Multi-user Diversity Systems with Application to Cognitive Radio

by

Ruochen Zeng

A Thesis Presented in Partial Fulfillment
of the Requirements for the Degree
Master of Science

Approved June 2012 by the
Graduate Supervisory Committee:

Cihan Tepedelenlioğlu, Chair
Tolga Duman
Antonia Papandreou-Suppappola

ARIZONA STATE UNIVERSITY

August 2012

ABSTRACT

This thesis aims to investigate the capacity and bit error rate (BER) performance of multi-user diversity systems with random number of users and considers its application to cognitive radio systems. Ergodic capacity, normalized capacity, outage capacity, and average bit error rate metrics are studied.

It has been found that the randomization of the number of users will reduce the ergodic capacity. A stochastic ordering framework is adopted to order user distributions, for example, Laplace transform ordering. The ergodic capacity under different user distributions will follow their corresponding Laplace transform order. The scaling law of ergodic capacity with mean number of users under Poisson and negative binomial user distributions are studied for large mean number of users and these two random distributions are ordered in Laplace transform ordering sense. The ergodic capacity per user is defined and is shown to increase when the total number of users is randomized, which is the opposite to the case of unnormalized ergodic capacity metric. Outage capacity under slow fading is also considered and shown to decrease when the total number of users is randomized.

The bit error rate (BER) in a general multi-user diversity system has a completely monotonic derivative, which implies that, according to the Jensen's inequality, the randomization of the total number of users will decrease the average BER performance. The special case of Poisson number of users and Rayleigh fading is studied. Combining with the knowledge of regular variation, the average BER is

shown to achieve tightness in the Jensen's inequality. This is followed by the extension to the negative binomial number of users, for which the BER is derived and shown to be decreasing in the number of users.

A single primary user cognitive radio system with multi-user diversity at the secondary users is proposed. Comparing to the general multi-user diversity system, there exists an interference constraint between secondary and primary users, which is independent of the secondary users' transmission. The secondary user with highest transmitted SNR which also satisfies the interference constraint is selected to communicate. The active number of secondary users is a binomial random variable. This is then followed by a derivation of the scaling law of the ergodic capacity with mean number of users and the closed form expression of average BER under this situation. The ergodic capacity under binomial user distribution is shown to outperform the Poisson case. Monte-Carlo simulations are used to supplement our analytical results and compare the performance of different user distributions.

Dedicated

To my family and Voita.

ACKNOWLEDGMENTS

I cannot thank enough my mentor and guide, Dr. Cihan Tepedelenlioglu, without whose support it would not have taken this form. His unlimited enthusiasm in helping me understand new concepts, which play a significant role in shaping this work. Special thanks to his extraordinary patience and encouragement, proved to be an immense help to me, all the times. Once again, my heartfelt thanks to Dr. Tepedelenlioglu.

I am grateful to Dr. Tolga Duman and Dr. Antonia Papandreou-Suppappola for their precious time in serving on my thesis committee member and for their valuable feedback. I would like to extend my appreciation to the School of Electrical, Computer and Energy Engineering at Arizona State University for providing me this opportunity to pursue my Master's degree. I also thke this opportunity to thank Esther Korner for helping me with all the official documents.

I would like to thank all my friends and colleagues in the Signal processing and Communication group, especially Sivaraman Dasarathan, who help me cope with all kinds of difficulties in my research.

Last but not least, I am totally indebted to my family and Voita, for their unconditional love, support and immense faith in my abilities without which I could not have completed this work.

TABLE OF CONTENTS

Page

LIST OF FIGURES viii

CHAPTER

1 INTRODUCTION 1

1.1. Wireless Communications 1

1.2. History of Multi-user Diversity 3

1.3. Multi-user Diversity With Random Number of Users 7

1.4. Cognitive Radio System 8

1.5. Contributions of this Thesis 11

1.5.1. Capacity Metric 11

1.5.2. BER Metric 12

1.5.3. Cognitive Radio with Multi-user Diversity 13

1.6. Outline of the Thesis 13

2 CAPACITY PERFORMANCE IN MULTI-USER DIVERSITY SYSTEMS 15

2.1. System Model and Channel Distribution 15

2.2. Mathematical Preliminaries 16

2.2.1. Completely Monotonic Functions 17

2.2.2. Laplace Transform Ordering 17

2.2.3. Regular Variation 18

2.3. Poisson and Negative Binomial Distributed Number of Users 19

2.3.1. Ergodic Capacity 20

2.3.2. Comparison of Poisson and Negative Binomial Number of Users 26

2.4. Ergodic Capacity Per User 28

2.4.1. Tightness In Jensen's Inequality 28

2.4.2. Optimal Distribution of Number of Users 31

2.5. Outage Capacity and Outage Probability 33

2.5.1. Outage Capacity 33

2.5.2. Outage Probability 36

2.6. Simulation Results 37

2.7. Summary 46

3 AVERAGE BER PERFORMANCE IN MULTI-USER DIVERSITY SYSTEMS 47

3.1. Background 47

3.2. Average Bit Error Rate 48

3.2.1. Representation of Average BER 48

3.2.2. Average BER for Poisson Number of Users 49

3.2.3. Special Cases: Rayleigh Faded Channel 51

3.3. Simulation Results 54

3.4. Summary 57

4 COGNITIVE RADIO SYSTEM WITH MULTI-USER DIVERSITY . . . 58

4.1. Cognitive Radio Systems With Multi-user Diversity 58

4.2. System Model 59

4.3. A Special Case: Binomial Number of Users and Rayleigh Fading . . 62

4.3.1. Asymptotic Scaling of Capacity with λ 63

4.3.2. Average Error Rate 66

4.3.3. Comparison of Binomial Distribution and Poisson Distribution 67

4.3.4. Multiple Interference Constraints 68

4.4. Simulation Results 69

4.5. Summary 72

5 CONCLUSIONS AND FUTURE RESEARCH 73

LIST OF FIGURES

Figure Page

2.1. System Model of Multi-user Diversity System 15

2.2. Ergodic Capacity under Different User Distributions (Rayleigh Fading). 38

2.3. Ergodic Capacity under Different User Distributions (Rician Fading). 39

2.4. Ergodic Capacity under Poisson and Negative Binomial $\mathcal{N}$. 40

2.5. Outage Capacity under Different User Distributions. 41

2.6. Outage Capacity under Poisson and Negative Binomial $\mathcal{N}$. 42

2.7. Ergodic Capacity Per User under Different User Distributions. . . . 43

2.8. Ergodic Capacity Per User under Poisson and Negative Binomial $\mathcal{N}$. 44

2.9. Optimal Probability Mass Function 45

3.1. Average BER under Different User Distributions. 55

3.2. Average under Poisson and Negative Binomial $\mathcal{N}$. 56

4.1. System Model of Cognitive Radio System 60

4.2. Ergodic Capacity and Average BER of Cognitive Radio System. . . 70

4.3. Ergodic Capacity Under Different User Distributions. 71

CHAPTER 1

INTRODUCTION

1.1. Wireless Communications

With the rapid development of wireless communication techniques, wireless networks are widely used in the world. Since Marconi and Tesla investigated radio telegram in 1890s, wireless communications have experienced several stages of evolution, especially for mobile wireless communications [1].

Until the 1980s, Frequency Modulation (FM) technique, an analog telecommunications standard, was used to ensure the reliability of voice communication, which is termed as *First Generation* (1G). The first commercially automated cellular network for 1G was launched in Japan by NTT (Nippon Telegraph and Telephone) in 1979, and was followed by the launch in 1983 in the United States, using Motorola DynaTAC mobile phone [2]. *First Generation* devices used the analog technology for the communication which includes *Frequency Division Multiple Access* (FDMA). In 1G systems, the conversation was full duplex, meaning both the persons can talk and listen at the same time. Since the whole technology was based on the analog system, noise introduced into the signal during communication was a disadvantage. Also, there was very little security as the data was transferred and the voice could be eavesdropped by a third party. Along with other disadvantages and its limit of usage, 1G was soon replaced by more advanced techniques.

After a decade since the appearance of 1G, with the explosion of number of clients, *Time Division Multiple Access* (TDMA) and *Code Division Multiple Access* (CDMA) were introduced to support more users with efficient use of spectrum. This is called the *Second Generation* (2G) in wireless communications. While radio signals on 1G networks are analog, radio signals on 2G networks are digital. *Second Generation* cellular networks were firstly commercially launched on the *Global System for Mobile Communications* (GSM) standard in Finland [3]. The data was transferred in discrete form and hence could be coded or encrypted. It offered greater privacy, efficient data transfer with robustness to noise and implementable with less expensive devices. *Second Generation* networks also introduced data services for mobile, starting with text messages, and is still a major part in the global market today.

In the past ten years, *Third Generation* (3G) tried to use broadband approach to complete fast data transmission business [4]. Application services included wide-area wireless voice telephone, mobile Internet access, video calls and mobile TV, all in a mobile environment. With its killer applications of *General Packet Radio Service* (GPRS), clients can download desired application. Japan's NTT Docomo launched world's first 3G network in the year 2001 [5]. It is a trend that a new generation of cellular standards has appeared approximately every tenth year since 1G systems were introduced in 1980s. Each generation is characterized by new frequency bands, higher data rates and non backwards compatible transmission technology.

The future of communication technology which is now coming on the horizon is the *Fourth Generation* (4G) technologies. 4G is defined as the peak rate requirements for service at 100 mega bits per second (Mbps) for high mobility communication (such as from trains and cars) and 1 giga bit per second (Gbps) for low mobility communication (such as pedestrians and stationary users) [6].

1.2. History of Multi-user Diversity

No matter under which generation of communication systems, channel fading and large number of users are always obstacles to reliable communication. System designers employ different kinds of modulation code schemes to improve the quality of communication in the fading scenario [7]. However, instead of avoiding fading, one can exploits it by using *Multi-user Diversity* techniques to deal with these two issues at the same time. In the system with many users with fading channels, at any time slot, there is always the case that some users have better channels than others. By choosing the best one or best several users to transmit, one can allocate the system resources to good users to achieve capacity and good bit error rate performance. Compared with single-user systems, multi-user systems not only choose which time to transmit, but choose which user to transmit as well, so that it can get extra diversity gain. Multi-user diversity gain depends on the dynamic range of fading and number of users. When total number of users is large and their channels fade rapidly, the effective channel gain is improved, so that with high probability there is a user which has a very good channel at any given time. This is exactly how multi-user diversity exploit fading and large number of users.

On the other hand, multi-user diversity has to face some challenges that will influence the system performance. As we describe above, at each time slot, good users are chosen to transmit. This raises the first critical issue, which is fairness among users. In practical systems, not all the user channels are statistically symmetric, which means that not all the users experience the same fading scenario.

A simple example is that of the non-light-of-sight cellular system, where the signal highly depends on the specific location of the base station so that the channel quality varies significantly among users [8]. Generally speaking, users near the base station are likely to have better channel quality than those far from the base station [9]. This means that the near users will have more chance to communicate than the far users. By using proportional fair scheduling strategy [10], the system can achieve asymptotic fairness in the long term. The scheduler tracks the user's rate normalized by its average throughput instead of tracking the rate only, so that all the users at their own peak can be arranged to transmit. Proportional fair scheduling has also been extended to the mobile clients moving at pedestrian speeds with multiple antennas [11], as well as OFDM systems [12]. Moreover, it has been stated that using proper dynamic resource allocation strategies on Rayleigh fading channels can yield a remarkable power boost which can be as high as 5 or 6 dB, and this gain will grow logarithmically with the number of users [13]. Second issue is as we introduced before, multi-user diversity depends on the rate and dynamic range of channel fluctuation. In other words, larger tail probability in the fading channel distribution yields, better channel quality for selected users.

The last issue is prediction error due to the feedback quality. Performance of a multi-user diversity system is analysed, taking into account the feedback errors due to channel variability [14], and the trade off between the multi-user diversity gain and feedback quality has been well established in [15]. Especially in downlink systems, base station and users can share a strong pilot, so the prediction error will be mainly caused by the feedback delay rather than channel estimation errors.

The upper bound of the achievable data rate is primarily defined by the feedback delay and its dependency to the delay factor has been studied in [16]. All the above points make reducing feedback delay to one of the most important challenges in designing future wireless systems. One can reduce the delay by firstly shortening the scheduling time slot; then setting a threshold of required rate and let all users below this threshold remain silent to decrease the feedback frequency. As a combination of all the issues we mentioned, different fair scheduling algorithms are compared and also it is shown that space-time coding makes the system more robust and gives performance gain in the case of feedback errors [17].

Capacity is one of the most important performance measures in multi-user diversity systems. The downlink system of multi-user diversity scheme is considered as a broadcast channel. Different types of capacity regions for fading broadcast channels are studied and their corresponding optimal resource allocation strategies are obtained in [18, 19]. In the uplink system, multi-user diversity scheme is exactly a *multiple access channel* (MAC), and its capacity region is known in [20]. For this case, FDMA, TDMA and CDMA techniques can be exploited where CDMA is more suitable for bursty traffic. The performance of two users with hierarchical modulations is analysed in [21]. Both in the uplink and downlink system, by choosing the single best user at each time slot, the Shannon capacity could be achieved [22,23]. This aroused the interest to learn how to maximize the system throughput with respect to the number of users in a single cell [24].

Moreover, multi-user diversity can be combined with multiple antennas and power adaptation [25, 26]. Ergodic capacity and outage capacity region of M-user fading MAC under the assumption that both transmitter and receiver have multiple antennas. A unified capacity analysis for wireless systems with joint multi-user scheduling and antenna diversity in Nakagami fading channels is derived in [27]. More analysis of capacity in the Rayleigh fading channel with MIMO antenna scheme can be found in [28, 29]. Reference [30] considers a look at the throughput when a SIMO scheme is employed. Also, adaptive modulation technique is developed to help the system to achieve the capacity. Adaptive modulation system dramatically enhances system robustness to multipath fading and transmission quality control [31–33]. Adaptive modulation technique for multi-user MIMO systems with multi-user diversity is considered in [34] and the proposed scheme maximizes the sum of the instantaneous bit rate under a target BER constraint. One critical issue is that this technique strongly relies on the accurate estimation of channel and a reliable feedback path [35].

Along with capacity, bit error rate (BER) is also an important performance metric in multi-user systems. In [36], a closed-form expression for the average BER is derived, and how the average BER goes to zero asymptotically as the number of users increases for a given SNR is analysed. Similarly to capacity, the influence of feedback delay on BER performance has been well established in [37, 38]. A lot of work has been done to investigate the BER performance when the spatial diversity schemes like *Maximum Ratio Combining* (MRC), *Selection Combining* (SC) are combined with multi-user diversity in MIMO system in [39–41]. It has been shown in [42] that the antenna correlation can surprisingly improve the BER with large number of users.

1.3. Multi-user Diversity With Random Number of Users

Unlike the early work which is based on the deterministic number of total users, multi-user diversity with random number of users has been studied recently, since the number of users is randomly varying in practice. For example, the probability of certain types of data request such as a cell phone call, stocks, weather and email are bursty in nature leading to very short channel access times. This implies that the number of users actively competing for channel access is a random variable across time. Additionally, schemes like cognitive radio system in which a user needs to follow instructions from the base station to request channel access, only if its transmitting SNR is larger than a predefined threshold, also leads to a random number of users across time. This scheme will be discussed in Chapter IV.

Ergodic capacity performance under multi-user diversity system with random number of users is studied in reference [43], in which all users are assumed to have symmetric independent Rayleigh fading channels and only single user with highest instantaneous SNR is picked to transmit at each time slot. The general instantaneous SNR distribution of the best user is derived for arbitrary fading and user distributions, which can be expressed in the form of the probability generation function of the user distribution. The ergodic capacity of the multi-user diversity system with deterministic number of users is shown to have a completely monotonic derivative with respect to number of user. Based on this, by using Jensen's Inequality, it is proved that randomization of number of users will decrease the ergodic capacity. Moreover, different user distributions can be compared with and ordered in the

Laplace Transform ordering sense, which means that we can compare the ergodic capacity under a variety of user distributions. At last, for a special case when the number of users is Poisson distribution and user channel is Rayleigh fading, a closed form expression of the ergodic capacity is given in [43].

1.4. Cognitive Radio System

It is a fact that most radio frequency spectrum is inefficiently utilized, and that spectrum utilization depends strongly on time and place [44, 45]. The concept of cognitive radio was first proposed by Joseph Mitola in a seminar at KTH, the Royal Institute of Technology in Stockholm, in 1998, and has become very popular since it is considered as an ideal way to use spectrum resources. Basically, cognitive radio is an opportunistic spectrum usage solution, in which the frequency bands are not used only by their licensed users (primary users), who own the right to get access to the channel in an arbitrary time as needed. Many unlicensed users (secondary users) continuously monitor the available spectrum holes and the activities of the primary users. Once the primary user is silent, the secondary users are allowed to transmit. By doing that the spectrum resources can be shared by large number of users and the primary users also get rid of the interference issue from secondary users [46]. In this approach, secondary users have to satisfy a strict interference constraint and its transmission power should be below a certain threshold at all times [47].

In the literature, a widely used way to solve the interference constraint is to treat it as an optimization problem using beamforming techniques [48]. The aim is to maximize the transmit SNR of the secondary users and meanwhile minimize the interference to the primary users. Publications describing the underlying cognitive radio systems with antenna beamforming can be found in [49–51]. In [49], an algorithm which computes the power control values and the beamforming weights in turn, is proposed to minimize the total transmission power of the CR system and to satisfy the *signal-to-interference plus noise power ratio* (SINR) requirement for both the primary and secondary users. In [50], the problem of joint power control and beamforming is studied to achieve the same goal, subject to the constraints that interference power to the primary user is below a threshold and SINR of the secondary users is above an acceptable value. In addition, a zero-forcing beamforming scheme incorporated with a user selection algorithm is proposed in [51] to maximize the sum rate, while satisfying the SINR requirements for the secondary users as well as the limited interference power to the primary users.

Another way to overcome the interference constraint is to incorporate relay techniques to increase their area of coverage. There exist various techniques and protocols for relaying a signal in cooperative networks among which *amplify and forward* (AF) is the most popular one due to its simplicity [52]. A general advantage of relaying is to improve diversity order by using multiple relays in the system [53,54]. Recently, few papers studied selective relaying in cognitive networks. A relay selection scheme for cognitive networks is described in [55]. A relay selection and power allocation scheme with limited interference to the primary users is proposed in [51]. A modified relay selection criterion is proposed in [56] which takes into account the interference constraint and the relays in the network are assumed to be operating in *decode and forward* (DF) mode. Main contribution in [56] is the derivation of the outage probability.

