PRODUCTION FUNCTIONS IN MARICULTURE

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Dedicated

To

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CERTIFICATE

Certified that the dissertation entitled **"PRODUCTION FUNCTIONS IN MARICULTURE"** is a record of independent bonafide research work carried out by **Ms. Neetha Susan David** (MC- 76) during the period of study from September 2001 to August 2003 under our supervision and guidance for the degree of **Master of Fisheries Science (Mariculture)** at the Central Marine Fisheries Research Institute, Kochi, and that the dissertation has not previously formed the basis for the award of any degree, diploma, associateship, fellowship or any other similar title.

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DECLARATION

I hereby declare that the thesis entitled "**PRODUCTION FUNCTIONS IN MARICULTURE** " *is an authentic record* of my own research work and that no part thereof has been presented for the award of any degree, diploma, associateship, fellowship or any other similar title.

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सारांश

जलजीव कृषि के क्षेत्र में उत्पादन फलन निदर्श का ज्यादतर ध्यान नहीं दिया गया था. समुद्री संवर्धन व्यवस्थाओं में उत्पादन का निदर्श विकसित करने का एक प्रयास किया गया है. साधारणतया उत्पादन फलन बढ़ती और मृत्युता पर आधारित है या गणितीय रूप से यह $P_t = \int_0^t N_t.dW_t.dt$ है. पशु एवं मुर्गी पालन में बढ़ती - वज़न के बीच का संबंध व्यक्त करने के लिए कई गणितीय फलन उपयुक्त किए गए हैं लेकिन जलजीव कृषि में जीवों की बढ़ती का निदर्श विकसित करने के लिए नगण्य प्रयास किया गया है. इसी प्रकार मूल्यांकन किए गए फलन थे गोम्पेट्र्स, लोजिस्टिक, वोन बेर्टलन्फी, रिचाड्र्स मोडिफाइड जानोसचेक तथा बहुपद बढ़ती समीकरण. अलग अलग रूप से वज़न - आयु वक्र के हर फलन के प्राचल आकलित करने के लिए SYSTAT के अरैखिक समाश्रयण प्रक्रिया का न्यूनतम वर्ग तरीका उपयुक्त किया गया. मृत्युता पर बिना अनुमान और रैखिक निदर्श और चरघातांकी निदर्श जैसे मृत्युता के अनुमानों के साथ उत्पादन फलन के निर्वचन के लिए बढ़ती नमूनों को उपयुक्त किया गया. ग्रे मल्लट मुजिल सेफालस में लोजिस्टिक बढ़ती नमूना अत्यंत बेहत्तर साबित हुआ बल्कि क्रासोस्ट्रिया माड्रासेन्सिस और पेर्ना विरिडिस जैसे मोलस्कों में गोम्पेट्र्स बढ़ती वक्र बेहत्तर देखा गया. शुक्तियों के लिए आकलित उत्पादन निदर्शन में यह व्यक्त हुआ कि मृत्युता में गोम्पेट्र्स बढ़ती वक्र के साथ रैखिक तथा चरघातांकी अनुमान से बेहत्तर परिणाम निकल गए.

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ABSTRACT

Production function in aquaculture has received very little attention. An attempt has been made to evaluate the production function of mariculture systems. Production functions are generally based on growth and mortality or mathematically $P_t = \int_0^t N_t.dw.dt$. Numerous mathematical functions have been used to describe the age-weight relationship in cattle and poultry; however, little work has been done in modeling growth of organisms in aquaculture. The functions evaluated were Gompertz, Logistic, von Bertalanffy, Richards, modified Janoschek and polynomial growth equations. The Gauss-Newton and Quasi-Newton method of the nonlinear regression procedure of SYSTAT was used to estimate parameters of each function for individual weight-age curves. The growth models were used for the derivation of production functions along with no assumption on mortality and different assumptions of mortality like linear and exponential model. In the case of grey mullet *Mugil cephalus* logistic growth model gave the best fit while that for molluscs *Crassostrea madrasensis* and *Perna viridis* Gompertz growth curve was the best. The production function was estimated for the oyster data. It was found that the linear and exponential assumptions in mortality along with the Gompertz growth curve gave better results.

CONTENTS

LIST OF TABLES

LIST OF FIGURES

1. INTRODUCTION

1. INTRODUCTION

During the past decade, aquaculture has been the world's fastest growing food production system. This spectacular growth has been fuelled by a steadily increasing demand for seafood and a leveling of production from capture fisheries throughout the world. Food and Agriculture Organisation (FAO, 1984) has defined aquaculture production as that portion of fisheries production achieved through human intervention involving physical control of the organism at some point in its life cycle other than at harvest. This intervention can range from minimal, such as the trapping of shrimp seed stock in coastal lagoons to cause an eventual increase in harvest, to maximal, such as the production of trout in closed systems.

Even though aquaculture is as old as first century B.C., it is still an emerging science. The practice of aquaculture is varied and based on different components. It is based on scientific knowledge – a formal knowledge contained in books and papers giving a rationale for taking decisions, then on folk wisdom – where things are done because it is known why and that they work to a certain degree and finally on conjecture where the situation is novel and there is no guidance from knowledge and tradition but one has to do something. The purpose of aquaculture research is to increase the percentage of the first component - scientific knowledge - at the expense of the other two components thereby increasing the efficiency of production.

Scientific knowledge is not just about observational data but it includes a conceptual scheme or hypothesis that corresponds with those data and it is the continual interaction between hypothesis and observational data that leads to progress. Any branch of science as it progresses from the qualitative to the more quantitative, reaches a point, where mathematics becomes essential for connecting theory to experiment. At this context emerges the science of mathematical modeling.

In mathematical modeling, mathematics is used as a language for expressing the numerical observations so that it can be properly connected with the hypothesis. A mathematical model is a simple description of a physical, chemical or biological state or process. Hypothesis expressed in mathematics can provide a quantitative description and understanding of biological process. Models are used to imply the existence of an *apriori* logical identification between the equations, variables, parameters and analytical behaviour on one hand and biological phenomena on the other hand. Three components or steps underlie the specification and use of a model;

1 *Biological component* :- The biological component must provide at least one measurable quantity.

2 *Mathematical component* :- Investigation of the properties of the model is the mathematical component.

3 *Statistical component* :- The estimation of the parameters and testing of the fit constitute the statistical component.

Growth is a factor of prime importance in aquaculture since every cultured organism demands a stipulated size in the market to fetch optimum price. From the farmer's point of view this should be attained in the shortest time for attaining maximum profit. So understanding the growth characteristics of a species is fundamentally important in aquaculture. Growth of any organism depends on intrinsic factors typical of the genetic constitution of the population as well as extrinsic environmental factors acting on the individual throughout its ontogenic development. Growth is the increase in length, volume or weight over time (Nikolskii, 1969; Hartnoll, 1983) and is the result of balance between the process of anabolism and catabolism that occur in an individual (von Bertalanffy, 1938).

Various attempts at mathematically describing the growth of animals have been made over the centuries. Much research has been devoted to modeling growth processes, and there are many ways of doing this like mechanistic models, time series modeling, stochastic differential equations etc. Research has continued in recent years with most of this effort aimed at establishing comparative curves for different animals. Due to the complexity of the various issues, models could not always be applied to specific situations. A better understanding of growth could result in significant benefits in terms of productivity, sustainability and profitability for aquaculture operations, provided this greater understanding is translated into relevant and simple applications.

