PRE-PRINT

Teaching and learning mathematics with technology.

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Introduction

The arrival of digital technologies such as computers, mobile phones, tablets (like iPads and Androids) is revolutionising the opportunities for both teachers and learners alike within education in general and within mathematics education in particular. All over the world teachers are looking to develop their personal skills and expertise with technology. 'Technology' includes many things – from computers to mobile devices, from software and applications to web-sites. There is a lot of technology out there and it keeps changing.

In this chapter we are hoping to point you in the right direction so that you can begin to explore a range of technologies that you can use in your mathematics teaching career and beyond. Technology can support both teachers *and* learners.

For *teachers* of mathematics, technology enables us to create (and display or print) our own teaching resources that include accurate mathematical text and diagrams - and share these with other teachers. We are no longer wholly reliant on buying published textbooks and worksheets - we can create our own and adapt them year-by-year to build our own bank of teaching resources. Technology can also help us to keep a digital record of learners' test scores and help us to analyse learners' progress over time – we can use the information to inform what topics we might need to revisit or when to begin to teach the next topic.

For *learners* of mathematics, technology offers an opportunity for them to be introduced mathematical ideas and concepts in completely new ways. Mathematical software and

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applications are available that enable learners to play with mathematical objects in a way that is simply not possible using traditional paper and pencil methods.

In this chapter you will learn about a wide range of technologies that are available to support teaching and learning mathematics.

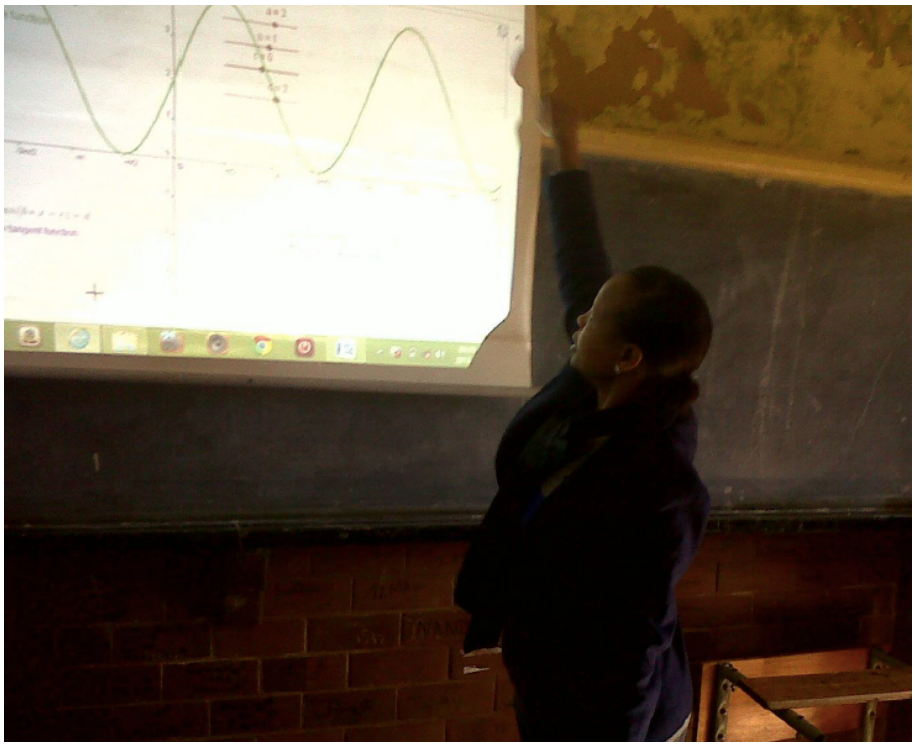


Fig. 1 Maki Nekhavhambe using a data projector for the first time in her classroom. IMAGE1.jpg

Of course, it may not be possible for you to try some of these ideas right away. However as your personal access to technology increases, you can return to this chapter when you are able to try some of the approaches in your own classroom.

Different types of technology

Technology includes a very wide range of devices, software (or applications or 'apps'), websites etc. that have all been designed for many different purposes and audiences. To help you with some of the terminology, we have included a Glossary at the end of the chapter - so if you come across something that is new to you, look it up!

Also, because there are many different technology products that are available, we cannot provide the specific instructions for all of the ideas we offer – you will need to use the individual 'Help' resources and product manuals that accompany the different

technology products - but we hope that you will see the value in making this time investment for yourself.

In addition, there are many online communities and a quick Internet search will often take you to the help you are looking for. The millions of Internet users around the world means that usually someone else has already asked (and published an answer) to the exact question you are asking yourself!

We have organized the chapter by dividing technology into the technologies that might be useful *inside the classroom* with learners and technologies that can be used by you and/or your learners *away from the classroom*.

Technology in the classroom looks at technology for

1) **exploring mathematics** in a) number, b) algebra, c) geometry and d) statistics and probability;
2) **projecting mathematics** such as a) data projectors, b) visualisers and c) webcams.

Technology away from the classroom looks at *creating*

1) **worksheets** by inserting a) symbols and equations, b) tables, c) graphs and d) geometric images.
2) **resources** to project using a) presentation software and b) interactive whiteboard software
3) **electronic markbooks** by a) entering data and then b) making sense of assessment data

and at *online*

4) **communities** for teachers and learners hosted by a) Ministry of education sites b) AimingHigh and c) the African Mathematics Initiative
5) **resources** such as a) Microsoft Maths, b) Everything Maths and Science, c) Khan Academy, d) MathsExcellence and e) Math-Aid.

