# Constructive Interference Based Constant Envelope Precoding

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***Abstract*—We present a new multiple-input-multiple-output (MIMO) transmission scheme for generic phase-shift-keying (PSK) modulations in the multi-user (MU) downlink channel, where Constant Envelope Precoding (CEP) is combined with concepts of interference exploitation. In the proposed approach, multi-user-interference (MUI) is treated as a resource for increasing the signal-to-interference-and-noise-ratio (SINR) at the receiver side, in contrast with conventional precoding schemes from the literature which aim to minimize MUI. Two different CEP schemes are presented: a first technique, based on the application of the cross-entropy solver, and a two-step approach, based on an initial relaxation of the power constraints and a subsequent enforcement of per-antenna power constraints. The benefits of the proposed algorithms are evaluated in terms of computational costs and achievable symbol error rate (SER) in a perfect channel state information (CSI) scenario for different modulation orders. The analytical and numerical results show that interference-exploitation concepts are able to further extend the benefits of classical CEP.**

***Index Terms*—Constant-Envelope Precoding, Multiuser MIMO, Massive MIMO**

## I. Introduction

In recent years, Massive Multi-Input-Multi-Output (M-MIMO) technology has become a promising candidate for future wireless communications, thanks to its ability to achieve higher spectral efficiency than classical MIMO approaches [1]. In fact, it was proven in [2] that scaling-up multiuser MIMO paradigm by equipping the base station (BS) with a very high-dimensional antenna array provides several benefits, such as very high throughput values and low radiated energy requirements. Moreover, simple linear precoding techniques, such as matched filtering and zero-forcing precoding [3], are asymptotically optimal [4] for massive systems, thanks to the favorable propagation effects introduced by infinitely large arrays. Finally, physically constrained M-MIMO systems [5] have shown that transmit antenna correlation and mutual coupling at the base station can be effectively exploited to obtain higher capacities.

For linear precoding, it is common to apply power constraints over the average or instantaneous total transmitted power [6]. The assumption of sum-power constraints is normally driven by their innate simplicity in modelling. However, in realistic scenarios, power constraints have to be specifically applied to each transmitting antenna, which is typically connected to its own power amplifier (PA) with its specific power constraint. These considerations become critical in large-scale systems, since very-large arrays lead to an equally large number of radio-frequency (RF) chains. For this, the possibility to employ power-efficient PAs becomes essential, as inefficient PAs are accountable for $\sim 40-50\%$ of the total power consumption [7] in a BS. In order to tackle the problem, low peak-to-average power-ratio (PAPR) precoding techniques have shown to be beneficial for the power efficiency of M-MIMO [8]–[10] when combined with non-linear RF components. More specifically, [8] proposes a CEP technique, i.e., with a unitary PAPR, to minimize the error norm in a single user scenario, while [9] expands the concept of CEP to multi-user systems with a MUI minimization approach. Finally, the authors in [10] showed that cross-entropy optimization can increase the performance of interference reduction CEP.

In this paper, we present two novel CEP techniques based on concepts of constructive interference [11]–[13] for PSK-modulated signals in the multiuser downlink. For the proposed techniques, conditions over interference are relaxed to allow the use of the interfering signal as a green source of additional power to increase the received SINR. Our numerical results show that the newly introduced constructive interference-based optimization region for CEP is able to increase the performances at the receiver side in terms of SER. Finally, we derive the computational costs of the proposed techniques in terms of floating-point operations (flops), showing that they require equivalent costs when compared with the classical CEP approach from the literature.

**Notation:** Upper case boldfaced letters are used for matrices (i.e., $\mathbf{X}$), lower case boldfaced letters denote vectors (i.e., $\mathbf{x}$), subindices in vectors are used to identify rows of a matrix (i.e., $\mathbf{x}_m$ is the $m$-th row of $\mathbf{X}$), $tr[\cdot]$ represents the trace of the argument and superscripts $(\cdot)^H$ and $(\cdot)^*$ stand for Hermitian transpose and complex conjugate, respectively. Hadamard product is represented via $\circ$. Operators $\Im(\cdot)$ and $\Re(\cdot)$ respectively represent the imaginary and real part of the argument.

