

# Money and Asset Liquidity in Frictional Capital Markets<sup>†</sup>

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One important function of financial intermediaries and markets is liquidity provision. Transaction costs and information frictions may prevent private agents from contracting and trading with each other in order to channel resources from investors with excess liquidity to financing constrained firms with funding needs. Financial intermediaries and dealers on financial markets offer specialized services to facilitate transactions of privately issued financial assets, thus providing liquidity and affecting asset prices. In view of frictional asset markets, publicly created liquidity, such as fiat money, provides an alternative—albeit low yielding—hedge against financing constraints.

Nevertheless, the macroeconomic literature rarely studies private liquidity provision jointly with its asset pricing implications. This paper explores the macroeconomic impact of endogenous liquidity provision through the financial sector. We model financial intermediation as a specific competitive search process (see, e.g., Moen 1997 on competitive search in labor markets and Rocheteau and Weill 2011 for a survey of search frictions and asset liquidity). Importantly, search frictions generate endogenous asset liquidity and financing constraints. Liquidity conditions, in turn, affect asset prices. Therefore, asset liquidity and prices vary with aggregate conditions and feed into real allocations. We show, in particular, that less liquid private capital markets are associated with stronger demand for public liquidity and tighter financing constraints, depressing real economic activities.

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## I. The Model

Consider an economy consisting of a continuum of households (with a continuum of members), firms, and financial intermediaries, each with measure one. Household members are subject to idiosyncratic investment risks as in Kiyotaki and Moore (2012) and Shi (2015). Time is discrete and denoted by  $t = 0, 1, 2, \dots$ . Each period is divided into four subperiods.

*The Household's Decision Period.*—The aggregate productivity shock  $A_t$  is realized. All members in a representative household *equally divide* the household's financial assets consisting of money and privately issued financial claims. The household instructs its members on the optimal type-specific choices to be carried out after types have been realized.

*The Production Period.*—Each member receives a status draw, becoming an entrepreneur (type  $i$ ) with probability  $\chi$  and a worker (type  $n$ ), otherwise. Workers supply labor, and only entrepreneurs have access to investment projects. Competitive firms rent aggregate capital stock  $K_t$  and labor  $N_t$  from households to produce numeraire consumption goods  $Y_t = A_t F(K_t, N_t)$ , where  $F(K_t, N_t) = K_t^\alpha N_t^{1-\alpha}$  and  $\alpha \in (0, 1)$ . The rental rate of capital and the wage rate are

$$(1) \quad r_t = A_t F_K(K_t, N_t) \quad \text{and} \quad w_t = A_t F_N(K_t, N_t).$$

*The Investment Period.*—There is no insurance among household members as they keep separated until the consumption stage. Entrepreneurs seek financing and undertake investment projects, transforming consumption goods into capital stock one-for-one. Capital markets open in which entrepreneurs offer financial claims for sale and workers purchase claims. Asset transactions are implemented by financial intermediaries and subject to costly search and matching. Financial intermediaries charge a fee for their

search technology. Private financial claims are thus only partially liquid. Money, in *fixed supply*  $\bar{B}$  and traded on a frictionless spot market, is fully liquid. Throughout the paper, we focus on the type of equilibrium where there are gains from trading both private claims and money.

*The Consumption Period.*—Entrepreneurs and workers reunite again in their respective households, pool all assets together, and *equally* share consumption goods across all members.

A. A Representative Household

*Preferences.*—The household derives per-period utility  $u(C_t)$  from total household consumption  $C_t$ , where  $u'(\cdot) > 0$  and  $u''(\cdot) < 0$ . The household's preferences are represented by  $E_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$ , where  $E$  is the expectation operator and  $\beta \in (0, 1)$  is the discount factor.