Technology in the classroom

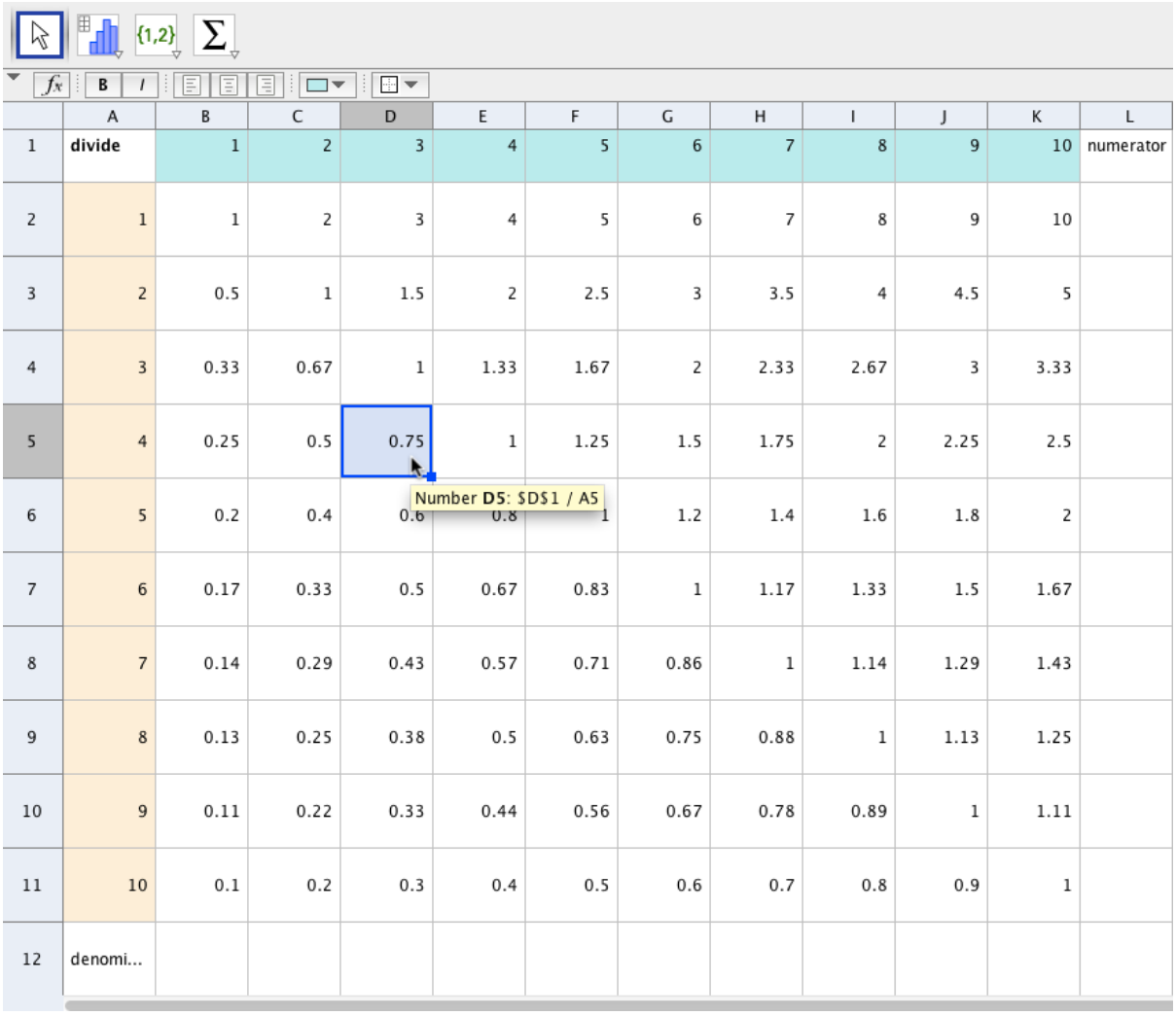
1) Technology for exploring mathematics.

   This is technology that has been created to change the way that we engage in mathematics altogether. Such technologies let us get to the 'nuts and bolts' of mathematics and have the potential to engage many more learners in the more challenging mathematical concepts.

   a) Number

      Spreadsheet software such as a *GeoGebra* spreadsheet or *Microsoft Excel* (which was originally designed for business and accountancy) can be used very creatively to explore number properties. The basic idea is that the columns and rows are labeled with letters and numbers and you use the 'Input' bar to enter text or numbers into the spreadsheet 'cells'. You can then enter data more efficiently by setting up some mathematical rules or relationships! You are not expected to enter each number one by one!

      For example, in Fig. 2 below, which shows a 'division table' for numerators and denominators from 1 to 10, only the values in cells A2 and B1 were entered manually. All of the other numbers were made by entering a formula, which was then copied – either across a row or down a column of data. In Fig. 2, you can see that the value of 3 ÷ 4 (in cell D5) was calculated using the formula = D1/A5. In spreadsheet software the '$" notation is called absolute referencing – this is important if you want to fix the value of a number in a formula.



	A	B	C	D	E	F	G	H	I	J	K	L
1	**divide**	1	2	3	4	5	6	7	8	9	10	numerator
2	1	1	2	3	4	5	6	7	8	9	10	
3	2	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	
4	3	0.33	0.67	1	1.33	1.67	2	2.33	2.67	3	3.33	
5	4	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	
6	5	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	
7	6	0.17	0.33	0.5	0.67	0.83	1	1.17	1.33	1.5	1.67	
8	7	0.14	0.29	0.43	0.57	0.71	0.86	1	1.14	1.29	1.43	
9	8	0.13	0.25	0.38	0.5	0.63	0.75	0.88	1	1.13	1.25	
10	9	0.11	0.22	0.33	0.44	0.56	0.67	0.78	0.89	1	1.11	
11	10	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	
12	denomi...											

Fig. 2 A division table created using a GeoGebra spreadsheet

IMAGE2_Spreadsheet_Division.jpg

The power of a spreadsheet is that, by changing just one number in the spreadsheet (the number in cell A2 from a 1 to a 2), a whole new set of decimal numbers can be created. This creates a very motivating space for pupils to explore fraction to decimal conversions and answer questions such as:

- How many times does the decimal number 0.5 appear? Why?
- Can you predict five different fractions that would have a decimal equivalence of 0.33?

b) Algebra

Many of the 'big ideas' and key concepts in mathematics are related to important generalisations, such as that all functions of the type f(x)=$\boldsymbol{a}$*x+$\boldsymbol{b}$, when plotted on a Cartesian (x-y) graph plane, produce graphs that have a particular geometric feature (a straight line). Most of us will have learned this by rote and believed it

because our teachers (and the text book) told us that it was true. However, with technology, it is possible for the learners to observe the graph features for themselves and, by changing the values of ***a*** and ***b***, discover much more besides. Within a software packages such as GeoGebra, many linear graphs can be explored very quickly, without the rather slower process of plotting the individual functions by hand using paper and pencil. This does not mean that the learners' graph plotting skills are no longer important – but it does mean that the more important knowledge about the effect that varying the vales of ***a*** and ***b*** has on the appearance of the linear graphs can be accessed by all learners. In Fig. 3 shows an exploratory task in GeoGebra that uses 'sliders' to change the values of ***a*** and ***b***.

