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Hiccups within technology mediated lessons: A catalyst for mathematics teachers' epistemological development

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Abstract

The notion of the lesson 'hiccup', defined as the perturbation experienced by a teacher during teaching that has been triggered by the use of mathematical technology, was first proposed in Clark-Wilson (2010). Hiccups which are both unanticipated and unplanned, emerged from a study that sought to make sense of the process of secondary mathematics teachers' situated learning as they began to use a particular new technological tool (TI-Nspire handheld devices and software) in their classrooms. The high frequency of the resulting hiccups enabled a categorisation of seven hiccup types that were shown to have influenced the development of teachers' mathematical, pedagogic and technological knowledge. This paper first reports and then extends this earlier work by articulating the design principles for a professional development approach within the Cornerstone Maths project that uses hiccups to try to address professional development 'at scale' concerning student use of dynamic digital technologies in mathematics classrooms.

Keywords: Mathematics teacher knowledge, hiccup, dynamic technology

Introduction

The gap between the potential of digital technologies that have been thoughtfully designed to support (or even transform) students' experiences of mathematical learning (Artigue, 2002; Hoyles & Noss, 1993; Kaput & Roschelle, 1998; Laborde & Laborde, 1995; Papert, 1980) and its wide use in classrooms has led many researchers to write of their disillusion as the initial optimism for fundamental changes in mathematics and its practices was not manifested. This has prompted a refocusing of the research lens onto classrooms in an effort to understand how teachers integrate digital technologies into their classroom practices in ways that add value to the perceived mathematical outcomes (Clark-Wilson, Robutti, & Sinclair, 2014; Hoyles & Lagrange, 2009). It is widely acknowledged that technology use disrupts the prevailing classroom practices and therefore the technology must act as a

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stimulus for unplanned and unanticipated events, the theme of this special issue. Of particular interest is the nature of such events in the technology-enhanced classroom and the role that they might play in the development of mathematics teachers' knowledge.

Although there are a number of significant studies that conclude *that* teachers' mathematical, pedagogical and technological knowledge develops as they integrate well-designed technological tools (Artigue, 1998; Gueudet, Pepin, & Trouche, 2012; Noss, Sutherland, & Hoyles, 1991; Roschelle & Shechtman, 2013), far less is known about the processes through which teachers come to know new mathematical technological tools and how these are assimilated into their professional practices.

The term technology has a wide range of interpretations and manifestations within secondary school education. Therefore it is important to offer a clear definition of the type of mathematical digital tools that are central to the research described in this paper. A key aspect of such tools is that they are transformative in the way that "students and teachers (re-)express their mathematical understandings, which are themselves simultaneously externalised and shaped by the interactions with the tools" (Clark-Wilson, Hoyles, Noss, Vahey, & Roschelle, 2015). Consequently, such tools are expressive as they give a particular view of a mathematical object/situation but they must also enable the user to interact and edit this view in ways that prompt them to reflect productively on the underlying mathematics. Furthermore, the tools present a mathematical view that is rich in its representational infrastructure (i.e. multiple, dynamic, linked) and prompts users to interact by changing variables (i.e. dragging particular objects or changing particular numeric values) in order to highlight mathematical variants and invariants. The above description of technology use is only present in classrooms where the students are the predominant users of the technology - although of course the teachers also make decisions about when and how to use the technology themselves during whole-class teaching or individual/small group work with students.

This paper begins by outlining the theoretical ideas, methodology and findings of a study (the TI-Nspire™ Evaluation Research Project) that led to the construct of the *"hiccup"* within technology mediated mathematics lessons (Clark-Wilson, 2008, 2010). This construct is then elaborated and used to inform the design of innovative professional development activities within an ongoing project (Cornerstone Maths, Hoyles, Noss, Vahey, & Roschelle, 2013) that shares a similar overarching aim (to transform mathematics classroom practices through the use of dynamic digital technologies), but adopts a more systemic approach as it strives to impact upon hundreds of key stage 3 mathematics classrooms in England.

Interpreting mathematics teachers' knowledge – constructs and trajectories

What follows is a summary of the theoretical ideas that underpin the interpretation of mathematics teachers' professional knowledge, which have not traditionally been examined from the specific context of the technology-enhanced classroom and a consideration of the role of unanticipated/unplanned classroom events in the development of such knowledge.

Teachers' mathematical knowledge

This research has its roots in Shulman's theory of content knowledge with its underlying constructs *subject matter content knowledge*, *pedagogical content knowledge (PCK) and curricular knowledge* (Shulman, 1986), where subject matter content knowledge extends beyond facts and concepts to encompass: *substantive structures* (the organisation of concepts and principles to incorporate facts); *syntactic structures* ("the set of ways in which truths, falsehoods, validity and invalidity are established… …like a grammar" (Shulman, 1986, p.9); an understanding of the importance of the substantive and syntactic structures within the discipline; and a knowledge of why particular topics are central to the discipline.
Shulman's second form of content knowledge, PCK "embodies the aspects of its content most germane to its teachability" (Shulman, 1986, p.9) and is defined by:

- The most useful forms of representations for a given topic idea and the explanatory structures that support others to understand the topic through these representations. In the technology-enhanced mathematics classroom this includes dynamic, digital representations for which a teacher would require a knowledge of the mathematical progression through a topic in the particular environment.
- Aspects of the topic that make it easy or difficult for students at different levels of progress (to include those commonly accepted or misunderstood) and teaching strategies for responding to these. Again, an awareness of technological tools that might be incorporated in teaching strategies for particular students at particular points within a topic is an aspect of such PCK.

