European Economic Review 87 (2016) 1–25

Contents lists available at ScienceDirect

European Economic Review

journal homepage: www.elsevier.com/locate/eer

Monetary–fiscal interactions with endogenous liquidity frictions

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ARTICLE INFO

Article history:
Received 7 August 2014
Received in revised form
9 March 2016
Accepted 10 March 2016
Available online 14 April 2016

JEL classification:
E20
E40
E50
E60
G10

Keywords:
Liquidity frictions
Monetary and fiscal interactions

ABSTRACT

I develop a tractable macro model with endogenous asset liquidity to understand *monetary–fiscal interactions* with liquidity frictions. Agents face idiosyncratic investment risks and meet financial intermediaries in competitive search markets. Asset liquidity is determined by the search friction and the cost of operating the financial intermediaries, and it drives the financing constraints of entrepreneurs (those who have investment projects) and their ability to invest. In contrast to private assets, government bonds are fully liquid and can be accumulated in anticipation of future opportunities to invest. A higher level of real government debt enhances the liquidity of entrepreneurs' portfolios and raises investment. However, the issuance of debt also raises the cost of financing government expenditures: a higher level of distortionary taxation and/or a higher real interest rate. A long-run optimal supply of government debt emerges. I also show that a proper mix of monetary and fiscal policies can avoid a deep financial recession.

1. Introduction

Asset liquidity captures the ease with which financial assets can be traded without strongly affecting their prices. The recent long-lasting world-wide financial crisis has shown that liquidity fluctuations in asset markets can have a huge impact on asset prices and the real economy.[1] In fact, empirical evidence points to procyclical variation in the liquidity of a wide range of financial assets.[2] Understanding the frictions that generate these fluctuations and the possible policy responses seems thus crucial.

This paper studies monetary–fiscal interactions when movements in asset prices and the real economy are driven by endogenous fluctuations in liquidity. Although the research on monetary–fiscal interactions at least go back to Sargent and Wallace (1981), Leeper (1991), and Sims (1994),[3] the consideration of *endogenous* liquidity frictions is usually missing.

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[1] For example, Dick-Nielsen et al. (2012) identify a structural break in the market liquidity of corporate bonds at the onset of the crisis. The liquidity component of spreads of all but AAA rated bonds increased and turnover rates declined, leading to a more difficult refinancing. Commercial papers, mostly traded on a search market, experienced pronounced illiquidity reported by Anderson and Gascon (2009). The main investors in the commercial paper market, money market mutual funds, shifted to highly liquid government securities.

[2] Studies by Huberman and Halka (2001), Chordia et al. (2001, 2005) and Naes et al. (2011) show that market liquidity is procyclical and highly correlated with GDP across asset classes such as bonds and stocks.

[3] Schmitt-Grohé and Uribe (2006) study the interactions in a new Keynesian framework.

http://dx.doi.org/10.1016/j.euroecorev.2016.03.007
0014-2921/

To fill this gap, I incorporate these frictions into an almost standard New Keynesian model with sticky prices. In the model, government bonds are fully liquid and represent public liquidity, while privately issued claims, traded on competitive search markets, are only partially saleable and have a bid-ask spread. Private claims represent private liquidity and carry a liquidity premium, a premium that buyers will demand when the assets cannot be easily converted into consumption goods at the fair market value.

I show that the government faces a tradeoff between the benefit of public liquidity provision and the cost of financing government expenditures. Therefore, an optimal long-run debt-to-GDP ratio can exist. When the economy is hit by large adverse financial shocks, a proper mix of monetary and fiscal policies is the key to stabilize the economy.

In the model, private agents face idiosyncratic investment opportunities and there is no insurance market for idiosyncratic risks. When an investment opportunity arrives, an agent becomes an entrepreneur. Otherwise, she is a worker earning wages. The entrepreneur can finance investment by issuing financial claims to the new capital stock and/or by liquidating existing claims. Issuing or reselling private claims needs to go through competitive financial intermediaries, which implement a costly search and matching technology.

The model's innovative aspect is the asset market structure. Asset liquidity is endogenously determined in a competitive search environment where agents meet financial intermediaries. Two important results emerge by endogenizing asset liquidity. First, the model generates the positive co-movement between asset saleability and asset prices, as observed in the data. The key aspect is that asset demand is endogenous and determines saleability, tightness of financing constraints, and asset price jointly. Second, government policies can affect the market structure and asset liquidity.

As market- and bank-based financial intermediation both share the essential feature of matching savers and borrowers, our framework admits both interpretations of the intermediation process, as in De-Fiore and Uhlig (2011). The financial frictions, induced by costly search, reduce the amount of resources that are transferred to entrepreneurs with investment projects. Therefore, if the liquidity of private claims is too low, private agents cannot fully self-insure idiosyncratic risks. When fully liquid government bonds circulate, private agents will purchase them as a buffer stock in case a good investment opportunity comes up. Such precautionary savings reduce the real return from bonds and further raise liquidity premium.

I analyze a class of monetary and fiscal policy rules with long-run targets suggested by actual policy practice. The monetary authority sets the nominal interest rate as a function of current inflation and output. The fiscal authority chooses a level of tax rate that depends on the quantity of real government debt; it can also adjust government expenditures as a function of current debt and output.

I first study the long-run optimal supply of (real) government debt. The government faces a key tradeoff when supplying government bonds. On the one hand, more (real) government debt implies more public liquidity, which can be accumulated by agents awaiting investment opportunities and enhances entrepreneurs' ability to carry out investment when the opportunity arises. The financing constraints are less tight, thus reducing the liquidity premium of private claims and further encouraging buyers entering the financial market. On the other hand, more (real) government debt also implies a higher real interest rate and/or a higher tax rate, raising the cost of financing government expenditures.

After analyzing the long-run economy, I turn to equilibrium dynamics after financial shocks. First, I study a simple class of policy rules in which the monetary authority sets the nominal interest rate as an increasing function of inflation only and the fiscal authority only adjusts the tax rate but not government expenditures.

Adverse financial shocks, modeled as rises in the search costs, drive away the asset demand, reduce the saleability of private claims, and push down their price. The financial shocks affect the bid-ask spread, as in Ajello (2012) and Bassetto et al. (2015). Adverse financial shocks drive up the hedging value of liquid assets and push workers towards them. The liquidity premium rises and the demand for private claims falls: a flight to liquidity occurs. Entrepreneurs who issue private claims are even more financing constrained. Therefore, demand for goods drops and firms produce less. Such output contraction lowers agents' income expectation further, pushing them even more towards liquid assets. We thus see persistent falls in investment, consumption, and output.

Because of less demand for physical investment after adverse financial shocks, inflation falls and nominal interest rates drop. When the adverse financial shocks are large, the nominal interest rate could drop to the zero lower bound (ZLB). With nominal frictions, I show that large adverse financial shocks combined with the ZLB can generate a much deeper recession, compared to the case when the ZLB is not a hard constraint.

Finally, I consider a more sophisticated class of policy rules. I allow the nominal interest rate to be a function of both inflation and output, and allow the fiscal authority to adjust government expenditures. The optimal policy responses to financial shocks, within this class of monetary and fiscal rules, suggest an anticipated *expansionary fiscal contraction* under which the ZLB does not constrain nominal interest rates. In particular, an anticipated reduction of government expenditures "crowds in" resources, relaxes entrepreneurs' financing constraints, and raises aggregate demand immediately. Compared to the simple rule, such expansionary fiscal contraction (in response to financial shocks) implies that the fiscal authority should be more accommodating while the monetary authority needs to be more responsive to inflation.

Relationships to literature: This paper analyzes financial frictions based on Kiyotaki and Moore (2012) and Shi (2015).[4] The large family structure for simple aggregation in this paper also follows Shi (2015). The presence of liquidity constraints opens up the possibilities for government bonds or fiat money to circulate, which at least goes back to Holmström and Tirole

[4] Similar papers at least include Nezafat and Slavik (2010), Del Negro et al. (2011), Ajello (2012), and Bigio (2012).

(1998). That is, if private liquidity is not enough, public liquidity can be added to achieve efficiency.[5] This paper provides a novel channel in which public liquidity provision is costly due to distortionary taxation. Therefore, an optimal supply of public liquidity emerges.

Importantly, asset saleability and asset price are endogenously generated from costly search and matching, thus avoiding the (counterfactual) negative co-movement discussed in Shi (2015) and Bigio (2012) when asset saleability is exogenous. A drop in asset demand reduces saleability of assets and their prices, thus tightening financing constraints. This effect is similar to that from the random search framework of Cui and Radde (2015), who focus on the fragility of financial markets. For an survey of the literature using search in asset markets, see Rocheteau and Weill (2011) or Lagos et al. (2016).

Since the last financial crisis, nominal interest rates in many developed economies have stayed at the ZLB. I show that why with liquidity frictions adverse financial shocks can generate deflationary pressures and push nominal interest rates to the ZLB (when the monetary authority follows a simple Taylor rule). In this regard, this paper is a compliment to Eggertsson and Krugman (2012) and Buera and Nicolini (2014).

Two sets of policy tools have been proposed to deal with the ZLB. The first is unconventional monetary policy or "Quantitative Easing" (QE).[6] Del Negro et al. (2011) demonstrate the large impact of liquidity fluctuation on investment and asset prices and how unconventional monetary policies might stabilize the economy. The second proposal, such as in Christiano et al. (2011) and Woodford (2011), argues that raising government expenditures will not crowd out private consumption when the ZLB binds. I show that even without the consideration of "QE", the monetary–fiscal interactions are important when the economy features liquidity frictions. Further, after adverse financial shocks, fiscal expansion might *not* be better than fiscal contraction which crowds in resources, relaxes financing constraints of some private agents, and raises aggregate demand.

Government debt provides liquidity service and has the "crowding-in" effect, similar to Woodford (1990). This aspect is in contrast with Aiyagari and McGrattan (1998) in which government debt is a perfect substitute to private assets (or capital stock). Government debt relaxes agents' borrowing constraints but also has the "crowding-out" effect on capital accumulation. In Angeletos et al. (2013), government bonds also "crowd in" resources as they have a higher level of exogenous *pledgeability* as collateral than capital assets. This paper, however, features asset saleability (and a bid-ask spread) instead of asset pledgeability. In addition, I show that public liquidity provision can also alter the liquidity of privately issued claims through the endogenous search and matching.

2. The model

I construct a growth model with liquidity frictions and nominal price stickiness. The economy is populated by a continuum of similar households (with measure one), firms, financial intermediaries, and a government. Time is discrete and infinite. Each period is divided into four sub-periods.

(1) *The households' decision period.* The shocks to aggregate productivity A_t (TFP shocks) and to intermediation search cost κ_t (financial shocks) are realized. All members in a representative household *equally divide* the household's financial assets consisting of government bonds and privately issued financial claims on capital stock. The household instructs its members on the optimal type-specific choices to be carried out after individual types (explained below) have been realized.

(2) *The production period*. Each member receives a status draw, becoming an entrepreneur (type i) with a probability χ and a worker (type n), otherwise. Workers supply labor hours, while only entrepreneurs have access to investment projects transforming consumption goods into capital stock one-for-one. Intermediate goods firms, in a monopolistically competitive market, rent capital stock K_t and labor N_t from the household to produce intermediate products. The after-tax rental rate and wage rate are r_t and W_t. Final goods firms produce numeraire consumption goods by aggregating intermediate products. The profits (D_t) from intermediate goods firms are equally distributed among members.

(3) *The investment period*. There is no insurance among household members, and they keep separated until the consumption stage. Because of the equal division of assets in the beginning, entrepreneurs are financing constrained in investment. They seek outside financing. Financial markets open in which entrepreneurs offer financial claims for sale and workers purchase these claims through financial intermediaries, which implement a costly search and matching technology. Search frictions imply that private financial claims are only partially liquid. In contrast, government bonds are fully liquid as they can be traded on a frictionless spot market.

Financial intermediaries are competitive and search in financial submarkets. Entrepreneurs and workers choose the best financial market subject to the participation constraints of financial intermediaries. The competitive search process shares similar features with that in the labor search literature such as Moen (1997).

[5] There are thus both fully liquid government issued assets and partially liquid private claims. Changing the portfolio compositions of the two assets can potentially affect the real economy. More recently, financial intermediations are added and the policy affects the asset compositions held by intermediaries. See, for example, Gertler and Karadi (2011) and Gertler and Kiyotaki (2010).

[6] "QE" here stands for exchanging government issued liquid assets (money or short-term government bonds) for privately issued and partially liquid assets to increase liquidity in asset markets, i.e., unconventional monetary policy. The policy was implemented via various facilities in "Quantitative Easing I" and some parts of "Quantitative Easing III". See a recent summary by Blanchard (2013). Recently, Wen (2014) compares equilibrium responses when the exit strategy of "QE" is implemented differently.

(4) *The consumption period.* Entrepreneurs and workers reunite again in their respective households, pool all assets together, and share consumption goods across all members.

Throughout the paper, I focus on the type of equilibrium in which entrepreneurs are financing constrained and both private claims and government bonds circulate. I verify the existence of such type of equilibrium in the numerical analysis.

2.1. A representative household

Preferences. Let $\mathbb{E}_t$ be the mathematical expectations operator conditional on all information available at date t. The household's preferences on consumption and leisure can be represented by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}-1}{1-\sigma} - Z_t^{1-\sigma}\mu N_t + \psi_g \frac{G_t^{1-\sigma}-1}{1-\sigma} \right] \tag{1}$$

where $\beta \in (0,1)$ is a discount factor, $\sigma > 0$ is a risk-aversion parameter, C_t denotes private consumption, and μ and ψ_g are constants that scale the disutility from hours worked N_t and the utility of public spending G_t. The linear disutility of labor follows the employment lottery technology from Rogerson (1988) so that the micro-level labor supply elasticity is not a concern. To allow for a balanced growth path, the disutility from labor is adjusted by Z_t, the level of labor augmenting technology which grows at a constant rate $Z_t/Z_{t-1} = \gamma_z$. The scaling $Z_t^{1-\sigma}$ follows Mertens and Ravn (2011) and could be motivated by the fact that leisure time becomes more valuable as technology improves recreational activities.

Balance sheet. Households hold B_t units of nominal and fully liquid government bonds, earning a nominal interest rate R_{t-1}. The nominal price level is P_t. In addition, physical capital K_{t-1}, earning the rental return r_t, is owned by households and depreciates to $(1-\delta)K_{t-1}$ at the end of each period t, where $\delta \in (0,1)$. There is a financial claim to the future return of every unit of capital. For example, the owner of one unit of claims issued at time $t-1$ is entitled to r_t at t, $(1-\delta)r_{t+1}$ at time $t+1$, $(1-\delta)^2 r_{t+2}$ at time $t+2$, and so on. For simplicity, I follow Kiyotaki and Moore (2012), who normalize the claims by the capital stock, such that all claims depreciate at the same rate δ and earn a return r_{t+s} at any date $t+s$ ($\forall s \geq 0$).

Hence, the representative household owns a portfolio of bonds, private claims issued by other households, and the fraction of their own capital stock which has *not* been issued to others (Table 1). New claims have the same liquidity as claims already issued, since both new claims (to unissued capital) and existing claims would need to be traded on the same search market. Therefore, besides liquid assets B_t, we only need to keep track of net private claims defined as

$$S_t \equiv \text{claims on others' capital stock} + \text{unissued capital stock}$$

Note that there is no currency, which is another form of public liquidity in practice. One can give fiat money an extra role, for example, by having money in the utility of the household. Nevertheless, a higher level of money or government debt does not necessarily mean more liquidity. Public liquidity here means real liquidity which has to be eventually backed by the government's primary surplus. What is important is the real value of liquidity, whether it is in the form of government bonds or in the form of money. For this reason and for simplicity, I mix government bonds and money.

