The Fiscal Multiplier in a Liquidity Constrained New Keynesian Economy*

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Abstract

We study the effects of fiscal policy on the macroeconomy using a liquidity-constrained New Keynesian model in which government bonds are liquid and private financial assets are only partially liquid. We find that the fiscal multipliers in this economic environment are large enough for fiscal policy to be highly effective. In this model, a bond-financed fiscal expansion can stimulate output since higher public borrowing improves liquidity by increasing the proportion of liquid assets in private sector wealth.

Keywords: DSGE Models, Monetary Policy, Fiscal Policy, Liquidity Trap, Credit Constraints

JEL: E32, E52, E58, E62

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1 Introduction

Over the last decade, in many if not all developed countries, monetary policy has been the main instrument for managing the growth of aggregate demand and inflationary pressure. The chief monetary policy tool has been short-term interest rates. The response to the recent financial crisis has typically been lowering the nominal interest rate to its zero lower bound. As monetary policy loses its power at the zero lower bound, the conventional option of cutting interest rates is no longer available. This raises the question of whether fiscal policy is effective in mitigating the effects of the crisis.

Answering this question requires a model that can capture the key aspects of the crisis. As many noted, the realisation at the onset of the crisis that many private financial assets were of lower quality and therefore accompanied by higher default risks than previously assumed led to a flight to liquid assets. At the height of the crisis, the markets for private financial assets essentially froze. The drop in the resaleability of private assets diminished firms' ability to raise funds and use their assets as collateral for borrowing. The consequent decrease in investment led to substantial drops in output and inflation. To combat the recession, central banks lowered the nominal interest rate to its zero lower bound, generating a liquidity trap.

This paper studies the effectiveness of fiscal policy using the model proposed by Del Negro, Eggertsson, Ferrero and Kiyotaki (2011) (henceforth "DEFK"). This model reformulates the state-of-the-art version of New Keynesian economics, as in Christiano, Eichenbaum and Evans (2005) ("CEE") and Smets and Wouters (2007) ("SW"), by incorporating the liquidity frictions as described in Kiyotaki and Moore (2008) ("KM"). In the DEFK model, the economy is populated with a large number of identical households. Each household can save in two types of financial assets: government bonds and private equity. Government bonds are liquid, while private assets are not.[1] During each period, a randomly chosen fraction of household members becomes entrepreneurs. Entrepreneurs have the opportunity to invest

[1] As noted by DEFK, private equity has a broad definition in this model. It can be interpreted as privately issued paper such as commercial paper, bank loans, mortgages, and so on.

in new capital, which gives a better return than government bonds or private equity. Although investment opportunities are attractive, entrepreneurs are liquidity constrained: Entrepreneurs can borrow by issuing new equity, but the amount that they can issue in each period is limited; Private equity is illiquid, so entrepreneurs can sell only up to a certain portion of their equity holdings in each period. The rest of the household members are workers. They do not have the opportunity to invest in new capital and are not liquidity constrained. They work, consume and save by holding government bonds and private equity. Other features of the model are standard New Keynesian. Firms and workers enjoy some degree of monopoly power; prices and wages remain unchanged, on average, for several months. The central bank sets the interest rate following a Taylor-style rule. The presence of liquidity frictions in the DEFK model allows us to simulate the recent financial crisis. Comparison of the empirical data and the model's projections shows that the DEFK model performs well in explaining the responses of the key macroeconomic variables to the recent crisis.[2]

We introduce a role for government spending in the DEFK model. In our experiments, we consider two different kinds of fiscal expansion: a government spending rise and a tax cut. In the former case, the government buys more goods and services from firms and therefore stimulates aggregate demand. In the latter case, the government carries out a lump-sum tax cut which in practice resembles a lump-sum transfer to households. In both cases, we assume that the fiscal expansion is financed mainly by bonds - the government issues bonds to households to be repaid by tax rises at a later date.

In our study, we consider two scenarios. In the first scenario, we look at the government spending multiplier using the version of the DEFK model in normal times (i.e., without liquidity shocks) when the zero lower bound on the nominal interest rate does not bind. We find that the size of the multiplier is much larger than that suggested by a standard DSGE model

[2]DEFK use their model to examine the effectiveness of quantitative easing and find it to be an effective policy. Ajello (2010), Driffill and Miller (2011) and Shi (2015) also use the DEFK/KM framework to study the current financial crisis.

without financial frictions. The cumulative government spending multiplier obtained using the DEFK model is 1.6, while the one in the standard model is 0.55. The intuition for this result is as follows. In both models, an increase in government spending leads to higher future tax burdens and rises in the real interest rate. Both of these factors cause households to postpone consumption and increase their government bond holdings. In the standard model, investment falls since the higher real interest rate on bonds increases the opportunity cost of investing in physical capital. The government spending multiplier is thus smaller than 1. In the DEFK model, the multiplier is large because, unlike in the standard model, a bond-financed government spending expansion improves liquidity by increasing the proportion of liquid assets in households' wealth, which in turn allows liquidity constrained entrepreneurs to increase investment. Increased economic activity then increases private consumption, leading to a large multiplier.

In the second scenario, we look at the government spending multiplier in a liquidity crisis caused by a fall in the resaleability of private equity, in which case the zero lower bound on the nominal interest rate becomes binding.[3] We find that, in the DEFK model and in the standard model, the government spending multiplier is much larger in a liquidity crisis than in normal times. Moreover, we find that in the crisis scenario the multiplier in the DEFK model is still larger than that in the standard model. The government spending multiplier suggested by the DEFK model is larger than 2 in crisis times. At the zero lower bound, an increase in government spending creates inflationary pressures which decrease the real interest rate and stimulate consumption. In the DEFK model, the stimulative effect of fiscal policy is even larger because the multiplier effect applies to both consumption and investment. Holding the persistence of government spending

[3] Erceg and Linde (2012) criticise the assumption of an exogenous zero-bound condition in the study of the fiscal multiplier. They point out that, as an increase in government expenditure may help push the economy out of a liquidity trap, the multiplier will be smaller if the zero-bound condition is endogenous. Mertens and Ravn (2010) warn that the value of the multiplier is sensitive to the type of shock that drives the economy into a liquidity trap. To address these issues, we examine the fiscal multipliers using the DEFK model, in which the liquidity trap is endogenously caused by a financial crisis.

constant, we show that the value of the government spending multiplier in the standard model tends to decrease as the crisis prolongs, whereas in the DEFK model it increases. Under the crisis scenario, we also examine the effects of the fiscal interventions in the US under the 2009 American Recovery and Reinvestment Act (ARRA). Our findings suggest that the fiscal interventions may have prevented a deeper recession.

We then study the tax multiplier in both the normal-times and the crisis scenarios. Our results obtained with the DEFK model show that the tax multiplier is smaller than the government spending multiplier. A cut in lump-sum taxes reduces the revenue of the government, causing it to increase bond issues. This improves the private sector's liquidity and leads to increases in investment, consumption and output. The tax cut is less effective than government spending in stimulating output since it does not directly generate aggregate demand. This result suggests that both an increase in aggregate demand and an improvement in liquidity are important in stimulating economic activity.

Finally, we test the sensitivity of the spending and the tax multipliers to the steady-state debt-to-output ratio. Our results suggest that fiscal policy is more effective in stimulating output when the initial debt-to-GDP ratio is low. The policy implication is that containing the debt level during normal times would allow governments to achieve more effective results of fiscal stimulus in times of crisis, when such results are most needed.

Before describing the model, let us briefly review the literature on this topic.[4] Most of the theoretical discussions on the effectiveness of fiscal policy have been based on the CEE/SW model (see, for example, Bilbiie, Monacelli and Perotti (2014), Christiano, Eichenbaum and Rebelo (2011), Cogan et al. (2010) and Woodford (2011)). The CEE/SW model assumes frictionless financial markets and therefore cannot provide a detailed account of the

[4] The majority of empirical research in this area seems to suggest that fiscal policy is not effective and that an increase in government spending does not have a significant effect on the economy (see, for example, Hall (2009), Ramey (2011b) and references therein). The government spending multiplier is typically estimated to lie between 0.6 and 1.2. However, some recent empirical studies show that the fiscal multiplier is much larger during a recession (see, for example, Auerbach and Gorodnichenko (2012)).

recent crisis. Our paper belongs to the recent literature that examines the effectiveness of fiscal policy in the presence of financial frictions. Important papers in this literature include Bilbiie, Monacelli and Perotti (2013), Carrillo and Poilly (2013), Eggertsson and Krugman (2012) and Fernandez-Villaverde (2010). Bilbiie et al. (2013) and Eggertsson and Krugman (2012) use a Borrower-Saver model in which some agents' ability to optimise intertemporally is limited by the borrowing constraints that they face. Both studies suggest that fiscal policy is more effective in stimulating output in the presence of borrowing constraints, although the value of the spending/tax multiplier depends heavily on the share of debt-constrained borrowers in the economy. Carrillo and Poilly (2013) and Fernandez-Villaverde (2010), on the other hand, use models that accommodate the form of liquidity frictions suggested by Bernanke, Gertler and Gilchrist (1999) ("BGG"), in which firms' ability to borrow is determined by the market value of their net worth. Fernandez-Villaverde (2010) finds that the value of the spending multiplier is around one upon impact and falls quickly thereafter. His multiplier is larger than that suggested by standard models but smaller than ours.[5] Carrillo and Poilly (2013) find that financial frictions have a greater contribution to the value of the multiplier in a liquidity trap than in normal times. Their cumulative multiplier in the liquidity-trap case is 3.7,[6] which is almost twice as large as ours. Our paper differs from previous studies in the way that financial frictions are introduced. While the Borrower-Saver model and the BGG model focus on borrowing constraints, the DEFK model accounts for both borrowing constraints and asset resaleability constraints.[7] To generate a liquidity trap, Carrillo and Poilly (2013) assume that the capital returns perceived by entrepreneurs are affected by a risk-premium shock similar to

[5] As shown later in our results, although our post-shock impact multiplier in normal times is smaller than 1, it increases gradually over time. As a result, the cumulative multiplier we obtain (1.6) is substantially larger than 1.

[6] See Table 1 in the online appendix that can be found as supplementary material at http://dx.doi.org/10.1016/j.red.2013.01.004.

[7] Although the DEFK model focuses mainly on resaleability constraints, borrowing constraints also play a significant role in generating large fiscal multipliers. If there are no borrowing constraints, as discussed in KM, new investment could be wholly financed by issuing new equity. In that case, shocks to resaleability would have negligible impacts.

the one in Smets and Wouters (2007). Since the empirical relevance of this kind of shock is uncertain (see Chari, Kehoe and McGrattan (2009) for a detailed discussion), the DEFK model offers an alternative way to generate a liquidity-trap crisis. Despite the difference in the approach, our findings are in line with these studies, strengthening the conclusion that the fiscal multiplier is larger under imperfect financial markets.

The presence of asset resaleability constraints in the DEFK model has new implications for the transmission mechanism of fiscal policy. In the Borrower-Saver model or the BGG model, fiscal expansion works by increasing debtors' income or net worth, hence relaxing their financing constraints. In the DEFK model, by contrast, fiscal expansion works by improving entrepreneurs' liquidity since government bonds are more liquid than private assets. There have been papers in the theoretical literature that propose the liquidity role of government bonds (see, for example, Woodford (1990), Holmstrom and Tirole (1998) and Aiyagari and McGrattan (1998)). In the empirical literature, Krishnamurthy and Vissing-Jorgensen (2012) suggest that the low yield on US Treasuries is due to the safety and liquidity that they offer. Using US data for the period from 1926 to 2008, these authors find that the yield spread between Treasury bonds and less liquid assets reduces when the supply of Treasury bonds is abundant, showing evidence of an improvement in market liquidity during such times.

