Potential uses of numerical simulation for the modelling of civil conflict

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Abstract

This paper explores ways in which civil conflict can be simulated using numerical methods. A general two-party model of conflict is developed by extending an approach proposed by Christia (2012), which is based on a metric of the 'relative power' that exists between the state and a rebel group. Various definitions of relative power are considered and one of these is chosen to illustrate different types of two-sided armed conflict, namely direct-fire, guerrilla and asymmetric warfare. The additional suggestion of Christia that random or stochastic events can lead to unexpected conflict outcomes is also further extended in this paper. The inclusion in the model of terms describing concurrent rebel recruitment of civilians and state deployment of troops are then described. Examples are presented for various hypothetical cases. It is demonstrated that numerical simulation techniques have great potential for modelling civil war. The Christia approach is shown to provide an excellent basis from which numerical models of civil conflict can be built and from which the progress of a conflict can usefully be visualised graphically.

Introduction

Numerical simulation (or computer-generated modelling) is a useful alternative approach for studying complex problems in addition to statistical modelling methods (Gilbert and Terna 1999; Ostrom 1988). Statistical methods involve the analysis of empirical data with the aim of uncovering correlations that are consistent with particular theories or mechanisms. But, since statistical approaches are correlational and not experimental, they cannot be used with confidence to demonstrate causality. Moreover, since many variables and constructs in civil war research are measured using proxy variables, it is difficult to isolate the role of particular mechanisms. In general, experimental research approaches allow for more control and reduce

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issues associated with validity, but are clearly not available, nor ethical, in conflict studies. Numerical simulation presents an alternative approach, which aims to uncover the underlying mechanisms by generating observed, macro-level phenomena (Smith and Conrey 2007). Unlike statistical modelling, where empirical data are processed via a statistical model and results produced that describe the relationships that exist within the data, simulation models are computer programs that incorporate the critical aspects of the phenomenon being studied (Groff 2007). As Doreian (2001) points out, simulation modelling tries to capture the generative mechanisms underlying the phenomena under study, whereas statistical modelling seeks a numerical summary of relationships between variables. Numerical simulations may therefore be regarded as simple representations of real-world systems (Gilbert and Troitzsch 2005) and a platform to test hypotheses and predict real-world outcomes in cases where data are sparse or non-existent.

Numerical models are generally used to test or falsify a theory or hypothesis and also to answer ‘what if?’ questions (Gilbert 2007). They can be used to test whether a theory is sufficient to generate an expected outcome, or if other processes are necessary for that outcome to be generated (Eck and Lui 2008). To build a numerical model, a theory has to be formalised by identifying the relevant parameters of the system, establishing the properties of the system and interactions within it and articulating the mechanisms underpinning these interactions in the form of programmed rules (Gilbert 2008). The model is validated by comparing generated outcomes to theoretical expectations or empirical data. Alternatively, numerical models can be used for prediction. In this case a *facsimile* model can be constructed which seeks to replicate the social system under study as accurately as possible by ‘tuning’ model parameters to match empirical data (Gilbert 2007). Different parameters within the model can then be manipulated so that their impact on model outcomes can be evaluated (Gilbert 2007).

The accuracy of numerical modelling methods is clearly a function of the validity of the assumptions on which the model is based and just like statistical models, they therefore reflect the quality of the theoretical ideas or empirical data available (Groff 2007). All numerical models remain approximations of the complexity of their real world counterpart, since certain features of human actors (such as irrationality, perceptions and other psychological or emotional factors) are difficult to quantify and incorporate into a numerical model (Keller *et al.* 2010). This means that care must be taken when analysing the findings from numerical models. The results obtained do not represent either an absolute empirical test of a theory nor produce guaranteed predictions of future events. Instead, the findings from numerical models should be used to indicate the plausibility of a theory or to highlight a range of potential outcomes given a certain set of assumptions and conditions (Groff 2007).

The aim of this paper is to assess the potential of numerical simulation techniques for the modelling of civil wars. Numerical approaches have not yet been fully utilised in conflict research because previous studies have favoured statistical modelling methods to study conflict dynamics such as duration and outcome (Hegre 2004, Karl and Sobek 2004, Cunningham 2006, Cunningham, Gleditsch and Salehyan 2009). But statistical approaches cannot provide information on the underlying generative mechanisms responsible for these macro-effects and they inadequately capture the complex, dynamic processes inherent in civil wars that are often qualitatively described in conflict studies. In contrast, numerical models have many features that are attractive for conflict modelling. They allow not only for the elucidation of bottom-up generative processes, but they also capture the dynamic nature of conflict and allow for the influence of stochastic processes to be studied.

The main contribution made by this paper is to highlight the utility of numerical simulation approaches for conflict research. A general two-party conflict model is proposed with the intention that it could act as a starting point from which more complex numerical simulations of conflict could be developed. The value of numerical models is that they can be validated using empirical data, allowing scholars to gain insight into the individual and group level generative mechanisms that drive different conflict dynamics. This aspect is particularly advantageous in a field, such as conflict research, where characteristics at the individual level are seldom specified in empirical models owing to the paucity of micro-level data.

Numerical models also have potential to assist with out-of-sample forecasting techniques, which are of increasing interest to conflict scholars (Hegre *et al.* 2013). For example, the parameters of numerical models of conflict can be ‘tuned’ to match empirical data and numerical experiments can be performed to generate predictions under various ‘what if?’ scenarios. The results of these experiments could be used as a complementary tool to evaluate the plausibility of out-of-sample predictions. An additional advantage of numerical models in comparison to statistical models is that stochastic events can be easily incorporated. For the purposes of assisting with out-of-sample forecasting, numerical models could be used to perform sensitivity studies to test how robust the out-of-sample predictions obtained from statistical models are to different degrees of randomness.

In this paper, one numerical approach is examined in particular; namely that proposed by Christia (2012), who suggests that two-sided conflict might be modelled by using the concept of the relative power that exists between the state and a rebel group. In the present paper, the relative power concept is explored and used to provide a basis for the development of a general simulation model of two-sided conflict, in which different mechanisms of conflict, such as asymmetric conflict, are modelled.

Previous empirical research on asymmetric conflict has focused on how particular strategies and conditions allow weaker rebels to prevail against stronger opponents (Arreguin-Toft 2001, Mack 1975). In such conflicts, the rebel side is at an advantage over the government, in spite of their smaller size, because the government must search for their opponents before they can make a kill, whilst the rebels are able to make a direct-kill by targeting government strongholds. This mechanism was apparent in the recent Afghan conflict, in which the rebels were difficult to locate because they were dispersed in mountains and villages but were able to make direct attacks on their opponents by placing improvised explosive devices on the routine patrol routes of Western forces. Some empirical studies have established a link between geographic terrain and conflict duration (Buhaug *at al.* 2009, Buhaug and Lujala 2005), but these studies have used statistical modelling methods, which are limited by the unavailability of data. The numerical model developed in this paper provides an obvious foundation from which the relationship between conflict duration and geographic terrain might be explored via bottom-up generative processes.

The numerical model proposed in this paper is extended to include terms for rebel group recruitment and the state deployment of troops. Previous empirical research has shown that rebel groups typically start out weak relative to the state and launch a rebellion with the expectation that they will be able to mobilise troops. If the government is unable to defeat the rebels in the early stages of conflict, the chances of a swift resolution are remote (Bapat 2005, Regan 2002) and if rebels have a high mobilisation capacity relative to the state's deployment of troops, rebels are more likely to win decisive victories (Cunningham, Gleditsch and Salehyan 2009). Conflict research has already started using survey data at the individual level to understand rebel group recruitment patterns, participation and attitudes, and how these influence civil war dynamics (Humphreys and Weinstein 2008). The present paper describes how these feature might be incorporated into numerical models, so that numerical experiments can be performed to test the effects of different rebel group mobilisation capabilities and strategies on civil conflict dynamics.

Previous numerical models of civil conflict

The first computational models of armed conflict appeared during the Cold war era (Cioffi-Revilla and Rouleau 2010). These early models were implemented using a system dynamics approach and utilised ordinary differential equations (ODEs) to model two-sided conflicts between a state and a rebel group (Forrester 1968; Hanneman 1988). These equations contain time derivatives (of troop numbers for example) and are used to describe dynamic phenomena, evolution and variation. ODE models have been used to model the progression of different types of two-sided conflict over time, including; *direct-fire* warfare (a type of battle in which each side shoots directly at the other), *guerrilla* warfare (where each side has to search for their enemy before attempting a kill) and *asymmetric* warfare (where one side adopts a *direct-fire* approach while the other adopts a *guerrilla* approach). See Lanchester (1956) for an early overview of these types of conflict. ODE models were often empirically informed by prominent insurgencies of the time, such as the Vietnamese War or the Soviet invasion of Afghanistan (Milstein and Mitchell 1969; Ruloff 1975; Allan and Stahel 1985) and they attained their greatest success in representing asymmetric two-sided conflict at a national level (Gilbert and Troitzsch 2005).

Mathematical and numerical modelling of armed conflict using a systems dynamics approach has remained dominant until the more recent introduction of *object-oriented* approaches such as *agent-based models* (ABM), a particularly useful method for representing for many real-world systems. ABM's involve the simulation of automated agents in the context of an artificial environment and the analysis of macro-level patterns emerging from these micro-level agent behaviours and interactions (Epstein and Axtell 1996; Gilbert and Troitzsch 1999). Agents are 'autonomous, goal-directed software entities' that engage in behaviours described as condition-action-rules (O'Sullivan and Haklay 2000). Agents commonly represent people, but may also represent groups, organisations or governments for example. The characteristics and behavioural rules of agents can be assigned according to agent-type (i.e. government agents, civilian agents), or they can be assigned according to a known distribution observed in the population, or at random, so that specific societal averages can be modelled. Thus, ABM's allow for heterogeneity among individuals that more closely approximates reality than many other computational modelling methods (Groff 2007).

Agents interact with each other on the basis of their condition-action-rules and their characteristics can be dynamically changed as a result of those interactions. In effect, agents are able to pass information to each other and react to that information on the basis of their programmed preferences, which may be fixed or acquired. This allows an examination of the evolution and history of the process under study, since information on the dynamics of the system can be collected as the ABM computer model runs (Axtell 2000). ABM is a bottom-up approach that models global behaviour as emergent properties of local interactions between agents. As such it is well suited for studying complex and non-linear social processes, such as armed conflict, where cumulative effects are produced over a protracted time scale (Keller e*t al.* 2010).

The SUGARSCAPE model of Epstein and Axtell (1996) was one of the first ABM simulations of social unrest. By formalising a few simple rules, SUGARSCAPE allowed for the experimental investigation of some of the theorised micro-level mechanisms responsible for the dynamics of civil war. Subsequently, Epstein (2002) produced the first ABM simulations of armed conflict. These simulations modelled the emergence of rebellion and ethnic-cleansing behaviour as a product of the rebel agent's perception of the police force numbers (i.e. the size of government forces) and intervention tactics at individual (civilian) level. This research was notable in that it was the first to examine the citizen-based impetus for rebellion and the model was generative, analysing civil conflict from an individual level up, as opposed to simply identifying state-level variables that increased the likelihood of civil war. Epstein's model of civil conflict has subsequently been extended (Ilachinski 2004). The ISAAC and EINSTEIN models use a similar technique to model combat at the level of individual soldiers and Doran (2005) has constructed the IRUBA model which represents a meso-scale replication of Epstein's model. The IRUBA model uses simple geographic features, such as terrain and the spatial distribution of rebel resources and troops, to test the impacts on model outcomes of various insurgency and counterinsurgency tactics. In a further model, Bennett (2008) omits variations in terrain but instead includes social emotions in the civilian population such as fear and anger felt against the state. This was an attempt to model the US military's strategy of appealing to the 'hearts and minds' of the US population.

