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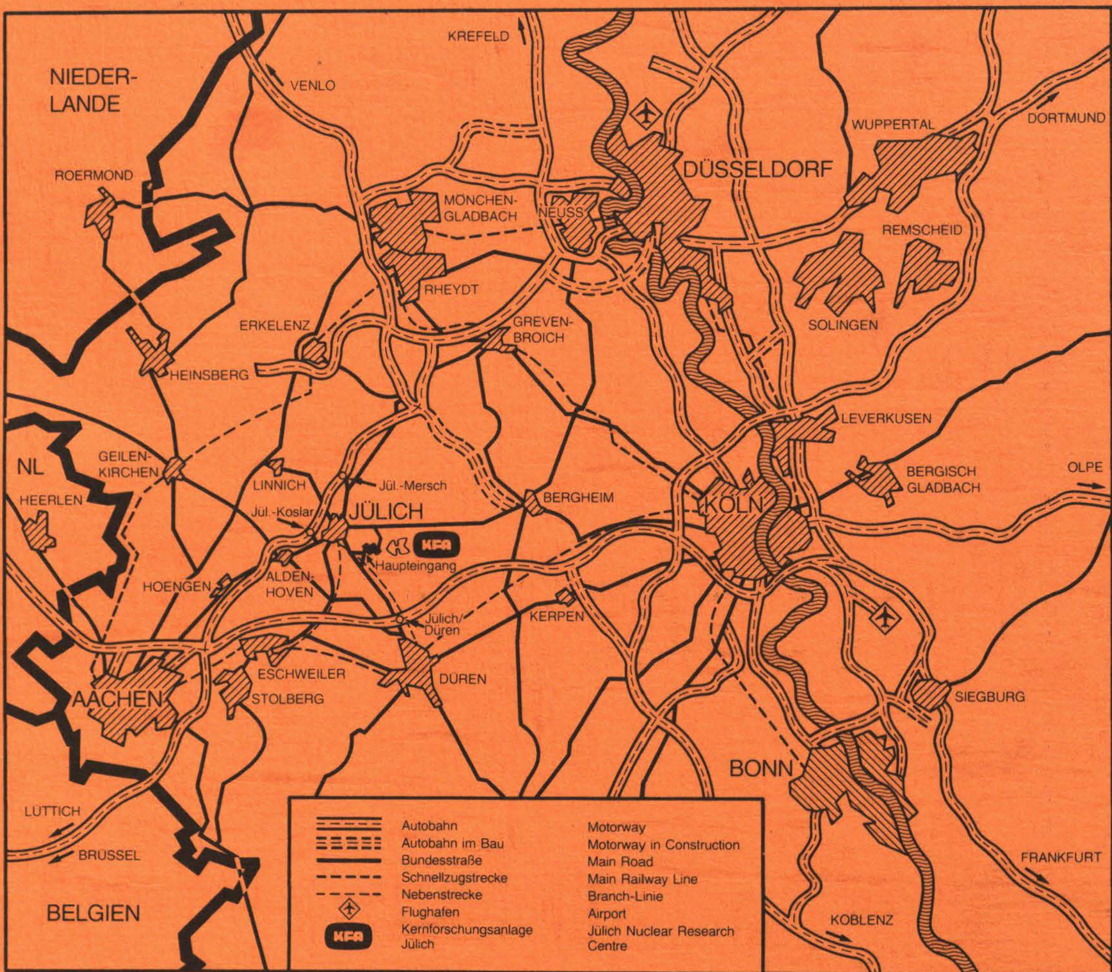

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Position Determination in Scintillation Detectors

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Abstract

Three different methods are described to determine the 'true' positions of a scintillation event in one- and two-dimensional position-sensitive scintillation detectors from experimental calibration data. Under certain assumptions the methods are shown to yield mathematically exact results. Computer-simulated results are presented to demonstrate the validity of the methods, to estimate the consequences of approximations to be made, and to provide figures for the required accuracy of experimental data. Results concerning spatial resolution and linearity of the response are discussed for the case that scintillation events are stored according to the pre-calculated and tabulated 'true' positions.

I. Introduction

A linear position-sensitive scintillation detector consists of a stack of photo-multiplier optically coupled to a light disperser and a scintillation strip with an air gap between the scintillator and disperser. Such an one-dimensional detector has been constructed for neutron detection in a neutron diffractometer at the FRJ-2/11. The scintillation light produced by a neutron capture event is confined to a 90° cone and reaches essentially three photo cathodes. A light reflector on top of the scintillator strip assures that all light reaches the photo cathodes apart from light losses due to absorption and scattering. The neutron capture event at position x is characterized by three PM anode signals I_{k-1}, I_k and I_{k+1} and the number k with maximum anode signal I_k (Fig. 1).