Shulman's final category of subject content knowledge, namely curricular knowledge (CK), is concerned with a teacher's knowledge of the landscape from which teaching resources and approaches are selected and the resulting student pathways created (or inferred) by these

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actions. Such resources would typically include the teaching materials, practical resources, assessment approaches and in more recent years, digital resources to support both teaching and learning. Of particular interest to technology-mediated learning is the way in which digital resources influence student pathways as existing mathematical representations are presented in dynamic environments that might challenge previously establish practices. Shulman furthers his articulation of teacher knowledge by conceptualising the forms of representation of the content knowledge that have previously been described. He labels these as “propositional knowledge, case knowledge and strategic knowledge”. His definition of “strategic knowledge”, namely the knowledge the teacher draws upon when confronted by “particular situations or problems, whether theoretical, practical, or moral, where principles collide and no simple solution is possible” (ibid. p.13), offers the closest interpretation of the type of knowledge developed through situated experiential learning. In their work, Turner and Rowland describe this as “contingent action” (2011, pp 201-2).

Within the technology-mediated context of the secondary mathematics classroom, such strategic knowledge might include: knowledge of the technology constraints for a particular topic; an awareness of how students make sense of their interactions with the technology; and knowledge of the way in which students relate paper-and-pencil techniques to those experienced through technology use. Therefore, it seems understandable that, at the point where a teacher is using a particular technology to teach a particular mathematical topic for the first time, they would encounter many unplanned or unanticipated events in the classroom as their students respond to tasks in multiple ways.

Unanticipated/unplanned events and their role in the development of teachers’ knowledge

The body of research within this special issue evidences a clear connect between teachers’ experiences with students in the mathematics classroom to potential or actual professional learning. Central to this idea is John Mason’s seminal work on “the discipline of noticing” whereby teachers’ mathematical attentions are tuned to the students’ frequencies so that the teachers see and hear the subtle actions (gestures, recordings and speech) that students make in response to mathematical tasks (Mason, 2002). The technology-enhanced classroom provides opportunities for additional student actions, such as the manipulation of on-screen objects and the ability to make a range of mathematical inputs, which places an additional demand on teachers as they strive to make sense of a diversity of student activity in real-time.

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In this context, there is the potential for multiple unanticipated/unplanned events. However, some pertinent questions arise:

- Who notices the unanticipated/unplanned event?
- What is the nature of the unanticipated/unplanned event?

Stockero and van Zoest describe the notion of a "pivotal teaching moment" (PTM) as "an instance in a classroom lesson in which an interruption in the flow of the lesson provides the teacher an opportunity to modify instruction in order to extend or change the nature of students' mathematical understanding" (Stockero & Van Zoest, 2013, p.127). In their study, the research team analysed video tapes of middle school mathematics practice to select a series of classroom events that had the potential to be a PTM. In this case, the 'noticing' was carried out by the research team (in collaboration with practicing teachers) and the subsequent analyses of the identified PTMs resulted in a framework that offered a typology of PTMs alongside a range of resulting teacher actions and the possible impact of these actions of student learning outcomes. In this research, the primary purpose for selecting the PTMs was not to develop the knowledge of the teachers whose classrooms had been filmed nor to use them as a construct to chart the trajectories of the novice teachers' professional learning.

An alternative idea considers events in the classroom as "teachable moments", which are defined as "the set of behaviors within a lesson that indicated students are ripe for, or receptive to, learning because they express confusion, misunderstanding, uncertainty, struggle, or difficulty with a mathematical problem, concept, or procedure" (Arafeh, Smerdon, & Snow, 2001, p.2-3). The construct of the teachable moment emerged from the analysis of over 225 Grade 8 mathematics and science lessons in Germany, Japan and the United States generated during the Third International Mathematics and Science Study (TIMSS) Videotape Classroom Study (Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999). However, the definition of a teachable moment is narrow as it is only concerned with whether, when given a mathematical problem, the students' "utterances" could be classified as: asking a question, making a statement, remaining silent or the response was inaudible or imperceptible. In this case, the teachable moments were identified and analysed by the large team of researchers for the purpose of contrasting teaching practices in the classrooms concerned. Of course a secondary purpose has been to inform the design of professional development resources for a much wider population of mathematics teachers. However, as in

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van Zoest and Stuckero's study, the teachers whose classrooms were videotaped were not the primary participants in this process.

In both of these studies, the teachers whose classrooms had been observed/recorded were not the primary identifiers of the unplanned/unanticipated events. Consequently it is not possible to establish if and how these events had any underlying impact on these teachers' knowledge. It could be argued that, if the teachers had not noticed, and if they had no subsequent opportunity to discuss and/or review the event through stimulated recall using audio/video of the lesson and accompanying artefacts, then the event may have passed them by. The study that let to the 'hiccup' theory was explicitly designed to position the participating teachers as central to the identification of any unplanned/unanticipated events within their planned lessons.