Asset accumulation. As mentioned before, only entrepreneurs can invest in capital stock. Investment X_t at time t will be available as capital stock in period $t+1$: $K_t = (1-\delta)K_{t-1} + X_t$.

No insurance market exists for idiosyncratic investment opportunities. To finance the investment, an entrepreneur can issue private claims to the future output from the investment. Alternatively, they can sell existing claims. Both new issuance and reselling go through financial intermediaries with a costly search and matching technology, and only an *endogenous* fraction $\phi_t \in (0,1)$ (i.e., *asset saleability*) of new or existing assets can be successfully sold with a price q_t^i. Buyers (who turn out to be the workers) need to pay a price $q_t^n > q_t^i$ because of the search costs.

Let S_{t-1}^j and B_{t-1}^j denote the total net private claims and bonds belonging to type $j \in \{i, n\}$ members at the beginning of period t. As all assets are equally divided among members,

$$S_{t-1}^i = \chi S_{t-1}, \quad S_{t-1}^n = (1-\chi)S_{t-1}, \quad B_{t-1}^i = \chi B_{t-1}, \quad \text{and} \quad B_{t-1}^n = (1-\chi)B_{t-1} \tag{2}$$

by the law of large numbers. Let S_t^j and B_t^j denote the end-of-period total net private claims and bonds for type j members. As household members pool assets together at the end of period t, we know that

$$S_t = S_t^i + S_t^n \quad \text{and} \quad B_t = B_t^i + B_t^n \tag{3}$$

Table 1
Balance sheet of a household.

Asset	Liability + net worth
Government bonds	Claims issued to others
Claims on others' capital stock	
Own capital stock	Net worth

Household members face two financing constraints. First, no private agent can issue government bonds, i.e.,

$$B_t^j \geq 0 \tag{4}$$

The second constraint relates to the accumulation of private claims. For each group j, the position of net private claims evolves according to

$$S_t^j = (1-\delta)S_{t-1}^j + X_t^j - M_t^j \tag{5}$$

where X_t^j is the level of investment, and M_t^j denotes the units of private claims sold on the financial market. Due to search frictions, agents need to retain at least a fraction $(1-\phi_t)$ of their existing private claims and of new investment, thus limiting the external funding for new investment. Then, (5) implies the second financing constraint:

$$S_t^j \geq (1-\phi_t)\left[X_t^j + (1-\delta)S_{t-1}^j\right] \tag{6}$$

The workers' flow-of-funds constraint. All workers are the same, and the worker group does not invest ($X_t^n = 0$). They accumulate financial assets ($M_t^n < 0$ and $B_t^n > 0$) to implement their household's intertemporal consumption smoothing plan. As a result, neither of their financing constraints is binding. They use labor income $W_t N_t$, the return on private claims (S_{t-1}^n) and bonds (B_{t-1}^n), and profits from intermediate goods firms $D_t^n = (1-\chi)D_t$, to finance consumption C_t^n and the end-of-period portfolio of private claims (S_t^n) and bonds (B_t^n):

$$C_t^n + q_t^n S_t^n + \frac{B_t^n}{P_t} = W_t N_t + \left[r_t + q_t^n(1-\delta)\right]S_{t-1}^n + \frac{R_{t-1}B_{t-1}^n}{P_t} + D_t^n \tag{7}$$

where private claims are purchased at the price q_t^n, while government bonds are valued in real terms by $1/P_t$.

The entrepreneurs' flow-of-funds constraint. The entrepreneur group needs to finance new investment $X_t^i > 0$. They can use return on private claims (S_t^i) and bonds (B_{t-1}^i), the issuance (or reselling) of private claims $M_t^i = X_t^i + (1-\delta)S_{t-1}^i - S_t^i$, and dividends from firms $D_t^i = \chi D_t$, to finance consumption C_t^i, new bonds B_t^i, and physical investment X_t^i:

$$C_t^i + \frac{B_t^i}{P_t} + X_t^i = r_t S_{t-1}^i + \frac{R_{t-1}B_{t-1}^i}{P_t} + q_t^i\left[X_t^i + (1-\delta)S_{t-1}^i - S_t^i\right] + D_t^i \tag{8}$$

where private claims are issued or resold at the price q_t^i. It is worth noting that q_t^i is also equal to Tobin's q: the ratio of the market value of capital to the replacement cost (i.e., unity). As long as $q_t^i > 1$, entrepreneurs will use all available resources to create new capital. We shall see that $q_t^i > 1$, when ϕ is sufficiently larger than zero and sufficiently smaller than one. This is realistic, as privately issued assets such as corporate bonds and equity claims are neither sufficiently illiquid nor sufficiently liquid.

Then, both financing constraints (4) and (6) bind, and entrepreneurs do not bring consumption goods back to their household, i.e., $C_t^i = 0$. Hence, $S_t^i = (1-\phi_t)\left[X_t^i + (1-\delta)\chi S_{t-1}\right]$ according to (6), and investment can be written as $X_t^i = S_t^i/(1-\phi_t) - (1-\delta)\chi S_{t-1}$. Then, the entrepreneurs' flow-of-funds constraint (8) becomes

$$q_t^r S_t^i = \frac{R_{t-1}B_{t-1}^i}{P_t} + \left[r_t + (1-\delta)\right]S_{t-1}^i + D_t^i \tag{9}$$

$$\text{where} \quad q_t^r \equiv \frac{1-\phi_t q_t^i}{1-\phi_t} < 1 \tag{10}$$

The right-hand side of (9) is entrepreneurs' total net worth. On the left-hand side, the end-of-period private claims are valued at q_t^r, which is the *effective* replacement cost of capital: for every unit of new investment, a ϕ_t fraction is issued at the price q_t^i; entrepreneurs need to make a "down-payment" $(1-\phi_t q_t^i)$ and retain a fraction $(1-\phi_t)$ as inside equity claims. The lower q_t^r is, the larger S_t^i is and entrepreneurs can bring more private claims back to the household.

Once we know S_t^i from (9), aggregate investment $X_t = X_t^i = S_t^i/(1-\phi_t) - (1-\delta)\chi S_{t-1}$ can be backed out as

$$X_t = \frac{\left[r_t + (1-\delta)\phi_t q_t^i\right]\chi S_{t-1} + \chi R_{t-1}B_{t-1}/P_t + \chi D_t}{1-\phi_t q_t^i} \tag{11}$$

Eq. (11) says that entrepreneurs' *liquid net worth*, including return from private claims and bonds, dividends, and the saleable part of existing claims $(1-\delta)\phi_t q_t^i \chi S_{t-1}$ can be "leveraged" with a factor $(1-\phi_t q_t^i)^{-1}$ to invest in new capital. One can see that a drop of asset saleability ϕ_t or asset price q_t^i reduces investment. Investment also falls when the (real) public liquidity $R_{t-1}B_{t-1}/P_t$ drops.

2.2. The household's problem

Define ρ_t as the ratio of the purchasing price q_t^n and the effective replacement cost q_t^r

$$\rho_t \equiv \frac{q_t^n}{q_t^r} = \frac{(1-\phi_t)q_t^n}{1-\phi_t q_t^i} \tag{12}$$

This ratio measures the degree of financial frictions. $\rho_t = 1$ in a standard real business cycle model or a standard New Keynesian model since $q_t^n = q_t^i = q_t^r = 1$. With financial frictions, $q_t^n > 1$ and $q_t^r < 1$ such that $\rho_t > 1$. Then, using (2) and $S_t = S_t^i + S_t^n$ and $B_t = B_t^n$ from (3), I derive a household budget constraint by using (7) $\times\ \rho_t +$(9)

$$C + q^n S + \frac{B}{P} = WN + (\chi\rho + 1 - \chi)\left(rS_{-1} + \frac{R_{-1}B_{-1}}{P} + D\right) + [\chi\rho + (1-\chi)q^n](1-\delta)S_{-1} \tag{13}$$

where the time subscript t of each variable is omitted. Note that ρ indeed captures financial frictions caused by liquidity risks. When $\rho = 1$, (13) is a standard budget constraint. When $\rho > 1$, it is a liquidity-adjusted budget constraint.

From now on, I stick to this recursive formulation without the time subscript t. Let Γ be the collection of aggregate state variables and the government policy variables (specified later). Given Γ, a representative household chooses consumption C, hours worked N, and a portfolio of private claims S and government bonds B, to maximize the expected utility. Specifically, given Γ, let $J(S_{-1}, B_{-1}; \Gamma)$ be the value (excluding the value of public spending) of a household with private claims S_{-1} and bonds B_{-1}. Let $u(C) \equiv \frac{C^{1-\sigma}-1}{1-\sigma}$, and the household solves the following Bellman equation

$$J(S_{-1}, B_{-1}; \Gamma) = \max_{C,N,S,B}\left\{u(C) - Z^{1-\sigma}\mu N + \beta\mathbb{E}[J(S, B; \Gamma_{+1},)|\Gamma]\right\}, \quad \text{s.t.} \quad (13)$$

I now derive the optimal choices of the household. The following first-order necessary conditions are also sufficient due to the concavity of the objective function. The first-order condition for labor is

$$Wu'(C) = Z^{1-\sigma}\mu N \tag{14}$$

which implies a standard intra-period tradeoff: the marginal gain from working which brings more consumption need to be equalized with the marginal cost from disutility of working.

The first-order condition for S_t is

$$\mathbb{E}\left[\Delta_{+1}\left[\chi\rho_{+1}r_{+1}^{ni} + (1-\chi)r_{+1}^{nn}\right]|\Gamma\right] = 1, \tag{15}$$

$$\text{where } \Delta_{+1} \equiv \frac{\beta u'(C_{+1})}{u'(C)}, \quad r_{+1}^{\text{ni}} \equiv \frac{r_{+1} + (1-\delta)}{q^n}, \quad \text{and} \quad r_{+1}^{\text{nn}} \equiv \frac{r_{+1} + (1-\delta)q_+^n}{q^n}$$

Δ_{+1} is the stochastic discount factor, and r_{+1}^{ni} and r_{+1}^{nn} resemble the workers' internal returns on private claims, if they (who are not financing constrained at date t) become entrepreneurs and workers at date $t+1$, respectively. From the flow-of-funds constraints at date $t+1$, the entrepreneurs' valuation of 1 unit of private claims is 1, while the workers' valuation is q_+. Since $\chi\rho_{+1}r_{+1}^{ni} + (1-\chi)r_{+1}^{nn}$ is the household's internal return with the adjustment of ρ_{+1} representing liquidity risks, Eq. (15) is a liquidity-adjusted asset pricing formula for private claims.

Following similar steps, I derive another asset pricing formula for government bonds. Let inflation be defined as

$$\Pi_{+1} \equiv \frac{P_{+1}}{P}$$

Then, R/Π_{+1} is the real return on bonds, and the first-order condition for B is

$$\mathbb{E}\left[\Delta_{+1}\frac{R(\chi\rho_{+1} + 1 - \chi)}{\Pi_{+1}}\middle|\Gamma\right] = 1 \tag{16}$$

where similarly $\frac{R(\chi\rho_{+1}+1-\chi)}{\Pi_{+1}}$ is the household's internal return on government bonds. With the adjustment of ρ_{+1}, Eq. (16) is a liquidity-adjusted asset pricing formula for government bonds.

2.3. *Asset price and asset liquidity*

Search and matching. There are capital submarkets, denoted by superscripts $m = 1, 2, 3, \ldots$. As we shall see, the number of submarkets is not important. On each submarket, entrepreneurs and workers post U^m units of sell quotes and V^m units of buy quotes, respectively. If matched, intermediaries ensure that buyers have enough resources to fill buy quotes and sell quotes U^m need to be backed by private claims (i.e., $\sum_m U^m \le X + (1-\delta)\chi S_{-1}$).

There is a continuum of competitive financial intermediaries, each of which chooses on which submarket to collect and match quotes at per-quote costs of κ units of consumption goods. The probability of filling a buy quote is f^m, while the probability of filling a sell quote (or asset saleability) is ϕ^m.

On each submarket m, financial intermediaries' gross profit amounts to the difference between the competitive buy price $q^{n,m}$ collected from workers and the sell price $q^{i,m}$ paid to entrepreneurs on the fraction of successfully matched quotes. Notice that workers direct their quotes to the submarket with the lowest buy price $q^{n,m} = q^n$, which is taken as given by intermediaries.

Since financial intermediaries operate in a competitive environment, they earn zero (net) profit from each transaction, i.e., $\kappa\left(\frac{1}{f^m} + \frac{1}{\phi^m}\right) = q^{n,m} - q^{i,m}$. In light of this zero-profit condition, intermediaries are indifferent between all submarkets and we can omit the superscript m:

$$\kappa\left(\frac{1}{f} + \frac{1}{\phi}\right) = q^n - q^i \tag{17}$$

The matching probabilities depend on intermediaries' matching technology. This technology is characterized by a matching function

$$M(U,V)=\xi U^{\eta}V^{1-\eta}$$

where ξ captures matching efficiency and η is the matching elasticity with respect to sell quotes U. Then, asset saleability and the probability of filling buy quotes are

$$\phi\equiv\frac{M(U,V)}{U},\quad f\equiv\frac{M(U,V)}{V}=\xi^{\frac{1}{1-\eta}}\phi^{\frac{\eta}{\eta-1}} \tag{18}$$

To maximize external funding via the financial market, entrepreneurs post quotes amounting to $U=X+(1-\delta)\chi S_{-1}$, of which a fraction $\phi U=M$ is sold. In this sense, ϕ indeed captures asset saleability. Similarly, workers post total quotes $V=f^{-1}\left[S^n-(1-\chi)(1-\delta)S_{-1}\right]$ and they have enough resources to fill matched buy quotes.

Asset price. While the matching function links ϕ and f through condition (18), the zero-profit condition (17) links q^i and q^n. Given these constraints, each submarket is characterized by its saleability-sell-price combination (ϕ,q^i). Accordingly, entrepreneurs choose the submarket in which to post their sell offers, which minimizes the effective replacement cost q^r, subject to the zero-profit condition and the relationship between f and ϕ:

$$\min_{\{0\le\phi\le1,\,q^i\ge1\}}\quad q^r=\frac{1-\phi q^i}{1-\phi},\quad \text{s.t. (17) and (18)}$$

This maximizes the end-of-period S^i, according to the entrepreneurs' flow-of-funds constraint (9). If we assume an interior solution, the optimal solution (see Appendix B) yields

$$q^i=1+\kappa\left[1+\frac{\eta(1-\phi)}{(1-\eta)f}-\frac{1}{\phi}\right] \tag{19}$$

Notice that $q^i>1$, if $\kappa>0$ and ϕ is neither too close to zero nor too close to one. Intuitively, private claims cannot be too illiquid, otherwise entrepreneurs do not enter the financial market; private claims cannot be too liquid, otherwise q^i becomes one and entrepreneurs are not financing constrained.

2.4. Firms

The production sector consists of intermediate goods firms and final goods firms, as in a standard New Keynesian model such as in Leeper et al. (2012) and references therein. To simplify, I only incorporate sticky prices using the Calvo type of price adjustment.