2 The Model with Liquidity Frictions

This section describes the special features of our model. The model that we use is proposed by DEFK, in which households are liquidity constrained and face shocks that tighten their liquidity. Government expenditure is absent in the original DEFK model. We introduce a role for government spending in the model for our study of the fiscal multiplier.

2.1 Households

The economy consists of a continuum of identical households. Each household consists of a continuum of members $j \in [0,1]$. In each period, members have an i.i.d. opportunity $\varkappa$ to invest in capital. Household members ($j \in [0,\varkappa)$) who receive the opportunity to invest are "entrepreneurs", whereas those who do not ($j \in [\varkappa, 1]$) are "workers". Entrepreneurs invest and do not work. Workers work to earn labour income. Each household's assets are divided equally among its own members at the beginning of each period. After members find out whether they are entrepreneurs or workers, households cannot reallocate their assets. If any household member needs extra funds, they need to obtain them from external sources. At the end of each period, household members return all their assets plus any income they earn during the period to the household.[8]

The representative household's utility depends on the aggregate consumption $C_t \equiv \int_0^1 C_t(j)\, dj$ as consumption goods are jointly utilised by its members. Each member seeks to maximise the utility of the household as a whole, which is given by:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{1}{1+\upsilon} \int_{\varkappa}^{1} H_s(j)^{1+\upsilon}\, dj \right], \tag{1}$$

where β is the discount factor, σ is the coefficient of relative risk aversion, and υ is the inverse Frisch elasticity of labour supply. Labour supply for entrepreneurs is zero (i.e. $H_t(j) = 0$). Each period, household members choose optimally among non-durable consumption, saving in bonds or equity and, if they are entrepreneurs, investment in capital. Details of their saving and investment options are as follows: (i) Entrepreneurs have the opportunity to invest in new capital (I_t) which costs p_t^I per unit. Each unit of capital goods generates a rental income of r_t^k, depreciates at a rate of δ and has a market value of q_t. The return on new capital is therefore

[8] The assumption that entrepreneurs and workers belong to the same household is based on Shi (2015). This is different from the setting in KM (2008), in which entrepreneurs and workers are two separate entities. As noted by DEFK, adopting this assumption increases the flexibility of the model to incorporate various modifications for sensitivity analysis.

$\frac{r^k_{t+1}+(1-\delta)q_{t+1}}{p^I_t}$. Entrepreneurs can borrow to invest. Borrowing is in the form of issuing equity, N^I_t, that entitles the holder to claim the future returns on the underlying capital goods. (ii) Household members can save in risk-free government bonds, L_t, which have a unit face value and pay a gross nominal interest rate, R_t, over the period t to $t+1$. (iii) Household members can also purchase the equity issued by other households, N^O_t, at the market price of q_t. As equity holders receive income from the underlying capital goods, the return on equity over t to $t+1$ is $\frac{r^k_{t+1}+(1-\delta)q_{t+1}}{q_t}$. The household's net equity is defined as $N_t \equiv N^O_t + K_t - N^I_t$.

At the beginning of each period, the household also receives dividends from intermediate-goods and capital-goods firms amounting to D_t and D^K_t respectively. The household pays lump-sum taxes, τ_t, to the government. The intertemporal budget constraint is:[9]

$$\begin{aligned} C_t + p^I_t I_t + q_t\left[N_t - I_t\right] + L_t &= \left[r^k_t + (1-\delta)\,q_t\right] N_{t-1} + \frac{R_{t-1}}{\pi_t} L_{t-1} \\ &\quad + \int_{\varkappa}^{1} \frac{W_t\,(j)}{P_t} H_t\,(j)\,dj + D_t + D^K_t - \tau_t \qquad (2) \end{aligned}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate at t and $W_t\,(j)$ is the nominal wage earned by type-j workers. Entrepreneurs and workers face different problems as explained below.

2.1.1 Entrepreneurs

In the steady state and the post-shock equilibria, the market price of equity q_t is always greater than the investment cost of new capital p^I_t. Hence, the return on new capital is strictly greater than those on equity and on government bonds. Entrepreneurs are rational and would invest all their available resources in new capital. To spare more funds for investment, entrepreneurs do not spend on consumption goods, i.e., $C_t(j) = 0$ for

[9] In this paper, stock variables at t show the amounts of stocks at the *end* of the period. This is different from the timing convention in DEFK, where the stock variables at t are defined as the amounts at the *beginning* of the period.

$j \in [0, \varkappa)$. They would also sell all their bond holdings so that $L_t(j) = 0$ for $j \in [0, \varkappa)$.[10] There are, however, borrowing and resaleability constraints if entrepreneurs want to obtain funds through equity: Entrepreneurs can borrow by issuing equity of only up to $\theta \in (0, 1)$ fraction of their new investment. Also, in each period, entrepreneurs can sell only up to $\phi_t \in (0, 1)$ fraction of their net equity holdings. Since borrowing and resaleability constraints are both binding, entrepreneurs' net equity evolves according to $N_t(j) = (1 - \phi_t)(1 - \delta) N_{t-1}(j) + (1 - \theta) I_t(j)$. Combining entrepreneurs' first order conditions for $C_t(j)$, $L_t(j)$ and $N_t(j)$ with the intertemporal budget constraint (2) gives the aggregate investment function:

$$I_t = \int_0^{\varkappa} I_t(j)\, dj = \varkappa \frac{\left[r_t^k + (1 - \delta) q_t \phi_t\right] N_{t-1} + \frac{R_{t-1}}{\pi_t} L_{t-1} + D_t + D_t^K - \tau_t}{p_t^I - \theta q_t} \tag{3}$$

2.1.2 Workers

Workers' consumption and saving decisions can be derived by considering the household as a whole. Workers choose C_t, L_t and N_t to maximise the household's utility (1), subject to the intertemporal budget constraint (2) and the investment decision of entrepreneurs (3). The first-order conditions give the respective Euler equations for bonds and equity:

$$C_t^{-\sigma} = \beta E_t \left\{ C_{t+1}^{-\sigma} \left[\frac{R_t}{\pi_{t+1}} + \frac{\varkappa \left(q_{t+1} - p_{t+1}^I\right)}{p_{t+1}^I - \theta q_{t+1}} \frac{R_t}{\pi_{t+1}} \right] \right\} \tag{4}$$

$$C_t^{-\sigma} = \beta E_t \left\{ C_{t+1}^{-\sigma} \left[\begin{array}{c} \frac{r_{t+1}^k + (1-\delta) q_{t+1}}{q_t} \\ + \frac{\varkappa\left(q_{t+1} - p_{t+1}^I\right)}{p_{t+1}^I - \theta q_{t+1}} \frac{r_{t+1}^k + (1-\delta) q_{t+1} \phi_{t+1}}{q_t} \end{array} \right] \right\} \tag{5}$$

These Euler equations reduce to the standard ones when $\varkappa = 0$. In the DEFK model, there is a premium on top of the standard returns on bonds and equity because households are liquidity-constrained. By choosing to

[10] Following DEFK, we assume that entrepreneurs cannot take negative positions in their government bond holdings.

hold one extra unit of government bonds at t instead of consumption, the bond-holder gains $\frac{R_t}{\pi_{t+1}}$ extra units of liquidity at $t+1$. Similarly, by choosing to hold one extra unit of equity at t instead of spending, the equity-holder receives $\frac{r^k_{t+1}+(1-\delta)q_{t+1}\phi_{t+1}}{q_t}$ extra units of liquidity at $t+1$. The extra liquidity allows them to profit from an investment opportunity if they become entrepreneurs at $t+1$.

2.2 Government Policies

The government's budget constraint is:

$$G_t + \frac{R_{t-1}L_{t-1}}{\pi_t} = \tau_t + L_t, \tag{6}$$

In addition, the fiscal rule requires that:

$$\tau_t - \tau = \psi_\tau \left[\left(\frac{R_{t-1}L_{t-1}}{\pi_t} - \frac{RL}{\pi} \right) \right] + \xi^\tau_t, \tag{7}$$

where the policy parameter $\psi_\tau > 0$. Variables without the time subscript represent steady-state values. The value of ψ_τ is low to reflect that the adjustment on taxes is slow compared to bond issue, so the government has to obtain funds for fiscal expansion mainly by issuing bonds. ξ^τ_t is an exogenous tax shock.

The central bank adopts a generalised Taylor rule similar to the one in SW (2007):

$$R_t = \max \left\{ R^{\rho_R}_{t-1} \left(R\pi^{\psi_\pi}_t \left(\frac{Y_t}{Y} \right)^{\psi_Y} \right)^{1-\rho_R} \left(\frac{Y_t}{Y_{t-1}} \right)^{\psi_{\Delta Y}},\ 1 \right\} \tag{8}$$

where ρ_R is the interest rate smoothing parameter, $\psi_\pi > 1$, and ψ_Y and $\psi_{\Delta Y}$ are both between zero and one. The zero lower bound on the nominal interest rate requires that R_t cannot be lower than 1.[11] The gross real interest rate is obtained by $r_t = \frac{R_t}{E_t(\pi_{t+1})}$.

[11] In the DEFK model, unlike in the standard model, the zero lower bound is not a constraint but an equilibrium condition. Households in this model are willing to hold bonds even if the nominal interest rate is negative because of the liquidity advantage that bonds provide.

2.3 Equilibrium and Solution Strategy

Other assumptions in the model are standard New Keynesian. In this paper, we study the policy multipliers for a government spending expansion and a lump-sum tax cut respectively. A government spending shock is measured as a percentage of GDP, $\widehat{G}_t \equiv \frac{G_t - G}{Y}$. We assume an AR(1) evolution of government spending: $\widehat{G}_t = \rho_G \widehat{G}_{t-1} + e_t^G$, where ρ_G is the persistence parameter. Similarly, a tax shock ξ_t^τ to the fiscal rule (7) is also measured as a percentage of GDP and evolves according to an AR(1) process: $\xi_t^\tau = \rho_\tau \xi_{t-1}^\tau + e_t^\tau$. Using the DEFK model, we study the fiscal multiplier under two scenarios: in normal times and in times of a liquidity crisis. We define normal times as the times when the fiscal policy shock is the only source of disturbances, whereas crisis times are when the economy is also struck by a liquidity shock. A liquidity shock refers to a sudden drop of private assets' resaleability, expressed by a fall in the value of the resaleability parameter ϕ_t from steady state. Evolution of $\widehat{\phi}_t \equiv \frac{\phi_t - \phi}{\phi}$ follows $\widehat{\phi}_t = e_t^\phi < 0$. In a liquidity crisis, large falls in output and inflation push the nominal interest rate to its zero lower bound.

We retain the nonlinear nature of the model in our simulation experiments. Since the competitive equilibria achieved following a liquidity shock can stay far away from the steady state for a long time, applying log-linearisation may lead to inaccurate results. Given the fact that, as it was under the 2009 American Recovery and Reinvestment Act (ARRA), the path of government spending is often expected for some periods after its announcement, we carry out deterministic simulations using Dynare based on the assumption of perfect foresight. Under this assumption, agents have perfect foresight on the paths of shocks and expect with certainty that no subsequent shock will follow in the future. In a deterministic simulation, Dynare generates the responses of variables from the realisation of a shock in the first period until the economy goes back to the steady state. To achieve this, Dynare solves a nonlinear system of simultaneous equations for every period by adopting a Newton-type method. We refer interested readers to Adjemian et al. (2011) for a detailed description of the algorithm.