## II. System Model

We consider a MU downlink scenario where the BS is equipped with an $N$ dimensional antenna array to communicate with $M$ single-antenna users. Given the channel gain between the $n$-th transmit antenna and the $m$-th user $h_{m,n}$, we can define the received signal for the $m$-th user as

$$y_m = \sum_{n=1}^{N} \sqrt{P_n} h_{m,n} e^{j\theta_n} + w_m, \quad (1)$$

where $P_n$ is the power transmitted from the $n$-th antenna, so that the total transmitted power from the BS $P_t = \sum_{n=1}^{N} P_n$, $\theta_n$ represents the precoding phase of the $n$-th transmitted signal $x_n = \sqrt{P_n} e^{j\theta_n}$ and $w_m$ is the zero mean additive white Gaussian noise form the $m$-th user, i.e., $w_m \sim \mathcal{CN}(0, \sigma^2)$. Channel gains $h_{m,n}$ in a M-MIMO scenario can be modeled as follows [2]

$$h_{m,n} = g_{m,n} \sqrt{\beta_m}. \tag{2}$$

where $g_{m,n}$ represents the complex small-scale fading between the $n$-th antenna and the $m$-th user and $\beta_m$ identifies the real large-scale fading coefficient experienced by the $m$-th user. Throughout this work, a single cell scenario is considered and channel gains are modeled by independent Rayleigh fading [14]. Without loss of generality, throughout this work we also consider a unitary transmitted power scenario, with equally distributed power among the $N$ antennas at the BS, i.e., $P_n = 1/N, \forall n \in \{1, ..., N\}$.

We can rearrange the first term of the received signal $y_m$ in order discriminate between the desired constellation point for the $m$-th user and the interference. Analytically we have

$$\sum_{n=1}^{N} \frac{1}{\sqrt{N}} h_{m,n} e^{j\theta_n} = t_m + d_m e^{j\phi_m} \tag{3}$$

where $d_m e^{j\phi_m}$ is the PSK desired symbol for the $m$-th user, with magnitude $d_m$ and phase $\phi_m$, and $t_m$ represents the interfering signal for the $m$-th user.

Accordingly, the total MUI energy can be evaluated as as

$$E_{MUI} = \sum_{m=1}^{M} |t_m|^2 = \sum_{m=1}^{M} \left| \left( \sum_{n=1}^{N} \frac{1}{\sqrt{N}} h_{m,n} e^{j\theta_n} - d_m e^{j\phi_m} \right) \right|^2. \tag{4}$$

As previously mentioned, first multi-user CEP approaches were based on MUI minimization [9], [10]. This can be achieved by the base station, by identifying the $N$ dimensional transmit phase angle vector $\boldsymbol{\theta} = [\theta_1, ..., \theta_N]$ that leads to the lowest MUI energy. Accordingly, the MUI minimization CEP algorithm can be formulated as the following optimization problem [9], [10]

$$\begin{aligned} \mathcal{P}_1 : \quad & \underset{\boldsymbol{\theta}}{\text{minimize}} \quad \sum_{m=1}^{M} \left| \left( \sum_{n=1}^{N} \tfrac{1}{\sqrt{N}} h_{m,n} e^{j\theta_n} - d_m e^{j\phi_m} \right) \right|^2 \\ & \text{subject to} \quad |\theta_n| \leq \pi, \forall n \in \{1, ..., N\}, \end{aligned} \tag{5}$$

which represents a non-convex nonlinear least squares (NLS) problem, affected by local minima. The optimization problem (5) was first solved in [9] with a gradient descent (GD) based approach, and further improved in [10] with a direct application of cross-entropy method [15].