Bhavnani *et al.* (2008) have created the first simulations of civil war involving state repression and Cioffi-Revilla and Rouleau (2010) have devised the MASON model which considers how political freedom within a state (i.e. the polity index) results in different emergent conflict outcomes. All of these numerical simulations examine the macro-level conflict dynamics emerging from rebel and civilian agent behaviours in two-sided conflicts.

The present paper builds on these previous insights by extending the approach proposed by Christia (2012) which is based on the metric of the 'relative power' between two sides. Terms for rebel recruitment of civilians and state deployment of troops are incorporated and ways in which stochastic processes can be implemented into the model are explored.

A general numerical model of two-sided conflict

Modelling relative power between the state and a rebel group

Christia (2012) has proposed that the progress and outcome of a civil war can be modelled using a metric p, which relates to the *relative power* between the groups involved. In Christia's model it is assumed that there are two sides at war; the government and a rebel group. Rebel groups are assumed to have two objectives: (*i*) to win the war (or at least sustain conflict against the state) and (*ii*) to maximise their returns. The metric of interest is relative, rather than overall, power and the basis of the model is that the evolution of relative power over time defines the progress of a conflict. The conflict between the government and a rebel group starts at $t = 0$, when the relative power of the government is p_0 and the relative power of the rebel group is $1 - p_0$. Christia proposes that the change in the value of p throughout a conflict can be described by an expression that contains the sum of a deterministic component $f(p)dt$ and a stochastic component $\Psi(p,t)$. Thus;

$$dp = f(p)\,dt + \Psi(p,t) \qquad (1)$$

where dp is the change in p over a time increment dt and $f(p)$ is a drift term that depends on the current level of p, which in the absence of the stochastic term, is simply the derivative of p with respect to time dp/dt. Based on this deterministic component in equation (1) alone, if the level of p at time t_0 were known, it would be possible to ascertain how p would evolve over time. In the model, a rebel group with an initial value $p > 0.5$, would eventually be expected to win because it would keep increasing its relative power over time eventually reaching $p = 1$. Conversely, if a rebel group were to start with $p < 0.5$, the rebels would eventually lose with the relative power reaching a value $p = 0$. Evidently, the rebel group would prefer to have a higher p at the start of the conflict because that would ensure a quicker victory. The deterministic function proposed by Christia (2012) must be continuously increasing and symmetric about $p = ½$ such that $f(p) > 0$ for $p > ½$, $f(p) < 0$ for $p < ½$ and $f(p) = 0$ for $p = ½$.

An important aspect of the concept is that the evolution of p over time is not just deterministic. If it were (assuming complete and symmetric information), then rational parties would never go to war simply because they would be able to predict the outcome of the conflict in advance and act accordingly. But, as represented by the second term in equation (1), there must also a stochastic component. This is intended to capture the inevitable randomness that arises in conflicts and might include battlefield mistakes, or other exogenous factors such as changes in external support, unexpected weather conditions, disease or other factors beyond the control of the actors involved. The random change in p over some time interval, dt, is represented by the term $\Psi(p,t)$, where Ψ determines the amount of randomness in the relative power change dp. Lower values of Ψ correspond to a lower random component, with no randomness if $\Psi = 0$. Thus, for the rebel side with $p > 1/2$ at t_0 with the deterministic component only, rebel victory would always be expected. But, if a stochastic component is included, victory need not necessarily occur. If victory does occur, it may be either quicker, or slower to attain than in the deterministic case alone, or indeed it is possible for the side that starts out weaker to emerge victorious. Christia (2012) suggests that the trajectory of a conflict resembles one of biased random walk, where at each time interval, there is a step in a direction of increased or decreased relative power that is random in part, but also influenced by the initial value of the relative power.

It is thus clear that equation (1) has attractive general features that make it suitable for modelling conflict, namely (*i*) it can be used to model a range of different scenarios relating to power-based theories of conflict, (*ii*) it models civil war as a dynamic process, (*iii*) it allows randomness to be implemented in the simulation of conflict and (*iv*) the expression is highly amenable to graphical representation which means that simulated conflict processes can be visualised to great effect.

If the expression is to be used effectively, the general concept proposed by Christia (2012) needs to be extended to include more specific aspects as follows (*i*) the development of mathematical descriptions of the deterministic function and the selection of the most promising possibilities, (*ii*) defining relative power in terms of actual parameters that could be used in a numerical model and that might also be amenable to statistical analysis of empirical conflict data, (*iii*) exploring the ability and utility of the expression to illuminate different types of warfare and (*iv*) investigating how terms might be included in the expression that account for other conflict characteristics such as government troop deployment and rebel group growth. These aspects are considered in the following sections.

Mathematical descriptions of the deterministic function

A specific mathematical description of the deterministic function is not given by Christia (2012) but various functions can be chosen that satisfy the required properties of the deterministic component $f(p)$. One obvious possibility is the relationship:

$$f(p) = \frac{dp}{dt} = \beta(2p - 1) \tag{2}$$

where β is a constant having units $time^{-1}$. The function $(2p - 1)$ has the properties required by Christia, namely that it is zero at $p = ½$, positive when $p > ½$ and negative when $p < ½$. For victory to occur when $p = 1$ and defeat when $p = 0$, the function takes the values $+ 1$ and $- 1$ respectively. It is required dimensionally to insert the term β into equation (2). This gives equivalent values of dp/dt of $+ \beta$ and $- \beta$ respectively. Victory or defeat then corresponds to the situation where the critical rates of change $\pm\beta$ are reached. Although the inclusion of the function $(2p - 1)$ into the expression is in accord with Christia's proposal and provides a promising way forward, the symmetry of the function about the value $p = ½$ is arbitrary. An alternative and simpler representation may be useful as described by the relationship:

$$f(p) = \beta' p \tag{3}$$

Unlike equation (2), which is symmetrical about $p = ½$, this function is symmetrical about $p = 0$, such that $dp/dt = 0$ when $p = 0$ with $dp/dt > 0$ for $p > 0$ and $dp/dt < 0$ for $p < 0$.

Physical interpretations of 'relative power'

The general concept of relative power is not defined by Christia (2012) in terms of actual parameters that could be used in conflict modelling, but an obvious first approach is to assume that relative power is related simply to the numbers of group members. Thus if there are N_1 and N_2 members in group-1 and group-2 respectively, the relative power may then be defined in several ways, such as the fractional values $f_1 = N_1/(N_1 + N_2)$ and $f_2 = N_2/(N_1 + N_2)$, the differences $\Delta_1 = N_1 - N_2$ and $\Delta_2 = N_2 - N_1$, or the ratios $R_1 = N_1/N_2$ and $R_2 = N_2/N_1$. It may also be useful to normalise the Δ terms as follows: $(N_1 - N_2)/(N_1 + N_2)$ and $(N_2 - N_1)/(N_1 + N_2)$, where the reference value is the total number $N_1 + N_2$. The implications of these various definitions of p are now given. If p is defined as the number of group members represented as a fraction of the total number of troops in the conflict, then:

$$p_1 = \frac{N_1}{N_1 + N_2}, \quad p_2 = \frac{N_2}{N_1 + N_2} \tag{4}$$

These definitions have the properties required by suggestion of Christia, namely that $p_2 = 1 - p_1$, there is symmetry about $p = ½$ (i.e. when $N_1 = N_2$ then $p_1 = p_2 = ½$). The victory/defeat criteria are also identical. Thus, when $N_2 << N_1$ then $p_1 \to 1$, $p_2 \to 0$ and group-1 wins. On the other hand when $N_1 <<$ N2 then $p_2 \to 0$, $p_2 \to 1$and group-2 wins.

The second representation of the relative power is in terms of the (normalised) difference:

$$p_1 = \frac{N_1 - N_2}{N_2}, \quad p_2 = \frac{N_2 - N_1}{N_1} \tag{5}$$

In this case, $p_2 \neq 1 - p_1$ and there is symmetry about $p = 0$ (i.e. when $N_1 = N_2$ then $p_1 = p_2 = 0$). When $N_2 << N_1$ then $p_1 \to \infty$ and $p_2 \to -1$and group-1 wins. When $N_1 << N_2$ then $p_1 \to -1$and $p_2 \to \infty$and group-2 wins.

The third representation of the relative power is in terms of the difference $(N_1 - N_2)$, normalised instead by the total number $(N_1 + N_2)$. Thus:

$$p_1 = \left(\frac{N_1 - N_2}{N_1 + N_2}\right), \quad p_2 = \left(\frac{N_2 - N_1}{N_1 + N_2}\right) \tag{6}$$

In this case, $p_1 + p_2 = 0$ and there is symmetry about $p = 0$ (i.e. when $N_1 = N_2$ then $p_1 = p_2 = 0$). When $N_2 << N_1$ then $p_1 \to +1$ and $p_2 \to -1$and group-1 wins. When $N_1 << N_2$ then $p_1 \to -1$and $p_2 \to +1$and group-2 wins.

Finally, a simple ratio may be used to represent the relative power between groups. Thus:

$$p_1 = \frac{N_1}{N_2}, \quad p_2 = \frac{N_2}{N_1} \tag{7}$$

In this case, $p_2 \neq 1 - p_1$ and there is symmetry about $p = 1$ (i.e. when $N_1 = N_2$ then $p_1 = p_2 = 1$). When $N_2 << N_1$ then $p_1 \to \infty$ and $p_2 \to 0$and group-1 wins. When $N_1 << N_2$ then $p_1 \to 0$ and $p_2 \to \infty$and group-2 wins.

Use of 'relative power' in the theoretical analysis of conflict

In a two-sided conflict, fatalities are incurred by both sides and if there is no concurrent replenishment of forces, each side must suffer a decrease in their number of troops over time. If the attrition rate of one group depends only on the size of the other, this is known as a *'direct-fire'* conflict. In its simplest form this involves each side firing directly at its adversary, for example in a long-bow battle with lines of opposing archers, or a modern direct-fire tank battle (Lanchester 1956). Direct-fire conflict is governed by the coupled solution of the two differential equations:

$$\frac{dN_1}{dt} = -B_2 N_2 \qquad (8)$$

$$\frac{dN_2}{dt} = -B_1 N_1 \qquad (9)$$

where (dN_1/dt) and (dN_2/dt) are the rates of change in numbers in groups 1 and 2 respectively. The killing rate experienced by one group depends only on the number in the opposing group. The proportionality constants B_1 and B_2 relate to the respective killing effectiveness of each side. Coupled differential equations are often difficult to solve analytically and commonly require numerical methods of solution. For this specific case however, the analytical solution of a similar system of differential equations does exist[§].