The final goods firms produce final consumption goods Y by assembling intermediate products Y_i ($i\in[0,1]$, with a slight abuse of notation) into $Y=\left[\int_0^1 Y_i^{1/(1+\zeta)}\,di\right]^{1+\zeta}$, where ζ is related to the elasticity of substitution among intermediate goods. Denote P_i as the nominal price of intermediate good i. The demand for intermediate inputs is then $Y_i=Y(P_i/P)^{-(1+\zeta)/\zeta}$, which gives rise to the aggregate price level $P=\left(\int P_i^{-1/\zeta}\,\mathrm{d}i\right)^{-\zeta}$.

Each intermediate firm i produces Y_i by renting capital k_i and employing labor hours n_i according to a constant-return-to-scale (CRS) production technology

$$Y_i=Ak_i^{\alpha}(Zn_i)^{1-\alpha},\quad \frac{Z}{Z_{-1}}=\gamma_z$$

where capital share $\alpha\in(0,1)$. A is the stationary aggregate productivity (TFP), while Z is again the labor augmenting technology that grows at a rate γ_z.

Each firm i solves the cost-minimization problem in renting capital and labor inputs

$$\min_{k_i,n_i}\{\tilde{r}k_i+\tilde{W}n_i\}\quad \text{s.t. } Y_i=Ak_i^{\alpha}(Zn_i)^{1-\alpha}$$

where $\tilde{r}$ and $\tilde{W}$ are the before-tax rental rate and wage rate. Let λ be the Lagrangian multiplier associated with the constraint, which is also the marginal cost of producing Y_i. Then, firm i that rents capital k_i hires labor hours $n_i=\left[\frac{\lambda(1-\alpha)AZ}{\tilde{W}}\right]^{\frac{1}{\alpha}}k_i/Z$. Given the aggregate capital stock K_{-1}, the aggregate labor demand is $\left[\frac{\lambda(1-\alpha)AZ}{\tilde{W}}\right]^{\frac{1}{\alpha}}K_{-1}/Z$. Since the aggregate labor supply is N, the before-tax wage rate and rental rate are

$$\tilde{W}=\lambda(1-\alpha)AZ\left(\frac{K_{-1}}{ZN}\right)^{\alpha}\quad\text{and}\quad \tilde{r}=\lambda\alpha A\left(\frac{K_{-1}}{ZN}\right)^{\alpha-1} \tag{20}$$

Thanks to the CRS technology, the capital-labor ratio in each intermediate firm is the same. This fact implies that the final output and total before-tax dividends (from the intermediate producers) are

$$Y=\left[\int_0^1 Y_i^{1/(1+\zeta)}\,di\right]^{1+\zeta}=AK_{-1}^{\alpha}(ZN)^{1-\alpha}\quad\text{and}\quad D=(1-\lambda)AK_{-1}^{\alpha}(ZN)^{1-\alpha}$$

In the monopolistically competitive market, each intermediate firm i can change price with a probability $1-\psi$, where $\psi\in(0,1)$. Otherwise, the price P_i will be indexed to the (gross) trend inflation rate $\overline{\Pi}$, i.e., $P_i=P_{i,-1}\overline{\Pi}$. Following the tradition, I look for a symmetric equilibrium in which firms that can adjust prices set the same P^*. Then, from $P=\left[\int P_i^{-1/\zeta}\,di\right]^{-\zeta}$, I can rewrite the aggregate price level P as

$$P=\left[(1-\psi)(P^*)^{-1/\zeta}+\psi(P_{-1}\overline{\Pi})^{-1/\zeta}\right]^{-\zeta}\tag{21}$$

When setting P^*, each firm i maximizes profits conditioning on the future dates when it cannot adjust. The optimal P^* satisfies the following recursive conditions (see Appendix B)

$$\frac{P^*}{P}=(1+\zeta)\frac{H^1}{H^2},\quad H^1=\lambda Y+\psi\mathbb{E}\left[\Delta_{+1}\left(\frac{\Pi_{+1}}{\overline{\Pi}}\right)^{\frac{1+\zeta}{\zeta}}H^1_{+1}\middle|\Gamma\right],\quad\text{and}\quad H^2=Y+\psi\mathbb{E}\left[\Delta_{+1}\left(\frac{\Pi_{+1}}{\overline{\Pi}}\right)^{\frac{1}{\zeta}}H^2_{+1}\middle|\Gamma\right]$$

where Δ_{+1} again is the household's stochastic discount factor. By using (21), I further eliminate P and P^* and obtain the *New Keynesian Phillips curve* with forward-looking inflation

$$1=(1-\psi)\left[(1+\zeta)\frac{H^1}{H^2}\right]^{-1/\zeta}+\psi\left(\frac{\Pi}{\overline{\Pi}}\right)^{-1/\zeta}\tag{22}$$

2.5. Government policies and the transformed economy

The government follows policy rules which are taken as given by private agents. I restrict to a class of policy rules that have long-run targets and induce a balanced growth path. For this reason, the "long-run equilibrium", the "steady state", and the "balanced-growth path" are used interchangeably.

Along the balanced growth path, all quantity variables (except labor hours N) and wage rate grows at the same rate γ_z, while interest rate, inflation, and asset price are constants. Since Z grows, I deflate all quantity variables (except N) by Z. That is, $c=C/Z$, $g=G/Z$, $x=X/Z$, $y=Y/Z$, and so on. Asset position variables are predetermined, and the detrended asset positions are $s_{-1}=S_{-1}/Z$ and $k_{-1}=K_{-1}/Z$. For convenience, I define the real value of government debt as

$$B_{r,-1}\equiv\frac{B_{-1}}{P_{-1}}$$

and similarly $b_{r,-1}=B_{r,-1}/Z$. Let the steady-state level of an arbitrary variable z be $\overline{z}$. In particular, the long-run policy targets are $\overline{g}$, $\overline{\tau}^y$, $\overline{R}$, and $\overline{\Pi}$.

Monetary and fiscal policies. The monetary authority sets the predetermined nominal interest rate R for $t+1$. The fiscal authority spends g and sets a value-added tax rate τ^y on total output y. This tax policy is equivalent to taxing capital returns, wages, and dividends at the common flat rate τ^y. The after-tax rental rate, wage rate, and dividends are

$$r=(1-\tau^y)\tilde{r},\quad w=(1-\tau^y)\tilde{w},\quad\text{and}\quad d=(1-\tau^y)(1-\lambda)y\tag{23}$$

Importantly, I do not allow lump-sum taxes and transfers which might potentially eliminate liquidity frictions. In practice, it is perhaps hard to verify investment opportunities, and that is why the government cannot simply eliminate liquidity frictions by taxes and transfers.

Institution settings. I consider policy rules (with long-run targets) which respond to economic fluctuations. Specifically,

$$\frac{R}{\overline{R}}=\left(\frac{\Pi}{\overline{\Pi}}\right)^{\psi_{R\Pi}}\left(\frac{y}{\overline{y}}\right)^{\psi_{Ry}},\quad\frac{g}{\overline{g}}=\left(\frac{b_r}{\overline{b}_r}\right)^{\psi_{gb}}\left(\frac{y}{\overline{y}}\right)^{\psi_{gy}},\quad\text{and}\quad\frac{\tau^y}{\overline{\tau}^y}=\left(\frac{b_r}{\overline{b}_r}\right)^{\psi_{\tau b}}\tag{24}$$

The long-run targets $\overline{g}$, $\overline{\tau}^y$, $\overline{R}$, and $\overline{\Pi}$ maximizes the steady-state social welfare, which will be discussed later. Note that (24) seems to be a class of rules suggested by actual policies. The monetary authority reacts to inflation and output fluctuations, while the fiscal authority can adjust government expenditures according to the current level of (real) debt and output. Finally, the tax rule mainly reflects the stabilizing role.

2.6. Recursive equilibrium

Now, I close the model by defining the recursive equilibrium. Readers who are not interested in the technical definition can skip this subsection.

I impose the equity market clearing $s=k$ and avoid discussing s. It is helpful to implement a few transformations before the equilibrium definition. First, the household's optimal labor supply and portfolio choice (14)–(16) can be transformed to

$$c^{-\sigma}w=\mu\tag{25}$$

$$\mathbb{E}\left[\frac{\beta\gamma_z^{-\sigma}c_{+1}^{-\sigma}}{c^{-\sigma}}\left[\chi\rho_{+1}\frac{r_{+1}+(1-\delta)}{q^n}+(1-\chi)\frac{r_{+1}+(1-\delta)q_+^n}{q^n}\right]\middle|\Gamma\right]=1 \tag{26}$$

$$\mathbb{E}\left[\frac{\beta\gamma_z^{-\sigma}c_{+1}^{-\sigma}}{c^{-\sigma}}\frac{R(\chi\rho_{+1}+1-\chi)}{\Pi_{+1}}\middle|\Gamma\right]=1 \tag{27}$$

where I have used the properties of $u(.)$. Further, the household budget constraint becomes

$$c+q^n\gamma_z k+\gamma_z b_r=wN+[\chi\rho+(1-\chi)]\left[rk_{-1}+\frac{R_{-1}b_{r,-1}}{\Pi}+(1-\tau^y)(1-\lambda)y\right]+[\chi\rho+(1-\chi)q^n](1-\delta)k_{-1} \tag{28}$$

and aggregate investment in (11) becomes

$$x=\chi\frac{[r+q^i\phi(1-\delta)]k_{-1}+R_{-1}b_{r,-1}/\Pi+(1-\tau^y)(1-\lambda)y}{1-\phi q^i} \tag{29}$$

where I have transformed the dividends by using (23). Second, I further transform the optimal nominal price setting in (22) to

$$1=(1-\psi)\left[(1+\zeta)\frac{h^1}{h^2}\right]^{-1/\zeta}+\psi\left[\frac{\overline{\Pi}}{\Pi}\right]^{-1/\zeta} \tag{30}$$

$$h^1=\lambda y+\mathbb{E}\left[\frac{\beta\psi\gamma_z^{-\sigma}c_{+1}^{-\sigma}}{c^{-\sigma}}\left(\frac{\Pi_{+1}}{\overline{\Pi}}\right)^{\frac{1+\zeta}{\zeta}}h_+^1\middle|\Gamma\right],\quad \text{and}\quad h^2=y+\mathbb{E}\left[\frac{\beta\psi\gamma_z^{-\sigma}c_{+1}^{-\sigma}}{c^{-\sigma}}\left(\frac{\Pi_{+1}}{\overline{\Pi}}\right)^{\frac{1}{\zeta}}h_+^2\middle|\Gamma\right] \tag{31}$$

where the total output satisfies $y=Ak_{-1}^{\alpha}N^{1-\alpha}$. Finally, notice that the total transaction of new issuance and reselling in the asset market is $\phi[x+(1-\delta)\chi k_{-1}]$; for each unit of transaction, the cost is $\kappa(\phi^{-1}+f^{-1})=q^n-q^i$. Then, the social resources constraint can be written as

$$c+g+x+(q^n-q^i)\phi[x+(1-\delta)\chi k_{-1}]=y \tag{32}$$

Now, I define the recursive equilibrium:

Definition 1. A *recursive competitive equilibrium* is an allocation in the private sector (k, b_r, c, x, N, y) and a price system $(w, r, \lambda, \phi, \rho, q^n, q^i, \Pi, h^1, h^2)$ as a function of $(k_{-1}, b_{r,-1}, R_{-1})$, given the exogenous stochastic processes $(\kappa, A)\rightarrow(\kappa_{+1}, A_{+1})$, the policy rules (24), and the long-run policy targets $(\overline{g}, \overline{\tau}^y, \overline{R}, \overline{\Pi})$, such that $\Gamma\equiv\{Z,\kappa,A,k_{-1},R_{-1},b_{r,-1},g,\tau^y,R\}$ and

1. the household's optimality conditions (25)–(27) and the budget constraint (28) hold; physical investment x is given by (29);
2. firms' pricing behaviors satisfy (30) and (31), with the after-tax rental rate and wage rate given by $r=(1-\tau^y)\lambda\alpha y/k_{-1}$ and $w=(1-\tau^y)\lambda(1-\alpha)y/N$;
3. the financial market equilibrium conditions (17) and (19) are satisfied, with ρ given by (12);
4. the goods market clearing condition (32) and the capital market clearing condition $x=\gamma_z k-(1-\delta)k_{-1}$ are satisfied, with output y given by $y=Ak_{-1}^{\alpha}N^{1-\alpha}$.

To check that the market for government bonds clears, i.e., Walras law holds, one should derive the government budget constraint from the equilibrium conditions. I use the investment equation (29), and I subtract the household budget constraint (28) from the social resources constraint (32). Then, I reach the government budget constraint

$$g+\frac{R_{-1}b_{r,-1}}{\Pi}=\tau^y y+\gamma_z b_r\quad \text{or}\quad G+\frac{R_{-1}B_{-1}}{P}=\tau^y Y+\frac{B}{P} \tag{33}$$

That is, the government uses taxes and the new debt issuance to finance the public spending and the repayment of existing debt.

3. Liquidity premium and the cost of public liquidity provision

In this section, I characterize the equilibrium analytically.

First, I show that in equilibrium in which $\kappa>0$ and both private claims and government bonds circulate, private claims carry a liquidity premium. Intuitively, private claims are subject to search costs while government bonds are not.

The government can provide public liquidity (real government debt) to mitigate the liquidity frictions. When private liquidity is in shortage due to the search frictions, public liquidity can be used as an alternative. A higher level of public liquidity means that entrepreneurs with government bonds become richer and can invest more, according to the investment equation (11). It can also reduces the liquidity premium through the endogenous search market structure, as entrepreneurs rely less on the outside financing.

Then, I show that public liquidity provision is, however, not free because of the distortionary taxation. When the government sets long run targets, it faces the tradeoff between the benefit of public liquidity provision and the cost of government financing.

3.1. Liquidity premium and asset price

Private claims are not as liquid as government bonds, since $0 < \phi_t < 1$ and $q_t^n > q_t^i$. Government bonds provide a liquidity service if liquidity frictions matter, i.e., $\rho_t > 1$. Taking government bonds as the benchmark assets, I compute the premium of private claims as

$$\Delta_t^{LP} \equiv \mathbb{E}_t\left[\chi\frac{r_{t+1}+(1-\delta)}{q_t^n}+(1-\chi)\frac{r_{t+1}+(1-\delta)q_{t+1}^n}{q_t^n}\right]-\mathbb{E}_t\left[\frac{R_t}{\Pi_{t+1}}\right]$$

which is the difference between expected returns of the two assets. In the steady state, Δ^{LP} only contains liquidity premium and is positive.

Proposition 1. *Suppose $\kappa > 0$ and both private claims and government bonds circulate. Government bonds provide a liquidity service in a neighborhood around the steady state. The steady-state liquidity premium of private claims relative to government bonds amounts to*

$$\overline{\Delta}^{LP} = (1-\chi)\left(1-\overline{\rho}^{-1}\right)(1-\delta)\frac{\chi\overline{\rho}}{\chi\overline{\rho}+1-\chi}\left(1-\frac{1}{\overline{q}^n}\right) > 0 \tag{34}$$

Proof. See Appendix B. □

One can see that when $\overline{q}^n$ is fixed, the liquidity premium $\overline{\Delta}^{LP}$ in (34) is an increasing function of $\overline{\rho}$. When there are no liquidity frictions on the balanced growth path, $\overline{q}^n = \overline{q}^i = \overline{q}^r = 1$ and thus $\overline{\rho} = 1$. That is, the liquidity premium becomes zero, and government bonds and private claims can act as perfect substitutes.