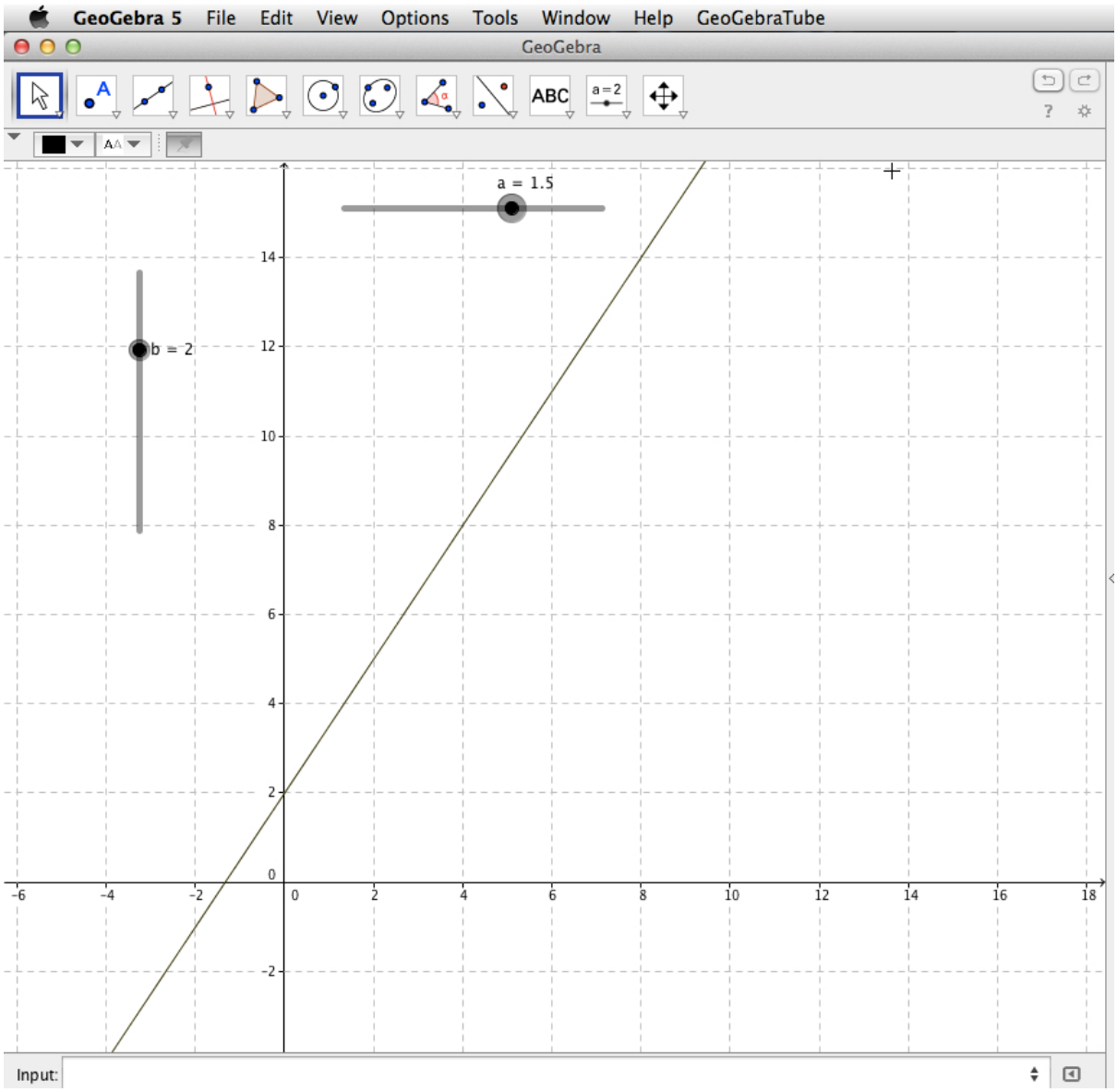


Fig. 3 A GeoGebra file that uses sliders to explore the effect of changing the values on 'a' and 'b' in linear functions given by f(x) = $\boldsymbol{a}$*x+$\boldsymbol{b}$

IMAGE3_Geogebra_Linear_functions.jpg

It is usually not enough to *only* present this file to the learners and 'show' them what happens. It is better to arouse the learners' curiosity by asking questions such as:

- What do you notice about all graphs for which the value of $\boldsymbol{b}$=0? (or 3? Or 5? Or…)
- What do you notice about all graphs for which the value of $\boldsymbol{a}$=3? (or 1.5? or -2? Or…)
- Can you predict what the graph of y = -1x+2 will look like? How do you know?

c) Geometry

Dynamic geometry software has revolutionised the way in which we can understand and explore ideas from geometry. This is because any diagram can be 'interactive' as we can change its appearance by dragging - and explore what happens. We can create mathematically 'robust' 2-D shapes that keep their mathematical properties – even when its vertices and side are transformed by dragging. Figs. 4 and 5 show a set of triangles, each of which has been created in the software such that it retains a particular set of properties. Can you spot which triangle is which (and why?).

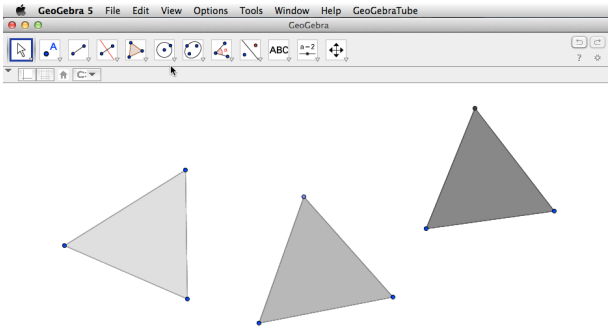


Fig. 4 Three triangles created in Geogebra

IMAGE4_Three_triangles1.jpg

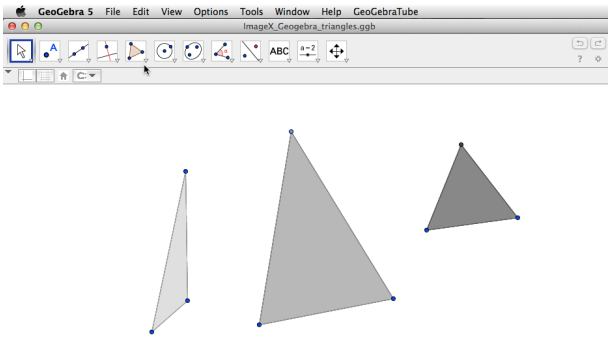


Fig. 5 The three triangles have been transformed by dragging only one of each triangle's vertices!

IMAGE5_Three_triangles2.jpg

You really do need to play with constructing these triangles in the software to appreciate how dynamic geometry environments are different – the equilateral and isosceles triangles were creating using a digital set of geometric construction tools. The 'Circle by center and radius' replaces the protractor and mid-points and perpendicular bisectors are built-in digital tools. As with the graphing example, dynamic geometry software does not replace paper and pencil construction techniques. However it does provide a stimulating environment in which to explore geometric constructions and theorems.