Information on the growth of animals is important for studying their population dynamics, physiology and biochemistry (Peters, 1983; Calder, 1984; Schmidt-Nielsen, 1984; Reiss, 1989; Xiao, 1998). Growth models can be of great value from production planning and management points of view (Iwama and Tautz, 1981). The use of growth models offers an objective and practical way of describing pattern of growth and predicting production. It also forms the basis for understanding mortality or survival and other characteristics that determine yield. Proper models can allow intrapolation of growth data to estimate weight between sampling intervals. This may be very helpful for the accurate estimation of the standing stock and the amount of feed to be distributed. Growth models can also be used to forecast (extrapolate) weight at a certain point in time past sampling interval. This may be helpful for example, to estimate time required to achieve a given target weight (e.g. market weight). The accuracy and utility of growth models have improved with the ability of modern computers to calculate complex concepts rapidly.

Aquaculture is a promising feature, which is expected to improve sustainability as well as the economic situation on small-scale rural farms through diversification and nutrient recycling. To determine the carrying capacity of a water body it is necessary to evaluate its production. In general, production is not synonymous with yield. Production is defined as the total amount of tissue elaborated in the population or community under study during a given period. It is a dynamic quantity that can rarely be measured directly. Its measurement or estimation calls essentially for knowledge of the biomass of the population or community at the beginning and end of the period and of the mass of living components that have been lost by death or emigration during the period (Allen, 1971).

Production Models are just a more complex series of calculations based on growth models. To manage a fish production facility or site, it is desirable to be able to model the individual fish stocks and to project fish stocks for an extended period. Production models, which simulate the performance or growth of aquatic animals in an aquaculture grow-out facility, can be practical and valuable tools for both the researcher and the pond manager. For the aquaculture researcher, such a model facilitates comparison of quantitative relationships between biological variables and animal growth with the actual growth data. Consequently this can further the understanding of the grow-out system biology and can result in determination of the key variables that determine growth. For the pond manager, such a model provides an inventory system under current management procedures and by simulating the grow-out system over different management strategies such as different stocking and harvesting schedules and stocking densities the model assists in selecting optimum management strategies. Once production has been quantified, it is possible to improve quantitative analysis by comparing production outputs varying one parameter at a time. To optimize profits, the optimum harvest time can also be determined by linking biomass models to total cost.

It is well known that mathematical modeling and simulation have numerous applications in describing animal growth and biological systems. Most of the models are excellent planning and management tools, but like all tools their effectiveness has as much to do with operator's experience, as it deals with the structure and design of the model. The field of modeling has expanded in the past decade, and should continue to do so, considering the costs involved with animal experimentation and the advancement of microcomputers.

Growth modeling and production modeling in aquaculture has not received its requisite status in India. This work was undertaken as an attempt to fill in the void. In the present study, an attempt has been made to model the growth of various organisms of mariculture importance like mussels, oysters and fish. For the estimation of production based on these growth models and assumptions made on mortality, production functions were derived. These were applied to the data set available and the production estimated.

The objectives of the study were

1) Mathematical modeling of growth of different organisms of mariculture importance.

2) Derivation of production functions based on different assumptions on growth and mortality.

3) Evaluation of production functions.

2. REVIEW OF LITERATURE

2. REVIEW OF LITERATURE

Webster's New World Dictionary defines growth as "the process of growing or developing, - gradual development towards maturity". In reality, growth does not have such a simple and constant meaning. Beever *et al.* (1992) defined animal growth as the increase in size and the change in functional capabilities of the various tissues and organs within the animal. Beever also provided a simpler definition of growth as the net accretion of protein and fat in respective tissues, controlled by nutrition, environment and the genetic capacity to grow.

Growth of crustaceans and other invertebrates differs significantly from that of vertebrates, such as fish. Although tissue growth is essentially a continuous process in crustaceans, the accompanying increase in external dimensions is discontinuous. This proceeds by a series of molts or ecdyses, when the old integument is cast off and a rapid increase in size occurs before the new integument hardens and becomes inextensible (Hartnoll, 1983).

Growth of fish is in many ways similar to growth of higher vertebrates. Fish can survive starvation for extended periods of time (days to months) and lose very significant amount of weight and remain able to resume their growth, without ill-effect, when conditions become more favorable again (Weatherley and Gill, 1987).

There have been many attempts to mathematically describe growth of fish using a large diversity of approaches and concepts. It is common to find growth expressed as centimeters per month, instantaneous growth rate, and percentage of change in weight (Iwama and Tauz, 1981; Muller-Feuga, 1990).

Since growth rate is highly dependent on species, genetics, nutrition, environment, husbandry and other factors, it is essential to calculate growth rate for a given aquaculture condition. Production records are very valuable starting points when trying to determine most appropriate growth model and consequently, predicting growth of the fish (Cho and Bureau, 1998).

Mathematical Models of Growth

A model is an abstraction of reality. It is a formal description of the essential elements of a problem (Jeffers, 1978). "A Mathematical model is an equation or set of equations which represents the behaviour of a system (Thornley, 1984). There are different type of models; physical versus abstract, dynamic versus static, empirical (correlative) versus mechanistic (explanatory), deterministic versus stochastic and simulation versus analytical.

Many mathematical equations representing growth have been developed (Brody, 1945; Guttman and Guttman, 1965; Fabens, 1965; Heinichs and Hargrove, 1987; Laird, 1966; Richards, 1959). A growth curve is simply a mathematical relation between weight of an animal and time (Fabens, 1965). Growth curves relate the interrelationships of the genetic ability of an animal to grow and mature and the environment in which the animal grows (Fitzhugh, 1976).

One of the earliest works in mathematically describing the growth of animals was reported by Gompertz (1825) as cited by Parks (1982) for the calculation of mortality rates. It is still one of the most frequently used curves in modeling growth and is given by: $W_t = W_0 e^{k(1-e^{(-gt)})}$,where W_t is the corresponding weight at time t, W_0 is the value at time, t=0, g is the instantaneous rate of growth and k is a dimensionless parameter.

Verhulst (1838) proposed a model for population growth, which can be used for somatic growth as well. It is defined by: $w_t = \frac{A}{1+be^{-ct}}$ where A is the adult value, both b and c are free parameters that adjust the shape of the curve. The autocatalytic or logistic curve is symmetrical around its inflection point. Growth rate increases with age till A or until the inflection point, where it decreases until maximum age is reached.

The Gompertz curve has an inflection point earlier than a logistic curve, *precisely when W = A/e. In addition, the Gompertz curve is asymmetrical about its inflection point.*

Ludwig von Bertalanffy (1957), in his studies of growth functions across many species, attempted to relate growth to metabolic rate, thus providing a physiological basis for growth functions. He theorized that, anabolic rate and body weight relationships are similar to metabolic rate and body weight relationships, implying that metabolic rate can predict growth type $w_t = w\alpha(1-e^{-kt})^3$ where w_α is the asymptotic weight or adult value, the value that is attained as t approaches infinity.

