

Applications and Calculation of a Distribution Class Locational Marginal Price

by

Nick Steffan

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Graduate Supervisory Committee:

Gerald Heydt, Chair
K. Hedman
G. Karady

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ABSTRACT

This thesis presents an overview of the calculation and application of locational marginal prices in electric power systems particularly pertaining to the distribution system. The terminology proposed is a distribution locational marginal price or DLMP. The calculation of locational process in distribution engineering is conjectured and discussed. The use of quadratic programming for this calculation is proposed and illustrated. A small four bus test bed exemplifies the concept and then the concept is expanded to the IEEE 34 bus distribution system. Alternatives for the calculation are presented, and approximations are reviewed. Active power losses in the system are modeled and incorporated by two different methods. These calculation methods are also applied to the 34 bus system. The results from each method are compared to results found using the PowerWorld simulator.

The application of energy management using the DLMP to control load is analyzed as well. This analysis entails the use of the DLMP to cause certain controllable loads to decrease when the DLMP is high, and vice-versa. Tests are done to illustrate the impact of energy management using DLMPs for residential, commercial, and industrial controllable loads. Results showing the dynamics of the loads are shown.

The use and characteristics of Matlab function FMINCON are presented in an appendix.

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NOMENCLATURE

A	Coefficients of the state vector in inequality constraints
A_{eq}	Coefficients of the state vector in equality constraints
b	Right hand sides of inequality constraints of the form $Ax \leq b$
B_{eq}	Right hand sides of equality constraints of the form $A_{eq}x = b_{eq}$
c	Coefficients of the linear terms in a cost function
C	Cost
CAISO	California ISO
d	Right hand sides of equality constraints of the form $Ex = d$
DAM	Day ahead market
DLMP	Distribution Locational Marginal Price
E	Coefficients of the state vector in equality constraints
EMS	Energy management system
ERCOT	Electric Reliability Council of Texas
F	Function to minimized
FERC	Federal Energy Regulatory Commission
FREEDM	The Future Renewables Electric Energy Delivery and Management
FUN	Call function for FMICON in MATLAB
IEEE	Institute of Electrical and Electronics Engineers
IFM	Integrated Forward Market
ISO	Independent system operator
ISO-NE	ISO New England
KKT	Karush-Kuhn-Tucker
L	Lagrangian function to be minimized
LB	Lower bound
LMP	Locational Marginal Price
LP	Linear programming
μ	Lagrangian multiplier
MRTU	Market Redesign and Technology Upgrade

NYISO	New York Independent System Operator
NYPP	New York Power Pool
P	Power
PJM	Pennsylvania-Jersey-Maryland Interconnection
PLL	Phase locked loops
PWM	Pulse width modulated
Q	Coefficients of the quadratic terms in a cost function
QP	Quadratic programming
r	Resistance
SMD	Standard market design
SQP	Sequential quadratic programming
UB	Upper bound
x	A state vector that gives the system operating point
Y	Admittance
Z	Impedance

Chapter 1: Locational marginal prices and their application and calculation in distribution systems

1.1 Motivation for thesis

Locational marginal pricing has been used in transmission systems for over a decade [1]. It has served as an adept measurement of different variations in prices and bottlenecks within the transmission system. There is no measurement like this currently in the system. As government and industry influences push towards a “smarter” grid, the application of a distribution based locational marginal price (DLMP) could be very useful. The DLMP would be very similar to that of the transmission system in that it would as accurately as possible display the cost for one additional unit of energy to be supplied to a particular bus. Where the transmission system is commonly defined as the cost for one additional megawatt to be supplied, it might make more sense to define the DLMP as the cost of one additional kilowatt.

The DLMP could then serve many purposes. Most obviously it could show price variations throughout the distribution grid, but others as well. The application of a DLMP could identify which components cause high prices within the system. With this information, it could be determined where improvements in the system would be most beneficial. Improvements such as additional lines, distributed generation or even distribution level storage could be implemented to lower costs of the system. Additionally the DLMP could serve as a pricing structure for consumer level energy management. Consumers could obtain information from the DLMP and react to the prices, choosing whether to continue consuming or to reduce load. This control would not only save consumers money, but it could also help reduce the peak load in times of high prices.

1.2 Objectives of the study

Recently, a variation of the LMP concept has been proposed for distribution systems, e.g., [2]. In distribution engineering, a pricing signal could be used for local control [3,4]. As an example, when pricing signals are high, local energy storage (e.g., in electric vehicles [5]) could be controlled to ‘discharge’ alleviate the high energy cost condition. Conversely, when the pricing signal is low, energy storage elements could be controlled to ‘store’ energy. Such a mechanism has the advantage of leveling distribution system demand and concurrently increasing load factor.

As stated previously, the DLMP would be defined as the cost to produce on additional kilowatt to a particular bus. This research aims to show the application of the DLMP in multiple sized systems. A very small system will be used as well as a slightly larger system from IEEE. For these two systems, different optimizing methods will be used to calculate the DLMP to illustrate:

- The inclusion of active losses
- To evaluate accuracy of the solution
- To include or exclude such phenomena as reactive power limits, PV versus PQ buses, and voltage controlled buses
- To show illustrations of the calculation method.

1.3 Literature review

1.3.1 Locational marginal prices

The present structure of LMPs was originally developed by Hogan of Harvard University in the early and mid 1990s [6]. Over a decade ago numerous markets around the world adapted location marginal pricing for their grids. The issues related to locational marginal prices, their calculation and use include

“adequacy of models and tools being used for economic dispatch, unit commitment and the calculation of the LMP; addressing infeasibilities; interpreting LMP components; physical and marginal loss pricing; recovering ‘as bid’ costs for the generators etc.” [7]

Reference [1] outlines the history of how LMPs have been applied to contemporary power marketing:

- 1992: The Energy Policy Act is passed. FERC initiates the transition to competitive bulk energy markets
- April 1997: PJM becomes the first association of interconnected electric systems, or power pool, to officially operate as a regional transmission organization and independent system operator (RTO/ISO)
- July 1997: New England (ISO-NE) is declared an ISO.

- July 1999: ISO-NE implements wholesale energy markets.
- December 1999: New York Independent System Operator (NYISO) formally takes over control and operation of bulk transmission and generation dispatch in New York from New York Power Pool (NYPP).
- 2003: ISO-NE adopts an LMP scheme as part of its transition to a so-called SMD.
- April 2009: California ISO (CAISO) goes live with a fully nodal LMP market. The Market Redesign and Technology Upgrade (MRTU) project establishes an LMP real-time market and a day-ahead market (DAM). This combination, known as the Integrated Forward Market (IFM), is designed to co-optimize energy, reserves, and capacity, balancing supply and demand.
- December 2010: ERCOT goes live with a fully nodal LMP market and DAM.

1.3.2 Contemporary applications of LMPs

Locational marginal prices conventionally are the cost to deliver one additional megawatt hour to a given bus within a power system. The calculation includes optimal dispatch, line and generation constraints, and potentially additional equality and inequality constraints. Locational Marginal Prices are a pricing method used to establish the price for energy purchases and sales at specific location and under a specific operating regime. Contemporary practice is that LMPs are calculated through a linear programming (LP) process. The LP minimizes the total energy cost for the entire area subject to constraints that represent the physical limitations of the power system. For example, at the New England ISO, the linear programming process yields three portions of the LMP corresponding to the energy component, the loss component and congestion component [8] as seen in (1.1). The energy component does not depend on the physical location in the system, while the loss and congestion components are uniquely calculated at each specific system bus,

$$LMP = LMP_{energy} + LMP_{congestion} + LMP_{loss}. \quad (1.1)$$

A common calculation method to obtain the LMPs is linear programming because (1) can be written in approximate terms by linearizing the power flow equations. However, the loss term in (1.1) is often omitted. In transmission systems, the active power losses are in the range of 2 to 5%. The unique characteristics of each individual bus are what cause price differentiation between each bus.

In transmission systems, LMPs have value in revenue pricing and identification of bottlenecks in the system. Since their implementation they have become one of the most popular methods for congestion management in many markets worldwide. As a result of their current structure, LMPs not only reveal current energy prices, but help price other ancillary services as well. [7].

1.4 Energy management systems

Numerous companies offer home energy management systems. Companies like General Electric, Schneider Electric, Hitachi and others provide a system that a home owner can use to control their home. [9] In the case of the Schneider electric product it “allows homeowners to reduce or shift energy use during peak times and helps electricity providers improve grid efficiency and network reliability.” [10] When referring to load control the device can be used for remote monitoring and management of HVAC compressors, water heaters, pool pumps and other power circuits.

These types of systems however are manually driven where a more automatically driven system is based on the DLMP is proposed later in this thesis. With the high probability of increased distributed generation and storage there is a need to have a more automatic energy management system. Other ‘automatic’ systems have been proposed. In [11] a system based on control through cloud computing is proposed. In this system the load is controlled based on peak power times and mitigates power based on connected appliances. In addition to [11], more proposals for energy management are listed in table 1.1 as well as different projects by the Electric Power Research Institute (EPRI) in Table 1.2.

Table 1.1 Energy management proposals

Load(s) Controlled	Basic Strategy	Reference
Home appliances based on a given schedule; has distributed generation and storage capabilities	Assigns dynamic priority to a household appliance according to the type of appliance and its current status. In accordance with the assigned priority, the use of household appliances is scheduled considering renewable energy capability	[11]
Controls household load based on appliances and cost	Consists of price prediction, a load scheduler and energy consumption monitor. The electricity pricing models provide the price prediction capability. The load scheduler is used to control the residential load with an aim at reducing the total energy cost and the smart meter and smart switchers are utilized to collect and monitor the energy consumption in the house	[28]
Individual homes major appliances and lighting	A smart home control system that can assign tasks to suitable components. It can automatically gather physical sensing information and efficiently control various consumer home devices.	[29]

Table 1.2 EPRI projects on Energy Management

Title	Abstract [31]
Energy Management Systems for Commercial Buildings:	Approximately 25,000 commercial buildings in the United States have energy management systems. Planners estimate that by 1990 another 80,000 systems will be in use. This primer on commercial building energy management systems describes their functions, components, and design options
Assessment of Commercial Building Automation and Energy Management Systems for Demand Response Applications:	An overview of commercial building automation and energy management systems with a focus on their capabilities (current and future), especially in support of demand response (DR). The report includes background on commercial building automation and energy management systems; a discussion of demand response applications in commercial buildings, including building loads and control strategies; and a review of suppliers' building automation and energy management systems
Commercial Building Energy Management Systems Handbook: Opportunities for Reducing Costs and Improving Comfort:	This document is written for the commercial building owner, manager, or developer without a technical background but wanting to understand and evaluate recommendations for energy savings or comfort made by energy consultants and/or building engineers. It provides an overview of commercial building heating, ventilating, air-conditioning (HVAC), and lighting systems, and of the energy management systems (EMSs) that control comfort and provide energy savings.
Integration of Utility Energy Management Technologies into Building Automation Systems:	The challenges with managing peak demand are expected to worsen as decarbonization, plant retirement, renewable integration, and electric vehicle rollouts unfold. One solution to this problem is in better management of the demand side. This study is focused on commercial buildings, which account for approximately 27% of all electricity used in the United States and have a large impact on demand since much of the consumption falls during business hours, which tend to correspond with peak demand windows
Standard Interfaces for Smart Building Integration:	Electricity systems in the United States are changing to accommodate increasing levels of distributed energy resources and demand responsive loads. Commercial buildings are positioned to play a central role in this change. With advances in energy generation and storage technologies, process management, and controls, commercial buildings are increasingly able to provide a range of grid supportive functions

1.5 Organization of this thesis

The remainder of this thesis will be organized into five additional chapters. Chapter two will discuss the process of using quadratic programming to minimize a cost. In addition it will discuss how the 'FMINCON' function in Matlab can potentially be used to approximate the losses. Chapter three will take the theory in chapter two and apply it to calculating the DLMP using quadratic programming. Both a small example and larger example will be analyzed. Chapter four is a continuation of chapter three but with the application of losses. Comparison of the three methods proposed in this thesis will be made against PowerWorld simulator. Chapter five discusses the role of the DLMP and it uses for energy management.

In this chapter, the DLMP is used in various ways to control the load of the system. Chapter six ends the thesis with a conclusion, recommendations and future work.

An appendix contains comments on Matlab function FMINCON. Execution time and convergence is discussed and illustrated.

Chapter 2: The theory and application of quadratic programming in power distribution engineering

2.1 Definition of the quadratic programming problem

Quadratic programming (QP) is the optimization of a quadratic function. Mathematically, consider the extremization of the scalar function $c(x)$,

$$f(x) = \frac{1}{2}x^T Qx + c^T x \quad (2.1)$$

where the objective function has a vector valued argument x , a vector of n rows, Q is a constant n by n matrix, and c is a constant n -vector. In the case where $Q = 0$, the problem is solved by LP. The constraints of (2.1) are

$$Ax \leq b \quad (2.2)$$

$$Ex = d \quad (2.3)$$

where A is an m by n matrix and E is a k by n matrix.

Quadratic programming is commonly solved by the Karush-Kuhn-Tucker (KKT) method. This method entails the creation of the Lagrangian function,

$$L(x, \mu) = cx + \frac{1}{2}x^T Qx + \mu(Ax - b) \quad (2.4)$$

where μ is a m -dimensional row vector. The conditions for a local minimum are as follows [10],

$$\begin{aligned} \frac{\partial L}{\partial x_j} &\geq, & j = 1, \dots, n & & c + x^T Q + \mu A &\geq 0 \\ \frac{\partial L}{\partial \mu_j} &\leq, & i = 1, \dots, m & & Ax - b &\leq 0 \\ x_j \frac{\partial L}{\partial x_j} &= 0, & j = 1, \dots, n & & x^T(c + x^T Q + \mu A) &= 0 \\ \mu_i g_i(x) &= 0, & i = 1, \dots, m & & \mu(Ax - b) &= 0 \\ x_j &\geq 0, & j = 1, \dots, n & & x &\geq 0 \\ \mu_j &\geq 0, & i = 1, \dots, m & & \mu &\geq 0. \end{aligned}$$

Reference [13] also discusses this formulation. In essence, the KKT method causes the last term in (2.4) to be zero. This happens by virtue of either the term μ as zero, or the coefficient of μ as zero (this occurs

row by row when the elements in (2.4) are vectors). To solve, rearrange the inequality constraints with nonnegative slack variables y , v inserted, and the KKT conditions can now be written as follows,

$$xQ + \mu^T A^T - y = c^T \quad (2.5)$$

$$Ax + v = b \quad (2.6)$$

$$x \geq 0, \quad \mu \geq 0, \quad y \geq 0, \quad v \geq 0 \quad (2.7)$$

$$y^T x = 0, \quad \mu v = 0. \quad (2.8)$$

Eqs. (2.5-2.8) are linear and LP is applied to obtain a solution [12]. This is the method used in Matlab.

