Online Appendix for: Wealth Inequality, Family Background, and Estate Taxation

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1 Appendix

1.1 A Graphic Illustration of the Model's Demographics

Figure 1 displays the demographic structure of our overlapping generations model.

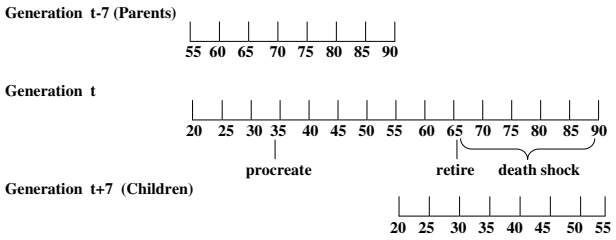

Figure 1: Model Demographics.

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1.2 Definition of Stationary Equilibrium

We focus on a stationary equilibrium concept in which factor prices and age-wealth distribution are constant over time. The collection of all of the agents' possible states is denoted by x. An equilibrium is described as follows.

Definition 1 *A stationary equilibrium is given by government tax rates, transfers, and spending* $(\tau_l, \tau_s, \tau_a, \tau_b,\ x_b, P(\widetilde{y}), G)$*; an interest rate* r *and a wage rate* w*; value functions* $V(x)$*, allocations* $c(x)$*,* $a'(x)$*; and a constant distribution of people* $m^*(x)$*, such that the following conditions hold:*

(i) Given government tax rates and transfers, the interest rate, the wage rate, and defined benefit policies, the functions $V(x),\ c(x)$ *and* $a'(x)$ *solve the described maximization problem for a household in state* x*.*

(ii) m^* *is the invariant distribution of households over the state variables for this economy.*[1]

(iii) All markets clear.

$$C = \int c\, m^*(dx),\ K = \int a\, m^*(dx),\ L = \int \epsilon y\, m^*(dx),$$

$$C + (1+n)K - (1-\delta)\, K + G = F(K; L).$$

(iv) The price of each factor is equal to its marginal product.

$$r = F_1(K, L) - \delta,\ w = F_2(K, L).$$

(vi) The government budget constraints are balanced at each period.

$$G = \tau_a r \int a\, m^*(dx) + \tau_l w L + \int \tau_b (1 - p_t) I_{t>9} \max(a' - x_b, 0) m^*(dx) + \tau_l \int I_{t>9} P(\widetilde{y}) m^*(dx),$$

$$\int I_{t>9} P(\widetilde{y}) m^*(dx) = \tau_s w L.$$

[1] We normalize m^* so that $m^*(X) = 1$, which implies that $m^*(\chi)$ is the fraction of people alive that are in state χ.

1.3 Earnings and Earnings Persistence Calibration

The transition matrix for Q_y is:

$$\begin{bmatrix} 0.8239 & 0.1733 & 0.0027 & 0.000070 \\ 0.2171 & 0.6399 & 0.1428 & 0.000196 \\ 0.0067 & 0.2599 & 0.7332 & 0.000198 \\ 0.1117 & 0.0000 & 0.0794 & 0.808958 \end{bmatrix}.$$

The transition matrix for Q_{yh} is

$$\begin{bmatrix} 0.8272 & 0.1704 & 0.0024 & 0.0000000000 \\ 0.5748 & 0.4056 & 0.0196 & 0.0000000000 \\ 0.2890 & 0.6173 & 0.0937 & 0.0000000005 \\ 0.0001 & 0.0387 & 0.9599 & 0.0012647506 \end{bmatrix}.$$

The transition matrices induce an initial distribution of earnings with probability masses over the respective earnings levels, given by [64.76% 32.80 % 2.44% 0.00006595%].

Table 1 reports the percentage of earnings earned at selected percentiles as a fraction of total earnings generated by the model and also displays the corresponding figures computed from the 1992 SCF observed data for the adult population (calculated by [1], Table 7, p. 845). The model earnings process produces a cross-sectional earnings distribution that is very close to that computed from the SCF data.

1.4 The Social Security Formula and Its Calibration

The Social Security benefit $P(\widetilde{y})$ mimics the Old Age and Survivor Insurance component of the Social Security system and is set as in this formula:

$$P(\widetilde{y}) = 0.9\min(\widetilde{y}, 0.2) + 0.32\max(0, \min(\widetilde{y}, 1.24) - 0.2) + 0.15\max(0, \min(\widetilde{y}, 2.47), 1.24).$$

The bend points are expressed in terms of average earnings and the Social Security earnings cap is $\widetilde{y_c} = 2.47$. The marginal rates of Social Security benefits are taken from [2].

More specifically, their formula applies to an economy with average earnings of one. The bend points are multiplied by average earnings in our model economy to make the formula consistent with our model economy.