Asset saleability ϕ and the purchase price q^n determine the liquidity premium. To illustrate, when Tobin's q or the selling price $\overline{q}^i > 1$, we know that $\overline{q}^n > \overline{q}^i > 1 > \overline{q}^r$. Entrepreneurs are financing constrained and $\overline{\rho} > 1$. Liquidity frictions matter, and an additional unit of government bonds can relax entrepreneurs' constraints by raising their net worth according to (11). This allows them to leverage their physical investment and eventually their future equity positions. In sum, government bonds provide liquidity service, and private claims carry a liquidity premium.

Now, I study the relationship between asset saleability and asset price. One shall see that whether a lower level of the equilibrium saleability ϕ is associated with a higher or a lower asset price q^i depends on the relative strength of asset supply and demand effects.

On one hand, a lower level of ϕ implies tighter financing constraints and less supply relative to demand on the asset search market as shown by Shi (2015). Then, the shadow value of private claims rises. As intermediaries have to offer better conditions to attract scarce supply, this should be reflected in a higher equilibrium asset price. On the other hand, lower equilibrium asset saleability implies that private claims are less effective investments to hedge future funding needs, which would reduce demand, increase the equilibrium liquidity premium, and compress the asset price.

Which effect dominates depends on the parameters of the economy. But I can show that q^i must fall, if the spread $q^n - q^i$ is higher when asset saleability ϕ drops.

Proposition 2. *Suppose $\kappa > 0$ and both private claims and government bonds circulate. If we have the negative co-movement between asset saleability and bid-ask spread, i.e., $\partial(q^n - q^i)/\partial\phi < 0$, then we have the positive co-movement of asset saleability and Tobin's q, i.e., $\partial q^i/\partial\phi > 0$.*

Proof. See Appendix B. □

The above proposition is a partial equilibrium result, but carries the insight from asset market structure into general equilibrium analysis later. That is, when a drop of asset saleability ϕ increases the spread, the liquidity premium channel dominates the drop of supply. Tobin's q falls in response. To generate the fall in asset price, one can see that the crucial component is the spread $\kappa\left(f^{-1}+\phi^{-1}\right)$ which is linked with asset saleability. This component, which is endogenous, allows demand to play an important role to affect asset prices. In contrast, ϕ is an exogenous parameter in the Kiyotaki and Moore (2012) model and asset demand is fixed to clear the asset market.

3.2. Costly public liquidity provision

Since private claims do not have enough liquidity, the government can provide public liquidity to mitigate liquidity frictions. But public liquidity provision is costly, as there is a tradeoff between the benefit of public liquidity provision and the cost of government financing.

First, from the asset pricing formula for government bonds (27), I compute $\overline{\rho}$ given a steady-state real interest rate $\overline{R}/\overline{\Pi}$

$$\overline{\rho}=1+\chi^{-1}\left[\left(\beta\frac{\overline{R}}{\overline{\Pi}}\right)^{-1}\gamma_z^{\sigma}-1\right]$$

A higher real interest rate implies a lower $\overline{\rho}$. Since $\rho\equiv q^n/q^r$, we know that if $\overline{\rho}$ falls, the difference between the values of private claims from the perspective of workers and entrepreneurs shrinks. That is, the degree of financial frictions is smaller. However, the real interest rate cannot be arbitrarily large. As $\overline{\rho}>1$ when $\kappa>0$, the real interest rate is bounded above

$$\frac{\overline{R}}{\overline{\Pi}}<\beta^{-1}\gamma_z^{\sigma}$$

To be specific, due to liquidity frictions, the monetary authority prefers a lower ρ, or a higher real interest rate $R\Pi^{-1}$ up to $\beta^{-1}\gamma_z^{\sigma}$. Then, entrepreneurs who have accumulated government bonds for precautionary saving purposes can invest more according to (29).

Intuitively, since private claims carry a liquidity premium and government bonds are scarce, precautionary entrepreneurs tend to push down the real interest rate compared to the case without liquidity frictions. Consequently, the (real) price of bonds are unnecessarily high, and the government needs to supply more real government debt. Given a q^n, this public liquidity provision also reduces the liquidity premium in (34), as entrepreneurs rely less on the financial market but more on public liquidity. In later numerical analysis, I will show that these effects still hold in general equilibrium.

The above discussion suggests that liquidity premium has a negative impact on financing investment. On the contrary, if government expenditures need to be financed at the same time, some degree of liquidity premium is preferred by the fiscal authority. One possible way to see this is that, a higher real interest rate implies that government expenditures become more expensive to finance. With a higher real interest rate, the government with a high level of real government debt will have to either cut government expenditures (which reduces the social welfare that depends on public spending), or raise more distortionary taxation.

To simplify, consider the economy where the tax rate $\overline{\tau}^y$ is fixed. Along the balanced growth path, how should the government choose the optimal level of debt, or the debt-to-GDP ratio $(\overline{b}_r/\overline{y})$? Given a tax rate $\overline{\tau}^y$, the planner faces a key tradeoff. On the benefit side, government bonds enhances the liquidity of entrepreneurs' portfolio. A higher level of (real) debt can raise investment from entrepreneurs who sell liquid assets to invest. On the cost side, a higher interest rate pushes up the cost of financing government expenditures which can be seen from the government budget constraint (33) on the balanced growth path:

$$\frac{\overline{g}}{\overline{y}}=\overline{\tau}^y+\left(\gamma_z-\frac{\overline{R}}{\overline{\Pi}}\right)\frac{\overline{b}_r}{\overline{y}}, \tag{35}$$

when $\overline{b}_r/\overline{y}$ increases, the real interest rate $\overline{R}\overline{\Pi}^{-1}$ rises to clear the market for government bonds, which mitigates liquidity frictions. If $\overline{R}\overline{\Pi}^{-1}$ is higher than the growth rate γ_z, government expenditures have to be cut with a further rise of $\overline{b}_r/\overline{y}$. As a result, there might exist an optimal debt-to-GDP ratio to balance the benefit and the cost of public liquidity provision.

As illustrated before, there is no money (which is another form of public liquidity in practice). It should be clear now that printing money or printing government bonds is *not* equivalent to providing liquidity. Public liquidity here means real liquidity which has to be eventually backed by primary surplus (the difference between $\tau^y y$ and g) suggested by the government budget constraint (35). If the present value of primary surplus is not enough, then either the nominal price level or the real interest rate adjust which is essentially captured in this paper. For this reason and for simplicity, I mix government bonds and money.

4. The optimal quantities of debt in the long run

The net effect of government debt on welfare is hard to be shown analytically. This section illustrates the optimal choice of government debt by numerical simulations. In particular, I calibrate the model to the data with long run targets $\overline{g}$, $\overline{\tau}^y$, $\overline{R}$, and $\overline{\Pi}$ which give rise to a particular debt-to-GDP ratio. Then, I discuss the welfare gains/losses with different debt-to-GDP ratios.

4.1. Calibration

The model is calibrated in quarterly frequency to match several US long-run statistics. The calculation of steady state can be found in Appendix A.

The relative risk-aversion, the depreciation rate, and the discount factor are standard (see Table 2). Capital share α targets 16% investment-to-GDP ratio. Investment includes physical investment and intermediation search costs. I choose μ such that the hours worked are 25% of total hours. 6% of household members are entrepreneurs ($\chi=0.06$) and have investment projects every quarter, which is the number to match investment spikes observed from U.S. manufacturing plants in Doms and Dunne (1998) and Cooper et al. (1999).

Table 2
Baseline calibration.

Parameter		Baseline value	Target/source
Preferences and production technology			
Household discount factor	β	0.9900	Exogenous
Relative risk aversion	σ	2	Exogenous
Utility weight on leisure	μ	6.1555	Working time: 25%
Utility weight on *G*	ψ_g	0.4586	Maximizing the steady-state welfare
Mass of entrepreneurs	χ	0.0600	Doms and Dunne (1998)
Depreciation rate of capital	δ	0.0250	Exogenous
Capital share of output	α	0.3880	Investment-to-GDP ratio: 16%
Growth rate	γ_z	1.0045	Annualized growth rate: 1.8%
Search and matching			
Supply elasticity	η	0.5000	Tobins $q=1.0600$
Matching efficiency	ξ	0.1146	Saleability $\overline{\phi}=0.1904$
Search costs	κ	0.0080	Liquidity premium = 91.6 basis points
Price stickiness			
Markup	ζ	0.1667	Elasticity of substitution: 7
Calvo probability	ψ	0.7500	Frequency of adjusting price: 4 quarters
Government financing			
Government expenditures	$\overline{g}$	0.1521	$\overline{g}/\overline{y}=0.22$
Government tax	$\overline{\tau}^y$	0.2203	$\overline{b}_r/\overline{y}=0.60$
Inflation	$\overline{\Pi}$	1.0025	Annualized inflation = 1%
Nominal interest rate	$\overline{R}$	1.0075	Annualized real interest rate = 2%

$\zeta=1/6$ such that the elasticity of substitution among intermediate goods is 7, a common value used in the New Keynesian literature. I set the Calvo parameter $\psi=0.75$, which is another conventional value and implies that on average firms change price every year.

The annual growth rate of the US per capita GDP is about 1.8% so that $\gamma_z=1.0045$. I set the steady-state government expenditures share of GDP to be 22%, which is in line with the post-war sample. I use 2% annualized real return on government bonds because I do not distinguish government bonds with different maturities. Further, inflation is set to 1% which implies a nominal interest rate of 3% in the steady state.

Debt in the data corresponds to government bonds held by the private sector. I follow the calculation method in Del Negro et al. (2011), and the US flow-of-funds data points to about 60% debt-to-GDP ratio from 1950 to 2015. I also follow their target $\overline{\phi}=0.19$ which seems to match the average turnover of non-government issued assets in the flow-of-funds data. The steady-state intermediation cost κ generates an annual liquidity premium (Δ^{LP}) of 91 basis points, in line with previous studies of non-defaultable components of corporate spreads in Longstaff et al. (2005). I set $\eta=1/2$ so that the asset price is $\overline{q}^i=1.06>1$ and entrepreneurs are indeed financing constrained. Then, $\overline{\tau}^y=22.03\%$ is pinned down by the government budget constraint (33).

The last parameter ψ_g is related to maximizing the social welfare. Note that the steady-state social utility is proportional to $U(\overline{c})-\mu\overline{N}+\psi_g U(\overline{g})$. The parameter ψ_g is set such that the calibrated $\overline{g}$ maximizes the social welfare with the calibrated debt-to-GDP ratio. I discuss the details in the following.

4.2. The optimal debt-to-GDP ratio

To illustrate the optimal level of debt, I fix the tax rate $\overline{\tau}^y$ and plot the welfare gains in terms of equivalent consumption goods relative to the economy with $b_r/y=0$ (Fig. 1). I also plot consumption, investment, government expenditures, and output as a function of b_r/y. These variables are also normalized to be 100% when $b_r/y=0$.

A higher debt-to-GDP ratio (public liquidity provision) drives up the real interest rate and reduces the return difference between liquid and illiquid assets (see the liquidity premium). Note that the "percentage spread", $\left(\overline{q}^n-\overline{q}^i\right)/\overline{q}^i$, in general also decreases with the public liquidity provision. That is, public liquidity provision mitigates the financing constraints faced by the entrepreneurs, and capital accumulation thus increases with a higher debt-to-GDP ratio.

With more public liquidity, workers search less private liquidity such that the equilibrium asset saleability falls. But at the same time entrepreneurs are less financing constrained, and the representative household is thus more willing to consume. As consumption rises with the debt-to-GDP ratio, leisure increases (hours worked fall) because they are complements.

One interesting feature is that output is non-monotone. The drop of labor hours reduce output initially, but output rises with the debt-to-GDP ratio because the cut in expenditures "crowds in" investment and production.

Importantly, although the public liquidity provision mitigates financial frictions, the cut in government expenditures will eventually reduce the social welfare with the rise of debt-to-GDP ratio (for a given ψ_g). The consumption-equivalence welfare gains are about 1.1% if the debt-to-GDP ratio is 0.6 (the calibrated level) and about 0.6% if the debt-to-GDP ratio is 1.0. Therefore, the parameter ψ_g is chosen such that welfare is maximized at the calibrated debt-to-GDP ratio.

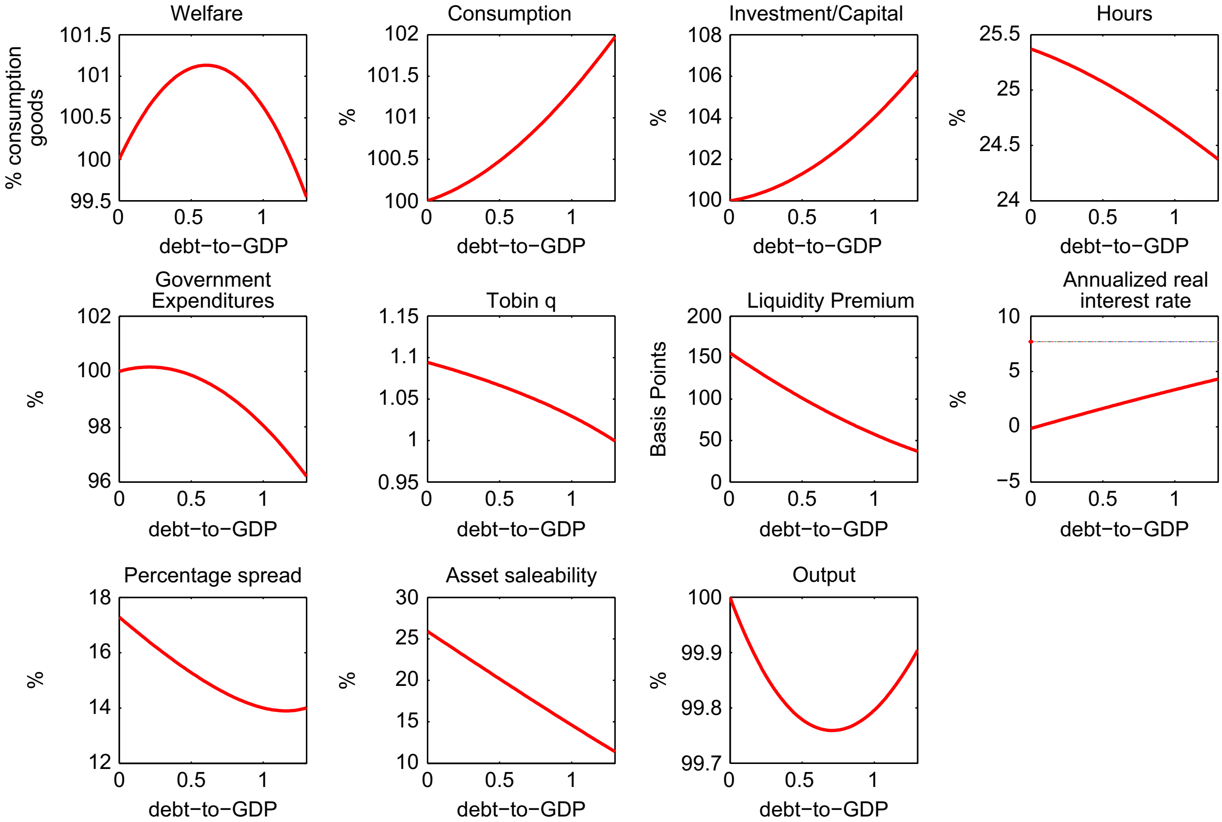

Fig. 1. Welfare and debt-to-GDP ratios.