The most frequently used model nowadays was developed by Richards (1959) and is defined by: $W = A(1+e^{-kt})^{-M}$ where A is asymptotic (mature) weight, k is rate of approach to mature weight and M is a shape parameter that allows for a variable inflection point. The Richards function is essentially a modification of the monomolecular curve, with M added as an exponent to adjust the proportion of mature size at the *inflection point.* The Richards' equation allowed flexibility by adding M *to vary to account for an inflection point but the others treat* M *as a given fixed constant,* hence increasing its usefulness. This high flexibility is, however, combined with disadvantages as well. The parameters (b, k, M) have a high covariance, which can produce problems during nonlinear regression.

Janoschek (1957) proposed growth curve that is nearly as flexible as the Richards function. It is defined by: $w = (W_0)e^{-kt^p}$ where W_0 is the adult value or asymptote, because this value is attained as t approaches infinity. The parameters k and p adjust both slope and point of inflection of the curve.

Many other models were also developed based on a linear relationship between age and weight; weight = a + b *(age); where a = intercept and b = slope (Russel, 1969). The models are generally empirical, static or deterministic in form. Therefore, the application of each model across a population under varied environmental conditions is limited.

The previously described functions (monomolecular, logistic, Gompertz, Bertalanffy, and Richards) all assume growth is a continuous process resulting in a smooth shaped growth curve. These nonlinear equations of animal growth have a physiological basis, similar to the approach of Bertalanffy, implying that the parameters have biological meaning.

The main criteria for choosing a growth curve are quality of fit and convenience. Ideally, growth models for fish and shrimp should offer possibility for comparing growth rates of animals of various sizes reared at various temperatures and culture conditions (Iwama and Tautz, 1981).

Application Of Growth Curves

Many equations have been used to predict growth in animals, which include Gompertz, Robertson's logistic, Brody, Bertalanffy, Feller, Weiss and Kavanau, Fitzhugh, Richards, Laird, and Parks equations (Brown *et al.*, 1976; Fitzhugh Jr., 1976; DeNise and Brinks, 1985; Johnson *et al.*, 1990; Beltran *et al.*, 1992; Lopez de la Torre *et al.*, 1992; Mezzadra and Miquel, 1994). Summarized descriptions can be found in Parks (1982).

According to Hartnoll (1983), most crustacean species appear to have a finite size or grow towards an asymptotic weight. He presented a series of curves representing the growth (in size) of various crustaceans throughout their life cycle.

The Chapman-Richards model has been applied successfully by Siqueira *et al.* (1989) to the penaeid shrimp *Penaeus subtilis* (Perez Farfante) in rearing ponds in a marine farm in north-east Brazil. Jackson and Wang (1998) adapted the Gompertz growth model to *Penaeus monodon* under aquaculture conditions. This model accounts for the effects of temperature, mortality and pond age.

Weymouth and Thompson (1930) applied the Gompertz curve to the growth of a bivalve. Thiesen (1973) generalized and suggested that growth of all lamellibranchs are sigmoidal and recommended a combined Gompertz von-Bertalanffy model. Gallucci & Quinn (1979) suggested a new parameter for Bertalanffy's model to enable growth properties for the bivalve, *Macoma balthica* (Linnaeus) in different spatial regions.

A study was performed by Shin *et al.* (1995) to determine the growth and production of *Mactra veneriformis* on the Songdo tidal flat off Inchon, west coast of Korea. They found that the growth in shell length fitted well to the von Bertalanffy model and the meat weight to the Gompertz model.

In the works conducted by Devillers *et al.* (1998) shell length (SL) records of known-age northern quahogs *Mercenaria mercenaria* over a 12-year period were used to compare Richards, Gompertz, logistic, and von Bertalanffy growth models. The Richards model gave the most accurate prediction of biologically and economically important ages.

A modified Gompertz function was fitted bv nonlinear numerical methods to the absolute growth rates (mm/day) of the marine gastropod, *Concholepas concholepas.* The modified Gompertz function provided a better fit than the bertalaffy function to the growth rate data (Rodriguez, 2001).

Jorg Urban (2002) fitted Gompertz, Special von Bertalanffy, Richards, Logistic and Generalized von Bertalanffy models to growth data of the Caribbean pearl oyster, *Pinctada imbricata.* Gompertz, Logistic and the Generalized von Bertalanffy model underestimated asymptotic length. Of all models, the Generalized von Bertalanffy model yielded the best fit.

Fish growth data are usually fitted through mathematical functions appropriate to generalize the process to predict and compare growth patterns within (or between) population or species (Chen *et al.,* 1992). The von Bertalanffy model is one of the functions mostly applied to fish growth (Beverton and Holt 1957; Ricker 1975,1979 and Santos 1978). Springborn (1991) reported that the initial value solution, a modification of von Bertalanffy's equation gave improved growth parameter estimates for aquaculture. However, this growth model could not explain variable rates of fish growth. He developed an initial value solution of von Bertalanffy's equation, which used the size of fish stocked as a model starting point, instead of a theoretical age of the fish at zero size.

Springborn *et al.* (1994), working in aquaculture experiments, showed that fish could increase dramatically in asymptotic length and growth rate. A variable growth model was developed to describe fish growth oscillations observed in aquaculture experiments. This provided an improved estimate of von Bertalanffy equation in aquaculture and can be used for an efficient evaluation of fish production.

Many researchers have used polynomial equations to fit data of fish growth, while others suggested replacing Bertalanffy's growth curve with polynomial models (Rafail1971, 1972; Ricker 1975; Roff 1980).

However, Chen *et al.* (1992) compared Bertalanffy's model and polynomial equations to fit growth data of sixteen populations, including six freshwater fish species. They concluded that Bertalanffy's model was more flexible for all populations than the polynomial equations with three and four parameters.

However according to Melo (1993) there are cases where fish growth is not well described by the Von Bertalanffy model, confirming Richard's (1959) and Chapman's (1961) assertions. It is therefore useful to test other mathematical models, as good data fitting is one of the criteria proposed by Ricker (1979) for choosing growth curves. Silliman (1967) used Gompertz model for fishes and Zweifel and Lasker (1976) argue that it should be the preferred model for fishes –including larval growth.

Rosa *et al.* (1997) compared different growth models like Chapman-Richards, Gompertz, Bertalanffy, Silva, Brody, Monomolecular and Logistic in cultures of *Oreochromis niloticus* L. and *Cyprinus carpio* L. The Chapman-Richards growth model was shown to be the most appropriate for *Oreochromis niloticus* L., while Silva's model best expressed growth for *Cyprinus carpio* L.

Walia *et al.* (1998) conducted study on three species of inland fish, viz., rohu, mrigal and common carp. Non-linear statistical models were fitted to forecast fish weight at the time of harvest after 12 months of stocking fish. Results revealed that forecast of fish weight can be made three months before harvest. For indigenous species like rohu and mrigal, logistic growth model can be used while that for exotic species, like common carp Gompertz model can be used to forecast the fish weight.

Fitting of nonlinear equations and goodness of fit

A procedure was advocated by Rao (1958) for growth comparison in which he suggested that efficient comparisons are possible if the data can be reduced to a few parameters, which summarize the aspects of growth. This is the most commonly used and powerful technique available to an investigator.

Most non-linear regression programmes calculate asymptotic standard errors and correlations for the parameter estimates. One approach is to use these statistics along with the parameter estimates in hypothesis test. Kingsley (1979) Galucci and Quinn (1979) and Misra (1980) have carried out univariate comparisons based on either the T or chi-square test for simultaneous comparisons of two or all three of the von Bertalanffy's parameters. The other approach advocated by Kimura (1980) is based on likelihood ratio statistic. Kingsley (1979) and Bernard (1981) suggested a procedure based on Hotelling T^2 statistic.