1.5 The Calibration of the Exogenous Parameters in the Model

Table 2 reports the exogenous parameters that are not used to match any aspects of the model-generated data.

1.6 Interpreting the Size of the Bequest Motive

To get a sense of the size of the bequest motive, consider a person who starts the period with cash on hand x and dies for sure next period. The budget constraint for such a person is given by $a' = (x - c)$, where a' is savings. The estate net of taxes b is given by

$b = a'$ if $a' < x_b$

$b = (1 - \tau_b)(a' - x_b) + x_b$, if $a' \geq x_b$.

The first-order condition for an interior solution implies that the marginal utility of consumption today equals the appropriately discounted marginal utility of bequests and solves the following problem.

For the case of net bequests in the utility function, the following maximization problem applies

$\max \frac{c^{1-\gamma}-1}{1-\gamma} + \phi_1 \left[(b + \phi_2)^{1-\gamma} - 1 \right]$ with

$b = a'$ if $a' < x_b$ and

$b = (1 - \tau_b)(a' - x_b) + x_b$ if $a' \geq x_b$, which we can solve for b. First, consider the case in which the $a' < x_b$, or in the case of gross bequests in the utility function, then we have

$b = \frac{x - f\phi_2}{1+f}$

with $f = \left(\phi_1 (1 - \gamma) \right)^{-\frac{1}{\gamma}}$.

Then consider the case in which $a' \geq x_b$ and bequests are net in the utility function to obtain

$b = \frac{(1-\tau_b)(x-x_b-f\phi_2)+x_b}{1+f(1-\tau_b)}$.

Since bequests cannot be negative, the expression for b reveals that x has to be large before the person will leave any bequests. If x is not sufficiently large, then $c = x$ and the solutions just derived do not apply. Assuming that x is in fact large enough, the marginal propensity to bequeath out of an extra dollar today is

$\frac{\partial}{\partial x}(b) = \frac{1}{1+f}$ when the estate is below the exemption level, and is

$\frac{\partial}{\partial x}(b) = \frac{(1-\tau_b)}{1+f(1-\tau_b)}$, when the estate is above the exemption level.

For the case in which gross bequests enter the utility function, the following maximization problem applies

$\max \frac{c^{1-\gamma}-1}{1-\gamma} + \phi_1 \left[(x - c + \phi_2)^{1-\gamma} - 1 \right]$.

Hence, we have

$a' = \frac{x-f\phi_2}{1+f}$

with $f = \left(\phi_1(1-\gamma) \right)^{-\frac{1}{\gamma}}$.

In a dynamic model, where the odds of dying in any given period are low, x should be interpreted not as the total stock of wealth, but as its annuity or consumption value. Regarding the point at which the bequest motive kicks in, take ϕ_2, divide it it by the income normalization (which is now 0.2 yearly), and multiply it by \$57,135 to express it in year 2000 dollars.

1.7 Tax Incidence in the Benchmark Economy

To evaluate the distribution of the tax burden and to better understand how tax reforms change it, Tables 3 and 4 report some figures corresponding to people who are, for example, at the top 1%, 1-5%, and so on of the wealth distribution. First, we report the average age in each wealth quantile; because age is an important variable affecting earnings and capital

accumulation, looking at average age helps us to understand the mechanisms behind a given tax burden. Second, in Table 3, we report the fraction of the total revenue from a given tax that is paid by the wealthiest in a given group. Table 3 shows that the wealthiest 1% of people are on average 64.3 years old, hence many of them are retired, and that they pay 35.8% of the total amount of capital income taxes in the economy. Given their age, they only pay 7.7% of the total labor income taxes. Given the high estate taxation threshold that we calibrate, the wealthiest pay 98.8% of the estate taxes and, finally, they pay 15.7% of the total taxes in our economy.

These numbers are interesting also because they indicate that, should we reduce estate taxation and raise earnings taxes to make up for the lost revenue, very little of the earnings tax increase would come from the pockets of the wealthiest. Since some households can become richer because of large labor earnings, as in [1] (and in the data), this is not a foregone conclusion, as they would be faced with a higher tax rate on very large earnings.

The second line of data in this table reports the corresponding figures for households who belong to the wealthiest 1-5%. The households in this group are slightly younger, they pay a much lower, but still large fraction of total capital taxes (18.1%), a very similar fraction of labor income tax, but close to none of the estate taxes because they are younger and less likely to die but also because they are more likely to leave estates smaller than the estate exemption level. The top 5-10% wealthiest households are quite similar to the latter group in terms of the tax burden.

