Spectrum and Energy Efficiency in Massive MIMO Enabled HetNets: A Stochastic Geometry Approach

Anqi He, Lifeng Wang, Maged Elkashlan, Yue Chen, and Kai-Kit Wong

***Abstract*—This letter focuses on the spectrum efficiency (SE) and energy efficiency (EE) of *K*-tier heterogeneous networks (HetNets), in which massive multiple-input multiple-output (MIMO) is employed in the macro cells. We consider the impact of massive MIMO on flexible cell association. We develop an analytical approach to examine SE and EE of HetNets. We confirm that using flexible cell association can improve the EE of HetNets by offloading data traffic to small cell. This is due to an increase of EE in the macro cell. Moreover, we show that serving moderate number of users in the macro cells with massive MIMO can boost both SE and EE.**

***Index Terms*—Energy efficiency, flexible cell association, HetNets, massive MIMO, spectrum efficiency.**

I. Introduction

Although there is no preliminary 5G standard yet, heterogeneous networks (HetNets) and massive multiple-input multiple-output (MIMO) are two key enablers for 5G [1], [2]. The deployment of small cells improves the coverage and spectrum efficiency (SE), since the mobile devices are closer to the base stations. In addition, base station equipped with large number of antennas achieves high SE and energy efficiency (EE) [3]. Therefore, HetNets with massive MIMO is viewed as a powerful architecture to enable 5G.

Recent works [4]–[6] have shown the significance of massive MIMO enabled HetNets. In [5], a distributed user association scheme was proposed to maximize the energy efficiency. In [6], inter-tier interference coordination was considered for throughput improvement in HetNets with massive MIMO.

Different from the aforementioned literature, we focus on the SE and EE in K-tier massive MIMO aided HetNets, where the massive MIMO macro cell BS adopts linear zero-forcing beamforming (ZFBF) to serve multiple users over the same time and frequency band. We adopt a stochastic geometry approach to analyze the effect of massive MIMO on the SE and EE of K-tier HetNets, which to the best of our knowledge, is not investigated in the existing works. We study the impact of massive MIMO on the user association, and consider flexible cell association that allows tier selection for cell load balancing, and potential SE and EE improvement. We derive analytical expressions for SE and EE. Our results show that offloading suitable data traffic to small cells can still improve the EE of HetNets. However, more macro cell users associated with the small cell will degrade the HetNets' SE and EE. Furthermore, serving moderate number of users in the massive MIMO macro cell improves both SE and EE of HetNets.

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A. He, M. Elkashlan, and Y. Chen are with the School of Electronic Engineering and Computer Science, Queen Mary University of London, London E1 4NS, U.K. (e-mail: a.he@qmul.ac.uk; maged.elkashlan@qmul.ac.uk; yue.chen@qmul.ac.uk).

L. Wang and K.-K. Wong are with the Department of Electronic and Electrical Engineering, University College London, London WCIE 71E, U.K. (e-mail: lifeng.wang@ucl.ac.uk; kai-kit.wong@ucl.ac.uk).

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II. System Description

We consider time-division duplex (TDD) downlink transmission in K-tier HetNets consisting of macro cells and small cells such as picocells and femtocells. We assume that the first tier represents the class of macro cell base stations (MBSs), where each MBS is equipped with a large antenna array. The positions of the MBSs are modeled following a homogeneous Poisson point process (HPPP) Φ_{M} with density λ_{M}. The positions of the small cell base stations (SBSs) in the i-th tier ($i = 2, \ldots, K$) are modeled following an independent HPPP Φ_i with density λ_i. Massive MIMO is applied in the macro cells [2], where each MBS has N antennas and simultaneously communicates with S users over the same time and frequency band, $N \gg S \geq 1$, while each SBS and user are single antenna nodes. Each MBS uses linear ZFBF to transmit S data streams with equal power assignment. We consider the perfect downlink CSI and the universal frequency reuse such that all of the tiers share the same bandwidth. All the channels undergo independent and identically distributed (i.i.d.) quasi-static Rayleigh fading.