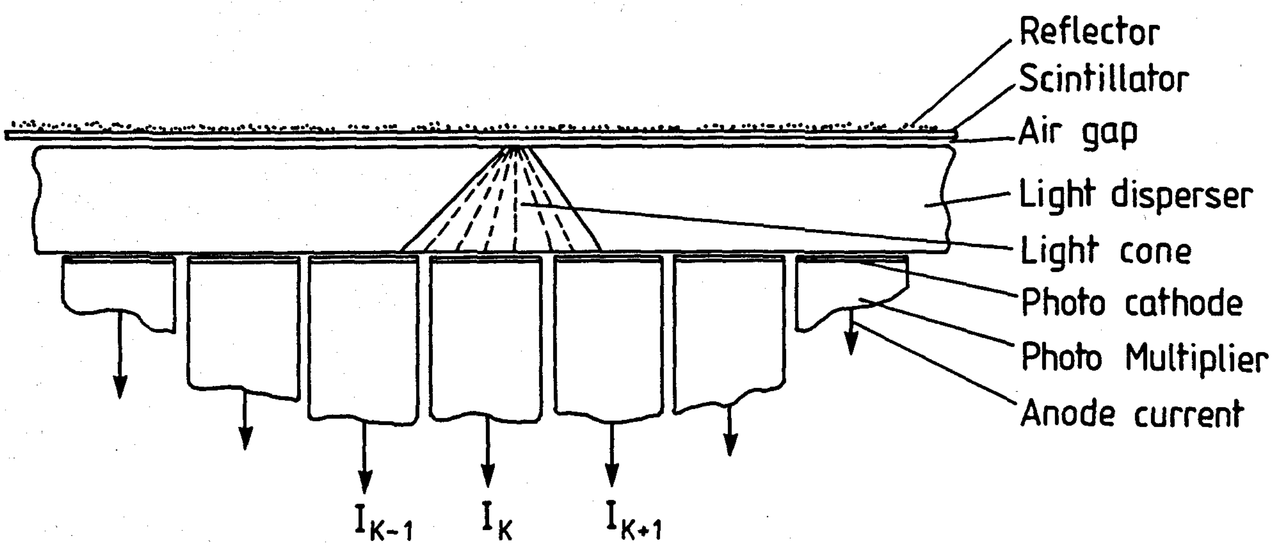

Fig. 1 Schematic view of a one-dimensional scintillation detector

Position information may be obtained from the quotient

$$(1) \quad Q = \sum_i \alpha_i I_i / \sum_i I_i$$

where the coefficients α_i increase monotonically with the PM number i. The quotient Q is then a monotonically increasing function with position. However, Q(x) is by no means a linear function of position x. This implies that the intensity spectrum of Q is not homogeneous if the detector is homogeneously illuminated with neutrons. Another method to define an encoded position is discussed in /2/. For the PM number k of maximum anode signal the ratio q_k is calculated,

$$(2) \quad q_k = \frac{I_{k+1} - I_{k-1}}{I_{k+1} + S_k \cdot I_k + I_{k-1}}$$

and with appropiately chosen coefficient A_k and B_k a continuous Q(x)-function is constructed

$$(3) \quad Q = A_k + B_k \quad q_k$$

This method of a coarse selection of three PM signals has the advantage that the integral linearity becomes independent of the length of the detector and that low resolution ADC's are needed irrespectively how long the detector is. The S_k factor may be adjusted to minimize differential non-linearities. A method has been developed to adjust the A and B values. Again the function Q(x) is not linear in x and therefore the intensity spectrum with respect to this Q is again not homogeneous when the detector is homogeneously illuminated with neutrons.

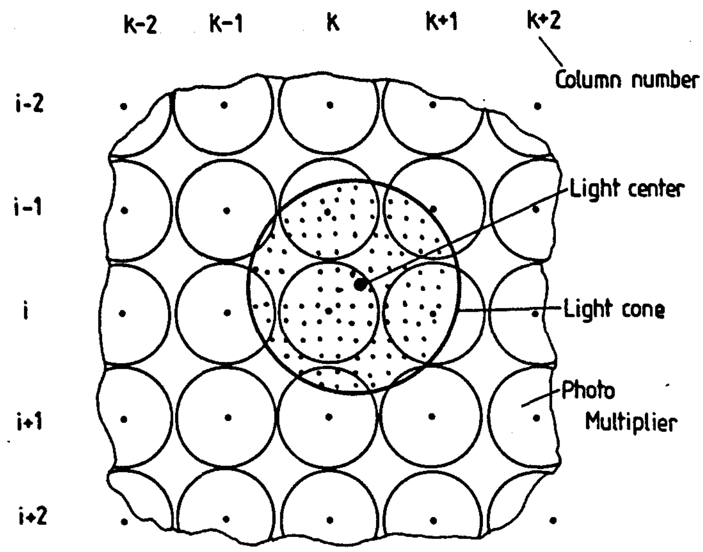

Fig. 2 Top view of a two-dimensional scintillation detector with a square arrangement of PMs

A two-dimensional scintillation detector consists of an array of photo multipliers optically coupled to a light disperser plate and of a scintillator plate. Again an air gap may be between the scintillator and disperser. Such a two-dimensional neutron detector is under construction for the small angle neutron scattering instrument at the FRJ-2. The scintillation light from a neutron capture event is again confined to a 90° cone and reaches essentially nine PM cathodes (Fig. 2).

From the nine PM signals one determines three column signals I_{k-1}, I_k, I_{k+1} by adding column-wise and three line signals I_{i-1}, I_i, I_{i+1} by adding line-wise. These column and line triplets are used to obtain an x and y information, respectively. They can be treated in the same way as described for the one-dimensional detector, to determine the encoded positions Q_x and Q_y.

Experiences with the constructed one-dimensional detector indicated that inhomogeneities of the photo-cathodes cause untollerable non-linearities in addition to the much smaller inherent non-linearities for ideal photo-cathodes. Thus, more thoughts were given to the problems of finding the neutron capture position x from the PM signals I_{k-1}, I_k, I_{k+1}. Because of statistical uncertainties of the PM signals one should look after a method of finding the most likely position for a neutron capture event. If this problem is solved the events would be stored in memory locations addressed by the most likely position which implies vanishing non-linearities and a constant intensity distribution for homogeneous illumination.

In the following, three different methods are described. Under certain assumptions they are exact, i.e. they result the true most likely positions. All three methods need additional information which has to be determined experimentally. For all three methods computer-simulated data are generated in order to demonstrate the power of the method, to estimate the consequences of approximation which had to made and to provide some figures for the required accuracy of experimental data.