A numerical solution of equations (8 and 9) can be achieved by using the following computational scheme. Values of the constants B_1 and B_2 and the initial numbers in both groups $(N_1)_0$ and $(N_2)_0$ are user-defined and the numbers in each group can be set to these values. Using a small time increment Δt, the incremental changes in the numbers in each group during this interval can be computed as follows:

$$(N_1)_{t+\Delta t} = (N_1)_t - B_2 (N_2)_t \, \Delta t \qquad (10)$$

$$(N_2)_{t+\Delta t} = (N_2)_t - B_1 (N_1)_t \, \Delta t \qquad (11)$$

[§] The solutions adapted from Kreysig (1983) are: $N_1(t) = m(\dot{N}_1)_0 \left[\sinh(mt) + (N_1)_0 \cosh(mt)\right]$ and $N_2(t) = m(\dot{N}_2)_0 \left[\sinh(mt) + (N_2)_0 \cosh(mt)\right]$, where $m = (B_1 B_2)^{-1/2}$ and $(N_1)_0$, $(N_2)_0$ are the initial values of group size and $(\dot{N}_1)_0$, $(\dot{N}_2)_0$ are the initial rates.

These expressions are sequentially iterated repeatedly in the computer program and the elapsed time is calculated by the continuous summation of the values of Δt. The program output gives the time-dependence of N_1 and N_2 and the computation is terminated when either N_1 or N_2 reaches zero (or any other pre-defined value). The total summed time represents the duration of the conflict. The computational flow diagram is shown in Appendix I.

To validate the above numerical method, the predicted variation of N_1 and N_2 with time were compared to the values obtained by evaluation of the analytical expression given in the Footnote. Excellent agreement was obtained. This validation of the numerical method, gives confidence when extending it to other types of conflict. Note that, although an analytical solution is available in this direct-fire case, it is regarded that the numerical scheme is preferable for the present purpose, since it is more amenable to modifications such as the inclusion of stochastic terms and the inclusion of other terms describing government troop deployment and rebel group recruitment. Both of these features are considered later in this paper.

To investigate the utility of the various definitions of relative power given by equations (5 – 7), the time dependence of N_1 and N_2 determined from numerical method can be used to calculate the values and time dependences of p_1 and p_2 as follows:

$$p_1(t) = \left(\frac{N_1(t) - N_2(t)}{N_1(t) + N_2(t)}\right), \qquad p_2(t) = \left(\frac{N_2(t) - N_1(t)}{N_1(t) + N_2(t)}\right) \tag{12}$$

$$p_1(t) = \frac{N_1(t)}{N_1(t) + N_2(t)}\ , \qquad p_2(t) = \frac{N_2(t)}{N_1(t) + N_2(t)} \tag{13}$$

$$p_1(t) = \frac{N_1(t) - N_2(t)}{N_2(t)}\ , \qquad p_2(t) = \frac{N_2(t) - N_1(t)}{N_1(t)} \tag{14}$$

$$p_1(t) = \frac{N_1(t)}{N_2(t)}\ , \qquad p_2(t) = \frac{N_2(t)}{N_1(t)} \tag{15}$$

The values of $p_1(t)$ and $p_2(t)$ for these four definitions were generated to simulate a simple direct-fire conflict using the computational method described above. The initial size of the government side was taken to be $N_2 = 1000$ in all four cases and the initial size of the rebel side (N_1) was varied for each computation. The progress of each hypothetical conflict is shown graphically in Figures 1 – 4 using the four definitions of $p(t)$ from equations (12 – 15).

It was assumed that $B_1 = B_2 = 0.01$ and time is represented in the normalised form (mt), where $m = (B_1B_2)^{-1/2}$, to ensure consistency with the analytical solution in the previous Footnote.

Figure 1 Graphical representation of a direct-fire conflict using the variation of $p_1(t)$ and $p_2(t)$ with time from equation (12). Values for initial group sizes are $N_2 = 1000$ and $N_1 = 800, 600, 400$ & 200 respectively and the curves are labelled as values of N_1/N_2

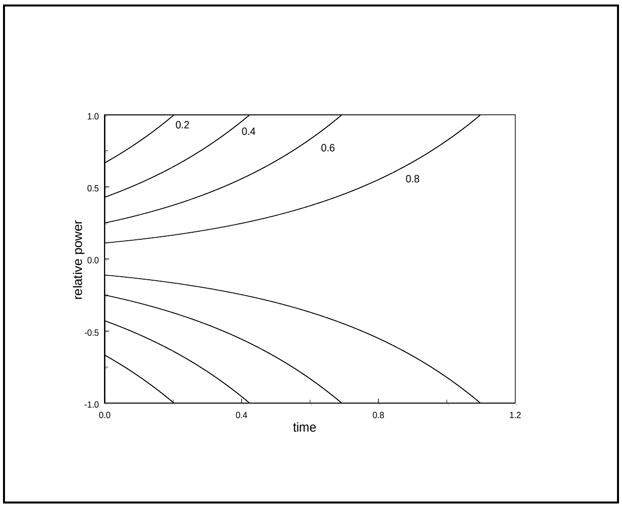


Figure 2 As for the previous figure but using $p_1(t)$ and $p_2(t)$ values given by equation (13)

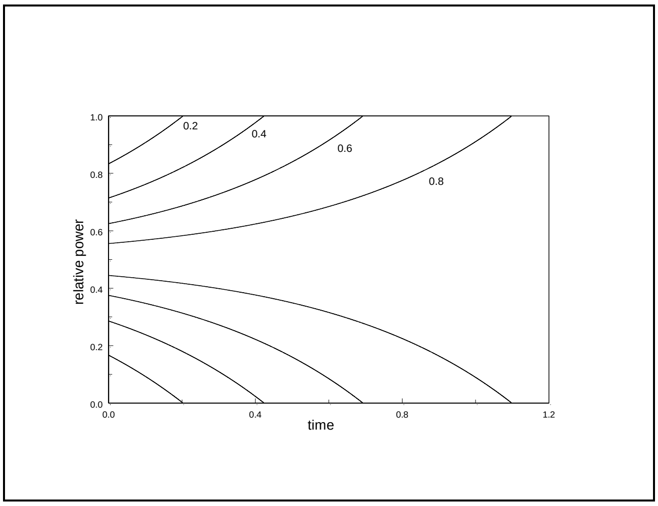


Figure 3 As for the previous figure but using $p_1(t)$ and $p_2(t)$ values given by equation (14)

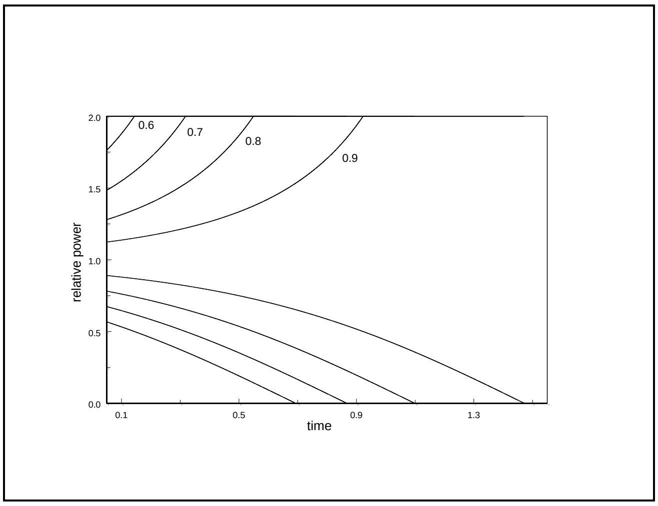
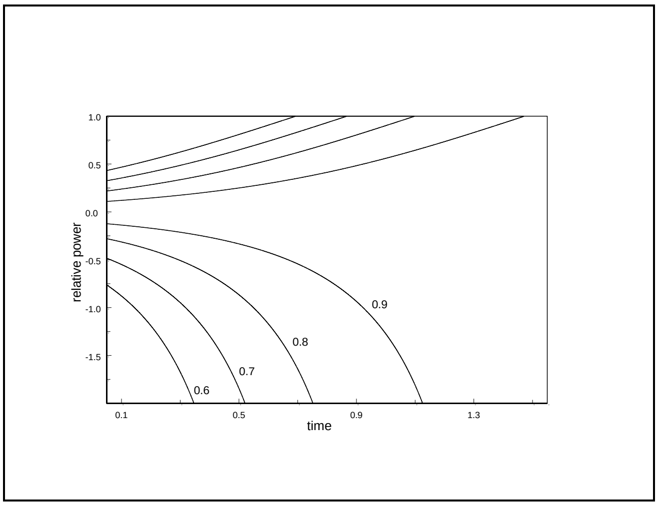


Figure 4 As for the previous figure but using $p_1(t)$ and $p_2(t)$ values given by equation (15)



Figures 1 and 2 are similar in shape, but differ in the range and symmetry of *p*. The range is between *+1* and *-1* with symmetry about the value $p = 0$ in Figure 1, but the range is between *0* and *1* with symmetry about the value $p = ½$ in Figure 2. Both figures show that rebel groups with a large initial size (e.g. troop ratio *0.8*) are able to sustain conflict longer than rebel groups with a small initial size (e.g. troop ratio *0.2*). The rebels suffer a defeat in all cases because they start the conflict weaker than the government.

Figures 3 and 4 show illustrations of the same conflict but with *p* values defined by equations 14 and 15 respectively. Inspection of the graphs suggests that these are less satisfactory for illustrating conflicts. In Figure 3, *p* ranges between *0* and *2*, but the curves are asymmetric around the value *1*. In Figure 4, *p* ranges between *1* and *-2*, again with the curves being asymmetric about the value *0*. This asymmetry arguably makes this representation less desirable because the curves do not convey the progress of relative power during a conflict in an intuitive or aesthetic manner. The definitions given by equations (12 and 13) are thus considered more suitable for modelling relative power and as such, the definition given by equation (13) is used for all subsequent simulations performed in this paper.

Interpretation of the rate constants B$_1$ and B$_2$

The constants B_1 and B_2 in equations (8 and 9) relate to the fighting capabilities (expressed in terms of the killing 'effectiveness') of each side in a direct-fire conflict. The products B_1N_1 and B_2N_2 may thus be regarded as *weighted* numbers. Thus, for the example given by equations (8 and 9), the rates of loss dN_1/dt and dN_2/dt are equal, not when $N_1 = N_2$, but rather when $B_1N_1 = B_2N_2$. The implication is that a small group with a high B-value can emerge victorious over a larger group with a low B-value. In the initial proposal by Christia (2012), war is assumed to be equally costly for both sides and indeed in the above initial numerical simulations, this was implicit, in that the same killing effectiveness was used for both sides in all calculations ($B_1 = B_2 = 0.01$). In general this will not be the case in real conflicts and in this respect the present approach extends that of Christia (2012). Amongst other possibilities the terms B_1 and B_2 may also contain a time dependence reflecting the actor's learning and perceptions over the course of conflict.

In cases where $B_1 \neq B_2$, the relative power can be represented in a modified form of equation (6) as follows:

$$p_1 = \left(\frac{B_1 N_1 - B_2 N_2}{B_1 N_1 + B_2 N_2} \right), \quad p_2 = \left(\frac{B_2 N_2 - B_1 N_1}{B_1 N_1 + B_2 N_2} \right) \qquad (16)$$

The other representations of relative power given by equations (4, 5 and 7) can also be modified in a similar way. For simplicity in all subsequent simulations, the values are kept constant at $B_1 = B_2 = 0.01$ and the definition of normalised time is mt, where $m = (B_1B_2)^{-1/2}$. The next section considers how the relative power concept may be utilised for illustrating other types of conflict, namely *guerrilla* and *asymmetric* warfare.