Appendix A contains a more detailed discussion of the Matlab implementation of QP, namely FMINCON.

2.2 Formulation of DLMP using quadratic programming

The application of quadratic programming is a process of taking real world functions and constraints and applying them to the process above. The function that will be minimized is associated with costs of the system. Those costs are the result of fuel (generation) and system related costs (congestion and losses). The equality and inequality constraints are derived from the characteristics of the system. Line data such as resistances, impedances and thermal limit, as well as load data and generation capacity all can contribute to these constraints. So to follow the formulas above, Q and c in equation (2.5) are derived from costs and A , b , E and d are due primarily line and load data.

2.3 Inclusion of losses

The inclusion of line losses in the above formulation is problematic because quadratic programming, at least in the classic formulation shown in section 2.1, does not permit nonlinear constraints. Losses are generally not negligible in distribution systems (estimates vary as to the percentage of losses, but generally 3 to 7% active power losses are reasonable estimates). Additionally, there are losses in the distribution transformers at the points of common coupling.

There are many approaches to the inclusion of losses one being linearization. However, in this paper another approach is offered: relaxation of the loss term by inclusion in the objective function. If the function $f(x)$ is augmented with an additive term that captures the cost of the losses, the minimization of

$f(x)$ (i.e., calculation of $f^* = f(x^*)$) will give an approximate solution to the constrained optimal dispatch. Subsequently, the loads specified can be increased by a small amount to calculate the change in f^* . Then the LMP at the bus at which the load was increased is calculated as the change in f^* .

To effectuate the approach outlined above, modify the $f(x)$ formulation as used in the lossless case,

$$f(x) = C(x) + \frac{f(x^*)}{P_{load_1} + \dots + P_{load_i}} [P_{loss_1} + \dots + P_{loss_i}] \quad (2.9)$$

where $C(x)$ is the total cost of generation without considering losses. In (2.9), the coefficient of the second term is the approximate generation cost expressed in \$/MWh, the sum term at the end of (2.9) represents the total system-wide active power losses. Therefore the entire second term in (2.9) is the approximate cost of active power losses. Assuming $f^* = f(x^*)$ to be the optimum (i.e., minimum operating cost) solution, and the total cost calculated including losses, then

$$f(x) = \frac{C(x)}{1 - \frac{P_{loss_1} + \dots + P_{loss_i}}{P_{load_1} + \dots + P_{load_i}}} \quad (2.10)$$

It is interesting to note that in (2.9), the term $f(x^*)$ is taken to be a constant, and differentiation of (2.9) then treats $f(x^*)$ as a constant. Of course, if $f(x)$ on the right hand side of (2.9) were taken as a variable, the derivative of $f(x)$ with respect to x would nonetheless be zero because $f(x)$ is being extremized (minimized in this case).

As the LMP is defined as the cost to deliver one more megawatt for one hour for a given location, then the DLMP can be formulated as,

$$DLMP = \frac{f(x(t+1)) - f(x(t))}{P_{load}(t+1) - P_{load}(t)} \quad (2.11)$$

where $f(x(t+1))$ is the new total cost due to load change $P_{load}(t+1)$, $f(x(t))$ is the total cost determined previously, i.e., at load $P_{load}(t)$. This is the approach taken to include line losses. It is noted, however, that the model for the losses is approximate.

Note that the proposed formulation as given in (2.11) disallows the use of classically formulated QP because of the nonlinear term in $f(x)$. This term occurs due to the inclusion of losses in the model. Modern commercially available software is used to solve the lossy case shown here. For example, Matlab uses function FMINCON which can be used in this application. FMINCON uses Sequential Quadratic Programming (SQP) [13] which is a variant of the Kuhn-Tucker approach. The basis of SQP is to model the minimization problem at x_k by a quadratic sub-problem and to use the solution to find a new point x_{k+1} . The explanation of FMINCON and SQP in general appears in [14,15].

The Kuhn-Tucker approach uses the exclusion conditions: in (2.4), the $\mu(AX-b)$ term must be such that either the μ_i term is zero or the $(AX-b)_i$ term is zero. There is one such exclusion condition for each inequality constraint, and therefore one assumes that the number of cases to be checked is proportional to 2^d where d is the number of inequality constraints and d is the dimension of μ . The actual Matlab code uses various procedures to reduce the dimensionality of the problem, but nonetheless, the execution speed is *not an advantage* of FMINCON. Also, in large scale problems, excessive memory requirements have been reported [16]. In distribution system applications, neither the execution time nor the memory requirements were found to be problematic, but these are marked for potential problems in some large scale applications. Note that in typical distribution engineering applications, the number of line limits to be applied as constraints is not large, and many could be dropped from consideration because of the robustness of the systems (i.e., the line limits are not reached in any credible steady state system operating condition).

Chapter 3: Calculation of DLMPs

3.1 Locational marginal prices for power distribution systems

The process to calculate the distribution based LMP will be explained in this chapter. Using the concepts explained in Chapter 2 an example using a small four bus system will be used to display the DLMP concept. The concept will then be expanded and applied to the IEEE 34 bus test bed. Both the lossless and lossy cases are illustrated. Figure 3.1 shows the general approach taken in these examples.

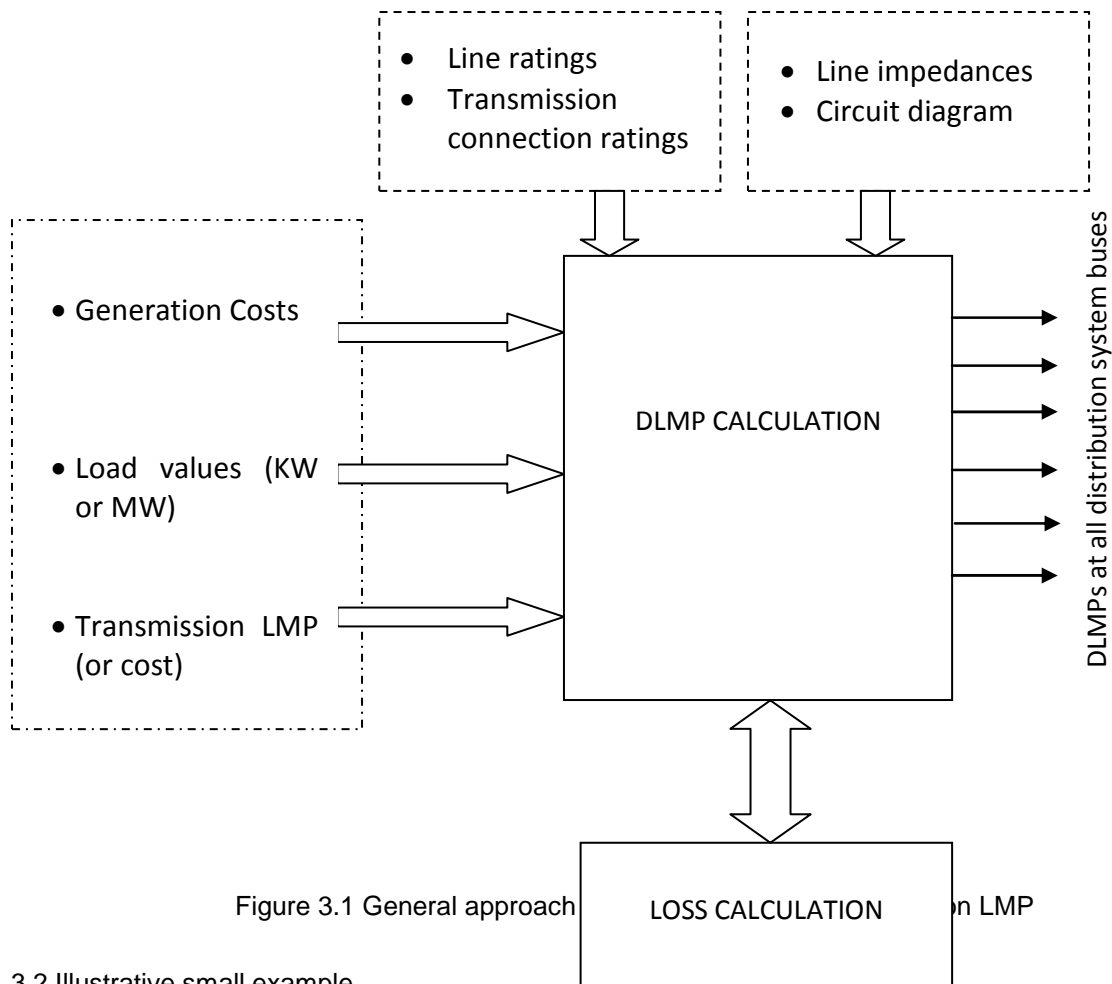


Figure 3.1 General approach

3.2 Illustrative small example

For purposes of illustrating the algorithms in Chapter 2, a small example is offered. Figure 3.2 shows a four-bus networked system with line data shown in Table 3.1. In the system, there are two sources P_1 and P_2 with corresponding cost functions (P_i in MW, C_i in \$/h),

$$C_1 = 2P_1 + 0.1P_1^2 \quad (3.1)$$

$$C_2 = 1.5P_2 + 0.12P_2^2 \quad (3.2)$$

Two cases are considered: the lossless case, and the lossy case.

3.2.1 Lossless case

The lossless case is considered first by assuming that the line impedances shown in Table 3.1 are all reactive (i.e., $R = 0$). The loads P_3 and P_4 both are less than or equal to 30 MW. It is desired to obtain the constrained economic dispatch for this system. Using a base power of 10 MW, let the bus loads be represented as $P_i > 0$. Then the problem is,

$$\text{Min } \frac{1}{2} X^T \begin{bmatrix} 20 & 0 & \dots & 0 \\ 0 & 24 & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & 0 \end{bmatrix} X + [20 \quad 15 \quad \dots] X$$

$$X = [P_1 \quad P_2 \quad P_3 \quad P_4 \quad P_{l1} \quad P_{l2} \quad P_{l3} \quad P_{l4} \quad \delta_2 \quad \delta_3 \quad \delta_4]^T.$$

The conservation of active power at each bus is,

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_{l1} \\ P_{l2} \\ P_{l3} \\ P_{l4} \end{bmatrix} = 0$$

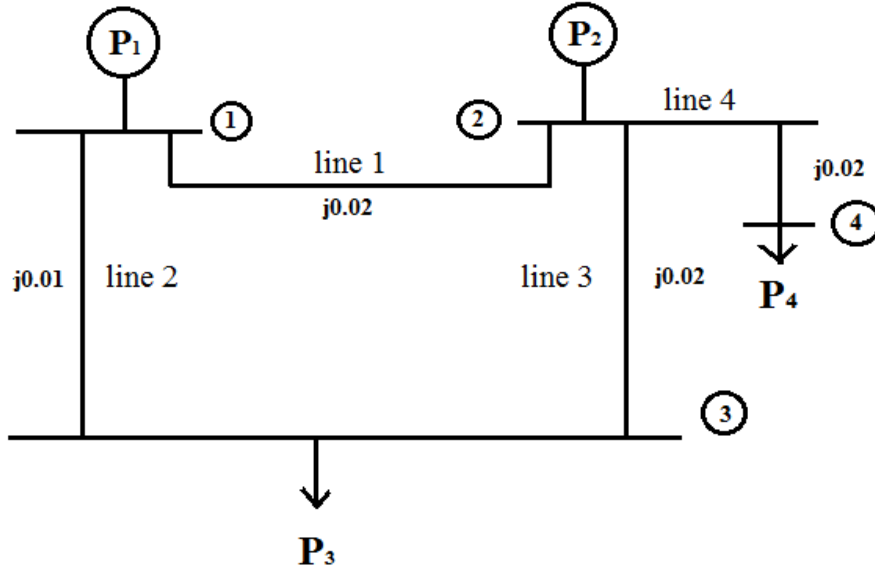


Figure 3.2 An illustrative example of quadratic programming to calculate the DLMP

Table 3.1 Line ratings for example system shown in Figure 3.2

Line	Line impedance (p.u., 10 MVA base)	Rating (MW)
1-2	$0.0025 + j0.02$	15
1-3	$0.0013 + j0.01$	15
2-3	$0.0030 + j0.02$	25
2-4	$0.0040 + j0.02$	30

The line active power flows are,

$$P_{l1} = \frac{\delta_1 - \delta_2}{x_{12}} \quad P_{l2} = \frac{\delta_1 - \delta_3}{x_{13}} \quad P_{l3} = \frac{\delta_2 - \delta_3}{x_{23}}$$

$$P_{l4} = \frac{\delta_2 - \delta_4}{x_{24}} \quad \delta_1 = 0.$$

The line constraints are (in per unit),

$$|P_{l1}| \leq 1.5 \quad |P_{l2}| \leq 1.5 \quad |P_{l3}| \leq 2.5 \quad |P_{l4}| \leq 3.0$$

Using the QUADPROG function in Matlab, the optimum operating cost results are obtained and shown in Figs. 3.3 and 3.4. Using (2.11), the DLMP for bus 3 is calculated and shown in Figure 3.4.