Unlike DEFK, who assume that the resaleability parameter $\widehat{\phi}_t$ follows a two-state Markov process, we assume that $\widehat{\phi}_t$ stays below zero after a liquidity shock for a deterministic number of periods. In view of the findings by Carlstrom, Fuerst and Paustian (2012), our main conclusion that the fiscal multipliers are large in the DEFK model would not be affected if we assume a stochastic exit for the liquidity-trap crisis rather than a deterministic exit. Carlstrom et al. (2012) find that the fiscal multiplier can be unboundedly large in a liquidity-trap crisis with a stochastic exit because when the end date of the crisis is uncertain, the value of the fiscal multiplier can be inflated by the low probability event of the pegged interest rate lasting for a very long time. Although in reality it is hard to assess people's expectations on the probability distributions of shocks, our deterministic-exit assumption can nevertheless provide a lower-bound estimate of the value of the fiscal multiplier under a certain expected duration of the crisis. If we instead assume a stochastic exit, the fiscal multipliers we obtain would have been even larger.

3 Calibration

Most of the calibration in this paper is drawn from the estimations of SW, except for the parameters related to liquidity frictions, which largely follow DEFK. The calibrated values are summarised in Table 1. Two important parameters related to the borrowing constraint (θ) and the resaleability constraint (ϕ_t) jointly determine the amount of liquidity in the economy. DEFK use US data for the period from 1952 to 2008 to obtain the steady-state values of θ and ϕ at 0.185, meaning that entrepreneurs can sell up to 56% ($= 1 - 0.815^4$) of their equity holdings in one year. We follow DEFK in our calibration of θ and ϕ. A similar calibration is used by Shi (2015).

Other parameters related to capital investment are $\varkappa$, κ, γ and δ. Consistent with DEFK, we calibrate the i.i.d. opportunity to invest in each quarter ($\varkappa$) to 0.05, which equals to a 19% ($= 1 - (1 - 0.05)^4$) chance to invest in one year.[12] The capital adjustment cost parameter (κ) is set to

[12] As noted by DEFK, 5% is a conservative estimate of the investment opportunity in

Structural parameters:		
β	0.99	Discount factor
σ	1.39	Relative risk aversion
δ	0.025	Depreciation rate
γ	0.36	Capital share
κ	1	Capital goods adjustment cost parameter
ν	1.92	Inverse Frisch elasticity of labour supply
λ_f	0.11	Price mark-up
λ_ω	0.11	Wage mark-up
ζ_p	0.65	Price Calvo probability
ζ_ω	0.73	Wage Calvo probability
Parameters related to liquidity constraints:		
$\varkappa$	0.05	Probability of an investment opportunity
θ	0.185	Borrowing constraint at steady state
ϕ	0.185	Equity resaleability constraint at steady state
Policy parameters:		
ψ_π	2.03	Taylor-rule coefficient on inflation
ψ_Y	0.08	Taylor-rule coefficient on output
$\psi_{\Delta Y}$	0.22	Taylor-rule coefficient on change in output
ρ_R	0.81	Interest rate smoothing
ρ_G	0.80	Persistence of government spending
ρ_τ	0.80	Persistence of a tax shock
ψ_τ	0.1	Fiscal rule parameter

Table 1: Calibration

1 as in DEFK. γ and δ take on the conventional values of 0.36 and 0.025 respectively.

For the parameters that are standard in a DSGE model such as σ and ν, we assign values mainly by referring to the mode of the posterior estimates obtained by SW. The Calvo probabilities for prices (ζ_p) and wages (ζ_w) are 0.65 and 0.73 respectively. Following Chari, Kehoe and McGrattan (2000), we assume the curvature parameters of the Dixit-Stiglitz aggregators in goods and labour markets to be 10, meaning a markup of 0.11 in both markets.

We also adopt the estimates of SW for the values of the parameters governing the conduct of monetary policy. For the fiscal rule parameter (ψ_τ), we assign the value of 0.1 as in DEFK, implying that the adjustment of taxes to the government's debt position is gradual. We follow Christiano, Eichenbaum and Rebelo (2011) to set the persistence of government spending (ρ_G) at 0.8. The persistence of a lump-sum tax cut (ρ_τ) is set at 0.8.

Two steady-state ratios are exogenous: the public debt-to-GDP ratio ($L/4Y$) and the government spending share in GDP (G/Y). Following DEFK, we set the former to 40%. The latter takes the average value of government consumption share observed in the post-war United States of 18%. Inflation is zero at the steady state.

4 How Large Is the Government Spending Multiplier?

In the literature, studies of the fiscal multiplier usually focus on the impact multiplier which is defined as $\frac{dY_t}{dG_t}$, where dY_t and dG_t are the respective differences of output and government spending from their steady state at period t. As noted by Woodford (2011), this way of calculating the multiplier requires the output rise to follow the same shape of time path as that

the literature. We thus carried out numerical experiments to increase the value of $\varkappa$ and found that even a slight increase of $\varkappa$ to 5.5% would cause the condition that $q_t > p_t^I$ not to hold. Since such condition is crucial in deriving the first order conditions of entrepreneurs, we stick with DEFK's calibration to set $\varkappa$ at 5%.

of the government spending rise for the multiplier to be meaningful. We recognise in our simulations that the effects of fiscal stimulus on GDP are often delayed, so the time paths of the two can differ from each other substantially. For this reason, we instead focus on the cumulative multiplier, defined as $\frac{E_t \sum_{t=0}^{\infty} dY_t}{E_t \sum_{t=0}^{\infty} dG_t}$. If it is greater than one, it implies that any change in government spending has a spillover effect on GDP. We examine the value of the multiplier in normal times and in times of crisis. We define "normal times" as the cases where the economy is in the vicinity of the steady state. Liquidity frictions are present in the DEFK model even in normal times due to the borrowing and the resaleability constraints facing households. As noted in the previous section, we follow DEFK in our calibration of the liquidity-constraint parameters, θ and ϕ, at steady state. Since DEFK calibrate these parameters using US data for the period from 1952 to 2008, the amount of liquidity in our model in normal times reflects the average condition for that period.[13] In the DEFK model, a liquidity crisis occurs when the resaleability constraint on equity tightens, simulating the condition when the financial crisis started in 2008.

4.1 The Multiplier in Normal Times

We use the DEFK model to calculate the government spending multiplier in normal times by giving the steady state a positive government spending shock of 1% of GDP. Government spending follows an AR(1) process with a persistence of 0.8. We obtain the cumulative multiplier on output at 1.61.

How does this result compare with that obtained using a standard New Keynesian DSGE model? We carry out a control experiment by stripping all liquidity-constraint features from the DEFK model.[14] With the same gov-

[13] In a speech in 2005, Alan Greenspan suggested that access to credit had become unproblematic to the vast majority of households. Specifically, he noted that "[w]ith these advances in technology, lenders have taken advantage of credit-scoring models and other techniques for efficiently extending credit to a broader spectrum of consumers...". The period that Greenspan was referring to is the time just before 2005, when the subprime bubble was forming. Arguably, that should not represent the liquidity in "normal times".

[14] In this standard DSGE model, investment opportunities are not scarce. Investing in

ernment spending shock, the model without liquidity frictions (henceforth the "standard model") predicts the cumulative multiplier on output to be 0.55. In Rows 2 - 4 of Table 2, we summarise the cumulative government spending multipliers obtained using the two models in the normal-times scenario.

Figure 1 reports the impulse-response functions (IRFs) of the key macroeconomic variables to a government spending shock. In the standard model, both investment and consumption are crowded out and the increase in output is moderate and short-lived. The IRFs generated by the DEFK model are very different for some variables, especially investment. Following the government spending shock, private investment falls slightly but then rises in a hump-shaped manner after two quarters. The positive effect on investment peaks around ten quarters after the shock and persists until thirty quarters after the shock.[15] Consumption shows a similar hump-shaped pattern, rising above the steady state from the 10th quarter onwards. It returns to its steady-state value only after about 80 periods from the shock. Accordingly, the increase in output in the DEFK model is larger and more persistent. As consumption and investment decrease in both models upon impact, the impact multipliers on output are not too different (0.70 in the DEFK model vs. 0.58 in the standard model). However, the cumulative multiplier on output obtained using the DEFK model (1.61) is almost three times that obtained using the standard model (0.55).

Our impulse response analysis suggests that government spending expansion has positive spillover effects on consumption and investment in the DEFK model. We also compute the cumulative multipliers on consumption and investment in both the standard and the DEFK models. These multipliers measure the expected cumulative increases in consumption and investment respectively, given a one-dollar cumulative increase in government

capital is not more profitable than holding other assets. The investment function hence reverts to a standard Euler equation. We use the calibration shown in Table 1 with the exception of β, which is adjusted to 0.9943 to keep the steady-state interest rate in line with that in the DEFK model.

[15] Upon impact, investment decreases slightly because an increase in bond holdings in period t only has an effect on investment in $t + 1$.

		Standard model	DEFK model
1	Impact multiplier on output	0.58	0.70
	Cumulative multiplier on:		
2	Output	0.55	1.61
3	Consumption	-0.35	0.27
4	Investment	-0.11	0.39
	Cumulative multiplier due to liquidity effect on:		
5	Output	-	0.89
6	Consumption	-	0.54
7	Investment	-	0.41
	Cumulative multiplier on output:		
8	(*i*) $\zeta_{p,w} = 0$	0.09	0.90
9	(*ii*) $\zeta_w = 0$	0.16	0.97
10	(*iii*) $\zeta_p = 0$	0.51	1.59

Table 2: Government spending multipliers in the DEFK model and in the standard model in normal times under different scenarios

Notes: Rows 1-7 report the multipliers in the baseline case: Row 1 reports the impact multipliers on output. Rows 2-4 report the cumulative multipliers on output, investment and consumption, while Rows 5-7 report the same only due to the liquidity effect. Rows 8-10 report the cumulative multipliers on output under different degrees of nominal rigidities: fully flexible prices and wages (Row 8); sticky prices and flexible wages (Row 9); and flexible prices and sticky wages (Row 10).

spending. As shown in Table 2, both the investment and the consumption multipliers are positive in the DEFK model. In the standard model, both of these multipliers are negative, so the cumulative multiplier on output is smaller than one.

The consumption behaviour predicted by the DEFK model is in line with the findings of the empirical studies based on standard VAR, which typically suggest that government spending crowds in consumption (see Blanchard and Perotti (2002) and Gali, Lopez-Salido and Vallés (2007), among others). On the other hand, empirical analyses that identify fiscal policy shocks using war dates usually suggest that consumption falls in response to a government spending rise (see, for example, Burnside, Eichenbaum and Fisher (2004) and Ramey (2011a, b)). Turning to the behaviour of investment, empirical evidence tends to suggest that government spending crowds out private investment. While Burnside, Eichenbaum and Fisher (2004) find a small, transitory increase in investment in response to a positive government spending shock, Blanchard and Perotti (2002), Gali, Lopez-Salido and Vallés (2007) and Ramey (2011a, b) find the opposite. The difference between our results obtained using the DEFK model and those of the empirical literature may be due to how fiscal expansion is financed. The empirical studies tend to focus on samples in which much of the spending was financed by distortionary tax increases, whereas in our paper, we focus on debt-financed expansion.