A. Cell Association

We consider the flexible cell association based on maximum received power. Since users in the macro cell have large array gains, the long-term average receive power at a user which is connected with the MBS ℓ ($\ell \in \Phi_{\mathrm{M}}$) is expressed as

$$P_{r,\mathrm{M}} = G_a \frac{P_{\mathrm{M}}}{S} L\left(|X_{\ell,\mathrm{M}}|\right), \tag{1}$$

where G_a is the array gain, P_{M} is the MBS's transmit power, $L\left(|X_{\ell,\mathrm{M}}|\right) = \beta |X_{\ell,\mathrm{M}}|^{-\alpha_{\mathrm{M}}}$ is the path loss function, β is the frequency dependent constant value, $|X_{\ell,\mathrm{M}}|$ is the distance, and α_{M} is the path loss exponent. The array gain G_a obtained by the ZFBF transmission is $N - S + 1$ [7], [8]. We see that the long-term average receive power is scaled by $(N - S + 1)/S$, compared to the single-antenna systems.

In the small cell, the long-term average receive power at a user which is connected with the SBS j ($j \in \Phi_i$) in the i-th tier is expressed as

$$P_{r,i} = P_i L\left(|X_{j,i}|\right) B_i, \tag{2}$$

where P_i is the SBS's transmit power in the i-th tier and $L(|X_{j,i}|) = \beta(|X_{j,i}|)^{-\alpha_i}$ with distance $|X_{j,i}|$ and path loss exponent α_i, and B_i is the biasing factor, which is useful for offloading the data traffic to small cells in conventional HetNets [9].

B. Channel Model

We assume that a typical user is located at the origin o. The receive signal-to-interference-plus-noise ratio (SINR) of a typical user at a random distance $|X_{o,\mathrm{M}}|$ from its associated MBS is given by

$$\mathrm{SINR_M} = \frac{\frac{P_\mathrm{M}}{S} h_{o,\mathrm{M}} L\left(|X_{o,\mathrm{M}}|\right)}{I_1 + \delta^2}, \tag{3}$$

where $I_1 = I_{\mathrm{M},1} + I_{\mathrm{S},1}$, $I_{\mathrm{S},1} = \sum_{i=2}^{K} \sum_{j \in \Phi_i} P_i h_{j,i} L(|X_{j,i}|)$, $I_{\mathrm{M},1} = \sum_{\ell \in \Phi_\mathrm{M} \setminus B_{o,\mathrm{M}}} \frac{P_\mathrm{M}}{S} h_{\ell,\mathrm{M}} L(|X_{\ell,\mathrm{M}}|)$, $h_{o,\mathrm{M}} \sim \Gamma(N-S+1, 1)$ [8] is the small-scale fading channel power gain between the typical user and its associated MBS, $h_{j,i} \sim \exp(1)$ and $|X_{j,i}|$ are the small-scale fading interfering channel power gain and distance between the typical user and SBS j in the i-th tier, respectively, $h_{\ell,\mathrm{M}} \sim \Gamma(S, 1)$ and $|X_{\ell,\mathrm{M}}|$ are the equivalent small-scale fading interfering channel power gain and distance between the typical user and MBS $\ell \in \Phi_\mathrm{M} \setminus \mathrm{B}_{o,\mathrm{M}}$ (except the serving MBS $B_{o,\mathrm{M}}$), respectively, and δ^2 is the noise power.

The SINR of a typical user at a random distance $|X_{o,k}|$ from its associated SBS $B_{o,k}$ in the k-th tier is given by

$$\mathrm{SINR}_k = \frac{P_k g_{o,k} L\left(|X_{o,k}|\right)}{I_k + \delta^2}, \tag{4}$$

where $I_k = I_{\mathrm{M},k} + I_{\mathrm{S},k}$, $I_{\mathrm{M},k} = \sum_{\ell \in \Phi_\mathrm{M}} \frac{P_\mathrm{M}}{S} g_{\ell,\mathrm{M}} L(|X_{\ell,\mathrm{M}}|)$, $I_{\mathrm{S},k} = \sum_{i=2}^{K} \sum_{j \in \Phi_i \setminus B_{o,k}} P_i g_{j,i} L(|X_{j,i}|)$, $g_{o,k} \sim \exp(1)$ is the small-scale fading channel power gain between the typical user and its serving SBS, $g_{\ell,\mathrm{M}} \sim \Gamma(S, 1)$ and $|X_{\ell,\mathrm{M}}|$ are the equivalent small-scale fading interfering channel power gain and distance between the typical user and MBS ℓ, respectively, and $g_{j,i} \sim \exp(1)$ and $|X_{j,i}|$ are the small-scale fading interfering channel power gain and distance between the typical user and SBS $j \in \Phi_i \setminus B_{o,k}$, respectively.