Many countries have GeoGebra Institutes that offer face-to-face meetings/courses and online support for teachers. (see www.geogebra.org/institutes)

d) Statistics and probability

Any data that has been imported into a mathematical tool such as Excel or Geogebra can be analysed. A good source of international data on learners' lifestyles and interest is provided by the Census at School website (http://www.censusatschool.org.uk/). The international section of the website shows the many countries that have taken part – and it is possible to request a randomised data set with which to contrast your own learners' data.

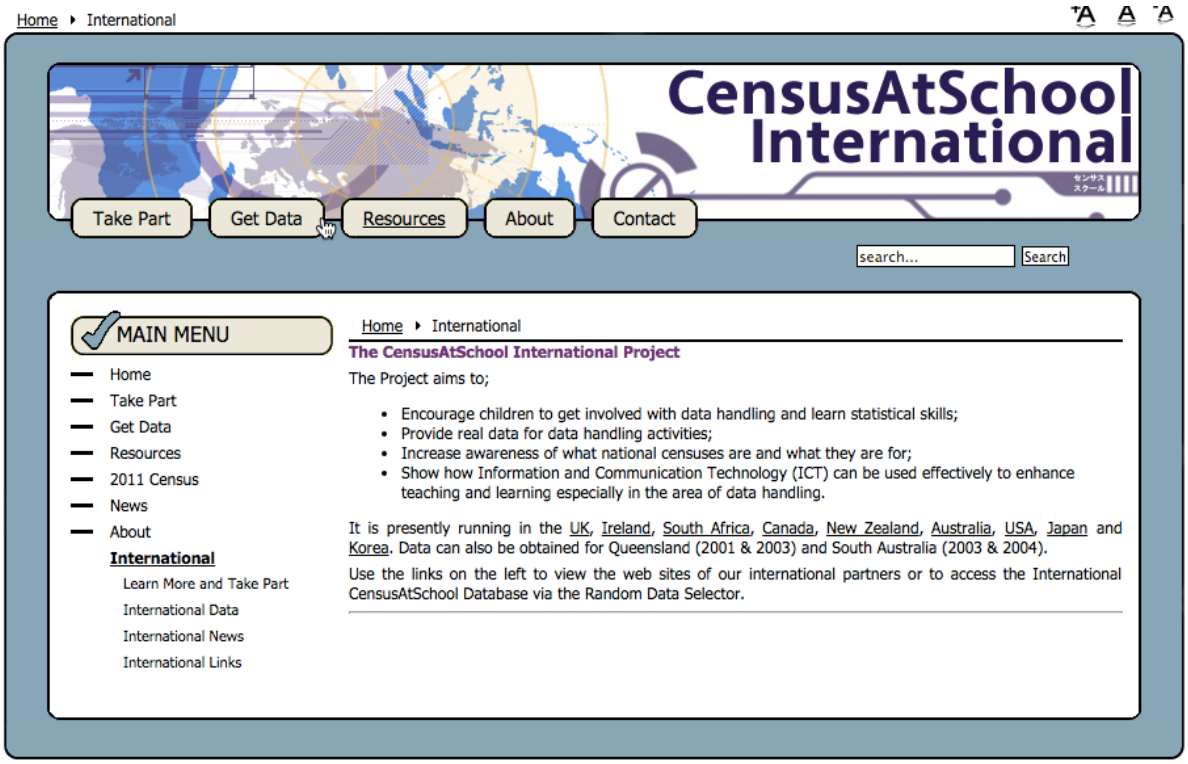


Fig. 6 The international page of the Census at School website.

IMAGE6_Census_at_School.jpg

2) Technology for projecting mathematics

This technology is designed to allow you to project images from your tablet or computer to the class. It is not limited to projecting mathematics. You can use it to project any content.

a) Data projectors

A data projector connects to your computer or tablet and projects its display onto a wall or screen. If you had an interactive whiteboard, it would be possible to operate the software on your computer or tablet by touching this projected image and annotate on top using 'electronic writing'.

b) Visualisers

A visualiser is a high resolution camera on a stand that connects to a data projector and, when you place objects, learners' work or a tablet screen underneath it, this image is projected onto the wall display. This is particularly useful when you are doing practical demonstrations as the learners can see your hands!

c) Web cams

Some computers and most tablets have a built-in webcam, which is a tiny pin-sized camera, usually found on the side of the screen. When the camera is switched on and you are connected to the Internet (i.e. using Skype, FaceTime or similar), this image (usually of your face!) can be seen by others. However, if you point the camera towards some learners' work, you can project this image to a data projector for all in the class to see. It is also possible to buy a low-cost webcam with a bendy arm that plugs into your computer/tablet – and makes it easier to place learners work underneath. This is a low cost visualiser!

Technology away from the classroom

1) Creating worksheets and tests with accurate mathematical text and diagrams in word processing software has the distinct (and time-saving) advantage in that you can edit the document again and again to make many different versions.
One of the first IT applications that many people use is a word processing package such as 'Word' to produce typed text that can be saved, edited and printed as and when you need it. This means you can produce your own tasks and tests, write your own helpful notes for your learners and even, by choosing a large clear text (or font), produce posters to display in the classroom.
   a) Inserting symbols and equations.
   The basic letters, numerals and simple symbols such as + and – are all on the keyboard. When you need to use more mathematical symbols such as ÷, ×, and √ you will usually go to a drop down menu such as 'Insert' and choose 'Symbols' to get a palette of other common symbols.

$	¢	£	¥	€	₾
½	⅓	⅔	¼	¾	⅕
⅖	⅗	⅘	⅙	⅚	⅛
⅜	⅝	⅞			
+	−	×	÷	±	/
<	>	≤	≥	=	≠
≅	≈	∧	∨	∞	√

Fig. 11 A typical palette of more mathematical symbols IMAGE 11a_Symbols.jpg

To write more complex numerical and algebraic expressions and equations, and writing fractions vertically you will need to use an 'equation editor' which is normally an optional feature of a word processing application. As before, you would select an 'Insert' menu and choose 'Equation', which would reveal a toolbar of different types.

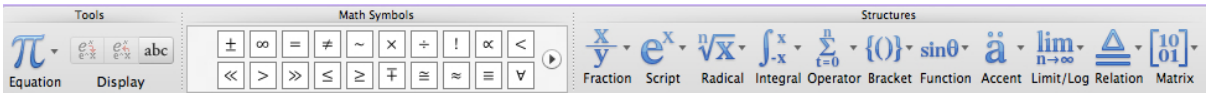


Fig. 12 A toolbar of choices of different types of equations mathematical symbols IMAGE 12_Equations.jpg

b) Inserting tables

There will be many occasions when you want to produce tables and, with your imagination, these can be quite creative!

All of the images in Fig. 13 below have been created as tables within a word processing package – the secret is to learn how to control the size, positioning, text and shading of each cell in the table.