In the works conducted by Cerrato (1990) the equations were fitted to the surf clam data using the iterative Gauss-Newton algorithm and the goodness of fit was checked by likelihood ratio, t-statistic, univariate chi-square and T^2 test. Out of which, likelihood ratio test was found to be the most appropriate. He also showed that maximum likelihood estimation for the von Bertalanffy curve is equivalent to finding least-square estimates of model parameters.

Rosa *et al.* (1997) fitted the growth models after transforming, using the differences of equations method as in Clutter, Forston, Pienaar, Brister & Bailey (1983). The models were fitted by the quasi-Newton and simplex methods, using SYSTAT software (Wilkinson, 1990). Transformations were made to allow the incorporation of desirable characteristics such as convergence and invariance into the growth models. Fitted models were compared through respective values of index of fit (IF), which resembles the coefficient of determination (R^2), because the dependent variable is the same in all models (Schlaegel, 1981). The difference between IF and R^2 is related to the methodology employed to estimate the parameters of the models. For the least square method, model is adjusted through the linearization of the observed data and R^2 is calculated with these linearized data. For the quasi-Newton and simplex methods, the parameters of a model are estimated through iteration processes, without any change in the observed data, and the IF is calculated directly on the observed data, relating them to the estimated data.

Production Function

Production is defined as the total elaboration of fish tissue during any time interval Δt including what is formed by individuals that do not survive till the end of Δt (Ivlev, 1945,1964). According to Ricker (1971) production can be defined as the increase in biomass in a given time including the growth of those which die or which are caught during this interval.

Estimation of production involves indirect methods and those so far developed seem to fall into three principal categories: those in which production is estimated from knowledge of the number and size of the individuals in the population at a series of points in time; those based on estimates of the amount of food consumed by the population and the efficiency of its conversion into the tissues of the population components; and those based on estimates of the food consumption of a predator population, which is assumed to take up most of the production to be estimated (Allen,1971).

Ricker (1946) first devised a way in which production can be estimated given that, data on instantaneous growth rate (G) and instantaneous mortality rate (Z) is also constant and known $\frac{dB}{dt} = (G-Z)B$. This follows the assumption that number decreases and weight increases exponentially. Integrating the above equation $B_t = B_0 e^{(G-Z)t}$ where B_0 = biomass at time = 0. The average biomass over year is $\overline{B} = \int_0^1 B_0 e^{(G-Z)t}.dt = \frac{B_0(e^{(G-Z)}-1)}{G-Z}$

Generally, Z and particularly G rarely remain constant during the life span of an animal. In such cases, this technique can still be applied to a succession of sub periods within which it can reasonably be assumed that both G and Z are constant. It may, however, be more satisfactory and less laborious to use a formula for calculating production based on a growth curve that more closely resembles those found in nature and thus avoid the sub divisions of the life span that are otherwise required. Formulae suitable for this approach have been developed for the negative exponential growth curve by Allen (1950) and for the von Bertalanffy growth curve by Gulland in Chapman (1968).

The graphical method of estimating production (Allen,1951) is, the solution by mechanical means of the equation in which progressive values for number of fish are plotted against mean weight of individuals. The area beneath such a curve represents production, which can be determined for any part of the year which can numerically be given as $P = \int_{W_1}^{W_2} N_t dw_t = \int_{T_1}^{T_2} N_t \frac{dW_t}{dt} dt$.

He computed production in a trout population in this manner where N_t and W_t are the number of animals and the mean individual weight respectively at time t.

Ricker (1946) and Allen (1950) have found the production function $P = G\overline{B}$ where P is the production; G is the instantaneous growth rate in weight and $\overline{B}$ the average biomass during the period of reference. It is assumed here that G does not vary during the period and the growth in weight follows exponential law with time viz., W = Wo e ^{Gt} whereas change in numbers may follow any law. Chapman (1968) considered this to be a realistic formula, if G is known to be constant.

Beverton and Holt (1957) have evaluated the production function on the assumption that, growth in weight follows Von Bertlanffy's model and change in numbers follows exponential law with time. For annual production by a fish population, Chapman (1968) developed a formula based on exponential rates did not include the assumptions that either individual fish or fish populations grow or die in an exponential manner.

A series of mathematical models representing various combinations of several different simple growth models like exponential growth, simple asymptotic growth, von Bertalanffy growth, linear growth in length and linear growth in weight and mortality functions like simple exponential mortality, multiple exponential mortality, linear model and fixed life span model was developed by Allen (1971) and was used to investigate the ratio between mean biomass and production over unit time.

Gulin and Rudenko (1973) have estimated production of lake Demenets by using $P_t = a_t N_t (1 - e^{-Zt}) / z_t$ assuming that growth in weight is linear and change in numbers is exponential with time.

Huang *et al.* (1976) used the dynamic production modeling approach, which considered the prawn population in a pond as a system, where the number of prawn at any given time can be estimated when the relationship between survival and growth can be determined.

To quantify the production of commercial aquaculture, Santos (1978) used mathematical models such as weight or length growth rate, weight and length growth models and survival models, all serving as the basis for biomass models.

Polovina and Brown (1978) used a stochastic population model to simulate the prawn size distribution in the grow-out ponds as a function of time. The model allowed growth and mortality parameters, which were dependent on size class and pond biomass.

Different types of production functions were derived by Alagaraja (1986) based on linear and exponential models on both growth and mortality. In certain models no assumption was taken either for growth or mortality. From these he concluded that model based on linear relationship on numbers over time and growth over time were considered superior for its simplicity, theoretical soundness and practical applicability.

3.MATERIALS AND METHODS

3. MATERIALS AND METHODS

3.1 GROWTH MODELS

Growth is one of the most complex activities of an organism and is a factor of utmost importance in aquaculture. The weight of any organism can be expressed as a function of time (age) and so growth models are a standard product of weight at age data. The different growth models fitted to the data are described below.

3.1.1 Gompertz growth model

This model was proposed by Gompertz (1825) to describe a portion of the ages in human population. It is the most used model for describing growth and is given by

$$W_t = W_0 e^{k(1-e^{(-gt)})}$$

where

W_t is the weight at time t

W_0 is the initial weight i.e. weight at time, t=o

g is the instantaneous rate of growth .

k is a dimensionless parameter .

3.1.2 Logistic growth model

The integral form of the logistic equation introduced by Verhulst (1838) for modeling growth. The logistic law of growth assumes that systems grow exponentially until an upper limit or "carrying capacity" inherent in the system is approached, at which point the growth rate slows and eventually saturates, producing the characteristic S-shape curve. It is given by

$$W_t = \frac{\alpha}{1+\beta e^{-kt}}$$

where

W_t is the weight at time t.

α is the asymptotic weight

k is the instantaneous rate of growth .

β is the free parameter that adjust the shape of the curve.