To understand why the DEFK model generates different results to the standard model's, let us first consider the mechanism at work in the standard model. In the standard model, while an increase in government spending creates aggregate demand which increases in output, it also creates inflation pressures, causing the central bank to tighten monetary policy. Both investment and consumption are crowded out by the rising interest rate. In addition, forward-looking households anticipate future tax increases and react by reducing consumption. The negative wealth effect induces workers to work more, leading to increases in labour supply. However, the overall increase in output is smaller than the increase in government spending.

The mechanism at work in the DEFK model is different from the one in the standard model in that an increase in government spending in the DEFK model also affects liquidity through an increase in the supply of government bonds, which we define as the "liquidity effect" of fiscal expansion. In the DEFK model, households are liquidity constrained in a way that entrepreneurs want to obtain funds to make profitable investments but cannot. The government, on the other hand, is not bound by liquidity constraints. As the government issues a bond to a household to be repaid by higher taxes on the household in the future, the government is in effect borrowing on behalf of the household at the risk-free interest rate. For this reason, a fiscal expansion financed mainly by bonds generates extra liquidity to the households. The improvement in liquidity is reflected in the reduction in the spread between liquid and illiquid assets, defined as $E_t\left[\frac{r^k_{t+1}+(1-\delta)q_{t+1}}{q_t}-\frac{R_t}{\pi_{t+1}}\right]$. Our model shows that the quarterly spread reduces by 3 basis points following the government spending expansion.

We carry out an experiment to isolate the liquidity effect of the government spending rise in the DEFK model. We consider the hypothetical case where government spending does not use output, so that aggregate demand is immune to any changes in government spending. Given the same amount of government bonds issued as in the baseline case, we obtain the cumulative multipliers solely due to the liquidity effect, which are reported in Rows 5 - 7 of Table 2. Both the consumption and the investment multipliers due to the liquidity effect are positive, suggesting that consumption and investment are crowded in by an improvement in liquidity. The intuition is as follows: A government spending expansion in the DEFK model is financed mainly by public debt since tax adjustments are slow. As the government increases their spending, higher real interest rates and future tax burdens cause households to increase their bond holdings, thus improving households' liquidity since government bonds are liquid. When an attractive investment opportunity arrives, rational entrepreneurs sell all their liquid assets to obtain funds to invest in new capital. Investment therefore increases following the government spending rise.[16]

[16] Following Shi (2015), DEFK assume that entrepreneurs and workers in a household

The increase in investment has a knock-on effect on consumption. The fact that consumption becomes positive later than investment reinforces this insight (see Figure 1). The intuition for the positive multiplier on consumption is as follows. Due to intertemporal substitution effects, rising interest rates cause workers to respond to the government spending shock initially by reducing consumption. As we assume that government spending follows an AR(1) process, the increase in government spending dissipates over time. As government spending falls, the real interest rate decreases. Workers then gradually increase their consumption. As capital is still being produced, reflected by the persistently higher-than-usual level of investment, the demand for labour is greater than steady state. A greater demand for labour translates into higher real wages, allowing workers to increase consumption spending. Indeed, as the IRFs show, consumption closely follows the dynamics of real wages.

4.2 Key Determinants of the Size of the Multiplier

Due to the presence of liquidity constraints, Ricardian equivalence does not hold in the DEFK model. Changes in taxes affect households' behaviour so the value of the multiplier should be sensitive to the fiscal rule. We carry out sensitivity analysis on the fiscal rule parameter, ψ_τ, which measures how quickly the government increases taxes following bond issues. In the baseline, ψ_τ is set to 0.1 following DEFK to reflect that a slow rise in taxes. If we increase ψ_τ to 1, the cumulative multiplier on output in the DEFK model reduces to 0.67. This result indicates that the government should delay increasing taxes to ensure effective expansionary policy. We also test our results by adopting one of the fiscal rules estimated by Leeper, Plante and Traum (2010). Different from our fiscal rule, Leeper et al. (2010)'s rule in-

pool their assets at the beginning of each period. When pooling is not allowed, as in KM (2008), entrepreneurs and workers are separate entities and the opportunity for entrepreneurs to invest is scarce. In that version of the model, an increase in government borrowing would increase the bond holdings of non-investing entrepreneurs. This would provide investing entrepreneurs with more liquidity when an investment opportunity arrives. Therefore, even without the asset-pooling assumption, the DEFK model still suggests a large multiplier effect on investment.

cludes output growth, which acts as an "automatic stabiliser" to the cyclical position of the economy. Following Leeper et al. (2010), we calibrate the coefficient of output growth in the rule at 0.13 and hold the coefficient of debt constant.[17] The results suggest that the inclusion of the automatic stabiliser in the rule does not affect the value of the fiscal multiplier significantly.

The stickiness of prices and wages also plays a role in generating a large fiscal multiplier. Rows 8-10 of Table 2 present the cumulative multipliers on output that we obtain with different degrees of nominal rigidities given the same government spending shock. Row 8 ($\zeta_{p,w} = 0$) shows the results under fully flexible prices and wages. Absent both price and wage stickiness, the standard model gives a very low cumulative output multiplier of 0.09. The DEFK model suggests a much larger multiplier (0.90), although it is small compared to the baseline case (1.61). Row 9 ($\zeta_w = 0$) shows the results obtained with fully flexible wages but sticky prices; whereas Row 10 ($\zeta_p = 0$) shows those obtained with sticky wages and fully flexible prices. With price stickiness alone, the multipliers are not too different from those obtained absent nominal rigidities ($\zeta_{p,w} = 0$). With wage stickiness alone, on the other hand, we are able to obtain multipliers similar to those in the baseline case. These results suggest that, in the DEFK model, both liquidity frictions and nominal rigidities play a key role in generating large fiscal multipliers.

To understand the reasons why nominal rigidities can lead to larger multipliers, we consider the IRFs of the key macroeconomic variables to a government spending shock in the DEFK model under different degrees of nominal rigidities (Figure 2). Let us first discuss the case with fully flexible nominal prices and wages. Although nominal rigidities are absent, government spending expansion leads to a negative wealth effect, inducing

[17] Leeper, Plante and Traum (2010) estimate the fiscal rules for various taxes using US quarterly data for the period from 1960 to 2008. Their estimation results imply that in a fiscal rule in the form of equation (7), the coefficient of government debt is 0.06 (compared to 0.1 in DEFK's calibration). This suggests that lump-sum taxes in reality are less responsive to changes in the level of government debt. Calibrating the coefficient of government debt at 0.06 gives a larger multiplier but does not change our main conclusions.

households to work more. Increased labour supply increases output, as indicated by the positive responses of output. If only prices are sticky but wages are flexible, the multiplier is larger than without nominal rigidities since the markup by firms becomes smaller. This is true because prices respond sluggishly in response to the increase in marginal cost caused by an increase in government spending. As noted by Christiano et al. (2011), a reduced markup leads to an outward shift of the labour demand curve. This increases employment and leads to a larger increase in output than in the case without nominal rigidities. On the other hand, if only wages are sticky but prices are fully flexible, the multiplier is even larger than in the case with price stickiness alone for the following reason. With wage stickiness alone, although the markup is constant as prices adjust immediately in full proportion to the increase in marginal cost, nominal wages do not increase as much as they do in the case with flexible wages. Muted wage responses in response to an increase in government spending allow firms to hire more, resulting in larger output rises. Indeed, as Figure 2 shows, real wages in the case with wage stickiness alone are much lower than in the case with price stickiness alone. The multiplier is largest in the case with both price and wage stickiness compared to all other cases considered here, since a fiscal expansion in this case results in a lower markup and also a higher labour demand by firms due to the sluggish adjustments in nominal wages.

The results reported in Christiano et al. (2011) and Woodford (2011) suggest that the government spending multiplier is smaller as the persistence of government spending (ρ_G) increases. We repeat our experiments by increasing ρ_G from 0.8 to 0.97, which is the estimate suggested by SW. The cumulative multiplier on output in the DEFK model reduces to 1.04 in this case, whereas the one in the standard model falls to only 0.27. The reason for this result is that as the government spending rise is more persistent, the present value of the associated tax rises also increases, causing larger negative wealth impacts on consumption. The rise in output is therefore much smaller, resulting in a much smaller government spending multiplier. However, our conclusion that the multiplier is larger in the DEFK model than in the standard model remains unchanged.

We also carry out sensitivity analysis on the monetary policy rule. Instead of (8), we assume that the central bank follows a standard Taylor rule with $\psi_\pi = 1.5$, $\psi_Y = 0.125$ and no interest rate inertia. In this case, the cumulative multiplier on output in the DEFK model is slightly higher at 1.8, whereas the one in the standard model (0.6) is almost the same as the baseline. These results seem to confirm that the multiplier is larger in the DEFK model regardless of the monetary policy rule.

4.3 The Multiplier in Times of Crisis

We now examine the value of the government spending multiplier in times of crisis. In the DEFK model, a liquidity crisis occurs when the value of the resaleability constraint parameter, ϕ_t, falls by 60% from steady state. The crisis brings about a liquidity trap. If the government decides to increase spending during a crisis, we assume that it happens in the same period as the arrival of the liquidity shock ($t = 1$). The cumulative government spending multiplier on output in a crisis is obtained by $\frac{E_t \sum_{t=0}^{\infty}(dY_t - dY_t^*)}{E_t \sum_{t=0}^{\infty} dG_t}$, where dY_t denotes the change in output due to the combined effects of the liquidity shock and the government spending shock, and dY_t^* denotes the same due to the liquidity shock alone by holding G_t constant. The difference between the two measures the output change that is due to fiscal stimulus. The multipliers on consumption and investment are calculated in the same way, with Y_t being replaced by C_t and I_t respectively.

Using the DEFK model, we simulate liquidity crises of various expected durations, and compute the cumulative multipliers in response to a government spending shock of 1% of GDP with $\rho_G = 0.8$.[18] This exercise cannot

[18] The size of the government spending shock is the same as that in the first section of Cogan et al. (2010). Erceg and Linde (2012) find that the value of the multiplier can be affected by the size of the fiscal stimulus when the liquidity trap is endogenous. The larger is the fiscal stimulus, the faster the economy exits the liquidity trap, causing a smaller multiplier. We test our results by increasing the size of the shock to 2% of GDP. We find that in normal times, the multipliers are unaffected; in times of crisis, the multipliers decrease only slightly (by around 0.1 on average).

Duration of crisis	Duration of liquidity trap	Cumulative multipliers on: *Output*	*Consumption*	*Investment*
1q	1q	2.00	0.68	0.32
4q	3q	2.09	0.78	0.32
8q	6q	2.17	0.86	0.32
12q	10q	2.22	0.91	0.34
16q	14q	2.27	0.95	0.34
20q	18q	2.28	0.97	0.34

Table 3: Government spending multipliers on output, consumption and investment in times of crisis in the DEFK model

be carried out using the standard model as it does not allow for financial frictions. Table 3 shows the cumulative multipliers and the number of periods in which the nominal interest rate falls to zero. Our results suggest that the longer is the liquidity crisis, the longer the liquidity trap is. In addition, the longer is the liquidity trap, the larger the fiscal multiplier is. The DEFK model implies the value of the cumulative multiplier on output ranges between 2.00 and 2.28 in the crisis state, which is much higher than that in normal times.