In cognitive radio communications, there are several major issues:

- **Spectrum Sensing**. Secondary users monitor the available spectrum resources and share the knowledge with limited interference with other peer users. In the literature, it is considered to have an allocated control channel to transmit this information [57]. In some works, it is proposed to have a centralized controller that gathers this information, determines spectrum availability, and allocates distinct bands to different cognitive users [58].

- **Spectrum Management**. Suitable spectrum holes are captured to meet unlicensed (secondary) user communication requirements while not creating harmful interference to licensed (primary) users. Cognitive radios should decide on the best spectrum band to meet the *Quality of Service* (QoS) requirements over all available spectrum bands. This process is what we are interested in and will be discussed in the Chapter IV.

- **Spectrum Mobility**. This is defined as the process when a secondary user exchanges its frequency of communications with time. Since the radio terminals always operated in the best frequency band, the cognitive radio networks target to use the spectrum in a dynamic manner.

- **Spectrum Sharing**. Since their exists multiple secondary users, like all the multi-user system, a proper spectrum scheduling method should be provided to ensure the fairness among all secondary users.

In our thesis, we will mainly focus on the cognitive radio system with multiple secondary users under both single and multiple primary users case, which will be detailed in Chapter IV. In the secondary users system, the multi-user diversity technique will be applied. A literature review of cognitive radio system with multi-user diversity scheme will be given in Chapter IV.

1.5. Contributions of this Thesis

In wireless communication systems, capacity and average BER are always two important metric to measure the system performance. In reference [43], the well known Shannon capacity, or ergodic capacity, of the multi-user diversity with random number of users is studied. In this thesis, we study three kinds of capacity metric and examines BER performance based on the result in [43] in single-selected-user multi-user diversity system with random number of users. Different kinds of discrete random variables for the user distribution are analysed. These random variables are compared in the Laplace transform ordering sense, which is a method of stochastic ordering to order different random variables. Hence, the contributions towards research in the area are categorized and summarized below.

1.5.1. Capacity Metric

1. Considering a multi-user diversity system with a large number of users and each user is active with a small probability, the distribution of the users is Poisson. Some properties of ergodic capacity under Poisson distributed users are

studied. Moreover, the ergodic capacity under negative binomial distribution, as an example of compound Poisson distribution, is studied.

2. Ergodic capacity for Poisson user distribution is compared with negative binomial user distribution in the Laplace transform ordering sense and the former outperforms the latter.

3. Outage probability is studied when the channel experiences slow fading scenario. Randomization of number of users will also reduce the performance of outage probability. Outage capacity is also studied in Rayleigh fading scenario.

4. In practical multi-user diversity systems, each user has a minimal rate requirement for reliable communications. To study how this rate get influenced by the number of users, a new metric named capacity per user, which is the ergodic capacity normalized by the number of users, is developed. The capacity per user performs better in the random number of users case than the deterministic number of users case.

1.5.2. BER Metric

1. In single-selected-user case, BER metric for Poisson user distributions is compared with negative binomial user distributions in the Laplace transform ordering sense and the former outperforms the latter.

2. Closed form expression of BER in negative binomial case is derived.

1.5.3. Cognitive Radio with Multi-user Diversity

1. Multi-user diversity with random number of users is combined with cognitive radio system with multiple secondary users, which is a practical and promising application.

2. Both single primary user and multiple primary users cases are considered and the performance analysis has been studied. With independent interference constraint to the primary receiver and choosing the best user to communicate, the secondary users transmission is exactly the multi-user diversity system with binomial distributed number of users.

3. Comparison between the ergodic capacity under Poisson and binomial distributed number of users are discussed and simulated in Matlab.

1.6. Outline of the Thesis

The remainder of this thesis is organized as follows. Chapter II begins with a system model for a mulit-user diversity system and some useful mathematical preliminaries including Laplace transform ordering and regular variation. Later in the chapter, ergodic capacity metric in different user distributions is discussed analytically in the single-selected-user case. Performance of other metrics like ergodic capacity per user and outage probability are also included in this chapter. This is followed by simulation results and a summary of the chapter. In Chapter III, still in the single-selected-user scenario, a closed form expression of average BER when N

is negative binomial distribution is derived and compared with Poisson N. Chapter IV discusses the multi-user diversity scheme in the cognitive radio system, which can be considered as an integrated application of the preceding chapters.

CHAPTER 2

CAPACITY PERFORMANCE IN MULTI-USER DIVERSITY SYSTEMS

2.1. System Model and Channel Distribution

This thesis extends the work done in [43], so we follow its system model. As shown in Figure 2.1, an uplink multi-user diversity system with one base station (BS) is considered. Both BS and users have only a single antenna.

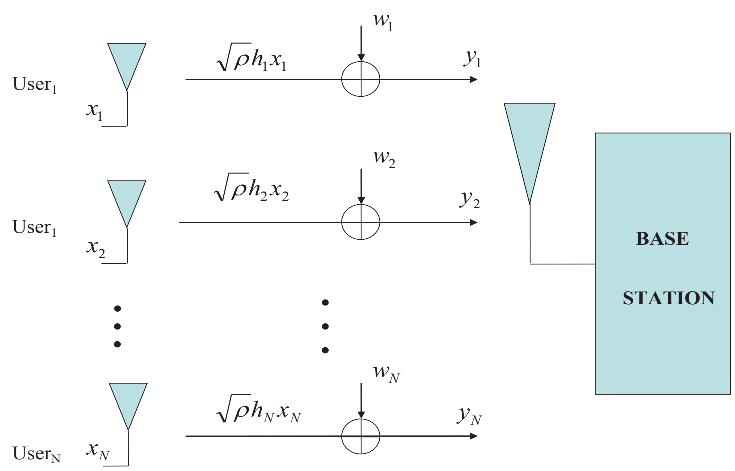


Figure 2.1. System Model of Multi-user Diversity System

The received signal at the BS from the n^{th} user can be expressed as,

$$y_n = \sqrt{\rho}h_n x_n + w_n, \qquad n = 0, 1, \ldots, \mathcal{N}, \tag{2.1}$$

where the number of users $\mathcal{N}$ is a a discrete non-negative integer random variable. When addressing the deterministic number of users case, $\mathcal{N}$ is set to be N, where N is a realization of the random variable $\mathcal{N}$. The average received power ρ at BS is identical so that this multi-user diversity system is homogeneous. h_n denotes the channel coefficient, x_n the transmitted symbol, w_n the additive white Gaussian noise (AWGN) corresponding to the n^{th} user and γ^* the channel. The channel is assumed to satisfy $\mathrm{E}[|h_n|^2] = 1$ for all n and to be independent and identically distributed (i.i.d.) across all users. The transmitted symbols satisfy $\mathrm{E}[|x_n|^2] = 1$. The channel gain of the n^{th} user at the BS can be expressed as $\gamma_n = |h_n|^2$. In the single user selected case, its channel gain is denoted by $\gamma^* = |h_*|^2$, where $|h_*|^2 = \max_n\{|h_n|^2\}$.

Define $F_\gamma(x)$ as the cumulative distribution function (CDF) of the channel gain of the i.i.d. fading channels across all users. Recalling that the total number of users $\mathcal{N}$ is a random variable, the CDF of the channel gain of the best user, conditioned on $\mathcal{N} = N$, can be written as:

$$F_{\gamma^*}(x|\mathcal{N} = N) = F_\gamma^N(x), \tag{2.2}$$

where the N^{th} power is obtained due to the i.i.d. Assumption of the N user channels. According to the total probability theorem, the CDF of the channel gain of the best user selected from a random set of users can be obtained by averaging (2.2) with respect to the distribution of $\mathcal{N}$:

$$F_{\gamma^*}(x) = \mathrm{E}_{\mathcal{N}}\left[F_\gamma^{\mathcal{N}}(x)\right] = \sum_{k=0}^{\infty} \Pr\left[\mathcal{N} = k\right] F_\gamma^k(x) = U_{\mathcal{N}}(F_\gamma(x)) \tag{2.3}$$

where $U_{\mathcal{N}}(t) = \sum_{k=0}^{\infty} \Pr\left[\mathcal{N} = k\right] t^k$, $0 \leq t \leq 1$, is the probability generating function (PGF) of random variable $\mathcal{N}$. From (2.3) it can be seen that for any fading channel distribution and any non-negative integer distribution on the number of users, the CDF of the best user's channel gain at the BS can be easily obtained.

2.2. Mathematical Preliminaries

In this section, we introduce some mathematical preliminaries that will be useful throughout the thesis.

2.2.1. Completely Monotonic Functions

A function $\tau(x) : \mathbb{R}^+ \to \mathbb{R}$ is *completely monotonic* (*c.m.*) if its derivatives alternate in sign [59], i.e.,

$$(-1)^k \frac{d^k \tau(x)}{dx^k} \geq 0, \quad \forall x, \quad k = 0, 1, 2, \ldots, \tag{2.4}$$

where $d^0\tau(x)/dx^0 = \tau(x)$ by definition. We are also interested in the functions whose first-order derivatives satisfy (2.4), which are said to have a completely monotonic derivative (*c.m.d.*). In practical systems, variable x denotes the number of user which is an integer, which are nothing but sequences obtained by sampling functions as defined by (2.4). Consequently, we will primarily study the asymptotic properties of $\tau(x)$. Due to the theorem by Bernstein [59], an equivalent representation for *c.m.* is that:

$$\tau(x) = \int_0^\infty e^{-sx} d\psi(s) \tag{2.5}$$

for some non-decreasing function $\psi(s)$. It has been proved in [59] that, for all function f of x that have a completely monotonic derivative, $\frac{f(x)}{x}$ is a completely monotonic function. This property will be useful in Section 2.4.

2.2.2. Laplace Transform Ordering

In this section we introduce *Laplace transform* (LT) ordering, a tool to compare how different user distributions affect the error rate, and ergodic capacity averaged across user and channel distributions. LT ordering, as a special case of stochastic ordering, deals with partial ordering of random variables.

Let $\mathcal{X}$ and $\mathcal{Y}$ be non-negative random variables. $\mathcal{X}$ is said to be less than $\mathcal{Y}$ in the LT order (written $\mathcal{X} \leq_{\mathrm{Lt}} \mathcal{Y}$), if $\mathrm{E}[\mathrm{e}^{-\mathrm{s}\mathcal{X}}] \geq \mathrm{E}[\mathrm{e}^{-\mathrm{s}\mathcal{Y}}]$ for all $s > 0$. An important theorem found in [59], and [60] is given next:

Theorem 1. *Let $\mathcal{X}$ and $\mathcal{Y}$ be two random variables. If $\mathcal{X} \leq_{Lt} \mathcal{Y}$, then, $\mathrm{E}\left[\psi(\mathcal{X})\right] \geq \mathrm{E}\left[\psi(\mathcal{Y})\right]$ for all c.m. functions $\psi(\cdot)$, provided the expectation exists. Moreover, the reverse inequality $\mathrm{E}[\psi(\mathcal{X})] \leq \mathrm{E}[\psi(\mathcal{Y})]$ holds for all $\psi(\cdot)$ with a completely monotone derivative, provided the expectation exists.*

This theorem implies that if the number of users is from a distribution that can be ordered in the LT sense, then both the average error rate and capacity can be ordered at every value of SNR ρ.

We will use another equivalent representation of LT ordering of discrete random variables to analyse the user distribution in terms of the ordering of their PGFs. By defining $t := e^{-s}$, one can rewrite $\mathrm{E}\left[\mathrm{e}^{-\mathrm{s}\mathcal{X}}\right] \geq \mathrm{E}\left[\mathrm{e}^{-\mathrm{s}\mathcal{Y}}\right]$ for $s \geq 0$ as $\mathrm{E}\left[\mathrm{t}^{\mathcal{X}}\right] \geq \mathrm{E}\left[\mathrm{t}^{\mathcal{Y}}\right]$ for $0 \leq t \leq 1$, which is the same as $U_{\mathcal{X}}(t) \geq U_{\mathcal{Y}}(t)$, $0 \leq t \leq 1$, where we recall that $U_{\mathcal{X}}(t) = \mathrm{E}[\mathrm{t}^{\mathcal{X}}]$ represents the PGF of the discrete random variable $\mathcal{X}$.

2.2.3. Regular Variation

A function $\psi(s)$ is *regularly varying* with exponent $\mu \neq 0$ at $s = \infty$ if it can be expressed as $\psi(s) = s^{\mu} l(s)$ where $l(s)$ is slowly varying and by definition satisfies $\lim_{s\to\infty} l(\kappa s)/l(s) = 1$ for $\kappa > 0$. Regular (slow) variation of $\psi(s)$ at $s = 0$ is equivalent to regular (slow) variation of $\psi(1/s)$ at ∞. The Tauberian theorem for Laplace transforms, whose proof can be found in [61], applies to *c.m.* functions of the form (2.5) and states that $\tau(x)$ is regularly varying at $x = \infty$ if and only if $\psi(s)$ is regularly varying at $s = 0$.

Theorem 2. *If a non-decreasing function $\psi(s) \geq 0$ defined on $s \geq 0$ has Laplace transform $\tau(x) = \int_0^\infty e^{-sx} d\psi(s)$ for $x \geq 0$, and $l(s)$ is slowly varying at $s = 0$ (or $s = \infty$), then $\psi(s)$ having variation exponent μ at ∞ (or 0) and $\tau(x)$ having variation exponent $-\mu$ at 0 (or ∞) imply each other.*

In this chapter, we are interested in the performance of capacities averaged across both the channel distribution, and the number of users. The expression $\overline{C}(\rho, N)$ represents the capacity of a multi-user diversity system with a deterministic N, that is averaged with respect to the distribution of the fading channel. The expression $\mathrm{E}_{\mathcal{N}}\left[\overline{C}(\rho, \mathcal{N})\right]$ represents the average error rate of a multi-user diversity system with a random number of users, which is averaged with respect to the distribution of the number of users and the fading channels.

2.3. Poisson and Negative Binomial Distributed Number of Users

In reference [43], a multi-user diversity system when $\mathcal{N}$ is Poisson distributed with parameter λ in i.i.d Rayleigh fading channel has been well studied. It has been stated that if the system contains a large number of users, and each user is active with a small probability independent of user numbers, the user distribution will be Poisson. Furthermore, parameter of Poisson distribution could be also random, which arises as compound Poisson distributions. This randomization could make the performance either better or worse than Poisson case. In this section, we will study negative binomial distribution as an example of compound Poisson distribution, where the mixing distribution of the Poisson rate is a gamma distribution. In this thesis, we denote Poisson distribution as $Pois(\lambda)$ where λ is the mean value of Poisson distribution. The details will be discussed in Section 2.3.1. Later in the chapter, we will show that the randomization of N will reduce the ergodic capacity performance.

2.3.1. Ergodic Capacity

Generally there are two scenarios in fading channel: 1)the channel remains constant over the transmission duration of the codeword, it is in slow fading scenario; 2)the codeword length spans many coherence periods, channel is in the so-called fast fading regime. Modelling by using the idea of parallel channel, the outage probability of the time diversity channel is

$$p_{\text{out}}(R) = \mathbb{P}\{\frac{1}{L}\sum_{l=1}^{L}\log(1+|h_l|^2\rho) < R\} \tag{2.6}$$

when $l = 1, \ldots, L$ represents a coherence period of symbols, and R is a rate with reliable communication. For fast fading channels, the ergodic capacity is a critical and important performance metric. In this scenario, as $L \to \infty$, the law of large numbers tells that

$$\frac{1}{L}\sum_{l=1}^{L}\log(1+|h_l|^2\rho) \to \mathrm{E}[\log(1+|h|^2)] \tag{2.7}$$

Now we can average over many independent fades of the channel by coding over a large number of coherence time intervals and a reliable rate of communication of

(2.7) can be achieved and defined as the ergodic capacity, or Shannon Capacity, to a fast fading channel.

Considering the single-user-selected multi-user system, the asymptotic average capacity and its scaling law with respect to λ has been derived in [43]. The approach is to first derive some useful properties of the ergodic capacity with deterministic users, and then see what is the influence when N is randomized.

The ergodic capacity for the deterministic number of users system can be expressed as,

$$\overline{C}(\rho, N) = \int_0^\infty \log(1+\rho x)\, dF_\gamma^N(x) = \rho \int_0^\infty \frac{1-F_\gamma^N(x)}{1+\rho x} dx. \tag{2.8}$$

where we use integration by parts, and assume that $F_\gamma(x)$ satisfies $\lim_{x\to\infty}\log(1+\rho x)(1-F_\gamma^N(x)) = 0$, for all $N \geq 0$. It can be seen that $\overline{C}(\rho, N)$ has a completely monotonic derivative since

$$\frac{\partial^{k+1}\overline{C}(\rho, N)}{\partial N^{k+1}} = -\rho \int_0^\infty \frac{F_\gamma^N(x)\left[\log(F_\gamma(x))\right]^{k+1}}{1+\rho x} dx. \tag{2.9}$$

alternates in sign as k is incremented. This implies that $\overline{C}(\rho, N)$ is a concave increasing function of N. Applying the well-known Jensen's inequality for concave functions, we have

$$\mathrm{E}_{\mathcal{N}}\left[\overline{C}(\rho, \mathcal{N})\right] \leq \overline{C}(\rho, \lambda). \tag{2.10}$$

Therefore, randomization of N will always deteriorate the average ergodic capacity of a multi-user diversity system.

A special case when users experience i.i.d Rayleigh fading channels and $\mathcal{N}$ is Poisson distributed is studied in [43]. First the distribution of channel gain is derived. For this case, the CDF of the channel gain of the best user chosen from the a Poisson random set of users in (2.3) can be expressed as,

$$F_{\gamma^*}(x) = \exp\left(-\lambda e^{-x}\right) \qquad \text{for} \quad x \geq 0. \tag{2.11}$$

The channel gain of the best user in (2.11) is identical to a truncated and shifted version of Gumbel distribution, which was seen in its untruncated form in [62]. Notice that for $x = 0$, (2.11) gives $e^{-\lambda}$. For $x > 0$, (2.11) has the form of the Gumbel distribution with $\alpha_\lambda = 1$ and $b_\lambda = \log(\lambda)$ corresponding to the parameters in [63]. The distribution in (2.11) is therefore of mixed type, mass of $e^{-\lambda}$ at the origin and the rest of the distribution has the form of a truncated Gumbel distribution.

After knowing all the knowledge above, the asymptotic average capacity and its scaling law with respect to λ can be derived. Using (2.8), the ergodic capacity averaged across the user distribution can be expressed as,

$$\begin{aligned} \mathrm{E}_{\mathcal{N}}\left[\overline{C}(\rho, \mathcal{N})\right] &= \mathrm{E}_{\gamma^*}[\log(1+\rho\gamma^*)] = \rho \int_0^\infty \frac{1 - e^{-\lambda e^{-x}}}{1+\rho x} dx \\ &= \log\left(1 + \rho \log(\lambda)\right) + O(1/\sqrt{\log(\lambda)}), \end{aligned} \tag{2.12}$$

as $\lambda \to \infty$.

