FROM TECHNOLOGICAL INNOVATION IN INDIVIDUAL CLASSROOMS TO LARGE-SCALE TRANSFORMATION OF TEACHING PRACTICES – MIND THE GAP

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The ICTMT conference proceedings chart the development of technology use in mathematics education from the 1990s to the current day. Throughout this period, the prevailing topics for plenaries, papers and workshops have been focused on the development of innovative classroom practices involving 'new' technologies. Alongside this, there has been a slow but emergent theme that has brought aspects of teachers' professional development to the fore - as attempts to scale the widespread use of technology by students have proved both challenging and expensive. In this plenary, I will draw on some personal contributions to ICTMT conferences from the past in order to highlight how my own work now focuses on the design and evaluation of technology-focused professional development for teachers of mathematics. This work is set against the backdrop of the Cornerstone Maths project in England, which is aiming to scale student access to dynamic mathematical technology in lower secondary mathematics in hundreds of schools.

Keywords: Mathematics education, Dynamic mathematical technology, Scaling professional development, Cornerstone Maths

INTRODUCTION

I begin by thanking the conference Chairs, Nélia Amado and Susana Carreira and the International Scientific Committee for the invitation to give a plenary at ICTMT, a series of conferences that I first attended in 2001 as a classroom teacher - and I have not missed an ICTMT conference since that date. I feel a real sense of community at these conferences, which I think is enhanced by the presence of practising teachers and lecturers, technology designers and researchers in both mathematics and mathematics education – a combination that is not often present at other academic conferences. At the heart of this keynote is the notion of innovation, meaning 'to introduce change and new ideas', which has been a common starting point for many of ICTMT's participants over the years since the very first conference in 1993. In this plenary, I chart my personal experiences of innovative technology in mathematics education through my role as a secondary school mathematics teacher, a Head of department and to my current role as an educational researcher whose work focuses on the iterative design of dynamic mathematical technologies, teaching resources and teacher professional development programmes. Of course I do not work alone and although my colleagues over the years are too numerous to mention, my own thinking and practice have been greatly influenced by the enthusiasm, knowledge and insights that Adrian Oldknow, Celia Hoyles and Richard Noss have stimulated.

For me, one innovation in technology that impacted on my mathematics classroom practices came in the form of dynamic geometry software (The Geometer's Sketchpad), that Warwick Evans and Adrian Oldknow had introduced me to in 1997 during my Master's degree course in mathematics education. In my role as a Head of Mathematics Department in an inner-city secondary school, I worked with my colleagues to devise tasks that encouraged students to work collaboratively in pairs at the computer to foster an inquiry-based approach to explore concepts in 2-D geometry (See Figure 1). At that time, I genuinely believed that I was at the beginning of an exciting rethinking of the school mathematics curriculum and, had I been asked to predict what pupils' mathematical experiences in classrooms and examinations would look like in 2015, I would have envisaged some form of dynamic geometry software in widespread use.

Figure 1. Exploring 'z-angles' – A task from my classroom in 1999.

DEFINING SOME IMPORTANT TERMINOLOGY

The research on technology use in mathematics education has tended to evaluate its impact based on one of three aspects: the affective - concerning ideas such as time on task, enjoyment, and motivation; the socio-cultural – including levels of collaboration, group work and the enabling of communities; and cognitive, which concerns the human interactions with mathematical objects such as sliders, syntax, geometric shapes etc. Of course these three aspects are not mutually exclusive and any attempt to evaluate the impact of an innovation is likely to touch on all three. However, in my own work I have always been drawn to the cognitive domain whereby the digital tools shape the mathematics that is learned whilst also opening a new landscape of 'learnable mathematics' (See Kaput, Hoyles, & Noss, 2002).