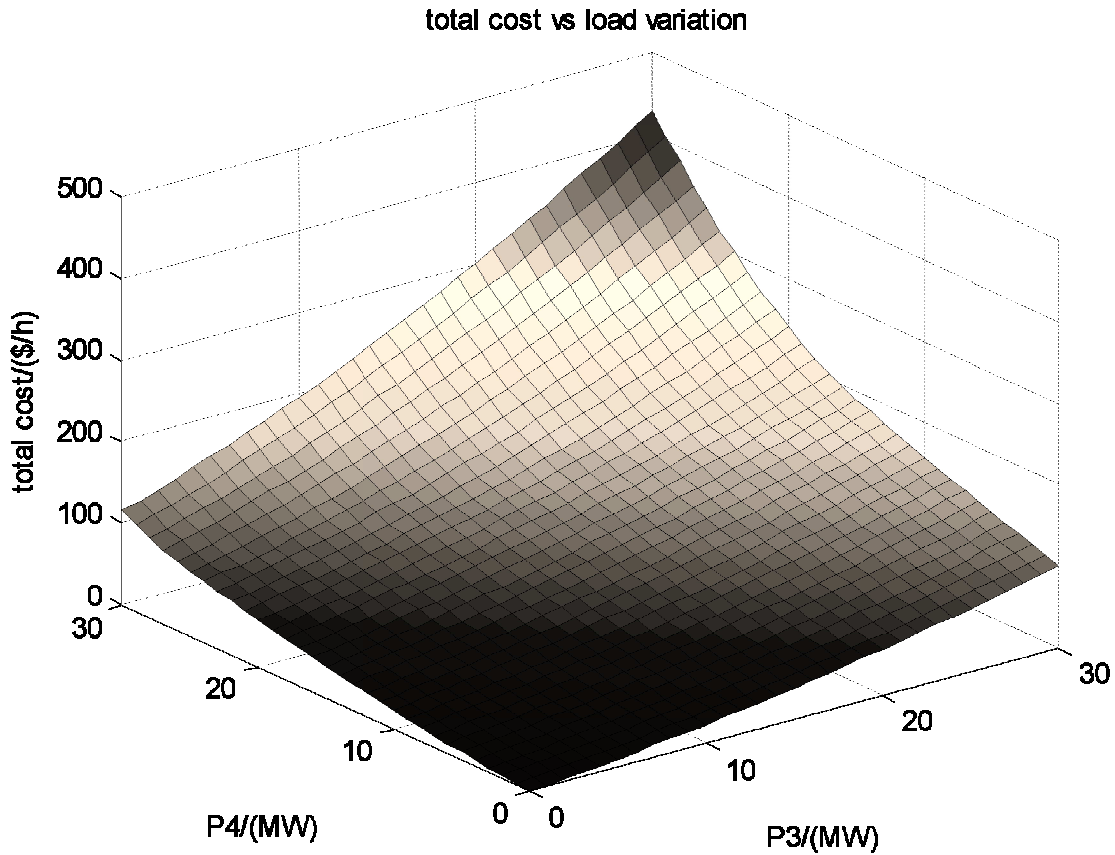


Figure 3.3 Results obtained for a constrained economic dispatch, P3 and P4 are loads – lossless case

The results in Figure 3.3 show a contoured map of the DLMP at buses three and four. As the load of each bus varies from zero to 30 MW, the DLMP at each bus is affected. The total load is set up in a way so that it is always equal to 30 MW. For instance if the load at bus three is five, bus four will be 25.

3.2.2 The lossy case

The same example shown in Figure 3.2 is reconsidered with losses included (i.e., $R \geq 0$ as shown in Table 3.1). The formulation is as in (2.10). For this case,

$$\underline{\hspace{10em}}$$

$$\underline{\hspace{10em}}$$

where

$$P_{loss(i)} = P_{li}^2 R_{li}$$

$$X = [P_1 \ P_2 \ P_3 \ P_4 \ P_{l1} \ P_{l2} \ P_{l3} \ P_{l4} \ \delta_2 \ \delta_3 \ \delta_4]^T$$

The formulation shown here is used in Matlab function FMINCON, and the resulting DLMP is shown in Figure 3.4. Abscissas and ordinates are superimposed so that quantitative comparisons can be made with lossy results. Bus P4 has been set to four different values and bus P3 is gradually increased.

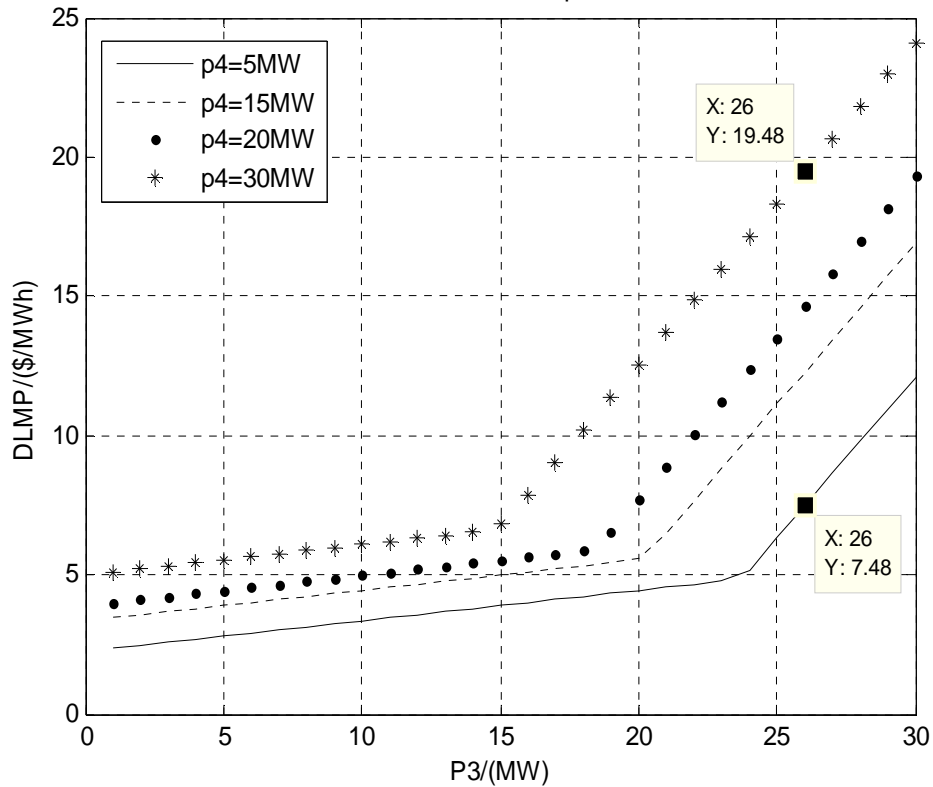


Figure 3.4 Result obtained for a constrained economic dispatch, P3 and P4 are loads – lossless case

The constraints are the same as those for the previous lossless example. Using the FMINCON function in Matlab, the optimum (minimum) total cost is obtained and shown in Figure 3.5 and the DLMP for bus 3 with loss are obtained and shown in Figure 3.6. Note that abscissa and ordinates are shown in the figure so that a comparison with Figure 3.6 can be made.

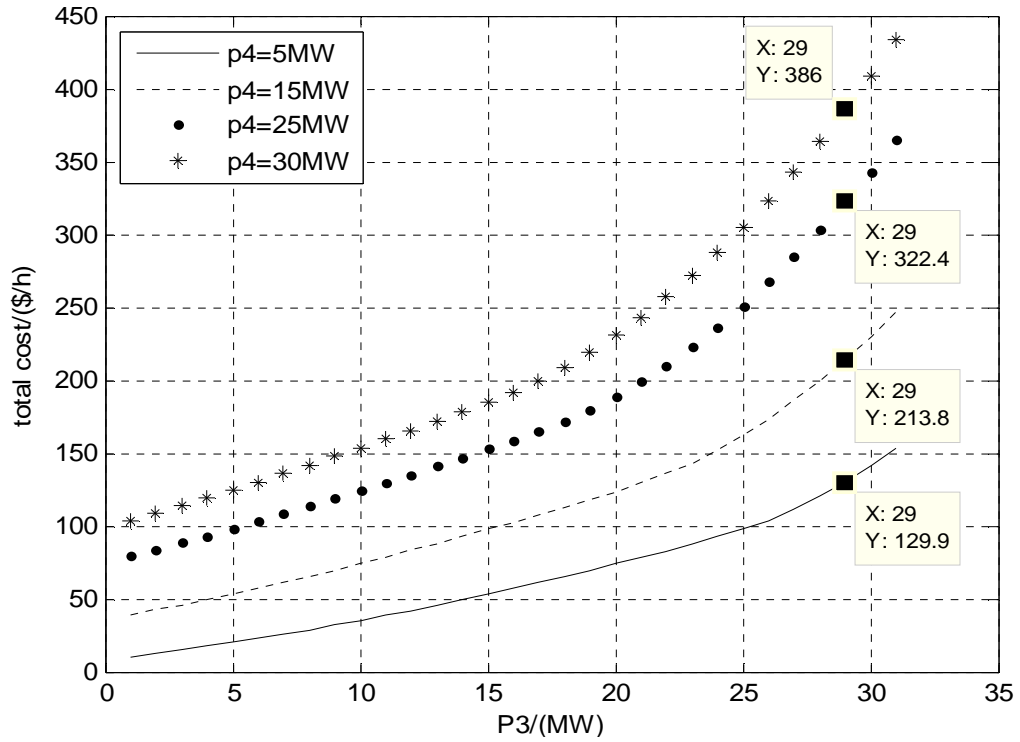


Figure 3.5 Result obtained for a system with constraint considering losses, P3 and P4 are loads

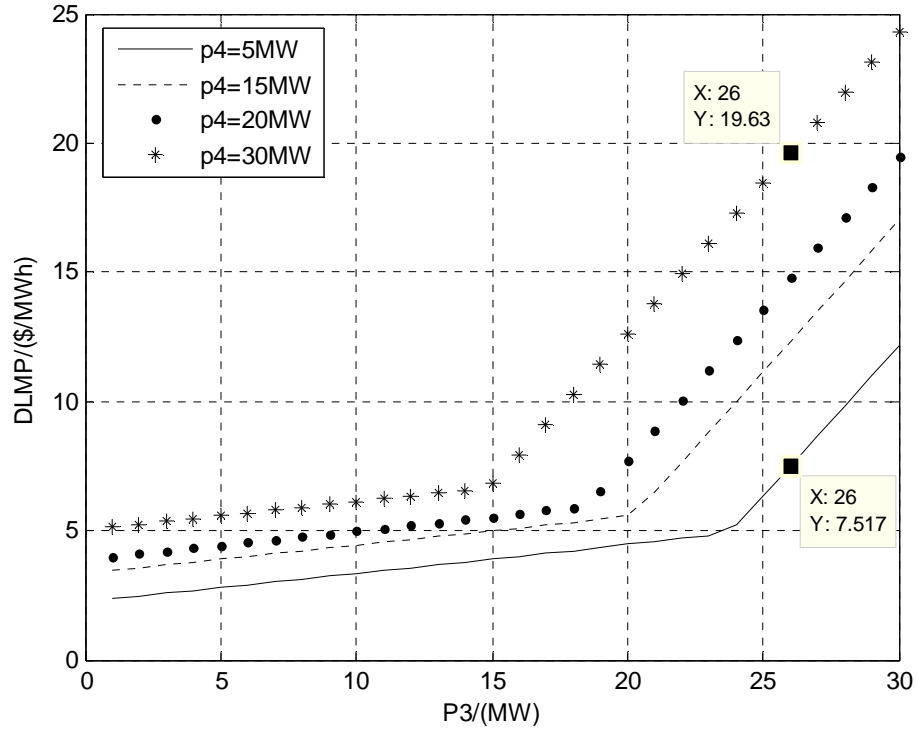


Figure 3.6 Result obtained for a system with constraint considering losses, P_3 and P_4 are loads

The results in Figure 3.6 show a slightly higher DLMP in the lossless case as compared to the results of the no loss case in Figure 3.4. The resistances of the system are low creating low losses. A system with higher resistance values should see higher losses.

3.3 Illustration using the IEEE 34 bus test bed

A test bed used is selected from an IEEE repository of test systems [17]. The intent is to demonstrate the DLMP calculation on a larger system for which some published results are available. This is a 34 bus distribution test bed and for purposes of this work the system has been modified as follows:

- Distributed generation was inserted at buses 800, 836 and 854, renamed 1, 2 and 3 respectively
- All single phase buses have been eliminated
- The symmetrical component transformation was used: $Z_{line} = T^{-1}Z_{abs}T = Z_{sc}$ for given data which give the three phase bus impedance matrix Z
- Unbalanced lines were ignored and only the positive sequence was considered
- Distributed loads (i.e. 802 to 806) were placed as spot loads at the bus which is 'upstream' in the feeder.
- Changes to the system have been reflected in Figure 3.7, and the system data are in Appendix A.