To better understand this, I discuss two cases. In the first case, the real interest rate $\overline{R\Pi}^{-1}$ is smaller than or equal to the growth rate γ_z; the other has the opposite feature. When $\overline{R\Pi}^{-1} \le \gamma_z$, government expenditures can be financed relatively easily (recall Eq. (35)), as the size of the economy grows at a faster speed than the real interest rate. Therefore, government expenditures increase with the debt-to-GDP ratio.

When $\gamma_z < \overline{R\Pi}^{-1} \le \beta^{-1}\gamma_z^{\sigma}$, a higher debt-to-GDP ratio implies that the government expenditures share of GDP has to fall. Note that the dotted line in the panel of (net) annualized real interest rate corresponds to $400(\beta^{-1}\gamma_z^{\sigma}-1)$, and the (gross) real interest rate never exceeds $\beta^{-1}\gamma_z^{\sigma}$. As the debt-to-GDP ratio rises, the government expenditures decrease at a faster speed and finally overturn the welfare improvement from public liquidity provision. Therefore, the social welfare peaks at the calibrated debt-to-GDP ratio (i.e., 0.60), while the expenditures peaks when the debt-to-GDP ratio is around 0.25.

In sum, facing liquidity frictions, the monetary authority prefers a higher real interest rate and public liquidity provision, as it will attenuate liquidity frictions. Nevertheless, financing government expenditures becomes more costly with a higher real interest rate. The fiscal authority thus prefers a lower interest rate. The conflicts then imply both an optimal real interest rate and an optimal level of government debt that balance the tradeoff.

5. Equilibrium dynamics and monetary–fiscal interactions

I have illustrated the monetary–fiscal interactions in the steady state. This section presents equilibrium responses to shocks under different policy rules and examines further the interactions. I use AR(1) processes for productivity shocks and financial shocks:

$$\ln(A_t) = \rho_A \ln(A_{t-1}) + \epsilon_{A,t}, \quad 0<\rho_A<1$$
$$\ln \kappa_t = \rho_\kappa \ln(\kappa_{t-1}) + (1-\rho_\kappa)\ln\kappa + \epsilon_{\kappa,t}, \quad 0<\rho_\kappa<1$$

where $\epsilon_{A,t} \sim N(0,\sigma_A^2)$ (IID over time), $\epsilon_{\kappa,t} \sim N(0,\sigma_\kappa^2)$ (IID over time), and the two innovations are uncorrelated. TFP shocks directly affect the efficiency of production, while financial shocks directly affect the cost of intermediation (search and matching) and the financial market.

First, I study a special and simple class of rules in (24), in which the monetary authority sets the nominal interest rate only as a function of inflation, while the fiscal authority adjusts taxes in response to the level of real government debt

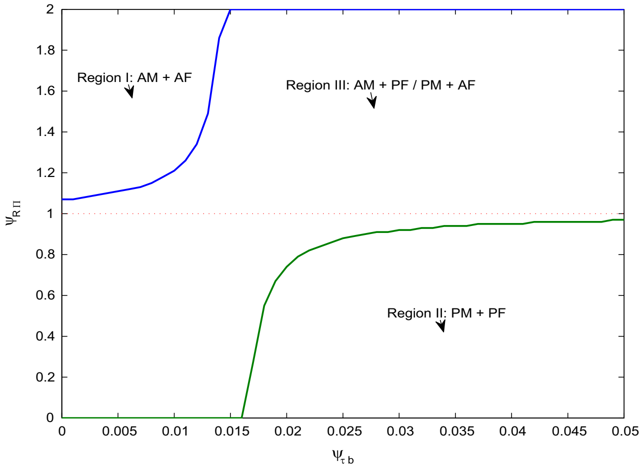

Fig. 2. Regime division: The division in policy parameter $(\psi_{\tau b}, \psi_{R\Pi})$ space. "A" and "P" stand for "active" and "passive", while "M" and "F" stand for "monetary policy" and "fiscal policy". In region I, no locally stable equilibrium exists. In region II, a locally stable equilibrium is indeterminate. In region III, a unique locally stable equilibrium exists.

outstanding. The simple rules are fairly standard in the literature. Then, I move to discuss the scenario when the government is able to follow the full set of rules in (24).

5.1. A simple class of monetary and fiscal rules

Suppose government expenditures are fixed at the steady-state level and the interest rate policy only responds to inflation. That is,

$$\psi_{gb} = 0, \quad \psi_{gy} = 0, \quad \psi_{Ry} = 0$$

and only $\psi_{\tau b}$ and $\psi_{R\Pi}$ are allowed to be non-zero. The policy parameter pair $(\psi_{\tau b}, \psi_{R\Pi})$ then determines whether a locally stable equilibrium around the steady state exists and whether it is unique. For discussion simplicity, I focus on $\psi_{\tau b} > 0$ and $\psi_{R\Pi} > 0$, which are suggested by actual policies.

I log-linearize the system,[7] and then I check the number of eigenvalues that lie outside the unit circle against the number of forward looking variables. When the two numbers are the same, a unique stable equilibrium exists (see Blanchard and Kahn, 1980). When the number of forward looking variables is larger, a locally stable equilibrium is not determined. When the number of eigenvalues that lie outside the unit circle is larger, a stable equilibrium does not exist. The policy parameter space $(\psi_{\tau b}, \psi_{R\Pi})$ can then be divided into three disjoint regions (Fig. 2).

In region III, we have a unique saddle-path equilibrium. When both $\psi_{R\Pi}$ and $\psi_{\tau b}$ are large, one can think of the case in which the monetary authority actively pursues price stability and the fiscal authority adjusts tax strongly to accommodate. That is, the fiscal policy obeys the constraints imposed by the economy and the monetary authority. This could generate a unique stable equilibrium. When both $\psi_{R\Pi}$ and $\psi_{\tau b}$ are small, one can think of the case in which the fiscal authority does not respond to the level of debt strongly, preventing deficit from being financed entirely with future taxes, and the monetary authority accommodates by not responding to inflation strongly. This could also generate a unique stable equilibrium.

In region I, $\psi_{R\Pi}$ is relatively large and $\psi_{\tau b}$ is relatively small (compared to those in region III). The monetary authority actively responds to inflation, while the fiscal authority does not generate sufficient tax revenues when the level of real government debt is high. This produces too many unstable roots in the log-linearized system, and we do not have a stable equilibrium. In contrast, in region II, $\psi_{R\Pi}$ is relatively small and $\psi_{\tau b}$ is relatively large (compared to those in region III). The monetary authority does not actively responds to inflation, while the fiscal authority generates relatively large tax revenues when the level of real government debt is high. This produces too few unstable roots, and the system is thus indeterminate.

This division is similar to Leeper (1991), who points out that both monetary policy and fiscal policy can be labeled as either *active* or *passive*. Being active means that the authority is forward looking and is not constrained by the current government budget constraint; it is free to choose a rule. Being passive means that it is backward looking and is constrained by the active authority and the private agents' behaviors. An economy system has a locally unique stable equilibrium if and only if one authority is active and the other authority is passive.

Therefore, region I is the active monetary–active fiscal (AM + AF) regime, while region II is the passive monetary–passive fiscal (PM + PF) regime. Region III combines both active monetary–passive fiscal (AM + PF) and passive monetary–active fiscal (PM + AF) regimes.

[7] Details available upon request.

The regime division in Fig. 2 features non-linear boundaries. When $\psi_{\tau b}$ increases, the boundary $\psi_{R\Pi}$ of region I also increases. That is, when the fiscal authority is more responsive to debt burden (as $\psi_{\tau b}$ increases), the monetary authority can set an even higher interest rate when inflation rises and still maintain a stable equilibrium. This intuition is also reflected in the boundary $\psi_{R\Pi}$ of region II, which also increases with $\psi_{\tau b}$. That is, when the fiscal authority is more responsive to debt burden, the monetary authority *has to* pay more attention to inflation; otherwise, both authorities balance the government budget such that a locally stable equilibrium is not uniquely determined.

The reason for such non-linear region boundaries is that liquidity frictions together with sticky prices generate non-trivial inflation and liquidity premium dynamics after shocks. The monetary–fiscal interaction is more complicated than a model without liquidity frictions. Next, I discuss the details.

5.2. Simple policy rules and shocks

I now assign values to productivity shocks and financial shocks. To calibrate the two shocks, it is reasonable to impose the economy under an active monetary–passive fiscal rule

$$\psi_{R\Pi} = 1.5, \quad \psi_{\tau b} = 0.015$$

As is common in the New Keynesian literature, $\psi_{R\Pi} = 1.5$ is a usual number which captures the experience of the post-war US economy. Given $\psi_{R\Pi} = 1.5$, the lowest possible $\psi_{\tau b}$ is about 0.015 in the AM + PF regime.

To compare productivity and financial shocks, the persistence and standard deviation of the underlying shock processes target the volatility (0.02) and the first-order correlation (0.91) of GDP's cyclical components (HP filtered with a smoothing coefficient of 1600). When using only productivity shocks, I obtain

$$\rho_A = 0.92, \quad \sigma_A = 0.0088$$

When focusing on shocks to intermediation costs only, the exercise yields

$$\rho_\kappa = 0.91, \quad \sigma_\kappa = 0.09$$

I use these parameters in the subsequent numerical simulations. By design, both shocks will generate very similar aggregate output dynamics, and one can thus focus on the differences in the paths of other variables. Because of the log-linearization, the risk premium is removed and the premium Δ_t^{LP} captures only liquidity premium.

Now, consider a one standard deviation drop in $\epsilon_{A,t}$ (negative TFP shocks) and a one standard deviation increase in $\epsilon_{\kappa,t}$ (adverse financial shocks that raise intermediation costs). Both shocks can generate many features of a recession: a drop of output, consumption, and investment. Compared to TFP shocks, financial shocks generate “flight to liquidity”, which is reflected in the rise of liquidity premium. Financial shocks imply that outside financing is more costly, and therefore having liquid assets (as a hedge for future investment) becomes more attractive. Then, demand for private claims falls, and entrepreneurs find it even harder to search for potential buyers.

On the contrary, when there are only negative TFP shocks, the return from investment is persistently low and the need for investment drops. Therefore, negative TFP shocks reduce liquid assets' hedging value for future investment (given that there are no financial shocks). The difference can be clearly seen in the liquidity premium dynamics after the two shocks. Adverse financial shocks push up liquidity premium, while negative TFP shocks do the opposite.

After the financial shocks, flight to liquidity pushes up liquidity premium which dominates the increase of marginal product of capital such that asset price falls. This channel avoids the rise of asset price generated by an exogenous fall of ϕ as noted by Bigio (2012) and Shi (2015). TFP shocks, however, generate smaller declines in asset saleability in the initial periods. This is because a persistent lower TFP, albeit pushing down liquidity premium, reduces rental rates of capital and also generates persistent falls in the demand for private claims and asset price.

Note that asset price and asset liquidity affect investment, since entrepreneurs use the financial market to leverage their net worth for investment projects. The drop of asset price q^i and saleability ϕ tightens entrepreneurs' financing constraints. That is why financial shocks have a significant impact on investment (with a 5% initial drop) compared to TFP shocks (with a 3% initial drop).

Together with the flight to liquidity after adverse financial shocks, inflation falls sharply to 0.15% and the monetary authority lowers the nominal interest rate from 3% to 1.8%. On the contrary, negative TFP shocks generate higher inflation rates (to 1.5% initially) like in many other New Keynesian models, as total resources become scarce and nominal prices have to go up. Observing a higher inflation rate, the monetary authority raises the nominal interest rate to 3.7%. Since the real interest rate rises and the total tax revenues fall with output persistently, the growth rate of government debt (see the “Debt Growth” panel in Fig. 3) is persistently higher than the steady-state level to finance government expenditures.[8]

Note that the consumption movement after financial shocks is affected by nominal rigidities. In the previous literature without sticky prices, investment and consumption usually move in opposite directions after negative financial shocks (Kiyotaki and Moore, 2012 and Shi, 2015). The drop of output is small and only limited to the forgone capital accumulation. With nominal rigidities, “flight to liquidity” reduces inflation and increases real wages. Intermediate goods firms thus find it

[8] The growth rate of government debt supply is $\frac{B_t}{B_{t-1}} = \frac{B_t/P_t}{B_{t-1}/P_{t-1}}\Pi_t = \frac{B_{r,t}}{B_{r,t-1}}\Pi_t = \frac{b_{r,t}}{b_{r,t-1}}\gamma_z\Pi_t$.

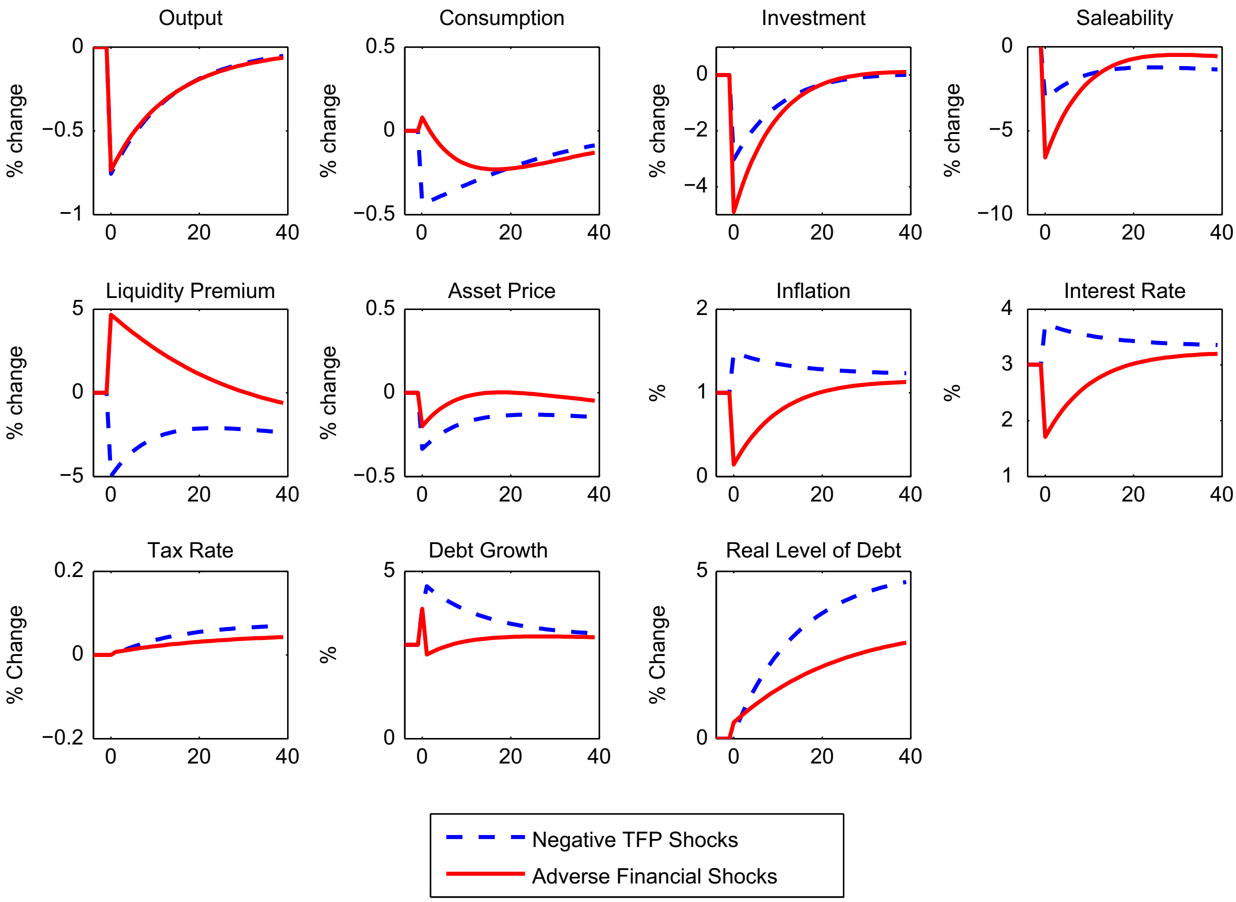

Fig. 3. Impulse responses after one standard deviation innovations in TFP and intermediation costs.

optimal to reduce labor hours. Then, output falls immediately together with a sharp decline of investment. That is why consumption does not need to rise and move in the opposite direction of investment (except for the first two periods as we have a moderate degree of price stickiness).