To determine the cause of a larger multiplier in the crisis state, we report in Figures 3 and 4 the IRFs to a liquidity shock that is expected to last for three years, for the cases with and without government spending expansion.[19] We first discuss the case without fiscal expansion. The liquidity shock leads to a large decrease in the resaleability of equity, so that entrepreneurs can obtain fewer funds for investment by selling their equity. Figure 3 shows that the fall in investment at $t = 1$ is as large as 19%. This substantial fall in investment seems to suggest that in the DEFK model, most new investment is financed by the sales of entrepreneurs' asset holdings, rather than the issues of new equity. Consumption, output and employment fall by significant amounts upon impact. Both output and consumption fall by around 10%, while labour hours fall by around 15%.[20] Reflecting the flight

[19] Note that the IRFs are not smooth in this case. Most of the lines bend upwards after 12 quarters from the shock, when the economy is expected to exit from the crisis.

[20] The fall in economic activity we obtain here is more severe than that suggested by

to liquidity, households' bond holdings increase by around 4% and continue to rise in a hump-shaped manner. The nominal interest rate falls to its zero lower bound in response to the liquidity shock and remains zero-bound for ten quarters. Inflation decreases by 3.7 percentage points, and because of the zero-bound nominal interest rate, the real interest rate increases by around 2 percentage points.

We now consider the case with government spending expansion. Similar to the case in normal times, the increases in public demand and liquidity lead to an increase in aggregate demand. As a result, inflation falls by less. Given the zero-bound nominal interest rate, the real interest rate increases by less relative to the case without fiscal stimulus, leading to smaller falls in consumption and hence in output. A natural question arises: why is the fiscal multiplier larger in the crisis state than in normal times? The reason is that the multiplier effect on consumption is larger at the zero lower bound. To confirm this, we also report in Table 3 the cumulative multipliers on consumption and investment in crisis times. Indeed, the consumption multiplier is larger than that in normal times and increases substantially as the liquidity trap lengthens, whereas the investment multiplier is similar to that in normal times (see Table 2). The positive responses of consumption and investment are consistent with the empirical findings reported by Auerbach and Gorodnichenko (2012), who show that fiscal expansion crowds in consumption and investment during recessions.

To gain an insight into the role that liquidity constraints play in generating large fiscal multipliers in crisis times, we also calculate the cumulative government spending multipliers on consumption, investment and output in the zero-bound state using the standard model. Since financial frictions are absent in the standard model, we cannot simulate a liquidity crisis in the same way as we do with the DEFK model. Instead, we follow Cogan et al. (2010) to assume that the nominal interest rate in the standard model

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DEFK. In DEFK, the government carries out quantitative easing in a liquidity crisis by buying private assets and selling government bonds in the open market. Such policy improves liquidity in the economy and helps alleviate the adverse effects of a liquidity shock. In this paper, we focus our study on the effectiveness of fiscal policy. Therefore, to simplify our model, we assume that no quantitative easing is carried out in a crisis.

Duration of	Cumulative multipliers on:		
liquidity trap	*Output*	*Consumption*	*Investment*
1q	1.65	0.47	0.18
4q	1.84	0.61	0.23
8q	1.66	0.48	0.18
12q	1.42	0.30	0.12
16q	1.24	0.17	0.08
20q	1.13	0.09	0.05

Table 4: Government spending multipliers on output, consumption and investment in the standard model with an imposed zero bound

remains constant at its steady-state value for various durations. The results are reported in Table 4.

The government spending multiplier is still larger in the DEFK model than that in the standard model when the nominal interest rate is constant due to the larger multipliers on both consumption and investment. In the standard model, the value of the output multiplier is driven mainly by the multiplier on consumption. The investment multiplier is very small. In addition, as the crisis prolongs, the output multiplier in the standard model increases in a hump-shaped manner, reaching its peak when the zero-bound state lasts for one year. This finding is related to the observation by Christiano et al. (2011) and Woodford (2011), who suggest that the fiscal multiplier is largest if the fiscal expansion lasts exactly as long as the zero-bound state. Since we assume that government spending evolves according to an AR(1) process with a persistence parameter of 0.8, the majority of the public spending rises in our model occurs within the first four quarters after the shock. The government spending multiplier is largest when the zero-bound state lasts for a similar duration. As the liquidity trap lengthens, the fiscal stimulus becomes less effective and the value of the multiplier decreases.

4.4 A More Realistic Path of Government Spending

Thus far, we have assumed that government spending follows an AR(1) process. While such a process is useful for understanding the possible effects of fiscal expansion on the economy, the path of government purchases under this assumption is inconsistent with the actual one implied by the 2009 American Recovery and Reinvestment Act (ARRA). We plot in Figure 5a the actual increase in government purchases as a share of GDP under the ARRA. While the AR(1) process suggests a large, immediate increase in government spending that dissipates over time, the increase in government purchases under the ARRA is more gradual and reaches its peak only after about a year.

To obtain the path of government purchases under the ARRA, we follow the same approach as in Cogan et al. (2010). Specifically, on top of the increase in federal purchases, we add 60% of the increase in transfers to states and localities. Following Romer and Bernstein (2009), Cogan et al. (2010) assume that 60% of intergovernmental transfers result in purchases of goods and services. We then divide the resulting sum by the actual US GDP data.[21]

We recalculate the fiscal multipliers in crisis times under this more realistic path of government purchases as implied by the ARRA, holding all other assumptions the same as in the previous subsection. The cumulative multipliers on output obtained in this case are almost the same as those under the assumption of an AR(1) process for the government spending shock. The output multiplier for a 3-year crisis obtained in this case is 2.34, compared to 2.22 under the AR(1) government spending shock (Table 3). Therefore, our conclusion that the multiplier is large in a liquidity crisis still holds. We also report in Figure 5b. the impulse-responses of output in a 3-year crisis with and without fiscal stimulus. As it is evident from the figure, without fiscal stimulus the fall in output would have been larger by around 1 percentage point at the early stage of the crisis, suggesting that the fiscal interventions

[21] The ARRA data is available at: http://www.bea.gov/recovery/pdf/arra-table.pdf. Total government purchases are obtained by adding 60% times the numbers in row 18 to those in row 6 of the table.

in the US during the recent financial crisis could have saved the economy from a deeper recession.

5 The Tax Multiplier

What if the government instead chose to stimulate growth by cutting taxes? In this section, we study the policy multiplier for a temporary cut in taxes with the DEFK model. We assume a lump-sum tax cut of 1% of GDP, which follows an AR(1) process with a persistence of 0.8. The cumulative tax multiplier, defined as the expected cumulative increase in output given a one-dollar cumulative cut in taxes, or $\frac{E_t \sum_{t=0}^{\infty} dY_t}{-E_t \sum_{t=0}^{\infty} d\tau_t}$, is obtained using the DEFK model. The multiplier we obtain in normal times is 0.84, while the one in a 3-year crisis is 1.41. The tax multiplier in the standard model, by contrast, is zero due to Ricardian equivalence. A tax cut in the DEFK model works mainly through the same liquidity effect as for a government spending expansion: a fall in tax revenue causes the government to issue more bonds, thereby increasing the proportion of liquid assets in households' portfolios. Improvement in liquidity increases investment, consumption and output. The reason why the tax multiplier is larger in crisis times than in normal times is the same as that for the case with government spending: an increase in economic activity due to liquidity improvement reduces deflation in a financial crisis. As the nominal interest rate is zero-bound, it causes a fall in the real interest rate and hence promotes consumption. To demonstrate the role of the liquidity effect in stimulating output after a tax cut, we also obtain the tax multipliers in the DEFK model by holding the amount of government bonds constant following the tax cut. In this case, the tax multiplier in normal times falls to almost zero, while the one in a 3-year crisis falls to only 0.26.

A comparison of the tax multiplier and the government spending multiplier suggests that government spending expansion is more effective in stimulating output. In the DEFK model, a government spending expansion works by increasing liquidity and creating aggregate demand. A tax cut, on the other hand, resembles a lump-sum transfer to households. While it relaxes households' liquidity constraints, it does not create aggregate demand directly.[22] Nevertheless, the tax multipliers that we obtain using the DEFK model are still much larger than those suggested by the standard model with frictionless financial markets.

6 Does the Initial Debt-to-GDP Ratio Matter?

Following DEFK, we calibrate the steady-state government debt-to-GDP ratio at 0.4. In this section, we perform a sensitivity analysis to see how the size of the fiscal multiplier depends on the steady-state debt-to-GDP ratio.[23] Figures 6a. and 6b. report the results from our analysis. Figure 6a. shows the value of the government spending multiplier as a function of the initial debt-to-output ratio under three scenarios: (a) during normal times, (b) during a 12-quarter liquidity crisis but without a zero lower bound ("ZLB"), and (c) during a 12-quarter liquidity crisis and with a ZLB. Figure 6b. reports the same for the tax multiplier. This exercise also helps us quantify how the spending and the tax multipliers depend on a liquidity crisis and a ZLB separately.

The results reported in Figures 6a. and 6b. suggest that the size of the multiplier is sensitive to the initial debt-to-GDP ratio. In both normal times and crisis times, the government spending multiplier becomes smaller as the initial debt-to-GDP ratio increases. The intuition of this result is that with a higher debt-to-GDP ratio at steady state, liquidity is more abundant to start with. The improvement of liquidity resulted from a fiscal expansion would therefore have smaller stimulative effects on output. During a liquidity crisis, the fiscal multiplier without a ZLB is smaller than that with a ZLB, but still larger than that in normal times, implying that both

[22] Using a standard DSGE model, Eggertsson (2011) also finds that fiscal policies that aim directly at stimulating aggregate demand are more effective.

[23] When changing the initial debt-to-GDP ratio, we adjust the value of beta to make sure that the steady-state interest rate falls within a reasonable range.

the presence of a ZLB and the deterioration of liquidity contribute to the larger multiplier in a crisis. Our results further suggest that, if the initial debt ratio is low (e.g. 0.2), the ZLB constraint will cause the multiplier to increase by more in a crisis than in the case with a high initial debt ratio. When the initial debt-to-GDP ratio is higher at 0.6, the effect of the ZLB on the size of the multiplier is smaller, probably because of the higher steady-state liquidity in that case. Figure 6b. shows that the main results for the government spending multiplier also hold for the tax multiplier. A comparison of Figures 6a. and 6b. shows that the multiplier resulting from an increase in government spending is larger than that from a tax cut, confirming our earlier findings.

Our results in this section have an important policy implication. Given the finding that fiscal policy becomes less effective with a higher initial debt-to-GDP ratio, policymakers may strive to keep the debt-to-GDP ratio low in normal times and use fiscal stimulus only in times of crisis in order to maximise the stimulative effects on output.

7 Summary and Conclusions

In this paper, we have extended the DEFK model by introducing a role for government spending. We use the resulting model to study the effects of fiscal policy shocks on the macroeconomy. The DEFK model accounts for liquidity constraints and generates a liquidity-trap crisis when the asset resaleability constraint tightens. Our main finding is that government spending expansion can be highly effective in an economic environment in which government bonds are liquid and private financial assets are only partially liquid. In this model, a bond-financed fiscal expansion increases the proportion of liquid assets in the private-sector wealth through an increase in the supply of government bonds. An improvement in liquidity has positive effects on private investment, consumption and output, therefore generating a large fiscal multiplier. Furthermore, using the DEFK model, we find that the tax multiplier is positive but smaller than the government spending multiplier since a lump-sum tax cut improves the private-sector liquidity but does not directly create aggregate demand.