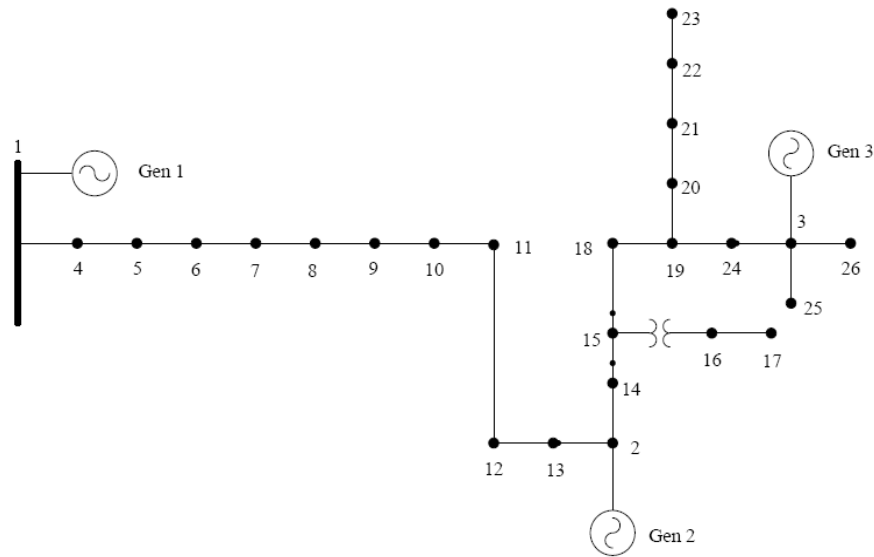


Figure 3.7 System diagram with single phase lines removed and generation inserted

The following cost data were used at buses 1, 2, and 3:

$$\begin{aligned}C_{P1} &= 3.75 * 10^{-2}P_1 + 9.38 * 10^{-5}P_1^2 \\C_{P2} &= 4.50 * 10^{-2}P_2 + 9.30 * 10^{-5}P_2^2 \\C_{P3} &= 4.00 * 10^{-2}P_3 + 9.90 * 10^{-5}P_3^2\end{aligned}$$

The analysis procedure is as follows:

- Convert line data and load data to per unit using a 500 kVA base, 24.9 kV
- Form the admittance matrix Y_{bus}
- Use QUADPROG in MATLAB solve for the LMP for the lossless case
- Use FMINCON to simulate case with losses.

The DLMP in the no loss case was found to be constant at all buses in the system. This ‘lossless’ DLMP was 0.0728 \$/kWh. The lossy case is solved and gives results shown in Table 3.2. Figure 3.8 shows contours of similarly valued DLMP regions superimposed on the system diagram.

3.4 Discussion: application of DLMPs

As the power system transitions into the future, the implementation of smart meters and distributed generation will create an application for DLMPs [18-21]. The DLMP could help support cost effective growth of new technologies and could be used as a road map for new renewable distributed generation. That is, in regions with high DLMP, greater investments could be made in distribution system assets. Another application of the DLMP might be in pricing energy and power differently at different buses. In [22], Heydt conjectures that a DLMP signal might be used in a future power distribution system control: the idea is to use a DLMP to control energy storage at the distribution level. This concept is being promoted as part of the Future Renewable Electric Energy Distribution Management (FREEDM) center (a National Science Foundation supported Engineering Research Center).

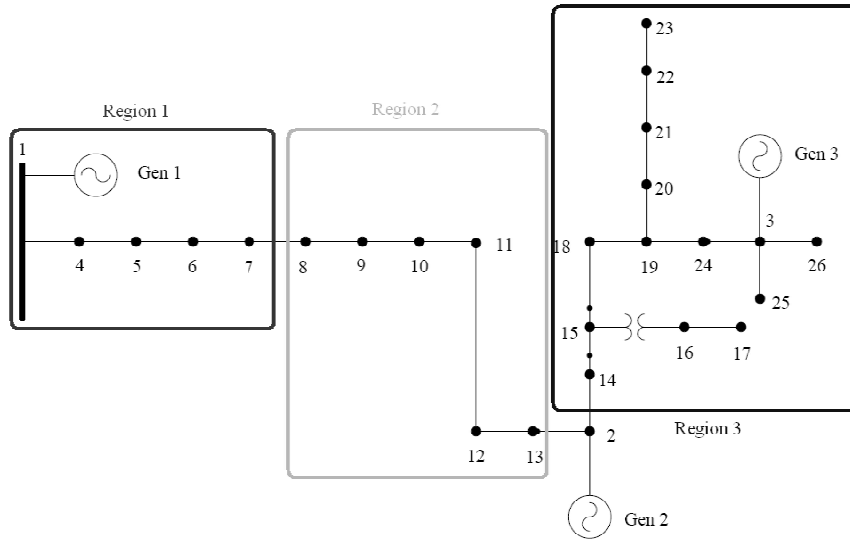


Figure 3.8 System diagram with similar DLMP regions. Region 1: 0.0726-0.0730\$/kW; Region 2: 0.0732-0.0735\$/kW; Region 3: 0.0742-0.0749\$/kW

Table 3.2 Results for a distribution LMP calculated for the IEEE 34 bus system, lossy case

Bus	DLMP \$/pu.h	DLMP \$/kWh	Bus	DLMP \$/pu.h	DLMP \$/kWh
1*	36.2829	0.0726	14	37.1001	0.0742
2*	36.7765	0.0736	15	37.1002	0.0742
3*	37.1253	0.0743	16	37.4300	0.0749
4	36.2922	0.0726	17	37.4300	0.0749
5	36.2979	0.0726	18	37.1206	0.0742
6	36.4028	0.0728	19	37.1431	0.0743
7	36.5249	0.0730	20	37.1446	0.0743
8	36.6218	0.0732	21	37.1517	0.0743
9	36.6218	0.0732	22	37.1548	0.0743
10	36.6233	0.0732	23	37.1551	0.0743
11	36.6370	0.0733	24	37.1371	0.0743
12	36.6771	0.0734	25	37.1253	0.0743
13	36.7742	0.0735	26	37.1256	0.0743

* indicates generation bus

Figure 3.9 shows a 'Generation II' control scheme for distribution systems. Various inputs are brought to a point of calculation of the DLMP, and the DLMPs are distributed to smart loads and other distributed controls. Intelligent fault management (IFM) may be integrated into the system. The building blocks of the concept illustrated are electronic controls, phase locked loops (PLLs) and pulse width modulated (PWM) controllers. The FREEDM center relates to the electronic control of power distribution systems.

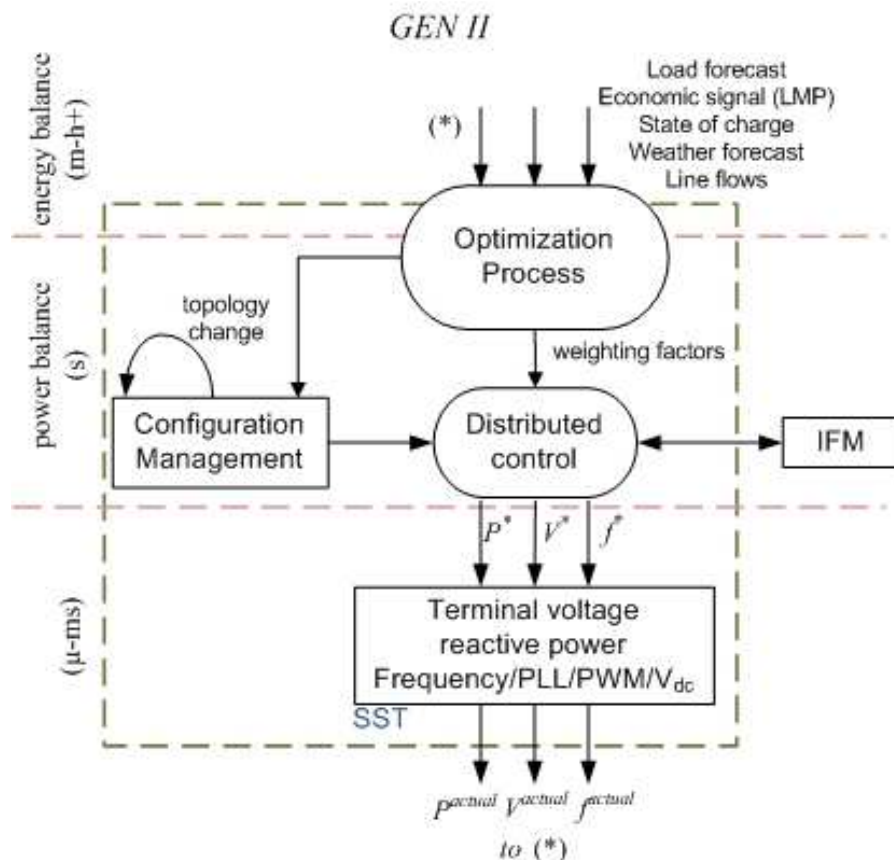


Figure 3.9 A conceptual picture of a 'Generation II' electronically controlled distribution system

3.5 Conclusions

The use of quadratic programming and the FMINCON application in common software has been demonstrated to find the DLMP within a small distributed grid. When comparing the results of the no losses case to the results found in the lossy case the effects of losses on the system can be viewed. The largest DLMP of the system is found to be about 6.45% higher than the value found in the lossless case

and 9.15% higher than the lowest value of the lossy case. According to [23] the average losses within the transmission and distribution systems are about 7%. More of the losses are found in the distribution system compared to the transmission system thus the increases found in the DLMP throughout the system are in line with what would be expected in real distribution system.

Chapter 4: DLMP calculation for the lossy case using loss factors

4.1 Motivation for the use of loss factors

In Chapter 3, a technique for the calculation of DLMPs for the lossy case was presented. That method was based on the Karush Kuhn Tucker method as implemented in FMINCON, a Matlab mainline program. Because of the significant run times of FMINCON, additional ways of calculating the DLMP have been explored. One technique is to use a concept from the method of B-coefficients [24] in which optimal dispatch is done for the lossy case using loss penalty factors. The central idea is to apply a pre-calculated penalty factor to generator incremental operating costs to model losses. That is, the higher the losses produced by a generator, the higher the penalty factor, and therefore the higher the penalized incremental operating cost. In the case of DLMP calculation, instead of calculating a penalty factor that is applied to the generation incremental cost, the penalty is applied to pre-calculated DLMPs at system buses, and the penalty is intended to capture the level of system losses due to loading at the specified bus. Figure 4.1 shows the general proposed concept.

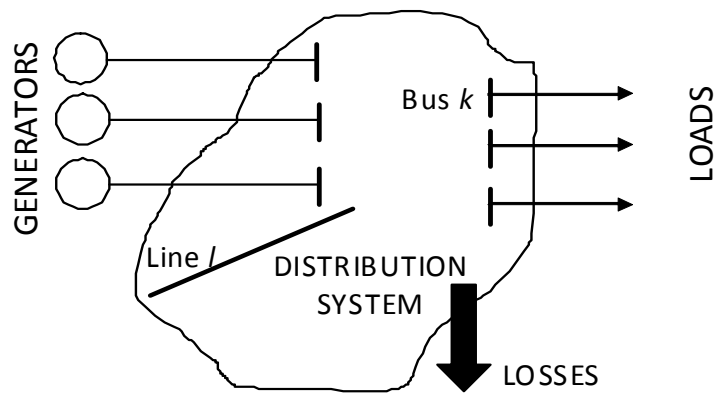


Figure 4.1 Concept of losses in a distribution system

4.2 Calculation of loss penalty factors

A way to calculate an appropriate loss factor for each bus is presented as follows:

- Run the lossless OPF using 'quadprog' as seen in Chapter 2 and calculate all the bus voltage phase angles (δ) in the distribution system.

- Calculate the total active power loss P_{loss} in the system by calculating the $I_\ell^2 R_\ell$ losses for every line.
- Calculate Y_{bus} and convert to Z_{bus} with all the generators grounded. Examine $(Z_{bus})_{kk}$ for load bus k . This is $r_{kk} + jX_{kk}$. Assume that at each load bus the contribution to the losses P_{loss} is proportional to approximately $r_{kk}(P_{kk})^2$.
- The fractional loss at each load bus k is,

$$P_{loss \text{ due to load } k} = P_{Lk} = \frac{r_{kk}(P_k)^2}{\sum_\ell P_\ell^2 r_{\ell\ell}} P_{loss}$$

where k is the specific load bus for which the DLMP is sought, and ℓ refers to the system lines.

- Add P_{Lk} to the original precalculated DLMP (i.e., the precalculated DLMP using quadprog),

$$DLMP'_k = DLMP_k \left(1 + \frac{P_{Lk}}{P_k} \right).$$

4.3 Application of the loss factor method to the 34 bus test bed

An example using the methods of Section 4.2 has been applied to the IEEE 34 bus system. The system diagram for the 34 bus system is reproduced in Figure 4.2. The original QP DLMP was calculated then each calculated loss factor was appropriately applied to each non-generation bus. Table 4.1 shows the common (data given) load profile. Table 4.2 shows the load at bus 17 equally distributed between buses 16 and 17. The results are shown in Table 4.1.

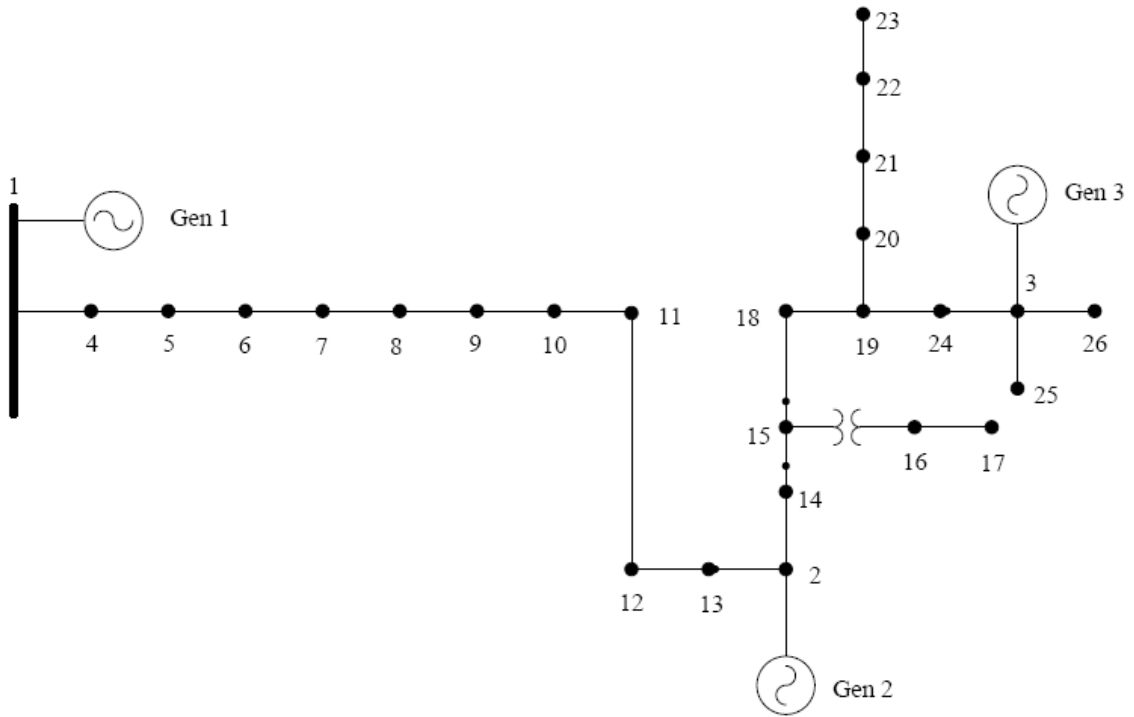


Figure 4.2 Test bed: 34 bus system [17]

A few noticeable characteristics are displayed in the results. Most notably is that load immediately adjacent to the non-generation side of the system transformer experiences the highest increases in the DLMP. Bus 16 reflects that in Table 4.1 and in Table 4.2. The next largest contributor to a higher DLMP is load size. Buses with a large load see a much greater percent change in comparison to smaller loads. The final factor that influences the DLMP is distance from generation. While not as impactful as load size, it clearly contributes to the DLMP change.