*Households' Wealth.*—Households hold fully liquid money with the nominal price  $P_t$ . In addition, physical capital ( $K_t$ ), earning the rental return  $r_t$ , is owned by households and depreciates to  $(1 - \delta)K_t$  at the end of each period where  $\delta \in (0, 1)$ . There is a financial claim to the future return of every unit of capital. For example, the owner of one unit of claims issued at time  $t - 1$  is entitled to  $r_t$  at  $t$ ,  $(1 - \delta)r_{t+1}$  at time  $t + 1$ ,  $(1 - \delta)^2 r_{t+2}$  at time  $t + 2, \dots$  For simplicity, we follow Kiyotaki and Moore (2012) and normalize the claims by the capital stock, such that they depreciate at the same rate  $\delta$ , but earn a return  $r_{t+s}$  at any date  $t + s (\forall s \geq 0)$ .

Hence, each household owns a portfolio of money, private claims issued by other households, and the fraction of their own capital which has *not* been issued. The latter has the same liquidity as claims previously issued, since claims to unissued capital would need to be traded on the same capital markets as existing claims. Therefore, besides liquid assets  $B_t$ , we only need to keep track of net private claims defined as  $S_t = \text{claims on others' capital} + \text{unissued capital}$ .

*Asset Accumulation.*—Let  $S_t^j$  and  $B_t^j$  denote the *total* net private claims and money belonging to household members of type  $j \in \{i, n\}$  at the beginning of  $t$ . As all assets are equally divided among members, we have  $S_t^i = \chi S_t$ ,

$S_t^n = (1 - \chi)S_t$ ,  $B_t^i = \chi B_t$ , and  $B_t^n = (1 - \chi)B_t$  by the law of large numbers.

Let  $S_{t+1}^j$  and  $B_{t+1}^j$  denote the total end-of-period net private claims and money for type  $j$  members. Then,  $S_{t+1} = S_{t+1}^i + S_{t+1}^n$  and  $B_{t+1} = B_{t+1}^i + B_{t+1}^n$  as household members pool assets together at the end of  $t$ . Further, household members face two financing constraints. First, no private agent can issue money, i.e.,

$$(2) \quad B_{t+1}^j \geq 0.$$

Second, for each group, the net private-claims position evolves according to

$$(3) \quad S_{t+1}^j = (1 - \delta)S_t^j + I_t^j - M_t^j,$$

where  $I_t^j$  is physical investment and  $M_t^j$  is the quantity of private claims sold. Due to search frictions on private-claims markets, only an endogenously determined fraction  $\phi_t \in (0, 1)$  of new or existing assets can be issued or resold.  $\phi_t$  thus captures *asset saleability*. Then, (3) implies the second financing constraint:

$$(4) \quad S_{t+1}^j \geq (1 - \phi_t)[I_t^j + (1 - \delta)S_t^j].$$

That is, agents need to retain the non-saleable fraction  $(1 - \phi_t)$  of their existing private claims and claims issued against new capital, thus limiting the external funding for new investment.

*The Workers' Flow-of-Funds Constraint.*—For simplicity, labor supply  $N_t$  is fixed at  $\bar{N}$ . Workers do not invest ( $I_t^n = 0$ ). They accumulate financial assets ( $M_t^n < 0$  and  $B_{t+1}^n > 0$ ) to implement their household's consumption smoothing plans. Hence, neither of their financing constraints is binding. They use labor income ( $w_t N_t$ ) and the return on money ( $B_t^n = (1 - \chi)B_t$ ) and private claims ( $S_t^n = (1 - \chi)S_t$ ) to finance consumption ( $C_t^n$ ), new money holdings ( $B_{t+1}^i$ ), and the purchase of private claims ( $-M_t^n$ ):

$$(5) \quad C_t^n + \frac{B_{t+1}^n}{P_t} - q_t^n M_t^n = w_t N_t + \frac{B_t^n}{P_t} + r_t S_t^n,$$

where private claims are purchased at the price  $q_t^n$ , while money is valued in real terms at  $1/P_t$ .