C. Power Consumption Model

Based on [10], the total power consumption at the MBS in each channel use is given by

$$P_\mathrm{M}^{total} = P_\mathrm{M}^0 + \frac{P_\mathrm{M}}{\varepsilon_\mathrm{M}} + \sum_{t=1}^{3} \left((S)^t \Delta_t + (S)^t N \Lambda_t\right), \tag{5}$$

where P_M^0 is the MBS's static hardware power consumption, $\varepsilon_\mathrm{M} (0 < \varepsilon_\mathrm{M} \le 1)$ is the efficiency of the power amplifier, the parameters Δ_t and Λ_t depends on the transceiver chains, coding and decoding, precoding, etc. [10], which are detailed in the Section IV.

In the small cell, the total power consumption at the SBS $j\, (j \in \Phi_i)$ in each channel use is given by

$$P_i^{total} = P_i^0 + \frac{P_i}{\varepsilon_i}, \tag{6}$$

where P_i^0 is the static hardware power consumption of the SBS in the i-th tier, and ε_i is the efficiency of the power amplifier.

III. Performance Evaluations

Based on the user association described in Section II-B, we first provide new probability density function (PDF) of the distance between a typical user and its serving base station. Using this statistical property, we present analytical expressions for SE and EE in massive MIMO enabled HetNets.

A. User Association Probability

The PDF of the distance between a typical user and its serving base station is derived as the following two lemmas.

Lemma 1: The PDF of the distance $|X_{o,\mathrm{M}}|$ between a typical user and its serving MBS $B_{o,\mathrm{M}}$ is given by

$$f_{|X_{o,\mathrm{M}}|}(x) = \frac{2\pi\lambda_\mathrm{M}}{\mathcal{A}_\mathrm{M}} x \exp\left\{-\pi\lambda_\mathrm{M} x^2 - \pi \sum_{i=2}^{K} \lambda_i \left(\frac{P_i B_i S x^{\alpha_\mathrm{M}}}{P_\mathrm{M}(N-S+1)}\right)^{2/\alpha_i}\right\}. \tag{7}$$

Here, $\mathcal{A}_\mathrm{M}$ is the probability that a typical user is associated with the MBS, which is given by

$$\mathcal{A}_\mathrm{M} = 2\pi\lambda_\mathrm{M} \int_0^\infty r \exp\left\{-\pi\lambda_\mathrm{M} r^2 - \pi \sum_{i=2}^{K} \lambda_i \left(\frac{P_i B_i S r^{\alpha_\mathrm{M}}}{P_\mathrm{M}(N-S+1)}\right)^{2/\alpha_i}\right\} dr. \tag{8}$$

Lemma 2: The PDF of the distance $|X_{o,k}|$ between a typical user and its serving SBS in the k-th tier $\mathrm{B}_{o,k}$ is given by

$$f_{|X_{o,k}|}(x) = \frac{2\pi\lambda_k}{\mathcal{A}_k} x \exp(x) \left\{-\pi\lambda_\mathrm{M} \left(\frac{P_\mathrm{M}(N-S+1) x^{\alpha_k}}{P_k B_k S}\right)^{2/\alpha_\mathrm{M}} - \pi \sum_{i=2}^{K} \lambda_i \left(\frac{P_i B_i x^{\alpha_k}}{P_k B_k}\right)^{2/\alpha_i}\right\}. \tag{9}$$

Here, $\mathcal{A}_k$ is the probability that a typical user is associated with the SBS in the k-th tier, which is given by

$$\mathcal{A}_k = 2\pi\lambda_k \int_0^\infty r \exp\left\{-\pi\lambda_\mathrm{M} \left(\frac{P_\mathrm{M}(N-S+1) r^{\alpha_k}}{P_k B_k S}\right)^{2/\alpha_\mathrm{M}} - \pi \sum_{i=2}^{K} \lambda_i \left(\frac{P_i B_i r^{\alpha_k}}{P_k B_k}\right)^{2/\alpha_i}\right\} dr. \tag{10}$$

Note that **Lemma 1** and **Lemma 2** can be easily obtained following the approach in Lemma 3 of [9].