II. Position determination based on the light distribution functions

For a single neutron capture event at a certain position x one observes photo anode signals $I_l(x)$ which are uncorrelated in the following sense. If $g_l(x)$ is the average value of $I_l(x)$ the deviations $I_l(x) - g_l(x)$ fluctuate randomly for different l, i.a.

(4) $$(I_l(x)-g_l(x)\ (I_{l'}(x)-g_{l'}(x)) \quad = \sigma_l^{2}(x)\ \delta_{ll'}$$

with $\sigma_l(x)$ the variance of the statistical fluctuation of the anode signal $I_l(x)$ and $\delta_{ll'}$, the Kronicker symbol.

The average values $g_l(x)$ and the variances $_l(x)$ can be measured as a function of position x by scanning a narrow and intense neutron beam across the detector. Under the assumption above and for a neutron capture event with anode signals I_k the probability for x being the position of the scintillation event is given by the product

(5) $$p(x) = \prod_k \frac{1}{2\pi\,\sigma_k(x)} \quad \exp\left\{- \frac{(I_k-g_k(x))^2}{2\ \ _k 2(x)}\right\}$$

if the correct Poisson statistic is substituted by the approximately correct gaussian statistic.

From (5) the most likely position $\hat{x}$ of the particular neutron capture event with anode currents I_k can be calculated. $\hat{x}$ is defined by

(6) $$p(\hat{x}) = \underset{x}{\mathrm{MAX}} \left\{ p(x) \right\}$$

There is certainly a true position at which the scintillation light originated. Nevertheless, the most likely position is the only information one can deduce from the anode signals $\{ I_k \}$ assuming that the light distribution functions g(x) and the variances σ are given. The major drawback of this method which allows to determine the 'correct' position is the fairly large computer time which is needed for each scintillation event to calculate the most likely position. There seems to be no chance to accomplish the required calculation in less than 500 µsec. To circumvent the calculation for each scintillation event a look-up table must be provided containing the most likely position for any possible anode signal combination. If the three anode signals I_{k-1}, I_k and I_{k+1} of a scintillation event are represented by 8 bit ADC words the look-up table is extremely large. Its size is $256 \times 256 \times 256 = 1.7 \; 10^7$ computer words per photomultiplier. An acceptable look-up table size is obtained if one deals only with two relevant currents, I_k and I_{k+1} with $I_k + I_{k+1}$ maximal, and by reducing the accuracy to that of 7 bit words. The table size is in this case 16 k per photo multiplier. This situation has been computer-simulated for a stack of five PM's and with light distribution functions g(x) as shown in Fig. 3. For the variances it has been assumed that $\sigma_l(x) \propto g_l(x)$ 1/2 with $\sigma_l/g_l = 0.1$ at the maximum of g_l.

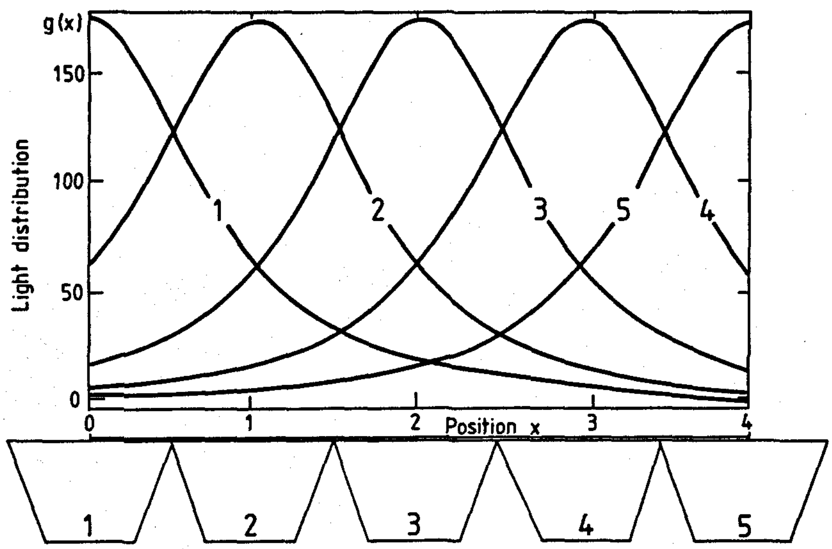

Fig. 3 Light distribution functions g(x) for a stack of five PMs versus the position x. Scintillation and PMs are drawn schematically below the x axis

Thus, a linear scintillation detector is simulated with the disperser thickness equal to the PM diameter and with 100 photo electrons generated at a photo-cathode if the scintillation event occures in front of a center of a PM. The functions $g_l(x)$ are generated stepwise at 40 positions per PM and interpolated by spline functions. For any signal combination k,

I_k, I_{k+1} with a total number of 5x128x128 = 82000 the most likely positions x are precalculated according to (6) and tabulated. The look-up table is then used to store a scintillation event given by the anode signals I_k and I_{k+1} with the PM number k chosen such that $I_k + I_{k+1}$ is maximum, by incrementing a memory location addressed by the looked-up $\hat{x}$ position. The intensity distribution for 10^7 scintillation events randomly distributed along the scintillator strip reaching the midpoints of PM number one and five is shown in Fig. 4. The detector response is indeed perfectly linear over the entire range. The drop of intensity on both ends