The ICTMT12 abstracts reveal a wide range of mathematical technological environments in use: Mathematica, GeoGebra, Geometer's Sketchpad, MathPen, CATO, Cubes & Cubes, Dessiner les formes, Gummii, Moodle, Graphing Calculator, TouchCounts, ALEX, Cabri, WinPlot, Wiimotes, Cabri Elem, Tinkerplots, Simply Geometry, Cornerstone Maths, Rhinoceros, R code, spreadsheet… and also some more general environments: Mobile devices, e-learning environment, intelligent support system, blended learning, YouTube clips, applet, widget, Web 2.0, computer games, c-book, CD-ROMs, Khan Academy… This diversity of interpretations of 'technology' can be problematic when we fail to define clearly the characteristics of our chosen technological tools, tasks, and pedagogic approaches – so I begin with some characteristics that underpin my own work.

My colleagues and I feel it necessary to define the technology that we use as 'dynamic mathematical technology' thus:

> transformative computational tools through which students and teachers can (re-)express their mathematical understandings, understandings which are simultaneously externalised and shaped by the interactions with the tools. (Clark-Wilson, Hoyles, Noss, Vahey, & Roschelle, 2015, See also Hoyles and Noss (2003))

So by default, if such technology is transformative, it <u>has</u> to disrupt knowledge and practice!

Alongside this, sits 'innovation'. Although it is easy to make a personal claim that something is innovative, this judgement is actually made by the receiver of the innovation. This explains in part why some many 'innovations' seem to appear and reappear albeit in slightly different guises at subsequent ICTMT conferences – sometimes to the annoyance of more knowledgeable or experienced conference participants. The need for all participants to substantiate any claims of innovation by paying attention to the content of past ICTMT conference proceedings, referencing the wider literature base and building on existing knowledge and practices seems an important one! However, this demand should not be at the expense of encouraging innovation at grass roots level by classroom teachers and lecturers who may not be yet engaged with the more-established research community!

A LONGITUDINAL PROGRAMME OF DESIGN BASED RESEARCH - CORNERSTONE MATHS

The Cornerstone Maths Project is a collaborative design-based-research project between colleagues at SRI, US and London Knowledge Lab, UCL Institute of Education that begun in 2011 with generous funding from the Li Ka Shing Foundation (LKSF). It aimed to capitalise on the outcomes of a number of programmes of research to exploit the dynamic and visual nature of digital technology (DT) in hundreds of mathematics classrooms to stimulate engagement with mathematical ways of thinking by:

- focusing on the 'big mathematical ideas' (linear function, algebraic variable, geometric similarity) that are often considered hard to teach;
- making links between key representational forms;
- providing an environment for students to explore and solve problems within guided structured activities;
- embedding activities within realistic contexts.

The resulting technology-enhanced curriculum units combine specially designed software, pupil workbooks, teacher guides and accompanying synchronous and asynchronous professional development.

The iterative design and evaluation processes have been undertaken in several phases[1]. The research reported in this plenary refers to outcomes of the final phase of the LKSF-funded work and some early findings from the current Nuffield Foundation funded research, which take place in England.

In both cases the theoretical foundations for the Cornerstone Maths curriculum and its accompanying models for teachers' professional development are reported in publications by the respective project teams (Clark-Wilson, Hoyles, Noss, et al., 2015; Geraniou, Mavrikis, Hoyles, & Noss, 2011; Hoyles, Noss, Vahey, & Roschelle, 2013; Mavrikis, Noss, Hoyles, & Geraniou, 2012; Roschelle & Shechtman, 2013; Phil Vahey, Roy, & Fueyo, 2013).

EVIDENCE FROM 'AT-SCALE' USES OF DYNAMIC MATHEMATICAL TECHNOLOGY

The products and processes of scaling a technological innovation

Since 2010, the LKL project team has worked with 18 PD 'multipliers' organised into 13 project networks. These networks have involved at least 417 teachers from 183 secondary schools and over

9500 pupils. This context has enabled us to develop our understand of the process of scaling student access to dynamic mathematical technology.