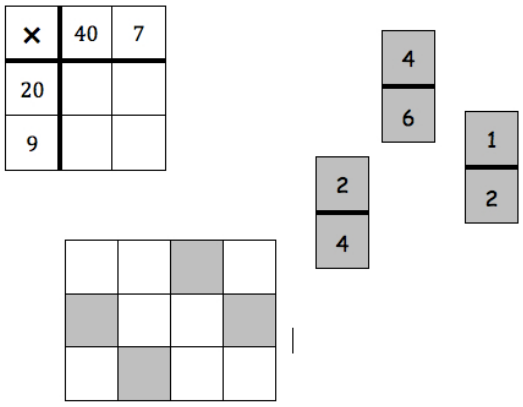


Fig. 13 A selection of different graphics – all created using tables in word processing software. IMAGE13_Tables.jpg

Here are some top tips:

- When you choose to insert a table, once you have created it with the number of rows and columns that you need, select the 'Table properties' to take control of the size of the rows and columns. Choosing a width and height of 1 cm is very useful for producing accurate images!
- Use border lines creatively – for, example to create the 'fraction cards' above, which can be cut out for learners to match and sort.

c) Inserting graphs

Firstly, you must have a Word document open.

In GeoGebra, use the Pointer tool to drag a box around the graph that you want to copy. Open the File menu and choose 'Export graphics view to Clipboard' (See Fig. 14 below).

(Nothing much will seem to happen)

Open your Word document and click on Paste.

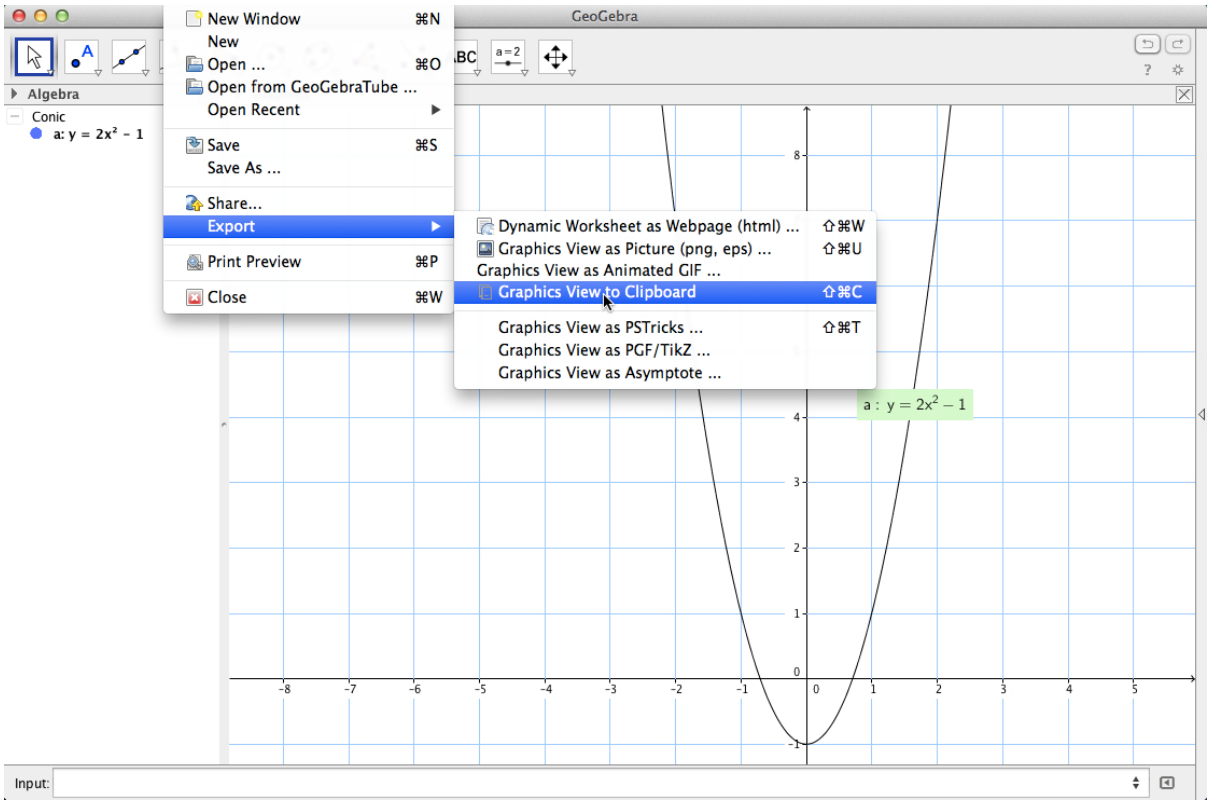


Fig. 14 Exporting a graph from GeoGebra Image14_Geogebra_export.jpg

You can then write the text around your picture.

d) Inserting geometric images

There are a limited range of basic geometric shapes available in most word processing applications when you choose Insert and Shape. However, although you can change the size of them by dragging, you have very little control over the sizes of perimeters, areas and angles. It is much better to create the accurate shape that you want in a mathematical application such as GeoGebra, and then drag around your image to select it and then use the Edit menu to copy it from GeoGebra - and paste into your word processing application, using the same steps for 'Inserting graphs'.

2) Creating resources to project to the class

As soon as you can use a data projector, it is likely that you will want to make presentation 'slides' that can include images from other software (such as graphing and geometry software) to support you to introduce your lesson and provide stimulating and mathematically correct diagrams, symbols and text. You can copy and paste images from other software into such software in the same way as described earlier in this chapter.

a) Presentation software

The most commonly used presentation software is Microsoft PowerPoint but other free software with similar features is available for download from the Internet.