B. Spectrum and Energy Efficiency

SE and EE are principal performance metrics in HetNets, since higher SE means lower spectrum consumption and higher EE means lower energy consumption. As such, in the following theorem, we first derive a tractable lower bound on the SE when a typical user is associated with MBS.

Theorem 1: The lower bound on the SE when a typical user is associated with MBS is given by

$$\tau_\mathrm{M}^\mathrm{L} = \log_2\left(1 + (N-S+1)\frac{P_\mathrm{M}}{S}\beta\Upsilon^{-1}\right), \tag{11}$$

where

$$\Upsilon = \int_0^\infty x^{\alpha_\mathrm{M}} \left(\overline{I}(x) + \delta^2\right) F_{|X_{o,\mathrm{M}}|}(x) dx$$

with $\overline{I}(x) = \frac{P_\mathrm{M}\beta 2\pi\lambda_\mathrm{M}}{\alpha_\mathrm{M}-2} x^{2-\alpha_\mathrm{M}} + \sum_{i=2}^{K} \frac{P_i \beta 2\pi\lambda_i}{\alpha_i - 2} \left(R_i^\mathrm{M}(x)\right)^{2-\alpha_i}$, $R_i^\mathrm{M}(x) = \left(\frac{P_i B_i S}{(N-S+1)P_\mathrm{M}}\right)^{\frac{1}{\alpha_i}} x^{\frac{\alpha_\mathrm{M}}{\alpha_i}}$.

Proof: The SE is defined as $\mathbb{E}\{\log_2(1+\mathrm{SINR_M})\}$. Using Jensen's inequality, we have the lower bound [3][1]

$$\tau_{\mathrm{M}}^{\mathrm{L}} = \log_2\left(1+\frac{1}{\mathbb{E}\left\{(\mathrm{SINR_M})^{-1}\right\}}\right). \quad (12)$$

Based on (3), $\mathbb{E}\{(\mathrm{SINR_M})^{-1}\}$ is calculated as

$$\begin{aligned}
&\mathbb{E}\left\{(\mathrm{SINR_M})^{-1}\right\} \\
&= \mathbb{E}\left\{\frac{I_1+\delta^2}{\frac{P_{\mathrm{M}}}{S}h_{o,\mathrm{M}}\beta x^{-\alpha_{\mathrm{M}}}}\right\} \\
&\overset{(a)}{\approx} \left(P_{\mathrm{M}}\frac{N-S+1}{S}\beta\right)^{-1}\mathbb{E}\left\{(I_1+\delta^2)x^{\alpha_{\mathrm{M}}}\right\} \\
&= \left(P_{\mathrm{M}}\frac{N-S+1}{S}\beta\right)^{-1}\int_0^\infty x^{\alpha_{\mathrm{M}}}\left(\bar{I}(x)+\delta^2\right)f_{|X_{o,\mathrm{M}}|}(x)dx,
\end{aligned} \quad (13)$$

where (a) is obtained by using the law of large numbers, i.e., $h_{o,\mathrm{M}} \approx N-S+1$ as N is large. In (13), $\bar{I}(x)=\mathbb{E}\{I_1\}$ is the expectation of the sum of interference, which can be derived as