We also study the effectiveness of fiscal stimulus in a liquidity crisis. In the DEFK model, a negative shock to liquidity reduces the resaleability of private assets and brings about a liquidity trap. As the multiplier effect on consumption is larger when the nominal interest rate is bound at zero, the fiscal multiplier we obtain is even larger than that in normal times. This result is consistent with previous research findings which suggest that, relative to the case without fiscal expansion, an increase in public demand at the zero lower bound pushes up prices, lowers the real interest rate and stimulates consumption (see, for example, Christiano, Eichenbaum and Rebelo (2011)).

Using the DEFK model in crisis mode, we also study the effects on output of the fiscal interventions under the 2009 American Recovery and Reinvestment Act (ARRA). The fiscal multiplier is large upon a shock that simulates the path of government purchases increase under the ARRA, suggesting that the fiscal interventions by the US government during the recent crisis may have prevented a deeper recession. This finding may explain why the economic downturn during the Great Recession was less severe in the US than in countries such as Germany and Sweden, where the government strived to contain their deficits and to keep their debt-to-GDP ratio constant. In 2009, the fall in GDP relative to the previous year was around 3% in the US, while in Germany and Sweden, the falls were larger at around 5%.

Finally, we find that the effectiveness of fiscal policy is sensitive to the steady-state debt-to-GDP ratio. Fiscal stimulus becomes less effective as the initial debt-to-GDP ratio increases. This finding has an important policy implication: Governments may want to contain the public debt ratio in normal times to obtain more effective results from fiscal stimulus during deep recessions, when the stimulative effects are most needed.

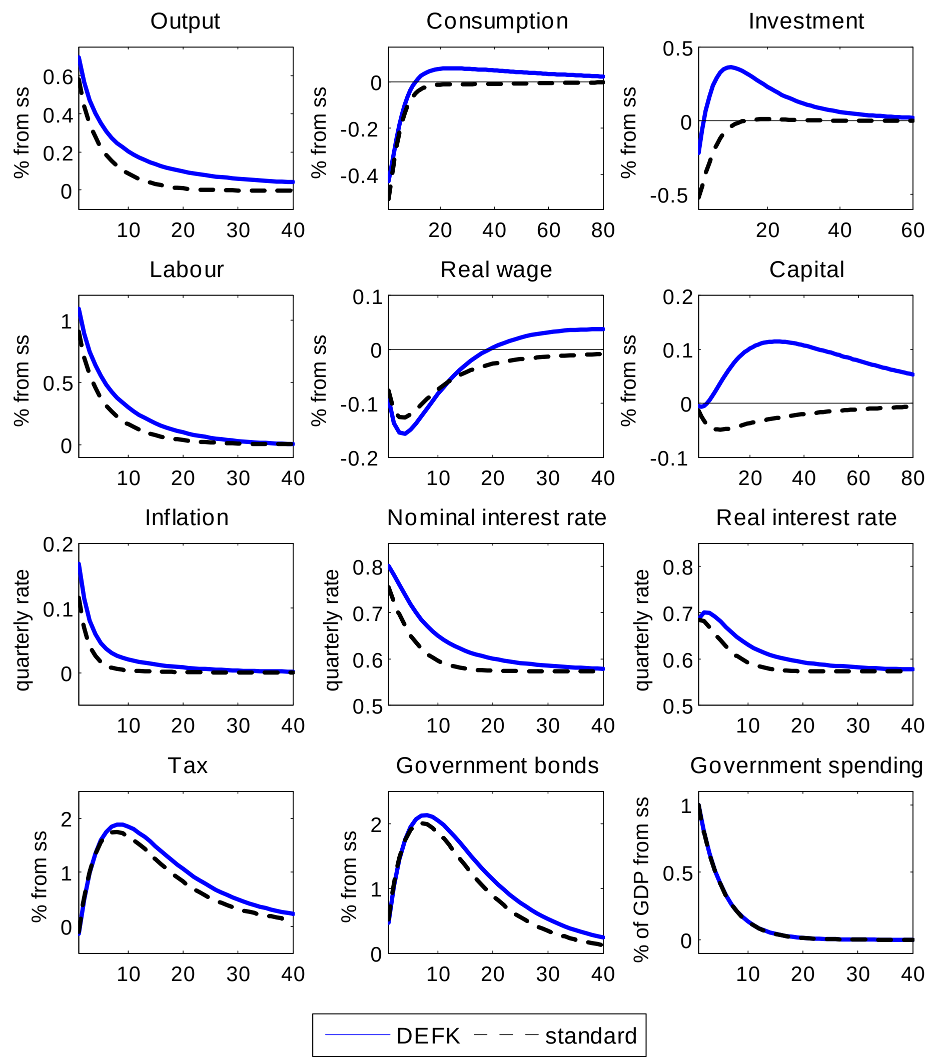

Figure 1: IRFs to a government spending shock in normal times: the DEFK model vs. the standard model

Notes: The dotted lines show the IRFs generated by the standard model, while the solid lines show the ones generated by the DEFK model.

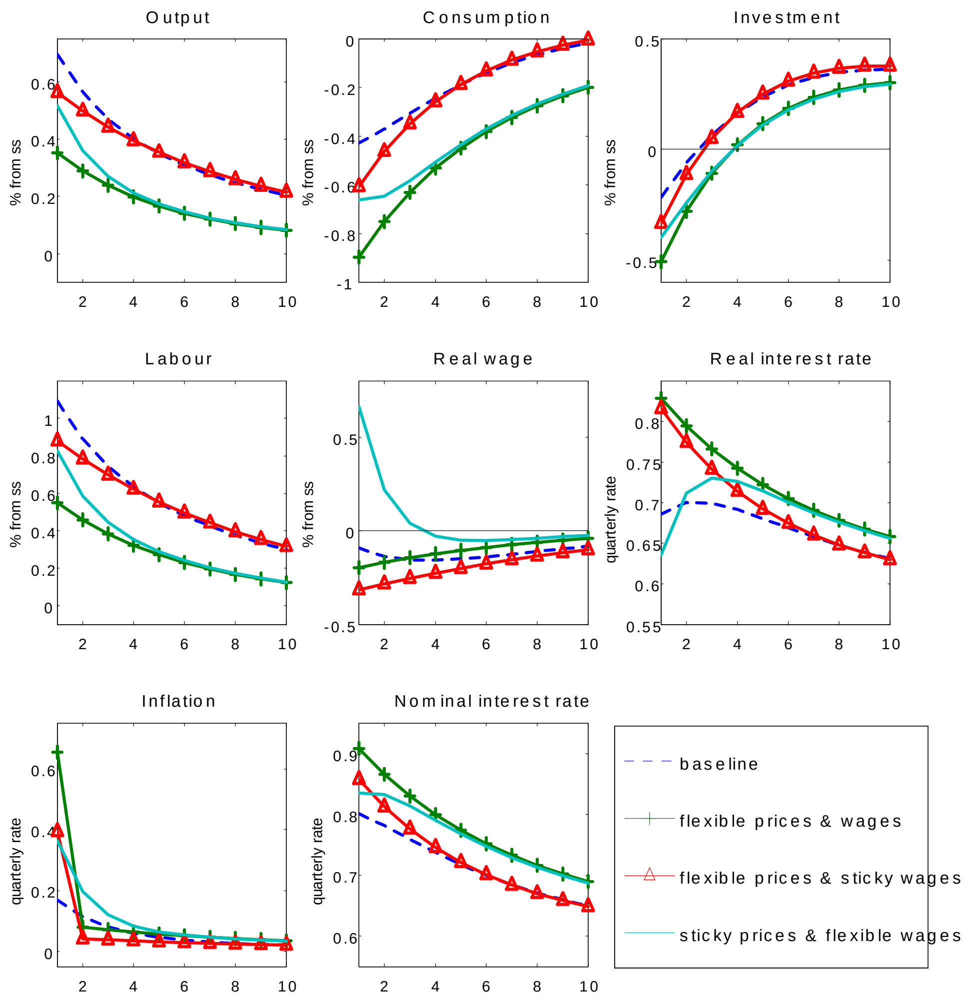

Figure 2: IRFs to a government spending shock in the DEFK model under different degrees of nominal rigidities

Notes: The crossed lines show the IRFs when both prices and wages are fully flexible, while the dotted lines show the ones when both of them are sticky. The lines with triangles show the IRFs with wage stickiness alone, while the solid lines show the IRFs with price stickiness alone.

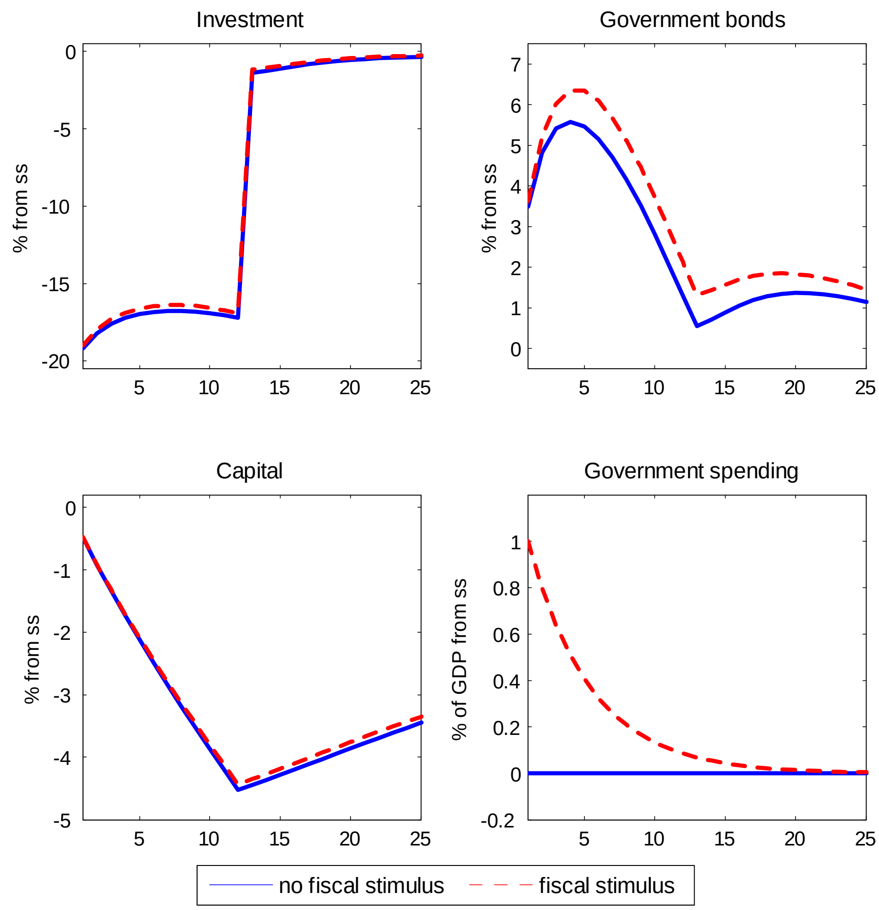

Figure 3: IRFs to a three-year liquidity crisis in the DEFK model: Effects of fiscal stimulus

Notes: The solid lines show the IRFs to a three-year crisis in the DEFK model without fiscal stimulus, while the dotted lines show the ones with fiscal stimulus.

