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SCOUR AROUND MARINE FOUNDATIONS IN LAYERED SEDIMENTS: A MATHEMATICAL MODELLING APPROACH

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Abstract

In order to improve the understanding of scour processes in layered conditions for offshore foundations, a laboratory study was conducted regarding equilibrium scour depth vs. time curve and its adjustment to hyperbolic and exponential/linear combined functions. The present paper provides a mathematical study describing a set of scour tests in order to clarify the scour depth evolution for complex, non-cohesive soil configurations.

Keywords: Scour Curve; Foundations; Layered Sediments; Linear, Exponential and Hyperbolic fitting.

1. Introduction

Local scour phenomena in offshore foundations may not only depend on the hydrodynamic conditions in the vicinity of structural element, but also on the sediment characteristics of the foundation's soil. Offshore structures are often founded in non-uniform soils which present different types of sediments along the length of the pile (Ferradosa, T. 2012a). Offshore structures are subjected to scour processes which can lead to stability and security problems.

For complex soil configurations such as layered sediments it is hard to predict and to model the scour phenomenon with a high level of certainty. Therefore it is crucial to increase the available knowledge of this process in such geotechnical conditions, in order to provide a better assessment of scour severity and achieve good prediction methods for the design of offshore structures (Ferradosa, T. 2012b). The costs associated with foundations in this type of structure can reach about 30% of total investment (Matutano, C. et al., 2013). An increasing comprehension of the local scour process around marine foundations will enable more favourable cost/benefit ratios in offshore investments, hence leading to considerable optimizations in the currently available design methodologies (De Vos, L. et al., 2012).

The present study encompassed both laboratory and mathematical modeling work on scour. Tests were performed in a unidirectional current flume with a slender pile, founded in layered sediment configurations representative of typical marine foundations such as the offshore windfarms located in Kentish Flats, Robbin Rigg or North Hoyle (T. Ferradosa, 2012a).

The mathematical modeling performed was mainly focused on the scour depth curve versus time aiming for an accurate description and comparison of the data acquired regarding scour depth and its equilibrium values in the several tests.

The series of scour tests include two uniform tests with fine and coarse sand and five layered ones with fine sand overlying coarse sand. The top fine sand layer depth was varied between experiments. The scour depth development with time was measured; and a hyperbolic function was adjusted to the acquired data. Comparisons were also made with the exponential and linear combined functions. The present paper shows that the combination of a linear component and exponential/hyperbolic fitting is important for removing the function-related error existing between the fitted values and the experimental data's curve, in multilayer sediment setups.

The research aimed to provide a specific relationship between the development of scour depth and the thickness of sediment layers, with the purpose of reaching a better understanding of local scour phenomena in complex non-cohesive soil configurations. In order to achieve the intended goals several mathematical functions were used to describe the maximum scour depth. The analysis performed intended to compare the results between the scour tests and to clarify the evolution of the scour pattern after the layer transition. The conclusions reached in this publication aimed to contribute to future numeric modeling in scour investigation for marine foundations.

2. Theoretical Concepts

Whenever a structure is placed in a certain flow, such as currents, waves or currents and waves combined, the flow-structure interaction tends to generate the scour phenomenon due to the contraction of the available section for the water to pass through or due to an increase in the bed shear-stress field near the foundation (Melvile B. *et al*, 2000). If a certain mass of water approaches the structure, an alteration of the velocity fields occurs. The oncoming flow, which constantly goes towards the structure, tends to go down towards the foundation soil (downflow) due to the pressure gradient. Near the surface, lower pressure values are linked to higher velocities; on the other hand, on the bottom part near the sediment bed, the friction forces are more intense and therefore lower velocities are presented. This section of the flow is the so called bed-boundary layer (Sumer, M. & Fredsøe, J. 2002).