$$\begin{aligned}
\bar{I}(x) &= \mathbb{E}\left\{\sum\nolimits_{\ell\in\Phi_{\mathrm{M}}\setminus \mathrm{B}_{o,\mathrm{M}}}\frac{P_{\mathrm{M}}}{S}h_{\ell,\mathrm{M}}\beta x^{-\alpha_{\mathrm{M}}}\right\} \\
&\quad + \mathbb{E}\left\{\sum_{i=2}^{K}\sum\nolimits_{j\in\Phi_i}P_i h_{j,i}\beta x^{-\alpha_i}\right\} \\
&\overset{(b)}{=} \frac{P_{\mathrm{M}}\beta 2\pi\lambda_{\mathrm{M}}}{\alpha_{\mathrm{M}}-2}x^{2-\alpha_{\mathrm{M}}} + \sum_{i=2}^{K}\frac{P_i\beta 2\pi\lambda_i}{\alpha_i-2}\left(R_i^{\mathrm{M}}(x)\right)^{2-\alpha_i},
\end{aligned} \quad (14)$$

where (b) is obtained by using Campbell's theorem [13]. In (14), $R_i^{\mathrm{M}}(x)$ is the closest distance between the interfering SBS in the i-th tier and the typical user. Substituting (13) and (14) into (12), we obtain (11). □

From **Theorem 1**, we find that the SE per user in the macro cell increases with increasing the number of antennas. However, it decreases with increasing number of users served by the MBS, since the transmit power assigned to each user is reduced, and the intra-cell interference is cancelled at the cost of decreasing array gain.

We then derive the SE τ_k when a typical user is associated with SBS in the k-th tier, which is given by

$$\tau_k = \mathbb{E}\left\{\log_2(1+\mathrm{SINR}_k)\right\} = \frac{1}{\ln 2}\int_0^\infty \frac{P_{\mathrm{cov}}^k(\gamma)}{1+\gamma}d\gamma, \quad (15)$$

where $P_{\mathrm{cov}}^k(\gamma)=\int_0^\infty P_{\mathrm{cov}}^k(x,\gamma)f_{|X_{o,k}|}(x)dx$. Here, $P_{\mathrm{cov}}^k(x,\gamma)$ is the conditional coverage probability given a distance x between a typical user and its serving SBS in the k-tier, which is given by

$$\begin{aligned}
P_{\mathrm{cov}}^k(x,\gamma) &= \Pr\left(\frac{P_k g_{o,k}\beta x^{-\alpha_k}}{I_k+\delta^2} > \gamma\right) \\
&= \exp\left(-\frac{\gamma x^{\alpha_k}\delta^2}{P_k\beta}\right)\mathcal{L}_{I_k}\left(\frac{\gamma x^{\alpha_k}}{P_k\beta}\right),
\end{aligned} \quad (16)$$

where $\mathcal{L}_{I_k}(\cdot)$ is the Laplace transform of the PDF of I_k. Using the generating functional of PPP, we derive $\mathcal{L}_{I_k}(\cdot)$ as

$$\begin{aligned}
\mathcal{L}_{I_k}(s) = \exp\Bigg\{&-\lambda_{\mathrm{M}}2\pi\sum_{\mu=1}^{S}\binom{S}{\mu}\left(s\frac{P_{\mathrm{M}}}{S}\beta\right)^{\mu}\frac{\left(-s\frac{P_{\mathrm{M}}}{S}\beta\right)^{-\mu+\frac{2}{\alpha_{\mathrm{M}}}}}{\alpha_{\mathrm{M}}} \\
&\mathrm{B}_{\left(-s\frac{P_{\mathrm{M}}}{S}\beta\left(R_{\mathrm{M}}^k(x)\right)^{-\alpha_{\mathrm{M}}}\right)}\left[\mu-\frac{2}{\alpha_{\mathrm{M}}},1-S\right] - \sum_{i=2}^{K}\lambda_i 2\pi s P_i\beta \\
&\frac{\left(R_i^k(x)\right)^{2-\alpha_i}}{\alpha_i-2}{}_2F_1\left[\frac{\alpha_i-2}{\alpha_i},1;2-\frac{2}{\alpha_i};-sP_i\beta\left(R_i^k(x)\right)^{-\alpha_i}\right]\Bigg\},
\end{aligned} \quad (17)$$

where $R_{\mathrm{M}}^k(x)=\left(\frac{P_{\mathrm{M}}(N-S+1)}{P_k B_k S}\right)^{1/\alpha_{\mathrm{M}}}x^{\alpha_k/\alpha_{\mathrm{M}}}$ and $R_i^k(x)=\left(\frac{P_i B_i}{P_k B_k}\right)^{1/\alpha_i}x^{\alpha_k/\alpha_i}$, $\mathrm{B}_{(\cdot)}[\cdot,\cdot]$ is the incomplete beta function [14, (8.391)], and ${}_2F_1[\cdot,\cdot;\cdot;\cdot]$ is the Gauss hypergeometric function [14, (9.142)].