3.1.3 von Bertalanffy growth model

Von Bertalanffy (1957), derived a function for body weight growth based on the principles of Putter (1920). The Von Bertalanffy growth curve assumes that fish grows towards some theoretical maximum weight and as they get closer to the maximum, the slower the rate of change of size. It has become one of the corner stones in fishery biology as it is used as a sub model in more complex models describing the dynamics of fish population. It assumes isometric relationship between length and weight and is given by

$$w_t = w_\alpha(1-e^{-kt})^3$$

w_α is the asymptotic weight or adult value,

k is the instantaneous rate of growth .

3.1.4 Richards growth model

In 1959, Richards developed a function, which is the most frequently used growth model. It is defined by:

$$W_t = A(1+e^{kt})^{-M}$$

where

A is asymptotic (mature) weight

k is rate of approach to mature weight

M is a shape parameter that allows for a variable inflection point.

The Richards growth curve is the generalization of all the growth curves mentioned above. At M=1 it changes into the Logistic function and at M against +/- infinity into the Gompertz function.

3.1.5 Modified Janoschek growth model

This model was introduced by Janoschek and a modified form of the model is given by

$$W_t = W_0 e^{-kt^p}$$

where W_0 is the adult value or asymptote

The parameters k and p adjust both slope and point of inflection of the curve.

3.1.6 Polynomial growth model

The polynomial functions evaluated is

$$w_t = a + bt + ct^2$$

where w_t is the weight at time t

a, b and c are the constants

3.1.7 Power Function

$$w_t = a + bt^c$$

where w_t is the weight at time t

a, b and c are the constants

3.2 Fitting of nonlinear models

Nonlinear Estimation is a general fitting procedure that will estimate any kind of relationship between a dependent (or response variable), and a list of independent variables. In general, all models may be stated as:

$$y = F(x_1, x_2, \dots, x_n)$$

In most general terms, we are interested in whether and how a dependent variable is related to a list of independent variables; the term *F(x...)* in the expression above means that *y*, the dependent or response variable, is a function of the x's, that is, the independent variables.

3.2.1 Least squares estimation.

In the most general terms, least squares estimation is aimed at minimizing the sum of squared deviations of the observed values for the dependent variable from those predicted by the model. The term *least squares* was first used by Legendre, 1805.

Minimization of residual sum of squares yield normal equations, which are nonlinear in the parameters. Since it is not possible to solve nonlinear equations exactly, the next alternative is to obtain approximate analytic solutions by employing iterative procedures. Three main methods of this kind are:

I. Linearization (or Taylor series) method

II. Steepest Descent method

III. Levenberg-Marquardt's method

The most widely used method of computing nonlinear least squares estimators is the Levenberg-Marquardt's method. This method represents a compromise between the other two methods and combines successfully the best features of both and avoids their serious disadvantages. The Levenberg-Marquardt (LM) algorithm allows for a smooth transition between these two methods as the iteration proceeds. It is good in the sense that it almost always converges and does not 'slow down' at the latter part of the iterative process.

3.2.2 Loss Functions.

Any deviation of an observed score from a predicted score signifies some *loss* in the accuracy of prediction, for example, due to random noise (error). Thus the goal of least squares estimation is to minimize a *loss function*; specifically, this loss function is defined as the sum of the squared deviation about the predicted values. When this function is at its minimum, then we get the same parameter estimates (intercept, regression coefficients), because of the particular loss functions that yielded those estimates, we can call the estimates *least squares estimates*.

3.2.3 Quasi-Newton Method.

The *slope of a function* at a particular point can be computed as the *first*- order derivative of the function (at that point). The "slope of the slope" is the second-order derivative, which tells how fast the slope is changing at the respective point, and in which direction. The quasi-Newton method will, at each step, evaluate the function at different points in order to estimate the first-order derivatives and second-order derivatives. This information is used to follow a path towards the minimum of the loss function.

3.2.4 Simplex Procedure.

This algorithm does not rely on the computation or estimation of the derivatives of the loss function. Instead, at each iteration the function will be evaluated at $m+1$ points in the m dimensional parameter space. An additional strength of this method is that when a minimum appears to have been found, the Simplex will again be expanded to a larger size to see whether the respective minimum is a local minimum. Thus, in a way, the Simplex moves like a smooth single cell organism down the loss function, contracting and expanding as local minima or significant ridges are encountered.

3.2.5 Choice of Initial values.

All the procedures for nonlinear estimation require initial values of the parameters and the choice of good initial values is very crucial. However, there is no standard procedure for getting initial estimates. The most obvious method for making initial guesses is by the use of prior information. Estimates calculated from the previous experiments, known values for similar systems, values computed from theoretical considerations all these form ideal initial guesses.

3.3 Data treatment

Data was fitted using the least square method in Microsoft excel as described in Draper and Smith (1981). *Non-linear regression algorithm of SYSTAT* 7.0 used to estimate the parameters.

Fitting of the growth equation was also done after transforming equation using the difference of equation method. An example for transformation of data is given below.

We have the von Bertalanffy growth equation

$$w_n = w_\infty (1 - e^{-kt_n})^3$$

w_α is the asymptotic weight or adult value, value is attained at t approaches infinity

k *is the instantaneous rate of growth .*

At time $t=t_1$ we have weight for the time as

$$w_1 = w_\infty (1 - e^{-k t_1})^3$$

rearranging we get

$$w_\infty = \frac{w_1}{(1 - e^{-kt_1})^3}$$

we also have at time $t=t_2$

$$w_2 = w_\infty (1 - e^{-kt_2})^3$$

Substituting for w_∞ $$w_2 = \frac{w_1}{(1 - e^{-kt_1})^3}(1 - e^{-kt_2})^3$$

Generalizing $$w_n = \frac{w_{n-1}}{(1 - e^{-kt_{n-1}})^3}(1 - e^{-kt_n})^3$$ Transformations were made to allow the incorporation of desirable characteristics such as convergence and invariance into the model. After transforming, the data was fitted using the non-linear regression method in SYSTAT 7.0 software.

3.4 Goodness of fit

After estimating the parameters, an essential aspect of the analysis is to test the appropriateness of the overall model.

3.4.1 Index of fit (IF)

Fits of models were also compared using the index of fit as in Rosa *et al.* (1997) which is defined as

$$IF = 1 - \left\{\frac{[\sum_{i=1} (y_i - \hat{y}_i)^2]}{[\sum_{i=1} (y_i - \overline{y}_i)^2]}\right\}$$

where

y_i is the observed values

$\overline{y_i}$ is the mean of the observed values

$\hat{y}_i$ represents estimated values of y_i

The difference between *IF* and R^2 (coefficient of determination) is related to the methodology employed to estimate the parameters of the models. For the least square method, a model is adjusted through the linearisation of the observed data and R^2 is calculated with the linearised data. For Qausi-Newton and simplex method, the parameters of the model are through iteration processes, without any change in the observed data, and the *IF* is calculated directly on the observed data relating them to the estimated data.

3.5 Production functions

Production is an increase in biomass over a given period of time. Production may be estimated either numerically or graphically.

Ricker (1946) and Allen (1950) have found the production function

$$P = G\overline{B}$$

Where P is the production, G the instantaneous growth rate in weight and $\overline{B}$ the average biomass during the period of reference.Numerically Production can be formulated as $P_t = \int_0^t N_t . dw . dt$, assuming without loss of generality that the entire period is divided into segments as months, year, seasons etc .

where P_t is the production in (t, t+1) segments

N_t is the number of organisms in t time segment

In the development of the model, different assumptions were put forward:

1) The individual organism under consideration is assumed to be biologically uniform i.e., both in terms of age, weight and length.
2) Operation is restricted to one group of organism i.e., organism of the same year class

3) The external factors which determine the growth of the organism and that can be controlled such as water temperature, P_H and level of dissolved oxygen, feeding rate, etc are considered to be constant over time.