*The Entrepreneurs' Flow-of-Funds Constraint.*—Entrepreneurs need to finance new investment ( $I_t^i > 0$ ). They can use the return on

money ( $B_t^i = \chi B_t$ ) and private claims ( $S_t^i = \chi S_t$ ), and the proceeds from the issuance (or reselling) of private claims ( $M_t^i = I_t^i + (1 - \delta)S_t^i - S_{t+1}^i$ ) to finance consumption ( $C_t^i$ ), new money holdings ( $B_{t+1}^i$ ), and physical investment ( $I_t^i$ ):

$$(6) \quad C_t^i + \frac{B_{t+1}^i}{P_t} + I_t^i = q_t^i M_t^i + \frac{B_t^i}{P_t} + r_t S_t^i,$$

where private claims are issued or resold at the price  $q_t^i$ . Note  $q_t^i$  is Tobin's  $q$ , i.e., the ratio of the market value of capital to the replacement cost (i.e., unity).  $q_t^i \geq 1$ , otherwise entrepreneurs prefer self-financing. If  $q_t^i > 1$ , entrepreneurs will use all available resources to create new capital. We assume and later verify that  $q_t^i > 1$ . That is, both financing constraints (2) and (4) bind, and entrepreneurs do not bring consumption goods back to their household ( $C_t^i = 0$ ).

Hence,  $S_{t+1}^i = (1 - \phi_t)[I_t^i + (1 - \delta)S_t^i]$  according to (4), and we can express investment as  $I_t^i = [S_{t+1}^i - (1 - \phi_t)(1 - \delta)S_t^i]/(1 - \phi_t)$ . Then, (6) becomes

$$(7) \quad q_t^r S_{t+1}^i = \frac{B_t^i}{P_t} + [r_t + (1 - \delta)] S_t^i,$$

where the right-hand side is total net-worth, and  $S_{t+1}^i$  on the left-hand side is valued at  $q_t^r$ , the effective replacement cost of private assets:

$$(8) \quad q_t^r \equiv \frac{1 - \phi_t q_t^i}{1 - \phi_t} < 1.$$

For every unit of new investment, a fraction  $\phi_t$  is issued at price  $q_t^i$ ; entrepreneurs need to make a "down-payment"  $(1 - \phi_t q_t^i)$  and retain a fraction  $(1 - \phi_t)$  as inside equity claims. The lower  $q_t^r$  is, the more claims to capital entrepreneurs can return to their household.

Once we know  $S_{t+1}^i$  from (7), aggregate investment  $I_t = I_t^i$  can be backed out as

$$(9) \quad I_t = \frac{[r_t + (1 - \delta)\phi_t q_t^i] \chi S_t + \chi B_t / P_t}{1 - \phi_t q_t^i}.$$

As  $(1 - \delta)\phi_t q_t^i \chi S_t$  is the saleable fraction of old claims, entrepreneurs can "leverage" their liquid net-worth with a factor  $(1 - \phi_t q_t^i)^{-1}$  to invest in new capital.

*The Household's Problem.*—For convenience, let  $\rho_t$  be the ratio of the purchasing price to the effective replacement cost:

$$(10) \quad \rho_t \equiv \frac{q_t^n}{q_t^r} = \frac{(1 - \phi_t)q_t^n}{1 - \phi_t q_t^i}.$$