Table 4.1 Table with loss factors applied to DLMP

Bus Number	Load (pu)	QP LMP	Loss Factor DLMP	% Change	Multiplier
4	0.1	0.0883	0.0883	0.00%	1.0000
5	0	0.0883	0.0883	0.00%	1.0000
6	0	0.0883	0.0883	0.00%	1.0000
7	0	0.0883	0.0883	0.00%	1.0000
8	0	0.0883	0.0883	0.00%	1.0000
9	0	0.0883	0.0883	0.00%	1.0000
10	0.007	0.0883	0.0883	0.00%	1.0000
11	0.01	0.0883	0.0883	0.00%	1.0000
12	0.008	0.0883	0.0883	0.00%	1.0000
13	0.03	0.0883	0.0883	0.00%	1.0000
14	0	0.0883	0.0883	0.00%	1.0000
15	0.015	0.0883	0.0883	0.00%	1.0000
16	0	0.0883	0.089	0.79%	1.0079
17	0.5	0.0883	0.0883	0.00%	1.0000
18	0.05	0.0883	0.0895	1.36%	1.0136
19	0.15	0.0883	0.0883	0.00%	1.0000
20	0.006	0.0883	0.0883	0.00%	1.0000
21	0.3	0.0883	0.0885	0.23%	1.0023
22	0.0233	0.0883	0.0884	0.11%	1.0011
23	0.06	0.0883	0.0883	0.00%	1.0000
24	0.14	0.0883	0.0883	0.00%	1.0000
25	0.08	0.0883	0.0883	0.00%	1.0000
26	0.018	0.0883	0.0886	0.34%	1.0034

Table 4.2 Table with calculated loss factors applied to DLMP and load at bus 17 distributed to buses 16 and 17

Bus Number	Load (pu)	QP LMP	Loss Factor DLMP	% Change	Multiplier
4	0.1	0.0896	0.0896	0.00%	1.0000
5	0	0.0896	0.0896	0.00%	1.0000
6	0	0.0896	0.0896	0.00%	1.0000
7	0	0.0896	0.0896	0.00%	1.0000
8	0	0.0896	0.0896	0.00%	1.0000
9	0	0.0896	0.0896	0.00%	1.0000
10	0.007	0.0896	0.0896	0.00%	1.0000
11	0.01	0.0896	0.0896	0.00%	1.0000
12	0.008	0.0896	0.0896	0.00%	1.0000
13	0.03	0.0896	0.0896	0.00%	1.0000
14	0	0.0896	0.0896	0.00%	1.0000
15	0.015	0.0896	0.0896	0.00%	1.0000
16	0.25	0.0896	0.0902	0.67%	1.0067
17	0.25	0.0896	0.0896	0.00%	1.0000
18	0.05	0.0896	0.0908	1.34%	1.0134
19	0.15	0.0896	0.0896	0.00%	1.0000
20	0.006	0.0896	0.0896	0.00%	1.0000
21	0.3	0.0896	0.0898	0.22%	1.0022
22	0.0233	0.0896	0.0897	0.11%	1.0011
23	0.1	0.0896	0.0896	0.00%	1.0000
24	0.14	0.0896	0.0896	0.00%	1.0000
25	0.08	0.0896	0.0896	0.00%	1.0000
26	0.018	0.0896	0.0897	0.11%	1.0011

4.4 Proportioning active power losses in the penalty factor loss approximation

In the optimization of generation sources as described above, a method has been proposed based on penalty factors (see Sections 4.2 and 4.3). The loss penalty factors account for the impact of losses on cost; but, as stated above, there is no inclusion in the model for the need for generation to produce power to balance and accommodate active power losses.

At this point, the concept is to attribute losses to each system generator based on the “distance” from each generator. For this purpose, consider a general distribution system as shown in Figure 4.3. In this figure, three generation sources are depicted. Regions I, II, and III are established based on the

electrical distance (i.e., point to point impedance) from a load bus to a generator. Thus the load at bus L in Figure 4.3 is evaluated to determine whether the point to point impedance L – N, L to T, or L to U is smallest. Then L is taken to be in the region associated with the smallest impedance. In Figure 4.3, this is shown as bus L in region I. This procedure is repeated for all the load buses. The result is exemplified by Figure 4.3.

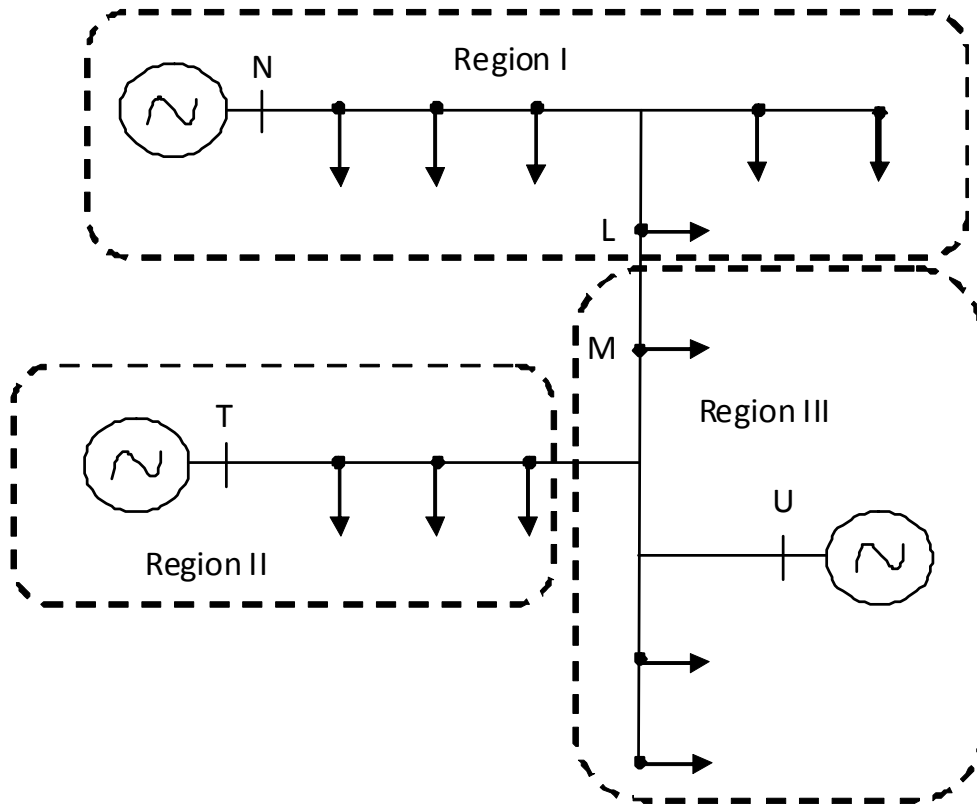


Figure 4.3 General distribution system

The foregoing procedure is applied to the IEEE 34 bus system shown in Figure 4.2. The result is shown in Figure 4.4. Generator #1 lies in region I which contains buses 4-8; generator #2 lies in region II which contains buses 9-17; and generator #3 lies in region #3 which contains buses 18-26.

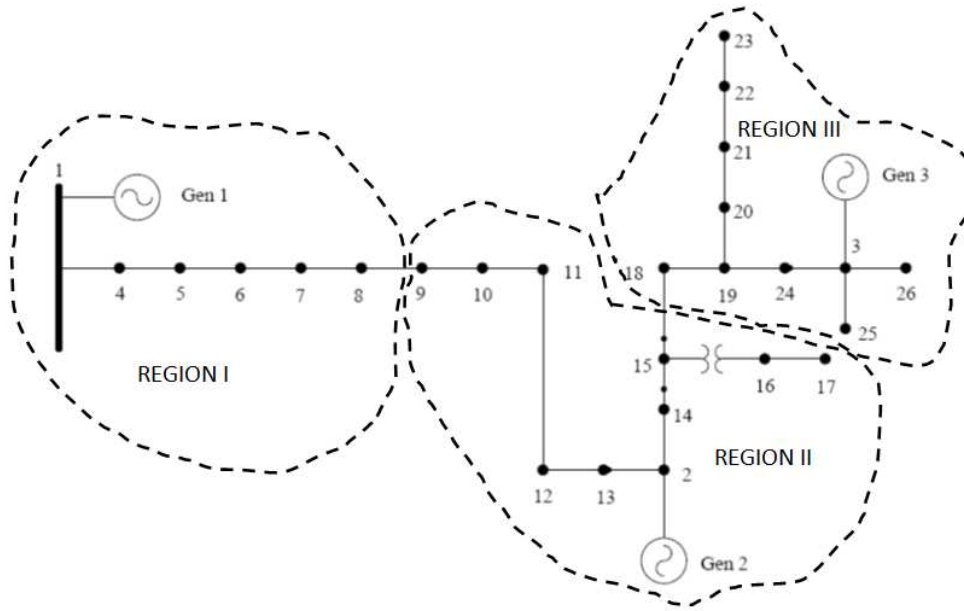


Figure 4.4 The 34 bus test bed with regions I, II, and III superimposed, used to identify attribute losses to generation.

In order to apportion losses among the several generators, the total load inside each identified region is divided the total load of the entire system. The resulting percentage is used to proportion the losses to each region. The proportioned loss is added to the generation in that region. The basic concept is written for the generalized case as,

$$P_{\text{attributed to generator } k} = \frac{\text{Total load in region } k}{\text{Total system load}} * [\text{Total system active power losses}].$$

The loss apportioning method is applied to the 34 bus test bed and results are shown in Table 4.3.

Table 4.3 Example of proportioning losses for the IEEE 34 bus test bed

Region	No loss generation level (kW)	Buses	Load (kW)	% of total load	Load added to generation* (kW)	New generator setting (kW)
1	0.543	4-8	0.1	6.68%	0.001108662	0.54411
2	0.4671	9-17	0.57	38.07%	0.006319375	0.47342
3	0.4893	18-26	0.8273	55.25%	0.009171963	0.49847

* The total loss found by this method is 0.0166 kW.

The loss apportioning method shown here has the following disadvantages and weaknesses:

- Reactive power is not modeled
- The load flow equations are not modeled.

The advantages of the approach are:

- The method is simple and completely repeatable
- The calculation is fast – even for large distribution systems.

Of course, the salient question relates to the accuracy of the apportioning method. This is discussed using the 34 bus test bed as an example below.

4.5 Optimal dispatch using PowerWorld

In PowerWorld, optimal power flow studies (OPFs) are solved using a linear programming (LP) approximation. In the standard mode, 'Simulator' solves the power flow equations using a Newton-Raphson power flow study algorithm. With the optimal power flow enhancement (an 'add on'), 'Simulator OPF' in PowerWorld can also solve many of the system control equations using an Optimal Power Flow algorithm. Specifically, Simulator OPF uses a linear programming OPF implementation. In the Simulator OPF, the LP OPF determines the optimal solution by iterating between a solved case that was obtained using a standard power flow algorithm, and then solving a linear programming problem to change the system controls. The latter is done to remove any limit violations [25].

The results of the base case using PowerWorld can be seen in Figure 4.3 and the load data in Table 4.4. The PowerWorld solution clearly displays the generation in MW at P1, P2 and P3 as well as the marginal cost in \$/MWh. The loads are very small so they are depicted as zeros on a MW scale in Figure 4.3.