For a multi-user diversity system with deterministic number of users, it has been shown in [10] that the ergodic capacity grows as $\log(\log(N))$. From (2.12) it is seen that for a multi-user diversity system with random number users with mean value λ, the ergodic capacity grows as $\log(\log(\lambda))$. This implies that when average number of users λ is equal to N of the deterministic number of users case, the ergodic capacity for both cases grow identically.

Furthermore, as we discussed at the beginning of Section 2.3, when λ is random, $\mathcal{N}$ could become a compound Poisson version, negative binomial distribution for instance. There are two representations of negative binomial distribution:

1. Suppose there is a sequence of independent Bernoulli trials, each trial having two potential outcomes called "success" and "failure". In each trial the probability of success is p and of failure is $1-p$. We are observing this sequence until a predefined number r of failures has occurred. Then the random number of successes we have seen, $\mathcal{X}$, will have the negative binomial (or Pascal) distribution and can be denoted as $NB(r,p)$.

2. First we denote gamma distribution as $Gamma(k,\theta)$ with a shape parameter k and a scale parameter θ. Then negative binomial distribution arises as a continuous mixture of Poisson distributions (i.e. a compound probability distribution) where the mixing distribution of the Poisson rate is a gamma distribution. That is, we can view the negative binomial as a $Pois(\lambda)$ distribution, where λ itself is a random variable, distributed according to $Gamma(r, p/(1-p))$.

We can easily observe a negative binomial distribution by simply randomizing λ yielding the gamma distribution. Suppose that $\mathcal{X}$ is a negative binomial random variable, the probability mass function (PMF) of $\mathcal{X}$ is

$$\begin{aligned} P_x[k] &= \int_0^\infty e^{-\lambda}\frac{\lambda^k}{k!}\lambda^{(r-1)}\frac{\exp(-\lambda(1-p)/p)}{(\frac{p}{1-p})^r\Gamma(r)}\mathrm{d}x \\ &= \frac{\Gamma(r+k)}{\Gamma(r)k!}(1-p)^r p^k \qquad (2.13) \end{aligned}$$

where $\Gamma(n)$ is the gamma function, and when n is integer, it reduces to $\Gamma(n) =$

$(n-1)!$. To study the behaviour of the ergodic capacity, we will first derive the CDF of the channel of the best user selected from a negative binomial random set of users, by simply taking an expectation of $F_{\gamma^*}(x)$ with respect to the distribution of λ:

$$\begin{aligned}F_{\gamma^*}(x) &= \int_0^\infty \exp(-\lambda e^{-x}) f(\lambda) d\lambda \\ &= \int_0^\infty \exp(-\lambda e^{-x}) \lambda^{r-1} \frac{e^{-\lambda/u}}{\Gamma(r)} d\lambda \\ &= \frac{1}{(1+e^{-x}u)^r}, \end{aligned} \tag{2.14}$$

where where $u = p/(1-p)$, $ru = \lambda$ and $f(\lambda)$ is the PDF of gamma distribution we have introduced with parameter r and $p/(1-p)$. Recall that we can also obtain $F_{\gamma^*}(x)$ by plugging the PGF of negative binomial distribution into (2.3), which will give us exactly the same answer as (2.14). Following the methods of (2.12) we have the following theorem:

Theorem 3. *For negative binomial distributed $\mathcal{N}$ with mean λ and Rayleigh faded channels, as $\lambda \to \infty$, we have*

$$E_{\mathcal{N}}\left[\overline{C}(\rho, \mathcal{N})\right] = \log\left(1 + \rho \log(\lambda)\right) + O(1/\log(\lambda)). \tag{2.15}$$

Proof. By substituting (2.14) into (2.8) and defining $e^{-x} = y$, the ergodic capacity of negative binomial $\mathcal{N}$ can be expressed as:

$$\begin{aligned}\mathrm{E}_{\mathcal{N}}[\overline{C}(\rho, \mathcal{N})] &= \int_0^\infty \log\left(1+\rho x\right) \mathrm{d}\mathrm{E}_{\mathcal{N}}[F_\gamma^N(x)] = \rho \int_0^\infty \frac{1 - F_{\gamma^*}(x)}{1+\rho x} \mathrm{d}x \\ &= \int_0^{1/\lambda} \frac{1-(1+uy)^{-r}}{1-\rho\log(y)} \left(\frac{\rho}{y}\right) \mathrm{d}y + \int_{1/\lambda}^\infty \frac{1-(1+uy)^{-r}}{1-\rho\log(y)} \left(\frac{\rho}{y}\right) \mathrm{d}y \end{aligned} \tag{2.16}$$

For the first term after the second equality in (2.16), we have

$$
\begin{aligned}
0 < \int_0^{1/\lambda} \frac{1-(1+uy)^{-r}}{1-\rho\log(y)}\left(\frac{\rho}{y}\right)dy &< \int_0^{1/\lambda} \frac{ruy}{1+\rho\log(\lambda)}\left(\frac{\rho}{y}\right)dy \\
&= \frac{\lambda\rho}{1+\rho\log(\lambda)}\left(\frac{1}{\lambda}\right)
\end{aligned}
\tag{2.17}
$$

by replacing the numerator of the integrand with its upper bound and the denominator of the integrand with its lower limit. It can be seen that the upper bound after the equality in (2.17) yields $O(1/\log(\lambda))$ and has limit 0 as $\lambda \to \infty$, implying that the first term should have limit 0. The second term in (2.16) has the bounds given by,

$$
\int_{1/\lambda}^{1} \frac{1-\left(1+\frac{u}{\lambda}\right)^{-\frac{\lambda}{u}}}{(1-\rho\log(y))}\left(\frac{\rho}{y}\right)dy < \int_{1/\lambda}^{1} \frac{(1+uy)^{-r}}{(1-\rho\log(y))}\left(\frac{\rho}{y}\right)dy < \int_{1/\lambda}^{1} \frac{1-(1+u)^{-\frac{\lambda}{u}}}{y(1-\rho\log(y))}dy
\tag{2.18}
$$

in which the lower and upper bounds are obtained by bounding the numerator, and they turn out to be $\int_0^{1/\lambda} \frac{1}{1-\rho\log(y)}\left(\frac{\rho}{y}\right)d\lambda = \log(1+\rho\log(\lambda))$. Consequently, combining the two upper bounds of two terms in (2.16), we prove the result of (2.15). $\square$

It can be seen that the first term in (2.16) is $\log\log(\lambda)$, i.e.,

$$
\lim_{\lambda\to\infty} \frac{\mathrm{E}_{\mathcal{N}}\left[\overline{C}(\rho,\mathcal{N})\right]}{\log(\log(\lambda))} = 1.
$$

This implies that the growing rate of ergodic capacity is maintained when $\mathcal{N}$ change from Poisson distribution to negative binomial distribution as $\lambda \to \infty$. Moreover, as $\lambda \to \infty$, the gap between the ergodic capacity averaged across the user distribution and the ergodic capacity at the average number of users vanishes. On the other hand, the second term in (2.15) can not be the evidence of Poisson $\mathcal{N}$ outperforming negative binomial $\mathcal{N}$ under the same mean value since the O notation stands for an approximation so that it is actually a very lose bound. The capacity under Poisson and negative binomial distribution $\mathcal{N}$ will be ordered from the perspective of LT ordering in the next section.

2.3.2. Comparison of Poisson and Negative Binomial Number of Users

In Section 2.3.1, we observed that Poisson distributed $\mathcal{N}$ performs better than negative binomial $\mathcal{N}$ when ergodic capacity is taken into account. In this section, we will use a general approach of stochastic ordering to compare two random variables.

As we discussed in Theorem 1, since $\mathrm{E}_{\mathcal{N}}\left[\overline{C}(\rho, \mathcal{N})\right]$ has a completely monotonic derivative with respect to N, once the distribution of $\mathcal{N}$ is ordered, $\mathrm{E}_{\mathcal{N}}\left[\overline{C}(\rho, \mathcal{N})\right]$ will follow the same order of $\mathcal{N}$. To compare two random variables, it is straight forward to use the equivalent interpretation of LT ordering in terms of the PGFs, by noting that $\exp(\lambda(t-1))$ is the PGF of Poisson distribution [64] while $\left(\frac{1-p}{1-pt}\right)^r$ is the PGF of negative binomial distribution where $0 \leq t \leq 1$. Since we compare them under the save mean value, then $\frac{pr}{1-p} = \lambda$. Consequently, we have the following theorem:

Theorem 4. *Let $\mathcal{X}$ denotes a Poisson random variable with parameter λ and $\mathcal{Y}$ denotes a negative binomial random variable with mean value $rp/(1-p)$. By assuming that $rp/(1-p) = \lambda$, we have $U_{\mathcal{X}}(t) \leq U_{\mathcal{Y}}(t)$, or equivalently,*

$$\mathcal{X} \geq_{\mathrm{Lt}} \mathcal{Y} \tag{2.19}$$

Proof. To show $U_{\mathcal{X}}(t) \leq U_{\mathcal{Y}}(t)$ first we take logarithm to $U_{\mathcal{X}}(t)$ and $U_{\mathcal{Y}}(t)$ and we get

$$\log(U_{\mathcal{X}}(t)) - \log(U_{\mathcal{Y}}(t)) = \frac{pr}{1-p}(t-1) - r\log\left(\frac{1-p}{1-pt}\right). \qquad (2.20)$$

By shuffling the terms we rewrite the problem as comparing $\frac{p}{1-p} - \log(1-p) + \log(1-pt)$ with 0. Taking the 1^{st} derivative with respect to t we get

$$\frac{\partial\left(\frac{p}{1-p} - \log(1-p) + log(1-pt)\right)}{\partial t} = \frac{p}{1-p} - \frac{p}{1-pt} \geq 0 \qquad (2.21)$$

for all $0 \leq t \leq 1$. This implies that (2.20) is an monotonically increasing function of t with the maximum value 0 at $t = 1$. Hence, we have $U_{\mathcal{X}}(t) \leq_{\mathrm{Lt}} U_{\mathcal{Y}}(t)$, completing the proof. □

From Theorem 1 and Theorem 4 we know that the Poisson $\mathcal{N}$ outperforms negative binomial $\mathcal{N}$ when considering the ergodic capacity. Moreover, (2.21) tells us the relationship between these two distributions. Specifically, considering a negative binomial distribution, the stopping parameter r goes to infinity, whereas the probability of success in each trial, p, goes to zero in such a way as to keep the mean of the distribution constant [65]. By doing that a negative binomial distribution can get closer to a Poisson distribution. In other words, the negative binomial $\mathcal{N}$ performance can be improved but never exceeds the Poisson $\mathcal{N}$ case. This fact can be justified by the simulation results in Section 2.6.

2.4. Ergodic Capacity Per User

In Section 2.3, we study the ergodic capacity when $\mathcal{N}$ is Poisson and negative binomial distributed respectively. Intuitively speaking, multi-user diversity systems aim to improve the sum rate of the system by increasing N. Apparently, one can not increase the total number of users to an unreasonable large number, since it will ruin the Quality of Service (QoS) of a single user in the system. In practical systems, designers of wireless communication systems are often required to design the system that have a certain rate constraint for individual user, in other words a lower bound, to ensure the reliability for each user in multi-user systems. In this section, we will take a look at the ergodic capacity per user in multi-user diversity systems with single-user-selected case.

2.4.1. Tightness In Jensen's Inequality

Define a new metric $\overline{C}_{\text{norm}}(\rho, N) = \overline{C}(\rho, N)/N$ as the ergodic capacity per user, which is simply the average ergodic capacity normalized by the total number of user N. Obviously $\overline{C}_{\text{norm}}(\rho, N)$ is an decreasing function of N, that's why N can not be arbitrarily increased. We are interested in how $\overline{C}_{\text{norm}}(\rho, N)$ grows as N increases and how randomization of N affects the system. Following the method we study ergodic capacity, we first check its monotonicity. Unfortunately, it is mathematically untrackable to directly take the n^{th} derivative of $\overline{C}_{\text{norm}}(\rho, N)$. Thanks to the celebrated properties of *c.m.* function [66], we can show that it is a *c.m.* function.

It has been shown in (2.9) that $\overline{C}(\rho, N)$ has a completely monotonic derivative with respect to N. According to the property of *c.m.* function, $\overline{C}_{\text{norm}}(\rho, N)$ is a completely monotonic function of N. This implies that $\overline{C}_{\text{norm}}(\rho, N)$ is a convex decreasing function of N. Applying Jensen's inequality for convex functions, we have

$$\mathrm{E}_{\mathcal{N}}\left[\overline{C}_{\text{norm}}(\rho, \mathcal{N})\right] \geq \overline{C}_{\text{norm}}(\rho, \lambda). \tag{2.22}$$

where λ is the mean value of random variable $\mathcal{N}$. Therefore, unlike the ergodic capacity metric, randomization of N will always help the average ergodic capacity per user of a multi-user system. To find out how much the normalized capacity under random N outperforms it under deterministic N, we will use the following theorem which has been proved in [67].

Theorem 5. *Let $f(x)$ be c.m. and regularly varying at $x = \infty$ and consider $E_{\mathcal{X}}[f(\mathcal{X})]$, where $\mathcal{X}$ is a Poisson distributed random variable with mean λ. Then,*

$$E_{\mathcal{X}}[f(\mathcal{X})] = f(\lambda) + O\left(f(\lambda)/\lambda\right) \tag{2.23}$$

as $\lambda \to \infty$.

This theorem is also referred as tightness in the Jensen's inequality. Since $\overline{C}_{\text{norm}}(\rho, N)$ is *c.m.*, by using Theorem 5, $\mathrm{E}_{\mathcal{N}}\left[\overline{C}_{\text{norm}}(\rho, \mathcal{N})\right]$ achieves tightness in the Jensen's inequality when $\mathcal{N}$ is Poisson distributed. Consequently, we have,

$$\mathrm{E}_{\mathcal{N}}\left[\overline{C}_{\text{norm}}(\rho, \mathcal{N})\right] = \overline{C}_{\text{norm}}(\rho, \lambda) + O\left(\overline{C}_{\text{norm}}(\rho, \lambda)/\lambda\right), \tag{2.24}$$

as $\lambda \to \infty$

Similar to ergodic capacity, (2.24) shows that the difference between the capacity per user averaged across the user distribution and its value at the mean of the users vanishes as λ tends to ∞.

To apply Theorem 5 we require $\overline{C}_{\text{norm}}(\rho, N)$ to be *c.m.* and regularly varying. We have already shown that it is always a completely monotonic function in N. Next we will show that $\overline{C}_{\text{norm}}(\rho, N)$ also satisfies regularly varying condition. To do this, we first rewrite $\overline{C}_{\text{norm}}(\rho, N)$ expression as,

$$\begin{aligned}\overline{C}_{\text{norm}}(\rho, N) &= \int_0^\infty \frac{\log(1+\rho x)}{N} dF_\gamma^N(x) \\ &= \left.\frac{\log(1+\rho x)F_\gamma^N(x)}{N}\right|_{x=0}^{x=\infty} - \int_0^\infty \frac{\rho}{(1+\rho x)N} F_\gamma^N(x)dx \end{aligned} \quad (2.25)$$

and define $B(\rho x) = -\frac{\rho}{(1+\rho x)N}$. Now setting $u = -\log(F_\gamma(x))$, and integrating by substitution we have,

$$\begin{aligned}\overline{C}_{\text{norm}}(\rho, N) &= \left.\frac{\log(1+\rho x)F_\gamma^N(x)}{N}\right|_{x=0}^{x=\infty} + \rho\int_0^\infty B(\rho x)e^{N\log(F_\gamma^N(x))}dx \\ &= \left.\frac{\log(1+\rho x)F_\gamma^N(x)}{N}\right|_{x=0}^{x=\infty} + \int_0^\infty \rho\frac{B(\rho F_\gamma^{-1}(e^{-u}))}{f_\gamma(F_\gamma^{-1}(e^{-u}))}e^{-uN}du \end{aligned} \quad (2.26)$$

The first term in (2.26) obviously regularly vary at 0. For the second term, under Rayleigh fading channel, $F_\gamma(x) = 1 - e^{-x}$. Then we have,

$$t(u) = \frac{B(\rho F_\gamma^{-1}(e^{-u}))}{f_\gamma(F_\gamma^{-1}(e^{-u}))} = \frac{\rho^{-u}}{N[1-\rho\log(1-e^{-u})](1-e^{-u})} \quad (2.27)$$

which satisfy $\lim_{u\to 0} t(ku)/t(u) = k^{-2}$, therefore proving the regular variation of $t(u)$ near its origin. It can been seen that $\overline{C}_{\text{norm}}(\rho, N)$ can be represented as the Laplace transform of $t(u)$. Using Theorem 2, $\overline{C}_{\text{norm}}(\rho, N)$ can be shown to be a regularly varying function of N as $N \to \infty$. To sum up, $\overline{C}_{\text{norm}}(\rho, N)$ is both *c.m.* and regularly varying at ∞, then its Jensen's inequality is tight.

