80 to 90 and 90 to 100+ age range.

Age band	α	α	α^{α}	α^{α}
60-70	1506.04	-1.53753	0.00039289	0.995
70-80	1790.56	-1.82854	0.00046682	0.995
80-90	1909.54	-1.95528	0.00050005	0.989
90-100	1934.80	-1.99756	0.00051432	0.974
100+	1246.23	-1.33228	0.00035287	0.947

(a) Men

Age band	α	α	α^{α}	α^{α}
60-70	830.93	-0.85020	0.00021814	0.971
70-80	630.84	-0.65222	0.00016874	0.980
80-90	327.28	-0.35180	0.000094043	0.985
90-100	0.80	-0.03391	0.000016223	0.974
100+	-540.43	0.49157	-0.00011207	0.947

(b) Women

Table 2: Fitted parameter values for the projection model applied to data for England and Wales for men and women

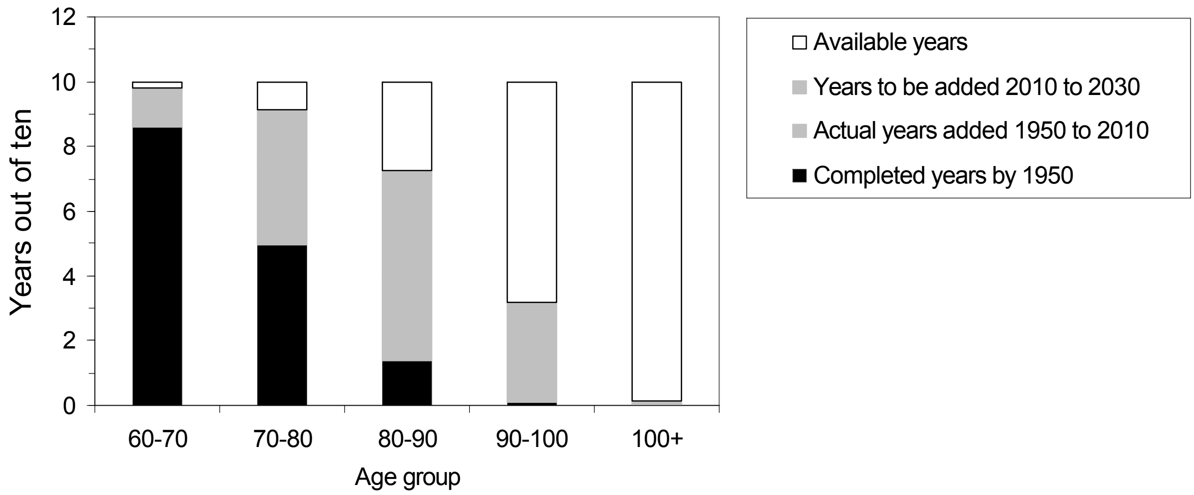

Figure 4: Jam-jar model of life expectancy at age 60 (England and Wales, men)

Age band	1950 (completed years)	added years between 1950 and 2010	added years between 2010 and 2030	available years
60-70	8.6	0.8	0.3	0.2
70-80	5.0	2.6	1.5	0.9
80-90	1.4	2.7	3.2	2.7
90-100	0.1	0.8	2.3	6.8
100+	0.0	0.0	0.1	9.8

Table 3: Completed and available years by age interval and calendar period (men)

The extrapolated trends in Tables 3 and 4 are obtained by adding together the forecast in each age bracket. They show that male (and female) life expectancy could continue to grow rapidly in coming decades particularly in certain age brackets. Clearly, as with any forecasting technique, the process of projection becomes more speculative the further ahead one looks, but we believe it is realistic for these trends to maintain their course until at least 2040 – not withstanding any unexpected shocks such as pandemics or wars.

Our extrapolations so far are clearly based on a normative assumption that a person can only have a maximum life expectancy of 110 years (i.e. 60 + 50 =110), but any presumed age horizon and whether people will survive beyond it remains an open question. Some argue for example the limit could be higher (e.g. Wilmoth 2000; Vaupel 2010). This uncertainty is plainly truer at higher ages, where ten-year life expectancy has advanced least, but it may take decades to establish if this view is correct or not which is why there is an interest in studying super centenarians (those aged 110 or older).