By multiplying (7) with  $\rho_t$  and adding (5), we then derive the household budget constraint

$$(11) \quad C_t + \frac{B_{t+1}}{P_t} + q_t^n S_{t+1} = w_t N_t + (\chi \rho_t + 1 - \chi) \left( \frac{B_t}{P_t} + r_t S_t \right) + [\chi \rho_t + (1 - \chi)q_t^n](1 - \delta)S_t,$$

where we have used the fact that  $S_{t+1} = S_{t+1}^i + S_{t+1}^n$ ,  $B_{t+1} = B_{t+1}^i + B_{t+1}^n$ ,  $C_t = C_t^i + C_t^n$ , and  $C_t^i = B_{t+1}^i = 0$ . Let  $J(S_t, B_t; \Gamma_t)$  be the value function of a household with private claims  $S_t$  and money stock  $B_t$ , given the aggregate state  $\Gamma_t \equiv (K_t, A_t)$ . Then,  $J$  satisfies the following Bellman equation

$$J(S_t, B_t; \Gamma_t) = \max_{\{C_t, S_{t+1}, B_{t+1}\}} \{u(C_t) + \beta E[J(S_{t+1}, B_{t+1}; \Gamma_{t+1}) | \Gamma_t]\}$$

s.t. (11).

### B. Capital Markets and Financial Intermediation

*Search and Matching.*—There are capital submarkets  $m = 1, 2, \dots$  with free entry. As we shall see, the number of submarkets is not important for our analysis. On each market, entrepreneurs post  $U_t^m$  units of sell offers backed by capital stock. Intermediaries screen submarkets for valuable projects to invest in. Posting buy quotes  $V_t^m$  on a particular submarket comes at a cost of  $\kappa$  per unit of quotes.

In order to match their buy quotes with suitable sell quotes in a submarket  $m$ , intermediaries operate a matching technology that determines the number of matched claims  $M_t^m$

$$M_t^m = M(U_t^m, V_t^m) = \xi (U_t^m)^\eta (V_t^m)^{1-\eta},$$

where  $\xi$  is matching efficiency and  $\eta$  is the matching elasticity. The matching technology endogenizes the probabilities of filling a sell quote,  $\phi_t^m \equiv M(U_t^m, V_t^m)/U_t^m$ , and, conversely, of filling a buy quote,  $f_t^m \equiv M(U_t^m, V_t^m)/V_t^m$ . Therefore,

$$(12) \quad f_t^m = \xi^{\frac{1}{1-\eta}} (\phi_t^m)^{\frac{\eta}{\eta-1}}.$$

Then,  $\theta_t^m \equiv V_t^m/U_t^m = \xi^{\frac{1}{\eta-1}} (\phi_t^m)^{\frac{1}{1-\eta}}$  is the search intensity of submarket  $m$  and positively co-moves with  $\phi_t^m$ . To maximize external funding, entrepreneurs post quotes amounting to  $U_t^m = I_t^i + (1 - \delta)\chi S_t$ , of which  $\phi_t^m U_t^m$  can be sold.  $\phi_t^m$ , indeed, captures asset saleability.

*Financial Intermediation.*—Matched claims are further intermediated. Financial intermediaries can sell one unit of claims, acquired from entrepreneurs at the price  $q_t^{i,m}$ , to other intermediaries on the same capital submarket  $m$  at the price  $q_t^m$ . Since only a fraction  $f_t^m$  of buy quotes is matched, the cost of selling one unit of claims is  $\kappa/f_t^m$ . Because of the competitive environment, the following zero-profit condition must then hold in each submarket:

$$(13) \quad \frac{\kappa}{f_t^m} = q_t^m - q_t^{i,m}.$$

Alternatively, intermediaries can sell claims trading at price  $q_t^m$  on submarket  $m$  to workers at price  $q_t^{n,m}$ . However, entrepreneurs must be monitored such that financial claims are eventually backed by capital stock. Only intermediaries have the capacity to monitor entrepreneurs at a cost of  $\kappa$  per unit of sell quotes posted by entrepreneurs. Since only a fraction  $\phi_t^m$  of sell quotes is matched, the cost of selling one unit of claims to workers is  $\kappa/\phi_t^m$ . We thus have another zero-profit condition:

$$(14) \quad \frac{\kappa}{\phi_t^m} = q_t^{n,m} - q_t^m.$$

Given these features of the intermediation process, each submarket is characterized by its saleability-sell-price pair  $(\phi_t^m, q_t^{i,m})$ . In light of the two zero-profit conditions above, intermediaries are indifferent between all submarkets. In addition, workers go to the submarket with the lowest  $q_t^{n,m}$ . We can thus omit the superscript  $m$ .