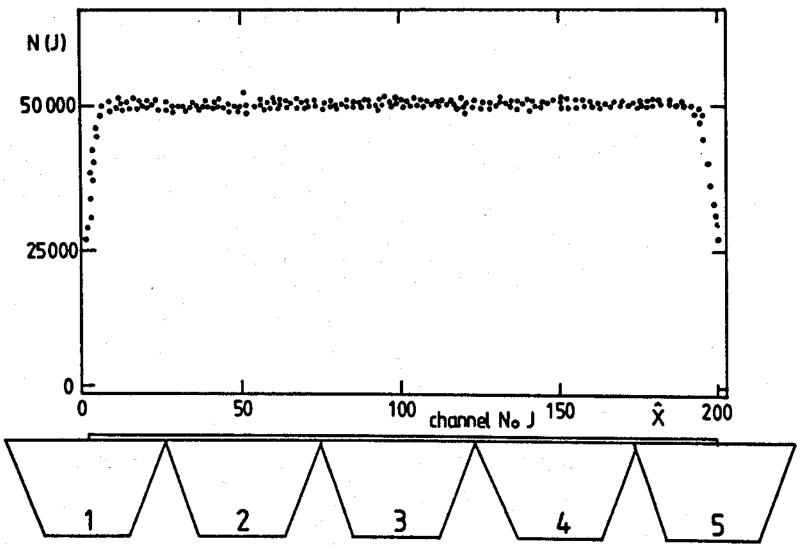

Fig. 4 Intensity distribution for randomly occurring scintillation events at the marked scintillator by addressing and incrementing 40 channels per PM according to precalculated and tabulated most likely positions $\hat{x}$.

is due to the lack of scintillation events beyond the scintillation range indicated in Fig. 4. The statistical scatter of the data is about $\pm$ 1% due to the limited accuracy of the 7 bit words for the anode signals.

The serious disadvantage of this method to determine the correct scintillation event from only two anode signals is shown in Fig. 5 where three spatial resolution peaks are presented. They are produced by simulating that narrow neutron beams hit the scintillator at the positions indicated by the arrows. The spatial resolution is strongly dependent on position, it is poor at a position in front of a midpoint of a PM and highest at a place between two PM's. This resolution behaviour can easily be understood by comparing the position dependence of the light distribution functions and of the statistical uncertainties of the anode signals.

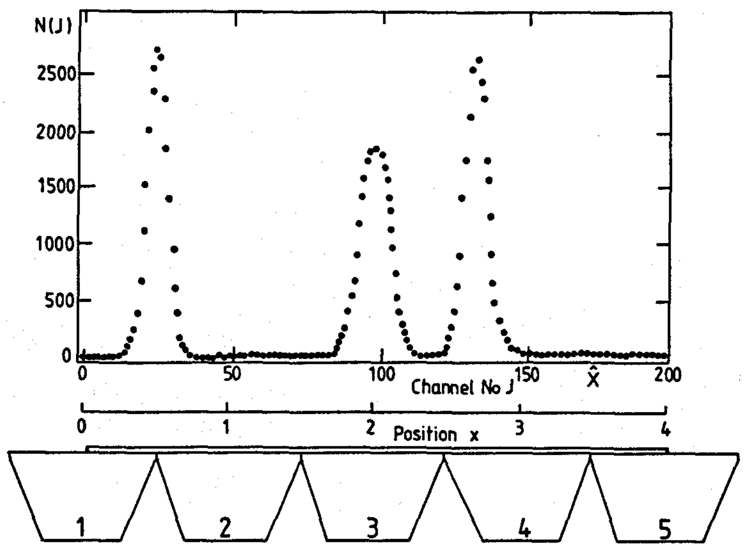

Fig. 5 Resolution peaks produced by 20 000 scintillation events at the positions marked by the arrows. Data storage as described in the legend of Fig. 4.

The attractive part of this method is that a linear response is achieved with the calculated $\hat{x}$ values irrespectively of inhomogeneities of the photo-cathode sensitivity, of the reflector on top of the scintillator, of the neutron absorption probability of the scintillator and of light losses in the disperser. It is even not necessary to adjust the PM gains properly, as long as the detector properties stay constant in time.

III. Position determination based on calibrated q_k values

From the previous section it became clear that a position should be determined via a look-up table and that the table variables cannot be the anode signals itself. In order to keep the size of the look-up table within practical and economical limits an attempt has been made to begin with the information (k, q_k) for a scintillation event which was originally characterized by the three major anode signals I_{k-1}, I_k and I_{k+1}. With these values the ratio q_k is calculated according to equation (2) with any S_k value, e.g. S_k=1. For calibration purposes q_k values are calculated not only for PM number k of maximum anode signal but also for number k-1 and k+1. In this case the anode signals I_{k-2} and I_{k+2} are needed. It is implied that the signals I_l with l not being a number of an existing PM are set to zero. Keeping in mind that a look-up table should be evaluated which allows to find a position for storage purposes for any scintillation event the q_k values must be restricted to certain digital values, e.g. $I_q = q\ M_q$. This is anyhow the case if the selection of the anode signals and the division to obtain (k, q_k) is done by hard-ware division. The calibration procedure to fill the look-up table requires that a narrow but intense neutron beam is scanned stepwise across the linear detector and that for each position a large number of scintillation events are evaluated. For each (k, q_k) value which occurs during the calibration procedure at least once an weighted average of positions is calculated according to

$$(7)\quad \bar{x}(k,\ q_k) = \frac{\sum_{j=1}^{N} \nu_j(k,\ q_k) \cdot x_j}{\sum_{j=1}^{N} \nu_j(k,\ q_k)}$$

where ν_j is the number of scintillation events at position x_j which lead to the particular (k, q_k) value and N is the number of positions during the scan. There are (k, q_k) values which occur frequently at a few x_j positions and rarely or even never at other positions. However, there are also k, q_k values which occur hardly at any x_j value although the number M of scintillation events per position may be very large. Finally, there are (k, q_k) values which never occur during the calibration procedure. There are in the mean N M events distributed on M_q values (k, q_k) per PM. With 50 steps per PM, 1000 events per step and 256 different q_k values the mean number of events per (k, q_k) value is $= \frac{1000 \times 50}{256}$ = 200. With this mean number of contributing events one can estimate the accuracy of the weighted average positions $\bar{x}(k, q_k)$. Its variance is