Methodology

The context for the study from which the hiccup theory emerged was a funded project, the TI-Nspire™ Evaluation Research Project, which took place between 2007 and 2009. The project involved the longitudinal study of 13 secondary mathematics teachers as they began to use a new technology (TI-Nspire™ handheld devices and software) in their classrooms with key stage 3 students (aged 11-14 years). During the project, the teachers designed and enacted 66 classroom activities[1] with their students, which were each accompanied by the teachers' written reflective accounts. The scrutiny of this data revealed evidence of their experiences of what seemed to be "contingent moments" in the classroom (Rowland, Huckstep, & Thwaites, 2005). Rowland et al describe these as "moments that are almost impossible to plan for" and define the components of contingent knowledge to be the "readiness to *respond to children's ideas* and a consequent preparedness to *deviate from an agenda* set out when the lesson was prepared" (Rowland et al., 2005, p.263). The teachers described incidents in their classrooms that had prompted them to reflect on their initial task designs – suggesting that their existing knowledge had been challenged in some way. However, a key difference in the nature of the unanticipated events that emerged from this study was that, as the technology acts as another agent in the mathematics classroom with its inherent mathematical affordances and constraints, the teachers' reported events that had been triggered in some way by students' interactions with the technology.

For example, one teacher reflected on an activity in which the students were required to create a *fan* of linear functions using the graphing application that satisfied the condition of a common point of intersection as a means to conclude variant and invariant properties about such families of functions:

> It was very difficult to get a generalisation from the pupils. I feel perhaps it was the way I questioned them. When looking at the 'fan' going through the point (0,5), there was little that they spotted apart from the fact that it looked like a fan! Perhaps they were more concerned with the 'm' changing than the 'c' remaining constant.

This suggests that the teacher was reconsidering aspects of his role as he sought to mediate between the students' individual experiences with the technology and the group's co-construction of the mathematical knowledge during his whole-class teaching.

The emergence of such events led to a second phase of work involving case studies of two of the participating teachers, Eleanor and Tim, who were subsequently observed during 6 and 8 one-hour lessons respectively. These teachers were selected as they had demonstrated a level of technical competency that showed they had grasped the skills needed to create activities using a range of applications within the chosen technology and they also adopted pedagogical approaches that placed the students' mathematical experiences at the centre of the classroom environment.

During the lessons each teacher wore a microphone and it was their actions during the lesson that were analysed. These actions incorporated their whole-class teaching, their interactions with individual and small groups of students and, on occasion, their interactions with the first author. In addition, the technology being used in the classroom enabled the students' work to be collected remotely from the handheld devices at any moment in time. Consequently, it was possible to record the exact status of the students' work at the point in time when the teacher intervened and, following the teacher-student dialogue, the outcomes as evidenced within the resulting dialogue and software file(s).

These multiple sources of data were imported into Nvivo as a synchronised set, which allowed the lesson to be 'replayed', transcribed and an initial analysis was conducted within a day or two of each lesson. This enabled a discussion with each teacher related to their experience of the lesson whilst it was still fresh in their mind. In addition, the teachers had access to the student files, which they reviewed in preparation for the next lesson.

A combination of the lesson observations and early analyses led the research lens to shift towards the existence of lesson 'hiccups'. These were the perturbations *experienced by the*

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teachers during the lesson, triggered by the use of the technology that seemed to illuminate discontinuities in their knowledge and offer opportunities for the teachers' epistemological development within the domain of the study. These perturbations were highly observable events as they often caused the teacher to hesitate or pause, before responding in some way. Occasionally the teachers looked across the classroom in surprise to make eye contact with the first author and, particularly in the case of hiccups relating to what they considered to be unhelpful technological outputs, they sometimes expressed their dissatisfaction verbally. Consequently, each lesson activity was coded for hiccups. An example of such coding for one of Tim's lessons, entitled *Equivalent quadratic equations* is shown in Figure 1.

[Figure 1. near here]

Interestingly, although the teachers' immediate reaction to the hiccups tended to be a negative one, our conception was that these provided an opportunity to focus upon the unanticipated elements of the lessons as a trigger for substantial professional discussions with each teacher.

The data analysis involved an initial coding of the lesson data (as in Figure 1) followed by a process of constant comparison, as elaborated within grounded theory (Corbin and Strauss, 2008) to generate a refined set of codes that categorised the hiccups according to the themes that had emerged.

The complete data set included post-lesson discussions and communications with the teacher about the lesson that were focused on the hiccups. Consequently, there was data that included the subtleties of each teacher's actions in response to the hiccup. These actions may have been observable during the lesson but they also occurred after the lesson, either as part of the teachers's reflective commentary on the lesson or in more incidental conversations or email exchanges. This data was identified and coded as 'Actions'. The coded actions from Tim's lesson *Equivalent quadratic equations* are shown in Figure 2.

[Figure 2 near here]

The relationship between the original hiccup and any resulting action(s) is not necessarily a one-to-one correlation. For example, in the lesson described above, the students had been

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asked to use the technology in a simple input-response mode to check their by-hand expansions of some quadratic expressions on a paper resource that Tim had prepared. The first of these was (x+2)(x+5). However, in his introduction, Tim read this to the class as “x plus two x plus five” as he simultaneously entered the key presses, which included brackets and a multiplication sign. When the students began to work independently on the task, Tim then noticed very quickly that students had neither noticed the brackets (TP9 Hiccup02) nor the invisible multiplication sign (TP9 Hiccup03) as he moved around the classroom observing the students at work and responding to their requests for help. He took direct actions in response to these hiccups by responding to students’ concerns at an individual and group level (TP9 Action07) and, after a few minutes, revising his language with the whole class to emphasise the importance of these aspects (TP9 Action09). Furthermore, after the lesson, Tim revised his worksheet for use by another class (TP9 Action10).