Table 4 looks at the tax burden from a different angle and reports the average amount of a given tax paid by a group, as a fraction of average income in our economy. The first line of data in this table shows that every year, on average, the wealthiest 1% pay an amount of capital income taxes that corresponds to 1.7 times average income, they also pay 1.3 times the average income in labor income taxes, and their estates pay, on average, 0.4 times average income in estate taxes. Scrolling down to the wealthiest 1-5%, we find that their

average tax burden is much lower across the board and totals 0.5 times average income in the economy. The wealthiest 5-10% pay 0.4 times average income in total taxes every year. The last line of data in the table shows the average tax burden of taxation in our benchmark economy. The average person is 51 years old and pays 0.1 times average income in capital taxes, 0.2 times average income in labor taxes, almost no estate taxes, and faces an average tax burden of 21% of average income in the economy.

These numbers also confirm and quantify the expectation that the burden of the labor income tax is more evenly distributed than the capital income tax, while the burden of the estate tax is the most unevenly distributed, with the top 1% paying 99% of the total taxes.

1.8 Distribution of the Tax Burden Effects When the Estate Tax Changes

We now report the tax burdens in the economy with a 55% tax rate on estates, an estate exemption level of $\$675,000$ (which corresponds to the the levels of statutory taxation in place in 2000), and a decreased capital income or labor income tax. All results are reported for the model with bequests net of taxes in the utility function. The tax burdens for the case of gross bequests in the utility function are very similar.

Table 5 reports the average amount of tax paid by a group in this economy with higher estate taxation and can be compared to Table 4, which refers to our benchmark economy. The biggest effect is that, due to a higher estate tax rate and a lower exemption level, the estate tax revenue on the estates left by the decedents in the richest 1% increases from 0.4 to 1.0 times average income, almost tripling. In contrast, the estate tax burden of everyone below the richest 1%, despite the lower estate tax exemption level, changes very little, due to the fact that very few people below the top 1% leave estates of taxable size. The second noticeable feature is that when the capital income tax is lowered, the average capital income tax for the richest 1% goes down from 1.7 to 1.4 times average income. Lastly, for this reform, despite the break coming from reduced capital income taxation, the average tax

burden for the richest 1% is 3.4 times average income in the benchmark economy, but goes up to 3.7 times average income when the capital income tax is lowered and up to 3.8 times when labor income taxation is lowered, due to the fact that the richest save more and thus benefit more from reduced capital income taxation.

1.9 Policy Results When Changing the Labor Income Tax

Tables 6, 7, 8, and 9 report the results for the case in which the labor income tax is adjusted to balance the government budget constraint when changing the estate tax.

References

[1] Ana Castañeda, Javier Díaz-Giménez, and José-Víctor Ríos-Rull. Accounting for U.S. earnings and wealth inequality. *Journal of Political Economy*, 111(4):818–857, 2003.

[2] Mark Huggett and Gustavo Ventura. Understanding why high income households save more than low income households. *Journal of Monetary Economics*, 45(2):361–397, 2000.

		Percentile (%)					
	Gini	1	5	20	40	60	80
SCF	0.63	14.76	31.13	61.39	84.72	97.21	100.00
All models	0.62	14.64	31.93	62.45	84.05	93.00	100.00

Table 1: Percentage of earnings in the top percentiles.

	Parameters		Value
Demographics	n	annual population growth	1.2%
	p_t	survival probability	see text
Preferences	γ	risk aversion coefficient	1.5
Labor earnings	ϵ_t	age-efficiency profile	see text
	ψ	labor earnings levels	see text
	Q_y	labor earnings transition matrix	see text
	ρ_h	AR(1) coef. of prod. inheritance process	0.40
	σ_h^2	innovation of prod. inheritance process	0.37
Production	α	capital income share	0.36
	δ	depreciation	6.0%
Government policy	τ_a	capital income tax	20%
	$P(\widetilde{y})$	Social Security benefit	see text
	τ_s	Social Security tax	12.0%

Table 2: Exogenous parameters used in the benchmark model.

Wealth Percentile	Age	Capital tax	Labor tax	Estate Tax	Total tax
0-1%	64.26	35.79	7.74	98.84	15.65
1-5%	61.80	18.14	7.66	1.16	9.86
5-10%	59.18	14.94	8.87	0.00	10.05

Table 3: Percentage of the total for a given tax paid by a selected wealth percentile.

Wealth Percentile	Age	Capital tax	Labor tax	Estate tax	Total tax
0-1%	64.26	1.69	1.26	0.395	3.35
1-5%	61.80	0.21	0.31	0.001	0.53
5-10%	59.18	0.14	0.29	0.000	0.43
average	50.70	0.05	0.16	0.004	0.21

Table 4: Average amount of a given tax paid by a group, as a fraction of average income in our economy.