3.5.2 *Evaluation of production function*

Production functions were derived using different assumptions for growth and mortality .

3.5.2.1 Growth models

The following growth models were taken into consideration.

1) Gompertz growth model
2) Logistic growth model
3) von Bertalanffy growth model
4) Richards growth model
5) Modified Janoschek growth model
6) Power function
7) Polynomial growth model

3.5.2 Mortality assumptions

Three types of mortality were taken into consideration for derivation of production function.

i. No assumptions on mortality were made.

ii. Linear model - mortality is assumed to have a linear relation with time.

$$N_t(t) = a + bt$$

Where $N_t(t)$ = number of animals at time = t , a = initial number of animals, b = rate of mortality

iii. Exponential model

$$N_t(t) = N_0 e^{-zt}$$

Where $N_t(t)$=number of animals at time=t

N_0 = initial number of animals

z = rate of mortality

3.5.3 Numerical integration

The Fundamental Theorem of Calculus gives an exact formula for computing $\int_a^b f(x).dx$, to find an integral for f. This method of evaluating definite integrals is called the analytic method. However, there are times when this is difficult or impossible. In these cases, an approximate, or numerical solution is obtained using the numerical integration process in MATLAB software.

3.6 Descriptions of Data used for analysis.

The analysis was carried out using secondary data sets on mariculture experiments.

3.6.1 Data –1

Edible oyster *Crassostrea madrasensis* is mainly cultured in temperate countries. In India oyster culture on commercial lines has not yet started. Experimental culture of edible oysters was done in Ashatamudi Lake to confirm its suitability for culture. The experiment was planned with the objective to collect the natural oyster spat from the extensive oyster beds in the Ashatamudi Lake and grow them. Monthly sampling was done and the weights were noted. Harvest was done after a period of 11 months (Velayudhan *et al.*, 1998).

3.6.2 Data-2

Culture of Green mussel *Perna viridis* is gaining importance in Kerala. An experimental culture was taken up the Molluscan Fisheries Division of CMFRI in Dalavapuram, Quilon district to test the suitability of the site for culture. The seed for culture were collected from the Kollam bay and were seeded in ropes. These were cultured in Dalavapuram. The culture was for a period of 6 months.

3.6.3 Data-3

Polyculture of mullets and shrimp are experimented in different systems in India and abroad. The feeding habits and its adaptation to varying salinity and availability of seed makes Mugil cephalus a good species for polyculture. In the current experiment polyculture of mullets were done in the experimental ponds of Narrakkal to estimate the production and survival in a short-term period. Monthly sampling was done and the environmental parameters were also noted. Culture was done for a period of 8 months (Imelda *et al.*, 2001).

3.6.4 Data –4

As a part of the *Integrated Village Linkage Programme* (IVLP) assessment of scientific monoculture of Mugil cephalus was under taken in tide- fed ponds of Elamkunnapuzha village of Ernakulam district. Monoculture of Mugil cephalus is the identified techno intervention to solve the intricate problem of low productivity in tide fed ponds. Sampling was done on a regular basis (Sathiadas *et al.*, 2003).

4. RESULTS

4. RESULTS

4.1 Fitting of Growth Models

The growth models were fitted to the data using Gauss-Newton method in *nonlinear regression*. The initial values of the parameters were made by making some guesses and using solver option in MS-EXCEL. The models were fitted both to the original and transformed data according to the *difference of equation method*.

The growth models fitted were

- Gompertz growth model
- Logistic growth model
- von Bertalanffy growth model
- Richards growth model
- Modified Janoschek growth model
- Power function
- Polynomial growth model

After fitting the models, the models were evaluated using the index of fit.

Data –1

The models were fitted to monthly growth data of Crassostrea *madrasensis*. Both the shell on weight and the meat weight were fitted. The estimated parameters and the index of fit are given in Table.1 for shell on weight and in Table2 and Table 3 for the meat weight. Logistic model gave the best fit compared to all other models for shell on weight while Gompertz model gave a better fit for meat weight (Fig.1. and Fig.2.)

Table 1. The parameters obtained for different growth models for the data-1(*Crassostrea madrasensis* shell on weight)

Gompertz Model							
Original Data				Transformed Data			
Parameters		Confidence Limits(95%)		Parameters		Confidence Limits(95%)	
		Upper	Lower			Upper	Lower
W0	1.5	-0.314	3.314	W0	-	-	-
k	3.364	2.245	4.484	k	3.3	1.845	4.755
g	0.339	0.219	0.458	g	0.3	0.12	0.48
Index of Fit			0.97	Index of Fit			0.94
Logistic Model							
Original Data				Transformed Data			
Parameters		Confidence Limits(95%)		Parameters		Confidence Limits(95%)	
		Upper	Lower			Upper	Lower
alpha	39.483	35.878	43.088	alpha			
b	14.347	5.15	23.545	b	14	-2.119	30.119
k	0.607	0.436	0.778	k	0.6	0.312	0.888
Index of Fit			0.98	Index of Fit			0.95
Polynomial Model (a+bt+ct^2)							
Original Data				Transformed Data			
Parameters		Confidence Limits(95%)		Parameters		Confidence Limits(95%)	
		Upper	Lower			Upper	Lower
a	-1.06	-5.464	3.344				
b	5.71	3.852	7.568				
c	-0.171	-0.332	-0.01				
Index of Fit			0.96	Index of Fit			

Table 2. The parameters obtained for Gompertz and Logistic models for the data-1 (*Crassostrea madrasens:'is* meat weight)

Gompertz Model							
Original Data				Transformed Data			
Parameters		Confidence Limits(95%)		Parameters		Confidence Limits(95%)	
		Upper	Lower			Upper	Lower
W0	0.172	-0.155	0.499	W0	-	-	-
k	3.385	1.523	5.248	k	3.3	1.943	4.657
g	0.594	0.354	0.833	g	0.59	0.229	0.951
Index of Fit			0.96	Index of Fit			0.875
Logistic Model							
Original Data				Transformed Data			
Parameters		Confidence Limits(95%)		Parameters		Confidence Limits(95%)	
		Upper	Lower			Upper	Lower
alpha	4.988	4.658	5.319	alpha			
b	13.21	1.727	24.709	b	13	0.021	25.979
k	0.921	0.609	1.234	k	0.92	0.516	1.324
Index of Fit			0.96	Index of Fit			0.873

Table 3. The parameters obtained for different growth models for the data-1 (*Crassostrea madrasensis* meat weight)

VBGF							
Original Data							
Parameters		Confidence Limits(95%)		Parameters		Confidence Limits(95%)	
		Upper	Lower			Upper	Lower
W0	5.051	4.613	5.488	W0			
g	0.596	0.474	0.718	g	2.43	0.677	4.183
Index of Fit			0.95	Index of Fit			0.815
Polynomial Model (a+bt+ct^2)							
Original Data				Transformed Data			
Parameters		Confidence Limits(95%)		Parameters		Confidence Limits(95%)	
		Upper	Lower			Upper	Lower
a	-0.12	0.32	-0.377				
b	1.195	0.894	1.495				
c	-0.06	-0.094	-0.042				
Index of Fit			0.95	Index of Fit			