Scaling (or widening/increasing use) involves both 'products' (or measurable outcomes) and 'processes' (the means through which these are achieved) (Hung, Lim, & Huang, 2010) and it is highly influenced by the context and culture in which it takes place. In England in 2015, this means:

- There are no recommendations for technology use in mathematics (5-16) – this is a 'pedagogical' decision for individual schools and teachers.
- Schools determine their own pathways through the curriculum (using a localised 'scheme of work').
- School inspection processes do not focus on the use of technology at a subject level.
- There is very little localised support for teachers of mathematics (i.e. mathematics advisers/consultants).

Our research has concluded a set of quantifiable outcomes that are key to understanding whether our innovation has indeed scaled that are shown in Figure 2.

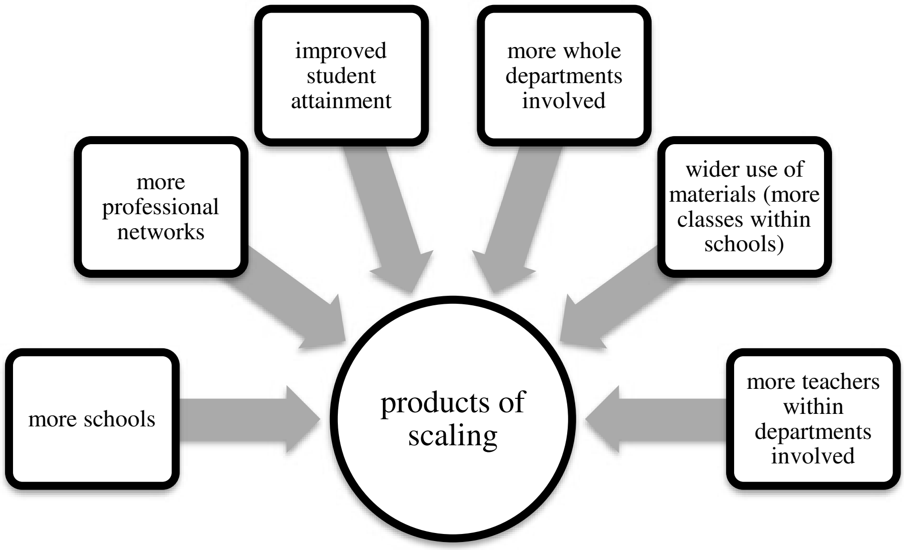

Figure 2 Quantifiable outcomes of scaling the Cornerstone Maths innovation

Alongside this, the all-important processes of scaling are shown in Figure 3. Some of these processes may be common across different countries and cultures; for example, the web-based curricular activity system has proved effective in US studies (P. Vahey, Knudsen, Rafanan, & Lara-Meloy, 2013) and the need for 'PD multipliers' (Rösken-Winter, Schüler, Stahnke, & Blömeke, 2015). Each of these processes is articulated in more detail in Clark-Wilson et al (2015).