Table 4.4 System load data (modified IEEE 34 bus test bed)

Bus Number	Load (pu)	Bus Number	Load (pu)
4	0.1	16	0.25
5	0	17	0.25
6	0	18	0.05
7	0	19	0.15
8	0	20	0.006
9	0	21	0.3
10	0.007	22	0.0233
11	0.01	23	0.06
12	0.008	24	0.14
13	0.03	25	0.08
14	0	26	0.018
15	0.015		

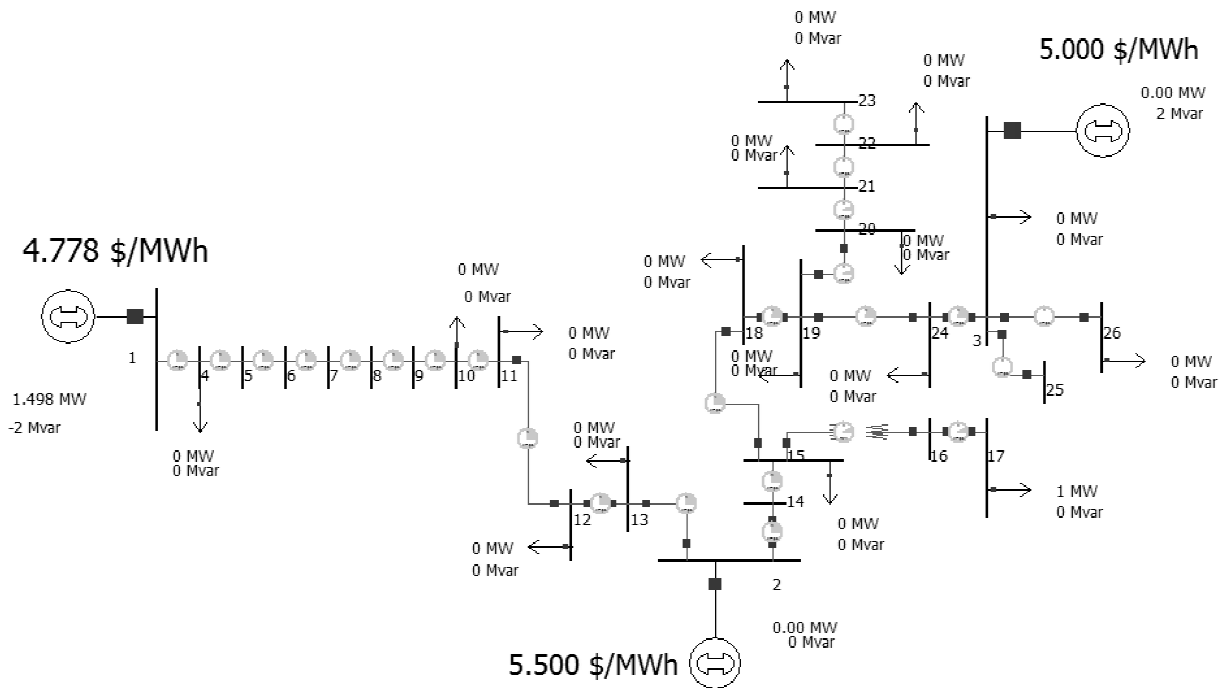


Figure 4.5 PowerWorld results (34 bus test bed)

4.6 Comparison of results of the several methods

The results from each method used to calculate the DLMP and the operating costs are shown in Tables 4.5-4.8 and graphically in Figures 4.6-4.8. Table 4.4 shows the linear and quadratic costs of each trial. Each trial has a different set of cost functions. Trial A uses an original set of cost functions used throughout the project. Trial B uses a set of cost functions that are very similar and trial C uses a set with

constant linear terms but varying quadratic terms. The first calculation method, namely ‘QuadProg no loss’, shows the results from a purely quadratic programming calculation only. There are no losses calculated in the system. The second method, labeled as ‘FMINCON’, uses the loss approximation method via the FMINCON function in Matlab. This method has been detailed in Sections 2.3 and 3.2.2, and in Appendix A. The third method, ‘QP with loss approximation’, uses the loss approximation method detailed in Sections 4.1 and 4.4. The final column shows the results calculated using PowerWorld.

Table 4.5 The coefficients of a quadratic cost function for three different test trials

Cost coefficients of each trial (\$/h)					
Trial A		Trial B		Trial C	
Linear*	Quadratic*	Linear*	Quadratic*	Linear*	Quadratic*
3.75	9.38	3.75	9.40	3.75	9.38
4.50	9.30	3.76	9.30	3.75	9.30
4.00	9.90	3.74	9.50	3.75	9.90

*Linear terms are multiplied by ($*10^{-3}$) and quadratic terms are multiplied by ($*10^{-6}$)

Table 4.6 Trial A – Original test case

Calculation Methods				
Solution	QuadProg no loss	FMINCON	QP with loss approximation‡	PowerWorld
P1 (kW)	0.543	0.5283	0.54411	1.498
P2 (kW)	0.4671	0.4689	0.47342	0
P3 (kW)	0.4893	0.5022	0.49847	0
Linear cost term**	0.06095	0.06100	0.06165	0.05618
Quadratic cost term**	0.00014	0.00014	0.00014	0.00014
Total calculated cost* **	0.0611	0.0611	0.06179	0.0563

*The total cost is the linear cost term plus the quadratic cost term

** In arbitrary consistent units, may be interpreted as \$/h

‡Losses served by generation as described in Section 4.4

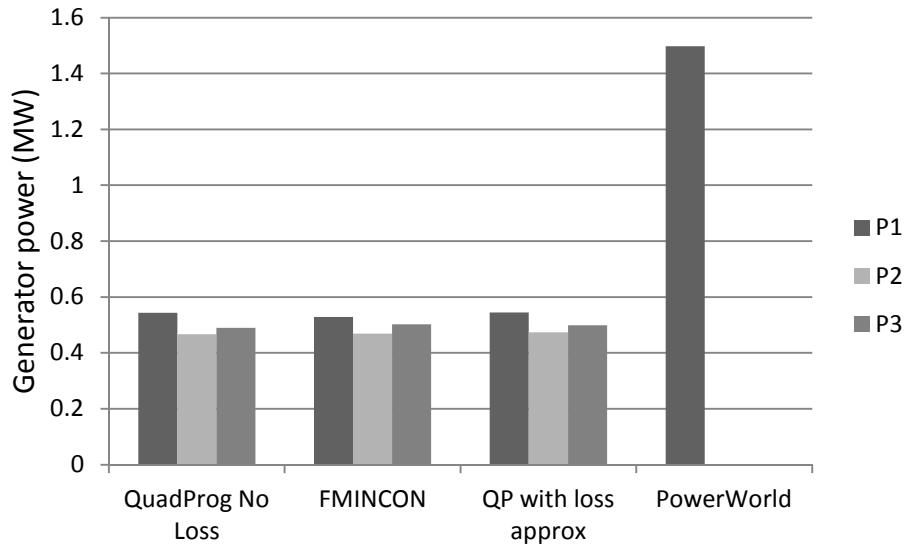


Figure 4.6 Graphical representation of trial A

Table 4.7 Trial B using similar cost functions

Calculation Methods				
Solution	QuadProg no loss	FMINCON	QP with loss approximation‡	PowerWorld
P1 (kW)	0.4997	0.5283	0.50081	1.498
P2 (kW)	0.504	0.4689	0.51032	0
P3 (kW)	0.4955	0.5022	0.50467	0
Linear cost term**	0.05622	0.05622	0.05684	0.05618
Quadratic cost term**	0.00014	0.00014	0.00014	0.00014
Total calculated cost* **	0.0564	0.0564	0.05699	0.0563

*The total cost is the linear cost term plus the quadratic cost term

** In arbitrary consistent units, may be interpreted as \$/h

‡Losses served by generation as described in Section 4.4

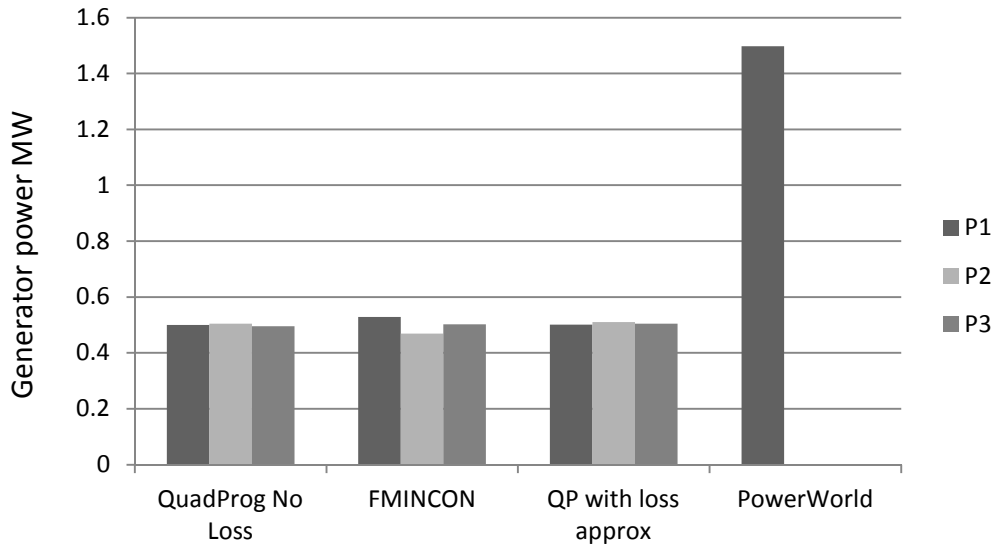


Figure 4.7 Graphical representation of trial B

Table 4.8 Trial C – test using constant linear term

Calculation Methods				
Value	QuadProg no loss	FMINCON	QP with loss approximation‡	PowerWorld
P1 (kW)	0.5072	0.4939	0.50831	0.489
P2 (kW)	0.5116	0.5128	0.51792	0.509
P3 (kW)	0.4806	0.4926	0.48977	0.48
Linear cost term**	0.05623	0.05622	0.05685	0.05543
Quadratic cost term**	0.00014	0.00014	0.00014	0.00014
Total calculated cost* **	0.0564	0.0564	0.05699	0.0556

*The total cost is the linear cost term plus the quadratic cost term

** In arbitrary cut consistent units, may be interpreted as \$/h

‡Losses served by generation as described in Section 4.4

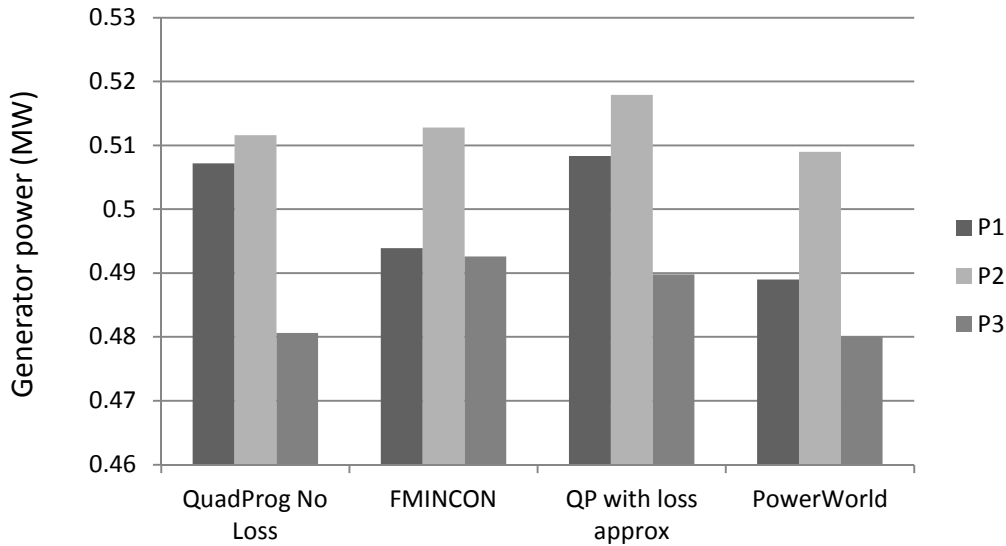


Figure 4.8 Graphical representation of trial C

The results from each trial show that the linear component of the cost function plays a large role in determining the dispatch. Table 4.9 shows a comparison of the assumed characteristics of each solution method. The variance in each method is likely due to the solution method each one takes. Not surprisingly the 'FMINCON' and the 'QP with loss approximation' are higher than the no loss QP method. Since these are just extensions to include losses of the QP method the results are reasonable. One potential reason for the difference between the three new proposed methods and PowerWorld is that PowerWorld solves the system by changing the quadratic costs into a piece-wise linear programming problem. This is less exact and could be the cause for the slightly lower total costs. There are also likely differences in how MATLAB and PowerWorld solve iteratively. Tables 4.10 and 4.11 show the percent difference in the results compared to PowerWorld regarding total load and total cost. Some other possible reasons for the variance in total load and total cost are:

- PW was created for large scale systems; does better with transmission system, megawatts not kilowatts
- In PW, DLMP (seen as bus marginal cost) is constant throughout system, while the other methods solve for the DLMP individually.
- The three new proposed methods don't take into account reactive power.

Table 4.9 A comparison of assumed characteristics of solution methods for the calculation of DLMPs

Method used	Models losses	Iterative	PQ and PV buses modeled	Linearize cost function	Load flow equations modeled	Quadratic cost function modeled
QuadProg no loss	N	Y	N	N	N	Y
FMINCON	Y	Y	N	N	N	Y
QuadProg with losses	Y	Y	N	N	N	Y
PowerWorld	Y	Y	Y	Y*	Y	N*

*Piecewise linear representation of the cost function is used
N = NO
Y = YES

Table 4.10 Percent difference from PowerWorld results regarding total load

Trial	QuadProg no loss	FMINCON	QP with loss approximation‡
A	0.09%	0.09%	1.20%
B	0.08%	0.09%	1.19%
C	1.45%	1.44%	2.57%

Table 4.11 Percent difference from PowerWorld results regarding total cost

Trial	QuadProg no loss	FMINCON	QP with loss approximation‡
A	8.53%	8.53%	9.75%
B	0.18%	0.09%	1.19%
C	1.44%	1.44%	2.50%

In each trial the total load wasn't too much higher than the results found from PowerWorld. A maximum of 2.57% larger in load was found. The proposed new systems were not as close to the results of PowerWorld when it came to total cost. When each cost had a similar or exactly the same linear cost as in trials B and C, the total cost was not much different varying as much as only 2.5%. However, in trial A where the linear cost of the system was much more diverse, the total cost of the system was approximately 9% higher than the PowerWorld cost.

Chapter 5: Energy management

5.1 Introduction

The previous chapter displayed the application and calculation of a distribution based locational marginal price. In this chapter, further applications involving energy management within the IEEE 34 bus test bed will be demonstrated. In most applications, energy management will primarily be based on the DLMP, that is, it will adjust based on original DLMP values. Different examples below will show how different factors affect the energy management.

Before going through the examples on how an energy management system (EMS) coinciding with a DLMP can be used, the possible applications for a realistic EMS should be discussed.

5.2 Example 1: the role of DLMP set points

Depending on the system parameters (costs, load values, line ratings, etc.) the DLMP will vary from load to load. The energy management system created looks at the initial values of the DLMP and applies energy management system 'multipliers'. In the real world this would be either load reduction or an increasing of the load with some sort of storage. Assuming there is some desired range of DLMP value, convergence to this value can vary depending location of the DLMP set points set up within the system. Figure 5.1 depicts a visualization of the ranges for a DLMP. Each range would have an associated multiplier. The EMS would view the previous DLMP data and apply the appropriate multiplier. The range where the multiplier is one or no change is the desired range of DLMP for the system to be in.