Wealth Percentile	Age	Capital tax	Labor tax	Estate tax	Total tax
Changing the capital income tax					
0-1%	64.52	1.39	1.25	1.034	3.67
1-5%	61.70	0.19	0.31	0.006	0.51
5-10%	60.20	0.13	0.28	0.000	0.40
average	50.70	0.04	0.16	0.011	0.21
Changing the labor income tax					
0-1%	64.36	1.60	1.21	1.020	3.83
1-5%	61.79	0.22	0.30	0.006	0.52
5-10%	59.67	0.15	0.28	0.000	0.42
average	50.70	0.05	0.16	0.010	0.21

Table 5: Tax burden (average amount of tax paid by each group, as a fraction of average income in our economy) effects of changing the estate tax rate and exemption level to the year 2000 statutory levels (the estate tax rate is raised to 55% and its exemption level is lowered to $\$675K$).

τ_b	ex_b	τ_l	K	Y	B	K/Y	B/Y	r	wage
Net bequest model, changing the estate tax rate									
0.00	–	0.200	3.148	1.006	0.00887	3.130	0.882	5.531	0.492
0.21	**756K**	**0.192**	**3.100**	**1.000**	**0.00880**	**3.102**	**0.880**	**5.622**	**0.489**
0.40	756K	0.189	3.061	0.996	0.00887	3.075	0.891	5.697	0.487
0.60	756K	0.185	3.039	0.993	0.00899	3.061	0.905	5.741	0.486
Net bequest model, changing the estate tax exemption level									
0.21	219K	0.191	3.085	0.998	0.00888	3.090	0.890	5.651	0.488
0.21	**756K**	**0.192**	**3.100**	**1.000**	**0.00880**	**3.102**	**0.880**	**5.622**	**0.489**
0.21	1095K	0.192	3.103	1.000	0.00879	3.102	0.879	5.617	0.489
Net bequest model, changing both the estate tax rate and the exemption level									
0.55	675K	0.186	3.040	0.993	0.00897	3.061	0.903	5.739	0.486
Gross bequest model, changing the estate tax rate and exemption level									
0.55	675K	0.187	3.023	0.991	0.00868	3.050	0.876	5.774	0.485

Table 6: Aggregate effects of changing the estate tax rate or exemption level, adjusting the labor income tax. The lines in bold refer to our benchmark economy.

				Percentile (%)			
τ_b	ex_b	τ_a	Gini	1	5	20	40
Net bequest model, change the estate tax rate							
0.00	–	0.196	0.811	36.92	53.32	67.22	83.54
0.21	**756K**	**0.192**	**0.804**	**35.15**	**51.90**	**66.09**	**82.89**
0.40	756K	0.189	0.799	33.79	50.78	65.20	82.36
Net bequest model, change the estate tax exemption level							
0.21	219K	0.191	0.805	35.51	52.24	66.39	82.92
0.21	**756K**	**0.192**	**0.804**	**35.15**	**51.90**	**66.09**	**82.89**
0.21	1095K	0.192	0.804	35.11	51.87	66.09	82.91
Net bequest model, change estate tax rate and exemption level							
0.55	675K	0.186	0.795	33.02	50.11	64.64	81.99
Gross bequest model, change estate tax rate and exemption level							
0.55	675K	0.187	0.794	32.52	49.78	64.41	81.84

Table 7: Distribution effects of changing the estate tax rate or exemption level, adjusting the labor income tax. The lines in bold refer to our benchmark economy.

	Moving to parent's earnings		
Parent's earnings	1st	2nd	3rd
Benchmark			
2nd	0.06	-	-
3rd	5.59	5.43	-
4th	35.71	35.50	28.41
Net bequest model, changing labor tax			
2nd	0.06	-	-
3rd	5.53	5.38	-
4th	34.15	33.98	27.12
Gross bequest model, changing labor tax			
2nd	0.06	-	-
3rd	5.49	5.35	-
4th	34.05	33.88	27.05

Table 8: Importance of parental background effects (both bequests and human capital) of changing the estate tax rate and exemption level to the year 2000 statutory levels (the estate tax rate is raised to 55% and its exemption level is lowered to $\$675K$). Asset compensation required for moving from a parental background level to another, normalized as a fraction of average income.

	Moving to parent's earnings		
Parent's earnings	1st	2nd	3rd
Benchmark			
2nd	0.06	-	-
3rd	0.57	0.42	-
4th	14.87	14.71	13.98
Net bequest model, changing labor tax			
2nd	0.06	-	-
3rd	0.56	0.41	-
4th	13.64	13.47	12.80
Gross bequest model, changing labor tax			
2nd	0.06	-	-
3rd	0.55	0.41	-
4th	13.66	13.49	12.82

Table 9: Importance of parental background effects (only bequests, no human capital) of changing the estate tax rate and exemption level to the year 2000 statutory levels (the estate tax rate is raised to 55% and its exemption level is lowered to $\$675K$). Asset compensation required for moving from a parental background level to another, normalized as a fraction of average income.