## III. Exploiting Constructive Interference in CEP Transmissions

Multi-user interference for PSK-modulated signals can be classified as constructive and destructive according to geometrical concepts, which can be found in detail in [11], [13], [16]. In fact, the interfering signal $t_m$ is beneficial for system performances when it pushes the noise free received symbol $r_m = u_m + t_m$ further away from the decision thresholds of the desired constellation symbol $u_m$, as visually presented in Fig.1 for the desired 8-PSK symbol $u_m = (1/\sqrt{2}, 1/\sqrt{2})$. Here, the blue shaded region identifies the constructive interference area, i.e., received symbols $r_m$ laying in this region are positioned further away from the decision thresholds when compared to the desired constellation point $u_m$.

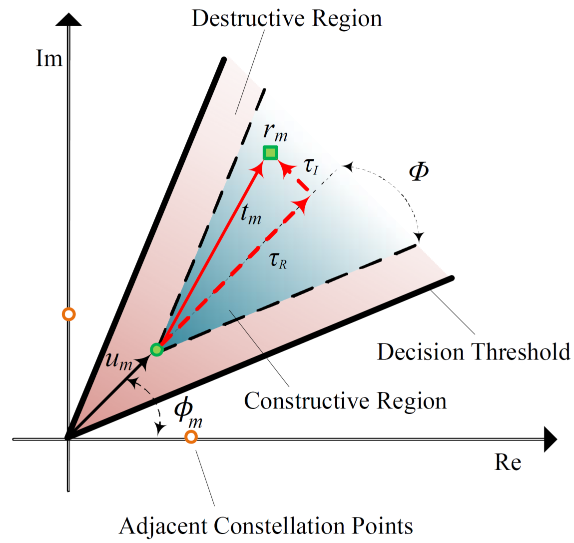


Fig. 1: Representation of the constructive interference region for 8-PSK modulated signal.

Following [13], the constructive interference condition for the $m$-th user can be derived from basic geometry properties as

$$\left| \Im \left( t_m \cdot e^{-j\phi_m} \right) \right| \leq \Re \left( t_m \cdot e^{-j\phi_m} \right) \tan \Phi, \tag{6}$$

where $\Phi = \pm\pi/L$ is the central angle of the constructive interference sectors and depends on the constellation order $L$ used for transmission. In (6) the interfering signal $t_m$ is phase-shifted according to the phase of the symbol of interest for the $m$-th user $\phi_m$. This operation is a fundamental step, as it isolates the amplitude and phase shift suffered by the desired symbol $u_m$ by means of the interference signal $t_m$, as shown in Fig.1. Here, $\tau_R = \Re \left( t_m \cdot e^{-j\phi_m} \right)$ and $\tau_I = \Im \left( t_m \cdot e^{-j\phi_m} \right)$ represent the real and imaginary components of the phase-shifted interfering signal $\bar{t}_m = t_m \cdot e^{-j\phi_m}$. As we can see, the real component $\Re(\bar{t}_m)$ can be regarded as the amplitude gain introduced by the interference, while the imaginary part $\Im(\bar{t}_m)$ represents a linear measure of the phase shift caused by $\bar{t}_m$.

Thanks to (6), we can identify a new constructive interference-based object function of the proposed optimization problems. This represents a key relaxation over the constraints normally imposed over CEP, as the constructive interference region is only constrained by the proximity to the decision thresholds and extends infinitely in the directions away from them. Analytically we can identify the new function as

$$\psi(\boldsymbol{\theta}) \triangleq \min_m \{\Re(\bar{t}_m)\tan\Phi - |\Im(\bar{t}_m)|\}, \quad (7)$$

which evaluates how constructive or destructive the interfering signal $t_m$ is for the desired constellation point $u_m$. From (6), we can infer that a negative value of $\psi(\boldsymbol{\theta})$ is characteristic of a destructive interfering signal, while a positive (7) implies that the interfering signal is constructive. Since $\Re(\bar{t}_m)$ represents the power gain introduced by the interfering signal, we can deduce that higher and positive values of $\psi(\boldsymbol{\theta})$ lead to stronger forms of constructive interference. Accordingly, we can define a new constructive interference-based optimization problem $\mathcal{P}_2$ as