Modelling other types of two-sided armed conflict

A second type of conflict is *guerrilla* warfare (Lanchester 1956; Deitchman 1962). This differs from direct-fire warfare in that each opposing side has to 'search' for their opponent before firing (e.g. in jungle warfare). This means that the overall killing rate must contain a term similar to that in a direct-fire type of conflict, but also reflect the fact that it must be moderated as the number of opponents decrease and therefore become progressively more difficult to find. This type of conflict is described by the following coupled equations:

$$\frac{dN_1}{dt} = -C_2 N_2 (D_1 N_1) \tag{17}$$

$$\frac{dN_2}{dt} = -C_1 N_1 (D_2 N_2) \tag{18}$$

These equations are similar to equations (8 and 9) in that the terms C_1N_1 and C_2N_2 are equivalent to B_1N_1 and B_2N_2. However, the equations contain the additional terms D_1N_1 and D_2N_2 to describe the decrease in killing rate as the remaining number decreases. The corresponding constants are D_1 and D_2 and may be regarded as 'search' probabilities. The analytical solution of these coupled equations is complex, but numerical analysis can be performed relatively easily by using a method similar to that described earlier for the direct-fire conflict. The modified forms of equations (10 and 11) shown below:

$$(N_1)_{t+\Delta t} = (N_1)_t - C_2 D_1 (N_2)_t (N_1)_t \Delta t \tag{19}$$

$$(N_2)_{t+\Delta t} = (N_2)_t - C_1 D_2 (N_1)_t (N_2)_t \Delta t \tag{20}$$

The computational flow diagram is similar to the previous case, except that equations (19) and (20) are used instead of equations (10) and (11).

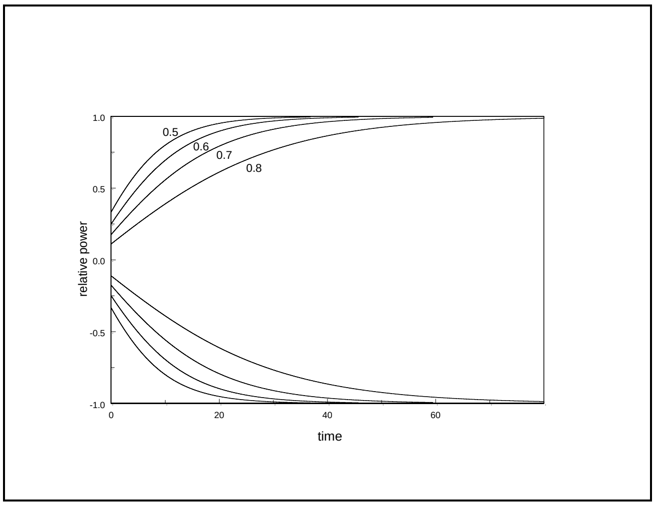
In the previous case of direct-fire war, ABM's have little advantage over numerical computational methods. In the case of guerrilla warfare however, ABM can be more suitable. One facility of ABM is that the virtual agents can be instructed to move around their environment randomly. They can then be instructed to engage in certain behaviours when they interact (collide) with other (specified) agents. Thus for a conflict involving two groups, government and rebel agents may be instructed to move randomly and on interaction, prescribed killing probabilities for both types of agent can be enabled, so that each side experiences a decrease in numbers of troops over time. ABM is an ideal method for modelling guerrilla conflict because the random motion of agents in two-dimensional space can be used as an analogue of the search-and-kill sequence that characterises guerrilla war.

Both the numerical computational method and the ABM method were used to model guerrilla conflict. The results of the numerical method based on equations (19 and 20) are shown in

Figure 5, using an assumed initial size of the government forces $N_2 = 1000$, with four different initial rebel sizes; $N_1 = 800, 700, 600$ & 500. For illustration purposes, the values of the *killing probability* constants C_1 and C_2 were taken to be identical to the constants B_1 and B_2 used earlier for the direct-fire case. In the numerical models the search parameter values $D_1 = D_2 = 0.0003$ were used. These were selected by performing sensitivity analyses until the numerical computational method and ABM were calibrated to give identical results.

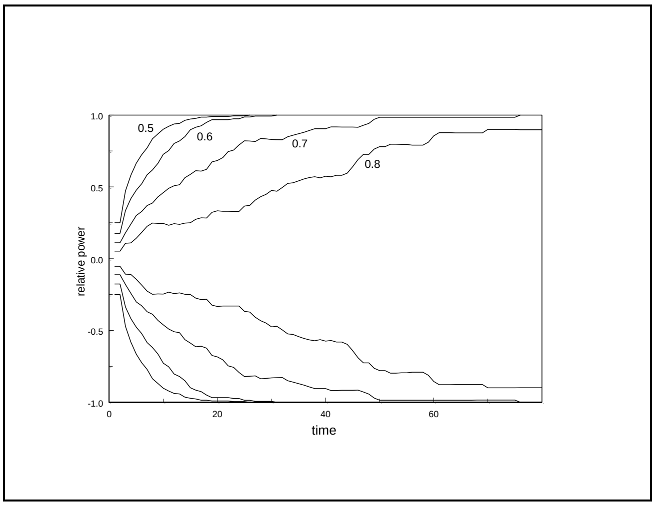
The variation of the relative power with normalised time predicted by the numerical method is illustrated in Figure 5. This reveals a striking difference in the shape of the curves to those in shown in Figure 2 for the direct-fire conflict. The curvatures of the lines are of opposite sign in these two figures and this characteristic emphasises the utility of the current approach in the visual representation of conflict type. The 'long-tailed' curves for guerrilla conflicts reflect strikingly the increasing difficulty of eliminating the last few rebels in this type of conflict and demonstrate clearly why such conflicts are protracted. This contrasts sharply with the abrupt termination revealed for the direct-fire conflicts.

Figure 5 Variation of $p_1(t)$ and $p_2(t)$ with normalised time for $N_2 = 1000$ and $N_1 = 800, 700, 600$ & 500 respectively during a guerrilla-conflict.



Values of $p_1(t)$ and $p_2(t)$ were also generated using an ABM. The flow diagram for the computation is shown in Appendix II. The variation of relative power with time for the rebel groups and the government is plotted in Figure 6, derived by assuming the same parameters as described in the above numerical model. Although the characteristic curvatures of these ABM plots are similar to those derived from the numerical model, a significant difference is that the curves derived from the ABM are 'noisy'. This feature results from the inherent stochastic nature of ABM and arises because the agents are programmed to move randomly and the collisions between agents from each side, is a proxy for killing, which occurs according to an input value of probability. Because one set of agents have to randomly 'search' for enemy agents before killing them, this effect naturally reflects guerrilla conflict. This 'noisy' nature of the curves illustrates this effect clearly. Thus, the 'built-in' stochastic effects of ABM modelling illustrate how the progress of conflicts with the same starting conditions can have different trajectories. Towards the end of the conflict when the numbers of troops are small, the level of noise increases. When values for the state and rebel groups sizes are small, it is clear that in some circumstances, the random features that result from ABM modelling, could lead to relative power curves for the state and rebel groups to 'cross over', so that an initially weaker side might emerge victorious.

Figure 6 Results generated from an Agent-Based Model for the variation of $p_1(t)$ and $p_2(t)$ with normalised time for $N_2 = 1000$ and $N_1 = 800, 700, 600$ & 500 respectively during a guerrilla conflict. Note that one run for each rebel size (N_1) is shown.



A third type of conflict is a*symmetric* conflict (Lanchester 1956; Schaffer 1968). Typically this might occur when a large force, such as the government, is matched against a smaller force such as a rebel group, with the latter employing guerrilla tactics and the former employing direct-fire tactics. In such asymmetric conflicts, the rebel side can have an advantage over the government, in spite of their smaller size, because the government must search for their opponents before they can make a kill, whilst the rebels use direct fire tactics. The extent of this rebel advantage depends on how difficult it is for the government to locate the rebels. The corresponding differential equations describing an asymmetric conflict are as follows:

$$\frac{dN_1}{dt} = -C_2 N_2 (D_1 N_1) \tag{21}$$

$$\frac{dN_2}{dt} = -C_1 N_1 \tag{22}$$

Once again, a numerical solution can be achieved by using a method similar to those used previously. The finite difference equations corresponding to equations (21 and 22) are:

$$(N_1)_{t+\Delta t} = (N_1)_t - C_2 (N_2)_t \{D_1 (N_1)_t\} \Delta t \tag{23}$$

$$(N_2)_{t+\Delta t} = (N_2)_t - C_1 (N_1)_t \Delta t \tag{24}$$

Values of relative power, $p_1(t)$ and $p_2(t)$, were generated to simulate asymmetric warfare using equations (23 and 24). The computational flow diagram is again similar to the previous case except for the use of these two equations. In this evaluation, the government side was assumed to adopt a *search-then-kill* strategy (with a search probability $D_1 = 0.0003$) and the rebels a direct-fire approach. The initial size of the government side was $N_2 = 1000$ and four different initial rebel sizes were assumed; $N_1 = 800, 600, 400, 200$. The results are plotted in Figure 7.

An important feature of asymmetric warfare (Figure 7) compared to direct-fire (Figure 1) and guerrilla warfare (Figure 5), is that the rebels can emerge victorious despite starting out weaker than the government. Figure 7 shows, for the present choice of parameters, that rebel victory ensues in all cases, even when they start the conflict with a troop size ratio of *0.2*. The victory of the initially weaker side also manifests itself in Figure 7 by the curves intersecting and crossing. This figure again illustrates the use of the present relative-power approach in producing highly visually-effective representations of conflict progress. The classic empirical case of asymmetric conflict is the Vietnamese-US war (Mack 1975; Paul 1994; Arreguin-Toft 2001). The present analysis produces a new representation of asymmetric concept based on the current extension to the relative-power approach proposed by Christia (2012).

Figure 7 Variation of $p_1(t)$ and $p_2(t)$ for $N_2 = 1000$ and $N_1 = 800, 600, 400$ & 200 during an asymmetric conflict with a 'search' probability $D_1 = 0.0003$. Each pair of lines cross in this example, indicating that rebel victory occurs in all cases

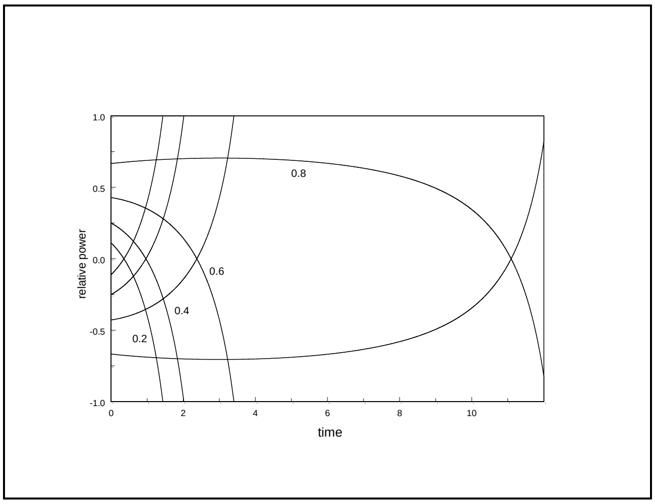
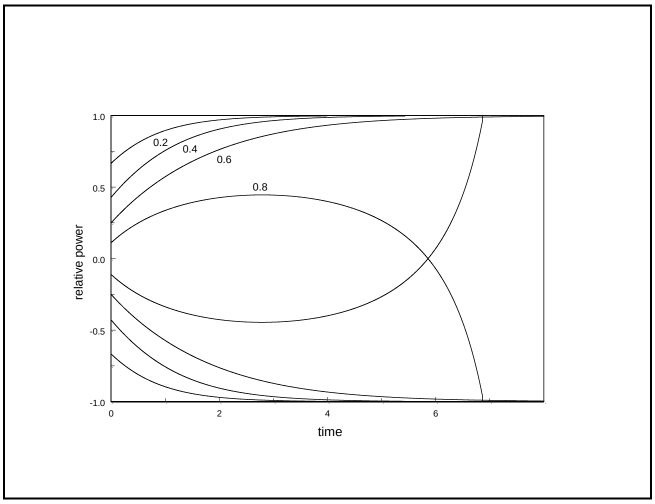


Figure 8 illustrates the results of a repeat evaluation of that represented in Figure 7 using identical parameters, except that D_1 is increased by a factor of *5*. This causes a major change of conflict outcome compared to that illustrated in Figure 7. Only the single case (where $N_1 = 800$) now results in a rebel victory. These curves clearly illustrate the expectation that when rebels are easier to find (i.e. for larger *D*), their advantage is diminished. A further important point is that the curves in Figure 8 for cases where rebels suffer defeat have the same sign for their curvature as those for guerrilla warfare (see Figure 5). Thus for cases where rebel defeat ensues, the curves for asymmetric warfare and guerrilla warfare are similar.