*Asset Price.*—Entrepreneurs choose the submarket in which to post their sell offers, by minimizing the effective replacement cost  $q_t^r$ , subject to the relationship between  $f_t$  and  $\phi_t$  and the zero-profit condition (13):

$$\min_{\{\phi_t, q_t^i\}} q_t^r = \frac{1 - \phi_t q_t^i}{1 - \phi_t}, \quad \text{s.t. (12), and (13),}$$

where again the superscript  $m$  is omitted. Doing so maximizes the end-of-period private claims  $S_{t+1}^i$ , according to (7). The optimal solution (see the proof in the online Appendix) yields

$$(15) \quad q_t^i = 1 + \frac{\kappa\eta}{1 - \eta} \frac{(1 - \phi_t)}{f_t} \geq 1.$$

We thus verify that  $q_t^i > 1$  when  $\kappa > 0$ . Using the two zero-profit conditions (13) and (14) together, we can eliminate  $q_t$  and obtain

$$(16) \quad q_t^n - q_t^i = \kappa \left( \frac{1}{f_t} + \frac{1}{\phi_t} \right).$$

Therefore, the spread  $q_t^n - q_t^i$  depends on both the search cost  $\kappa$  and the market structure.

## II. Equilibrium with Two Types of Assets

When  $\kappa \rightarrow 0$  private assets provide sufficient liquidity, as (15), (16), and (10) jointly imply that  $q_t^i = q_t^n = q_t^r = \rho_t = 1$ . Money is, therefore, not valued as a lubricant for investment financing, such that  $B_{t+1}/P_t = 0$ . The efficient level of investment can be implemented by issuing private claims only, and our model resembles a standard real business cycle model.

Instead, we focus on the particular type of equilibrium, in which money and private claims co-exist. This type of equilibrium will exist, whenever the intermediation cost  $\kappa$  is large enough for money to relax entrepreneurs' financing constraints, while at the same time being sufficiently small, such that the issuance of private claims remains profitable.

*Portfolio Choices.*—We can define the return from private claims by forwarding the budget constraints of workers (5) and entrepreneurs (7) by one period. Let  $r_{t+1}^{ni} \equiv [r_{t+1} + (1 - \delta)]/q_t^n$  be the return on private claims purchased at  $t$  from the point of view of a worker,

who becomes an entrepreneur at  $t + 1$ . Let  $r_{t+1}^m \equiv [r_{t+1} + (1 - \delta)q_{t+1}^n]/q_t^n$  be the corresponding return if the worker does not change her type. The household's optimal portfolio choices for money and private financial assets yield two liquidity-adjusted asset pricing formulae:

$$(17) \quad 1 = E_t \left[ \Delta_{t+1} \frac{\chi \rho_{t+1} + 1 - \chi}{P_{t+1}/P_t} \right] \\ = E_t \left[ \Delta_{t+1} [\chi \rho_{t+1} r_{t+1}^m + (1 - \chi) r_{t+1}^m] \right],$$

where  $\Delta_{t+1} \equiv \beta u'(C_{t+1})/u'(C_t)$  is the stochastic discount factor.

A *Recursive Competitive Equilibrium* is a mapping  $\Gamma_t \equiv (K_t, A_t) \rightarrow \Gamma_{t+1}$ , with consumption, investment, and portfolio choices  $\{C_t, I_t, S_{t+1}, B_{t+1}\}$ , asset market structures  $\{\phi_t, f_t\}$ , a collection of prices  $\{\rho_t, q_t^i, q_t^n, P_t, w_t, r_t\}$ , and with an exogenous process for  $\{A_t\}$ , such that: firms' optimality conditions in (1) hold; given prices, the policy functions solve the representative household's problem, satisfying (10), (11), and (17);  $I_t$  is determined by (9);  $K_{t+1} = (1 - \delta)K_t + I_t$ ,  $S_t = K_t$ , and  $B_t = \bar{B}$ ; the asset search market "clears": (12), (15), and (16) hold.