Based on the above analysis, using the law of total expectation, a tractable lower bound on the SE of K-tier HetNets with massive MIMO is given by

$$\mathrm{SE}_{\mathrm{HetNets}}^{\mathrm{L}} = \mathcal{A}_{\mathrm{M}}\times\mathrm{SE_M} + \sum_{k=2}^{K}\mathcal{A}_k\times\mathrm{SE}_k, \quad (18)$$

where $\mathrm{SE_M}=S\times\tau_{\mathrm{M}}^{\mathrm{L}}$ is the SE of macro cell, and $\mathrm{SE}_k=\tau_k$ is the SE of small cell in the k-th tier. The lower bound on the EE of transmitting one user data stream in K-tier HetNets with massive MIMO is given by

$$\mathrm{EE}_{\mathrm{HetNets}}^{\mathrm{L}} = \mathcal{A}_{\mathrm{M}}\times\mathrm{EE_M} + \sum_{k=2}^{K}\mathcal{A}_k\times\mathrm{EE}_k, \quad (19)$$

where $\mathrm{EE_M}=\frac{S\times\tau_{\mathrm{M}}^{\mathrm{L}}}{P_{\mathrm{M}}^{total}}$ is the EE of macro cell, and $\mathrm{EE}_k=\frac{\tau_k}{P_k^{total}}$ is the EE of small cell in the k-th tier.

IV. Numerical Results

We consider a two-tier network consisting of macro cells with density $\lambda_{\mathrm{M}}=(500^2\times\pi)^{-1}$ and picocells with density $\lambda_2=20\times\lambda_{\mathrm{M}}$ in a circular region with radius 1×10^4 m. Such a network is assumed to operate at a carrier frequency of 1 GHz and a bandwidth of BW = 10 MHz, the path loss exponents $\alpha_{\mathrm{M}}=3.5$ and $\alpha_2=4$, the transmit power at the MBS is $P_{\mathrm{M}}=40$ dBm, the transmit power at the picocell base station is $P_2=30$ dBm, and the noise figure is Nf = 10 dB, hence the noise power is $\delta^2=-170+10\times\log_{10}(\mathrm{BW})+\mathrm{Nf}=-90$ dBm. We set the coefficients for efficiency of power amplifier as $\varepsilon_{\mathrm{R}}=\varepsilon_{\mathrm{M}}=0.38$ and for power consumption under LZFBF precoding in (5) as $P_{\mathrm{M}}^0=4$ W, $\Delta_1=4.8$, $\Delta_2=0$, $\Delta_3=2.08\times10^{-8}$, $\Lambda_1=1$, $\Lambda_2=9.5\times10^{-8}$ and $\Lambda_3=6.25\times10^{-8}$ [10]. The static power consumption of PBS is $P_2^0=13.6$ W [15]. In the figures, Monte Carlo simulated lower bounds of the SE and EE marked by 'o' are numerically obtained to validate the analytical, and the red, blue and green curves represent the SE and EE achieved by the macro cell, picocell, and HetNets, respectively. Fig. 1 shows the SE and EE for different biasing factor B. Here, we set $N=200$ and $S=15$.

[1]We note that the exact expression for the SE can be written in a general-form following the approaches in [11] and [12], however, using these approaches for computing the SE will lead to intractable solutions in this work.