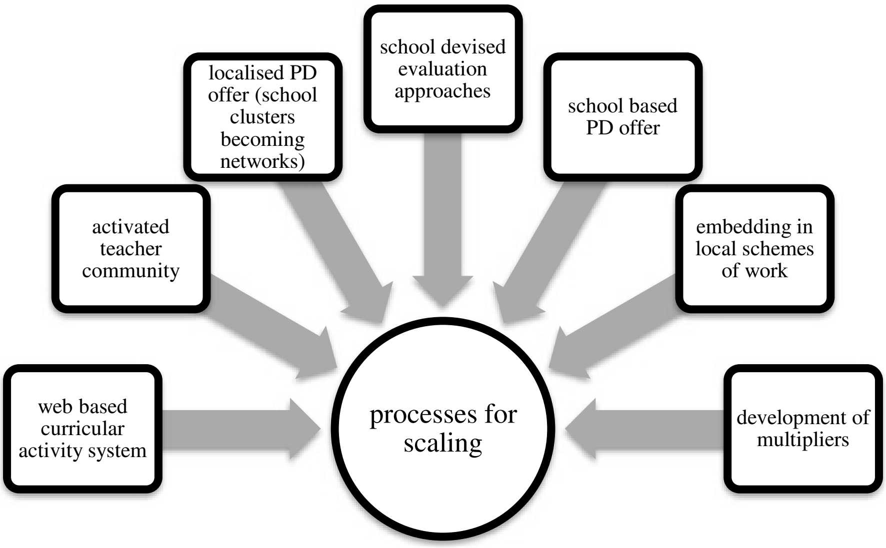

Figure 3 Processes of scaling the Cornerstone Maths innovation

Researching teacher development through a technological innovation

The current research project (funded by Nuffield Foundation[2]) aims to analyse the development of teachers' mathematical knowledge for teaching and associated mathematics pedagogical practice related to the three Cornerstone Maths topics; algebraic generalisation, geometric similarity and linear functions as they participate in professional development and classroom experimentation.

There are many theories that conceptualise aspects of teacher knowledge: pedagogical content knowledge (PCK, Shulman, 1986); it's elaboration as technological pedagogical content knowledge (TPaCK, Mishra & Koehler, 2006); the Knowledge Quartet (Turner & Rowland, 2011); Mathematical Knowledge for Teaching (MKT, Ball, Hill, & Bass, 2005); and Horizon Content Knowledge (Hill, Ball, & Schilling, 2008). The methodological challenge is to devise ways to access such knowledge using survey, interview, classroom observation, video analysis, using critical incidents etc.

In our methodology we begin with Hill et als' MKT: content knowledge relating to topic; teachers' understanding of students' topic-specific knowledge and we add, key representations within the technology and how these relate to each other (mathematically). We focus our research on teachers' (re)design of selected Cornerstone Maths' tasks, which stimulate 'landmark activities' (using a Lesson Study approach) to provide a window on teachers' knowledge and practice as they reflect on ***disruptions*** when embedding digital technology. (The construct of a 'landmark activity' is extended in Clark-Wilson, Hoyles and Noss (2015)).

EMERGING FINDINGS ON TEACHERS' MKT RELATING TO ALGEBRAIC VARIABLE

We report some early findings on teachers' MKT relating to algebraic variable for a cohort of 72 teachers who represent a group that is skewed towards younger (< 10 years) and less experienced teachers (mathematics teaching experience) teachers. (See Figures 4 and 5).

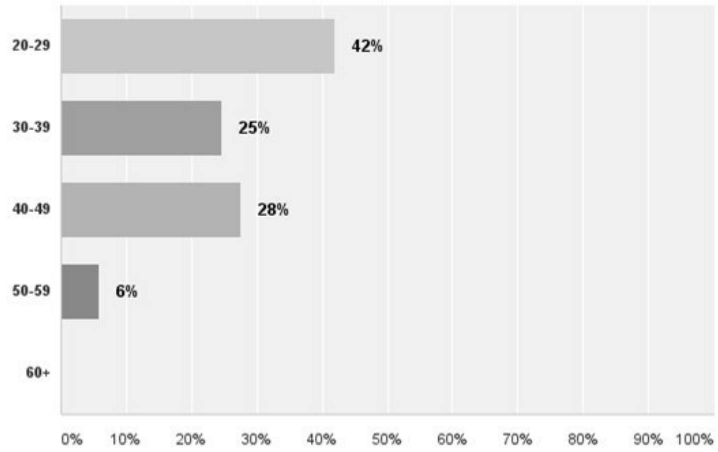

Figure 4 Age demographic of teachers (n=72)

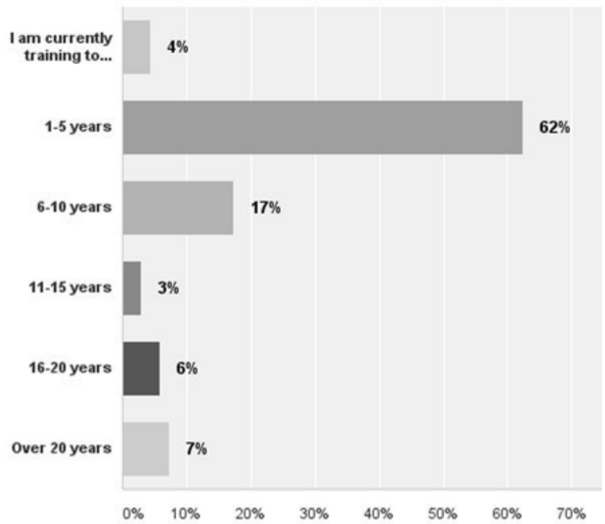

Figure 5 Years of mathematics' teaching experience (n=72)

Perhaps surprisingly for a younger group of teachers, they report little use of dynamic mathematic technology in their teaching, with 88.4% of the teachers reporting that they had *never* or *only occasionally* given their students such opportunities in lessons.