After financial shocks, workers invest more in liquid assets which are not backed by real investment. Therefore, capital accumulation significantly slows down and the total net worth of the economy falls, which further exacerbates the financing situation of entrepreneurs in the future. Anticipating this, workers further fly to liquidity which causes persistent reduction of investment, consumption, and output.

5.3. *The impact of zero lower bound after financial shocks*

The previous exercise suggests that the nominal interest rate could drop below zero, when adverse financial shocks are large enough to generate a high deflationary pressure. Nevertheless, the zero lower bound (ZLB) seems to be a crucial technical constraint, as agents will prefer to hold money if the nominal rate is below zero. In my model, the ZLB will be important as product prices are sticky. If constrained by the ZLB, which is a hard constraint,[9] the nominal interest rate can further affect policies and the real economy activities.

To illustrate, I modify the simple interest rate rule in the previous section to

$$R_t = \max\left\{0, \quad \overline{R}\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\psi_{R\Pi}}\right\}$$

When the financial shocks are large, i.e., a four standard deviation increase in $\epsilon_{\kappa,t}$, the ZLB constrains the nominal interest

[9] Several central banks, including the Swiss National Bank and the Bank of Japan, have recently implemented negative nominal policy rates. In practice, slightly negative nominal rates are possible due to reasons such as regulations on holding government bonds. Nevertheless, large negative policy rates are unlikely, and the ZLB should be interpreted as some negative lower bound that is not far away from zero.

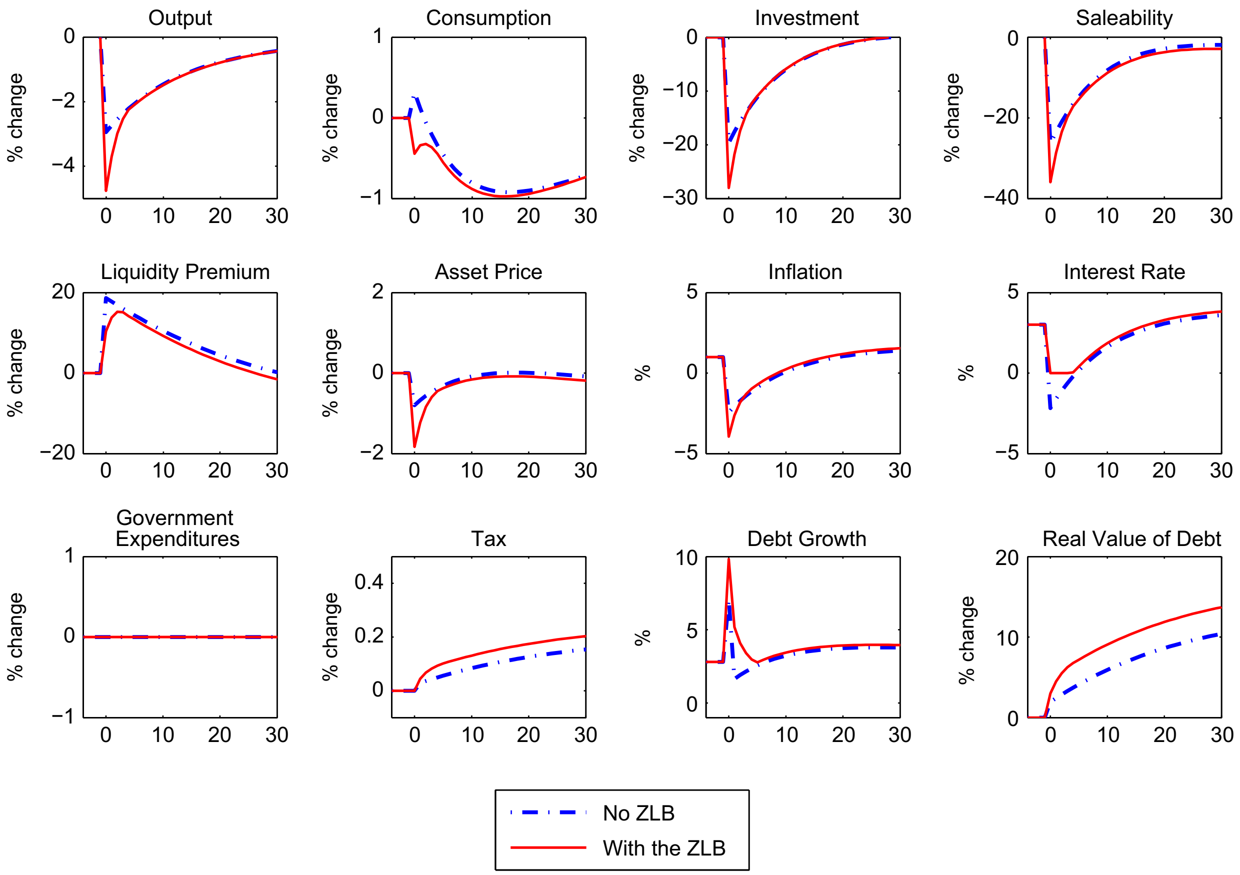

Fig. 4. Large financial shocks and the zero lower bound. Note: The figure shows equilibrium responses after a four standard deviation increase in $\epsilon_{\kappa,t}$.

rate at zero for almost one year after the shocks (Fig. 4).[10] The dash-dot lines are the equilibrium responses in the absence of ZLB, while the sold lines are the counterparts when the ZLB is respected.

A monetary authority following the simple Taylor-type rule in (24) would have lowered the nominal interest rate to almost -2%, thereby inducing a fall in the real interest rate from the steady-state level 2% into the negative territory. In contrast, when the ZLB is respected, the nominal interest rate is stuck at zero for one year. Consumption, investment, and output fall by another 0.75%, 11%, and 2% compared to the case when the ZLB is not in presence.

Importantly, the ZLB amplifies the effect of financial shocks: the constraint is binding in a given period, and agents expect it to be binding in the future. This belief lowers expected future income and net worth and generates deflationary expectations. Such expectation leads to a rise in real rates and a fall in demand. Therefore, workers prefer to save in liquid government bonds and reduce consumption further. This again reduces the demand for private claims and their saleability, and entrepreneurs find it even harder to finance investment projects. As a result, asset price drops significantly more if the ZLB binds.

Note that the ZLB brings up the real interest rate when inflation falls, and the fiscal authority needs to raise more taxes and issue more government debt to finance the interest payments. This effect further distorts the economy. Liquidity premium increases less when the ZLB binds, reflecting that the fiscal authority finds it more costly to finance government expenditures.

The above exercise complements the studies by Eggertsson and Krugman (2012) and Buera and Nicolini (2014) in which they argue that the hit of ZLB is because of tightening of borrowing constraints. Here, adverse financial shocks are similar to the tightening of borrowing constraints as issuing and/or reselling assets are harder and more costly. Agents fly to liquidity, reduce aggregate demand, and push down inflation. Then, the monetary authority significantly reduces the nominal interest rate, which could stay at zero for a long period of time.

5.4. Optimal simple and sophisticated policy rules

Finally, I also let government expenditures and the nominal interest rate to respond as specified in (24). I label this type of policy rules as the *sophisticated rules*, in order to distinguish from the *simple rules* in previous analysis in which $\psi_{gb}=\psi_{gy}=\psi_{Ry}=0$.

[10] I use the algorithm in Guerrieri and Iacoviello (2015) to solve the model with the ZLB.

Table 3
Optimal policy parameters in response to financial shocks, given $\psi_{\tau b} = 0.015$.

Rule\parameters	$\psi^*_{R\Pi}$	ψ^*_{Ry}	ψ^*_{gb}	ψ^*_{gy}
The simple rules	2.71	0	0	0
The sophisticated rules	3.10	0.21	−0.32	−0.24

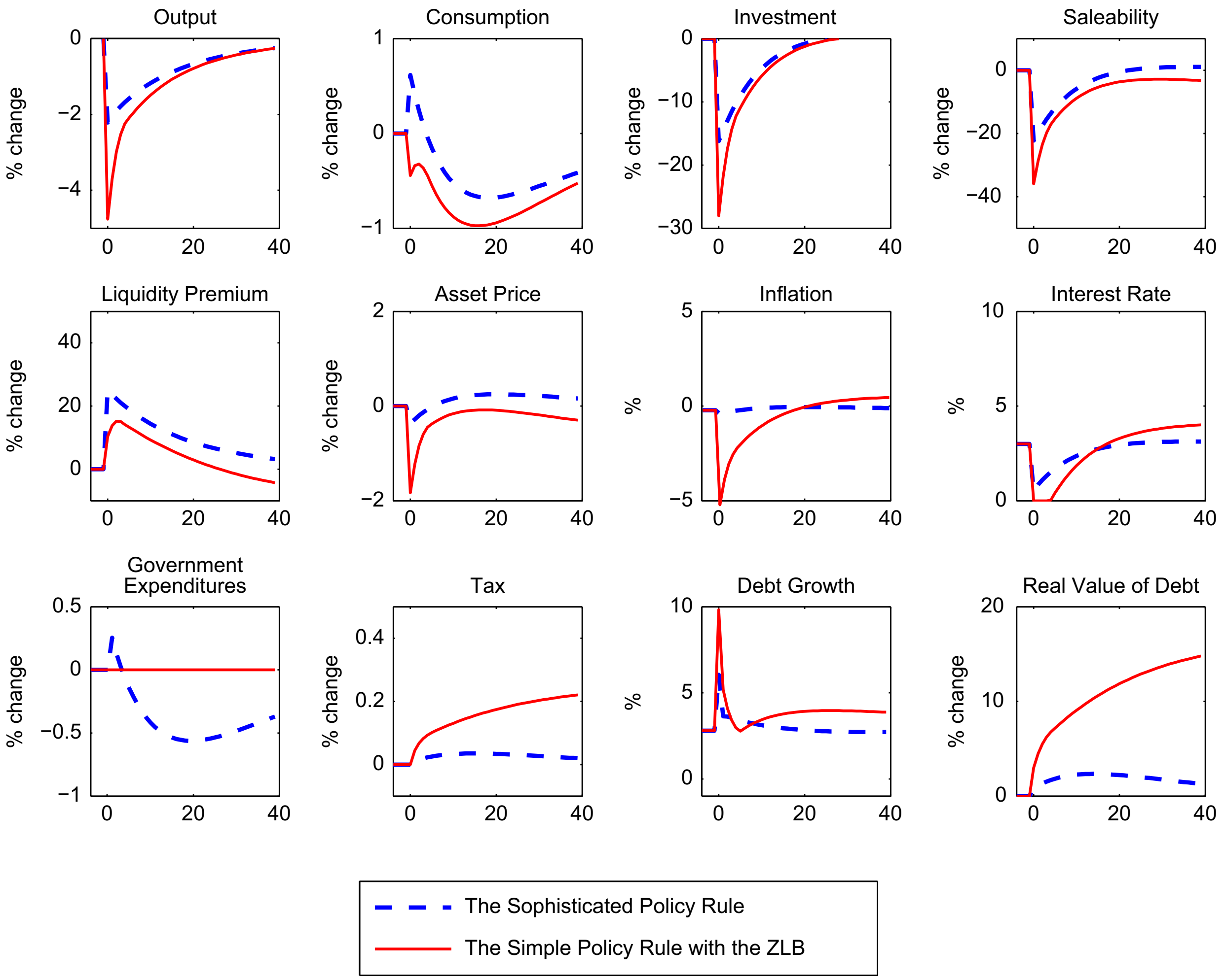

Fig. 5. Optimal mix of policies after large financial shocks. Note: The figure shows equilibrium responses after a four standard deviation increase in $\epsilon_{\kappa,t}$.

In particular, I search for the optimal sophisticated rules. In doing so, I first approximate the average household utility by a second-order Taylor expansion (see Appendix C), which depends on the deviation of output, inflation, consumption, investment, and government expenditures from their steady-state levels.[11] Then, I search the optimal policy parameters $\psi^*_{R\Pi}$, ψ^*_{Ry}, ψ^*_{gb}, and ψ^*_{gy}, which maximize the social welfare subject to the log-linearized system.

Since the unconditional expectation of average household utility can be expressed as the weighted unconditional variances of output, inflation, consumption, investment, and government expenditures, I search for the coefficients which minimize the unconditional variances. As a comparison, I also compute the optimal policy parameters when the government is constrained to the simple rules. The difference can be seen from the optimal policy parameters in Table 3.

When government expenditures can be adjusted in response to financial shocks, the fiscal authority increases spending when output falls ($\psi^*_{gy} = -0.24 < 0$) and reduces spending when the real level of debt becomes large ($\psi^*_{gb} = -0.32 < 0$). In

[11] When I approximate the utility function, I ignore the ZLB because of the complication of computing second-order derivatives. However, because of the optimal mix of monetary and fiscal policies, the nominal interest rates will not hit the ZLB even after a four standard deviation shock.

general, the fiscal policy becomes more passive or more accommodating, as the changes of government debt is much larger than the changes of output (see Fig. 5). That is why the monetary authority can be more responsive to inflation to reduce the interest payments on government debt (i.e., $\psi^*_{R\Pi}$ increases from 2.71 to 3.10).

Fig. 5 shows the equilibrium dynamics after a four standard deviation drop in $\epsilon_{\kappa,t}$, under the simple rules with $\psi_{R\Pi} = 1.5$ (solid lines) and the optimal sophisticated rules (dashed lines) respectively. The monetary authority under both policy rules react strongly to inflation. But the sophisticated policy rules, with more fiscal instruments, stabilize most macro variables. One also sees that inflation is stable under the sophisticated policy rules, which is particularly welfare-enhancing. As inflation is stabilized, nominal rates never touch the ZLB, and the distortion of nominal price dispersion arising from nominal price rigidity is thus negligible.

Under the sophisticated rules, the government implements fiscal contraction after adverse financial shocks, except for the fiscal expansion in the first two periods. To do so, government expenditures increase initially (as output falls on impact and $\psi^*_{gy} < 0$), and then decrease persistently below the steady-state level (as the real debt is higher than the steady-state level and $\psi^*_{gb} < 0$). The initial rise of government expenditures generates inflation due to the price stickiness environment through the standard New-Keynesian demand channel. The initial 0.4% increase of government expenditures avoids almost 3% drop of output under the simple rules.