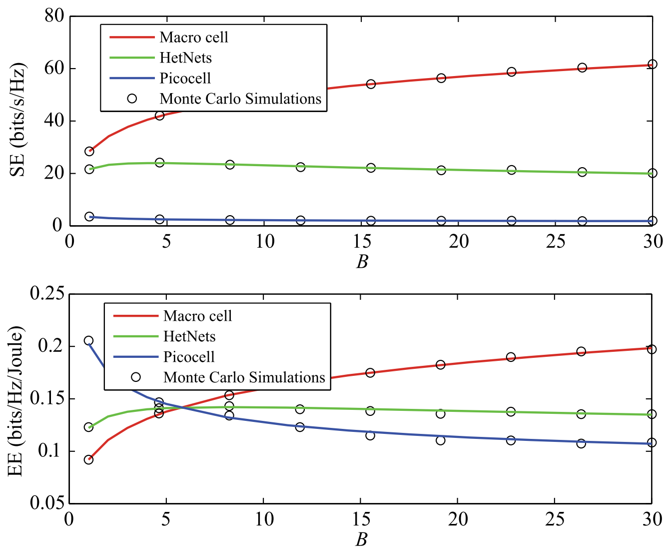

Fig. 1. SE and EE for different biasing factor B.

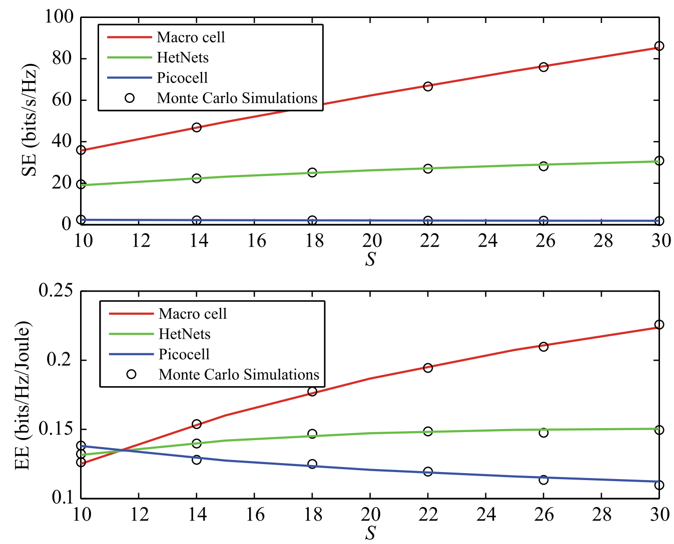

Fig. 2. SE and EE for different number of users.

The analytical SE and EE curves are obtained from (18) and (19), respectively. Evidently, the analytical curves have a good match with the Monte Carlo simulations, which validates the analysis. The effects of different B are:

- **SE**. As the picocell biasing factor B increases, the SE of picocell decreases and the SE of macro cell increases. This is attributed to the fact that more macro cell users with low SINR are associated with the picocell, which in turn improves the SE of the macro cell but degrades the SE of picocell. The SE of HetNets decreases with increasing B, which can be explained by the fact that when macro cell users with low SINR are associated with the picocell, they obtain lower SINR.
- **EE**. By increasing B, the EE of macro cell improves but the EE of picocell degrades, this is due to the fact that the SE of the macro cell increases and the SE of picocell decreases over the identical power consumption. With increasing B, the EE of HetNet first increases, then it converges to the constant value, which indicates that selecting a suitable biasing factor is still useful for improving the EE of HetNets.

Fig. 2 shows the SE and EE for different number of users S served by the MBS. Here, we set $N = 200$ and $B = 10$. The effects of different S are:

- **SE**. As the number of users served by the MBS increases, the SE of macro cell significantly increases, and the SE of HetNets also has a significant improvement. Meanwhile, increasing S has no big effect on the SE of picocell.
- **EE**. By increasing S, the EE of macro cell and HetNets also increases, because of increasing SE over the identical power consumption. The decrease of the EE in picocell can be explained by the fact that from (11), with increasing S, the SE per user in the macro cell decreases, which results in more macro cell users with low SINR being offloaded on the picocell.

V. Conclusion

We analyzed the downlink spectrum and energy efficiency in K-tier massive MIMO enabled HetNets. The impacts of flexible user association and massive MIMO were examined. Important guidelines are drawn from our results, which indicates that the mixture of flexible user association and massive MIMO can achieve significant improvements to both the spectrum efficiency and energy efficiency.

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