The descending movement of the oncoming flow generates a turbulent phenomenon near the base of the pile, hence leading to the formation of horseshoe vortices from the upstream to the downstream side of the pile. The horseshoe vortex is the major local scour mechanism for unidirectional current cases. On the downstream side, the Lee-wake vortices also contribute for the scour extent. Their influence in the present process tends to be increased with more complex hydrodynamic conditions, such as the existence of currents and waves combined (Sumer, M. & Fredsøe, J. 2002).

Such scour mechanisms, as well as the increasing velocities around the submerged part of the structure, lead to an increase of the bed shear-stress in the sediment bed (Whitehouse, R. 1998). The local scour mechanisms described before are represented in figure 1. The amplification of the bed shear-stress is usually expressed through the amplification factor (α). Once it overcomes a critical value, which depends on the diameter of the particles, the sediment tends to be dragged to the downstream side. However, if for a certain case the shear-stress is below the critical value, but approaching it, some sediment transport can occur only near the pile, particularly in the lee-wake side. This situation is called the clear-water regime. For these situations the transportation of solid matter on the downstream is not compensated by sediments dragged from the upstream zones. Hence a scour hole is generated putting at risk the stability of the structural element's foundation.

At a certain stage, the scour hole reaches equilibrium morphology (both in depth and extent) because the horseshoe vortex's intensity tends to diminish with the scour depth. It is due to this relation that the equilibrium scour morphology is achieved, generating an approximately exponentially shaped function for the scour depth development with time (Whitehouse, R. 1998), as can be seen in the following figure 2:

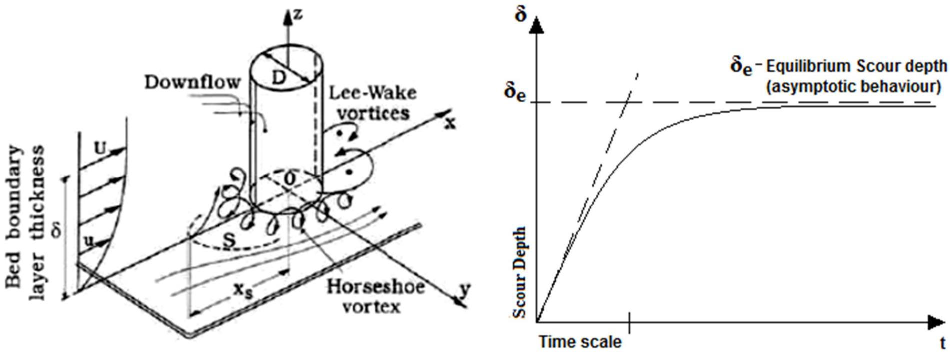

Figure 1 and 2. Local scour mechanisms (on the left, adapted from Sumer, M. & Fredsøe, J. 2002) and typical exponential shape for the scour depth versus time curve (on the right).

3. Methodology & Laboratorial Activities

For the scour tests a unidirectional current flume was used with a rectangular cross section of 0.36 m per 0.25 m and a total length of 8.59 m. The following figures 3 and 4 provide the setup of the sediment box zone and an overall layout of the flume:

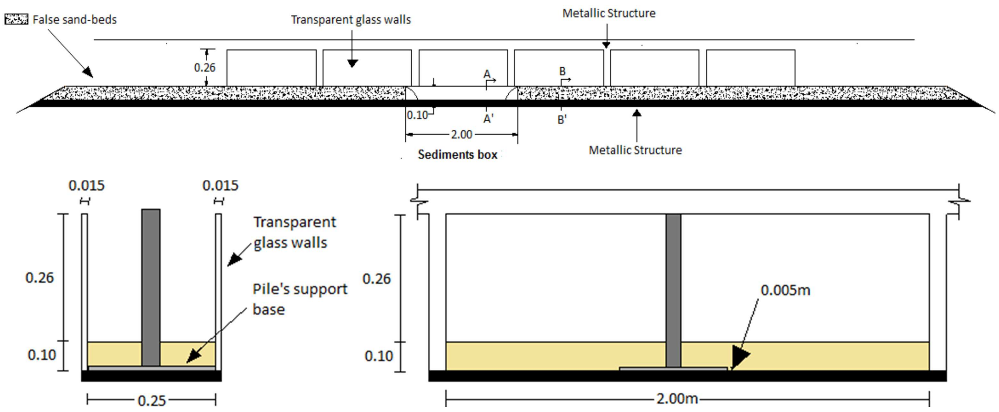

Figure 3 and 4. Overall layout of the flume (above) and cross-section and longitudinal section of the sediment box (below).