One might view that fiscal expansion is always helpful in stabilizing inflation, since it can raise aggregate demand when inflation is low. This point is argued by Christiano et al. (2011) and Woodford (2011) when nominal interest rates are at the ZLB. Nevertheless, anticipated fiscal contraction can also raise aggregate demand when entrepreneurs are financing constrained. Agents expect that future fiscal contraction gives back resources to entrepreneurs, thus relaxing financing constraints and raising the demand for investment in the future. Workers are more willing to search for investment projects, relaxing entrepreneurs' financing constraints today. Therefore, entrepreneurs can invest more today, and households accumulate more capital and find themselves richer in the future, which further encourages workers to search for investment projects today.

That is, in the simulation, the fiscal contraction's effect dominates the fiscal expansion's effect in raising demand, as endogenous financing constraints are powerful in amplification. The important consequence of this transmission is that inflation is stabilized even if the economy is hit by large financial shocks.

The monetary–fiscal interactions can be further seen from the difference in the dynamics of real debt. Under the simple rules, as the ZLB constrains nominal interest rates, the deflationary pressure drives up the real debt burden significantly. Recall the government budget constraint

$$g + \frac{R_{-1} b_{r,-1}}{\Pi} = \tau^y y + \gamma_z b_r$$

and we know that a fall in Π together with a fixed g implies that τ^y needs to increase. Although the fall of inflation implies that there is more public liquidity, the fall is unnecessarily large such that the distortionary taxation increases significantly as in Fig. 5. The rise of government debt is thus *involuntary*.

Under the sophisticated policy rules, however, inflation is stable and real interest rates drop after shocks. That is why liquidity premium increases more than that under the simple policy rules while the real debt burden only increases slightly. The tradeoff between the benefit of public liquidity provision and the cost of government financing is less severe when agents fly to liquidity and government expenditures fall. The increase of the real value of debt is mostly from the *voluntary* drop of government expenditures, instead of from the drop of nominal price levels. By using fiscal contraction, the government maintains a healthy increase of public liquidity provision. As a result, the rise in distortionary taxation is moderate (Fig. 5).

In sum, the monetary authority is more active than what the simple rules imply. However, it is unlikely constrained by the ZLB even if the economy is hit by very large financial shocks. Although the government is constrained by policy rules in the model, a careful mix of monetary and fiscal policies can avoid the deep recession generated by liquidity frictions and the ZLB.

Remark: I do not analyze the optimal taxation in these policy experiments. First, fixing the tax rule is relatively simple. Perhaps, in practice, it is indeed hard to change taxation within a short period of time, while adjusting government expenditures is easier. Second, if the fiscal authority can adjust tax rules, one might view that it is better to provide real liquidity right after shocks by promising higher tax rates in the future. However, it is unlikely that such policy is better than the sophisticated rules, since it requires a higher level of distortionary taxation and is subject to more severe commitment issues.

6. Conclusion

I illustrate the importance of monetary–fiscal interactions with endogenous liquidity frictions. In particular, the tradeoff between the benefit of public liquidity provision and the cost of financing government expenditures implies an optimal long-run supply of government debt. In combating large adverse financial shocks, the proper mix of monetary and fiscal policies can avoid the zero lower bound on nominal interest rates under a simple Taylor rule.

For simplicity, the paper assumes government debt as a blend of non-interest and interest bearing liabilities. Future research can give an explicit role for fiat currency and/or reserves of central banks. At the same time, I assume “private claims” as an amalgam of privately issued equity and debt. One could further analyze a more realistic capital structure and the implication for monetary–fiscal interactions. Finally, further work could also focus on the government commitment issues (for example illustrated in Bassetto, 2005) when government debt provides liquidity services.

Acknowledgment

I am deeply grateful to the editor Eric M. Leeper, one anonymous associate editor, and three anonymous referees for helpful comments. I thank Marco Bassetto, Nobu Kiyotaki, Morten Ravn, Ricardo Reis, Mirko Wiederholdt for helpful discussions, and participants at numerous seminars and conferences for useful comments. I also acknowledge financial supports from both the ESRC funded CfM (Centre for Macroeconomics, grant reference ES/K002112/1)) at University College London and the European Union's Horizon 2020 project ADEMU (A Dynamic Economic and Monetary Union, grant agreement N° 649396).

Appendix A. The transformed economy

Along the balanced growth path, all quantity variables (except N_t) and the wage rate grow at the same rate γ_z, while interest rate, inflation, and asset price are constants. Therefore, I deflate all quantity variables by Z_t. In the following, I collect all competitive equilibrium conditions for (c_t, x_t, N_t, y_t, k_t, $b_{r,t}$ ρ_t, ϕ_t, q_t^n, q_t^i, w_t, r_t, Π_t, λ_t, h_t^1, h_t^2), given state variables (k_{t-1}, $b_{r,t-1}$, R_{t-1}, κ_t, A_t, Z_t) and policy rules (g_t, τ_t^y, R_t).

1. Optimal conditions from households:

$$c+q^n\gamma_z k+\gamma_z b_r = wN+\left[\chi\rho+(1-\chi)q^n\right](1-\delta)k_{-1}+\left[\chi\rho+(1-\chi)\right]\left[rk_{-1}+\frac{R_{-1}b_{r,-1}}{\Pi}+(1-\tau^y)(1-\lambda)y\right] \tag{36}$$

$$c_t^{-\sigma}(1-\tau_t^y)\lambda_t(1-\alpha)y_t = \mu N_t \tag{37}$$

$$\mathbb{E}_t\left[\frac{\beta\gamma_z^{-\sigma}c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\frac{\left[\chi\rho_{t+1}+(1-\chi)\right]r_{t+1}+(1-\delta)\left[\chi\rho_{t+1}+(1-\chi)q_{t+1}^n\right]}{q_t^n}\right]=1 \tag{38}$$

$$\mathbb{E}_t\left[\frac{\beta\gamma_z^{-\sigma}c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\frac{R_t}{\Pi_{t+1}}(\chi\rho_{t+1}+1-\chi)\right]=1 \tag{39}$$

$$x_t=\chi\frac{\left[r_t+q_t^i\phi_t(1-\delta)\right]k_{t-1}+\dfrac{R_{t-1}}{\Pi_t}b_{r,t-1}+(1-\tau_t^y)(1-\lambda)y_t}{1-\phi_t q_t^i} \tag{40}$$

2. The financial market in equilibrium:

$$\rho_t=\frac{(1-\phi_t)q_t^n}{1-\phi_t q_t^i}, \tag{41}$$

$$q_t^i = 1+\kappa_t\left[1+\frac{\eta\xi^{\frac{1}{\eta-1}}}{1-\eta}\phi_t^{\frac{\eta}{1-\eta}}(1-\phi_t)-\frac{1}{\phi_t}\right] \tag{42}$$

$$\kappa_t\left[\phi_t^{-1}+\phi_t^{\frac{\eta}{1-\eta}}\xi^{\frac{1}{\eta-1}}\right]=q_t^n-q_t^i \tag{43}$$

3. Optimal conditions from firms:

$$r_t=\frac{(1-\tau_t^y)\lambda_t\alpha y_t}{k_{t-1}},\quad w_t=\frac{(1-\tau_t^y)\lambda_t(1-\alpha)y_t}{N_t} \tag{44}$$

$$h_t^1=\lambda_t y_t+\mathbb{E}_t\left[\frac{\beta\psi\gamma_z^{-\sigma}c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\left(\frac{\Pi_{t+1}}{\overline{\Pi}}\right)^{\frac{1+\zeta}{\zeta}}h_{t+1}^1\right] \tag{45}$$

$$h_t^2=y_t+\psi\mathbb{E}\left[\frac{\beta\psi\gamma_z^{-\sigma}c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\left(\frac{\Pi_{t+1}}{\overline{\Pi}}\right)^{\frac{1}{\zeta}}h_{t+1}^2\right] \tag{46}$$

$$1=(1-\psi)\left[(1+\zeta)\frac{h_t^1}{h_t^2}\right]^{-1/\zeta}+\psi\left[\frac{\Pi}{\Pi_t}\right]^{-1/\zeta} \tag{47}$$

4. Market clearing:

$$y_t=A_t k_{t-1}^{\alpha} N_t^{1-\alpha} \tag{48}$$

$$x_t=\gamma_z k_t-(1-\delta)k_{t-1} \tag{49}$$

$$c_t+g_t+x_t+\kappa_t\left(1+\xi^{\frac{1}{\eta-1}}\phi_t^{\frac{1}{1-\eta}}\right)[x_t+\chi(1-\delta)k_{t-1}]=y_t \tag{50}$$

Next, I illustrate how to compute the optimal steady-state debt-to-GDP ratio, by fixing the tax rate $\overline{\tau}^y$. Note that we do not need to know the steady-state inflation and nominal interest rate separately. Let $\tilde{R}_+\equiv R/\Pi_+$ be the real interest rate. Given a $\overline{b}_r/\overline{y}$ (which will be chosen optimally later), I guess an asset saleability $\overline{\phi}$ and know that $\overline{f}=\overline{\phi}^{\frac{\eta}{\eta-1}}\xi^{\frac{1}{1-\eta}}$. Then, I solve $\overline{q}^n$, $\overline{q}^i$, and $\overline{\rho}$ from (41), (42), and (43)

$$\overline{q}^i=1+\kappa\left[1+\frac{\eta\xi^{\frac{1}{\eta-1}}}{1-\eta}\overline{\phi}^{\frac{\eta}{1-\eta}}\left(1-\overline{\phi}\right)-\frac{1}{\overline{\overline{\phi}}}\right]$$

$$\overline{q}^n=\overline{q}^i+\kappa\left(\frac{1}{\overline{\overline{f}}}+\frac{1}{\overline{\overline{\phi}}}\right)$$

$$\overline{\rho}=\frac{\overline{q}^n}{\overline{q}^r}=\frac{\overline{q}^n(1-\overline{\phi})}{1-\overline{\phi}\overline{q}^i}$$

The steady-state real interest rate $\overline{\tilde{R}}$ and the (after-tax) capital return $\overline{r}$ are obtained from (38) and (39):

$$\overline{\tilde{R}}=\frac{\gamma_z^{\sigma}}{\beta[\chi\overline{\rho}+1-\chi]}$$

$$\overline{r}=\frac{\overline{q}^n\gamma_z^{\sigma}/\beta-(1-\delta)[\chi\overline{\rho}+(1-\chi)\overline{q}^n]}{\chi\overline{\rho}+(1-\chi)} \tag{51}$$

With the knowledge of $\overline{\tilde{R}}$, we know from the government budget constraint (implied by Walras law) that

$$\frac{\overline{g}}{\overline{y}}=\overline{\tau}^y+(\gamma_z-\overline{\tilde{R}})\frac{\overline{b}_r}{\overline{y}}$$

Using the knowledge of Cobb–Douglas production function, I compute the steady-state capital-to-output ratio

$$\frac{\overline{k}}{\overline{y}}=\frac{\alpha(1-\overline{\tau}^y)\lambda}{\overline{r}}$$

We are ready to check the initial guess of $\overline{\phi}$, since (40) and (49) imply that the capital-to-output ratio satisfies

$$\frac{\overline{k}}{\overline{y}}=\chi\frac{\overline{\tilde{R}}\frac{\overline{b}_r}{\overline{y}}+\chi(1-\lambda)(1-\overline{\tau}^y)}{\left[(1-\overline{\phi}\overline{q}^i)\tilde{\delta}-[\overline{r}+\overline{\phi}\overline{q}^i(1-\delta)]\chi\right]}$$

If yes, the correct $\overline{\phi}$ is found; if not, $\overline{\phi}$ should be adjusted.

Now, I compute $\overline{c}/\overline{y}$ from the goods market clearing condition (50)

$$\frac{\overline{c}}{\overline{y}}=1-\frac{\overline{g}}{\overline{y}}-\left[\tilde{\delta}+\kappa\left(\frac{\overline{\phi}}{\overline{f}}+1\right)\left[\tilde{\delta}+(1-\delta)\chi\right]\right]\frac{\overline{k}}{\overline{y}}$$

With the tax rate, the steady-state (after-tax) wage rate is $\overline{w}=(1-\overline{\tau}^y)\lambda(1-\alpha)\left[\frac{\overline{r}}{\alpha(1-\overline{\tau}^y)}\lambda\right]^{\frac{\alpha}{\alpha-1}}$, and from the labor supply condition (37) we know that

$$\overline{c}=\left[\frac{(1-\overline{\tau}^y)\lambda(1-\alpha)\left[\frac{\overline{r}}{\alpha(1-\overline{\tau}^y)}\lambda\right]^{\frac{\alpha}{\alpha-1}}}{\mu}\right]^{\frac{1}{\sigma}},\quad \overline{y}=\overline{c}/\frac{\overline{c}}{\overline{y}},\quad \overline{g}=\frac{\overline{g}}{\overline{y}}\overline{y}$$

Therefore, the labor supply is

$$\overline{N} = \frac{(1-\overline{\tau}^y)\lambda(1-\alpha)\overline{y}}{\overline{w}}$$

Finally, I compute the steady-state social welfare. The per-period utility is given by

$$Z^{1-\sigma}\left(\frac{c^{1-\sigma}}{1-\sigma} - \mu N + \frac{g^{1-\sigma}}{1-\sigma}\right)$$

Then, given an initial Z_0, one knows that $Z_t = Z_0(\gamma_z)^t$ and the steady-state welfare becomes

$$(1-\beta\gamma_z^{1-\sigma})^{-1} Z_0^{1-\sigma}\left(\frac{\overline{c}^{1-\sigma}}{1-\sigma} - \mu\overline{N} + \frac{\overline{g}^{1-\sigma}}{1-\sigma}\right)$$

I pick the $\overline{b}_r/\overline{y}$ that maximizes the above steady-state welfare.