2.4.2. Optimal Distribution of Number of Users

Since we have shown that the ergodic capacity for different distributions of $\mathcal{N}$ can be also ordered and follow the same order of $\mathcal{N}$, it is possible to find a proper user distribution under which $\overline{C}_{\text{norm}}(\rho, N)$ can be improved. Assuming that $\mathcal{N}$ is an arbitrary non-negative discrete random variable over an interval $[a, b]$ and the mean value λ is known, we have the following theorem:

Theorem 6. *In LT ordering sense, the PDF of $\mathcal{N}$ that has the smallest order has only two values at a and b, which is termed as the optimal PDF.*

Proof. To prove the theorem is equivalent to solve the following optimization problem:

$$\begin{aligned} \text{maximize} \quad & \sum_{i=a}^{b} t^i p_i \\ \text{subject to} \quad & \sum_{i=a}^{b} i p_i = \lambda \\ & \sum_{i=a}^{b} p_i = 1 \end{aligned}$$

Here we apply the equivalent interpretation of LT ordering in terms of PGF of different distributions. Assuming that the optimal PDF have two probability value p_1 and p_2, we have:

$$\begin{aligned} p_1 + p_2 &= 1 \\ a p_1 + b p_2 &= \lambda \end{aligned} \tag{2.28}$$

Assuming that a general discrete probability distribution have probability values q_i,

$i \in \{a, a+1, \ldots, b\}$, we have:

$$\sum_{i=a}^{b} q_i = 1$$
$$\sum_{i=a}^{b} i q_i = \lambda \tag{2.29}$$

From (2.28), we substitute $p_1 = 1 - p_2$ into the second equation and get

$$a(1 - p_2) + b p_2 = \lambda \Longrightarrow (b - a) p_2 = \lambda - 1. \tag{2.30}$$

Similarly we substitute $q_a = 1 - \sum_{i=a+1}^{b} q_i$ into the second equation and get

$$a(1 - \sum_{i=a+1}^{b} q_i) + \sum_{i=a+1}^{b} i q_i = \lambda \Longrightarrow \sum_{a+1}^{b} (i - a) q_i = \lambda - 1. \tag{2.31}$$

By substituting p_1 and q_a, the PGFs of the optimal PDF and the general case can be expressed respectively as:

$$t^a(1 - p_1) + t^b p_2 = (t^b - t^a) p_2 + t^a \tag{2.32}$$

$$t^a(1 - \sum_{i=a+1}^{b} q_i) + \sum_{i=a+1}^{b} t^i q_i = \sum_{i=a+1}^{b} (t^i - t^a) q_i + t^a \tag{2.33}$$

Since (2.30) is equal to (2.31), we have:

$$(b - a) p_2 = \sum_{i=a+1}^{b} (i - a) q_i \Longrightarrow p_2 = \frac{\sum_{i=a+1}^{b} (i - a) q_i}{b - a} \tag{2.34}$$

Now the problem is equivalent to compare $(t^b - t^a) p_2 + t^a - \sum_{i=a+1}^{b} (t^i - t^a) q_i - t^a$ with 0. Substituting (2.34) into the (2.32) and (2.33) we have:

$$(t^b - t^a) p_2 + t^a - \sum_{i=a+1}^{b} (t^i - t^a) q_i - t^a \Longrightarrow (t^b - t^a) \frac{\sum_{i=a+1}^{b} (i - a) q_i}{b - a} - \sum_{i=a+1}^{b} (t^i - t^a) q_i. \tag{2.35}$$

We expand (2.35) into $b - a + 1$ terms and pick up an arbitrary pair of counter terms from them:

$$\frac{t^b - t^a}{b - a}(i - a) - (t^i - t^a) = \frac{i - a}{b - a} - \frac{t^i - t^a}{t^b - t^a} \tag{2.36}$$

Since $b - i > t^b - t^i$ and $i < b$, from basic fraction knowledge we know

$$\frac{i-a}{b-a} - \frac{t^i - t^a}{t^b - t^a} < 0 \tag{2.37}$$

This shows that the PGF of the optimal PDF we proposed is larger than any general discrete random variable over the interval $[a, b]$ with the same mean value λ, which implies that the "two-value" PDF is the optimal PDF in LT ordering sense, completing the proof. □

This section illustrated the behaviour of a new performance metric, capacity per user. Theorem 6 gives us a new perspective in wireless communication systems design. As it is mentioned in Section 2.4 that there is a conflict between $\overline{C}(\rho, N)$ and $\overline{C}_{\text{norm}}(\rho, N)$ since one is increasing and the other is decreasing as N grows. When the system has specific requirements of both sum rate and capacity per user, it is capable to design the system by following two steps: 1)number of users can be increased to improve the sum rate performance; 2)by scheduling the activities of clients in the cell, the number of users can obey the optimal distribution to increase the ergodic capacity per user.

2.5. Outage Capacity and Outage Probability

2.5.1. Outage Capacity

In Section 2.3.1, fast ergodic fading channel was discussed so that ergodic capacity metric has been studied. However, in practical systems, the channel might not change rapidly enough during a coherence time, under which we can not directly average the capacity over the fading. In this section, we will look at the situation when the channel gain is random but remains constant for all time. This models the slow fading scenario where the outage capacity is considered. This is also called the quasi-static scenario [10].

Conditional on a realization of the channel gain, this is equal to an AWGN channel with received SNR $\rho|h|^2$. This channel can support a reliable communication with the maximum rate at $\log(1 + |h|^2\rho)$ bits/s/Hz. This quantity is a random variable because it is a function of the random channel gain h. According to the channel coding theorem, if transmitter encodes data at a rate R bits/s/Hz, which satisfy $\log(1 + |h|^2\rho) < R$, then no matter what kind of channel code used by the transmitter, the decoding error probability cannot be arbitrarily small. In this situation, the system is said to be in outage, and the outage probability is

$$p_{\text{out}}(R) = \mathbb{P}\{\log(1 + |h|^2\rho) < R\} \tag{2.38}$$

Thus, the best that the transmitter can do is to encode the data assuming that the channel gain is strong enough to support the desired rate R. Reliable communication can be achieved whenever that happens, and outage occurs otherwise. So far it is similar to the AWGN channel, but they have a conceptual difference. In the AWGN channel, one can send data at any rate less than the channel capacity while making the error probability as small as possible. This cannot be done for the slow fading channel, unless the probability of the channel in deep fade is non-zero. Thus, strictly speaking, the Shannon capacity of the slow fading channel is zero.

An alternative performance metric is the outage capacity, C_{out}. This is the highest rate one can transmit and meanwhile the outage probability is less than ϵ. Solving $p_{\text{out}}(R) = \epsilon$ in (2.38) we have

$$C_{\text{out}}(\rho, \epsilon, N) = \log\left(1 + \rho \overline{F}_{\gamma}^{-1}(1-\epsilon)\right) \tag{2.39}$$

where $\overline{F}_{\gamma}$ is the complementary CDF of the chosen user, i.e., $\overline{F}_{\gamma}(x) = 1 - F_{\gamma}(x)$. Since we are interested in F as a function of the number of users N. So we assume that ϵ is a predefined constant, and we can drop the variable ϵ in $C_{\text{out}}(\rho, \epsilon, N)$ and interpret it as $C_{\text{out}}(\rho, N)$.

From (2.39) we have,

$$C_{\text{out}}(\rho, N) = \log\left(1 + \rho \cdot F_{\gamma}^{-1}(\epsilon^{\frac{1}{N}})\right), \tag{2.40}$$

where $F_{\gamma}(x)$ is the CDF of i.i.d. channel gain over all users. It is mathematically intrackable to study the complete monotonicity of $C_{\text{out}}(\rho, N)$ by taking the k^{th} derivative. However, instead of proving the complete monotonicity of $C_{\text{out}}(\rho, N)$, we can show that outage capacity is not completely monotonic in N over the whole range of ρ or ϵ. Our approach is evaluating the second order derivative of $C_{\text{out}}(\rho, N)$ with respect to N at a certain value of ϵ and N under a specific fading scenario. Assuming that $\epsilon = 0.001$ and fading is Rayleigh, we have,

$$C_{\text{out}}(\rho, N) = \log\left(1 + \rho \cdot \log(1 - \epsilon^{\frac{1}{N}})\right), \tag{2.41}$$

and take the second derivative of (2.41) with respect to N we have,

$$\left.\frac{d^2C_{\text{out}}(\rho,N)}{dN^2}\right|_{\epsilon=0.001} = -\frac{47.7171\times 0.001^{2/N}}{(1-0.001)^2N^4(1-\log(1-0.001^{1/N}))^2} + \frac{47.7171\times 0.001^{2/N}}{(1-0.001)^2N^4(1-\log(1-0.001^{1/N}))} + \frac{47.7171\times 0.001^{1/N}}{(1-0.001)N^4(1-\log(1-0.001^{1/N}))} - \frac{13.8155\times 0.001^{1/N}}{(1-0.001)N^3(1-\log(1-0.001^{1/N}))}. \tag{2.42}$$

Equation (2.42) equals to 0.04 and -0.004 at $N = 2$ and $N = 4$ respectively. Consequently, the second derivative of $C_{\text{out}}(\rho, N)$ is not always positive or negative over N for all values of ϵ, which implies that $C_{\text{out}}(\rho, N)$ is not *c.m.* or has a *c.m.d.*. Furthermore, we can state that $C_{\text{out}}(\rho, N)$ is neither convex nor concave of N, so that randomization of N does not always increase or reduce $C_{\text{out}}(\rho, N)$.

2.5.2. Outage Probability

In Section 2.5.1, we study that outage capacity is neither a *c.m.* function of N nor having a *c.m.d.* in N. In this section, we will prove that *outage probability* is a completely monotonic function of N, which will be increased by randomizing the number of users.

From (2.38), the outage probability can be expressed,

$$\begin{aligned}\epsilon(N) &= \mathbb{P}\{\log(1+|h|_*^2\rho) < R\} \\ &= \mathbb{P}\left\{\gamma^* < \frac{2^R-1}{\rho}\right\} \\ &= F_{\gamma^*}\left(\frac{2^R-1}{\rho}\right) = F_\gamma^N\left(\frac{2^R-1}{\rho}\right)\end{aligned} \tag{2.43}$$

The k^{th} derivative of (2.43) can be written as,

$$\frac{d^k \epsilon(N)}{dN^k} = F_\gamma^N \left(\frac{2^R - 1}{\rho}\right) \cdot \left[\log\left(F_\gamma\left(\frac{2^R - 1}{\rho}\right)\right)\right]^k \tag{2.44}$$

In (2.44), since $\log\left(F_\gamma\left(\frac{2^R-1}{\rho}\right)\right) \leq 0$ we have,

$$(-1)^k \frac{d^k \epsilon(N)}{dN^k} \geq 0, \qquad k \geq 0. \tag{2.45}$$

which implies $\epsilon(N)$ is *c.m.*.

Using (2.45) for $k = 1$ and $k = 2$, it is seen that $\epsilon(N)$ is convex decreasing function of N. For when the number of users in the system is random, by applying Jensen's inequality for convex functions, we have,

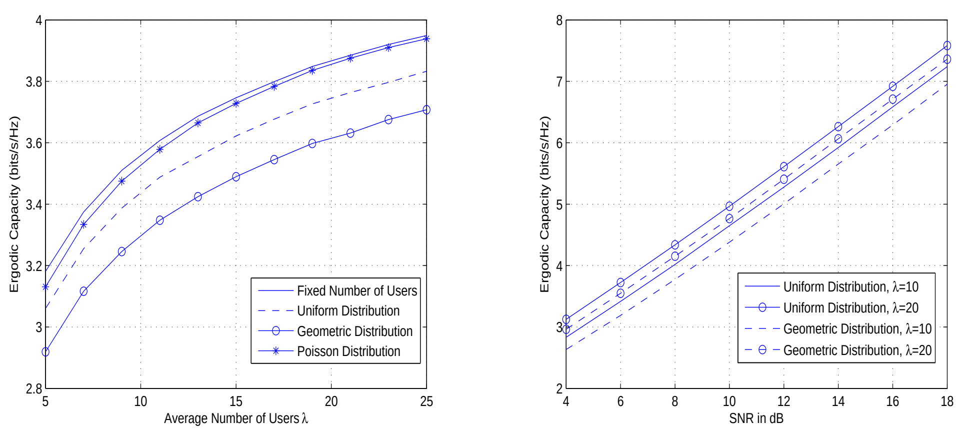
$$\mathrm{E}_{\mathcal{N}}\left[\epsilon(\mathcal{N})\right] \geq \epsilon(\lambda), \tag{2.46}$$

where $\lambda = \mathrm{E}(\mathcal{N})$. Therefore, randomization of the number of users will always reduce the outage probability performance of a multi-user diversity system since we hope the outage probability to be as small as possible.

2.6. Simulation Results

In this section, an uplink multi-user system where both the BS and users having a single antenna is considered. Using Monte-Carlo simulations, the ergodic capacity, outage capacity, capacity per user and optimal PDF are simulated to corroborate our analytical results. The wireless channel between any user and BS is assumed to be not only Rayleigh faded but also Rician faded.

In Figure 2.2(a), the ergodic capacity is plotted against λ for the random cases and $N = \lambda$ for the deterministic case. Following the result in Section 2.3.1, it is seen that the capacity of the deterministic number of users system is the highest while for all distribution of $\mathcal{N}$. This also reflects the fact that all these distributions can be ranked in the LT ordering sense. The average SNR is assumed to be 6 dB and the Rayleigh parameter is set to be 1. In Figure 2.2(b), the ergodic capacity is plotted against the channel gain (SNR) in dB. Similar result that ergodic capacity performances under different distributions follow their stochastic ordering can be observed. Average number of users $\lambda = 10$ and $\lambda = 20$ are simulated respectively with the Rayleigh faded parameter 1.

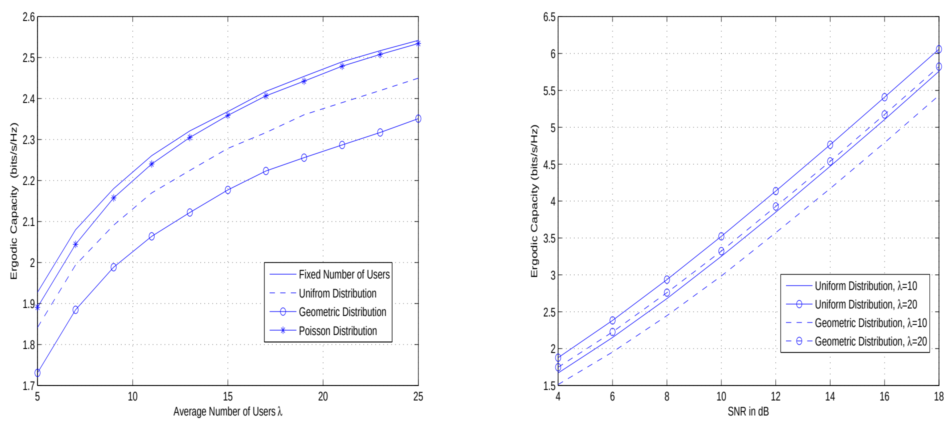


(a) Ergodic Capacity vs. λ

(b) Ergodic Capacity vs. SNR

Figure 2.2. Ergodic Capacity under Different User Distributions (Rayleigh Fading).

In Figure 2.3(b) and 2.3(b), we also simulate the ergodic capacity over average number of users and average SNR respectively under the Rician fading scenario. In this figure, fading parameter $K = 3$ which implies that the ratio of the power in the *Light-of-Sight* (LOS) component to the power in the non-LOS components is 3. Randomization of user number will reduce the ergodic capacity performance in the Rician fading case, which is exactly the same as in Rayleigh fading case. This is because the concavity of ergodic capacity as a function of the user number doesn't depend on the type of fading scenarios. To notice that with the same average SNR, Rayleigh fading case outperforms Rician fading case since because of the LOS component, the Rician fading distribution is less random and has a lighter tail than the Rayleigh distribution.



(a) Ergodic Capacity vs. λ

(b) Ergodic Capacity vs. SNR

Figure 2.3. Ergodic Capacity under Different User Distributions (Rician Fading).

In Section 2.3, we showed that Poisson distributed random variables and negative binomial distributed random variables can be LT ordered, which would also order there corresponding average error rate and capacity performance when averaged across the user distributions. In Figure 2.4, it can be seen that ergodic capacity metrics follow the LT order of the user distributions, and increase as the number of users is incremented. Since negative binomial distribution can be considered as a compound Poisson distribution, changing the negative binomial parameter p can control its approximation to Poisson distribution. In Figure 2.4, it can be observed that for small value p, ergodic capacity under negative distribution is quite close to the Poisson case.

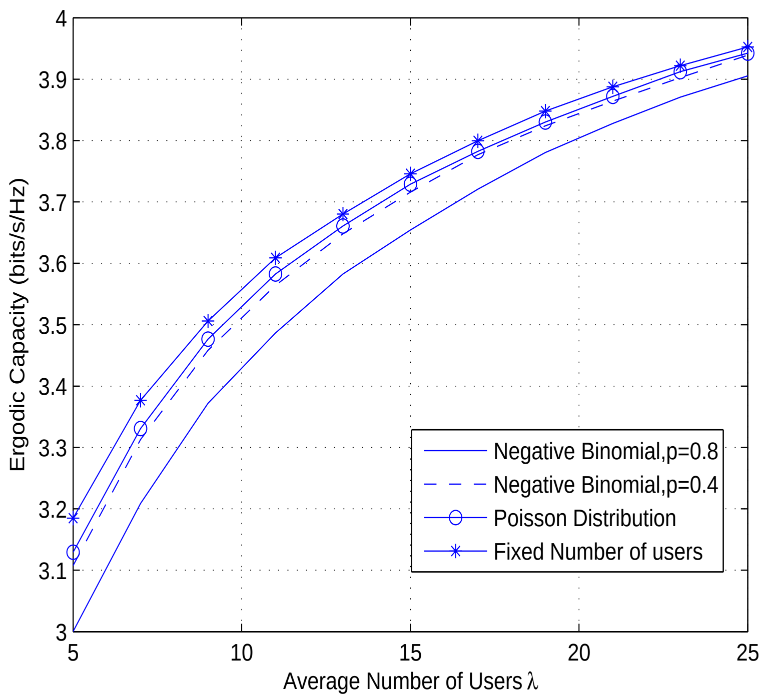
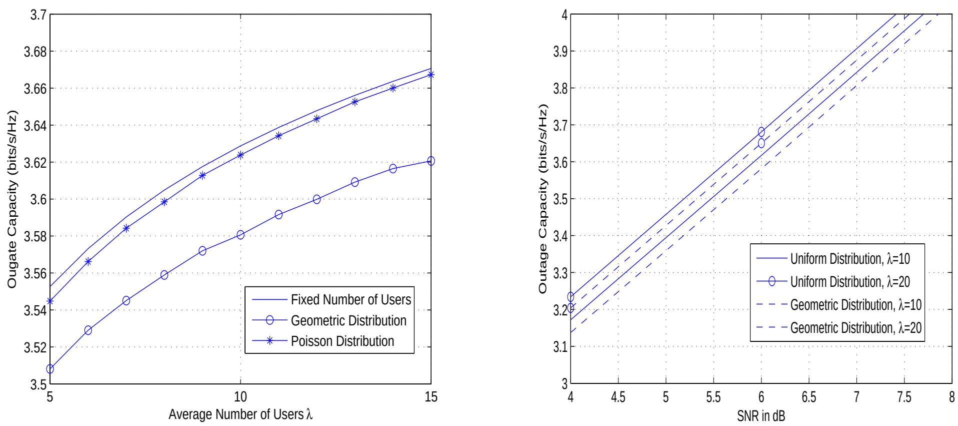


Figure 2.4. Ergodic Capacity under Poisson and Negative Binomial $\mathcal{N}$.

In Figure 2.5(a) and 2.5(b), outage capacity under different user distributions are simulated against mean number of users λ and SNR respectively. In Figure 2.5(a), average SNR is still 6 dB and Rayleigh faded parameter is also set to be 1. In Section 2.5.1, outage capacity is not always convex or concave function of N. However, from the numerical results that for large value of average SNR and small outage probability in Rayleigh fading case, randomization of number of users will reduce the outage capacity. Figure 2.5(a) and 2.5(b) verify our numerical analysis.