These experiments were performed with a circular pile with 40 mm diameter (*D*) and constant water depth (*h*) of 17 cm. The target flow velocity was 19 cm/s in a clear-water regime near to the critical velocity determined in preliminary tests for the fine sand (u_c=24cm/s). The main reason for not using a higher velocity than 19 cm/s was to make sure that no ripples on the upstream side of the pile would appear and generate geometrical scale issues in the physical modelling conditions.

The sediment characteristics are presented in table 1. Both sediments can be considered to be uniform, since their uniformity parameter (dimensionless), σ_D, is below the range of 2 (Ferradosa, T. 2012b).

Table 1. Characteristics of the sediments.

Sediment	d_{16} (mm)	d_{50} (mm)	d_{84} (mm)	ρ (kg/m^3)	σ_D
Fine Sand	0.16	0.23	0.32	2650	1.41
Coarse Sand	0.55	0.64	0.77	2650	1.18

In order to start a certain scour test, the pile was placed in the sediment box with the desired foundation soil configuration, and then the flow was gradually increased up to the target velocity. After this initial phase the test was carried until the equilibrium phase was reached. Cardoso, A. & Bettess, R. (1999) criterion was used to define the equilibrium phase During the experiments, 2D profiles of the sediment bed were taken by means of an echosounder profiler. The velocity values were monitored with two velocimeters: ADV – Acoustic Doppler Velocimeter and LDV – Laser Doppler Velocimeter.

Of the tests analysed in this paper, tests 4 and 6 (0 and 100 mm of layer height, respectively) are representative of single layered settings, therefore being the boundaries of the expected variability, in terms of maximum and minimum values of scour depth. Test 1 experienced an over-consolidation period of the sediment bed which led to lower values of scour depth than the expected. Due to the characteristics of its setup, test 1 functions as a control test and is expected to have lower quality (hard to fit to) behaviour. The top layer thickness (fine sand) and the corresponding test numbering are presented in Table 2. Top layer thickness for each TestTable 2. Other detailed conditions regarding the tests performed, including velocity profiles and measurements, compaction of sand, turbulence levels and tests procedures can be consulted in Ferradosa, T. (2012b).

Table 2. Top layer thickness for each Test.

Test Number	1	2	3	4	5	6	7
Top layer thickness (mm)	25	25	10	0	40	100	55

4. Mathematical functions and fitting

For the study of the mathematical fitting capability of the scour depth curve, three independent mathematical functions (relating scour depth and time) were considered, namely the two most commonly used – the Hyperbolic (Eq. 1) and the Exponential (Eq. 2; based on Franzeti et al., 1982) – and the Logarithmic (Eq. 3). The parameter *a, b, c* and *d* are the fitting parameters of the functions, *f(x)* and *f*(x)* stand for scour depth (*δ*) and *x* represents the time (*t*) in minutes.

Two variations (named options 2 and 3 in comparison with option 1) were considered for each of these functions, namely (2) a linear component added to the original function (Eq. 4) and (3) the same linear component added only after the layer transition (Eq. 5). The variations of the initial mathematical functions (options 2 and 3) were designed for adapting the idealized functions to the multilayer test settings.

$$f(x)=\frac{1}{x^a \times b}+c \quad [1]$$

$$f(x)=a\times\left(1-\frac{1}{e^{-b\cdot x^c}}\right) \quad [2]$$

$$f(x)=\log_a(x+b)+c \quad [3]$$

$$f^x(x) = f(x) + d \times x \quad [4]$$

$$\begin{cases} f^x(x) = f(x), & if\ x \leq \varkappa \\ f^x(x) = f(x) + d \times x, & if\ x > \varkappa \end{cases} \quad [5]$$

5. Analysis and Discussion

The following Figure Figure is provided by analysing the ordinate variation between tests (for the same abscissa), using test 6 (equivalent to 100 *mm* of fine sand) as the reference abscissa. A perfectly matching behaviour between tests will lead to the red 45º line.