This process of coding both the lesson hiccups and the list of any related teacher actions was repeated for each of Tim’s and Eleanor’s observed lessons, which led to a generalised set of definitions of seven hiccup types. A more extensive description of the methodology that led to the hiccup construct and several examples of each hiccup type is given in Clark-Wilson (2014).

Findings

A total of 63 hiccups were identified and classified from the combination of Eleanor’s and Tim’s lessons and the resulting set of categories, and the underlying conditions that described them more fully are shown in Table 1.

[Table 1 near here]

The categories of hiccups and their underlying conditions are a set of somewhat negative statements. This was because, in the moment, the teachers had tended to perceive the hiccups as a negative event – it *was* unwelcomed. Also, when reflecting on the hiccups that they had experienced, the teachers were most critical of their own activity designs and suggested modifications to tasks, which provided evidence of their cognitive challenge. However, this did not mean that the teachers were not positively surprised by their students’ responses to tasks. For example, in one of Eleanor’s lessons, she was extremely impressed by a student who, when invited to create a linear function that passed through a given coordinate point, responded with a quadratic function, although in the moment, Eleanor chose to dismiss this diversion and continue with her lesson as planned (Clark-Wilson, 2013). However, in the post-lesson discussion, Eleanor was very animated as she described another task that she wanted to design for a different class that used this student's response as the starting point.

It is important to state that the teachers in this study were learning to use a particular technological tool that was newly available at the time of the study and for which no previous lesson tasks had been developed. In addition the technology was particularly complex to learn to use as it combined spreadsheet, dynamic geometry, dynamic graphing, dynamic statistics, scientific calculator and note taking functionalities, all of which could be dynamically linked by creating local and global variables. Although most of the participating teachers had some previous experiences of using existing individual software for mathematics (e.g MS Excel, The Geometer's Sketchpad, Autograph), very few of the teachers had used a dynamically linked environment in the mathematics classroom. Hence, the high number of hiccups that were observed in Eleanor's and Tim's lessons (n=63) could be explained by a number of factors:

- The teachers were designing, trialling and evaluating new classroom tasks for which there were no historical patterns of use.
- The complexity of the technology required the teachers to learn about new functionality and features for themselves – and then incorporate these into their task designs within a short timescale.
- The pedagogical approach that was encouraged during the project (exploratory tasks that used multiple dynamic representations of mathematics to highlight variant and invariant properties) was unfamiliar to the students.

The distinctive nature of the construct of the hiccup

A subtle difference between the construct of the hiccup and other interpretations of unanticipated/unplanned events in the classroom is that a hiccup is by definition an event that is explicitly noticed *by the teacher*, although he/she may not necessarily act in response to it in the moment. So unlike Stockero and van Zoest's "pivotal teaching moment" or Arafeh, Smerdon and Snow's "teachable moment", which are concerned with the potential of unanticipated/unplanned events as a means for teacher educators to discuss potential improvements to classroom practices, a hiccup is a potential epistemological construct in that it provides a trigger for the teacher to activate untapped aspects of teacher knowledge.

Hiccups provide an insight into the processes through which a teacher's contingent knowledge develops – expanding Rowland et al's element of the knowledge quartet, "contingency", by positioning the technology as a source of contributions to the lesson. It does this in two ways. The technology informs and underpins the contributions made by students whilst also impinging directly on classroom practices when it is used as a mathematical authority within whole-class teaching episodes - to which the teacher is expected to respond.

Finally, although Table lists the types of hiccups that were observed, it masks some underlying features concerning the potential impact of each particular hiccup on the individual teachers' epistemological development. Some hiccups (mostly Types 4 and 5) required an immediate (re-)action by the teachers in order to maintain the mathematical progression of lesson activity. Other hiccups (Types 1 and 2) were more latent and therefore less critical to the flow of the lesson activity, but prompted deeper reflections by the teachers in their post-lesson evaluation as they rethought their activity designs. An example of this was where students were dragging sides and vertices of a triangle to explore variant and invariant properties but, due to insufficient labeling, the teacher faced a series of recurring hiccups as the students were unable to take part in a meaningful whole-class discussion.

The complete data analysis process also focused on each teacher's actions in response to individual hiccups as they arose (see Figure 2) leading to the concept of a *response repertoire* (Clark-Wilson, 2010, p.185), which was defined according to three modes of action:

- No immediate response repertoire: the teacher did not appear to have any immediate contingencies, as evidenced by offering a 'holding' response such as 'we'll look at that another time'.
- A developing response repertoire: the teacher, whilst obviously acting in the moment began to offer a response that involved dialogue, use of the technology or both.
- A well-rehearsed response repertoire: the teacher responded confidently and involved dialogue, use of the technology or both.

Hence, it was possible to evidence the teachers' epistemological developments over time as they reflected upon previous hiccups and rehearsed a malleable set of response repertoires for a range of possible future related contexts.

Applying the idea of a hiccup theory within professional development

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What follows is a description of how the construct of a hiccup theory is proving a useful tool in supporting the ongoing design of the professional development approach within a project that is seeking to address the reported underuse of dynamic digital technologies by students in secondary mathematics classrooms in England at scale (Bretscher, 2014; Office for Standards in Education, 2009, 2012).

Cornerstone Maths (CM) has evolved from the seminal work of Jim Kaput and his colleagues in the USA (Hegedus & Roschelle, 2013) through a “curriculum activity system” approach (Vahey, Knudsen, Rafanan, & Lara-Meloy, 2013) that integrates teaching materials, bespoke web-based software and teacher PD with a focus on *hard-to-teach* topics in key stage 3 mathematics (e.g. linear functions, geometric similarity and algebraic expressions).