### III. Asset Liquidity and the Macroeconomy

We focus on the long-run equilibrium featuring both private claims and money.

*Liquidity Premium.*—When money is valued by investors (workers), it relaxes their future financing constraints. To see this, consider the asset pricing formula for money (17). In the steady state, this condition implies that  $[\chi \rho + 1 - \chi] P_t/P_{t+1} = \beta^{-1}$ . As money is in fixed supply,  $P_t = P_{t+1}$  in the steady state. Therefore,

$$\rho = \rho^* \equiv 1 + (\beta^{-1} - 1)/\chi > 1,$$

or by definition,  $q^n/q^r = \rho^* > 1$ . This means that the cost of private claims for workers exceeds that of entrepreneurs, as the latter cannot issue as many private assets as they would desire and remain financially constrained.

In view of binding financing constraints, investors will demand a higher return from

holding only partially resaleable private claims relative to money. As a result, a positive liquidity premium emerges between the returns on private claims and money, defined as

$$\Delta_t^{LP} \equiv E_t [\chi r_{t+1}^m + (1 - \chi) r_{t+1}^m] - E_t \left[ \frac{P_t}{P_{t+1}} \right].$$

**PROPOSITION 1:** *Suppose  $\kappa > 0$  and private claims and money coexist. Then,  $r/q^n > \delta$  and money provides a liquidity service in a neighborhood around the steady state. The steady-state liquidity premium amounts to  $\Delta^{LP} = [1 - (\rho^*)^{-1}] (r/q^n - \delta)(1 - \chi) > 0$ .*

The proof (in the online Appendix) basically uses the two asset pricing formulae in (17), which imply that the two assets earn the same return after adjusting  $\rho$ . With money in fixed supply, we must have  $\rho = \rho^* > 1$ , and the equilibrium thus features a liquidity premium.

*Asset Saleability and Prices.*—The liquidity of private claims depends both on their price and their physical saleability. In fact, both dimensions are related as the steady-state asset price  $q^i$  is a function of asset saleability  $\phi$ . Specifically, we can rewrite (15) by using (12) as

$$(18) \quad q^i = 1 + \frac{\kappa \eta \xi^{\frac{1}{\eta-1}}}{1 - \eta} \phi^{\frac{\eta}{1-\eta}} (1 - \phi).$$

Further, since money is valued in the coexistence type of equilibrium,  $\rho = \rho^*$  is uniquely pinned down as discussed above. Using the definition  $\rho = q^n/q^r$ , we know that

$$(19) \quad (1 - \phi)q^n - \rho(1 - \phi q^i) = 0,$$

where  $q^i$  satisfies (18) and we know from (12) and (16) that  $q^n = q^i + \kappa [\phi^{-1} + \xi^{\frac{1}{\eta-1}} \phi^{\frac{\eta}{1-\eta}}]$ . Then, (19) determines  $\phi$ , as  $q^n$  and  $q^i$  are functions of  $\phi$  only.

Importantly, (19) could admit multiple solutions for  $\phi$ . For instance, if  $\eta = 1/2$ , (19) becomes a quartic equation of the unknown  $\phi$  (see the online Appendix). Then, multiple values of  $\phi \in (0, 1)$  could solve (19), implying multiple co-existence equilibria. Intuitively, the

competitive asset search process may generate multiplicity, as coordination between sellers and buyers on different submarkets of private claims may lead to different outcomes. As a comparison, the random search framework of Cui and Radde (2016) only features a unique co-existence equilibrium.