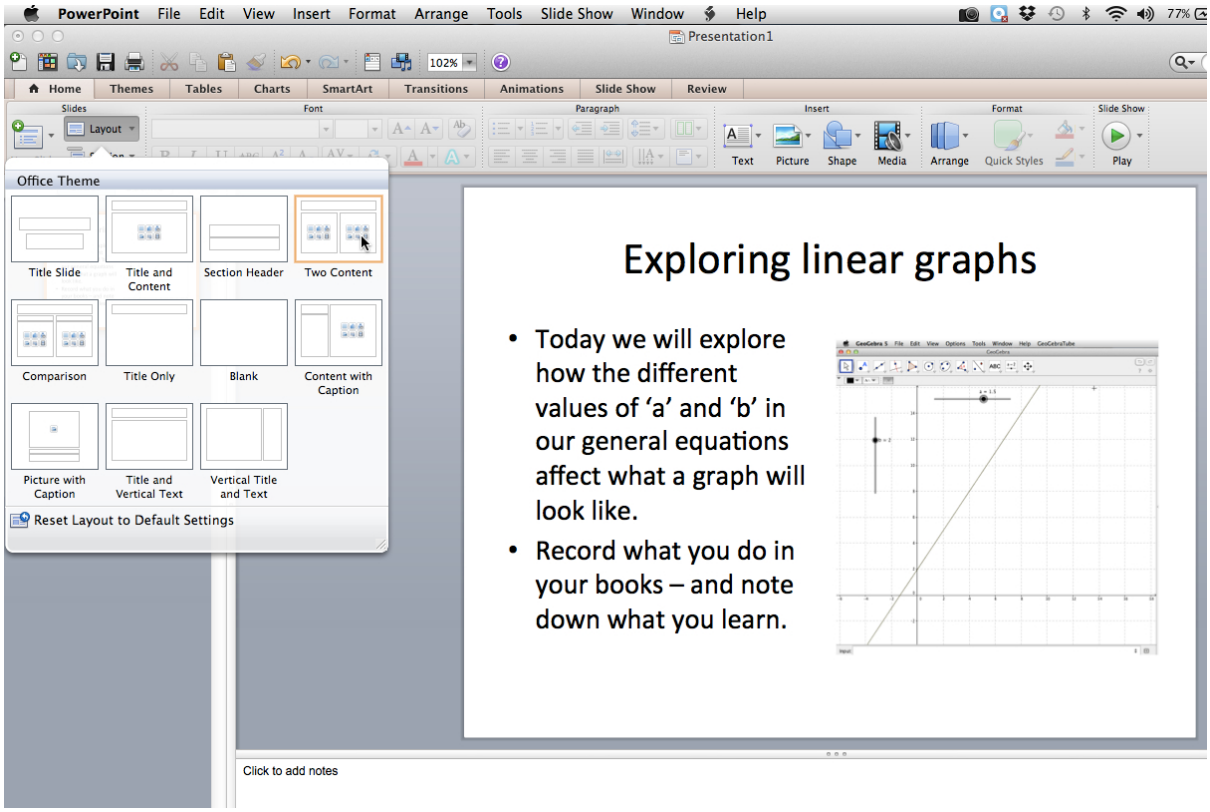


Fig. 15 Creating a slide in PowerPoint IMAGE15_PowerPoint_Presentation.jpg

As you insert each slide into your presentation, you can choose the layout that best suits your lesson. A combination of clear text in a large font size and images (which can be movies) works best.

b) Interactive whiteboard software

If you have access to an interactive whiteboard, which must always be connected to both your computer AND a data projector, you can use the special software that comes with the board to create presentation files. This software is usually freely

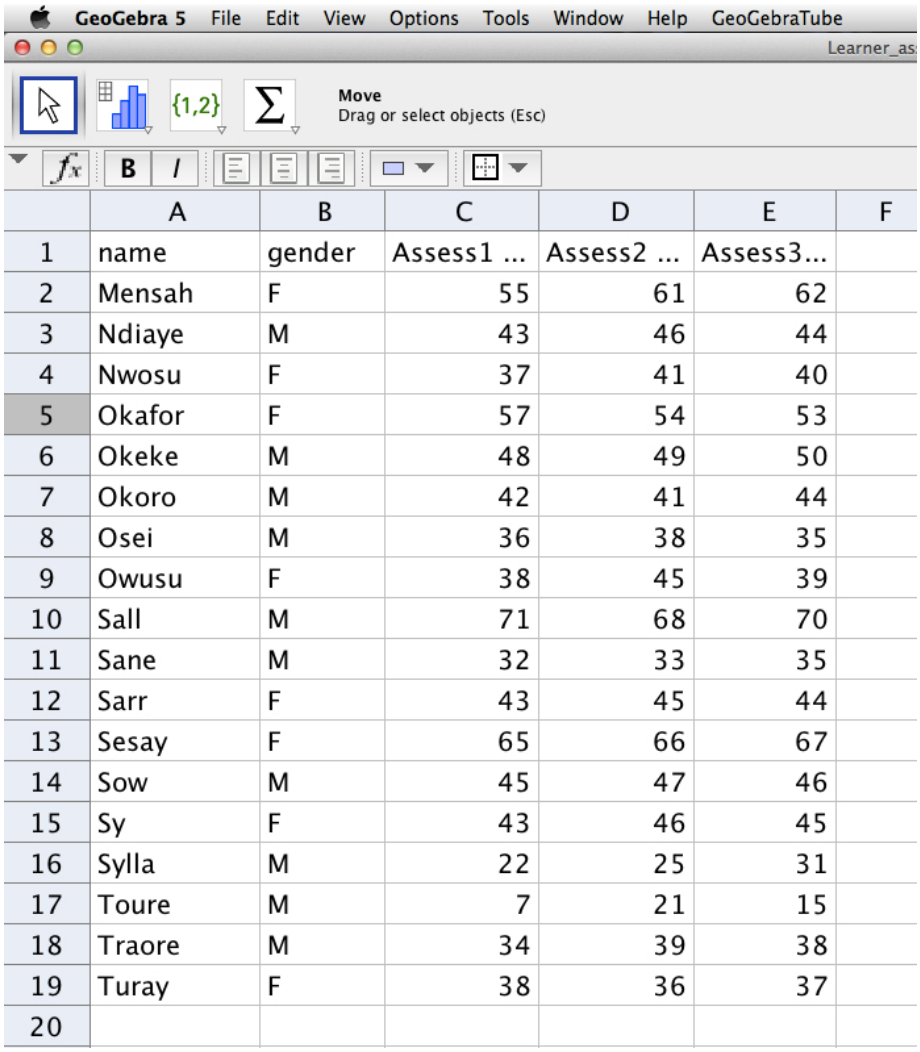
downloaded from the manufacturer's website, accompanied by tutorials and help resources to help you make the most of the 'finger touch' functionality.

3) Creating an electronic markbook

Many teachers have found spreadsheet software to be most useful for creating an electronic markbook in which they keep a record of learners' outcomes. The electronic nature of this book means that it is quick and easy to sort any numeric data from highest to lowest values, calculate mean. Mode and median averages and produce graphs and tables to show learners' progress over time. Of course this does rely on high quality assessment data in the first place!

a) Entering data (Names/dates/marks)

The starting point is to enter the students' names, gender and the assessment marks.