(a) Outage Capacity vs. λ (b) Outage Capacity vs. SNR

Figure 2.5. Outage Capacity under Different User Distributions.

In Figure 2.6, outage capacity under Poisson and negative binomial distributed users are simulated and compared with the deterministic number of users where $N = \lambda$. In the simulate the outage capacity in a high SNR Rayleigh fading scenario. Similar results to the ergodic capacity can be observed. Randomization of N will deteriorate the outage capacity performance, and the outage capacity of negative binomial user distribution get closer to the Poisson user distribution as parameter p decreases in this case.

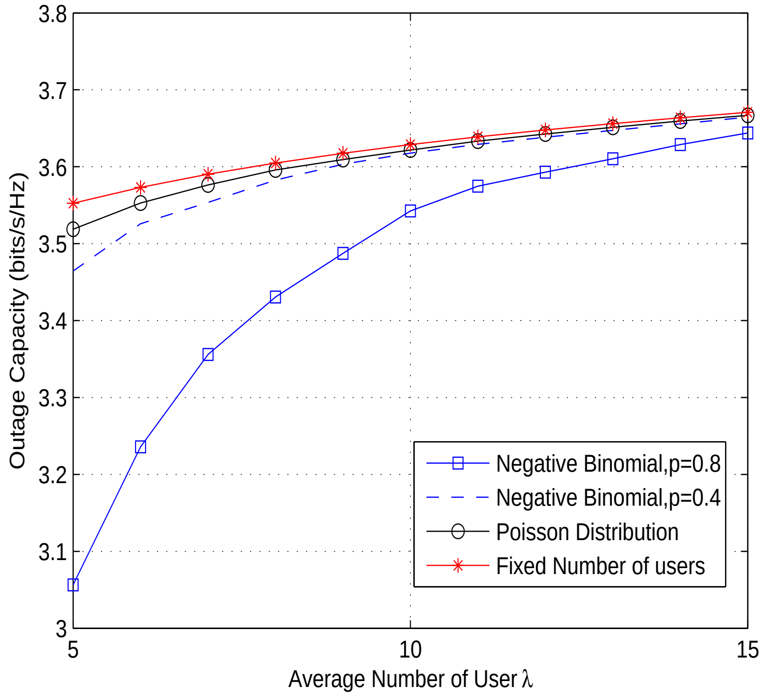
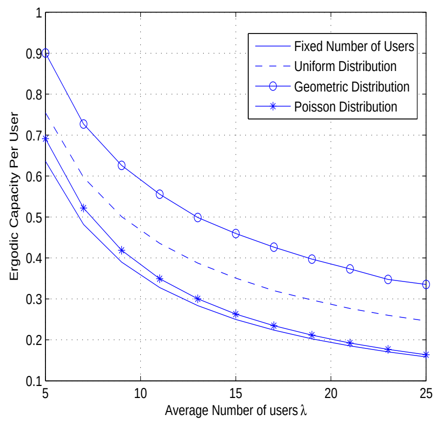


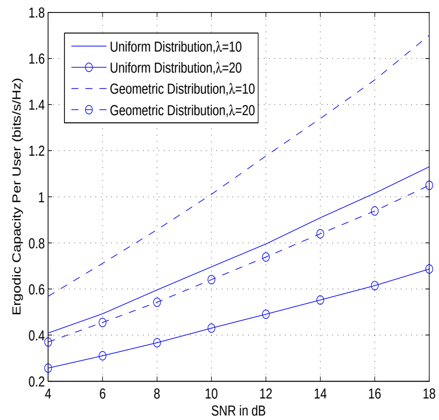
Figure 2.6. Outage Capacity under Poisson and Negative Binomial $\mathcal{N}$.

In Section 2.4, we investigate a new metric named ergodic capacity per user, which is the ergodic capacity normalized by the total number of users. It is shown that this metric is a decreasing convex function, which implies that randomization will make it increasing. Moreover, ergodic capacity per user under different user distributions follow the opposite LT order compared with ergodic capacity. In Figure 2.7(a) and 2.7(b), ergodic capacity per user under different distributions are simulated. In Figure 2.7(a), the average SNR is 6 dB and in Figure 2.7(b) the Rayleigh faded parameter is 1.

In Figure 2.8, ergodic capacity per user under Poisson and negative binomial distribution are simulated. Average SNR is assumed to be 6 dB and Rayleigh faded



(a) Ergodic Capacity Per User vs. λ



(b) Ergodic Capacity Per User vs. SNR

Figure 2.7. Ergodic Capacity Per User under Different User Distributions.

parameter is 1. Unlike the ergodic capacity, normalized capacity under negative binomial $\mathcal{N}$ outperforms the Poisson case. The similarity is the smaller native binomial parameter is, the closer the binomial performance gets to the Poisson case.

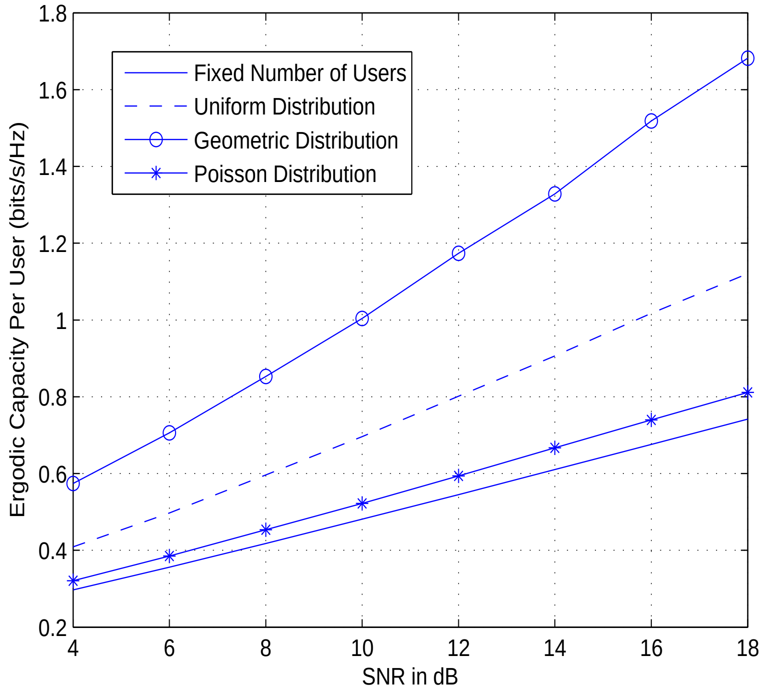


Figure 2.8. Ergodic Capacity Per User under Poisson and Negative Binomial $\mathcal{N}$.

In Section 2.4.2, with the knowledge of LT ordering and probability generating function, we derive the optimal PDF of $\mathcal{N}$ for normalized capacity. Assuming that the random variable $\mathcal{N}$ is in an arbitrary interval and the mean value is known, its optimal PDF behaves like a "two-value" distribution with values at the starting and ending point of the interval. In other words, this optimal PDF has the largest probability generating function over the interval. In Figure 2.9, we form this problem into an optimization problem and use CVX tool box in Matlab to solve it. $\mathcal{N}$ is assumed to be distributed over $[20, 50]$, $[25, 45]$ and $[30, 40]$ with mean value 30, 30 and 37 respectively. Simulation results verify our analytical derivation.

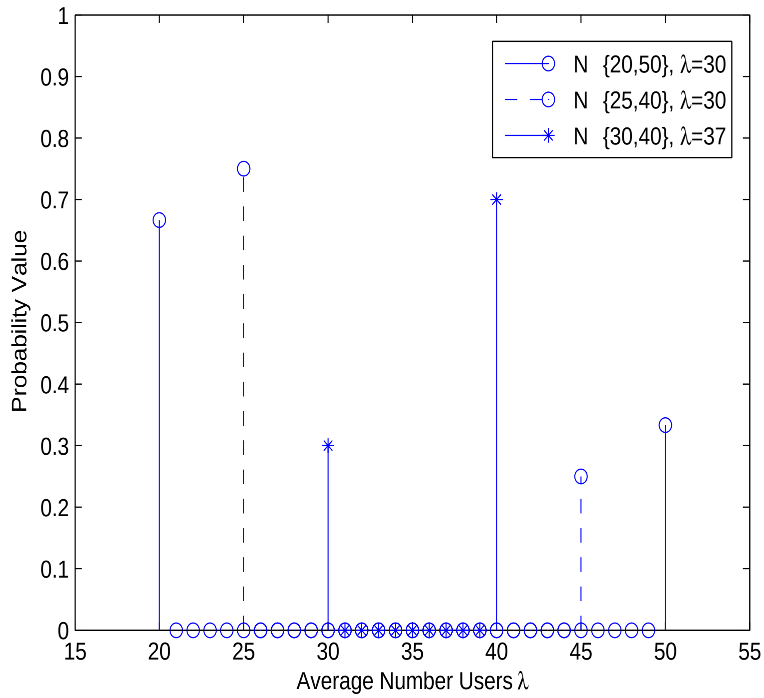


Figure 2.9. Optimal Probability Mass Function

2.7. Summary

We began this chapter by first introducing an uplink multi-user diversity, in which each user is equipped with a single antenna. This is then followed by a review of the properties of ergodic capacity in [43], in which it has been stated that the randomization of the user number will reduce the ergodic capacity. The ergodic capacity under different user distributions will follow their corresponding LT order. Moreover, a special case when $\mathcal{N}$ is Poisson was studied. The scaling law of ergodic capacity with respect to λ in this case obeys the fact that when mean number of users goes to infinity, the difference of the capacity between the random number of users and deterministic users vanishes. Then the results in [43] are extended in this chapter. The similar scaling law with λ can be observed when $\mathcal{N}$ is negative binomial distributed. Then a new performance metric, the ergodic capacity per user, was developed. Ergodic capacity per user stands for the average rate of each user and will be increased as the number of user is randomized, which is opposite to ergodic capacity. This contradiction should be considered when designing wireless communication systems. More specifically, both ergodic capacity and capacity per user can be controlled by arranging the random distribution of the number of users.

CHAPTER 3

AVERAGE BER PERFORMANCE IN MULTI-USER DIVERSITY SYSTEMS

3.1. Background

In digital transmission, the number of bit errors is the number of received bits of a data stream over a communication channel which is altered due to noise, interference, distortion, bit synchronization errors or other impairment of the channel. The bit error rate or bit error ratio (BER) is defined as the number of bit errors divided by the total number of transferred bits during a desired time interval. Average BER is a unit-less performance measure, often expressed as a percentage. Along with the capacity, average BER is a critical measurement to judge whether a channel can offer reliable communication. By using Monte Carlo simulation, BER can be calculated numerically through computer. If a channel model and data source model is assumed, it can also be studied analytically.

The definition and the general mathematical expression of the average error rate of a multi-user system with deterministic number of users N and average SNR ρ is given by,

$$\overline{\mathrm{P}}_e(\rho, N) = \int_0^\infty \mathrm{P}_e(\rho x) dF_\gamma^N(x) \tag{3.1}$$

where $\mathrm{P}_e(\rho x)$ is the instantaneous error rate over an AWGN channel for an instantaneous SNR ρx of the best user and $\mathrm{P}_e(\rho x)$ is the instantaneous error rate. For example, for binary phase-shift keying (BPSK) and quadrature phase-shift keying (QPSK) modulation scheme, $\mathrm{P}_e(\rho x)$ yields $\mathrm{P}_e(\rho x) = Q(\rho x)$, where $Q(x)$ is called the *right-tail probability* and is defined as $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}t^2)dt$. Combining with Q function: $Q(x) \leq (1/2)e^{-x^2/2}$, instantaneous error rate is often assumed to have the form $\mathrm{P}_e(\rho x) = \alpha e^{-\eta \rho x}$, where α and η can be chosen to capture different modulations [4]. For binary differential phase-shift keying (DPSK) this is exact with $\alpha = 0.5$ and $\eta = 1$. For Gray-coded M-level quadratic amplitude modulation (M-QAM), $\alpha = 0.2$ and $\eta = 1.5/(M-1)$ yields an error rate within 1 dB for $M \geq 4$ [4]. Sometimes, it is also written as $\mathrm{P}_e(\rho x) = \alpha Q(\sqrt{\eta\rho x})$. Since we are interested in the asymptotic behaviour of BER, we chose the former approximation of $\mathrm{P}_e(\rho x)$, which will leads to some close form expression. We will also consider (3.1) with N being a real number. To represent (3.1) in terms of the CDF $F_\gamma(x)$, rather than the probability density function (PDF), we expressed it in the way of a Stieltjes integral [68] even though it can also be expressed in terms of the PDF $f_\gamma(x)$ using $dF_\gamma^N(x) = NF_\gamma^{N-1}(x)f_\gamma(x)$.

3.2. Average Bit Error Rate

In this section, we first introduce the results in the paper [43] by proving that the average error rate of a multi-user diversity system, with a deterministic number of users N, is a *c.m.* function of N, under general conditions. This will be used to infer the behaviour of the average error rate of the multi-user system when a random number of users is considered in the later of this section. Then we will extend the results from the Poisson $\mathcal{N}$ to negative binomial $\mathcal{N}$.

3.2.1. Representation of Average BER

We begin by introducing that $\overline{\mathrm{P}}_e(\rho, N)$ is a completely monotonic function of N not only for $\mathrm{P}_e(\rho x)$ in the forms of exponential function and Q function, but for any arbitrary form of instantaneous error rate function as well. In other words, the only assumption of $\mathrm{P}_e(\rho x)$ is decreasing in x for any $\rho > 0$. After integrating (3.1) by parts, the k^{th} derivative of $\overline{\mathrm{P}}_e(\rho, N)$ can be written as,

$$\frac{\partial^k \overline{\mathrm{P}}_e(\rho, N)}{\partial N^k} = \rho \int_0^\infty B(\rho x) F_\gamma^N(x) \left[\log\left(F_\gamma(x)\right)\right]^k dx. \tag{3.2}$$

where we define $B(x) = -d\mathrm{P}_e(x)/dx$. Since $\mathrm{P}_e(\rho x)$ is decreasing, and $\log\left(F_\gamma(x)\right) \leq 0$ we see that (3.2) satisfies the definition in (2.4). In particular, $\overline{\mathrm{P}}_e(\rho, N)$ being a *c.m.* function means that (3.2) is negative for $k = 1$ and positive for $k = 2$, and consequently $\overline{\mathrm{P}}_e(\rho, N)$ is a convex decreasing function of N. For the case that the number of users in the system is random, by applying Jensen's inequality for convex functions, we have,

$$\mathrm{E}_{\mathcal{N}}\left[\overline{\mathrm{P}}_e(\rho, \mathcal{N})\right] \geq \overline{\mathrm{P}}_e(\rho, \lambda), \tag{3.3}$$

where $\lambda := \mathrm{E}[\mathcal{N}]$. Therefore, randomization of the number of users always deteriorates the average error rate performance of a multi-user diversity system.

The only fact we need to establish the complete monotonicity of $\overline{\mathrm{P}}_e(\rho, N)$ as a function of N, is that the instantaneous error rate $\mathrm{P}_e(\rho x)$ in (3.1) is a decreasing function of x for $\rho > 0$, which obviously holds for any multi-user system. Similar to the capacity metric, this *c.m.* property can be used to stochastically order user distributions as we did in Section 2.2.2.

3.2.2. Average BER for Poisson Number of Users

Following the approach in our study of the capacity metric, we have shown that average error rate is a completely monotonic function of N. According to Theorem 1, once the probability distributions of N are LT ordered, their BER performance can also be ordered for any value of ρ. From (3.3) we saw that randomization always deteriorates performance, however, similar to ergodic capacity, the reference [43] states that for large average number of users it should approximately yield the same performance as the deterministic case. This amounts to the tightness of Jensen's inequality for the Poisson users case. Moreover, reference [43] provides sufficient conditions for Jensen's inequality involving $\overline{\mathrm{P}}_e(\rho, \lambda)$ to be tight. Using Theorem 5, we have

$$\mathrm{E}_{\mathcal{N}}\left[\overline{\mathrm{P}}_e(\rho, \mathcal{N})\right] = \overline{\mathrm{P}}_e(\rho, \lambda) + O\left(\overline{\mathrm{P}}_e(\rho, \lambda)/\lambda\right) \tag{3.4}$$

as $\lambda \to \infty$. Equation (3.4) shows that as $\lambda \to \infty$, the gap between the error rate averaged across the user distribution and the error rate evaluated at the average number of users vanishes. This verifies the fact that for sufficiently large λ, the performance of the multi-user diversity systems with random number of users will closely approach to the performance of the multi-user diversity systems with a deterministic number of users with the number of users equal to λ.

Reference [43] shows that for the conclusions of (3.4) to hold (i.e., Jensen's inequality to be asymptotically tight), the CDF of the single-user channel $F_\gamma(x)$, and the error rate expression $\mathrm{P}_e(\rho x)$ have to jointly satisfy the regular variation condition given in Theorem 5. It has been examined that this condition holds for commonly assumed instantaneous error rates $\mathrm{P}_e(\rho x)$ with γ_n being exponentially distributed (i.e., under Rayleigh faded user channels). Also, both the choices of $\mathrm{P}_e(\rho x) = \alpha e^{-\eta\rho x}$ and $\mathrm{P}_e(\rho x) = \alpha Q(\sqrt{\eta\rho x})$ are proved regular variation as $N \to \infty$. Consequently, it is illustrated in [43] that when $\mathrm{P}_e(\rho x)$ is in the form of $\alpha e^{-\eta\rho x}$

or $\alpha Q(\sqrt{\eta\rho x})$ and the fading is Rayleigh (i.e. channel gain is exponential), the difference in error rate performance of a multi-user diversity system with a random number of users averaged over the number of users distribution and of a deterministic number users approaches zero for sufficiently large λ, as in (3.4).

3.2.3. Special Cases: Rayleigh Faded Channel

In Section 3.2.2, the average BER under Poisson $\mathcal{N}$ is well investigated and it has been stated that the it achieves tightness in Jensen's inequality. In this section, as we discussed in the capacity chapter, we will take a look at the special case that when the channel experiences Rayleigh fading, and under which, the closed form of BER behaviour with negative binomial N will be derived and compared to the Poisson case.