Figure 8 As for the previous figure but with D_1 increased by a factor of *5*. In this case, rebel victory now occurs only for the largest rebel group size



Inclusion of stochastic effects

Christia (2012) suggests that the time variation of the relative power *p* during a conflict is unlikely to be smooth, but may be likened to a biased random walk, where random changes occur in the direction of increased or decreased relative power. The general 'drift' direction is driven at the onset by the initial values of *p*, but random influences are likely to occur over time. The ABM for guerrilla conflict illustrated in Figure 6 clearly shows the importance of such stochastic effects, but the numerical evaluations performed so far have not included any attempts to model stochastic terms. It is important to re-emphasize the importance of the stochastic term in equation (1). In its absence, conflict outcomes would always be predictable if the initial conditions were known. In principle, there would be no need for a conflict.

Christia does not suggest how the stochastic term could be implemented practically for the purposes of illustrating conflict dynamics. This section examines some specific ways in which such a term might be applied to deterministic models of two-sided conflict. These are not intended to model actual physical processes that might occur, but are simply used to illustrate the above important point, namely that random events can produce unexpected outcomes. The next section considers how randomness might be introduced into the value of the probability parameters (B_1 and B_2 or C_1 and C_2) and is followed by a consideration of how randomness could be introduced via sudden changes that might occur in the number of troops.

In the first instance, random changes were incorporated into the term describing the 'killing' probability. This is achieved by putting a degree of randomness into the constants B_1 or B_2 in equations (8) and (9). Since these are related to the fighting capability of a group, the real-life interpretation of this might be the unexpected acquisition of military equipment and arms from an external source, or indeed the unexpected loss of equipment and arms as a result of bad weather, bombardment or looting. One way in which the randomness can be incorporated is by changing the values of B_1 and B_2 after each time step Δt in the numerical computation, using the uniform distributions defined by the relationships:

$$B_1 = (B_1)_{lower} + U\left[(B_1)_{upper} - (B_1)_{lower}\right] \tag{25}$$

$$B_2 = (B_2)_{lower} + U\left[(B_2)_{upper} - (B_2)_{lower}\right] \tag{26}$$

where $0 \leq U \leq 1$ is a random number and $(B_1)_{lower}$, $(B_2)_{\text{lower}}$, $(B_1)_{upper}$ and $(B_2)_{upper}$ are the lower and lower limits for the range of values for any given illustration.

Figures 9 – 11 show the effect of applying the above stochastic variation to the killing probability of *just the rebel side*, for the cases of direct-fire, guerrilla and asymmetric warfare respectively. A random increase or decrease in the killing probability of the rebels was introduced after each time step of the computation according to equation (25). In the previous simulations constant values $B_1 = B_2 = 0.01$ were assumed. In this simulation it was assumed that $(B_1)_{lower} = 0$ in all cases and $(B_1)_{upper} = 0.05$ for the direct-fire conflict (Figure 9) and $(B_1)_{upper} = 0.04$ for the guerrilla and asymmetric conflicts (Figures 10 and 11). Once again $N_2 = 1000$ and $N_1 = 800, 600, 400$ & 200.

Consider the results obtained for the specific evaluation illustrated in Figure 9. Comparing this to the equivalent but entirely deterministic case shown in Figure 1, reveals that the stochastic contribution described above does not change the rebel outcome for $N_1 = 600, 400$ & 200, but slightly increases the timescale over which the conflict is sustained. For the largest rebel group ($N_1 = 800$), inclusion of the stochastic term now results in rebel victory. Inspection of Figures 10 and 11 (compared with Figures 5 and 8) reveals a similar effect. For the case of guerrilla warfare shown in Figure 10, the rebels have emerged victorious for $N_1 = 800$ & 600, unlike the outcome illustrated in Figure 5, where all rebel groups suffered defeat. In Figure 11, for the case of asymmetric conflict, the rebels achieve victory for $N_1 = 800, 600$ & 400, unlike the outcome in Figure 8, where victory occurred only for the largest rebel group, $N_1 = 800$.

Figure 9 Variation of $p_1(t)$ and $p_2(t)$ for $N_2 = 1000$ and $N_1 = 800, 600, 400$ *and* 200 during a direct-fire conflict with a stochastic element applied to killing probability (B_1) of the rebel side, resulting in rebel victory for the largest group

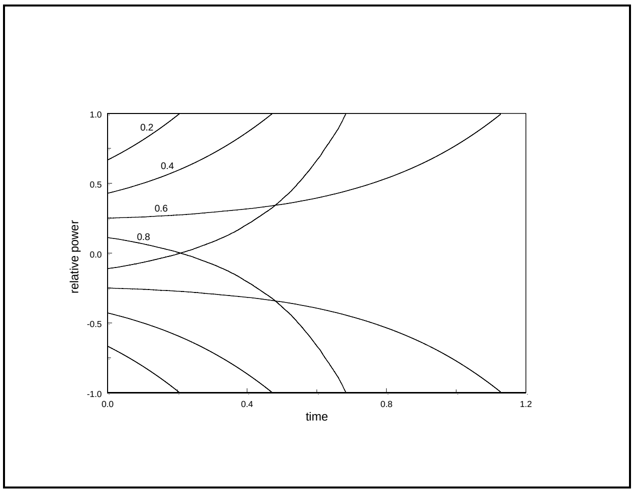


Figure 10 Variation of $p_1(t)$ and $p_2(t)$ for $N_2 = 1000$ and $N_1 = 800, 600, 400$ & 200 during a guerrilla conflict with a stochastic element applied to killing probability (C_1) of the rebel side only. Rebel victory occurs for the two largest groups

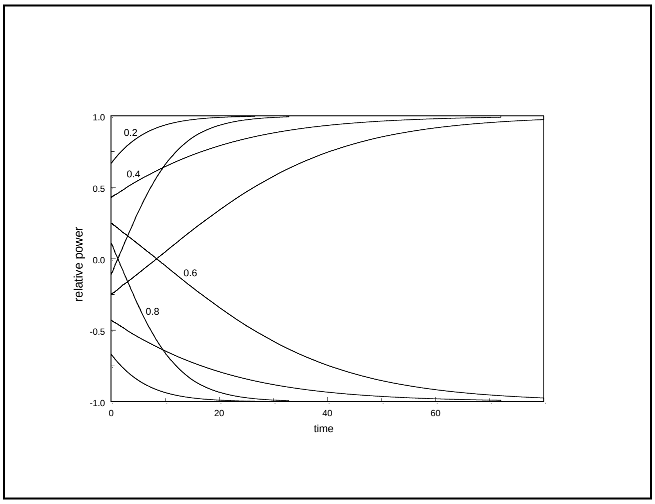
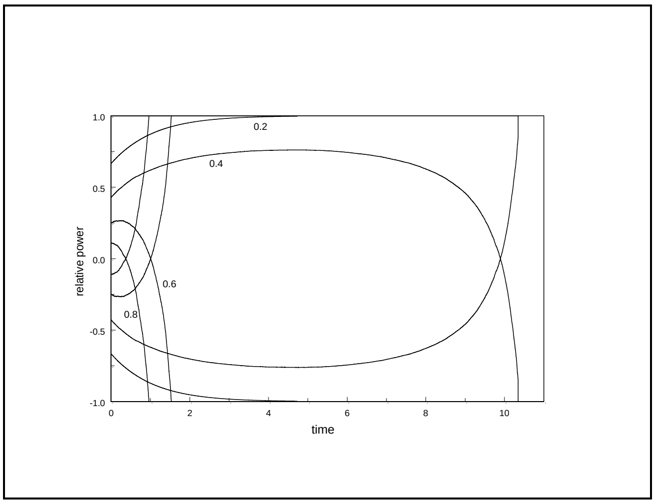


Figure 11 Variation of $p_1(t)$ and $p_2(t)$ for $N_2 = 1000$ and $N_1 = 800, 600, 400$ & 200 during an asymmetric conflict with a stochastic element applied to killing probability (C_1) of the rebel groups only. A search probability is applied to just the government side with the value 1.5×10^{-3}. Rebel victory occurs in three cases



An alternative approach to implementing stochastic effects is to introduce random changes to the number of troops of one or both sides (i.e. add randomness to either N_1 or N_2). The real-world interpretation of this might be a sudden increase in troops as a result of access to mercenaries, or a decrease in troops as a result of disease. Unlike the previous case, where randomness was applied to the search probability, a different strategy is necessary in the case of adding stochastic changes to group numbers. An obvious possibility is to simply add or subtract a fixed random component $\pm\Delta N$ to the troop and/or rebel group size after each time increment Δt of the computation. It is unlikely however that this 'Brownian-type' of randomness would occur in the real-world situation and a power-law relationship is more likely. A power law distribution is described by the function:

$$f(x) = \frac{\kappa}{x^{\alpha}} \tag{27}$$

where κ is a constant and α is the power law exponent. This type of distribution is common in nature and is characterised by a high frequency of small values, an intermediate frequency of medium-sized values, with large values occurring more rarely. Power-law distributed random values of a variable z in the range $z_{min} \leq z \leq z_{max}$ can be generated from the standard relationship:

$$z = \left(\frac{z_{\min}^{\alpha} z_{\max}^{\alpha}}{U z_{\min}^{\alpha} + (1-U) z_{\max}^{\alpha}} \right)^{\frac{1}{\alpha}} \tag{28}$$

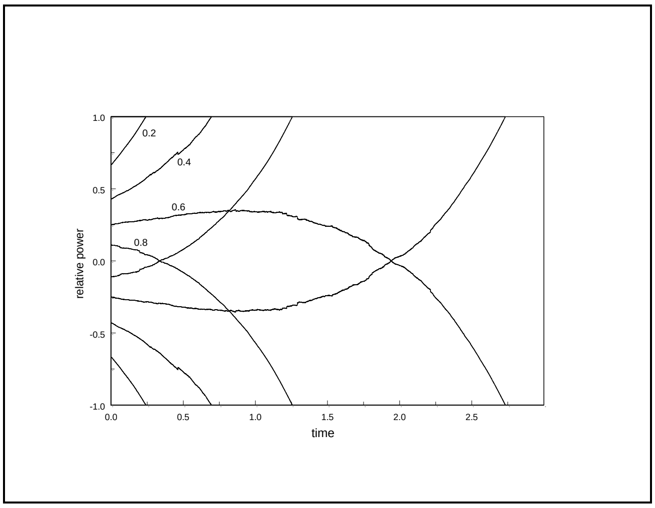
where $0 \leq U \leq 1$ is a uniformly distributed random number.