$$\begin{aligned}\mathcal{P}_2: \quad & \underset{\boldsymbol{\theta}}{\text{maximize}} \quad \min_m \{\Re(\bar{t}_m)\tan\Phi - |\Im(\bar{t}_m)|\} \\ & \text{subject to} \quad |\theta_n| \le \pi, \forall n \in \{1,...,N\},\end{aligned} \quad (8)$$

where $m \in \{1,...,M\}$ and the operator $\min_m\{\cdot\}$ represents the minimum value of the argument among all the $M$ values. The optimization problem $\mathcal{P}_2$ maximizes the minimum value of the constructive interference metric, so that positive values of $\psi(\boldsymbol{\theta})$ suggest that constructive interference condition is met and maximized for all the users. On the other hand, negative values imply that the precoding phases are designed to minimize the destructive interference as its least constructive components are maximized. Given that $\mathcal{P}_2$ is non-convex, we propose two different approaches to solve it: a cross-entropy method (CEM), where the equality power constraint are always respected, and a two-step approach, where they are firstly relaxed to allow convex optimization solution methods and then re-enforced for CEP transmission.

### *A. A CEM solution to Constructive Interference Precoding*

The cross-entropy method is an adaptive algorithm that identifies rare events through variance reduction. The algorithm is based on iterative approach [15], where for each iteration it proceeds in generating random samples, according to a specific distribution $f(\boldsymbol{\theta}, \mathbf{u})$, and updating the distribution parameters $\mathbf{u}$, according to the values of the cost function. Such approach allows to improve the random samples generation in the subsequent iterations.

CEM application to optimization problems starts with the association of the optimization problem with the estimation of the probability of a rare event. Given $\mathcal{P}_2$, we can estimate the probability of the equivalent rare event $\psi(\boldsymbol{\theta}) \ge \gamma$ as

$$\mathcal{L}(\gamma) = \mathbb{P}_{\mathbf{u}}(\psi(\boldsymbol{\theta}) \ge \gamma) = \int \mathcal{I}\{\psi(\boldsymbol{\theta}) \ge \gamma\} f(\boldsymbol{\theta}, \mathbf{u})\, d\boldsymbol{\theta}, \quad (9)$$

where $\gamma$ is a chosen performance threshold, the operator $\mathbb{P}_{\mathbf{u}}(\cdot)$ evaluates the probability of the event in argument and $\mathcal{I}\{\cdot\}$ is boolean indicator function that returns 1 or 0 values when its argument it true or false, respectively. The estimation of $\mathcal{L}(\gamma)$ is performed through Monte Carlo simulations by means of *importance sampling* [15]. Under importance sampling, the probability density function $f(\boldsymbol{\theta}, \mathbf{u})$ is replaced by a different function $g(\boldsymbol{\theta})$ that more frequently generates the chosen rare event. Commonly, $g(\boldsymbol{\theta})$ is chosen from the same family of the original distribution $g(\boldsymbol{\theta}) = f(\boldsymbol{\theta}, \mathbf{v})$, where $\mathbf{v}$ is the new parameters set, called *tilting parameters* [15].

Therefore, the importance sampling function can be readily derived by correctly identifying the tilting parameters. Estimation of $\mathbf{v}$ is performed by identifying the function with the minimum *Kullback-Leiber* distance from the ideal solution $g^*(\boldsymbol{\theta}) = \frac{\mathcal{I}\{\psi(\boldsymbol{\theta}) \ge \gamma\} f(\boldsymbol{\theta}, \mathbf{u})}{\mathcal{L}(\gamma)}$ [15]. Analytically, we have

$$\mathbf{v}^* = \arg\max_{\mathbf{v}} \int \frac{\mathcal{I}\{\psi(\boldsymbol{\theta}) \ge \gamma\} f(\boldsymbol{\theta}, \mathbf{u})}{\mathcal{L}(\gamma)} \ln f(\boldsymbol{\theta}, \mathbf{v}) d\boldsymbol{\theta}, \quad (10)$$