	A	B	C	D	E	F
1	name	gender	Assess1 ...	Assess2 ...	Assess3...	
2	Mensah	F	55	61	62	
3	Ndiaye	M	43	46	44	
4	Nwosu	F	37	41	40	
5	Okafor	F	57	54	53	
6	Okeke	M	48	49	50	
7	Okoro	M	42	41	44	
8	Osei	M	36	38	35	
9	Owusu	F	38	45	39	
10	Sall	M	71	68	70	
11	Sane	M	32	33	35	
12	Sarr	F	43	45	44	
13	Sesay	F	65	66	67	
14	Sow	M	45	47	46	
15	Sy	F	43	46	45	
16	Sylla	M	22	25	31	
17	Toure	M	7	21	15	
18	Traore	M	34	39	38	
19	Turay	F	38	36	37	
20						

Fig. 16 Entering assessment data in a GeoGebra spreadsheet

IMAGE16_Assessment_data.jpg

b) Making sense of assessment data

Within GeoGebra, once your data is entered, when you select the column of data you can generate a number of summary statistics and graphs. So for the data on the learners' achievements in Assessment 1, this analysis could be:

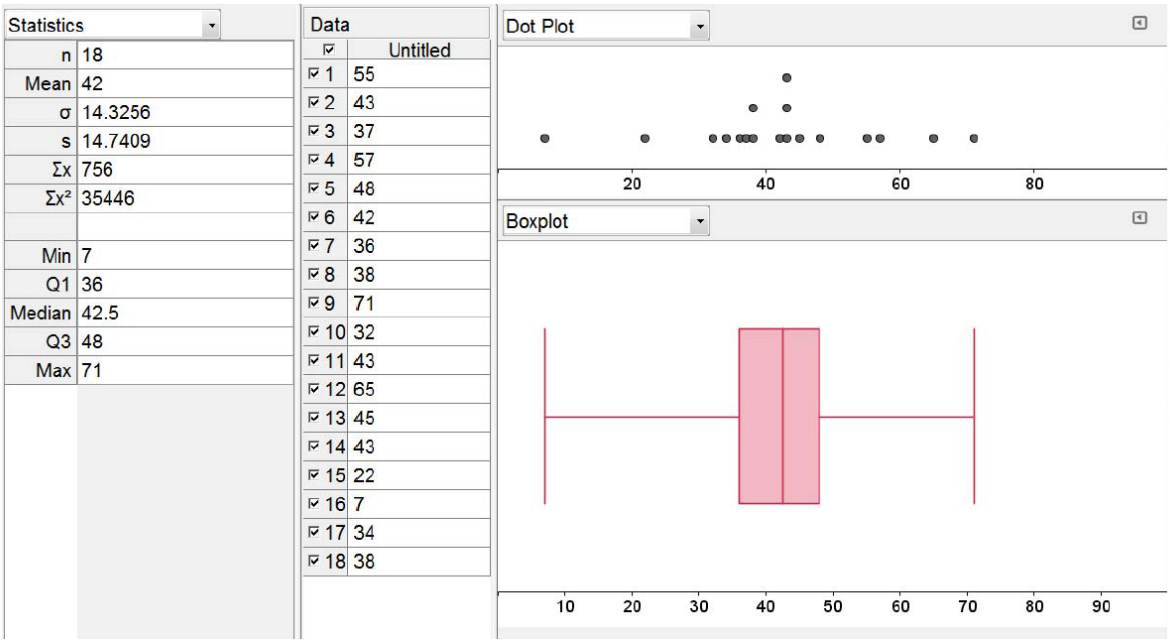


Fig. 17 Summary statistics and Box and Whisker plot for assessment 1.

IMAGE17_Box_whisker1.jpg

The Box and whisker plot is a useful graphic that shows the range of marks (71 – 7 = 64), and that 50% of the students were in the mark range from 36 to 48.

If the summary statistics for Assessment 2 are also generated, AS LONG AS the scales for the x-axis are the same, you get a useful graphic that helps you to assess whether the students are making progress over time.

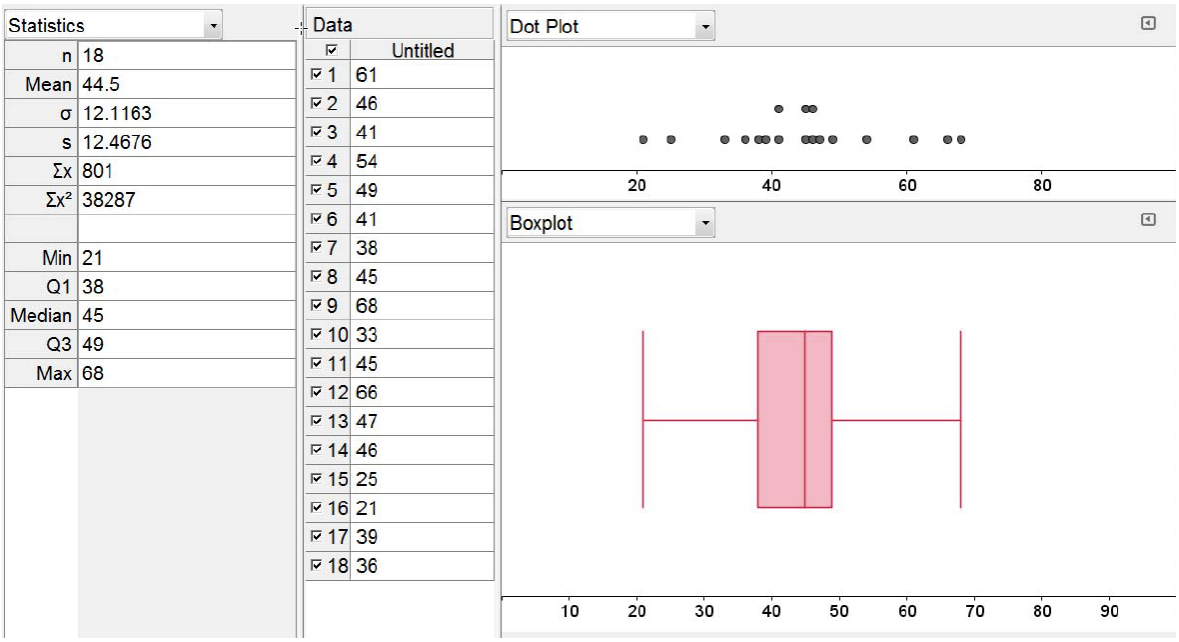


Fig. 18 Summary statistics and Box and Whisker plot for assessment 2.

IMAGE18_Box_whisker1.jpg

So it appears that the class is making some progress!!

4) Online communities for teachers and learners

a) Ministry of education sites

Many countries are developing their websites to include information about the curriculum, details of any regional and national testing arrangements and as a gateway or 'portal' for resources to support teaching and learning.

For example,

South Africa: the Thutong website www.thutong.doe.gov.za.

Zimbabwe: the Ministry of Primary and Secondary education website www.mopse.gov.zw

b) Aiming high

The Aiming High teacher network has grown from the hundreds of teachers who have attended the African Institute for Mathematical Sciences School Enrichment Centre's (AIMSSEC) courses in South Africa since 2005.