Reference [43] has already given the closed form of the Poisson case. In Section 2.3.1, the CDF of the channel gain of the best user chosen from random Poisson set of users has been derived. It is important to note here that unlike the usage of the Gumbel distribution in [42] where a deterministic but large number of users considered, in (2.11) the parameter λ is finite and hence (2.11) is not an asymptotic result in λ. The PDF of the channel gain of the best user chosen from a random set of users can be expressed as,

$$f_{\gamma^*}(x) = \lambda e^{-x} e^{-\lambda e^{-x}} + \delta(x) e^{-\lambda} \quad x \geq 0. \tag{3.5}$$

Assuming that the error rate has the form of $\mathrm{P}_e(\rho x) = \alpha e^{-\eta\rho x}$ as we mentioned, the average error rate achieved by the system across time, can be expressed as,

$$\mathrm{E}_{\mathcal{N}}\left[\overline{\mathrm{P}}_e(\rho,\mathcal{N})\right] = \lambda\alpha\int_0^{\infty} e^{-\eta\rho x}e^{-x}e^{-\lambda e^{-x}}dx + \alpha\int_0^{\infty}\delta(x)e^{-\lambda}e^{-\eta\rho x}. \tag{3.6}$$

By setting $y = \lambda e^{-x}$ and integrating by substitution, it can be expressed as,

$$\begin{aligned}\mathrm{E}_{\mathcal{N}}\left[\overline{\mathrm{P}}_e(\rho,\mathcal{N})\right] &= \alpha\int_0^{\infty}\frac{y}{\lambda}^{\eta\rho}e^{-y}dy + \alpha e^{-\lambda} \\ &= \alpha\lambda\cdot\gamma(\eta\rho+1,\lambda) + \alpha e^{-\lambda},\end{aligned} \tag{3.7}$$

where $\gamma(s,x)$ is the lower incomplete gamma function which is defined in [68] as:

$$\gamma(s,x) = \int_0^{x} t^{s-1}e^{-t}dt. \tag{3.8}$$

Obviously, the average error rate is a decreasing function of λ, as expected.

To observe the expression of BER for negative binomial case, we have two methods: 1)we can use (3.6) by substituting the PDF of Poisson case into its negative binomial version; 2)we could directly take an expectation of (3.7) with respect to λ where λ is Gamma distribution. Because of the mathematical intractability of the second methodf, the first choice is preferable. In Section 2.3.1, we derive the CDF of the channel gain of the best user chosen from a negative binomial distributed set of users by taking the expectation of (2.11). Here we take derivative with respect to x so that the PDF of the channel gain of the best user in the negative binomial case can be expressed as:

$$f_{\gamma^*}(x) = \frac{dF_{\gamma^*}(x)}{dx} = rue^{-x}(1+ue^{-x})^{-1-r}, \qquad x > 0. \tag{3.9}$$

where r is the parameter of the negative binomial distribution. Assuming the instantaneous error rate has the form $\mathrm{P}_e(\rho x) = \alpha e^{-\eta\rho x}$, substituting (4.10) into (3.7) we can get:

$$\mathrm{E}_{\mathcal{N}}\left[\overline{P_e}(\rho,\mathcal{N})\right] = \int_0^\infty \alpha e^{-\eta\rho x} r u e^{-x}(1+ue^{-x})^{-1-r} dx$$
$$= \frac{ru\alpha}{1+\eta\rho} \cdot {}_2F_1\left(1+r, 1+\eta\rho, 2+\eta\rho, -u\right) \tag{3.10}$$

where ${}_2F_1\left(1+r, 1+\eta\rho, 2+\eta\rho, -u\right)$ is the hyper geometric function${}_2F_1\left(a,b,c,z\right)$. According to its properties, hyper geometric function${}_2F_1\left(a,b,c,z\right)$ automatically evaluates to exact values, and also can be evaluated at arbitrarily any numerical precision. Since it is not very intuitive to observe the complete monotonicity or convexity by taking the k^{th} derivative to $\mathrm{E}_{\mathcal{N}}\left[\overline{P_e}(\rho,\mathcal{N})\right]$, we can only state that it is an decreasing function of r.

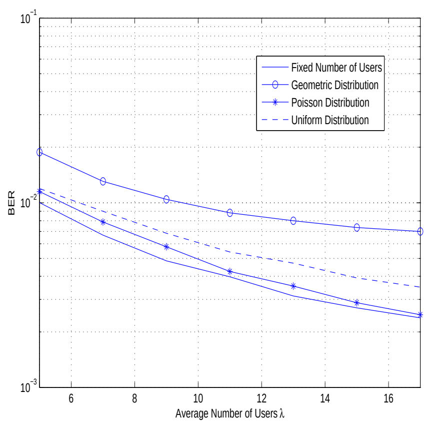
An alternative approach to study the hyper geometric function${}_2F_1\left(a,b,c,z\right)$ is to plot it in some mathematical software, for example Mathematica or Matlab, to see if it is convex. One can change the parameter to check the behaviour of $\mathrm{E}_{\mathcal{N}}\left[\overline{P_e}(\rho,\mathcal{N})\right]$. Moreover, although we can not plot the infinite order derivative of $\mathrm{E}_{\mathcal{N}}\left[\overline{P_e}(\rho,\mathcal{N})\right]$, one can plot the second order derivative of $\mathrm{E}_{\mathcal{N}}\left[\overline{P_e}(\rho,\mathcal{N})\right]$ to check whether it is positive, which implies $\mathrm{E}_{\mathcal{N}}\left[\overline{P_e}(\rho,\mathcal{N})\right]$ is convex. We have already proved in (3.2) that average error over both fading and user distribution under general fading assumption, rate will decrease when total number of users is randomized. This general assumption has no constraint of the type of user distribution. Therefore, although we still can not observe a strict analytical result, software is an intuitive and corroborative evidence of $\mathrm{E}_{\mathcal{N}}\left[\overline{P_e}(\rho,\mathcal{N})\right]$'s convexity. The numerical analysis will be discussed in the simulation section in detail.

It is important to notice that, average error rate and average capacity per user are all convex with respect to the number of users. This implies that the optimal PDF of the ergodic capacity per user in Section 2.4 also applies to average error rate. However, unlike the the normalized capacity, we hope the error rate to be as small as possible. Consequently, the “two-value” PDF of $\mathcal{N}$ introduced in Section 2.4 for the capacity per user is an undesired one for the error rate.

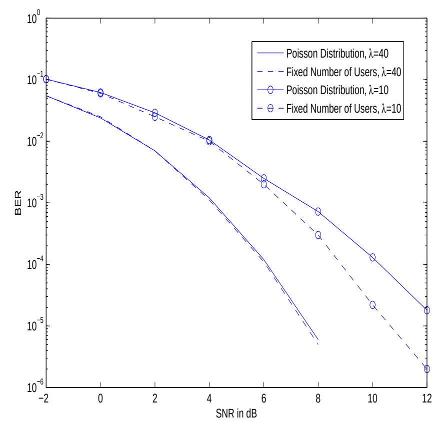
The conflict of ergodic capacity and normalized capacity should be considered when designing wireless communication systems. More specifically, any wireless communication system should have certain requirements on three major metrics as ergodic capacity, normalized capacity and bit error rate. There is a trade off between these metrics. For example, wireless communication systems frequently have a predefined upper bound of the average BER, equivalently an lower bound of total number of users, to ensure the reliable communication for any individual user. In this situation, one can increase the number of users to maximized the ergodic capacity and meanwhile choose a proper user distribution to avoid violating the normalized capacity, since it will decrease as the user numbers increase. This trade off should be taken into account during the designing of the system.

3.3. Simulation Results

In Section 3.2.1, we proved that averaged error rate is a completely monotonic function of N which implies that randomization of N will cause the increasing of BER metric. With the knowledge of LT ordering, different random distributions will order their respective error rate when averaged across the respective user distribution. In Figure 3.1(a) and 3.1(b), BER performance under different distributions are simulated and compared with the case of fixed number of user. It can be seen that error follow their corresponding ordering and average BER performance gets improved as the number of users increases. In Figure 3.1(a), averaged SNR is 0 dB and in both figures the Rayleigh parameter is 1.



(a) Average BER VS. λ



(b) Average BER VS. SNR

Figure 3.1. Average BER under Different User Distributions.

In Section 3.2.3, we showed that error rate under negative binomial and Poisson distribution can be ordered. In Figure 3.2, error rate is plotted against λ for negative binomial $\mathcal{N}$ with different parameter and Poisson $\mathcal{N}$. It can be seen that for small value of p, negative binomial distribution could be fairly close to the Poisson case, but never exceeds it. The averaged SNR is still 0 dB and Rayleigh parameter is 1.

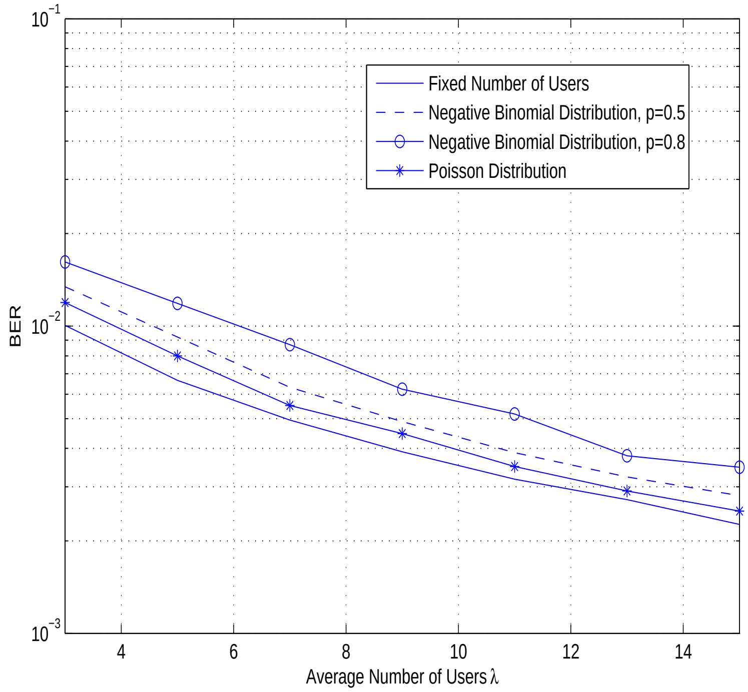


Figure 3.2. Average under Poisson and Negative Binomial $\mathcal{N}$.

3.4. Summary

We began this chapter by introducing the averaged error rate in the paper [43] that the BER in a general multi-user diversity system has a completely monotonic derivative, which implies that, according to the Jensen's inequality, the randomization of the total number of users will reduce the average BER performance. As we did in Chapter II, the special case of Poisson user distribution and Rayleigh fading channel was studied. Combining with the knowledge of regular variation, the average BER was shown to achieve tightness in the Jensen's inequality. This is followed by the extension to the negative binomial user distribution, for which the closed form expression of average BER was derived and shown to decrease in number of users.

CHAPTER 4

COGNITIVE RADIO SYSTEM WITH MULTI-USER DIVERSITY

4.1. Cognitive Radio Systems With Multi-user Diversity

There have been previous studies on spectrum sharing cognitive radio where the capacity of cognitive radio network has been analysed for Gaussian and fading channels [69–71]. Most of these studies assume a single primary and a single cognitive user. Cases where the presence of multiple primary and cognitive users are also interested. In these case, multi-user diversity, as a fundamental property of wireless networks, has been widely applied for opportunistic communications in cognitive radio system, which will bring new issues related to user scheduling and medium access control [72]. A popular form of multi-user diversity is usually exploited in a wireless system with multiple independent fading communication links by selecting one link with the best instantaneous channel condition to transmit at one time [73].

Multi-user diversity gains have been explored in conventional non-cooperative wireless networks by user selection, exploiting the fluctuations of fading channels of different users [23, 74]. In [75], authors adopt a cooperative sensing framework to overcome low SNR and shadowing. It is shown that the average throughput of a secondary network scales like $\log_2 \ln(N)$ and $\log_2(N)$ under finite and infinite peak transmit power constraints at the secondary transmitters, respectively, in [72, 73]. A scaling law of a cognitive ad hoc network is studied in [76], but it did not focus on the multi-user diversity gain by opportunistic user selection. Channel capacity is also studied by formulating it as a maximization problem with respect to the channel gain and solving for the optimum solutions. Closed form capacity formulas under different fading channels are provided where possible in [77]. Results suggest that a significant spectrum access gain may be achieved in fading environments. In [78], extending [76], authors cover cases considering a selection diversity scheme with multiple primary users and multiple secondary receivers. More generally, unlike non-cognitive-radio multi-user systems, the spectrum sensing reliability may degrade the achievable multi-user diversity gain, which is studied in [79].

Fairness for the secondary users system is another popular issue considered in [80, 81]. Fairness can be ensured among the secondary users by providing them with the same opportunity for accessing an available spectrum band. Authors of [82] extend results to non-independent and identically distributed (non-i.i.d.) channels by employing fairness scheduling and show that fairness scheduling in non-i.i.d. channels has the same asymptotic throughput characteristics as the best user selection in i.i.d. channels.

4.2. System Model

In this chapter, we consider an uplink cognitive radio system with multiple secondary users and single primary user and one base station (BS). Both the BS and users are assumed to have a single antenna.

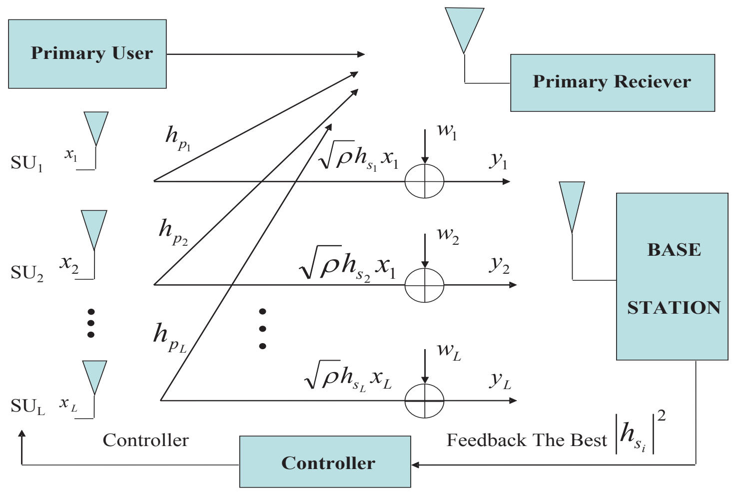


Figure 4.1. System Model of Cognitive Radio System

As shown in Figure 4.2, we consider a cognitive radio system with L secondary users and multi-user diversity scheme is applied to the secondary user system. “SU” in the figure stands for secondary users. A secondary user is allowed to share the spectrum with a primary link as long as an independent interference constraint to the primary receiver (PR) with a peak value Q is respected. Assuming that the secondary link is not affected by the primary transmission, we aim to increase the spectral efficiency of the secondary link by applying multi-user diversity technique. Equivalently to the uplink of a cellular system, the L secondary users and the secondary receiver (SR) is equipped with a single antenna. The received signal at the base station from the i^{th} secondary user can be expressed as,

$$y_i = \sqrt{\rho} h_{s_i} x_i + w_i, \qquad i = 0, 1, \ldots, L, \tag{4.1}$$

where h_{s_i} denotes the channel coefficient from the secondary user to base station, x_i the transmitted symbol, w_i the additive white Gaussian noise (AWGN) corresponding to the i^{th} secondary user. The average received power ρ at BS is identical so that this multi-user diversity system is homogeneous. The channel gain of the i^{th} secondary user at the BS can be expressed as $\gamma_{s_i} = |h_{s_i}|^2$. In the single user selected case, its channel gain is denoted by $\gamma^* = |h_{s_*}|^2$, where $|h_{s_*}|^2 = \max_i\{|h_{s_i}|^2\}$. Since all secondary users have *i.i.d.* fading channel, so we can drop the subscript i when we derive the CDF of γ_{s_i}.

Using *scan and wait* combining (SWC) [83], the secondary user cyclically switches between the L transmitter antennas in order to find the antenna with highest output SNR while also satisfies the interference constraint. Whenever the interference constraint is satisfied, the PR sends a binary ACK (Acknowledgement) to the secondary users through a reliable feedback channel, otherwise it sends a NACK. We also assume that there is a reliable feedback channel between the base station and the secondary user. This channel is implemented in a discrete-time fashion, more specifically, short guard periods are periodically inserted into the transmitted signal. During these guard periods, the SR selects a suitable user and the signal constellation to be used for transmission throughout the subsequent data burst. Under the assumption of frequency flat fading, we use a block-fading model assuming that different secondary users experience roughly the same fading conditions (or equivalently the same SNR) during the data burst and its preceding guard period. For our study, we assume that the received signal from each diversity branch experiences independent identically distributed (i.i.d.) Rayleigh fading. As such, the received SNRs from the i^{th} user at the base station and the primary receive, denoted by h_{s_i} and h_{p_i} respectively, $(i = 1, 2, \dots, L)$.

4.3. A Special Case: Binomial Number of Users and Rayleigh Fading

In Chapter II and III, we discussed how randomization of the total number of users affects the system performance. In the system model of this chapter, due to the interference constraint from the secondary user to the primary user, the number of active secondary user will be randomized. In probability theory and statistics, the binomial distribution is the discrete probability distribution of the number of successes in a sequence of n independent Bernoulli experiments, each of which yields success with probability p. We define $\mathcal{N}$ as the active number of users, the number of users after dropping those who fail to satisfy the interference constraints. Obviously, $\mathcal{N}$ is a binomial random variable. The success probability p of this binomial random variable is a function of Q which depends on the interference channel between secondary users and the primary receiver, which can be represented as $F_{\gamma_p}(Q)$, where $F_{\gamma_p}(x)$ is the CDF of $|h_p|^2$ which is $i.i.d.$ across all secondary users.

There are two modes of operation to pick up the desired user: 1)Choosing the user set among L total users which satisfy the interference constraint $S_1 = \{j \in 1, \ldots, L : \gamma_{p_j} < Q\}$, then choosing the user with highest γ_{s_i} from S_1; 2)Choosing the user with the highest γ_{s_i} among all L users and then checking if it satisfy the interference constraint. The value of success probability $F_{\gamma_p}(Q)$ determines the efficiency of two different modes. For large value of $F_{\gamma_p}(Q)$, with high probability that any arbitrary user will satisfy the interference constraint. In other words, a desired user will be found in a very limited number of search. On the other hand, for small value of $F_{\gamma_p}(Q)$, mode 1 is more proper. However, two modes result mathematically in the same closed form expression of ergodic capacity, since at last the user with highest γ_{s_i} which also satisfy the interference constraint will be chosen to transmit no matter which mode is applied. In this thesis, we will derive the expression of ergodic capacity and average error rate of the secondary users system.

4.3.1. Asymptotic Scaling of Capacity with λ

In Chapter II, we study the scaling law of the ergodic capacity in the multi-user diversity system when the total number of users are random. It has been stated that the ergodic capacity grows in the way of $\log\log\lambda))$ as the mean value of random variable $\mathcal{N}$ increases. Also, when $\lambda \to \infty$, the performance gap between the random $\mathcal{N}$ and deterministic N vanishes. In this chapter, we can observe the similar result when the multi-user diversity technique is applied to the secondary users in the cognitive radio system.