A baseline item to assess teachers' MKT related to algebraic variable asked for their personal definitions, which were categorised using Küchemann's interpretations of the use of letters in school algebra (Küchemann, 1981).

Of the 69 teacher respondents:

- 19% gave definitions that suggested that letters represented a specific unknown - 'An algebraic variable is when a letter is used to express a number we don't yet know. The variable is usually separated by an operation symbol such as plus, minus multiply or divide sign. Example x + 3 = 7 where x is the variable;
- 67% gave definitions that suggested that the value of a letter can vary – 'A real life value that can change (eg. Temperature, time, cost etc), represented by a letter';

- 6% gave richer definitions that considered the mathematical domain – 'An unknown that can take on a range of values'.
- 8% of the teachers gave incorrect or ambiguous responses – 'Algebraic variables consist of letters and numbers. Take for example 2a + 3b. 2a is an algebraic variable 3b is another algebraic variable'.
- Furthermore, when asked to rate their level of confidence in the response that they had provided for this item, 48% of the teachers stated that they were 'not at all confident' or only 'quite confident' with their definitions. One teacher clarified this by saying 'I feel much more confident in my understanding of how algebra works than my ability to explain it. There are so many different and interconnected ideas that I find it hard to choose an explanation, which is both accessible and correct'.

A key element of the initial face-to-face PD event for these teachers was discussion time during which a range of definitions were presented and discussed. This discussion was extended during a PD task where teachers worked within the Cornerstone Maths 'Patterns and expressions' software in an activity that required them to create a simple repeating pattern and 'unlock' the value of the 'No of (repeating) blocks' such that this number can be varied by using a dynamic slider, which again revealed uncertainties in the action of 'naming' of this algebraic variable and the teachers' interpretations of its mathematical meaning. Further more, we research how teachers lessons plan for the same task are conceived as a window on their developing pedagogic practices when using dynamic mathematical technology with pupils. Although these are tentative early findings, our early work is promising with respect to our project's aims and further outcomes from this project will feature in future publications.

TOWARDS ICTMT 22… NEW VISIONS FOR MATHEMATICAL TECHNOLOGY

In concluding my plenary paper, I return to my own starting point (some 14 years ago) when I first presented my own 'innovative' classroom practice to an ICTMT conference audience. In the intervening time my professional pathway has led me away from my own classroom practice to focus more on how large-scale teacher development projects and processes might provide the best conditions possible for more students of mathematics to experience a dynamic technology-enhanced mathematics curriculum.

I challenge the ICTMT audience to consider the following questions[3]:

What impact do you want your work (research, innovation, products) to have on learners'/teachers' mathematical experiences?

What will be <u>your</u> legacy at ICTMT22 in 2035? - and for the younger members of the ICTMT community, what will you be presenting?

Notes

1. **Li Ka Shing Foundation funded work**: Planning Phase 1 (Jun-Jul 2011); Pilot Phase 1 (Jul – Dec 2011); Pilot phase 2 (Jan – Jul 2012); Phase 3 (Dec 2012 – Nov 2014)

Nuffield Foundation funded work: Dec 2014 – Nov 2016 *Researching impact on teachers' mathematical knowledge for teaching (MKT) and practice.*

2. See http://www.nuffieldfoundation.org/developing-teachers-mathematical-knowledge-using-digital-technology.

3. You are welcome to contribute your own reactions by posting a comment to the web-page http://bit.ly/ictmt12Vision

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