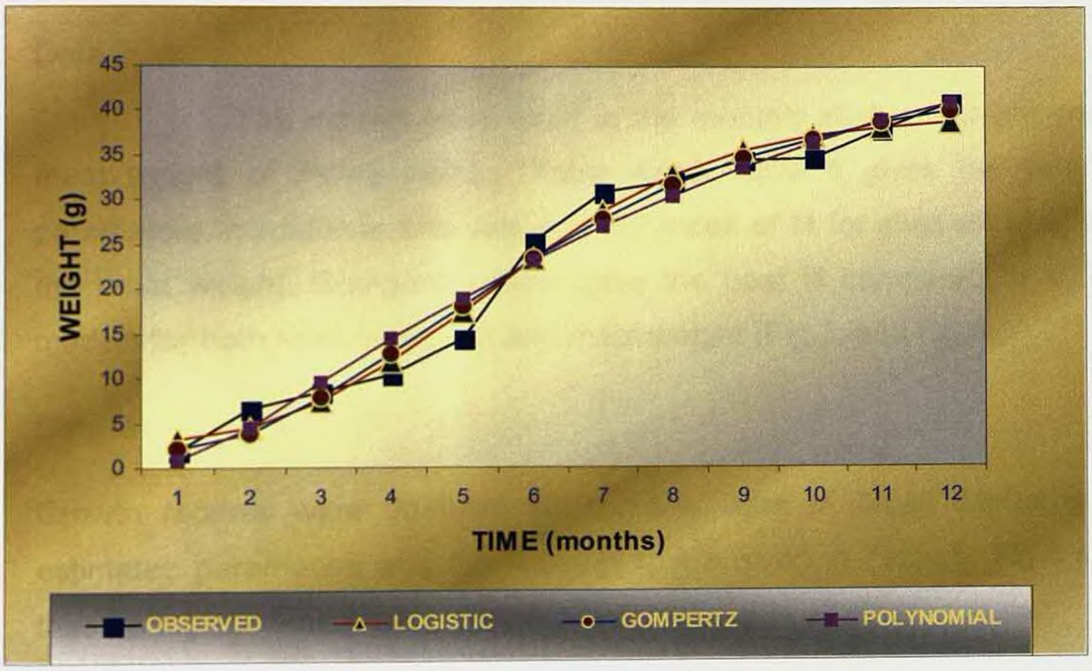

Fig.1. Different growth models fitted for data-1 (*Crassostrea madrasensis* shell on weight).

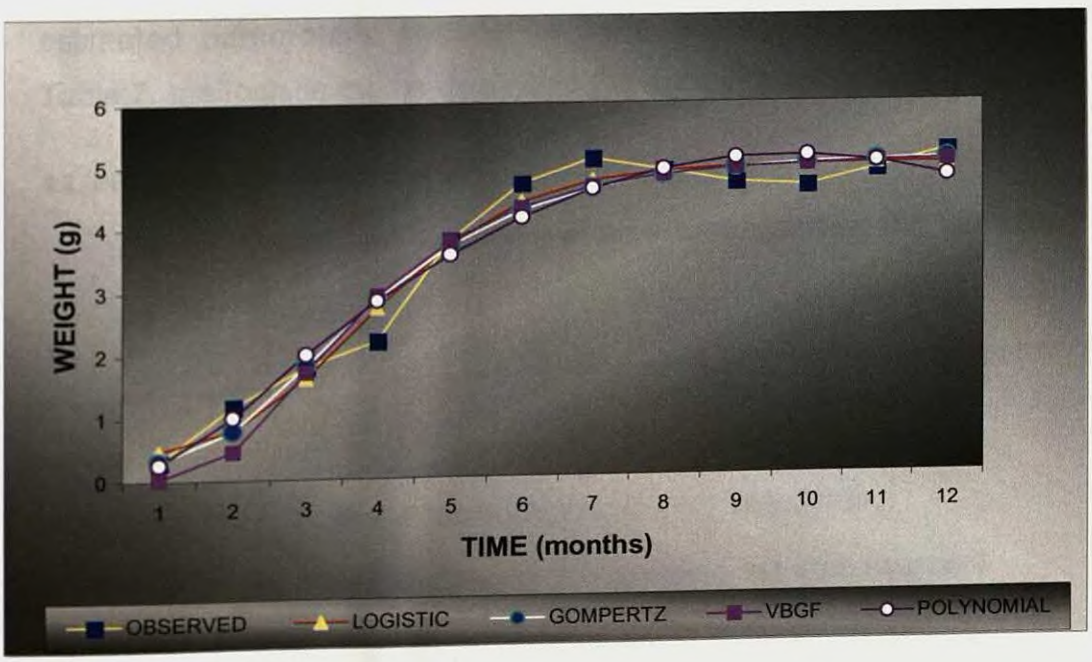

Fig.2. Different growth models fitted for data-1 (*Crassostrea madrasensis* meat weight).

Data-2

The models were fitted to the monthly shell on weight and the meat weight of *Perna viridis.* Table 4and Table 5 gives the estimated parameters, confidence intervals and the index of fit for shell on weight and the meat weight. Gompertz model gave the best fit compared to all other models for both shell on weight and meat weight (Fig.3. and Fig.4.)

Data-3

Growth models were fitted to the monthly data of *Mugil cephalus.* The estimated parameters and the index of fit are given in Table.6. Fig.5 shows that the logistic model gave the best fit compared to all other models.

Data –4

The models were fitted to growth data of *Mugil cephalus.* The estimated parameters, confidence limits and the index of fit are given in Table.7. the logistic model described the data in a better way .

4.2 PRODUCTION FUNCTION

Production function can be estimated using the formulae

$$P_t = \int_0^t N_t . dW_t . dt$$

where

P_t is the production in (t, t+1) segments .

N_t is the number of organisms in t time segment

$$dW_t = \frac{d(W_t)}{dt}$$

Table 4. The parameters obtained for different growth models for the data-2 (*Perna virdis* shell on weight)

Gompertz Model							
Original Data				Transformed Data			
Parameters		Confidence Limits(95%)		Parameters		Confidence Limits(95%)	
		Upper	Lower			Upper	Lower
W0	2.4	-1.6	6.4	W0			
k	4.078	3.008	5.147	k	3.92	-0.38	8.226
g	0.172	0.034	0.31	g	0.22	-0.146	0.6
Index of Fit			0.96	Index of Fit			.897
Logistic Model							
Original Data				Transformed Data			
Parameters		Confidence Limits(95%)		Parameters		Confidence Limits(95%)	
		Upper	Lower			Upper	Lower
alpha	96.46	50.468	142.462	alpha			
b	22.39	5.587	39.192	b	22	-27.47	71.47
k	0.402	0.2	0.604	k	0.43	0.006	0.872
Index of Fit			0.97	Index of Fit			0.893
Polynomial Model (a+bt+ct^2)							
Original Data				Transformed Data			
Parameters		Confidence Limits(95%)		Parameters		Confidence Limits(95%)	
		Upper	Lower			Upper	Lower
a	-0.58	-9.62	8.446				
b	3.811	0	7.621				
c	0.303	-0.027	0.633				
Index of Fit			0.965				