The research methodology for the Cornerstone Maths project has involved three staged online questionnaires for teachers before, after and during each CM unit, together with video and audio-recorded classroom observations and individual and focus group interviews. Schools that become part of CM each nominate two teachers to attend a one-day PD session (with some follow-up meetings in schools), followed by webinars and ongoing support through an online project community[2]. The face-to-face PD is designed to enable teachers to: see the ‘big picture’ of the mathematical progression embedded in the curriculum unit; experience a reflective hands-on experience in a range of roles (learner and teacher); envision new classroom practices involving student uses of dynamic technologies; plan and share their lesson designs (to include looking at students’ outcomes); learn of the expectations of their involvement in the project community; and learn of their role in the research process (which was primarily to teach the unit and provide feedback through online questionnaires).

It is within the face-to-face element of the PD that the opportunity arises to foreground “known” hiccups, whilst also preparing teachers to anticipate and discuss potential hiccups. It is hypothesized that an important contributory factor to the successful scaling of CM is that teachers can experience, notice and then act productively in response to hiccups.

We will now focus on a particular CM curriculum unit on linear functions and outline how aspects of its design principles relate to the construct of hiccups.

Figure 3 shows a screenshot of the software that accompanies one of the 14 activities within the unit.

[Figure 3 near here]

In this particular activity, as students ‘play’ the animation, the character moves along the number line and the synchronised dynamic representation of the graph and its corresponding table of values are highlighted on the screen. Within the activity, students are asked to edit the simulated motion to create different linear motion scenarios. The software design facilitates this in one of two ways. Students can either edit the values of the gradient and/or intercept within the function (the algebraic representation) or they can drag particular hotspots on the graph to alter its features (the geometric representation).
The curriculum unit has the following overarching design principles:

- Dynamic simulation and linking between representations.
- Simulation controlled by the graph or the mathematical function.
- Show/hide representations, as appropriate.

However, for teachers who are unfamiliar with teaching approaches that introduce the concept of linear functions using dynamic digital technologies in this way, making sense of the dynamic simulations and understanding how the dynamic representations relate to each other is non-trivial new content knowledge. This paper draws data from the research team’s experiences within the professional development sessions for participating teachers, alongside teachers’ responses to the final online questionnaire[3] to provide an insight into a particular hiccup. This is accompanied by a more detailed explanation of the software to help contextualize and make sense of the teachers’ comments.
In the unit of work the students are first asked to ‘edit the graph’ within an investigative task ‘Texas Road Rally’ about a fictitious car that travels from a given start position to a given end point, travelling at a constant speed throughout. The opening screen and accompanying task instructions are shown in Figure 4.

[Figure 4 near here]

When the students select the edit button, three ‘hotspots’ appear on the screen. Each of these hotspots is constrained to move in a particular way and drive a particular mathematical variable within the linear model.

Hotspot 1: This point varies the start position of the moving character/object, i.e. the value of the intercept parameter, '*c*', in the linear model. When dragged, it is constrained to move vertically on the y-axis.

Hotspot 2: This point varies the total time for the simulated motion. It is constrained on the x-axis and so, when moved it will extend the length of the line segment. It does not affect the speed of the motion, i.e the gradient of the line segment does not change

Hotspot 3: This point varies speed of the character/object, i.e. the value of the gradient parameter, '*m*', in the linear model. When dragged, it is constrained to move vertically.

The subtleties of this software design force the students to think about the different variables at play within a linear model of simulated motion.

During PD sessions we had observed teachers' frustrations as they began to edit the graph for the first time, suggesting an occurrence of *Hiccup Type 6.* These observations were also corroborated by two particular responses to the post-teaching questionnaire:

- *Students found moving the lines on the graph difficult at first. They wanted to touch the endpoint and move it to the place required.*
- *Pupils had to reset tasks when the graph went off the grid as it was difficult to find again. I think the axis should adjust automatically.*

Reviewing these two comments, it is likely that the teachers had observed their students' frustrations, as they could not drag the point at hotspot 2 arbitrarily on the screen. In the absence of observation, it is not possible to know if each teacher had experienced *Hiccup Type 5* or whether each teacher had a *response repertoire* to resolve the 'issue' they had reported, only that they had noticed the students' difficulties.

The second teacher's comment about "the graph going off the grid" relates to a phenomenon that has been observed in teachers' PD sessions as CM teachers have been introduced to the software. This has been replicated in Figure 5.

[Figure 5 near here]

This situation occurs when hotspot 3 is dragged to extend the time traveled such that (because the gradient is already fixed) the end-point will reflect the position of the character/object after this extended amount of travel time. However, because most users want to arrive at a situation where the final situation is at 350 miles (to satisfy the task instructions) they cannot

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see hotspot 2 on the screen in order to achieve this goal. Classroom observations and focus group interviews have revealed that teachers seem to have one of three responses to this particular perturbation. The first is to interpret this as a technical issue, and ask students to refresh the page, which resets the task to its opening state. The second encourages them to use the zoom functionality to adjust the axes and see more of their graph. The third prompts them to think about why the end point has disappeared and consider how fixing the gradient of the line first and then extending the journey time to achieve the required end point might be a better strategy.
Table articulates the strategic knowledge pertinent to this task described that the (potential) hiccup might provoke and suggests a professional development task that might explicitly expose the phenomena to teachers.