$$(8) \quad V(\bar{x}) = \frac{V_r}{\sqrt{\nu}}$$

where V_r is the spatial resolution variance of the detector. A typical value is $\pm$ 0.04 of the diameter of the detector if it is assumed that at the most 100 photo-electrons are produced at a photo-cathode. With these values the statistical uncertainty of $\bar{x}$ is still about 14% of the step size. The simple calculation demonstrates that one needs about 100 000 scintillation events per scan step during the calibration procedure in order to obtain an average position $\bar{x}$ with an accuracy of 1% of the step size. It is somewhat more practical to smooth the experimentally determined relation $\bar{x}(k, q_k)$ by a spline approximation /3/ which allows to gain smoothness by sacrificing data fidelity. Such a method can imply small systematic deviations of the approximated relation from the true one which leads in this case to non-linearities.

According to the statistical treatment of the previous section one should again calculate the most likely position $\hat{x}(k, q_k)$ for a given k, q_k value instead of the weighted average $\bar{x}(k, q_k)$. However, a precise determination of the most likely positions $\hat{x}$ would require a smaller scan step and a much larger number of events per step than it is required for a determination of $\bar{x}$. In the case of a symmetrical distribution of positions at a given (k, q_k) value there is no difference between the values $\bar{x}$ and $\hat{x}$, however this assumption should not be expected to be fulfilled.

When the calibration procedure is finished a look-up table with M_q memory locations per PM is filled with smoothed $\bar{x}(k, q_k)$ values which may be used for data storage. In order to avoid Moiré patterns one has to interpolate for a given k, q_k between the calculated positions before changing the position into an address of a memory location for incrementing the memory location.

In the following some computer-simulated results are shown. The light distribution functions of Fig. 6 are used in the following computer-simulations. They correspond to a light

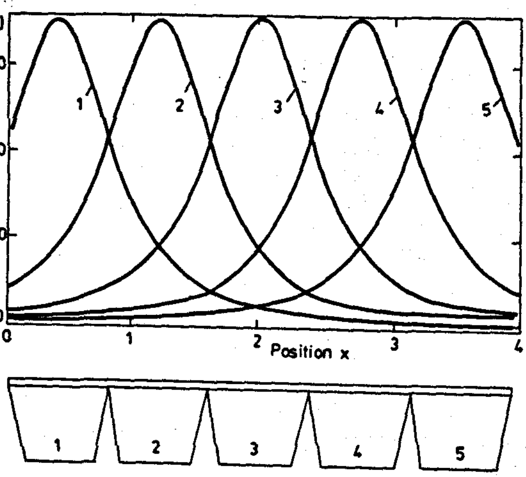

Fig. 6 Light distribution functions g(x) versus the position x for five PMs.

disperser thickness of 70% of the PM diameter in a real detector. The calibration of the x, (k, q_k) relation was done for M_q = 128 q_k values per PM in 40 steps per PM, and with 2000 neutrons per step. The calculated average positions $\bar{x}$ were thereafter spline approximated with experimentally determined variances $\nabla(x)$. The values depend on k, q_k. For (k, q_k) values which occur frequently the scatter of x is small while for rarely occurring (k, q_k) values the scatter is larger. A typical value of $\nabla(x)$ is 6% of the step size which is quite in accordance with the estimates given above if the new numbers are used. The $\bar{x}$ (k, q_k) relation as determined from equation (7) and their spline interpolations are shown in Fig. 7. Data for q-values larger than 0.85 and

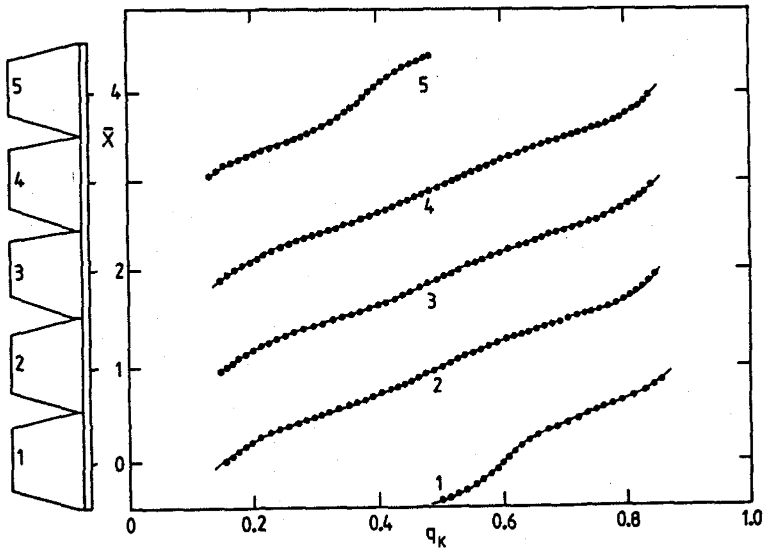

Fig. 7 Measured average position x for discrete q_k values of five PMs. The solid lines are spline approximations of the experimental x versus q_k relations for the five k values.

smaller than 0.15 are not shown. For these q values $\bar{x}$ values were obtained from the calibration measurement, however they are not used later to store scintillation events. This intensity distribution of the one-dimensional detector when flooded homogeneously with neutrons is presented in Fig. 8. The scintillation events are stored in 40 channels per PM, the content