To obtain the stochastic contribution for incorporation into the present model, values of z were calculated from equation (28) after each time step (Δt) in the computation. The random value $\Delta N = z(N_1)_{init}$ was then determined, where $(N_1)_{init}$ is the initial number in the rebel group. A second (uniformly distributed) random number in the range $0 \leq U \leq 1$ was then used such that if $U \geq 0.5$, then ΔN was added to N_1 or was subtracted otherwise. It was assumed that $\alpha = 2$, $z_{min} = 0.0001$ and $z_{max} = 1$.

Once again, it was assumed that the group sizes were $N_2 = 1000$ and $N_1 = 800, 600, 400$ & 200. A direct-fire conflict was assumed for illustration and results are shown for a typical evaluation in Figure 12. By comparison with the direct-fire conflict with no stochastic term illustrated in Figure 1, it is clear that inclusion of the stochastic term can lead to rebel victories (intersecting curves are observed for two cases) that would not be expected from predictions based on deterministic calculations alone. No equivalent illustrations are given here for guerrilla and asymmetric conflict, but the general message is clear, namely that stochastic effects can give rise to radically different conflict outcomes.

It is important to reiterate that the examples shown are driven by parameter values, which were selected by performing sensitivity studies. Their purpose is to demonstrate the scope of the various models, not to demonstrate what might happen over a number of experimental runs. That is, with the parameter values selected for the experiment shown in Figure 12, it is possible to observe a cross-over effect, where the group who starts off weaker (in this case, the rebels) may emerge victorious as a result of stochastic effects.

Figure 12 Variation of $p_1(t)$ and $p_2(t)$ for $N_2 = 1000$ and $N_1 = 800, 600, 400$ & 200 during a direct-fire conflict with a stochastic element applied to the rebel groups only (N_1). Rebel victory ensues for the two largest rebel groups



Inclusion of rebel recruitment and government deployment

The types of conflict described above, involve only the attrition of two opposing groups and in the absence of stochastic changes in troop numbers, it is assumed that there is no replenishment of state or rebel group members during the conflict. In most real-world cases however, numbers are likely to be replenished in various ways. This can be modelled by including one or more of the following possibilities: *i)* instantaneous increments of troop numbers occur, *ii)* time-dependent increases of troop numbers occur *iii)* increases of troop numbers occur in response to the size of the opposing group and *iv)* time-dependent increases of troop numbers occur either in response to (or independent of) the size of the opposing group, but with a time-lag.

In either ABM or ODE models, items *i)* and *ii)* can be modelled simply by allowing either an instantaneous step increase in numbers at specific times, or enabling a time-dependent increase as the computation progresses. Items *iii)* and *iv)* can be modelled by incorporating a term that allows for a rate of increase in group numbers that is driven by the number in the opposing group. Note that the inclusion of troop replenishment in conflict models represents an important novel contribution of the present work. This represents an important contribution that emphasises the efficacy of the Christia relative power concept in graphically representing the progress of conflicts.

Consider first the growth of a rebel group with no attrition by an opposing force. If the growth follows the 'limited growth equation' described in Appendix III, then the growth rate is given by the expression:

$$\frac{dN_1}{dt} = A_1 N_1 (N_{\max} - N_1) \qquad (29)$$

where N_{max} is the maximum number of potential rebel recruits in the population and A_1 is a recruitment rate constant. Thus, in order to model an asymmetric conflict with concurrent rebel recruitment, the growth rate can be incorporated into equation (21) to give the overall rate of change of rebel group numbers as follows:

$$\frac{dN_1}{dt} = A_1 N_1 (N_{\max} - N_1) - (C_2 N_2) D_1 N_1 \qquad (30)$$

During a conflict it is likely that the state will respond to increases in the rebel threat by increasing its own strength. One of the possible strategies listed at the beginning of this section is for the state to increase its troop numbers at a rate proportional to the current number of rebels (i.e. proportional to A_2N_1, where A_2 is a deployment rate constant). This response term can be incorporated into equation (22) to give the overall rate of change of state troop numbers as follows:

$$\frac{dN_2}{dt} = A_2 N_1 - C_1 N_1 \qquad (31)$$

Note that the 'loss' terms (C_2N_2), D_1N_1 and C_1N_1 in equations (30 and 31) are those described earlier in this paper for an asymmetric conflict. That is, the state troops are killed by a direct-fire process and the rebels by a search-and-kill, guerrilla strategy. Since asymmetric fighting is common to contemporary civil wars this is assumed for all subsequent numerical experiments.

The progress of a hypothetical conflict with concurrent recruitment of the rebel group and deployment of state troop numbers in response is shown in Figure 13. The various constants were chosen to illustrate the important case that even if the state initially outnumbers the rebels (in this case by a ratio *20:1*) the rebels can emerge victorious because of their high rate of concurrent recruitment. The following values were used: $N_{max} = 2000$, $N_1 = 100$, $N_2 = 2000$, $A_1 = 10^{-5}$, $A_2 = 5 \times 10^{-2}$, $C_1 = C_2 = 1 \times 10^{-2}$ and $D_1 = 3 \times 10^{-4}$. In Figure 14, the time-dependence of the state and rebel numbers during the course of the above conflict are shown represented as a fraction of their initial numbers.

Figure 13 Variation of relative power for the state and a rebel group in an asymmetric conflict with concurrent rebel recruitment and state deployment. In this example the rebel group gains victory even though it is outnumbered initially in the ratio *20:1*

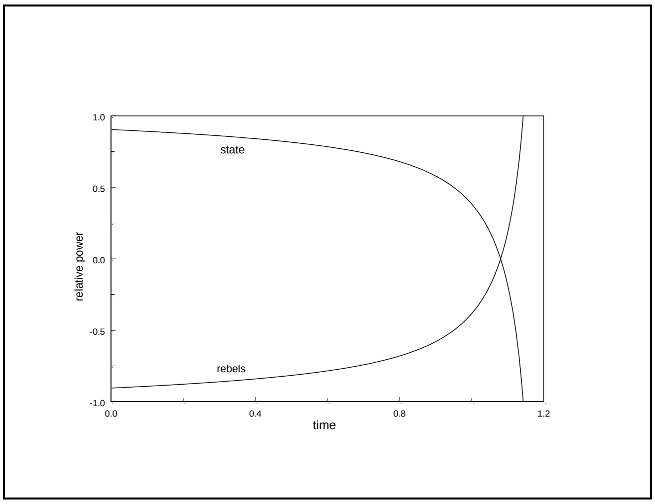
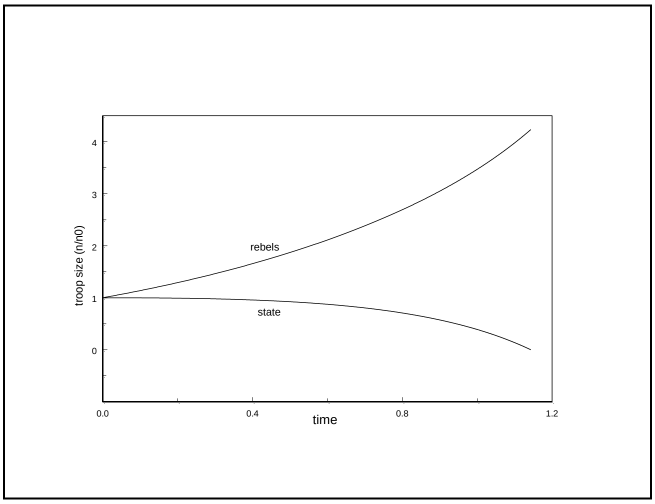


Figure 14 Time-dependence of the state and rebel numbers during the course of the conflict illustrated in the previous figure. Numbers are represented in terms of the fractions of their initial values



A further evaluation was performed, identical to that above, except that D_1 was increased by a factor of ten, which means that rebels are easier to locate by the state. Results are plotted in Figure 15, which shows that the outcome changes from a rebel victory to a state victory. In Figure 16, results are re-plotted in terms of the variation of troop numbers with time. In this particular case, state numbers remain almost unchanged throughout the conflict.

Figure 15 As for the hypothetical conflict illustrated in the previous two figures, except that the constant D_1 was increased by a factor of ten. This now results in state victory since rebel troops are now easier to find

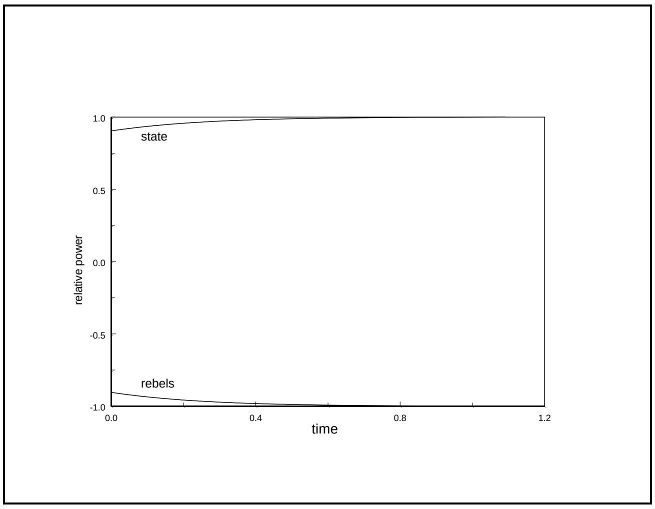
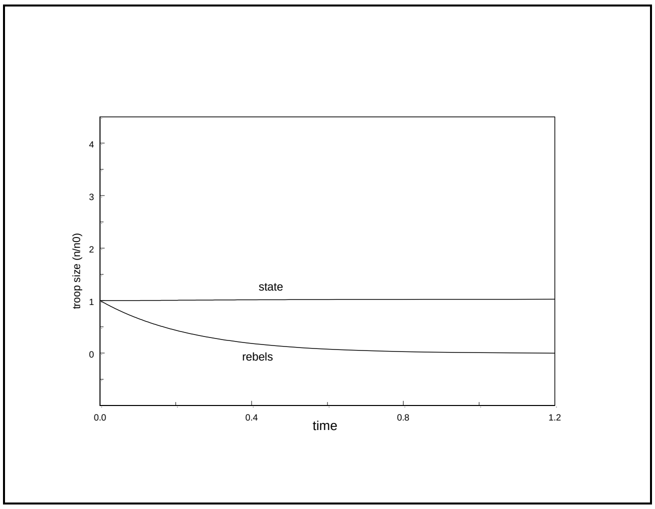


Figure 16 Results for the previous figure, plotted as the variation of troop numbers, normalised in terms of the initial number, as a function of time



The influence of reducing the initial state strength on the outcome of a conflict otherwise similar to that described above is illustrated in Figure 17, where the ratio of state to rebels was changed from *20:1* to *6:1*. The state still emerges victorious, but the conflict is more protracted than in the previous example. Figure 18 shows the variation of troop numbers, normalised in terms of the initial number, as a function of the normalised time. Figure 19 illustrates the effect on the outcome of the previous conflict if the value of the state deployment constant (A_2) is reduced by a factor of *10*. The rebels are now victorious because the state cannot deploy enough troops to successfully overcome the rebels. Figure 20 shows the variation of troop numbers with time for this conflict. Comparison with Figure 18 shows that the curves representing the state and for rebels become inverted.