[Table 2 near here]

The ongoing data from the teachers involved in the CM project is continuing to reveal a range of hiccups at particular points in the activities. Many of these hiccups do seem to coincide with the students' "instrumentation" phase as they learn to use a new mathematical technological tool, - an aspect that is well-documented in mathematics education research literature (Hoyles, Noss, & Kent, 2004; Trouche, 2004). Interestingly, it is a common reaction by teachers to suggest ways in which they think the software might be improved, which often involves the removal of functionality that was considered important within the original set of design principles. For example, for the "lost hotspot" hiccup described previously, several teachers have suggested a redesign of the software to automatically adjust the axes, which could remove opportunities to act directly on the software. However, in order that teachers can understand and appreciate the software design principles, the idea of this particular hiccup provides a useful opportunity to share and discuss the more nuanced functionalities and their potential positive impact on students' understanding.

Discussion and implications for the design of professional development 'at scale'

The original research that led to the development of the hiccup idea was highly contextualized in that it involved a close relationship between the two case study teachers and the first author (as the primary researcher) whereby significant conversations took place that undoubtedly shaped the teachers knowledge growth. From these case study teachers' perspectives there were some distinct advantages to having a ‘significant other’ in their classrooms with whom they could discuss their task designs and lesson outcomes in depth. In addition, this dialogue was highly personalised for each teacher as it was focused on the identification and validation of the hiccups that had emerged from the systematic collection and collation of the synchronised lesson data, which included their students’ outcomes. This prompts the question as to whether it is possible or indeed desirable to use a known hiccup within a pre-designed PD task with teachers who have not necessarily experienced that particular hiccup first-hand in their own classrooms.

We conjecture that it is both possible and desirable to prepare teachers for such occurrences during the one-day PD event (or during additional webinars and school-based meetings) through PD tasks that require teachers to experience the phenomena, make sense of it from an epistemological perspective and then discuss and rehearse possible “response repertoires” for a range of possible classroom contexts (Clark-Wilson, 2010, p185). Such contexts would consider different response repertoires for key stage 3 students of different: age; prior mathematical attainment; level of confidence/experience with technology; etc. Such PD tasks would aim to develop teacher knowledge for the particular scenario of technology enhanced learning, whilst also integrating aspects of more traditional teaching approaches as a means to understand the transformative contribution of the technology. This suggests that the design of a PD course that aims to prepare teachers for teaching with dynamic digital technologies at scale should prepare them for hiccups - phenomena that look highly likely to cause perturbations.

The next stage in our research will be to continue to investigate the potential for using hiccups productively within PD at scale, which involves three considerations. Firstly, the challenge to devise effective methodologies whereby teachers’ lesson hiccups are made visible at scale. Secondly to use this information to inform the design of new PD tasks that foreground these known hiccups. Finally, to consider the new methodologies necessary to capture data on the impact of these PD tasks on teachers’ knowledge and classroom practices in technology enhanced lessons.

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[1] Each lesson activity might include all or some of: a formal lesson plan, the teacher's hand-written personal notes; a lesson structure for use in the classroom (for example a Smart NoteBook or PowerPoint file); a software file developed by the teacher for use by the teacher (to introduce the activity or demonstrate an aspect of the activity); a software file developed by the teacher for use by the students that would normally need to be transferred to the students handhelds on advance of the lesson; a task or instruction sheet developed by the teacher for students' use; students' written work resulting from the activity; and students' software files captured during and/or at the end of the activity.

[2] The online project community was facilitated by the government funded National Centre for Excellence in the Teaching of Mathematics portal www.ncetm.org.uk.

[3] At the end of the 2013-14 academic year, 111 of the project teachers had completed the survey.

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Figure 1

Clark-Wilson, Alison, & Noss, Richard. (2015). Hiccups within technology mediated lessons: A catalyst for mathematics teachers' epistemological development. *Research in Mathematics Education, 17*(2). doi: 10.1080/14794802.2015.1046476

Name
TP9 Hiccup01 – Student's instrumentation issue entering X^2
TP9 Hiccup02 – Teacher's assumption that students 'notice' the brackets
TP9 Hiccup03 – Teacher's assumption that students 'see' the invisible multiplication sign
TP9 Hiccup04 – Students' task sequencing issue
TP9 Hiccup05 – Teacher's assumption that task sequencng will lead to counter-examples
TP9 Hiccup06 – Technology fails as teacher attempts to use 'Live presenter' in final plenary
TP9 Hiccup07 – Teacher noticing that the worksheet uses letters other than x
TP9 Hiccup08 – Student doubts the authority of the MRT 'Why did it say false'
TP9 Hiccup09 – Insufficient specificity for the labelled objects under discussion

Figure 1 The complete set of 9 lesson hiccups from Tim's lesson *Equivalent quadratic equations* as coded in Nvivo8 software. [Note: MRT means Multi-representational technology].

Figure 2

Name
TP9 Action01 – Appreciated 'hiccup' over choice of expressions
TP9 Action02 – Apprecoated that the MRT could have accepted any letter as a variable
TP9 Action03 – Made explicit link between 'Medley's rule' and grid multiplication
TP9 Action04 – Noticed the students appreciated being sent the same file to work on
TP9 Action05 – Noticed that students were highly motivated by MRT response 'true'
TP9 Action06 – Privileged 'press control z to undo'
TP9 Action07 – Responded to students' concerns about needing to type the brackets
TP9 Action08 – Reversed question order in second section of task 2 to force inverse strategies
TP9 Action09 – Revised language wrt needing brackets
TP9 Action10 – Revised the task to account for TP9Hiccup02
TP9 Action11 – Appreciated that the students had noticed a bold' graph
TP9 Action12 – Appreciated student's connection between 'Medley's rule' and grid multiplication

Figure 2 Tim's 12 actions (as identified from the research data) in response to the hiccups within his lesson *Equivalent quadratic equations* as coded in Nvivo8.