For super centenarians, the Human Mortality Database truncates data due to the tiny number of survivors and so no extrapolation is possible using our model at present. It is therefore important that the upper limit is not regarded as an absolute, but one which can be altered in the light of any new evidence and updated accordingly. The implication from Figure 5, with the data at our disposal, is that life expectancy at age 60 would start to level out after 2050, although this is clearly speculative as there are two possible scenarios. The first is that there will not be any appreciable growth in the number of centenarians and the second is that there will be! This results in two limits and not one.

Note also that if the question is 'what is life expectancy in the age interval 60 to 100?' rather than 'what is the upper limit?', then including occurrences of survival above 100 becomes redundant. Because of the paucity of data from age 100 and above, this is potentially a more reliable way of comparing future life expectancy in different countries (see below). Arguably, it is the forecasts of life expectancy in intermediate decades which are of more practical value for demographers and governments.

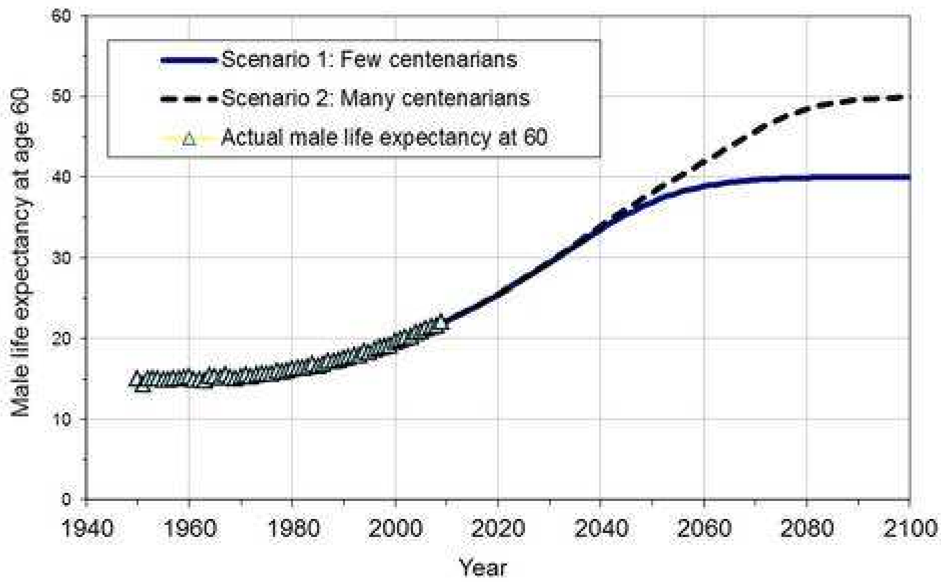

Figure 5: Projected male life expectancy at age 60 for England and Wales

4. Discussion

If the process of ageing described is correct then it will also be accompanied by a convergence in age of death. This will give individuals, government policy makers, pension providers and insurers more certainty with which to plan. This in turn could be arguably beneficial to individuals and to society as a whole, but there are downsides as well as upsides to this suggestion. For example, the extra years from age 80 to 90 are important because it depends on whether they are spent in good health or disability. The jury is still out whether extra years of life expectancy are being spent in good health or disability.

In fact the decade of life from 80 to 90 is fast becoming the new pivotal years, which we may characterise in terms of four types of older household. These depend on whether it is a couple household or single dweller household and on whether there are care needs or not. From our results, life expectancy in this age segment has grown by nearly three years for both genders and that the genders are starting to converge. Depending on the age difference we know that couple households will generally be more resilient than single dwellers, especially if the single dweller is disabled and hence more likely to require state support and intervention.

It follows that trends in the prevalence of each household type could become a key driver for determining future care needs. The fact that male life expectancy is catching up with female life expectancy, and that both are living longer, suggests that older couple households will grow in prevalence and remain in their homes for longer. In turn, this will impact on housing supply and hence house prices unless house building keeps abreast of demand. This could point to a new policy opportunity for Government, namely to provide financial inducements to downsize at younger ages, for example, when the children reach adulthood.