Figure 17 Effect of decreasing the initial state strength. The value of the rebel/state ratio is *1/6*, compared to the value *1/20* used for the previous conflict

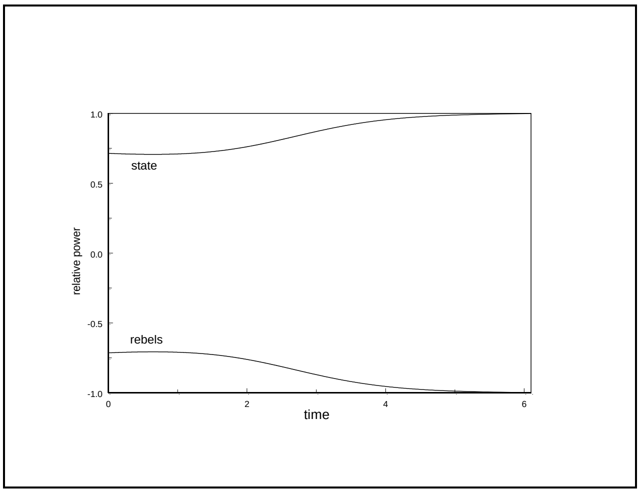


Figure 18 Results for the previous figure, plotted as the variation of troop numbers, normalised in terms of the initial number, as a function of time

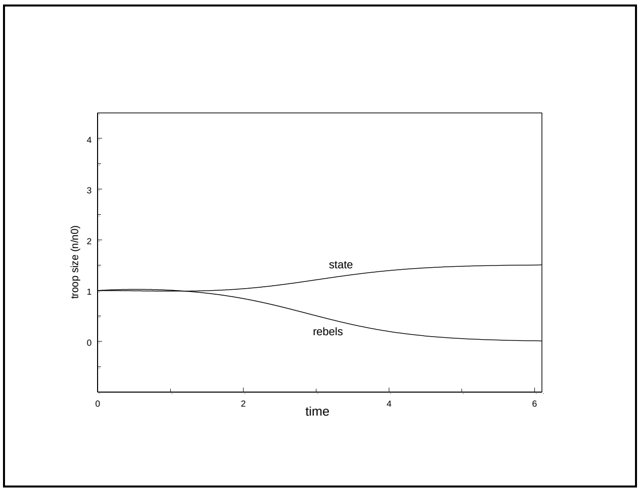


Figure 19 Influence of reducing the state deployment constant by a factor of *10*, in the previous conflict. Compared to Figure 17, the outcome is reversed; rebel victory occurs

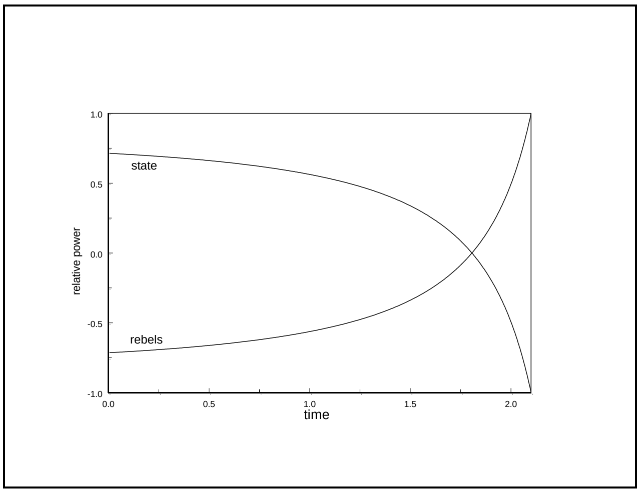
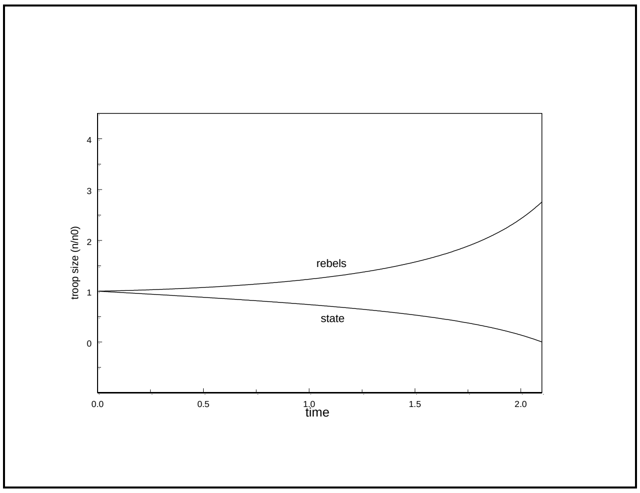


Figure 20 Variation of normalised state and rebel troop numbers with normalised time for the previous conflict



Concluding remarks

This paper has explored various ways in which two-sided civil conflict can be modelled using numerical simulation techniques. The approach proposed by Christia (2012) based on a metric of relative power has been extended. General numerical models of two-sided conflict were developed to test the efficacy of the Christia approach in illustrating the progress of conflict. Various aspects were investigated, including the optimal definition of relative power, the different mechanisms of two-sided conflict (direct-fire, guerrilla and asymmetric warfare), the inclusion of a stochastic element (both to the number of rebel troops and the rebel group killing constant) and inclusion of terms for rebel recruitment and the deployment of government troops.

A comparison of ordinary differential equation (ODE) modelling and agent-based modelling (ABM) allowed an assessment of the suitability of these two types in different contexts. ABM was found to be particularly suited to the modelling of guerrilla warfare due to an inherent feature of ABM programming software, where agents can be programmed to move randomly and collide with other agents (thus replicating the search-and-kill characteristic of guerrilla warfare). ABM was found to be less advantageous to modelling direct-fire and asymmetric warfare because both of these types of warfare involve a direct-fire mechanism on at least one side. Since a direct-fire mechanism does not require the agents to search for their opponent before more making a kill, the random movement of agents around an artificial environment (which is inherent in ABM) is superfluous.

Overall, this paper has shown that Christia's general concept provides a sound basis from which numerical models of conflict can be developed. The metric of relative power and how it varies throughout a conflict, provides a highly effective and clear visual representation of conflict dynamics, namely duration and outcome. Striking visual differences are apparent in the characteristic curves representing the different types of conflict.

Contemporary civil wars are often asymmetric; meaning that conflicts last longer because the rebel agents become increasingly difficult to locate as they decrease in number. This means that rebels are at a significant advantage despite their small number. This mechanism explains why some conflicts involving weak rebels become protracted. This paper has shown that implementing a random element to numerical models of conflict can dramatically change the duration and outcome of war. Similarly, the effects of rebel recruitment and government deployment are shown to have important influences on conflict dynamics in ways that might be expected intuitively.

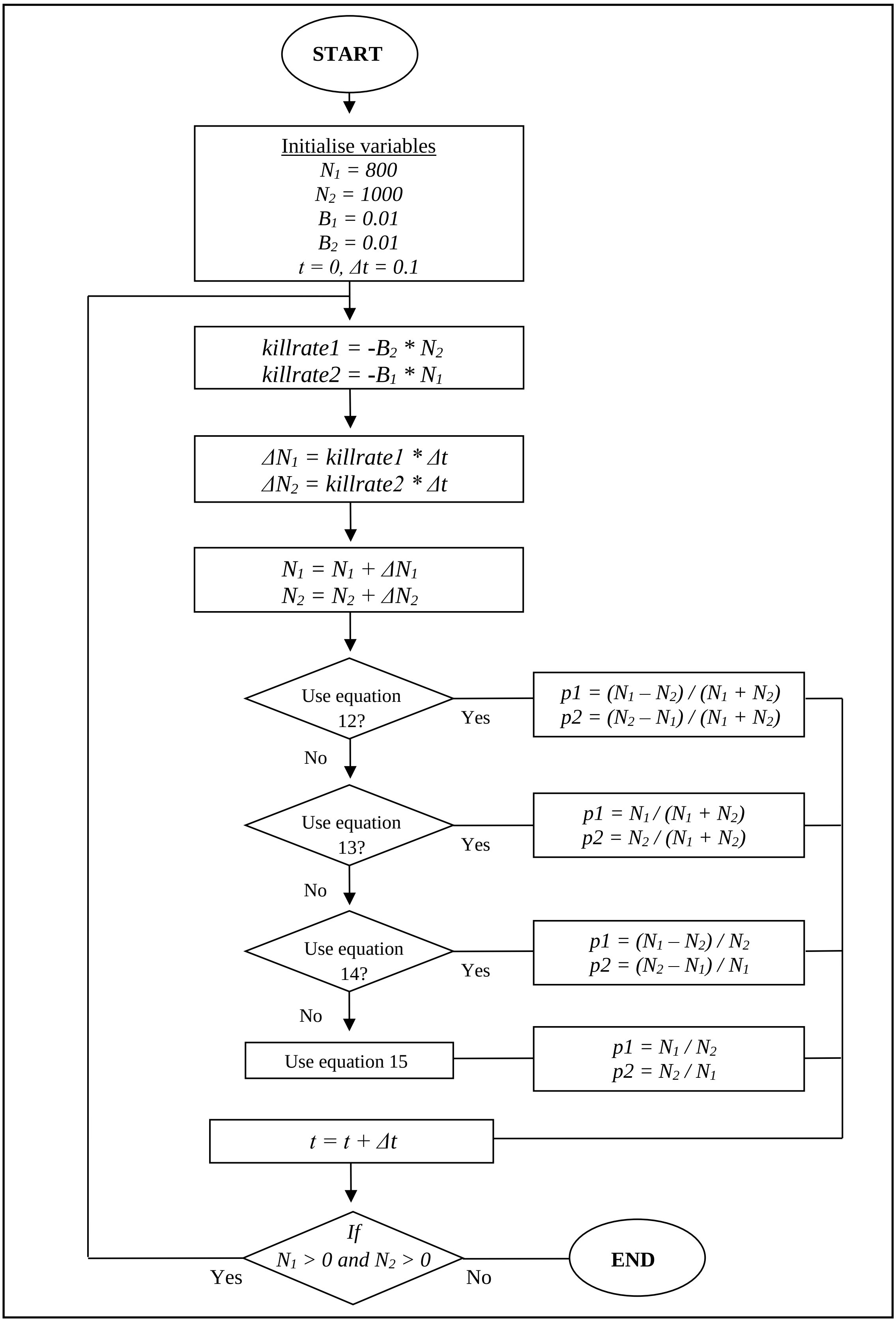
Future research should also utilise the models of two-sided conflict presented in this paper to uncover the micro-level mechanisms underlying real-life conflicts. To accomplish this, sensitivity studies could be performed and the various model parameters could be tuned, or set according to empirical distributions, so that the simulated conflict outcomes match those observed in empirical data. It might then be possible to gauge what micro-level factors lead to certain macro-level war dynamics. Future research could also utilise numerical models to assist with out-of-sample forecasting. In this case, numerical models could be used to assess the plausibility of predictions obtained from statistical models and to ascertain how robust predictions are to random events.