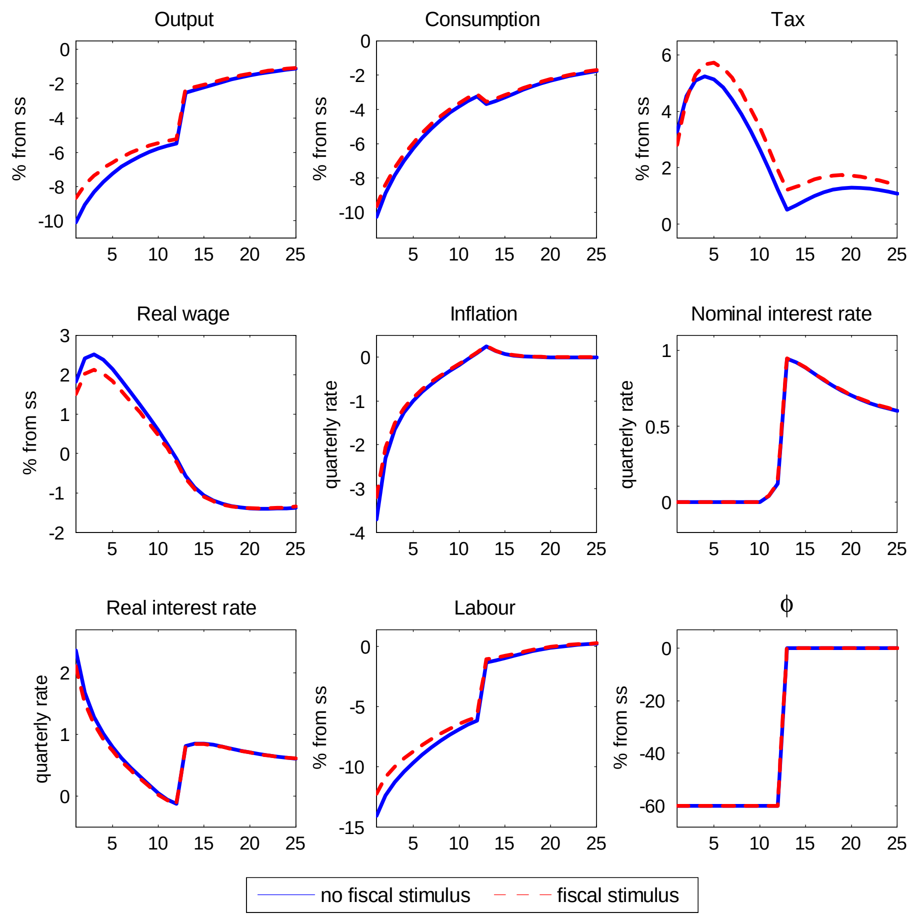

Figure 4: IRFs to a three-year liquidity crisis in the DEFK model: Effects of fiscal stimulus

Notes: The solid lines show the IRFs to a three-year crisis in the DEFK model without fiscal stimulus, while the dotted lines show the ones with fiscal stimulus.

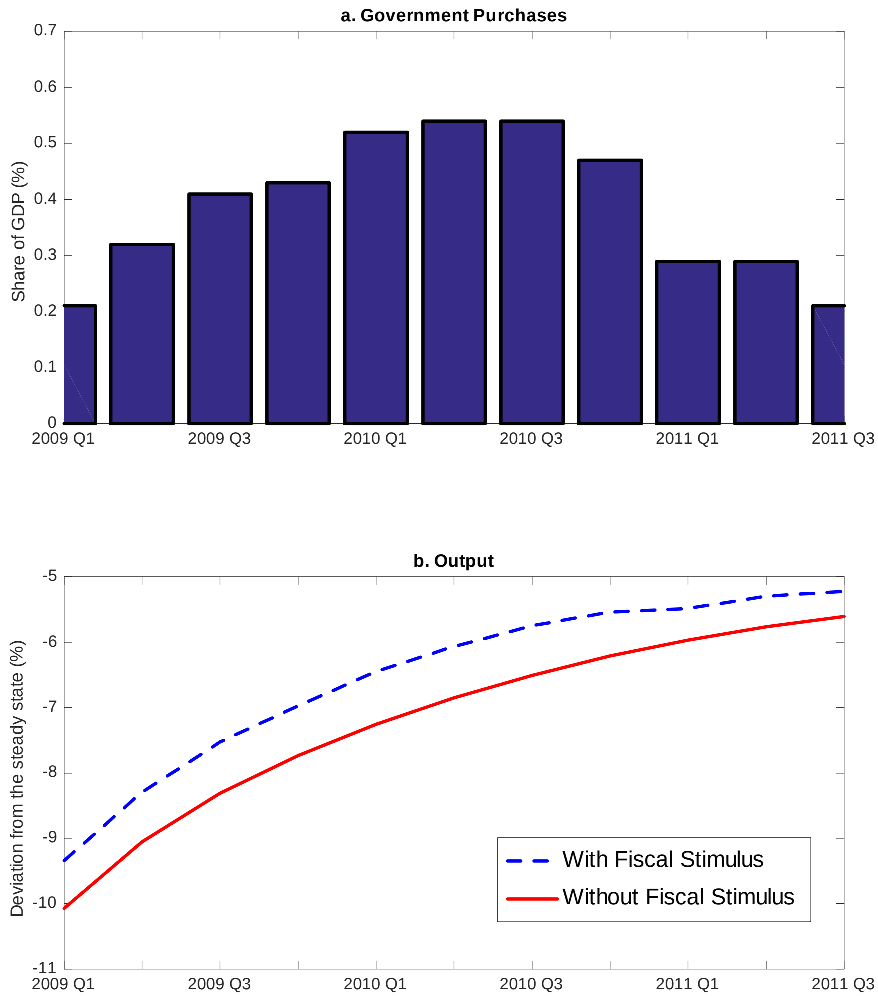

Figure 5: Increase in government purchases under the 2009 American Recovery and Reinvestment Act and its effects on output implied by the DEFK model

Notes: The solid line in panel b. shows the output response in the DEFK model in a three-year crisis without fiscal stimulus. The dotted line shows the one with fiscal stimulus under the ARRA.

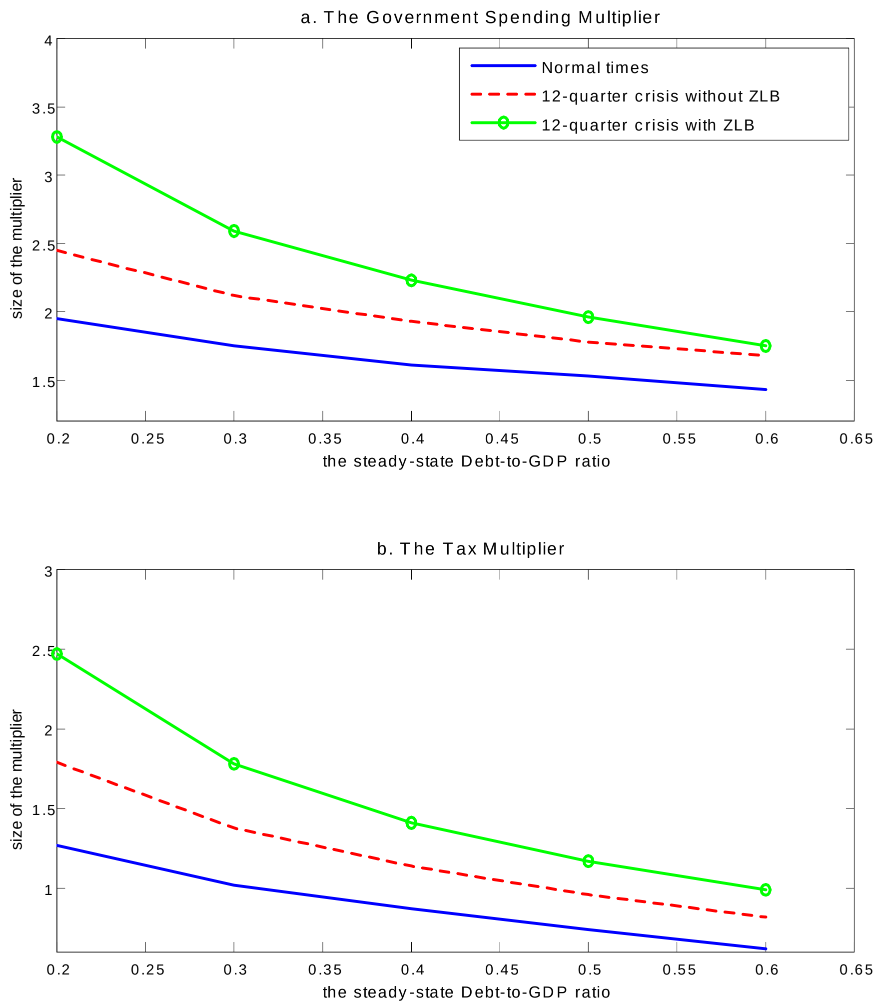

Figure 6: The sensitivity of the government spending multiplier and the tax multiplier to the initial debt-to-output ratio

Notes: The lines show the values of the multipliers under three different scenarios: (a) during normal times, (b) during a 3-year crisis (but without a ZLB), and (c) during a 3-year crisis and with a ZLB.

References

[1] Adjemian, Stéphane, Houtan Bastani, Michel Juillard, Frédéric Karamé, Ferhat Mihoubi, George Perendia, Johannes Pfeifer, Marco Ratto and Sébastien Villemot, 2011. "Dynare: Reference Manual, Version 4," Dynare Working Paper 1, CEPREMAP

[2] Aiyagari, S. Rao and McGrattan, Ellen R., 1998. "The optimum quantity of debt," *Journal of Monetary Economics*, vol. 42(3), pages 447-469.

[3] Ajello, Andrea, 2010. "Financial intermediation, investment dynamics and business cycle fluctuations," MPRA Paper 32447, University Library of Munich, Germany, revised Mar 2011.

[4] Auerbach, Alan and Yuriy Gorodnichenko, 2012. "Measuring the Output Responses to Fiscal Policy," American Economic Journal: Economic Policy, 4(2), pages 1-27.

[5] Bernanke, Ben, Mark Gertler and Simon Gilchrist, 1999. "The financial accelerator in a quantitative business cycle framework," *Handbook of Macroeconomics*, in: J. B. Taylor and M. Woodford (ed.), *Handbook of Macroeconomics*, edition 1, volume 1, chapter 21, pages 1341-1393.

[6] Bilbiie, Florin, Tommaso Monacelli and Roberto Perotti, 2013. "Public Debt and Redistribution with Borrowing Constraints," *The Economic Journal*, 123, F64 - F98.

[7] Bilbiie, Florin, Tommaso Monacelli and Roberto Perotti, 2014. "Is Government Spending at the Zero Lower Bound Desirable?," NBER Working Paper 20687.

[8] Blanchard, Olivier and Roberto Perotti, 2002. "An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output," *The Quarterly Journal of Economics*, vol. 117(4), pages 1329-1368.

[9] Burnside, Craig, Eichenbaum, Martin and Fisher, Jonas D. M., 2004. "Fiscal shocks and their consequences," *Journal of Economic Theory*, vol. 115(1), pages 89-117.

[10] Calvo, Guillermo, 1983. "Staggered Prices and in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12(3): 383-98.

[11] Carlstrom, Charles T., Timothy S. Fuerst and Matthias Paustian, 2012. "Fiscal Multipliers under an Interest Rate Peg of Deterministic vs. Stochastic Duration," Federal Reserve Bank of Cleveland Working Paper 1235.

[12] Carrillo, Julio and Celine Poilly, 2013. "How do financial frictions affect the spending multiplier during a liquidity trap?" *Review of Economic Dynamics*, vol. 16(2), pages 296-311.

[13] Chari, V. V., Patrick J. Kehoe and Ellen R. McGrattan, 2000. "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?" *Econometrica*, vol. 68(5), pages 1151-1180.