However, some further words of caution are required. Pensions Minister Steve Webb, in announcing the budget changes in 2014, suggested that retirees could be issued guidance on how long they are likely to live**.** He said: “Estimates of life expectancy would be based on factors such as gender, where they live, and whether they smoke.” It is maintained that this information would help them plan their finances in retirement more efficiently. However, this ignores the fact that life expectancy is only an average based on current mortality experience and due to continuing mortality improvements and selection effects, many will not die when they had expected to. This reinforces the need for improved forecasts of life expectancy for different sub-groups of the population on an annual basis.

Will decomposition be helpful for forecasting populations? Currently, our decomposition method produces a single forecast of life expectancy for each age band. This essentially deterministic approach could be supplemented using regression confidence intervals which would show ranges of uncertainty. This is likely to be most useful at higher age intervals for obvious reasons. Stochastic formulations are another possible direction for development. Because the method is new, it is too early to compare its forecasting potential with the greatly expanding literature on demographic forecasting based either on life expectancy or mortality (e.g. for reviews see Booth 2006 or Rafferty et al 2013). Conceptually, however, there are clear differences of approach.

Our model is based on trends in what were termed ‘partial life expectancy’ from which mortality rates may be derived as required. It requires a long-time series and unabridged life tables for best results and so far has only been tested on low mortality countries. In the widely used Lee-Carter model, it is the other way around with trends in mortality rates forming the starting point (Lee and Carter 1992). Whereas Lee-Carter is stochastic, our model is able to predict turning points using the logistic function and exploits natural age limits found in each age segment. This is an important difference since, as Bongaarts (2005) notes, the invariance in the rate of decline of mortality assumed by Lee-Carter produces implausible results after just a few decades.

Using the logistic function, Bongaarts is able to project the force of mortality more accurately than the Lee-Carter method. Nevertheless, drawing general conclusions of the relative merits of each approach, and comparing it with our own is somewhat complicated and also arguably premature because of the many Lee-Carter variants available (e.g. see Renshaw and Haberman 2003 and 2006 or the 'rotation' approach in Li et al 2013). In contrast to Lee-Carter, deterministic methods are widely used for forecasting life expectancy as opposed to mortality, with the United Nations, which produces world estimates by country, being a notable example of this practice (UN, 2009). More recently Rafferty et al (2013) have proposed a stochastic UN variant for which superior accuracy is claimed.

Both the UN and Rafferty et al base their forecasts on the double logistic function. This assumes life expectancy will decline over a period, before continuing asymptotically at a linear rate. Our method also uses a logistic variant to forecast life expectancy but within discrete ten-year age bands which are upper-bounded. In other words, we do not consider the possibility of ever-increasing life spans but nor do we rule it out. By working with, in this case ten-year age segments, we know there is a definite limit within each age interval and so we can concentrate on age intervals that are still showing growth.

Put another way, decomposition can address the question 'what is life expectancy in an arbitrary age interval, say, between 60-and 100?' rather than 'what is the upper limit to life expectancy?' The maximum attainable is 40 years in this particular example and so the question of life expectancy above 100 becomes redundant. Although the method is flexible in this respect, and works equally well for men and women, available data from which to establish trends is clearly more problematic and users can choose whether to include the highest age brackets in their reporting for which there may be a lack of obvious trend.

In conclusion, most future growth in life expectancy in retirement will come between ages 70 and 100. Life expectancy beyond 100 years of age is increasing very slowly and so will not contribute as much as was thought. Age at death will tend to increasingly cluster in early 90s as the age of death of men and women converge (Wilmoth and Horiuchi, 1999; Canudas-Romo, 2008; Mayhew and Smith 2014). Decomposition gives us more precision and flexibility in in identifying and unpicking these future trends, which is beneficial both for governments and individuals alike.

This brings us back to the more fundamental question of whether we are starting to plan for older age better than we were. Later retirement requires a capacity to work for longer and it may also mean downsizing one's home, at an earlier stage i.e. before a person has reached the nursing home stage requiring a forced home sale in order to pay for care. This could be to boost to the person's income in retirement or allow them to pay for luxury items like a new car or a world cruise. If this kind of planning gains traction it could lead to a significant transformation in the housing market as well as growth in the provision of professional financial advice including tax and inheritance planning.

In parallel, greater responsibility will fall on the individual to make choices, to pay for services and to seek help and advice. We already know that average pension pots are small and must last for longer so that using the value in the home becomes more important, especially if ill health increases living costs. Better planning tools including decomposition approaches to life expectancy as described here will help to anticipate and mitigate these effects.

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