The website includes Teaching resources, an online Forum and News about the AIMSSEC courses. AIMSSEC work closely with the Geogebra Institute in South Africa to offer courses and support for technology use.

aiminghigh.aimssec.ac.za

Fig. 20 The AimingHigh Teacher Network homepage

IMAGE20_Aiming_high.jpg

c) African Mathematics Initiative

The African Mathematics Initiative (AMI) is a Kenyan NGO formed by mathematicians and mathematics educators who are working to create a stronger mathematical community and culture of mathematics across Africa. AMI has produced resources to support the use of digital tools for mathematics such as GeoGebra and Scratch, which are available on its website.

www.africanmathsinitiative.net

5) Online resources for teachers and learners

a) Microsoft Math

Microsoft Math is a free resource that can be accessed by high school learners on their cellphones. It includes theory pages and quizzes. Learners can compete against each other. Teachers can create a group for their class and can then keep track of the number of points earned by each learner. To create an account, go to math.microsoft.com.

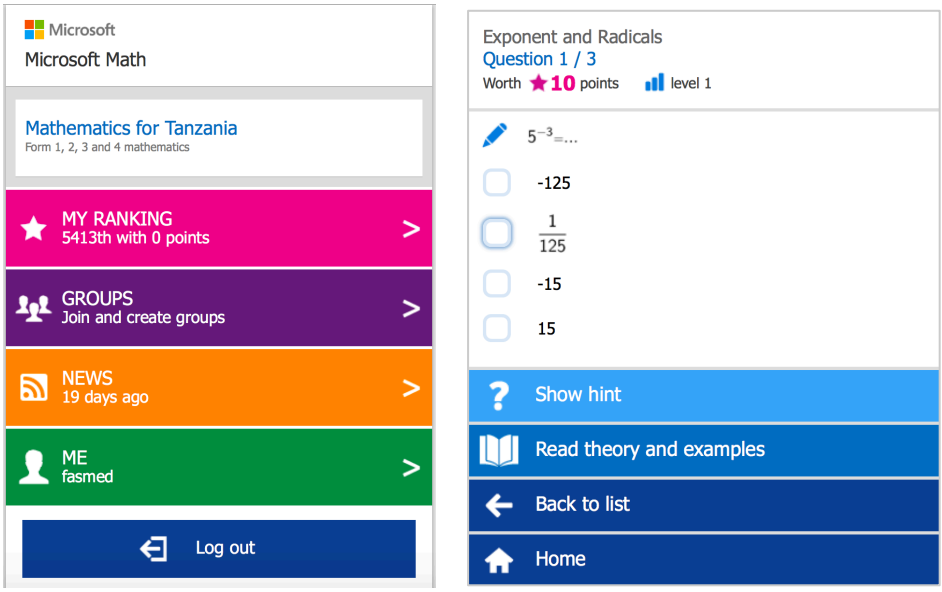


Fig. 21 MicrosoftMath menu and example of a practice question

IMAGE21a_MicrosoftMath_Menu.png IMAGE21b_MicrosoftMaths_Example.png

b) Everything Maths and Science

Everything Maths and Science has free high school textbooks with videos and simulations. The content is specifically created for the South African curriculum but can be used by anybody. The textbooks can be viewed online or downloaded and printed. The site also has a free online practice service that allows teachers to set problems for their learners. Teachers can create a group for their class, set problems and then keep track of the number of points earned by each learner. www.everythingmaths.co.za.

c) Khan Academy

The Khan Academy is an online resource that includes test questions and videos to support learners away from the classroom. You will need to watch the videos carefully too - as they may introduce your learners to a range of different problem solving approaches, mathematical vocabulary and notation. The resources are organised by mathematical topics and they are not aligned to any particular country's curriculum or examination system. If learners create an account, then their progress through the topics is recorded and they are awarded points and rewards for their work. www.khanacademy.org

d) MathsExcellence

MathsExcellence is a website started by South African teachers to improve the standard of Maths education and to make maths more accessible. It contains

many different free resources such as textbooks, videos, tutorials and exam papers.
www.mathsexcellence.co.za

e) Math Aids
Math Aids provides free maths worksheets for teachers and learners. There are many different topics to choose from and because the worksheets are randomly they are all different. The worksheets can be downloaded and printed.
www.math-aids.com

Next steps

Any changes to our teaching practices come at a personal cost. It takes time to learn to use new tools and resources; we often make a personal financial investment in technology that will support us in our work; and in the classroom things might not always go as expected. However, there are very few teachers who disagree with the need to develop personally and professionally as a means to supporting our learners to achieve mathematics to the very best of their abilities.

Glossary

Dynamic geometry: an interactive mathematics environment that enables geometric constructions to be created and manipulated. For example, 2-D shapes can be created and their angles, side lengths, areas and perimeters can be measured. Using the software, these shapes can be transformed by dragging and they retain their underlying mathematical properties, although the measurements may change. The accurate images can be copied and pasted into other software, such as MS Word.

Examples: GeoGebra, Cabri-geometry, The Geometer's Sketchpad, Cinderella

Dynamic graphing: an interactive mathematics environment that enables Cartesian and polar graphs to be plotted automatically by entering the function. The accompanying tables of values can be displayed and the functions can be transformed.

Examples: GeoGebra, Desmos, Autograph

Spreadsheet: an interactive environment where you can input, organise analyse and store text and numerical data as a table. This data can be displayed graphically and a number of statistical analyses can be used to query and interpret the data.

Examples: Excel, GeoGebra,

Application or App: These are usually digital tools that have been designed to *only* run on SmartPhones, iPads and Android tablets, although some people are now using this word to describe software. However, many Apps have limited editing and file-saving features. For example, GeoGebra is available as an App, but there is limited functionality when compared to the software version.

Software: A particular digital tool that has been designed for a specific purpose is called 'software'. For example 'word-processing software' allows you to create, edit, save and share files. This text has been written in word processing software.

Web-based: If a particular resource is web-based, you will need to have an Internet connection to be able to access and use the resource. Sometimes you can save the resource onto your own computer so that you can use it 'off-line'.

Search engine: When you have an internet connection, you use a search engine to look for resources and information on the internet. The most common search engines are Google, Internet Explorer and Yahoo.

Internet (or World Wide Web): This is a global network that connects everyone's computers to the resources that different individuals and communities have published on their websites.

Mobile messaging application: This is an instant messaging service that can be used on mobile 'phones and computers. It can be a quick way to communicate information in 'real-time'.

Data projector: A data projector projects the image from your computer screen onto a wall or screen so that everyone can see the image. It will usually need a power source and a connection cable from your computer or tablet. The more expensive versions have a more powerful bulb, which means they will project a brighter image.