As shown in figure 5, when the scour is limited to the top layer, the tests present similar evolution to test 6, within certain variability limits. However, when scour reaches the bottom layer (coarse sand), the curves veer from the 45º line (Test 6). Such variations suggest that the tests with multilayer setup do not behave has a simple overlap of a single layer test. When the transition section is reached the fine and the coarse sand get mixed, therefore the d_{50}, the σ_D and the interaction flow/sediments changes. It is hard to quantify and to account for these changes in prediction models. In order to model the scour after the transition layer it is important to account for other formulations (e.g. options 2 and 3) which are more complex than option 1 or other semi-empiric equations.

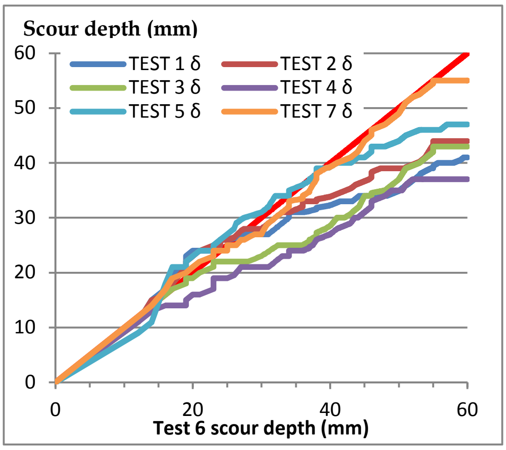

Figure 5. Ordinate comparison from each test with test 6 (100 *mm* of fine sand).

In the tests performed, the presence of fine sand mixed with coarse sand tended to increase the scour depths values in comparison with test 4 (0 *mm* of fines). For tests 1, 2, 3, and 5, the decrease rate of scour depth until the stabilization presented higher variations, than the ones showed in the single layer setup, hence corresponding to a higher flexibility of the scour hole. The exception of test 7 (55 *mm*) may be explained due to the fact that, after the 55 *mm* of scour depth, the strength of the horseshoe vortex was very low. Therefore there was no significant mixing of the sediments. Besides that, due to the coarse sand subjected to a weak horseshoe vortex no significant scour occurred in the bottom layer. For tests 1, 2, 3 and 5 the parameters *a, b, c* and *d* from equations 1, 2, 3 and 4 showed a good potential for scour development fitting.

The upgrade from option 1 to options 2 and 3 was meant to simulate the scour depth behaviour after the transition to the bottom layer. Whilst other, higher order, relationships could have been used, a more extensive series of tests is necessary for an accurate verification of their applicability. An example of the fitting performed, namely using option 1 hyperbolic function, is shown in Figure for all tests:

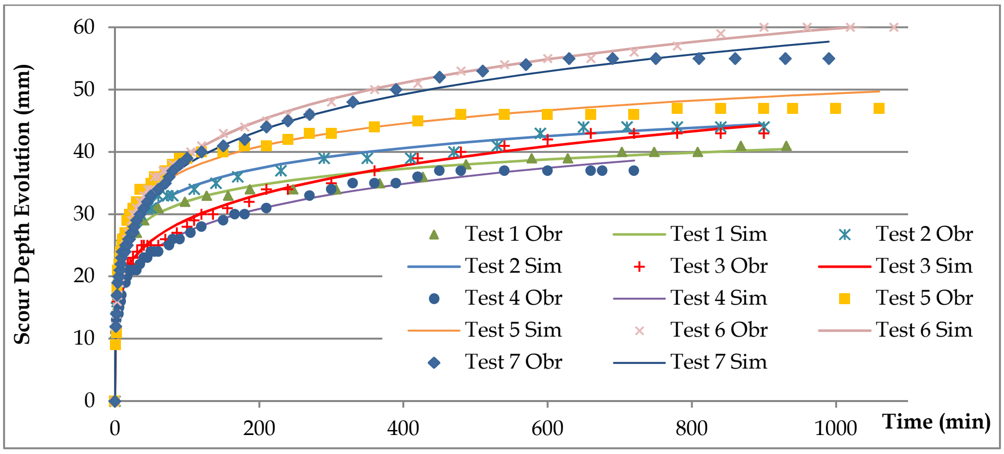

Figure 6. Ordinate comparison for simulated (Sim) and observed (Obr) values with option 1 Hyperbolic.

The total square errors resulting from the mathematical functions in equation 1 to 3 are displayed in Table 3. Options 2 and 3 presented smaller square errors in comparison with option 1, since this mathematical approach with the linear component provides a higher flexibility to the simulated curves. The multilayer setup was better fitted by these options. However since option 3 only adds the linear component after the transition, there was no improvement for tests 4 and 6 which are single layer. In test 7 almost no scour occurred after the transition layer therefore the improvement between option 1 and 3 for the 55 mm top layer was not significant.

Table 3. Total square error (mm^2) in the fitting of each function to the scour depth curve.

Option	Function	Test 1	2	3	4	5	6	7
	Log	62.5	41.9	155.0	70.6	119.0	106.2	138.4
1	Hip	63.7	46.9	42.3	24.8	124.1	21.1	42.6
	Exp	75.4	61.4	47.0	24.1	54.3	28.8	32.5
	Log + Lin	62.0	41.2	39.3	38.7	37.7	35.2	84.8
2	Hip + Lin	62.9	45.0	38.1	24.8	39.2	21.1	24.6
	Exp + Lin	23.1	25.7	39.4	22.5	41.0	15.6	24.7
	Log + Partial Lin	61.9	41.2	39.3	70.6	33.7	106.2	126.3
3	Hip + Partial Lin	62.9	45.0	38.1	24.8	34.9	21.1	25.8
	Exp + Partial Lin	23.7	26.0	39.4	24.1	40.6	28.8	25.6

The comparison of the application of the mathematical functions indicates that the hyperbolic and the exponential are the ones which present the smallest total square error. The hyperbolic function displays errors generally equal or below the ones presented by the exponential function (with the exception being test 1). The fitting performed is also important to identify strange behaviours in the experiments. Note that test 1 was the one with 25 *mm* of top layer suffered from an over consolidation period (difference of 15 days).

When this test is compared with test 2, which is the same test without the over consolidation, it can be seen that the exponential fitting from option 3 did not provided very different results. The over consolidation phenomenon was not detected by this fitting, in each option. However, by performing several mathematical approaches, namely the hyperbolic ad logarithmic ones, it was possible to detect the differences between the errors from test 1 to test 2.

Also regarding these functions, upon experimentation, it was concluded that the hyperbolic function requires the most complex and laborious fitting procedure of all function, as there is a considerable dependency between its parameters (requiring several iterations of the Newton-Raphson method). Note that for larger data sets and series of tests this consideration is quite important in terms of the optimization for the numerical modelling approaches.

Regarding the addition of the linear component, the fitting improvement is clear when compared with the initial functions. The partial linear solution (option 3) provides similar results as the full linear (option 2) and is therefore preferable due to the lesser increase of degrees of freedom introduced in the function. In terms of the option 1, the Hyperbolic function appears to be the most appropriate solution for the situation here presented, given its low error and proportion between the increase in the fitting error and the relevance of the multilayer setup.