which is obtained by minimizing the Kullback-Leiber distance [15]. In our studies, we assume the distribution $f(\boldsymbol{\theta}, \mathbf{v})$ to be Gaussian. This is a common assumption for continuous optimization problems [10] and allows to analytically estimate $\mathbf{v} = [\mu, \sigma]$ as

$$\widehat{\mu} = \frac{\sum_{k=1}^{K} \mathcal{I}\{\psi(\boldsymbol{\theta}) \ge \gamma\} \boldsymbol{\Theta}_k}{\sum_{k=1}^{K} \mathcal{I}\{\psi(\boldsymbol{\theta}) \ge \gamma\}} \quad (11)$$

$$\widehat{\sigma} = \sqrt{\left[\frac{\sum_{k=1}^{K} \mathcal{I}\{\psi(\boldsymbol{\theta}) \ge \gamma\} (\boldsymbol{\Theta}_k - \widehat{\mu})^2}{\sum_{k=1}^{K} \mathcal{I}\{\psi(\boldsymbol{\theta}) \ge \gamma\}}\right]}, \quad (12)$$

where $\widehat{\mu}$ and $\widehat{\sigma}$ respectively represent mean and standard deviation of the importance sampling distribution, i.e., $\widehat{\mathbf{v}}^* = [\widehat{\mu}, \widehat{\sigma}]$ and $\boldsymbol{\Theta}_k$ is the $k$-th random state from $f(\boldsymbol{\theta}, \mathbf{u})$. In order to avoid the convergence to suboptimal solutions [15], it is often recommended to use smooth updating procedures. Analytically

$$\mu^{(l)} = \alpha \widehat{\mu}^{(l)} + (1-\alpha)\mu^{(l-1)} \quad (13)$$

$$\sigma^{(l)} = \alpha \widehat{\sigma}^{(l)} + (1-\alpha)\sigma^{(l-1)}, \quad (14)$$

where the superscript $(\cdot)^{(l)}$ represents the $l$-th iteration of the value in argument.

### *B. Two-Step Convex CEP*

In addition to CEM, we propose a different approach to solve constructive interference CEP precoding, where power constraints are initially relaxed into inequality and then re-applied via equalization before transmission (i.e., antenna outputs that do not respect power constraints are divided by their absolute value). Accordingly, we can identify a convex relaxation to the optimization problem $\mathcal{P}_2$ by relaxing the conditions over the transmitted signal $x_n \in \mathbb{C}, \forall n \in \{1,...,N\}$ and its absolute value $|x_n| \le 1/\sqrt{N}, \forall n \in \{1,...,N\}$. Analytically, we can define the relaxed problem $\mathcal{P}_2^{'}$ as

$$\begin{aligned}\mathcal{P}_2^{'}: \quad & \underset{\boldsymbol{x}'}{\text{maximize}} \quad \min_m \{\Re(\bar{t}_m)\tan\Phi - |\Im(\bar{t}_m)|\} \\ & \text{subject to} \quad |x_n'| \le 1/\sqrt{N}, \forall n \in \{1,...,N\} \\ & \qquad \bar{t}_m = \left(\sum_{n=1}^{N} h_{m,n} x_n - u_m\right) e^{-j\phi_m}\end{aligned} \quad (15)$$

where the superscript $\{\cdot\}'$ represents the solution achieved through relaxation. The newly formulated problem is convex and can be effectively solved by means of standard convex optimization techniques [17]. Unfortunately, a direct usage of the solution to $\mathcal{P}_2'$ cannot guarantee CEP transmission because of the relaxed constraints. For this, in the second and final stage of the proposed scheme, we proceed in equalizing the elements of $\mathbf{x}' = [x_1', ..., x_N']^T$ which do not respect equality power requirements. The equalization is performed as follows

$$x_n = \begin{cases} x_n' / \left(\sqrt{N}|x_n'|\right) & \forall n \text{ where } |x_n'| \neq 1/\sqrt{N} \\ x_n' & \forall n \text{ where } |x_n'| = 1/\sqrt{N}. \end{cases} \quad (16)$$

## IV. Cost Function Computational Analysis

In this section, we compare the computational complexity of the cost function of the proposed CEM scheme with the cross-entropy optimization (CEO) approach to interference reduction (CEO-IR) precoding from [10]. The comparison is performed in terms of floating-point operations (flops), following the operational costs listed in the literature [17].