Whether a lower level of the steady-state equilibrium saleability  $\phi$  (or search intensity  $\theta$ ) gives rise to a lower asset price  $q^i$  depends on the relative strength of asset supply and asset demand effects: on the one hand, a lower level of  $\phi$  implies tighter financing constraints and less supply relative to demand on the asset search market. As intermediaries have to offer more attractive conditions to attract scarce supply, this raises the equilibrium asset price. On the other hand, lower equilibrium asset saleability implies that private assets are less effective investments to hedge future funding needs, which would reduce demand, increase the equilibrium liquidity premium, and compress the asset price.

Which effect dominates depends on the parameters of the economy. Notice that with  $\partial q^i / \partial \phi = \kappa \eta (1 - \eta)^{-2} f^{-1} \phi^{\frac{\eta}{1-\eta}} (\eta / \phi - 1)$ , we obtain a sufficient condition for  $q^i$  and  $\phi$  to positively comove:

**PROPOSITION 2:** *Suppose there are multiple values of asset saleability  $\phi \in (0, 1)$  that solve (19). If  $\phi < \eta$ ,  $\forall \phi \in (0, 1)$ , then  $\partial q^i / \partial \phi > 0$ .*

Intuitively, when  $\phi$  is sufficiently small, the low hedging value of private claims is associated with a high sensitivity of asset demand from workers to market conditions. As a result, the demand effect dominates the supply effect, such that a lower level  $\phi$  is associated with a higher liquidity premium and thus a lower  $q^i$ .

*The Macroeconomy.*—When both  $\phi$  and  $q^i$  fall, aggregate investment in (9) falls, because both liquid net-worth and leverage drop. That is, an economy with particularly illiquid asset markets simultaneously features a low asset price and tighter financing constraints—and thus less investment. This result illustrates the effect of asset illiquidity via financing constraints on real allocations.

#### IV. A Numerical Example

The following example highlights that coordination on financial markets strongly impacts asset liquidity and portfolio allocations between private and public financial assets, thus significantly affecting real economic activity.

Let  $\eta = 0.5$ . Set  $\beta = 0.99$ ,  $\delta = 0.03$ , and  $\alpha = 0.33$  as in a standard calibration for a quarterly macro model. Following Shi (2015) and interpreting  $\chi$  as the fraction of firms that has investment opportunities each quarter, we set  $\chi = 0.056$  in line with Doms and Dunne (1998). Finally, set  $\xi = 0.2$  and  $\kappa = 0.01$  such that  $q_i = 1.05$  and the annualized liquidity premium amounts to 50 basis points in the equilibrium with the most liquid asset markets. Then,

$$\phi_1 = 0.0685 \quad \text{and} \quad \phi_2 = 0.2828$$

solve (19) which is a quartic equation (see the online Appendix for details). That is, private financial markets are active, but exhibit different degrees of liquidity (or search intensity). The corresponding (real) value of money  $B/P$  is around 88.64 percent lower in the equilibrium with more liquid and active asset markets. In other words, as the liquidity of private financial markets improves, agents value public liquidity substantially less.

In both equilibria  $\phi < \eta$ , such that Proposition 2 implies that the equilibrium  $q^i$  will be higher if  $\phi$  is higher. In fact, when the steady-state saleability increases from  $\phi_1$  to  $\phi_2$ , the asset price  $q^i$  increases from 1.02 to 1.05, while the liquidity premium decreases by 5 basis points. Also note that  $\phi < 0.5$  seems empirically plausible according to Del Negro et al. (2011), as otherwise all claims would have a turn-over rate of more than 50 percent within a quarter.

By affecting asset liquidity, participation decisions in the financial market can, thus, have a strong impact on firms' financing constraints, capital accumulation, and output. In our numerical example, investment, consumption, and output increase by 7.02 percent, 3.06 percent, and 2.26 percent, respectively, as steady-state saleability switches from  $\phi_1$  to  $\phi_2$ .

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