Figure 3

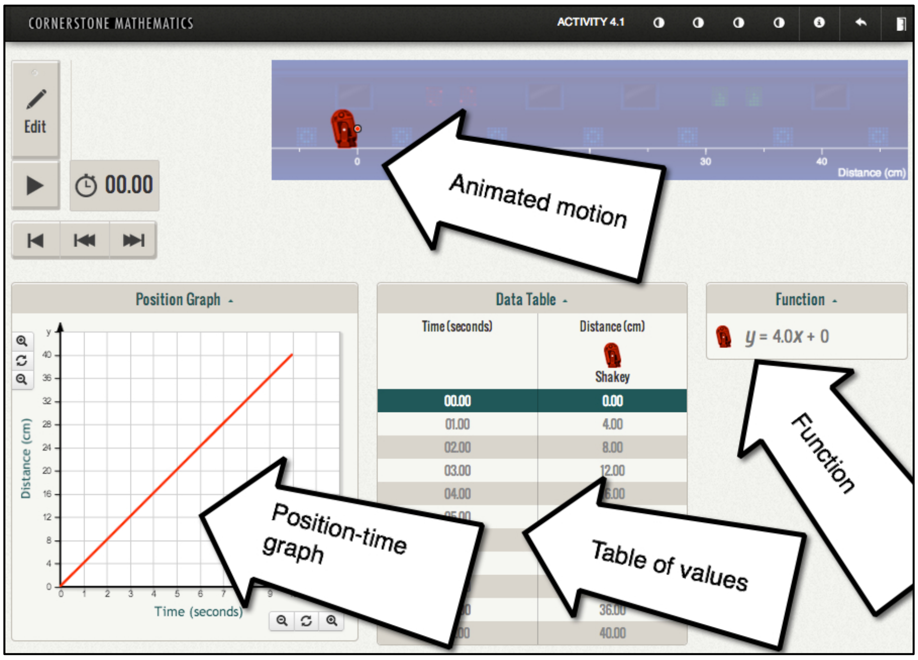

Figure 3 An activity from the CM curriculum unit on linear functions, selected as it displays the full range of dynamic mathematical representations.

Figure 4

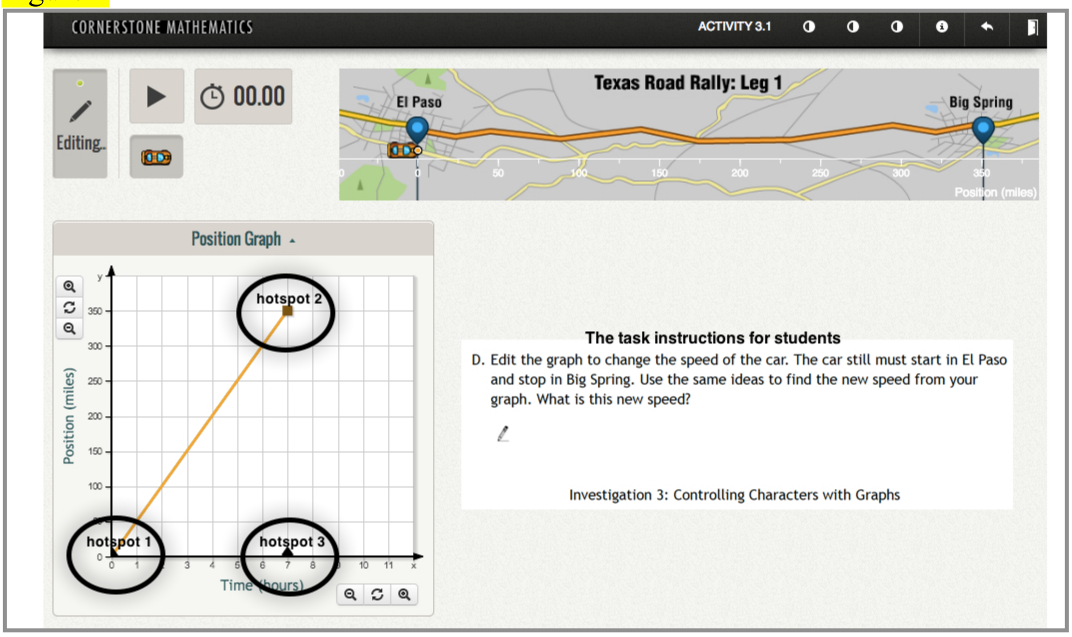

Figure 4 The opening screen of the software and the task instructions for 'Texas Road Rally'.