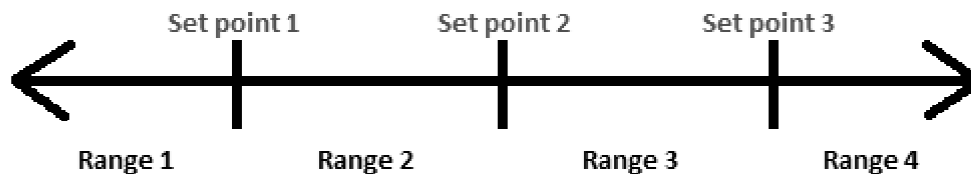


Figure 5.1. Visualization of different DLMP multiplier ranges.

Adjusting the location of the set points can have varying effects on the speed of the convergence of the DLMP. In Figure 5.2 the system converges to the desired range after 4 units of time. Figure 5.3 is

the result of reducing the size of the desired range of DLMP. In this scenario it takes the system much longer than 10 units of time and does not even show signs of convergence at all. It is clear the placement on the DLMP set points can have a significant impact on the speed and completion towards convergence of a desired DLMP.

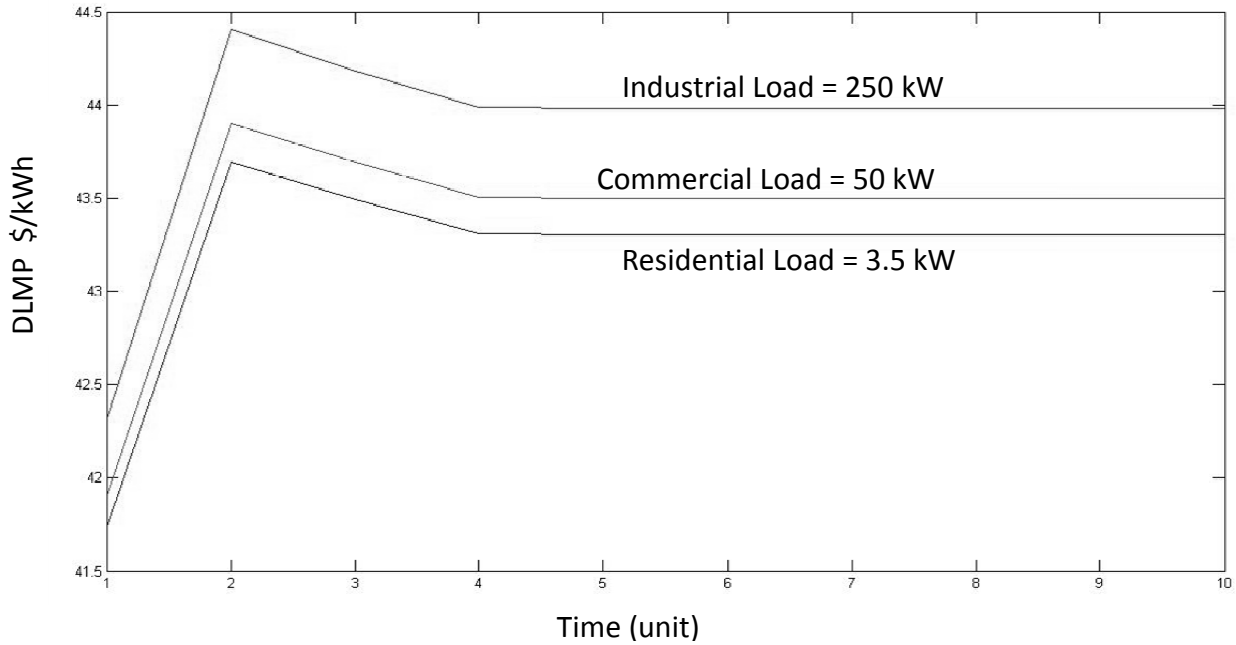


Figure 5.2 Faster DLMP convergence with a wider desired range

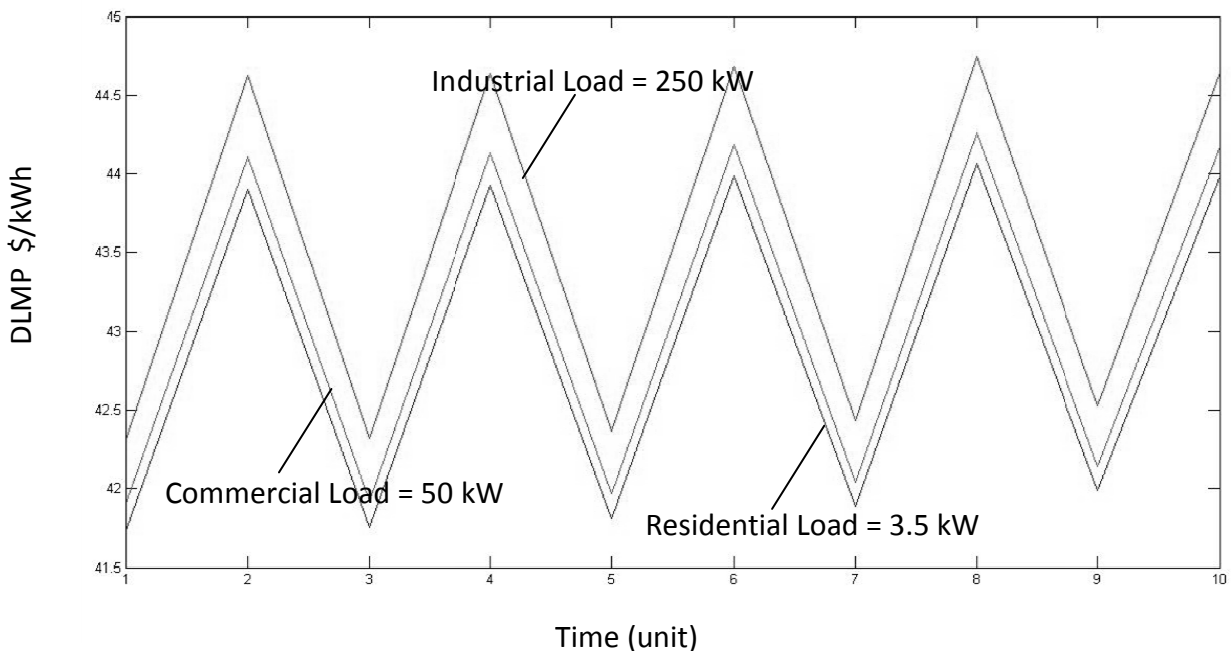


Figure 5.3 No DLMP convergence with a reduced desired range

5.3 Example 2: the role of energy management systems 'multipliers'

As said previously the 'multipliers' of the system would act as either load increase such as implementing storage or a load shedding. The severity of the load increase can drastically change the DLMP. Two different runs were performed. The first, "run A" had a large variance in load manipulation. As much as 50% of the load could be shed based on what range the DLMP fell in. The second trial, "run B", had far less variance. A maximum of 10% could be reduced at one time. The multipliers of each range are located in Table 5.1 and the results are in figures 5.3 and 5.4.

Table 5.1 Range multipliers for each run

Name	Range 1 multiplier	Range 2 multiplier	Range 3 multiplier	Range 4 multiplier
Run A	0.5	0.75	1	1.25
Run B	0.9	0.95	1	1.05

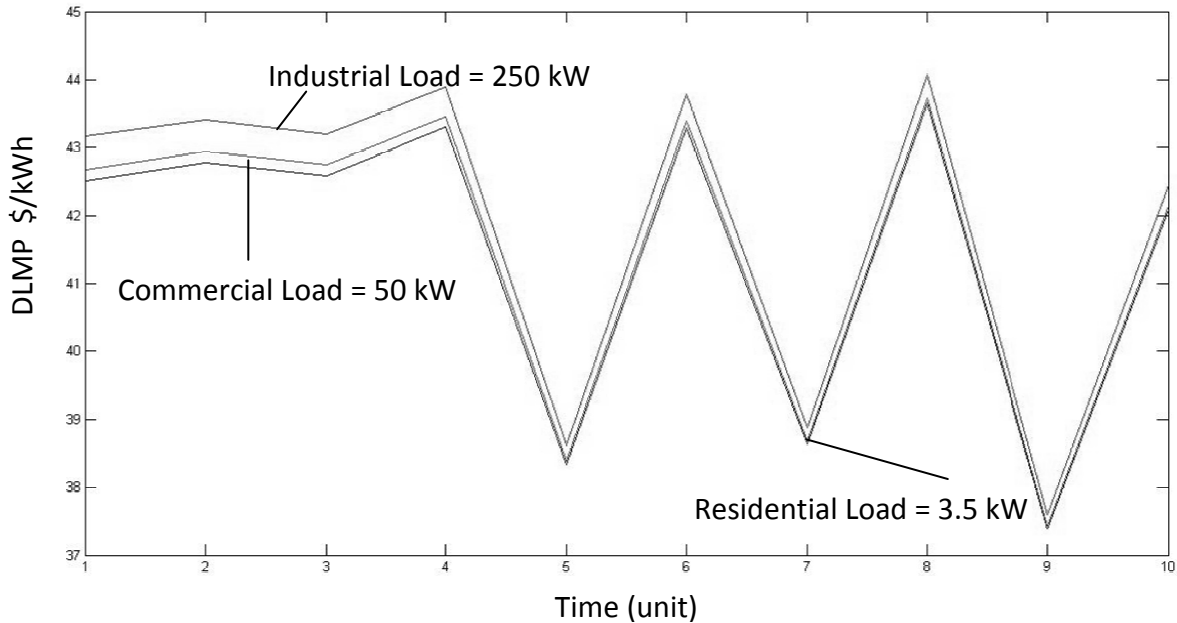


Figure 5.4 DLMPs with large variance in 'multipliers'

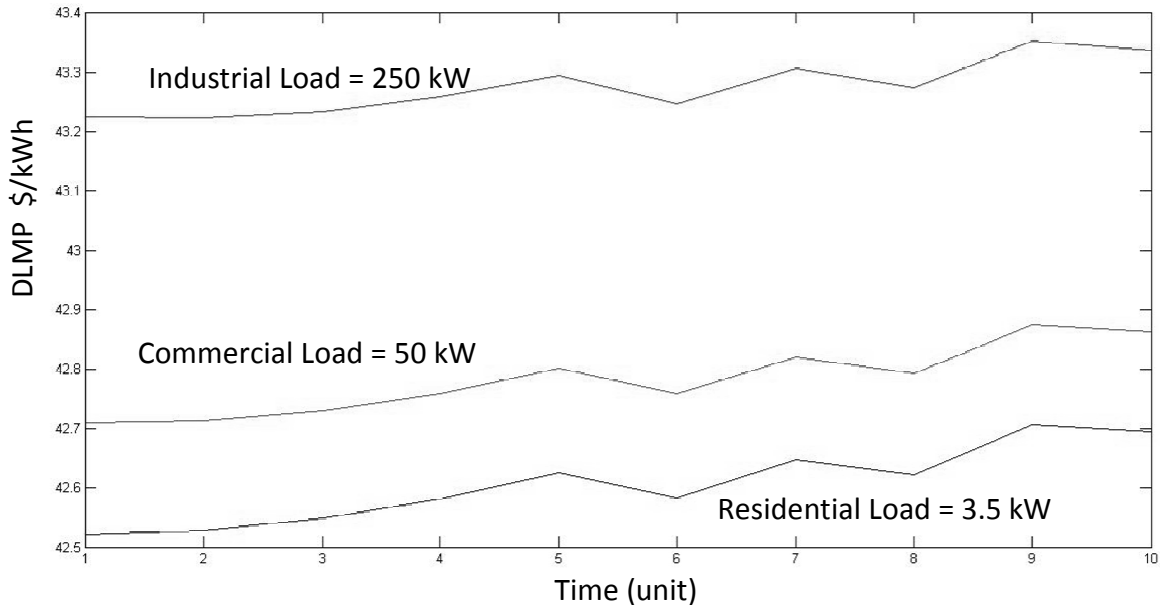


Figure 5.5 DLMPs with small variance in 'multipliers'

The figures show how much this can affect the DLMP. In the case of having large multipliers the industrial load DLMP varied from 44 to 38. However, in the run with smaller multipliers, the industrial load varied from less than 43.4 and greater than 43.2.

5.4 Example 4: load control using DLMP with a single load

In sections 5.2 and 5.3 each bus of the system was increased or decreased based on its own DLMP. Conversely, this section looks into how individual DLMPs change based a single load having an energy management system based on the DLMP. Buses 2, 8 and 26 of figure 4.2 are controlled in the system in three different trials and their load values and load type are seen in Table 5.2. The results of each run are shown in figures 5.6 and 5.7.