The 2D-profiles of the echosounder analysis showed that the cavity formed in the upper layer of the multi-layer test is wider than the one resulting from an equal layer of coarser sediments. Since the cavity has considerable dimensions it affects the flow in the vicinity of the pile, as well as the sediment composition in the scour hole. Therefore it is understandable that the scour evolution after the transition between layers does not correspond to a perfect match with the coarse sand test 4.

6. Error Distribution Analysis

Errors observed in the fitting of the mathematical functions may have a multitude of sources. However the ones whose analysis is most relevant at this stage are the random ones (e.g. derived from test setup, inaccuracy of measurements) and the systematic errors (mostly derived from the appropriateness of the fitted function) in the fitting quality.

Given that each function parameters are determined individually for each situation, other systematic errors are not supposed to be reflected in the fitting quality. In order to analyse this aspect, a moving average of the error between observed and simulated (fitted) values was computed.

The spread of the error is successively smaller starting from the Logarithmic and Exponential to the Hyperbolic function, therefore the later one presents the smallest systematic error. These calculations were consistent for options 1, 2 and 3. Figure 7 provides the example for option 1, the time-lapse considered only goes until 600 minutes corresponding to the time frame where all tests where measured (note that the tests had different durations).

A relevant point to note is the consistent error displayed by all the functions, which indicates the need of the addition of an extra component to option 1. For options 2 and 3 these errors decreased to 50% in comparison to the ones in Figure 7.

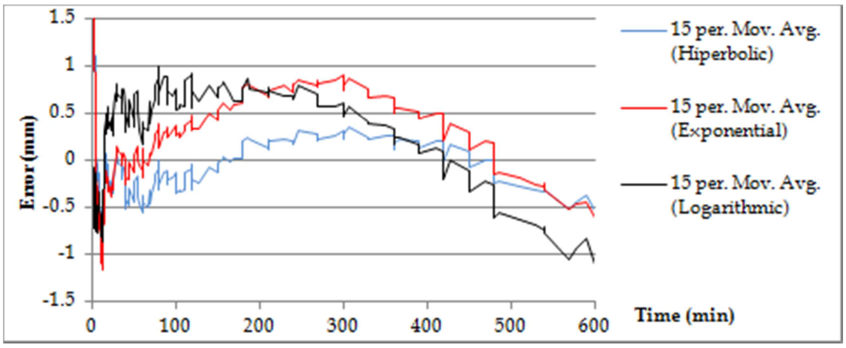

Figure 7. Error *versus* Time distribution for each function.

7. Equilibrium Scour Depth

The fitting of functions to scour evolution have several advantages, namely the removal/attenuation of errors of observation (outliers included) and the smoothing of the observed scour values for calculation purposes and the contribution provided for numeric prediction models of the equilibrium scour depth of a certain given test. Another important fact is that the fitting process, based in experimental data, also accounts for implicit effects such as the ones generated by multi-layer features. The possibility of creating a continuous function describing the scour process instead of a discrete one is also a key element for statistical purposes, such as risk and survival analysis for example.

At this point, given that the nature of the component to be added to the pure functions is still not fully defined (the linear component is just one possibility), an option was made for analysing equilibrium scour depth only for the functions in option 1. One of the reasons why the exponential function is often of interest in scour evolution fitting is that the parameter a in Eq. 2 corresponds approximately to the equilibrium scour depth (e.g. Alabi, P., 2009). In Table 4 are displayed the best fitted a parameters, the observed equilibrium scour depth and the increase in the fitting error due to the usage of an a parameter equal to the equilibrium scour depth (as a percentage of the best fitted function's error).

Table 4. Best fit a (mm), equilibrium scour depth (mm) and the increase in the error (%) from using the observed equilibrium scour depth as the a parameter.

	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Test 7
Best Fit a	42.7	47.3	2683.4	80.2	47.2	78.1	75.7
Equilibrium Scour Depth	40	44	43	37	46	60	50
% increase error	4%	6%	107%	106%	7%	101%	123%

As it can be observed, the increase in error is greater in the tests where the multilayer setup is of greater importance, thereby reducing the reliability of the exponential function. This fact displays the importance of a good fitting of the functions and also demonstrates the exposure of the exponential function to the possibility of an unreasonable (scour-wise) parameter fitting (see test 3, table 4).