### A. CEO-CIO Cost Function Complexity

The computational burden of the cost function for the proposed scheme can be evaluated by decomposing the steps required for its evaluation. More specifically, the proposed CEO-based constructive interference optimization (CEO-CIO) approach requires: the computation of the received signal vector in a noise free scenario (i.e., $\mathbf{r} = \mathbf{H}\mathbf{x}$), the identification of the interfering signal vector (i.e., $\mathbf{t} = \tilde{\mathbf{r}} - \mathbf{u}$), the rotation of the interfering signal (i.e., $\bar{\mathbf{t}} = \mathbf{t} \circ \mathbf{u}^*$) and, finally, the identification of the minimum value in a vector (i.e., $\min\{\Re(\bar{\mathbf{t}}) \tan\Phi - |\Im(\bar{\mathbf{t}})|\}$). The computational costs of each of the aforementioned operations can be found in the literature [17]. Following [17], the multiplication between a $M \times N$ matrix and an $N \times 1$ vector requires $M(2N-1)$ flops, while the computation of the interfering signal and its rotation can be performed with $M$ flops each, since they can be achieved by $M$ subtractions and multiplications, respectively. Finally, the costs of the identification of the minimum are equivalent to the costs of a search through an $M$-sized vector, which is characterized by $M$ flops. Accordingly, the cost function evaluation of the proposed approach is characterized by a total flop count of $M(2N-1)+4M$ flops, which includes the cost of the separation between the real and imaginary part of the rotated interfering signal.

### B. CEO-IR Cost Function Complexity

Cost function evaluation for conventional CEO-IR is characterized by similar computational burdens. In fact, both techniques require the identification of $\mathbf{t} = [t_1, ..., t_M]^T$ for each of the randomly generated samples. More specifically, the computational costs of CEO-IR can be decomposed in the derivation of received and interference vectors in a noise free scenario and subsequent computation of MUI energy (i.e.,$\sum_1^M |t_m|^2$). Since MUI energy can be seen as the inner product of two $M$-sized vectors, which requires $2M-1$ flops, the total computational complexity for the estimation of the cost function of the CEO-IR algorithm is represented by $M(2N-1)+3M-1$ flops.

Therefore, the computational costs of the proposed technique CEO-CIO are comparable to the ones of the CEO-IR approach from the literature, as the flop count difference is almost negligible. This result is particularly interesting, as it shows that the proposed scheme is able to achieve higher performances than the approaches from the literature, without affecting the overall computational complexity of the system.

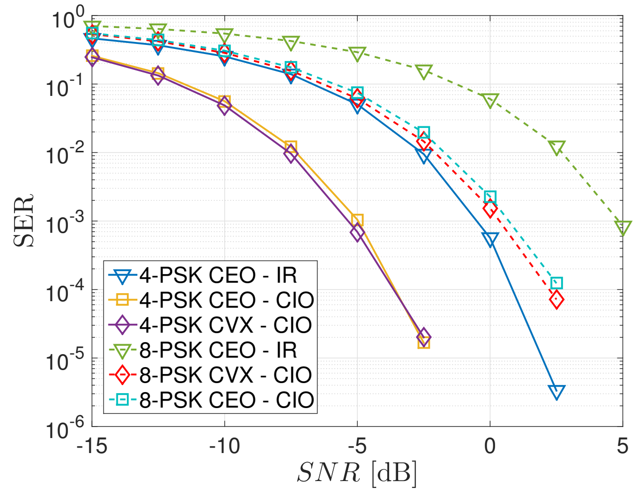


Fig. 2: Symbol Error Rate when $M = 12$, $N = 64$.