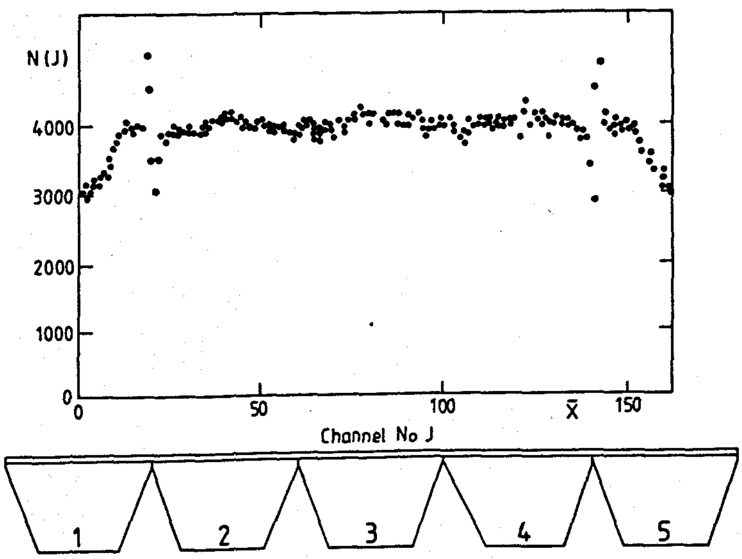

Fig. 8 Intensity distribution as determined for randomly occurring scintillation events at the indicated scintillator by addressing and incrementing 40 channels per PM according to precalculated, tabulated and interpolated average positions x.

of a channel is about 4000. For each event the three relevant anode signals I_{k-1}, I_k and I_{k+1} with number k such that I_k is maximum are substituted in (2) to calculate q_k. With the input value (k, q_k) an interpolated $\bar{x}$ position is determined from the tabulated $\bar{x}$ values to address a memory location for incrementing its content by one. The intensity distribution is apart from some statistical fluctuations and some systematic deviations due to the spline interpolation flat over the range of the three inner PM's. At the place where the transition from (1, q_1) to (2, q_2) and from (4, q_4) to (5, q_5) occurs the intensity distribution is strongly oscillating.

Over the half range of the outer two PM's the intensity distribution drops 25% although the scintillator spans the total range of all photo-cathodes. The difference between the detector responses at the inner and outer PM ranges is due to the drastic change of the expression in (2) since for k=1 and k=5 the anode signals I_{k-1} and I_{k+1} become zero, respectively. In addition the average value $\bar{x}$ instead of the most likely value $\hat{x}$ has been calculated during the calibration procedure. Without prove it is maintained that the detector response would be as in the previous section linear over the entire range if $\hat{x}$ could have been calculated with sufficient accuracy.

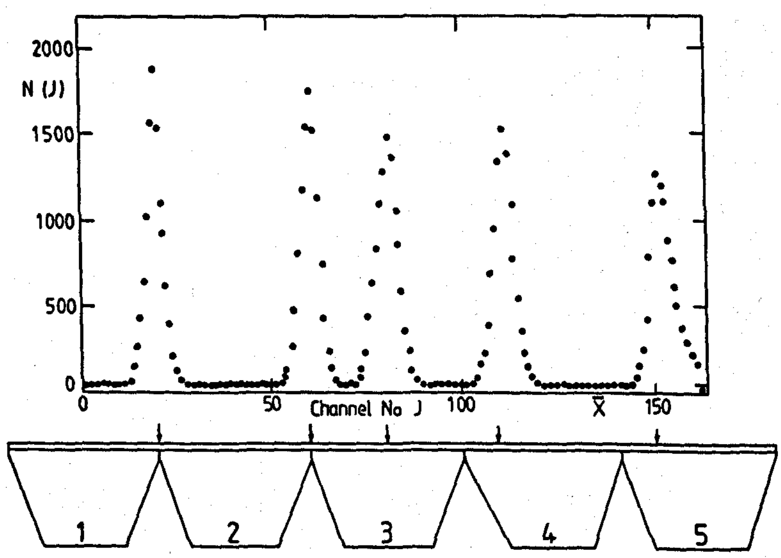

Fig. 9 Resolution peaks produced by 10 000 scintillation events at the positions marked by arrows. Data storage as described in legend of Fig. 8.

Five resolution peaks are shown in Fig. 9. They are measured by creating 10 000 scintillation events at the places marked by arrows. The full width at half maximum $B_{1/2}$ of the resolution curves is 0.12 times the PM diameter which compares well with a full width resolution calculated from the statistics of the anode signals. The result of such a calculation is

$$(9)\quad \frac{B_{1/2}}{D} = \frac{2.35}{N_e} \; p_x^{-1} \left\{ \frac{g_{k-1}(x)\;(1+p)^2 + g_k(x)\;p^2 + g_{k+1}(x)\;(1-p)^2}{g_{k-1}(x) \;+\; g_k(x) \;+\; g_{k+1}(x)} \right\}^{1/2}$$

where N_e is the mean number of photo-electrons per event created at a photo-cathode if the event occurs in front of a PM midpoint.

The $g_l(x)$ functions are light distribution functions normalized to 1 at there maxima. The ratio p is defined similarly as q_k with $S_k = 1$ by

$$(10)\quad p(x) = \frac{g_{k+1}(x) - g_{k-1}(x)}{g_{k+1}(x) + g_k(x) + g_{k-1}(x)}$$

and the p_x is the derivative of p with respect to x. The number 2.35 transfers from variance values to full width at half