Appendix B. Proofs

B.1. Price setting of intermediate goods firms

I follow the proof of Ascari (2004). When firm i is setting the price at time t, it takes into account the fact that its price grows with the steady-state inflation $\overline{\Pi}$, if it is not allowed to change the price in the future. Therefore, the per-period profit is $P_{i,t}\Pi^s Y_{i,t+s}/P_{t+s} - MC_{i,t+s}Y_{i,t+s}$ where $MC_{i,t+s}$ is the real marginal cost. By using the fact that $Y_{i,t+s} = Y_{t+s}\left(\frac{P_{i,t}\overline{\Pi}^s}{P_{t+s}}\right)^{-\frac{1+\zeta}{\zeta}}$, I write down the problem of the firm as

$$\max_{\{P_{it}\}} \mathbb{E}_t\left[\sum_{s=0}^{\infty}\psi^s\Delta_{t,t+s}\left[\left(\frac{P_{i,t}\overline{\Pi}^s}{P_{t+s}}\right)^{-\frac{1}{\zeta}} Y_{t+s} - MC_{i,t+s}Y_{i,t+s}\right]\right]$$

where $\Delta_{t,t+s}$ is the stochastic discount factor between time t and time $t+s$, and $MC_{i,t+s} = \lambda_{t+s}$ since we are looking at the symmetric equilibrium. The optimal price $P_{it}^* = P_t^*$ is

$$P_t^* = (1+\zeta)\frac{\mathbb{E}_t\sum_{s=0}^{\infty}\psi^s\Delta_{t,t+s}\lambda_{t+s}\left(\frac{P_{t+s}}{\overline{\Pi}^s}\right)^{\frac{1+\zeta}{\zeta}}Y_{t+s}}{\mathbb{E}_t\sum_{s=0}^{\infty}\psi^s\Delta_{t,t+s}\left(\frac{P_{t+s}}{\overline{\Pi}^s}\right)^{\frac{1}{\zeta}}Y_{t+s}}$$

Notice that $P_{t+s} = \frac{P_{t+s}}{P_{t+s-1}}\frac{P_{t+s-1}}{P_{t+s-2}}\dots\frac{P_{t+1}}{P_t}P_t$ and $P_{t+s}/P_t = \Pi_{t+s}\Pi_{t+s-1}\dots\Pi_{t+1}$, and I can simplify the above identity to

$$\frac{P_t^*}{P_t} = (1+\zeta)\frac{\mathbb{E}_t\sum_{s=0}^{\infty}\psi^s\Delta_{t,t+s}\lambda_{t+s}\left(\frac{\Pi_{t+s}\Pi_{t+s-1}\dots\Pi_{t+1}}{\overline{\Pi}^s}\right)^{\frac{1+\zeta}{\zeta}}Y_{t+s}}{\mathbb{E}_t\sum_{s=0}^{\infty}\psi^s\Delta_{t,t+s}\left(\frac{\Pi_{t+s}\Pi_{t+s-1}\dots\Pi_{t+1}}{\overline{\Pi}^s}\right)^{\frac{1}{\zeta}}Y_{t+s}}$$

Finally, let $H_t^1 = \lambda_t Y_t + \psi\mathbb{E}\left[\Delta_{t+1}\left(\frac{\Pi_{t+1}}{\overline{\Pi}}\right)^{\frac{1+\zeta}{\zeta}}H_{t+1}^1\middle|\Gamma_t\right]$ and $H_t^2 = Y_t + \psi\mathbb{E}\left[\Delta_{t+1}\left(\frac{\Pi_{t+1}}{\overline{\Pi}}\right)^{\frac{1}{\zeta}}H_{t+1}^2\right]$. I can rewrite the above equation recursively as $P_t^*/P_t = (1+\zeta)H_t^1/H_t^2$ as in the main text.

B.2. Proof of optimal (ϕ, q^i) pair

Since $\frac{\kappa}{f}+\frac{\kappa}{\phi} = q^n - q^i$, then $1-\phi q^i = \kappa\left(1+\frac{\phi}{f}\right)+1-\phi q^n$ and

$$q^r = \frac{1-\phi q^i}{1-\phi} = \frac{\kappa\left(1+\frac{\phi}{f}\right)+1-\phi q^n}{1-\phi}$$

To minimize q^r, I derive the first-order condition with respect to ϕ:

$$\frac{dq^r(\phi)}{d\phi} = \frac{(1-\phi)\left[\frac{\kappa}{(1-\eta)f} - q^n\right] + \left[\kappa\left(1+\frac{\phi}{f}\right)+1-\phi q^n\right]}{(1-\phi)^2} = 0 \tag{52}$$

where I have used the fact that $f = \xi^{\frac{1}{1-\eta}}\phi^{\frac{\eta}{\eta-1}}$. Rearranging, I obtain

$$q^n = 1+\kappa\left[1+\frac{1-\phi}{(1-\eta)f}+\frac{\phi}{f}\right] \tag{53}$$

which implies that

$$q^i = q^n - \frac{\kappa}{f} - \frac{\kappa}{\phi} = 1 + \kappa\left[1 + \frac{\eta(1-\phi)}{(1-\eta)f} - \frac{1}{\phi}\right]$$

as shown in the main text. I also check the second-order condition to ensure minimization (available upon request).

B.3. Proof of Proposition 1:

If $\kappa > 0$ and private claims and government bonds co-exist, $\overline{\rho} = 1 + \chi^{-1}\left[\left(\beta\overline{\tilde{R}}\right)^{-1}\gamma_z^{\sigma} - 1\right] > 1$ from the Euler equation for bonds (39). The steady-state liquidity premium can be written as

$$\overline{\Delta}^{LP} = \chi\overline{r}^{ni} + \frac{(1-\chi)\overline{r}^{nn}}{\overline{\rho}} + (1-\chi)\left(1-\overline{\rho}^{-1}\right)\overline{r}^{nn} - \overline{\tilde{R}} = \overline{\rho}^{-1}\beta^{-1}\gamma_z^{\sigma} - \overline{\tilde{R}} + (1-\chi)\left(1-\overline{\rho}^{-1}\right)\overline{r}^{nn}$$
$$= \overline{\rho}^{-1}(\chi\overline{\rho} + 1 - \chi)\overline{\tilde{R}} - \overline{\tilde{R}} + (1-\chi)\left(1-\overline{\rho}^{-1}\right)\overline{r}^{nn} = (1-\chi)(1-\overline{\rho}^{-1})\left(\overline{r}^{nn} - \overline{\tilde{R}}\right) = (1-\chi)(1-\overline{\rho}^{-1})\left(\overline{r}/\overline{q}^n + 1 - \delta - \overline{\tilde{R}}\right)$$

where I have used (38) in the second equality. Further using (26) and (27), we know that

$$(\chi\overline{\rho} + 1 - \chi)\overline{r} + [\chi\overline{\rho} + (1-\chi)\overline{q}^n](1-\delta) = \beta^{-1}\gamma_z^{\sigma}\overline{q}^n = \overline{\tilde{R}}(\chi\overline{\rho} + 1 - \chi)\overline{q}^n$$

Therefore, $\overline{r}/\overline{q}^n = \overline{\tilde{R}} - \frac{\left[\frac{\chi\overline{\rho}}{\overline{q}^n} + (1-\chi)\overline{q}^n\right]}{\chi\overline{\rho} + 1 - \chi}(1-\delta)$ and the steady-state liquidity premium becomes

$$\overline{\Delta}^{LP} = (1-\chi)\left(1-\overline{\rho}^{-1}\right)\left(1 - \delta - \frac{\left[\frac{\chi\overline{\rho}}{\overline{q}^n} + (1-\chi)\overline{q}^n\right]}{\chi\overline{\rho} + 1 - \chi}(1-\delta)\right) = (1-\chi)\left(1-\overline{\rho}^{-1}\right)(1-\delta)\frac{\chi\overline{\rho}}{\chi\overline{\rho} + 1 - \chi}\left(1 - \frac{1}{\overline{q}^n}\right) > 0.$$

B.4. Proof of Proposition 2:

The spread is $\Delta^s = q^n - q^i = \kappa\left(\frac{1}{\phi} + \frac{1}{f}\right)$ and the partial derivative

$$\frac{\partial\Delta^s}{\partial\phi} = \kappa\left[-\frac{1}{\phi^2} + \frac{\eta}{1-\eta}\frac{1}{\phi f}\right]$$

where I have used the fact that $f = \xi^{\frac{1}{1-\eta}}\phi^{\frac{\eta}{\eta-1}}$. Then, $\frac{\partial\Delta^s}{\partial\phi} < 0$ is equivalent to

$$\frac{1}{\phi} - \frac{\eta}{(1-\eta)f} > 0 \tag{54}$$

Since $0 < \phi < 1$, we know that if $\frac{\partial\Delta^s}{\partial\phi} < 0$ then

$$\frac{\partial q^i}{\partial\phi} = \frac{\eta}{1-\eta}\left[\frac{\eta}{1-\eta}\frac{1-\phi}{\phi f} - \frac{1}{f}\right] + \frac{1}{\phi^2} = \frac{\eta^2}{(1-\eta)^2}\frac{1-\phi}{\phi f} + \frac{1}{\phi^2} - \frac{\eta}{(1-\eta)f} > 0$$

by using (54).

Appendix C. Second-order approximation of welfare

Let $\tilde{z}$ denotes the log-deviation of z_t from its steady-state level $\overline{z}$. That is, $z_t = \overline{z}e^{\tilde{z}_t}$. Define

$$\tilde{\beta} \equiv \beta\gamma_z^{1-\sigma}$$

so that $\tilde{\beta}$ is the discount factor when we detrend all variables. Take the Taylor expansion of utility of consumption about its steady state

$$\frac{c_t^{1-\sigma} - 1}{1-\sigma} \approx \frac{\overline{c}^{1-\sigma} - 1}{1-\sigma} + \overline{c}^{1-\sigma}\left[\tilde{c}_t + \frac{(1-\sigma)}{2}\tilde{c}_t^2\right]$$

Similarly, I derive

$$\psi_g\frac{g_t^{1-\sigma} - 1}{1-\sigma} \approx \psi_g\frac{\overline{g}^{1-\sigma} - 1}{1-\sigma} + \psi_g\overline{g}^{1-\sigma}\left[\tilde{g}_t + \frac{(1-\sigma)}{2}\tilde{g}_t^2\right], \quad \mu N_t \approx \mu\overline{N} + \mu\overline{N}\left(\tilde{N}_t + \frac{1}{2}\tilde{N}_t^2\right)$$

Notice that labor demand from individual firm i is $l_{it}=\frac{\lambda(1-\alpha)Y_{it}}{\hat{W}_t}=\frac{y_{it}}{A_t(k_{t-1}/N_t)^{\alpha}}$ and $y_{it}=y_t\left(\frac{P_{it}}{P_t}\right)^{-\frac{1+\zeta}{\zeta}}$, then

$$N_t=\frac{y_t}{A_t(k_{t-1}/N_t)^{\alpha}}\int\left(\frac{P_{it}}{P_t}\right)^{-\frac{1+\zeta}{\zeta}}di,\quad (1-\alpha)\tilde{N}_t=\tilde{h}_t+\tilde{\Sigma}_t$$

where $h_t\equiv y_t/A_t/k_{t-1}^{\alpha}$ and $\Sigma_t\equiv\log\int\left(\frac{P_{it}}{P_t}\right)^{-\frac{1+\zeta}{\zeta}}di$. Therefore, I can rewrite the second order approximation of period utility as

$$U(c_t,N_t,g_t)\approx-\frac{\mu\overline{N}}{1-\alpha}\left[\tilde{h}_t+\tilde{z}_t+\frac{1}{2(1-\alpha)}\tilde{h}_t^2\right]+\overline{c}^{1-\sigma}\left[\tilde{c}_t+\frac{(1-\sigma)}{2}\tilde{c}_t^2\right]++\psi_g\overline{g}^{1-\sigma}\left[\tilde{g}_t+\frac{(1-\sigma)}{2}\tilde{g}_t^2\right]+t.i.p$$

where $t.i.p$ indicates terms that are independent of policy (note: the steady-state government targets are fixed).

Further, I define $y_t^e\equiv c_t+g_t$ as the total expenditures of private consumption and government expenditures, and take the second-order approximation of y_t^e

$$\theta_c\tilde{c}_t+\frac{1}{2}\theta_c(\tilde{c}_t)^2+\theta_g\tilde{g}_t+\frac{1}{2}\theta_g\tilde{g}_t^2=\tilde{y}_t^e+\frac{1}{2}(\tilde{y}_t^e)^2$$

where θ_c and θ_g denote the consumption share and the government-expenditure share of total expenditures. After substituting $\tilde{g}_t$, I rewrite $U_t=U(c_t,N_t,g_t)$

$$U_t\approx-\frac{\mu\overline{N}}{1-\alpha}\left[\frac{\left(\tilde{h}_t+1-\alpha\right)^2}{2(1-\alpha)}+\tilde{\Sigma}_t\right]+\overline{c}^{1-\sigma}\left[\tilde{c}_t+\frac{(1-\sigma)}{2}\tilde{c}_t^2\right]+\frac{\psi_g\overline{g}^{1-\sigma}}{\theta_g}\left[\frac{(1-\sigma)\theta_g}{2}\tilde{g}_t^2+\tilde{y}_t^e+\frac{1}{2}(\tilde{y}_t^e)^2-\frac{1}{2}\theta_g\tilde{g}_t^2-\theta_c\tilde{c}_t-\frac{1}{2}\theta_c\tilde{c}_t^2\right]+t.i.p$$

$$=-\frac{\mu\overline{N}}{1-\alpha}\left[\frac{\left(\tilde{h}_t+1-\alpha\right)^2}{2(1-\alpha)}+\tilde{\Sigma}_t\right]-\frac{\frac{\theta_c}{\theta_g}\psi_g\overline{g}^{1-\sigma}+\overline{c}^{1-\sigma}(\sigma-1)}{2}\left[\frac{\tilde{c}_t-\overline{c}^{1-\sigma}-\frac{\theta_c}{\theta_g}\psi_g\overline{g}^{1-\sigma}}{\frac{\theta_c}{\theta_g}\psi_g\overline{g}^{1-\sigma}}+\overline{c}^{1-\sigma}(\sigma-1)\right]^2$$
$$-\frac{\psi_g\overline{g}^{1-\sigma}\sigma}{2}\tilde{g}_t^2+\frac{\psi_g\overline{g}^{1-\sigma}}{2\theta_g}\left[\tilde{y}_t^i+1\right]^2+t.i.p$$

I approximate $\tilde{\Sigma}_t\approx\frac{(1+\zeta)}{2\zeta}var_i\{\log P_{it}\}$ by following the proof in Gali and Monacelli (2005). Further, following Woodford (2003), we have

$$\sum_{t=0}^{\infty}\tilde{\beta}^t var_i\{\log P_{it}\}=\frac{\psi}{(1-\psi)(1-\tilde{\beta}\psi)}\sum_{t=0}^{\infty}\tilde{\beta}^t\tilde{\Pi}_t^2$$

Therefore, the second-order approximation of total utility of the representative household is

$$\mathbb{E}_0\sum_{t=0}^{\infty}\tilde{\beta}^tU_t\approx-\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\tilde{\beta}^t\{\lambda_h\left(\tilde{h}_t+1-\alpha\right)^2+\lambda_{\Pi}\tilde{\Pi}_t^2+\lambda_c\left[\frac{\tilde{c}_t-\overline{c}^{1-\sigma}-\frac{\theta_c}{\theta_g}\psi_g\overline{g}^{1-\sigma}}{\frac{\theta_c}{\theta_g}\psi_g\overline{g}^{1-\sigma}}+\overline{c}^{1-\sigma}(\sigma-1)\right]^2+\lambda_g\tilde{g}_t^2+\lambda_y(\tilde{y}_t^e+1)^2\}+t.i.p.$$

$$\lambda_h=\frac{\mu\overline{N}}{(1-\alpha)^2},\quad\lambda_{\Pi}=\frac{\mu\overline{N}\psi(1+\zeta)}{(1-\alpha)(1-\psi)(1-\tilde{\beta}\psi)\zeta},$$
$$\lambda_c=\frac{\theta_c}{\theta_g}\psi_g\overline{g}^{1-\sigma}+\overline{c}^{1-\sigma}(\sigma-1)\quad\lambda_g=\psi_g\overline{g}^{1-\sigma}\sigma,\quad\lambda_y=-\frac{\psi_g\overline{g}^{1-\sigma}}{\theta_g}$$

A welfare-maximizing policy maximizes the utility function above. In addition, an optimal policy that commits to the rules specified in the main text minimizes the variance of $\tilde{h}_t$, $\tilde{\Pi}_t$, $\tilde{c}_t$, $\tilde{g}_t$, and $\tilde{y}_t^e$, with weights λ_h, λ_{Π}, λ_c, λ_g, and λ_y, subject to all equilibrium conditions in Appendix A (in log-linearized forms).

Appendix D. Supplementary data

Supplementary data associated with this paper can be found in the online version at http://dx.doi.org/10.1016/j.euroecorev.2016.03.007.

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