## V. Results

This section analyzes the performances of the proposed precoding techniques through Monte Carlo simulations over 50000 channel realizations. We consider a downlink transmission scenario, where the BS communicates with a population of $M = 12$ single-antenna mobile users through a large array of $N = 64$ radiating elements. We use the following notation for the legends: CEO-IR represents interference minimization CEO precoding, CEO-CIO and CVX-CIO identify the constructive interference optimization techniques with CEM-based solver and two-step convex approach, respectively. Considering that the proposed technique can be applied for any PSK modulation order, we here present the results for 4-PSK and 8-PSK. Both CEM-based techniques consider the same parameter settings: $T = 1000$, $\rho = 0.05$ and $\alpha = 0.08$ [10].

Figure 2 shows how the SER achieved by the presented schemes evolves as a function of the transmitted SNR for 4-PSK and 8-PSK modulation respectively. As we can see from Fig.2, the proposed approaches strongly outperform the classical CEO-IR precoding. This is due to the fact that CIO techniques wisely exploit the interference signal $t_m, \forall m \in \{1, ..., M\}$ to increase the received signal power, while CEO-IR aims to a direct minimization of the interference energy.

In our studies we consider a unitary energy constellation for all the streams, i.e., $d_m = d = 1, \forall m \in \{1, ..., M\}$.

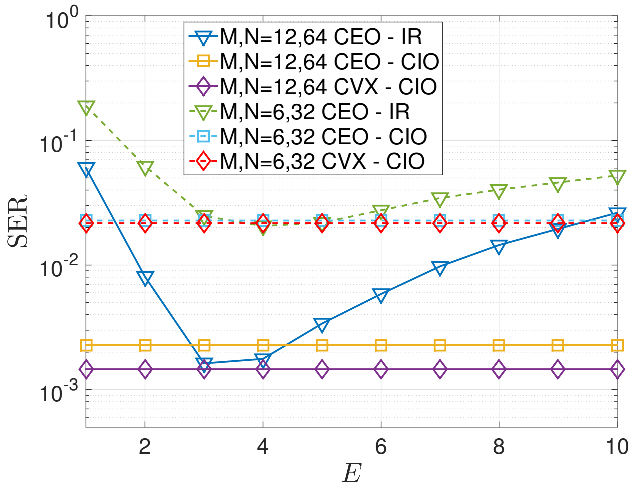


Fig. 3: Symbol Error Rate for 8-PSK as a function of $E = d^2$.

Whereas this is a common approach in CEP literature [8]–[10], [18], CEO-IR performances can be improved by increasing $d$. Nevertheless, the choice of the constellation energy represents a critical element of CEO-IR. This is shown in Fig. 3, where we can see that he performances of CEO-IR worsen as we incautiously increase the constellation energy $d$. This phenomenon is caused by the MUI-based metric used in CEO-IR, which focuses on the minimization of (4), without any control over the phase of the interfering signal. Moreover, we can see that the optimal constellation amplitude $d^*$ for CEO-IR changes as we consider different scenarios, proving how it is not possible to identify $d^*$ before transmitting. The dependence over the constellation energy represents one of the key drawbacks of the CEO-IR approach. In fact, since the expected value of (4) changes according to both topology (i.e., number of antennas at the BS and number of users) and modulation used in transmission [9], it is not possible to identify the optimal constellation amplitude $d^*$ a priori. Accordingly, deriving $d^*$ would require to dynamically estimate the SER at the transmitter side as a function of the constellation energy $E$, therefore causing a further increase of the computational complexity of the system.

## VI. Conclusions

This paper presents a new optimization metric for CEP-based systems where downlink multi-user interference is effectively exploited as a source of additional energy to increase the SER performances at the receiver. Two different techniques are proposed and prove that a constructive interference-based relaxation of the precoding optimization region is beneficial to achieve reliable communications. Moreover, the computational burden of the proposed CEM approach is analyzed in terms of flops, and compared with previous approaches from the literature, showing negligible differences. Finally, SER performances show the benefits of the proposed metric over interference reduction-based techniques from the literature.

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