The other two functions considered do not possess a parameter directly comparable with the equilibrium scour depth. In these situations, in order to evaluate the capacity of these functions to forecast the equilibrium scour depth, a comparison was made with the average scour evolution rate (derivate of the mathematical fitting function) when the observed scour is in equilibrium phase.

Table 5 displays a significant correlation between the hyperbolic derivate and the layer width, which otherwise cannot be observed for the logarithmic derivate. Further tests are presently being conducted, with the purpose of more precisely defining this relation of bed mobility and the hyperbolic derivate. Note that the hyperbolic derivate increases as the layer thickness decreases, therefore with a larger number of tests, it may be possible to deduce a relationship between these two that allows for the calculation of the equilibrium scour depth.

Table 5 – Layer width in comparison with the hyperbolic and logarithmic derivate.

Test	1	2	3	4	5	6	7
Layer Thickness (mm)	25	25	10	0	40	100	55
Hyp. Derivate	3.26E-04	3.10E-04	3.10E-04	4.12E-04	3.07E-04	1.79E-04	2.71E-04
Log. Derivate	6.70E-04	7.57E-04	1.46E-03	1.12E-03	8.14E-04	1.76E-03	4.68E-03

8. Conclusions

In the present paper, the hyperbolic, exponential and logarithmic functions were applied within three options which included an extra linear component, in order to account for scour depth behaviour after the transition layer. The calculations performed allowed for the following conclusions: without an extra linear component, the hyperbolic function was the one which presented the best overall fit, for both options 2 and 3, the exponential function combined with a linear extra component presented a better fitting than the hyperbolic one (also with the linear extra component). The hyperbolic function presented smaller systematic error in option 1. For situations where the multilayer setting is more relevant, the addition of an extra mathematical component, in order to compensate for the altered behaviour in the second layer, provided for a better description of scour depth evolution. This component was represented by a linear function (which provided a reasonable approximation) but, for application purposes, further studies on its definition are essential since only a small number of tests were considered. Options 2 and 3 have relatively similar errors, option 3 being therefore preferable (due to the smaller flexibility introduced in the functions).

Concerning the forecast of equilibrium scour depth, the direct usage of the exponential function's parameter is not completely free of risks, given its vulnerability to unreasonable parameter fitting (e.g. test 3 in Table 4). Comparatively, the analysis applied to the derivate of the hyperbolic function displayed a more clear relation with the equilibrium scour depth, which, upon further testing, may provide a reasonable approach for equilibrium scour depth estimation. This relation consisted of an inverse relationship between the derivate and the equilibrium depth.

In a more pragmatic analysis, this type of fittings provide a tool that allows for the verification of the tests results and for larger sets of tests it is even possible to correct measurements using this function as a reference situation. Additionally, in scour depth curves, the major models available are mainly semi-empiric and, given the complexity of some of the phenomena involved (particularly in multilayer setups) mathematical functions present a considerable contribution for estimation/definition of generalized numerical models for scour depth evolution. The conversion of the observed scour curves, from discrete to continuous, carries with it considerable advantages in terms of the application of many statistical methods and numerical models, which are the base for non-empiric prediction tools of scour depth.

The number of tests considered does not allow for the creation of a prediction model in itself but it helps to identify the type of functions and analysis that are inherent to it, thereby contributing to future computational models, suited for multilayer tests.

Since the existent theories of prediction do not account for complex soil configurations, this study was also important as a starting point to provide clues for future equations that attempt to estimate the equilibrium scour depth, even if those equations have a semi-empiric nature. Taking into consideration the complexity of scour phenomena, and the small number of tests considered, continuing to study these subjects would prove useful, namely regarding the consideration of a wider range of layers thicknesses and complex hydrodynamic conditions.

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