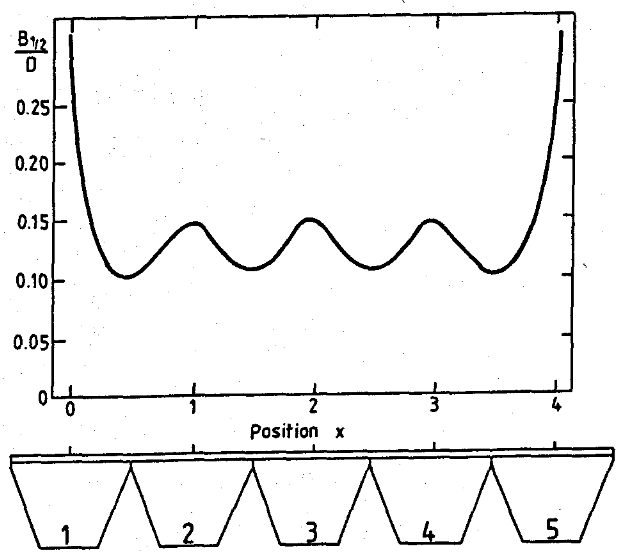

Fig. 10 Ratio of full width at half maximum $B_{1/2}$ and PM diameter D versus position calculated from the statistical fluctuations of the anode signals I_l according to equation (9) with $N_e = 100$, g(x) from Fig. 6 assuming gaussian statistics.

maximum assuming gaussian statistics. For the light distribution curves from Fig. 6 and with N_e = 100 as always assumed in this paper the relative half width is calculated with (9) and plotted in Fig. 10 as a function of position. The resolution is oscillating by $\pm$ 25%. Its mean value $B_{1/2}/D$ = 0.125. At the ends of the detector the resolution becomes rapidly very poor due to the lack of a third anode signal and because the two remaining signals become small.

This method of position determination by calibration with a narrow neutron beam scanned stepwise across the detector has the disadvantages that a high number of neutrons is required to calculate with sufficient accuracy the most likely position x for each k, q_k value and that the detector response is not completely linear if the mean position $\bar{x}$ is calculated.

IV. Position determination based on a measured intensity distribution

In this section an attempt is made to obtain from the encoded position Q as defined in (3) the correct position x by evaluating the non-linear response, i.e. the intensity distribution of data which are stored according to the encoded position Q. For any scintillation event the information (k, q_k) is selected and calculated from the anode signals and according to (3) an encoded position Q is determined by choosing appropriate values for the coefficients A_k and B_k. If the detector is homogeneously flooded by neutrons and the then randomly occuring scintillation events are stored according to the encoded position Q one obtains a non-uniform intensity distribution which reflects the deviations of the encoded positions from the true ones. In order to derive the relation Q = f(x) the number of events with a Q-value between Q and Q + dQ is given by the following expression

(11) $$N(Q) \propto \int_{-\infty}^{+\infty} dx \frac{1}{2\pi \sigma_f^2} \exp\left\{ -(Q-f(x))^2/2\sigma_f^2 \right\}$$

Again for simplicity gaussian statistics has been assumed to describe the probability that an event at position x leads to a Q value which falls into the mentioned interval. σ_f is the variance of the fluctuation of the encoded position Q about its mean value f(x) for events from the real position x. By changing variables

(12) $$N(Q) = c \int df \frac{1}{\frac{df}{dx}} \frac{1}{2\pi \sigma_f^2} \cdot \exp\left\{ - \frac{(Q-f(x))^2}{2\sigma_f^2} \right\}$$

Assuming that σ_f and $\frac{df}{dx}$ are changing slowly with respect to the exp-function which is particularly true if the resolution is high ($\sigma_f \to 0$) the integral can be evaluated

(13) $$N(Q) = c \frac{dx}{dQ}$$

With the knowledge of two Q-values at two x positions (x_1, Q_1) and (x_2, Q_2) one gets rid of the proportionality constant c and of the integration constant. The wanted relation is

(14) $$x^* = x_1 + \frac{x_2 - x_1}{\int_{Q_1}^{Q_2} N(Q')dQ'} \int_{Q_1}^{Q} N(Q')dQ'$$

Since (14) is only an approximation the calculated position is called x* to distinguish its value from the true position x. With this relation the x* position can be precalculated for a large number of Q values and tabulated in order to be able to store data with respect to the x* positions.

An even faster method to store data is to precalculate x* positions for a set of discrete (k, q_k) values via (3) and (14) and to tabulate the results accordingly. To avoid Moiré patterns one needs again an interpolation between consecutive positions before changing from x* to an address of a memory location. Since the intensity distribution N(Q) can be measured with only limited accuracy again a spline approximation is performed which is then used for the numerical integration according to (14).

Results from computer simulations are shown in the following figures. The calculated relation of the position x as a function of the encoded position exhibits already deviations from a straight line indicating that Q as defined by equation (2) and (3) is an encoded position which is not linearly related to x* position. In Fig. 11 the intensity distribution is shown which is obtained by storing data according to the encoded position Q. The spline approximated spectrum is used

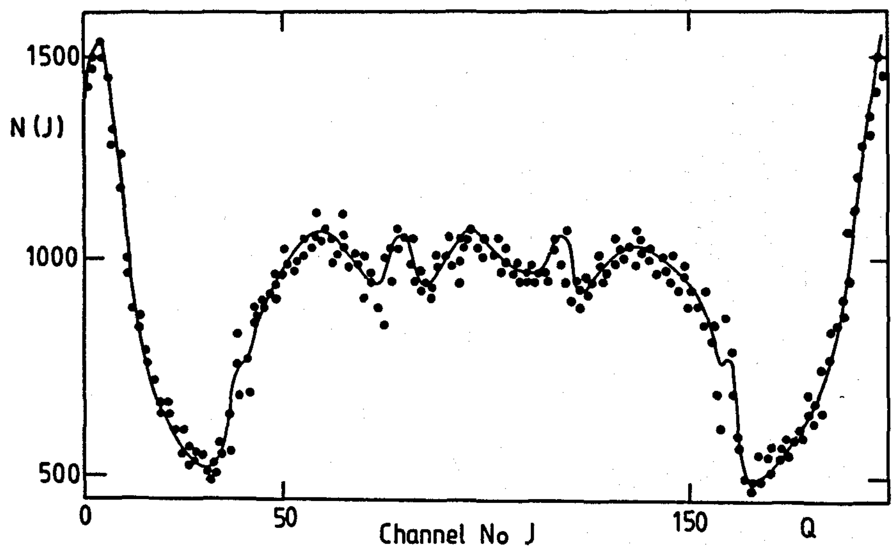

Fig. 11 Intensity distribution as determined for randomly accuring scintillation events at the indicated scintillator by addressing and incrementing 40 channels per PM according to the encoded position Q as defined in equation (3). The solid line is a spline approximation of the Q spectrum.