The numerical models presented in this paper are limited to the assumption of two-sided conflict, but they are highly amenable to be extended to consider the case of multi-party conflict, with multiple rebel groups fighting the state simultaneously (or sequentially). Such multi-party models can be extended further to simulate rebel group interaction strategies during conflicts. These include strategies such as alliance formation and inter-rebel violence. The influence of group interactions on conflict dynamics could then be compared to the results of statistical analyses of real conflicts.

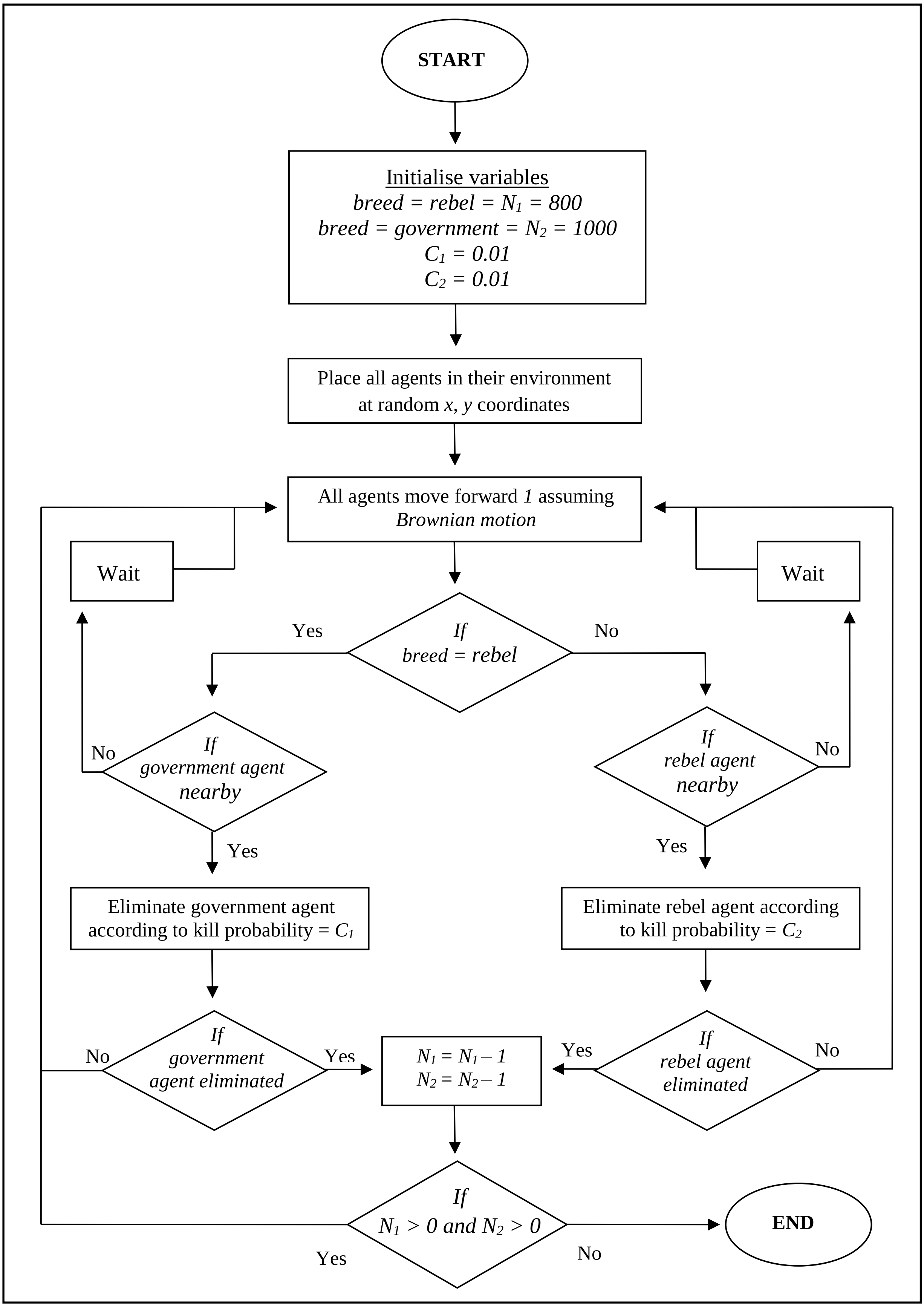
The current models are also limited to wars of total attrition and as such, they represent conflict progression in situations where bargaining has failed and negotiated settlements are not reached. However, the models have the flexibility to be easily adapted to explore bargaining theories of war. This presents obvious avenues for further research, where negotiation subgames between the actors could be incorporated with ease. These outcomes of these bargaining subgames could be explored by incorporating them with various assumed scenarios. For example, they could be assumed to occur at arbitrary time points, or when the relative power between the two sides reaches some defined threshold, or possibly when the rate-of-change of relative power exceeds a threshold. Probabilities within these bargaining subgames could then be tuned so that the occurrence of negotiated peace settlements predicted by the numerical models match those observed in empirical data.

It is clear that the models provide a firm foundation from which to build-in a wide variety of further adaptations and incorporate other aspects. The extension of the models to multi-party conflict is an obvious step forward, but amongst many other possibilities, the constants used in the models for illustration in this paper, may themselves have a time (or even group size) dependences. These may be associated with the change in perceptions, learning and skills of actors throughout the conflict. These features can be incorporated with ease and provide a further flexibility when tuning the model to real empirical data.

Appendix I. Flow diagram of computational scheme used to generate $p_1(t)$ and $p_2(t)$ values for equations (12 – 15) plotted in Figures 1 – 4.



Appendix II. Flow diagram for ABM used to generate $p_1(t)$ and $p_2(t)$ values for guerrilla warfare plotted in Figure 6.



Appendix III. The modelling rebel group growth

The growth of a rebel group can be likened to the spread of an infection through a population, such that existing group members (or infected individuals) recruit (or infect) others. Such a process can be described by the logistic (or limited growth) equation (Turner *et al.* 1976). The rate of growth of recruited members (*dN/dt*) might then be modelled by the equation:

$$\frac{dN}{dt} = A_1 (N_{\max} - N) N \qquad \text{(A.1)}$$

where N_{max} is the maximum number in the population available to be recruited and A_1 is a rate constant. In the early stages, when $N << N_{max}$, this equation takes the approximate form: $(dN/dt) \approx A_1 N_{\max} N$, which has the solution: $N = N_0 \exp\,(A_1 N_{\max} t)$, where N_0 is the group size at $t = 0$. This exponential growth continues until the term $(N_{max} - N)$ in equation (A.1) becomes increasingly important and the growth rate then decreases, eventually approaching zero as $N \rightarrow N_{\max}$. Note that when $N = N_0$ the initial growth rate is $(dN/dt)_0$ (= $\dot{N}_0$), so that the rate constant can be expressed in the form $A_1 = \dot{N}_0 (N_{\max} - N_0) N_0$ and equation (A.1) can then be rearranged to give:

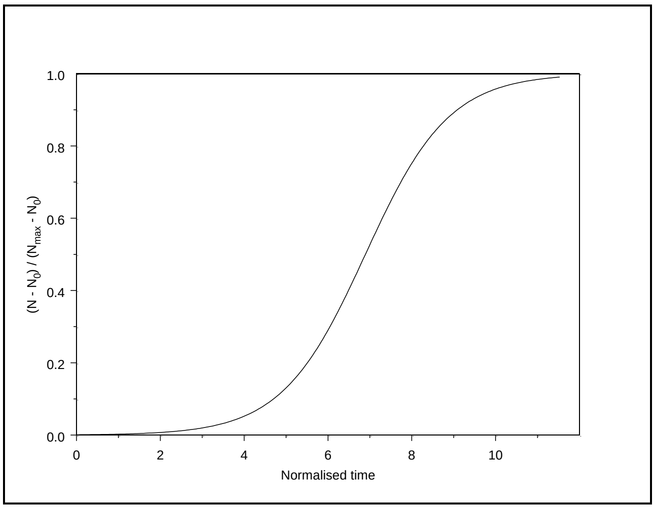
$$\frac{\dot{N}}{N_{\max}} = \frac{\dot{N}_0}{N_0} \left(\frac{1 - N / N_{\max}}{1 - N_0 / N_{\max}} \right) \frac{N}{N_{\max}} \qquad \text{(A.2)}$$

The solution of this equation gives the time dependence of the group size as follows (Turner *et al.* 1976):

$$\frac{N(t)}{N_{\max}} = \frac{1}{1 + (\frac{N_{\max}}{N_0} - 1) \exp - \left[\frac{\dot{N}_0 t}{N_0 (1 - N_0 / N_{\max})} \right]} \qquad \text{(A.3)}$$

This equation represents an S-shaped growth curve and is illustrated in Figure A.1. The group size and time are plotted in the normalised forms $(N - N_0)/(N_{\max} - N_0)$ and $(\dot{N}_0 t / N_0)/(1 - N_0 / N_{\max})$ respectively.

Figure A.1 Group size as a function of time according to equation (A.3). The size and time are expressed in the normalised forms as given in the text

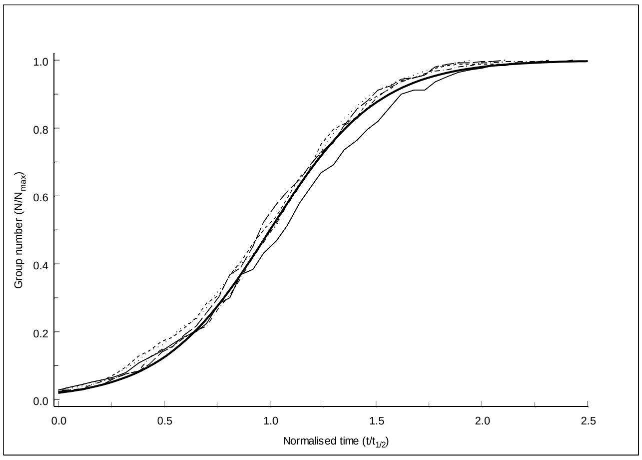


Agent-Based Modelling of Group Growth

Agent-based modelling (ABM) can be used as an alternative to the deterministic method described above. Here the growth of a single rebel group is modelled to reproduce and therefore validate the deterministic method described above. At the onset ($t = 0$), N_{max} agents were programmed to move by random motion over a two-dimensional surface. Of these agents, a number N_0 carried the attribute 'recruited'. On collision with a 'non-recruited' agent, that agent was recruited and therefore gained the attribute 'recruited' according to a user-input value of the probability $0 \leq P \leq 1$. The program produced an output of rebel number versus time.

A set of growth curves obtained by this ABM are shown in Figure A.2 using the values $N_{max} = 1000$ and $N_0 = 50$. The initial rate $\dot{N}_0$ was determined from the average initial rate determined from 15 runs of the program. The value of A_1 was then calculated from the expression given earlier, namely: $A_1 = \dot{N}_0 (N_{\max} - N_0) N_0$. The growth curve determined from deterministic methods, reproduced from Figure A.1, is shown for comparison. The curves are in good agreement, thus validating the compatibility of the two methods.

Figure A.2 Comparison of rebel group growth curves obtained by ABM and those predicted by equation (A.3)



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