Define $F_\gamma(x)$ as the cumulative distribution function (CDF) of the channel gain of the i.i.d. fading channels across all users. According to the total probability theorem, the CDF of the channel gain of the best user selected from a binomial random set of users can be obtained by,

$$F_{\gamma_s{}^*}(x) = U_{\mathcal{N}}(F_\gamma(x)) \tag{4.2}$$

where $U_{\mathcal{N}}(t) = \sum_{k=0}^{\infty} \Pr[\mathcal{N} = k]\, t^k$, $0 \le t \le 1$, is the probability generating function (PGF) of the binomial random variable $\mathcal{N}$. In this thesis, we use $bin(L, p)$ to denote the binomial distribution with L trails and success probability p. Since $\mathcal{N}$ users are chosen from L total users subjecting to an independent interference threshold Q, the random variable $\mathcal{N}$ yields the distribution of $bin(L, F_{\gamma_p}(Q))$, where $F_{\gamma_p}(x)$ is the CDF of the interference channel gain from the secondary users to the primary receiver. Consequently, the CDF of the channel gain of the desired user in our scheme can be expressed as,

$$\begin{aligned} F_{\gamma_s{}^*}(x) &= [1 - F_{\gamma_p}(Q) + F_{\gamma_p}(Q)F_{\gamma_p}(x)]^L \\ &= [1 - t + t(1 - e^{-x})]^{\frac{\lambda}{t}} \\ &= (1 - te^{-x})^{\frac{\lambda}{t}} \end{aligned} \tag{4.3}$$

where $t = F_{\gamma_p}(Q)$ and $\lambda = Lt$ is the mean value of random variable $\mathcal{N}$. Then we can observe the scaling law of the ergodic capacity with respect to λ in our scheme:

Theorem 7. *The ergodic capacity averaged across the user distribution and its scaling law with respect to λ can be expressed as,*

$$\begin{aligned} E_{\mathcal{N}}\left[\overline{C}_{\text{cog}}(\rho, \mathcal{N})\right] = E_{\gamma_s{}^*}[\log(1 + \rho\gamma_s{}^*)] &= \rho \int_0^\infty \frac{1 - (1 - te^{-x})^{\frac{\lambda}{t}}}{1 + \rho x} dx \\ &= \log(1 + \rho \log(\lambda)) + O(1/\log(\lambda)), \end{aligned} \tag{4.4}$$

as $\lambda \to \infty$.

Proof. Defining $y := e^{-x}$ and integrating by substitution,

$$\begin{aligned}
\mathrm{E}_{\mathcal{N}}\left[\overline{C}_{\mathrm{cog}}(\rho,\mathcal{N})\right] &= \int_0^1 \frac{1-[1-e^{-x}t]^{\frac{\lambda}{t}}}{1+\rho x}dx \\
&= \int_0^1 \frac{1-(1-yt)^{\frac{\lambda}{t}}}{1-\rho\log y}\left(\frac{\rho}{y}\right)dy \\
&= \int_0^{1/\lambda} \frac{1-(1-yt)^{\frac{\lambda}{t}}}{1-\rho\log y}\left(\frac{\rho}{y}\right)dy + \int_{1/\lambda}^1 \frac{1-(1-yt)^{\frac{\lambda}{t}}}{1-\rho\log y}\left(\frac{\rho}{y}\right)dy
\end{aligned} \tag{4.5}$$

For the first term after the second equality in (4.5), we have

$$0 < \int_0^{1/\lambda} \frac{1-(1-yt)^{\frac{\lambda}{t}}}{1-\rho\log y}\left(\frac{\rho}{y}\right)dy < \int_0^{1/\lambda} \frac{\lambda y}{1+\rho\log(\lambda)}\left(\frac{\rho}{y}\right)dy \tag{4.6}$$

$$= \frac{\rho}{1+\log(\lambda)} \tag{4.7}$$

This is so because the numerator of the integrand is replaced with its upper bound and the denominator of the integrand is replaced with its lower limit. It can be seen that the upper bound after the equality in (4.7) yields $O(1/\log(\lambda))$ and has limit 0 as $\lambda \to \infty$. This implies that the first term should have limit 0. The second term in (4.5) has the bounds given by,

$$\int_{1/\lambda}^1 \frac{\rho\left(1-(1-\frac{t}{\lambda})^{\frac{\lambda}{t}}\right)}{y(1-\rho\log(y))}dy < \int_{1/\lambda}^1 \frac{1-(1-yt)^{\frac{\lambda}{t}}}{1-\rho\log y}\left(\frac{\rho}{y}\right)dy < \int_{1/\lambda}^1 \frac{1-(1-t)^{\frac{\lambda}{t}}}{1-\rho\log y}\left(\frac{\rho}{y}\right)dy \tag{4.8}$$

in which the lower and upper bounds are obtained by bounding the numerator, and they turn out to be $\int_0^{1/\lambda} \frac{1}{1-\rho\log(y)}\left(\frac{\rho}{y}\right)d\lambda = \log(1+\rho\log(\lambda))$. Therefore, for a fixed ρ, and as $\lambda \to \infty$ we can express (4.5) as (4.4). Therefore we have

$$\mathrm{E}_{\mathcal{N}}\left[\overline{C}_{\mathrm{cog}}(\rho,\mathcal{N})\right] = \log\left(1+\rho\log(\lambda)\right) + O(1/\log(\lambda)) \tag{4.9}$$

as $\lambda \to \infty$. □

To notice that the first terms in (2.12), (2.15) and (4.9) are same because the capacity metrics under Poisson, negative binomial, and binomial distribution yield the same growing speed of $\log(\log(\lambda))$. On the other hand, the second terms in (2.12), (2.15) and (4.9) are different. Since $O(\cdot)$ is an approximation, so that we can not judge the growing speed of the capacity under the three distributions according to the second term, even the three expressions of capacity scaling law with respect to λ do have different speed of convergence. In fact, the difference of the all three second terms is caused by the choice of the integral limit during the proof. For example, in (4.5), if we choose another integral limit instead of $1/\lambda$, the proof will still work but the second term in (4.9) will end up with a different term in the form $O(\cdot)$.

4.3.2. Average Error Rate

In the previous section, the BER under Poisson and negative binomial $\mathcal{N}$ are well investigated and it has been stated that the it achieves tightness in the Jensen's inequality. In this section, we will take a look at the special case that when the channel experiences Rayleigh fading, and under which, the closed form of BER behaviour with binomial $\mathcal{N}$ will be derived.

In section 4.3.1, we derive the CDF of the channel gain of the best user chosen from a binomial distributed random set of users. Here we take derivative of (4.2) with respect to x so that the PDF of the channel gain of the best user in the binomial case can be expressed as:

$$f_{\gamma^*}(x) = \frac{dF_{\gamma^*}(x)}{dx} = \lambda e^{-x}(1-e^{-x}t)^{\frac{\lambda}{t}-1} \quad \text{for} \quad x>0. \tag{4.10}$$

where x is the variable. Assuming the instantaneous error rate has the form $\mathrm{P}_e(\rho x) = \alpha e^{-\eta\rho x}$, substituting (4.10) into (3.7) we can get:

$$\begin{aligned} \mathrm{E}_{\mathcal{N}}\left[\overline{P_e}(\rho,\mathcal{N})\right] &= \int_0^{\infty} \alpha e^{-\eta\rho x} e^{-x}(1-e^{-x}t)^{\frac{\lambda}{t}-1} dx \\ &= \alpha t^{-1-\eta\rho}\lambda \cdot \beta\left(t, 1+\eta\rho, \frac{\lambda}{t}\right) \end{aligned} \tag{4.11}$$

where $\beta\left(t, 1+\eta\rho, \frac{\lambda}{t}\right)$ is the incomplete beta function defined as $\beta\left(x,a,b\right) = \int_0^x t^{a-1}(1-t)^{b-1}dt$.

4.3.3. Comparison of Binomial Distribution and Poisson Distribution

From probability theorem, binomial distribution can converge to Poisson distribution as $p \to 0$ and Lp remains constant. In this section, binomial distribution will be proved to dominate Poisson distribution under LT ordering sense.

To compare two random variables, it is straight forward to use the equivalent interpretation of LT ordering in terms of the PGFs, by knowing that $\exp(\lambda(t-1))$ is the PGF of Poisson distribution [64] while $(1-p+pt)^L$ is the PGF of binomial distribution where $0 \le t \le 1$. Since we compare them under the save mean value, then $Lp = \lambda$. Then we have the following theorem:

Theorem 8. *Let $\mathcal{X}$ denotes a Poisson random variable with parameter λ and $\mathcal{Y}$ denotes a binomial random variable with mean value Lp. By assuming that $Lp = \lambda$,* *we have* $U_{\mathcal{X}}(t) \geq U_{\mathcal{Y}}(t)$*. Then we conclude that*

$$\mathcal{X} \leq_{Lt} \mathcal{Y} \tag{4.12}$$

Proof. To show $U_{\mathcal{X}}(t) \geq U_{\mathcal{Y}}(t)$ first we take logarithm to $U_{\mathcal{X}}(t)$ and $U_{\mathcal{Y}}(t)$ and we get

$$\log(U_{\mathcal{X}}(t)) - \log(U_{\mathcal{Y}}(t)) = Lp(t-1) - L\log\left(1-p+pt\right). \tag{4.13}$$

By shuffling the terms we rewrite the problem as comparing $s - \log(1+s)$ with 0, where $s = p(t-1)$. Taking the 1^{st} derivative with respect to s we get

$$\begin{aligned}\frac{\partial\left(s-\log(1+s)\right)}{\partial s} &= 1-\frac{1}{s+1}\\ &= \frac{s}{1+s} \leq 0\end{aligned} \tag{4.14}$$

for all $0 \leq t \leq 1$. This implies that (4.13) is an monotonically decreasing function of t with the minimum value 0 at $t = 1$. Hence, we have $U_{\mathcal{X}}(t) \geq_{\mathrm{Lt}} U_{\mathcal{Y}}(t)$, completing the proof. □

For the extreme case that $p = 1$, binomial user distribution converges to the deterministic number of users, which dominates any kind of random distributions under LT ordering sense. The numerical results will be shown in the later section.

4.3.4. Multiple Interference Constraints

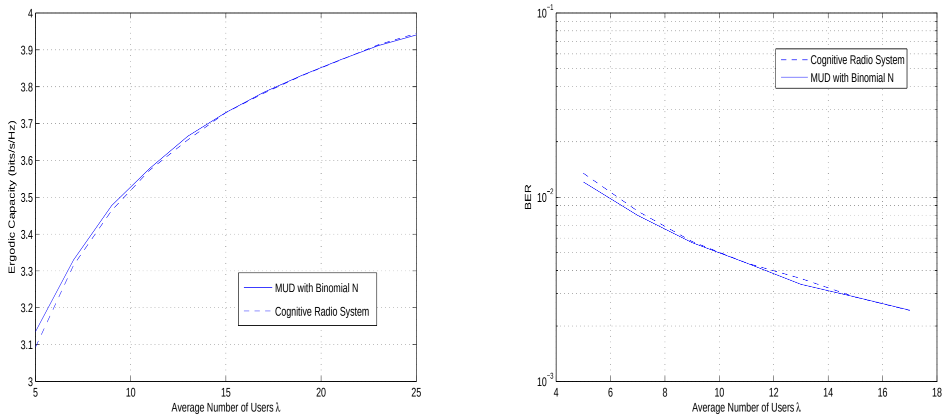
In Section 4.3.1, we discussed the situation when there exists only one interference channel and the success probability of the binomial distribution is $F_{\gamma_p}(x)$.

When multiple interference channels case is considered, since the interference channels are still independent to the secondary transmission, the active number of users still results in a binomial distribution, only the success probability changes. Assuming that there are K interference channels between each secondary and primary receivers, by using the basic probability knowledge, the success probability is $\mathbf{P}\{x_1 < Q_1; x_2 < Q_2; \ldots; x_K < Q_K\}$, where $Q_1, \ldots, Q_K$ are K interference constraints. When the K constraints are independent, the interference constraints probability $F_{\gamma_p}(x_1, x_2, \ldots, x_K) = F_{\gamma_p}(x_1)F_{\gamma_p}(x_2)_{\gamma_p}(x_K)$; when the K constraints are non-independent, the success probability is $F_{\gamma_p}(Q_1, Q_2, \ldots, Q_K)$ where F_{γ_p} is the joint CDF of the K interference channels. Consequently, all the analysis did in the previous sections apply for this case.

4.4. Simulation Results

In Section 4.3, we proposed a cognitive radio system with multi-user diversity scheme in the secondary users system. It has been shown that if the interference channel is independent with the secondary transmission, the secondary users system will be a multi-user diversity system with binomial $\mathcal{N}$. Further, in 4.3.1, we derive the similar scaling laws of ergodic capacity with respect to λ under binomial distributed $\mathcal{N}$ to the Poisson case. In Figure 4.2, ergodic capacity and average BER under the proposed cognitive radio system and multi-user diversity system with binomial $\mathcal{N}$ are simulated respectively. In Figure 4.2(a) and 4.2(b), averaged SNR is 6 dB and 0 dB respectively, and in both figures the Rayleigh faded parameter is 1.

From the figure we can see that the performance of our proposed scheme fit well with the binomial $\mathcal{N}$ multi-user diversity system, which verify our assumption.



(a) Ergodic Capacity VS. λ

(b) Averaged Error Rate VS. λ

Figure 4.2. Ergodic Capacity and Average BER of Cognitive Radio System.

In Section 2.3.1, ergodic capacity performance under negative binomial and Poisson user distributions are well established. According to the probability theory, negative binomial distribution can converge to the Poisson distribution as the trail probability $p \to 0$ and stopping parameter $r \to \infty$, which has been shown numerically in Section 2.6. On the other hand, the binomial distribution also converges towards the Poisson distribution as the number of trials goes to infinity while the product Lp remains fixed. Consequently, as the p parameter in binomial distribution goes to zero, performance of binomial distribution will also get close to the Poisson case. Moreover, binomial distributed users means that all the users are checked according to the interference, while negative binomial distributed users means that the interference checking stops when a certain number of users satisfy the interference constraint. Obviously, binomial distribution should outperform negative binomial distribution when ergodic capacity is considered. As shown in Figure 4.3, binomial and negative binomial performance converge to Poisson case as $p \to 0$ and binomial performance is always better than the negative binomial case.

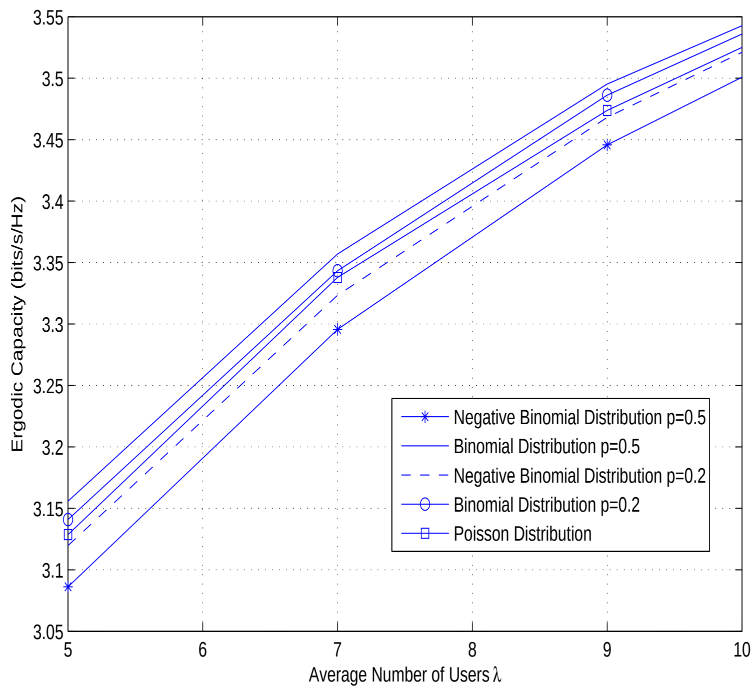


Figure 4.3. Ergodic Capacity Under Different User Distributions.

4.5. Summary

We began this chapter by a literature review of cognitive radio technique and it together with multi-user diversity system. We proposed a single primary user cognitive radio system multi-user diversity at the secondary users. Comparing to the general multi-user diversity system, there existed an interference constraint between secondary and primary users, which is independent of the secondary users' transmission. The secondary user with highest transmitting SNR which also satisfies the interference constraint is selected to communicate. Under all these assumptions, the active number of secondary users is a binomial random variable. This was then followed by a derivation of the scaling law of the ergodic capacity with respect to λ and the closed form expression of average BER under this situation. What is more, the binomial distribution was shown to dominate Poisson distribution under LT ordering sense.

CHAPTER 5

CONCLUSIONS AND FUTURE RESEARCH

This thesis introduces the multi-user diversity when total number of users is random and considers its application to cognitive radio system. The average error rate of multi-user systems implementing multi-user diversity is proved to be a completely monotonic function of the number of users in the system. Further, ergodic capacity is shown to have a completely monotonic derivative with respect to the number of users. Using the above properties along with Jensen's inequality, it is shown that the ergodic capacity performance averaged across the number of users distribution will always perform inferior to the corresponding performance of a system with deterministic number of users. Further, an approach to compare the performance of the system for different user distributions using Laplace transform ordering is studied. Moreover, the results of average BER and ergodic capacity under user distribution of Poisson are extended to negative binomial and compared in Laplace transform ordering sense. Outage capacity and outage probability is also been studied for the slow fading scenario. Outage probability is completely monotonic that the randomization of total number of users will deteriorate the performance. A new metric named ergodic capacity per user, which is the ergodic capacity normalized by the total number of users, is investigated to observe the property of individual rate in the system. It can be proved that this metric is a completely monotonic function of N, which also implies convexity. Consequently, the ergodic capacity per user will increase as the number of users is randomized. With the knowledge of Laplace transform ordering, one can control the user distribution to improve the system performance by comparing their probability generating function. A “two-value” distribution is shown to be an optimal user distribution when the ergodic capacity per user is taken into account when designing the system.

We also develop the multi-user diversity technique applied in the multiple secondary user cognitive radio system. The secondary user with best transmitted SNR, subjecting an interference constraint between the secondary users and the primary receiver simultaneously, is selected to transmit at each time slot. This interference constraint is assumed to be independent to the secondary users system, which will randomize the number of active secondary users. Under this situation, the scaling law of ergodic capacity and the closed form expression of the BER for a special case of binomial distributed users are derived. Due to the result, interference constraint will degrade the system performance by randomizing the number of users. Furthermore, binomial distribution is shown to dominate Poisson distribution under Laplace transform ordering sense.