for calculating the positions. The data are distributed among 40 channels per PM and about 1000 events are counted per channel. By storing data of randomly occurring scintillation events at any position along the detector with respect to the precalculated and tabulated positions x* one obtains a spectrum as shown in Fig. 12. The intensity distribution is flat apart from statistical fluctuations. The missing intensity in the very first and last few channels is due to a systematic error of the spline interpolation and due

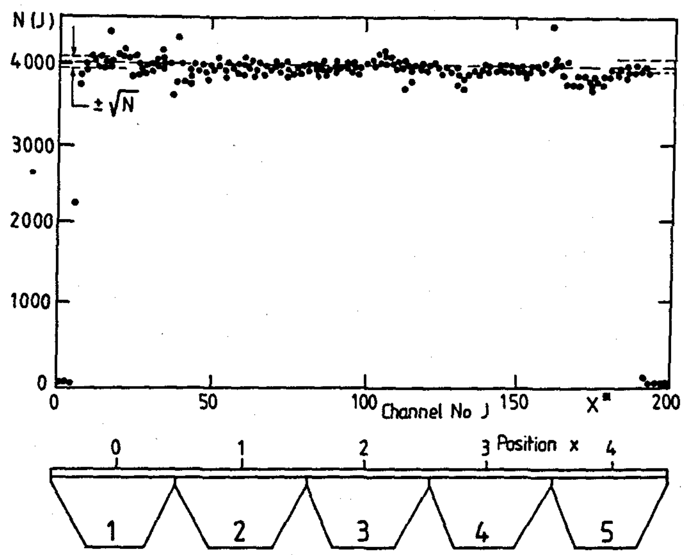

Fig. 12 Intensity distribution as determined for randomly occurring scintillation events at the indicated scintillator by addressing and incrementing 40 channels per PM according to precalculated, tabulated, and interpolated x* positions according to equation (14).

to missing scintillation events outside the drawn scintillator range. In Fig. 13 five resolution peaks are shown. The obtained resolution compares well with the spatial resolution as calculated from the statistical uncertainties in equation (3) and shown in Fig. 10.

The major advantage of this method which leads to satisfactory results is

1) that it is based on experimental results which are obtainable by homogeneously flooding the detector with neutrons. Such an illumination is easy to provide for instance with an incoherent scatterer in a neutron beam from a reactor or with a natural neutron source.

2) that a fairly small number of neutrons is required to achieve a satisfactorily linear response

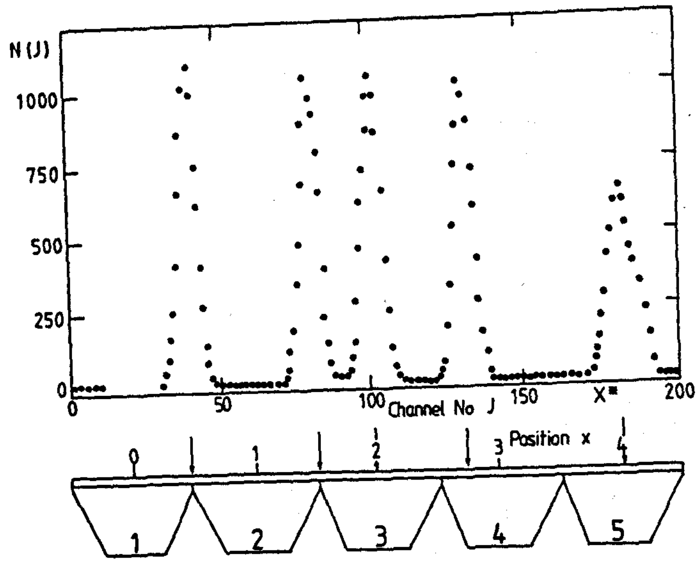

Fig. 13 Resolution peaks produced by scintillation events at the position marked by arrows. Data storage as described in legend of Fig. 12.

3) that a more linear response can be obtained by applying the method iteratively. The stepwise linearisation allows to start with very crude A and B values and avoids systematic errors of the spline approximation in progressive steps

4) that a small amount of memory space is required to tabulate the precalculated x* positions, e.g. 256 locations per PM

5) that a high data aquisition rate can be handled if the (k, q_k) values are selected and calculated per hard-ware. Thereafter merely an interpolation between two x positions from the look-up table is required

6) that the method can be directly applied to a two-dimensional scintillation detector with a quadratic arrangement of PMs as shown in Fig. 2.

The only disadvantage might be that the neutron absorption probability of the scintillator must be uniform. Otherwise the intensity modulations are misinterpreted by the expression of equation (14).

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References

/1/ W. Schäfer, I. Naday and G. Will, Conference Proceedings of the Position-Sensitive Detector Workshop in Grenoble, Oct. 1982.

/2/ J. Schelten, R. Kurz, I. Naday and W. Schäfer, Nucl. Instr. Meth. (in press).

/3/ The subroutine DSP from KFA program library was used. The routine finds the function S(x) of minimum $\langle [S'']^2 \rangle$ under the constraint of data fidelity controlled by : $\sum_j w_j \, (S(x_j) - y_j)^2$.