[14] Chari, V. V., Patrick J. Kehoe and Ellen R. McGrattan, 2009. "New Keynesian Models: Not Yet Useful for Policy Analysis," *American Economic Journal: Macroeconomics*, vol. 1(1), pages 242-66.

[15] Christiano, Lawrence, Martin Eichenbaum and Charles L. Evans, 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, vol. 113(1): 1-45.

[16] Christiano, Lawrence, Martin Eichenbaum and Sergio Rebelo, 2011. "When Is the Government Spending Multiplier Large?" *Journal of Political Economy,* vol. 119(1): 78 - 121.

[17] Cogan, John F., Tobias Cwik, John B. Taylor and Volker Wieland, 2010. "New Keynesian versus old Keynesian government spending multipliers," *Journal of Economic Dynamics and Control*, Volume 34, Issue 3: 281-295.

[18] Del Negro, Marco, Gauti Eggertsson, Andrea Ferrero and Nobuhiro Kiyotaki, 2011. "The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities," FRB of New York Staff Report No. 520.

[19] Dixit, Avinash K. and Joseph E. Stiglitz, 1977. "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, vol. 67(3), pages 297-308.

[20] Driffill, John and Marcus Miller, 2011. "Liquidity when it matters: QE and Tobin's q," CAGE Online Working Paper Series 67, Competitive Advantage in the Global Economy (CAGE).

[21] Eggertsson, Gauti B., 2011. "What Fiscal Policy Is Effective at Zero Interest Rates?" *NBER Macroeconomic Annual 2010*, Volume 25: 59 - 112.

[22] Eggertsson, Gauti B. and Paul Krugman, 2012. "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach," *The Quarterly Journal of Economics*, Oxford University Press, vol. 127(3), pages 1469-1513.

[23] Erceg, Christopher, Dale Henderson and Andrew Levin, 2000. "Optimal monetary policy with staggered wage and price contracts," *Journal of Monetary Economics*, vol. 46(2), pages 281-313.

[24] Erceg, Christopher and Jesper Linde, 2012. "Is there a fiscal free lunch in a liquidity trap?" International Finance Discussion Paper No. 1003r.

[25] Fernandez-Villaverde, Jesus. 2010. "Fiscal Policy in a Model with Financial Frictions," *American Economic Review*, 100(2): 35-40.

[26] Gali, Jordi, D. Lopez-Salido and J. Valles, 2007. "Understanding the effects of government spending on consumption," *Journal of the European Economic Association*, vol. 5, pp. 227–270.

[27] Hall, Robert E., 2009. "By How Much Does GDP Rise If the Government Buys More Output?" *Brookings Papers on Economic Activity*, 2: 183–231.

[28] Holmstrom, Bengt and Jean Tirole, 1998. "Private and Public Supply of Liquidity," *Journal of Political Economy*, 106 (1): 1-40.

[29] Kiyotaki, Nobuhiro and John Moore, 2008. "Liquidity, Business Cycles, and Monetary Policy," Mimeo, Princeton University.

[30] Krishnamurthy, Arvind and Vissing-Jorgensen, Annette, (2012). "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 120, issue 2, pages 233 - 267.

[31] Leeper, Eric M., Michael Plante, and Nora Traum, 2010. "Dynamics of fiscal financing in the United States," *Journal of Econometrics*, vol. 156(2), pages 304-321.

[32] Mertens, Karel and Morten O. Ravn, 2010. "Fiscal Policy in an Expectations Driven Liquidity Trap," CEPR Discussion Paper 7931.

[33] Ramey, Valerie, 2011a. "Identifying Government Spending Shocks: It's all in the Timing," *The Quarterly Journal of Economics*, vol. 126(1), pages 1-50.

[34] Ramey, Valerie, 2011b. "Can Government Purchases Stimulate the Economy?" *Journal of Economic Literature*, 49(3): 673-85.

[35] Romer, C. and Bernstein, J. 2009. The job impact of the American recovery and reinvestment plan, January 2009.

[36] Shi, Shouyong, 2015. "Liquidity, Assets and Business Cycles," *Journal of Monetary Economics*, 70: 116-132.

[37] Smets, Frank and Rafael Wouters, 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, vol. 97(3), pages 586-606.

[38] Taylor, John B., 1993. "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy,* 39: 195-214.

[39] Woodford, Michael, 1990. "Public Debt as Private Liquidity," *American Economic Review*, vol. 80(2), pages 382-88.

[40] Woodford, Michael, 2011. "Simple Analytics of the Government Expenditure Multiplier," *American Economic Journal: Macroeconomics*, vol. 3(1), pages 1-35.

A Appendix (not intended for publication)

A.1 Other Equilibrium Equations of the DEFK Model

Differentiated workers $j \in [\varkappa, 1]$ supply labour $H_t(j)$ to the production sector through the arrangement of employment agencies as in Erceg, Henderson and Levin (2000). Competitive employment agencies choose their profit-maximising amount of $H_t(j)$ to hire, taking nominal wages $W_t(j)$ as given. They combine $H_t(j)$ into homogeneous units of labour input according to $H_t = \left[\left(\frac{1}{1-\varkappa} \right)^{\frac{\lambda_\omega}{1+\lambda_\omega}} \int_\varkappa^1 H_t(j)^{\frac{1}{1+\lambda_\omega}} dj \right]^{1+\lambda_\omega}$. The demand for type-$j$ labour is therefore $H_t(j) = \frac{1}{1-\varkappa} \left[\frac{W_t(j)}{W_t} \right]^{-\frac{1+\lambda_\omega}{\lambda_\omega}} H_t$, where $\lambda_\omega \geq 0$ and W_t is the aggregate wage index. Each type-j labour is represented by a labour union who sets their nominal wage $W_t(j)$ optimally on a staggered basis. Each period, there is a history-independent probability of $(1-\zeta_\omega)$ for a union to reset their wage. Otherwise, they keep their nominal wage constant. The optimal wage-setting equation in real terms is:

$$E_t \sum_{s=t}^{\infty} (\beta\zeta_\omega)^{s-t} C_s^{-\sigma} \left\{ \frac{\widetilde{w}_t}{\pi_{t,s}} - (1+\lambda_\omega) \frac{\left[\frac{1}{1-\varkappa} \left(\frac{\widetilde{w}_t}{\pi_{t,s} w_s} \right)^{-\frac{1+\lambda_\omega}{\lambda_\omega}} H_s \right]^v}{C_s^{-\sigma}} \right\} \left(\frac{\widetilde{w}_t}{\pi_{t,s} w_s} \right)^{-\frac{1+\lambda_\omega}{\lambda_\omega}} H_s = 0, \tag{9}$$

where $\widetilde{w}_t(j) \equiv \frac{\widetilde{W}_t(j)}{P_t}$ is the optimal wage chosen by a labour union at t, $w_t \equiv \frac{W_t}{P_t}$ and $\pi_{t,s} \equiv \begin{cases} 1, \text{ for } s = t \\ \pi_{t+1}\pi_{t+2}...\pi_s, \text{ for } s \geq t+1 \end{cases}$. The dynamics of w_t follows:

$$w_t^{-\frac{1}{\lambda_\omega}} = (1-\zeta_\omega) \widetilde{w}_t^{-\frac{1}{\lambda_\omega}} + \zeta_\omega \left(\frac{w_{t-1}}{\pi_t} \right)^{-\frac{1}{\lambda_\omega}} \tag{10}$$

Final-goods firms produce homogeneous final goods Y_t by combining heterogeneous intermediate goods $Y_t(i)$ according to $Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_f}} di \right]^{1+\lambda_f}$, where $\lambda_f \geq 0$. Their profit-maximising condition implies that the demand for type-i intermediate good is $Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\frac{1+\lambda_f}{\lambda_f}} Y_t$, where $P_t(i)$ and P_t are the respective nominal prices for intermediate and final goods. Monopolistic competitive intermediate-goods firms produce according to the production function $Y_t(i) = A_t K_t(i)^{\gamma} H_t(i)^{1-\gamma}$, where A_t is productivity and γ is the capital share. Intermediate-goods firms maximise their real profits $D_t(i)$ by choosing the optimal capital and labour inputs, taking real wage and rental rate of capital as given. The cost-minimising conditions imply that their real marginal cost is:

$$mc_t = mc_t(i) = \frac{1}{A_t}\left(\frac{w_t}{1-\gamma}\right)^{1-\gamma}\left(\frac{r_t^k}{\gamma}\right)^{\gamma}, \tag{11}$$

which is universal across firms. Intermediate-goods firms also set nominal prices for their heterogeneous goods. In each period, each firm has a constant probability of $\left(1-\zeta_p\right)$ to reset their price. They keep their price unchanged otherwise. Firms who reset their price choose the one that maximises their expected future profits, giving the price-setting equation (in real terms):

$$E_t \sum_{s=t}^{\infty} \left(\beta\zeta_p\right)^{s-t} C_s^{-\sigma} \left\{\frac{\widetilde{p}_t}{\pi_{t,s}} - (1+\lambda_f)\, mc_s\right\} \left(\frac{\widetilde{p}_t}{\pi_{t,s}}\right)^{-\frac{1+\lambda_f}{\lambda_f}} Y_s = 0, \tag{12}$$

where $\widetilde{p}_t(i) \equiv \frac{\widetilde{P}_t(i)}{P_t}$ as the optimal price chosen at t. The zero-profit condition for final-goods firms give rise to the evolution of inflation:

$$1 = \left(1-\zeta_p\right)\widetilde{p}_t^{\,-\frac{1}{\lambda_f}} + \zeta_p\left(\frac{1}{\pi_t}\right)^{-\frac{1}{\lambda_f}} \tag{13}$$

Capital-goods firms convert final goods into capital goods. The adjustment cost is quadratic in aggregate investment in a way that $S(\frac{I_t}{I}) = \frac{\kappa}{2}\left(\frac{I_t}{I}-1\right)^2$, where I is the steady-state investment and $\kappa > 0$ is the adjustment cost parameter. Capital-goods firms choose the amount of I_t to produce which maximises their profits. The first-order condition is:

$$p_t^I = 1 + S(\cdot) + S'(\cdot)\frac{I_t}{I} \tag{14}$$

Upon aggregation, the market clears for both labour and capital so that $H_t = \int_0^1 H_t(i)di$ and $K_{t-1} = \int_0^1 K_t(i)di$. The capital-labour ratio is:

$$\frac{K_{t-1}}{H_t} = \frac{\gamma}{(1-\gamma)}\frac{w_t}{r_t^k}, \tag{15}$$

Capital evolves according to:

$$K_t = (1-\delta)\,K_{t-1} + I_t \tag{16}$$

and the aggregate production function is:

$$A_t {K_{t-1}}^{\gamma} {H_t}^{1-\gamma} = \int_0^1 Y_t(i)di \tag{17}$$

Capital is owned by households through their private equity holdings:

$$K_t = N_t \tag{18}$$

The profits for intermediate-goods and capital-goods firms are wholly distributed to households as dividends. Substituting for D_t and D_t^K, (3) becomes:

$$I_t = \varkappa \frac{\left[r_t^k + (1-\delta)\,q_t\phi_t\right] N_{t-1} + r_{t-1}L_{t-1} + Y_t - w_tH_t - r_t^k K_{t-1} + p_t^I I_t - \left[1+S(\cdot)\right] I_t - \tau_t}{p_t^I - \theta q_t} \tag{19}$$

Finally, the resource constraint of the economy requires that:

$$Y_t = C_t + \left[1+S(\cdot)\right] I_t + G_t \tag{20}$$