Further work in this area can be included the following: 1) extension from the single-user-selected multi-user diversity system to the multi-user-selected case; 2) properties of capacity and average BER when other fading scenario is considered; and 3) more general model of the cognitive radio system with non-independent interference constraint and fading other than Rayleigh.

[1] "Radio," Wikipedia term: http://en.wikipedia.org/wiki/Wireless.

[2] "1G," Wikipedia term: http://en.wikipedia.org/wiki/1G.

[3] "2G," Wikipedia term: http://en.wikipedia.org/wiki/2G.

[4] A. J. Goldsmith, *Wireless Communications*, 1st ed. Cam-bridge: Cambridge University Press, Aug. 2005.

[5] "3G," Wikipedia term: http://en.wikipedia.org/wiki/3G.

[6] "Itu global standard for international mobile telecommunications," *Circular letter*, March 2008.

[7] H. Bai and M. Atiquzzaman, "Error modeling schemes for fading channels in wireless communications: A survey," *Communications Surveys Tutorials, IEEE*, vol. 5, no. 2, pp. 2 –9, Quarter 2003.

[8] F. Niu and H. Bertoni, "Path loss and cell coverage of urban microcells in high-rise building environments," in *Global Telecommunications Conference, 1993, including a Communications Theory Mini-Conference. Technical Program Conference Record, IEEE in Houston. GLOBECOM '93., IEEE*, Nov-2 Dec 1993, pp. 266 –270 vol.1.

[9] K. Kansanen and R. Muller, "On near-far gain in multiuser diversity systems," in *Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks and Workshops, 2008. WiOPT 2008. 6th International Symposium on*, April 2008, pp. 92 –96.

[10] D. N. C. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, 1st ed. Cam-bridge: Cambridge University Press, Jun. 2005.

[11] "A proportional fair space time scheduling for wireless communications," *Communications, IEEE Transactions on*, vol. 53, no. 4, p. 747, April 2005.

[12] R. Almatarneh, M. Ahmed, and O. Dobre, "Performance analysis of proportional fair scheduling in ofdma wireless systems," in *Vehicular Technology Conference Fall (VTC 2010-Fall), 2010 IEEE 72nd*, Sept. 2010, pp. 1 –5.

[13] R. Knopp, "Achieving multiuser diversity under hard fairness constraints," in *Information Theory, 2002. Proceedings. 2002 IEEE International Symposium on*, 2002, p. 451.

[14] D. Piazza and L. Milstein, "Analysis of multiuser diversity in time-varying channels," *Wireless Communications, IEEE Transactions on*, vol. 6, no. 12, pp. 4412 –4419, December 2007.

[15] M. Castaneda and J. Nossek, "Multiuser diversity with limited feedback," in *Smart Antennas (WSA), 2011 International ITG Workshop on*, Feb. 2011, pp. 1 –7.

[16] S. Valentin and T. Wild, "Studying the sum capacity of mobile multiuser diversity systems with feedback errors and delay," in *Vehicular Technology Conference Fall (VTC 2010-Fall), 2010 IEEE 72nd*, Sept. 2010, pp. 1 –5.

[17] J. Hamalainen and R. Wichman, "Performance of multiuser diversity in the presence of feedback errors," in *Personal, Indoor and Mobile Radio Communications, 2004. PIMRC 2004. 15th IEEE International Symposium on*, vol. 1, Sept. 2004, pp. 599 – 603 Vol.1.

[18] L. Li and A. Goldsmith, "Capacity and optimal resource allocation for fading broadcast channels .i. ergodic capacity," *Information Theory, IEEE Transactions on*, vol. 47, no. 3, pp. 1083 –1102, Mar 2001.

[19] ——, "Capacity and optimal resource allocation for fading broadcast channels .ii. outage capacity," *Information Theory, IEEE Transactions on*, vol. 47, no. 3, pp. 1103 –1127, Mar 2001.

[20] E. Biglieri, J. Proakis, and S. Shamai, "Fading channels: information-theoretic and communications aspects," *Information Theory, IEEE Transactions on*, vol. 44, no. 6, pp. 2619 –2692, Oct 1998.

[21] M. Hossain, M.-S. Alouini, and V. Bhargava, "Two-user opportunistic scheduling using hierarchical modulations in wireless networks with heterogenous average link gains," in *Communications, 2008. ICC '08. IEEE International Conference on*, May 2008, pp. 1098 –1102.

[22] R. Knopp and P. Humblet, "Information capacity and power control in single-cell multiuser communications," in *Communications, 1995. ICC '95 Seattle, 'Gateway to Globalization', 1995 IEEE International Conference on*, vol. 1, Jun 1995, pp. 331 –335 vol.1.

[23] P. Viswanath, D. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *Information Theory, IEEE Transactions on*, vol. 48, no. 6, pp. 1277 –1294, Jun 2002.

[24] D. Mazzarese and W. Krzymien, "Throughput maximization and optimal number of active users on the two transmit antenna downlink of a cellular system," in *Communications, Computers and signal Processing, 2003. PACRIM. 2003 IEEE Pacific Rim Conference on*, vol. 1, Aug. 2003, pp. 498 – 501 vol.1.

[25] L. Li, N. Jindal, and A. Goldsmith, "Outage capacities and optimal power allocation for fading multiple-access channels," *Information Theory, IEEE Transactions on*, vol. 51, no. 4, pp. 1326 – 1347, April 2005.

[26] W. Yu, W. Rhee, and J. Cioffi, "Optimal power control in multiple access fading channels with multiple antennas," in *Communications, 2001. ICC 2001. IEEE International Conference on*, vol. 2, 2001, pp. 575 –579 vol.2.

[27] C.-J. Chen and L.-C. Wang, "A unified capacity analysis for wireless systems with joint multiuser scheduling and antenna diversity in nakagami fading channels," *Communications, IEEE Transactions on*, vol. 54, no. 3, pp. 469 – 478, March 2006.

[28] C. Mun, M.-W. Lee, J.-G. Yook, and H.-K. Park, "Exact capacity analysis of multiuser diversity combined with transmit diversity," *Electronics Letters*, vol. 40, no. 22, pp. 1423 – 1424, Oct. 2004.

[29] W. Rhee and J. Cioffi, "On the capacity of multiuser wireless channels with multiple antennas," *Information Theory, IEEE Transactions on*, vol. 49, no. 10, pp. 2580 – 2595, Oct. 2003.

[30] D. Mazzarese and W. Krzymien, "High throughput downlink cellular packet data access with multiple antennas and multiuser diversity," in *Vehicular Technology Conference, 2003. VTC 2003-Spring. The 57th IEEE Semiannual*, vol. 2, April 2003, pp. 1079 – 1083 vol.2.

[31] S. Sampei and H. Harada, "System design issues and performance evaluations for adaptive modulation in new wireless access systems," *Proceedings of the IEEE*, vol. 95, no. 12, pp. 2456 –2471, Dec. 2007.

[32] A. Goldsmith, "Adaptive modulation and coding for fading channels," in *Information Theory and Communications Workshop, 1999. Proceedings of the 1999 IEEE*, 1999, pp. 24 –26.

[33] X. Qiu and K. Chawla, "On the performance of adaptive modulation in cellular systems," *Communications, IEEE Transactions on*, vol. 47, no. 6, pp. 884 –895, Jun 1999.

[34] L. Tang, S.-X. Wang, and Y.-C. Liang, "Variable-rate adaptive modulation in mimo systems exploiting multiuser diversity," in *Acoustics, Speech, and Signal Processing, 2004. Proceedings. (ICASSP '04). IEEE International Conference on*, vol. 4, May 2004, pp. iv–693 – iv–696 vol.4.

[35] Q. Ma and C. Tepedelenlioglu, "Practical multiuser diversity with fair channel assignment," in *Acoustics, Speech, and Signal Processing, 2004. Proceedings. (ICASSP '04). IEEE International Conference on*, vol. 4, May 2004, pp. iv–685 – iv–688 vol.4.

[36] Y. Ko and C. Tepedelenlioglu, "Impact of the number of users on error probabilities in multiuser diversity systems," in *Information Sciences and Systems, 2007. CISS '07. 41st Annual Conference on*, March 2007, pp. 566 –569.

[37] Y. Ko, Q. Ma, and C. Tepedelenlioglu, "Comparison of adaptive beamforming and orthogonal stbc with outdated feedback," *Wireless Communications, IEEE Transactions on*, vol. 6, no. 1, pp. 20 –25, Jan. 2007.

[38] Q. Ma and C. Tepedelenlioglu, "Practical multiuser diversity with outdated channel feedback," *Vehicular Technology, IEEE Transactions on*, vol. 54, no. 4, pp. 1334 – 1345, July 2005.

[39] E. Larsson, "On the combination of spatial diversity and multiuser diversity," *Communications Letters, IEEE*, vol. 8, no. 8, pp. 517 – 519, Aug. 2004.

[40] R. Chen, J. Andrews, and J. Heath, R.W., "Transmit selection diversity for multiuser spatial multiplexing systems," in *Global Telecommunications Conference, 2004. GLOBECOM '04. IEEE*, vol. 4, Nov.-3 Dec. 2004, pp. 2625 – 2629 Vol.4.

[41] M. Torabi, D. Haccoun, and W. Ajib, "Ber performance analysis of multiuser diversity with antenna selection in mrc mimo systems," in *Global Telecommunications Conference, 2009. GLOBECOM 2009. IEEE*, 30 2009-Dec. 4 2009, pp. 1 –6.

[42] C. Tepedelenlioglu, Y. Zhang, and O. Rahman, "Asymptotic ber analysis of a simo multiuser diversity system," *Vehicular Technology, IEEE Transactions on*, vol. 58, no. 9, pp. 5330 –5335, Nov. 2009.

[43] A. B. Narasimhamurthy, C. Tepedelenlioglu, and Y. Zhang, "Multi-user diversity with random number of users," *Wireless Communications, IEEE Transactions on*, vol. 11, no. 1, pp. 60 –64, January 2012.

[44] V. Valenta, R. mars anda andlek, G. Baudoin, M. Villegas, M. Suarez, and F. Robert, "Survey on spectrum utilization in europe: Measurements, analyses and observations," in *Cognitive Radio Oriented Wireless Networks Communications (CROWNCOM), 2010 Proceedings of the Fifth International Conference on*, June 2010, pp. 1 –5.

[45] D. Cabric, S. Mishra, and R. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," in *Signals, Systems and Computers, 2004. Conference Record of the Thirty-Eighth Asilomar Conference on*, vol. 1, Nov. 2004, pp. 772 – 776 Vol.1.

[46] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proceedings of the IEEE*, vol. 97, no. 5, pp. 894 –914, May 2009.

[47] R. Ahuja, R. Corke, and A. Bok, "Cognitive radio system using ieee 802.11a over uhf tvws," in *New Frontiers in Dynamic Spectrum Access Networks, 2008. DySPAN 2008. 3rd IEEE Symposium on*, Oct. 2008, pp. 1 –9.

[48] W. Zong, S. Shao, Q. Meng, and W. Zhu, "Joint user scheduling and beamforming for underlay cognitive radio systems," in *Communications, 2009. APCC 2009. 15th Asia-Pacific Conference on*, Oct. 2009, pp. 99 –103.

[49] H. Islam, Y.-C. Liang, and A. T. Hoang, "Distributed power and admission control for cognitive radio networks using antenna arrays," in *New Frontiers in Dynamic Spectrum Access Networks, 2007. DySPAN 2007. 2nd IEEE International Symposium on*, April 2007, pp. 250 –253.

[50] H. Islam, Y. chang Liang, and A. Hoang, "Joint power control and beamforming for cognitive radio networks," *Wireless Communications, IEEE Transactions on*, vol. 7, no. 7, pp. 2415 –2419, July 2008.

[51] K. Hamdi, W. Zhang, and K. Ben Letaief, "Joint beamforming and scheduling in cognitive radio networks," in *Global Telecommunications Conference, 2007. GLOBECOM '07. IEEE*, Nov. 2007, pp. 2977 –2981.

[52] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *Information Theory, IEEE Transactions on*, vol. 50, no. 12, pp. 3062 – 3080, Dec. 2004.

[53] P. Anghel and M. Kaveh, "Exact symbol error probability of a cooperative network in a rayleigh-fading environment," *Wireless Communications, IEEE Transactions on*, vol. 3, no. 5, pp. 1416 – 1421, Sept. 2004.

[54] A. Ribeiro, X. Cai, and G. Giannakis, "Symbol error probabilities for general cooperative links," *Wireless Communications, IEEE Transactions on*, vol. 4, no. 3, pp. 1264 – 1273, May 2005.

[55] Y. Zou, Y.-D. Yao, and B. Zheng, "Cognitive transmissions with multiple relays in cognitive radio networks," *Wireless Communications, IEEE Transactions on*, vol. 10, no. 2, pp. 648 –659, February 2011.

[56] J. Lee, H. Wang, J. Andrews, and D. Hong, "Outage probability of cognitive relay networks with interference constraints," *Wireless Communications, IEEE Transactions on*, vol. 10, no. 2, pp. 390 –395, February 2011.

[57] X. Jing and D. Raychaudhuri, "Spectrum co-existence of ieee 802.11b and 802.16a networks using the cscc etiquette protocol," in *New Frontiers in Dynamic Spectrum Access Networks, 2005. DySPAN 2005. 2005 First IEEE International Symposium on*, Nov. 2005, pp. 243 –250.

[58] J. Hillenbrand, T. Weiss, and F. Jondral, "Calculation of detection and false alarm probabilities in spectrum pooling systems," *Communications Letters, IEEE*, vol. 9, no. 4, pp. 349 – 351, April 2005.

[59] M. Shaked and J. G. Shanthikumar, *Stochastic orders and their applications.* Academic Press, 1994.

[60] A. Muller and D. Stoyan, *Comparison methods for stochastic models and risks.* John Wiley and Sons, 2002.

[61] W. Feller, *An Introduction To Probability Theory And Its Applications, 2Nd Ed.* Wiley India Pvt. Ltd., 2009.

[62] E. Castillo, A. S. Hadi, N. Balakrishnan, and J. M. Sarabia, *Extreme value and related models with applications in engineering and science.* Wiley, 2005.

[63] H. A. David, *Order Statistics*, 2nd ed. New York: Wiley, 1981.

[64] D. V. Widder, *An introduction to transform theory.* Academic Press, 1971.

[65] "Negative-binomial distribution," Wikipedia term: http://en.wikipedia.org/wiki/Negativebinomialdistribution.

[66] R. L.Schilling and R. Song, *Bernstein Functions.* De Gruyter, 2010.

[67] P. J. Downey, "An Abelian theorem for completely monotone functions," 1993, available online at ftp://ftp.cs.arizona.edu/ reports/1993/TR93-15.ps.Z.

[68] M. Abramowitz and I. A. Stegun, *Handbook of mathematical functions with formulas, graphs, and mathematical tables.* Courier Dover Publications, 1964.

[69] M. Gastpar, "On capacity under receive and spatial spectrum-sharing constraints," *Information Theory, IEEE Transactions on*, vol. 53, no. 2, pp. 471 –487, Feb. 2007.

[70] L. Musavian and S. Aissa, "Fundamental capacity limits of cognitive radio in fading environments with imperfect channel information," *Communications, IEEE Transactions on*, vol. 57, no. 11, pp. 3472 –3480, Nov. 2009.

[71] R. Zhang, S. Cui, and Y.-C. Liang, "On ergodic sum capacity of fading cognitive multiple-access and broadcast channels," *Information Theory, IEEE Transactions on*, vol. 55, no. 11, pp. 5161 –5178, Nov. 2009.

[72] T. W. Ban, W. Choi, B. C. Jung, and D. K. Sung, "Multi-user diversity in a spectrum sharing system," *Wireless Communications, IEEE Transactions on*, vol. 8, no. 1, pp. 102 –106, Jan. 2009.

[73] R. Zhang and Y.-C. Liang, "Investigation on multiuser diversity in spectrum sharing based cognitive radio networks," *Communications Letters, IEEE*, vol. 14, no. 2, pp. 133 –135, February 2010.

[74] T. Yoo, N. Jindal, and A. Goldsmith, "Multi-antenna downlink channels with limited feedback and user selection," *Selected Areas in Communications, IEEE Journal on*, vol. 25, no. 7, pp. 1478 –1491, September 2007.

[75] T. Do and B. mark, "Exploiting multiuser diversity for spectrum sensing in cognitive radio networks," in *Radio and Wireless Symposium (RWS), 2010 IEEE*, Jan. 2010, pp. 228 –231.

[76] M. Vu and V. Tarokh, "Scaling laws of single-hop cognitive networks," *Wireless Communications, IEEE Transactions on*, vol. 8, no. 8, pp. 4089 –4097, August 2009.

[77] A. Ghasemi and E. S. Sousa, "Fundamental limits of spectrum-sharing in fading environments," *Wireless Communications, IEEE Transactions on*, vol. 6, no. 2, pp. 649 –658, Feb. 2007.

[78] H. Wang, J. Lee, S. Kim, and D. Hong, "Capacity of secondary users exploiting multispectrum and multiuser diversity in spectrum-sharing environments," *Vehicular Technology, IEEE Transactions on*, vol. 59, no. 2, pp. 1030 –1036, Feb. 2010.

[79] J. Kim, Y. Shin, and R. Schober, "Multi-user diversity gain in uplink cognitive radio systems with spectrum sensing reliability," in *Personal Indoor and Mobile Radio Communications (PIMRC), 2010 IEEE 21st International Symposium on*, Sept. 2010, pp. 1195 –1200.

[80] A. Tajer and X. Wang, "Multiuser diversity gain in cognitive networks," *Networking, IEEE/ACM Transactions on*, vol. 18, no. 6, pp. 1766 –1779, Dec. 2010.

[81] S. Ganesan, M. Sellathurai, and T. Ratnarajah, "Opportunistic interference projection in cognitive mimo radio with multiuser diversity," in *New Frontiers in Dynamic Spectrum, 2010 IEEE Symposium on*, April 2010, pp. 1 –6.

[82] J.-P. Hong and W. Choi, "Throughput characteristics by multiuser diversity in a cognitive radio system," *Signal Processing, IEEE Transactions on*, vol. 59, no. 8, pp. 3749 –3763, Aug. 2011.

[83] H.-C. Yang, M. Simon, and M.-S. Alouini, "Scan and wait combining (swc): a switch and examine strategy with a performance-delay tradeoff," *Wireless Communications, IEEE Transactions on*, vol. 5, no. 9, pp. 2477 –2483, September 2006.

[84] N. H. Bingham, C. M. Goldie, and J. L. Teugels, *Regular variation.* Cambridge University Press, Jul. 1989.