Table 5.2 Controlled load data

Bus Number	Load Value	Load Type
2	50 kW	Commercial
8	3.5 kW	Residential
26	250 kW	Industrial

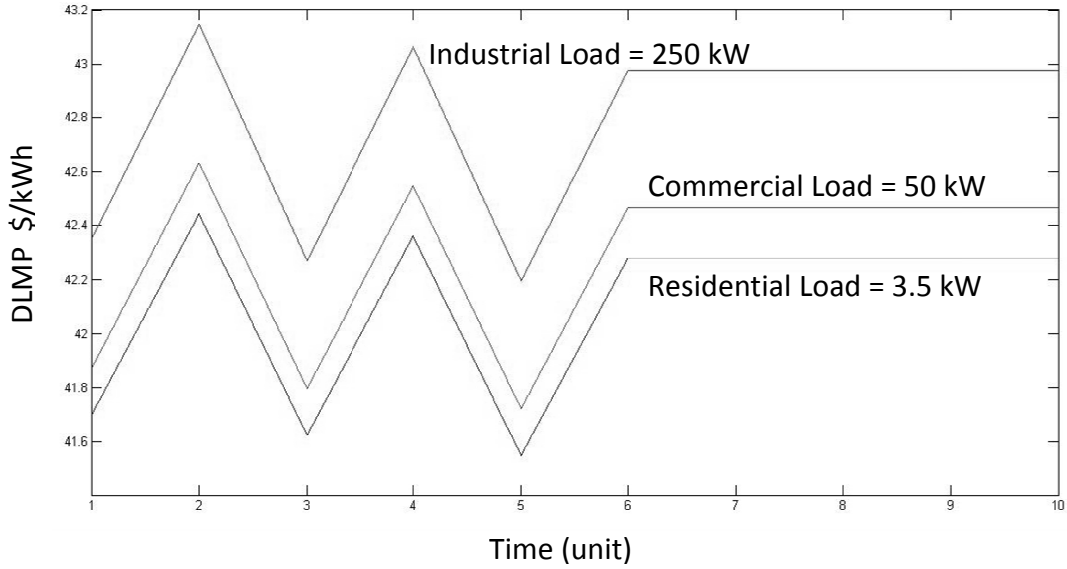


Figure 5.6 Single industrial load (bus 26) altering load based on DLMP

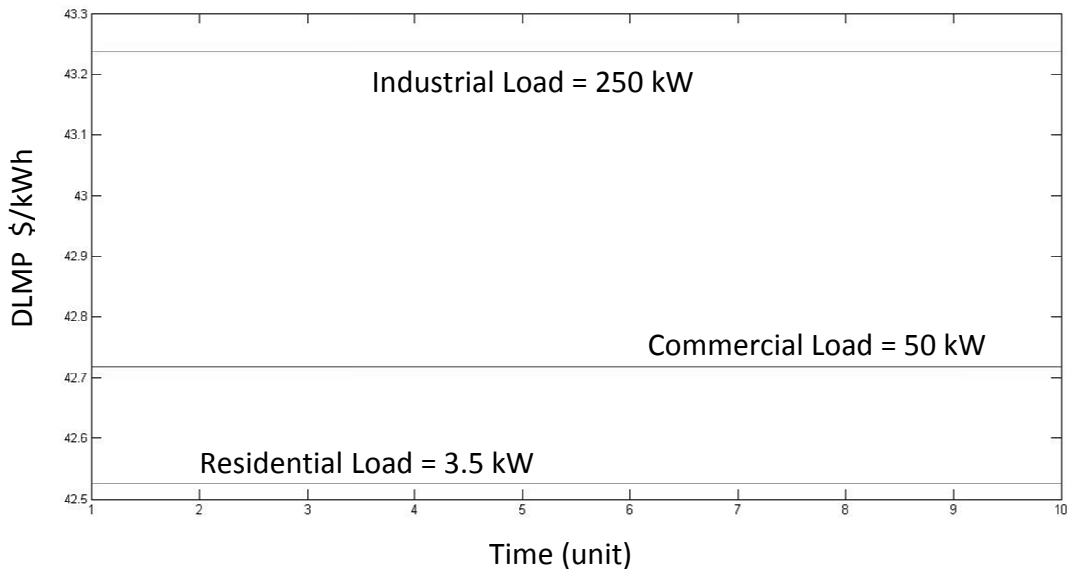


Figure 5.7 Results from residential and commercial loads (bus 2 and 8) altering load based on DLMP

When the industrial load is changed the system sees a variant DLMP. This is likely due to the larger percentage of total load this bus has compared to the others. However when in the case of the industrial and residential loads, the change is insignificant to the DLMP as it remains constant. This of course is also dependent on the settings of the 'set point' as discussed in section 5.2.

5.5 Conclusions

A few approaches to the use of the DLMP as a pricing based method for energy management have been discussed in this chapter. The underlying concept is that a single home, neighborhood or entire distribution network could use DLMP signals to control loads. The user could identify which loads are controllable and what strategy might be employed to control load based on price. This could be done either from the utility perspective or the homeowner depending on the deployment of certain technologies. Many contemporary smart meters are able to give present load values immediately, which is the primary real time data needed to calculate the DLMP. It is likely that only a few selective loads have the ability to be controlled. Any load with this type of control would require technologies such as Wi-Fi integrated appliances to control the load. As an example, ZigBee technology has been used for energy management [28, 29]. Deployment of energy management using DLMP to save money would likely be customer driven rather than utility driven.

An additional application relates to the use of the DLMP as a signal that can be used to identify heavily used assets in the distribution system. The potential utility company application would be that the DLMP is used to identify which distribution assets need to be enhanced.

The effects of DLMP set points in the controller and the 'multiplier' (in the form of load shedding and storage) have also been discussed in this section. The set points in the controller determine the time response of the load. The 'multipliers' discussed in this chapter also have a significant impact on the time response of the energy management system.

Chapter 6: Conclusions, recommendations, and future work

6.1 Conclusions

The main and secondary conclusions of this thesis are outlined in Tables 6.1 and 6.2. The tables include conclusions and application areas. The selected application areas are 'Calculation Methods', 'DLMP applications', 'DLMP results' and 'Energy Management'.

Table 6.1 Main conclusions of thesis

<i>Description</i>	<i>Application Area</i>
The application of present techniques for the calculation of transmission LMPs in distribution systems can be transported to distribution engineering.	DLMP applications
Quadratic programming is effective in finding the minimal operating cost (quadratic expressions for fuel costs assumed).	Calculation methods
FMINCON can be used for the indicated optimization and results compare favorably with PowerWorld.	DLMP results
Modeling active power losses using loss factors gives results that are close to PowerWorld results.	DLMP results
The use of DLMPs as a fundamental control signal for energy management could be effective.	Energy management
Distributed generation will have significant effects on the DLMP.	DLMP applications

6.2 Recommendations and future considerations

Table 6.3 contains topical areas for future work. Issues and recommendations are presented in the table.

Table 6.2 Secondary conclusions of thesis

<i>Description</i>	<i>Application Area</i>
Calculation time to obtain the DLMP is faster using the described method with loss factors – as compared to the use of FMINCON.	Calculation methods
Costs increase in moving from source to load in a radial distribution system.	DLMP applications
The use of FMINCON and the use of the method of loss factors for the calculation of DLMPs give results that do not agree well with PowerWorld (e.g., approximately 9% discrepancies). The discrepancy appears to exacerbate when the generation operating cost functions differ widely.	DLMP results
The DLMP is an indicator for assets (e.g., lines and transformers) needing improvement in the distribution system	DLMP applications

Table 6.3 Issues and recommendations for future work

Recommendations and Future Explorations	
Issue	Recommendation
DLMP should be applied to networked distribution systems	Explore test systems with a mesh configuration; note differences in cost and still include distributed generation. Could be usefully applied to new construction neighborhoods with smart meters that have implemented distributed generation and possibly storage.
Determining the best location for distributed generation	Develop software to identify optimal location of distributed generation; compare total cost and individual DLMP at each bus to determine optimal locations for the generation.
Consider time varying LMPs in the transmission system	Consider having system adapt to changing transmission LMP as well as time varying distributed generation.
Public acceptability of paying for electricity based on home location	Conduct market research to determine homeowner opinion of subject. Consider developing incentive programs if DLMP is found to be helpful in lowering utility total cost.
Methods have only been applied to 34 bus system	Apply to larger distribution network. Look at percent change in total cost and total load compared to PowerWorld as before. Note change in percentage differences based on each scenario.
Consider energy storage.	Model energy storage.
Utilization of more complex generation cost 'curves' (e.g., tabular costs)	Use of alternative optimization methods.

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Appendix A:

The utilization and performance of Matlab function FMINCON

A.1 Function FMINCON: a brief description

The Matlab function FMINCON is an optimization routine based on the ‘interior point’ method [30]. As stated in the Matlab ‘help’ command, the function “FMINCON attempts to find a constrained minimum of a scalar function of several variables starting at an initial estimate. This is generally referred to as constrained nonlinear optimization or nonlinear programming.”

A.2 Function FMINCON and its implementation in Matlab

Matlab function FMINCON is an automated optimization tool. Function FMINCON finds a constrained minimum of a nonlinear multivariable function of several variables. It attempts to minimize the function subject to linear and non-linear equality and inequality constraints. FMINCON uses Sequential Quadratic Programming (SQP) which is a variant of the Kuhn-Tucker approach. The basis of SQP is to model the minimization problem at x_k by a quadratic sub-problem and to use the solution to find a new point x_{k+1} -- this is a search. FMINCON has options for four different algorithms to solve the equation. They are ‘*sqp*’, ‘*active-set*’, ‘*interior-point*’ and ‘*trust region reflective*’. Figure A.1 shows the pseudocode for a quadratic programming problem.

The call for FMINCON is set up in the following:

$$X = \text{FMINCON}(\text{FUN}, X0, A, B, Aeq, Beq, LB, UB)$$

FMINCON starts at an initial point $X0$ and finds a minimum X to the function FUN . The function is subject to the linear inequalities $A \cdot X \leq B$ and linear equalities $Aeq \cdot X = Beq$. The function FUN accepts input X and returns a scalar function value F evaluated at X . Initial value $X0$ may be a scalar, vector, or matrix. LB and UB are a set of lower and upper bounds on the variables, X , so that a solution is found in the range $LB \leq X \leq UB$ [26].

$$\min_x f(x) \text{ such that } \begin{cases} c(x) \leq 0 \\ ceq(x) = 0 \\ A \cdot x \leq b \\ Aeq \cdot x = beq \\ lb \leq x \leq ub, \end{cases}$$




Figure A.1 A quadratic programming pseudocode taken from Matlab [27]

A.3 Execution time of FMINCON

The run time of FMINCON has been found to be long and variant based on the optimization set and starting point x_0 used by the program. Investigation into different parameters of the 'optimset' and starting point (x_0) of FMINCON was done to find the performance of the software. The approach taken is purely experimental. Consider two different test beds,

$$f_1 = (x_1 - 1)^2 + (x_2 - 12 + \dots + (x_{50} - 50))^2 \quad (\text{A.1})$$

$$f_2 = \sum_{i=1}^{50} (x_i - i)^2. \quad (\text{A.2})$$

In order to investigate the performance of FMINCON, parameters x_0 and 'optimset' are varied. Considering (A.1) first, varying the starting point x_0 , three possible variations are studied and listed below. The equations (A.3) – (A.5) define the X value for three tests. For convenience, the tests shall be denominated as run A.3, A.4, and A.5 respectively. The three cases studied are:

$$X_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad (\text{A.3})$$

$$X_0 = \begin{pmatrix} 1 \\ 2 \\ 3 \\ \vdots \\ 50 \end{pmatrix} - 0.1 * \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \quad (\text{A.4})$$

$$X_0 = \begin{pmatrix} 1 \\ 2 \\ 3 \\ \vdots \\ 50 \end{pmatrix} \quad (\text{A.5})$$

The timing difference when solving (A.1) was explored using the three different X_0 values that are shown in (A.3) – (A.5). No modification to the 'optimset' of FMINCON was done. Each initial value x_0 (A.3-A.5) was run twice using the 'sqp' algorithm type and twice using the 'active-set' type. The total run

time was recorded in Table A.1. Each initial value was run again 25 times in each algorithm type and the average time was recorded in Table A.2.

The results depicted in Tables A.1 and A.2 show that there is a benefit to initiating the search near the solution. The run times are about four times longer in A.3 as compared to starting *exactly* on the answer as in run A.5. The difference between the average run times of starting at the solution as compared to very close to the solution ranges from about 30-75% longer per run when starting at an X0 value displaced by 0.1 in each row of the vector X0 as indicated in (A.4).

Table A.1 Run times with different X0 values for examples (A.3) – (A.5)

X0 value*	Algorithm Type	Time (s)	fval
A.3	sqp	21.5737	1.6271×10^{-12}
A.3	sqp	19.5066	1.6271×10^{-12}
A.3	active-set	22.0704	5.4432×10^{-10}
A.3	active-set	20.9760	5.4432×10^{-10}
A.4	sqp	6.6423	2.3819×10^{-12}
A.4	sqp	6.6989	2.3819×10^{-12}
A.4	active-set	5.9303	2.3827×10^{-12}
A.4	active-set	6.1320	2.3827×10^{-12}
A.5	sqp	4.2784	0
A.5	sqp	3.6210	0
A.5	active-set	6.7066	0
A.5	active-set	5.0039	0

*The contents of this column show the 'run number' for tests performed

Table A.2 Average time after 25 runs for examples (A.3)-(A.5)

X0 value*	Algorithm Type	Time (s)
A.3	sqp	21.2264
A.3	active-set	21.9850
A.4	sqp	8.1359
A.4	active-set	6.7765
A.5	sqp	4.6500
A.5	active-set	5.211

*The contents of this column show the 'run number' for tests performed, the actual X0 values are shown in equations (A.3) – (A.5)

A.4 Solution accuracy for FMINCON

Using the run (A.2), the different 'Algorithms' of the optimset of FMINCON were varied to investigate the error and value obtained (F^*). The results are displayed in Table A.3

Table A.3 Solution error in FMINCON for run (A.2)

Algorithm Type	F^*	Error in x^*	ΔT (s)
Active-set	~2.4	~1.5	0.07-0.042
Interior-point	Bad away from x^*	Does not solve	0.026-0.075
sqp	~2.4	~1.5	0.034-0.072
Trust-region-reflective	~2.4 ‡	~1.5	0.04-0.05

‡ This method produces an automated warning that advises the user that the method can not be used.

A.5 OPTIMSET parameters for MATLAB

The parameters of OPTIMSET are used in MATLAB for various optimization parameters. They are:

- Display - Level of display [off | iter | notify | final]
- MaxFunEvals - Maximum number of function evaluations allowed [positive integer]
- MaxIter - Maximum number of iterations allowed [positive scalar]
- TolFun - Termination tolerance on the function value [positive scalar]
- TolX - Termination tolerance on X [positive scalar]
- FunValCheck - Check for invalid values, such as NaN or complex, from user-supplied functions [{off} | on]
- OutputFcn - Name(s) of output function [{[]} | function]
- All output functions are called by the solver after each iteration.
- PlotFcns - Name(s) of plot function [{[]} | function]
- Function(s) used to plot various quantities in every iteration [26]