Figure 5

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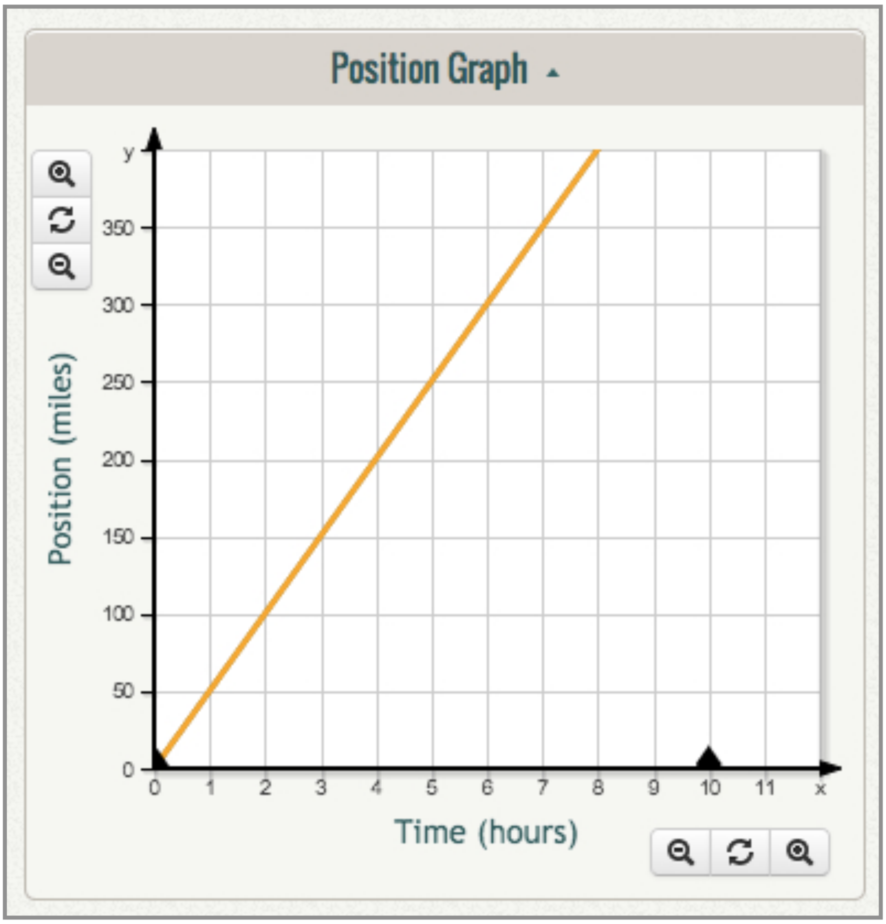

Figure 5 The line segment ‘disappears’ from the graph window.

Table 1

Hiccup type	**Underlying condition**
1. Aspects of the initial activity design:	• Teachers were dissatisfied with their choice of initial examples. • Teachers were dissatisfied with their sequencing of examples. • Teachers and students had insufficient definition of language (and labelling) to identify and discuss objects displayed by the technology. • Students were unfamiliar with the chosen pedagogical approach.

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2. Interpreting the mathematical generality under scrutiny:	• Students struggled to relate specific cases to the wider generality under scrutiny. • Students struggled to appreciate the permissible range of responses that satisfied the generality. • Students failed to notice the generality.
3. Unanticipated student responses as a result of using the technology:	• Students' prior understanding was below the teacher's expectation. • Students' interpretations of the activity objectives differed to those of the teacher. • Students developed their own instrument utilisation schemes for the activity.
4. Perturbations experienced by students as a result of the representational outputs of the technology:	• Students were unsure about particular syntactic or geometric outputs. • Students doubted the *authority* of the syntactic output (Stacey, 2008).
5. Instrumentation issues experienced by students when making inputs to, and actively engaging with the technology:	• Students were unsure about entering numeric and syntactic data. • Students were unsure about plotting free coordinate points. • Students were unsure about grabbing and dragging dynamic objects. • Students struggled to organise on-screen objects within the screen space. • Students were unsure about navigating between application windows. • Students enquired about a new instrumentation. • Students deleted on-screen objects accidentally.

6. Instrumentation issues experienced by teachers whilst actively engaging with the technology[3]:	• The teacher forgot how to display a particular representation during whole class teaching.
7. Unavoidable technical issues:	• The teacher could not transfer the necessary files to the students' handheld devices. • The teacher's software or handheld device screen could not be displayed to the class.

Table 1 The seven hiccup types and related underlying conditions that prompted the teacher's perturbation.

Table 2

Strategic knowledge (Shulman, 1986):	Knowledge of machine constraints when working on linear functions with technology. Knowledge of the way in which students make sense of their interactions with technology concerning linear functions. Knowledge of the way in which students relate the model of a linear function (i.e. $y = mx + c$) to the interactions they have with the software – and the reciprocal way that their interactions with the draggable objects relate to the linear model.
Related hiccup	The process of editing the graph of a given linear function may result in the end-point of the graph 'disappearing' from the graph window.

Design principles for PD approach	**Aim**: develop understanding of the way in which the draggable points relate to the variant properties of linear functions in a motion context. (i.e. contrasting traditional knowledge of 'm' and 'c' with the notions of start position, end position and speed.) **Task 1**: Following an interaction with the software when working on the investigation 'Texas Road Rally' the teachers are given a copy of the screenshot of the software (as in Figure 4) and informed that this is a common response to students' first attempts to edit the graph in order to answer question D within this investigation. Pose the following questions to the teachers for discussion in small groups. • What do you think the student(s) may have done that resulted in this screenshot? • What are the different actions that the students might take to enable them to achieve the task outcomes? • Are some of these actions more mathematical than others? • Plan how you might respond to the student(s). • How might your response differ for older students or students with different mathematical starting points?

Table 2. Design principles for PD task that address potential hiccups for the unit on linear functions.