Table 5. The parameters obtained for different growth models for the data-2 (*Perna virdis* meat weight)

Gompertz Model							
Original Data				Transformed Data			
Parameters		Confidence Limits(95%)		Parameters		Confidence Limits(95%)	
		Upper	Lower			Upper	Lower
W0	0.691	-0.66	2.043	W0			
k	4.497	3.177	5.817	k	4.4	0.247	8.553
g	0.179	0.043	0.316	g	0.221	-0.082	0.525
Index of Fit			0.976	Index of Fit			0.911
Logistic Model							
Original Data				Transformed Data			
Parameters		Confidence Limits(95%)		Parameters		Confidence Limits(95%)	
		Upper	Lower			Upper	Lower
alpha	41.68	23.952	59.408	alpha			
b	28.564	6.691	50.437	b	28	-33.894	89.894
k	0.425	0.233	0.617	k	0.451	0.035	0.867
Index of Fit			0.972	Index of Fit			0.907
Polynomial Model (a+bt+ct^2)							
Original Data				Transformed Data			
Parameters		Confidence Limits(95%)		Parameters		Confidence Limits(95%)	
		Upper	Lower			Upper	Lower
a	1.47	-0.374	3.314				
b	0.157	-0.003	0.317				
c	0.65	-3.722	5.022				
Index of Fit			0.96	Index of Fit			

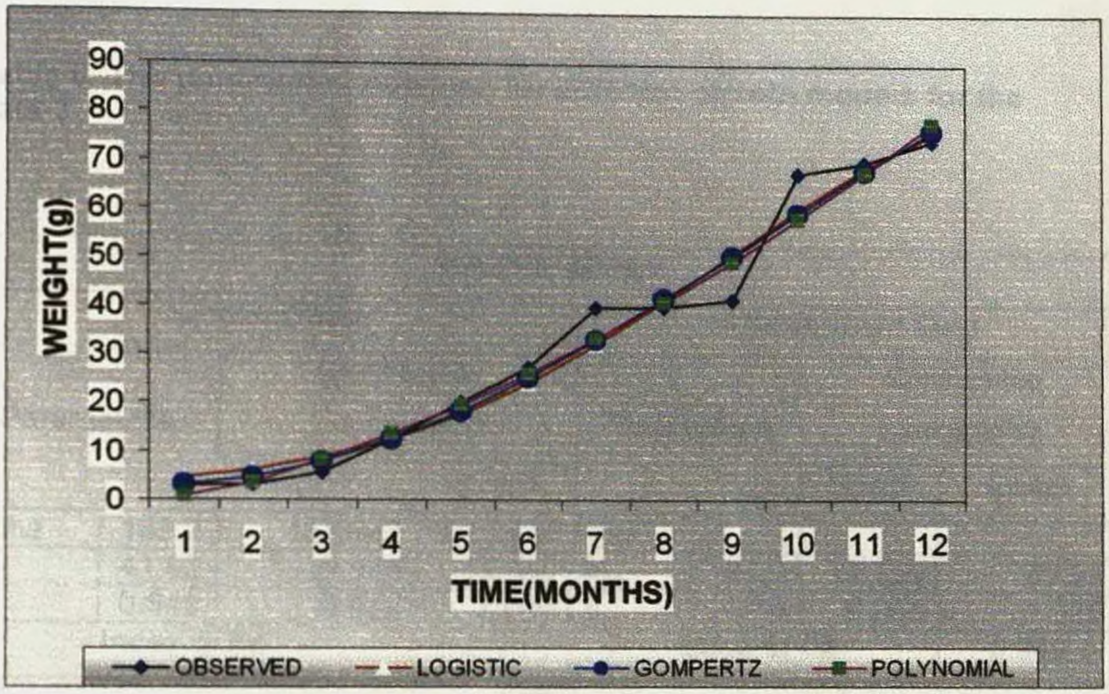

Fig.3. Different growth models fitted for data-2 (*Perna viridis* shell on weight).

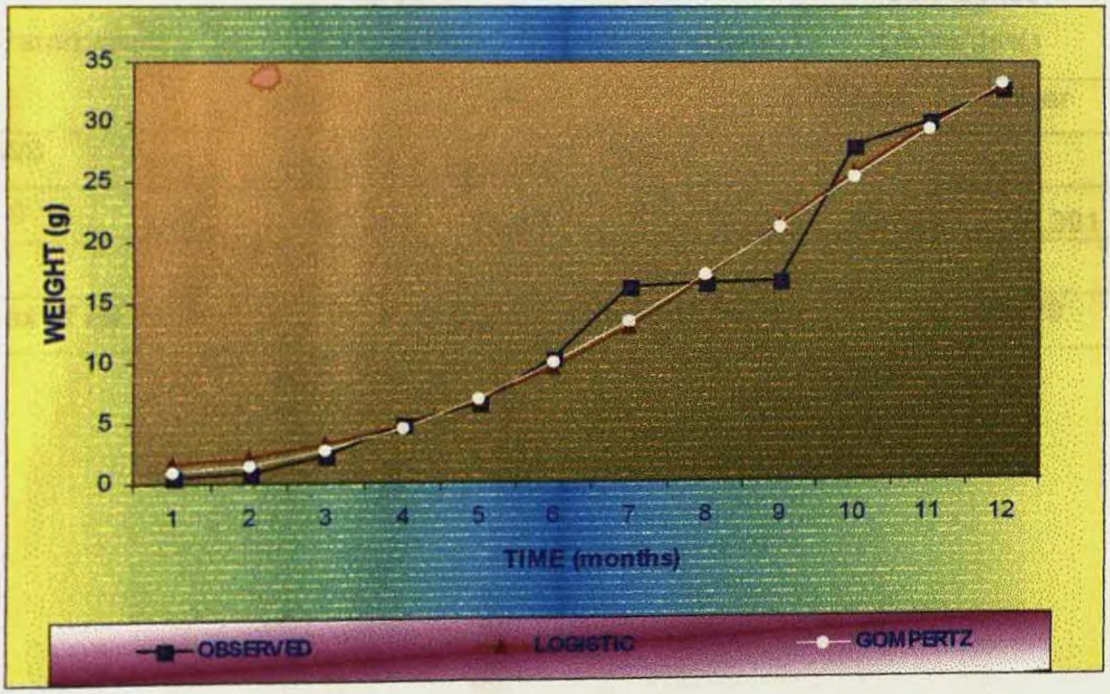

Fig.4. Different growth models fitted for data-2 (*Perna viridis* meat weight).

Table 6. The parameters obtained for different growth models for the data-3

Logistic Model							
Original Data				Transformed Data			
Parameters		Confidence Limits(95%)		Parameters		Confidence Limits(95%)	
		Upper	Lower			Upper	Lower
alpha	189.2	133.443	245.063	alpha			
b	41.02	9.573	72.481	b	38.2	-20.756	97.23
k	0.648	0.419	0.877	k	0.64	0.268	1.026
Index of Fit			0.99	Index of Fit			0.949
VBGF							
Original Data							
Parameters		Confidence Limits(95%)		Parameters		Confidence Limits(95%)	
		Upper	Lower			Upper	Lower
W0	316	191.685	440.315	W0			
g	0.2	0.145	0.255	g	0.23	0.073	0.391
Index of Fit			0.98	Index of Fit			0.939

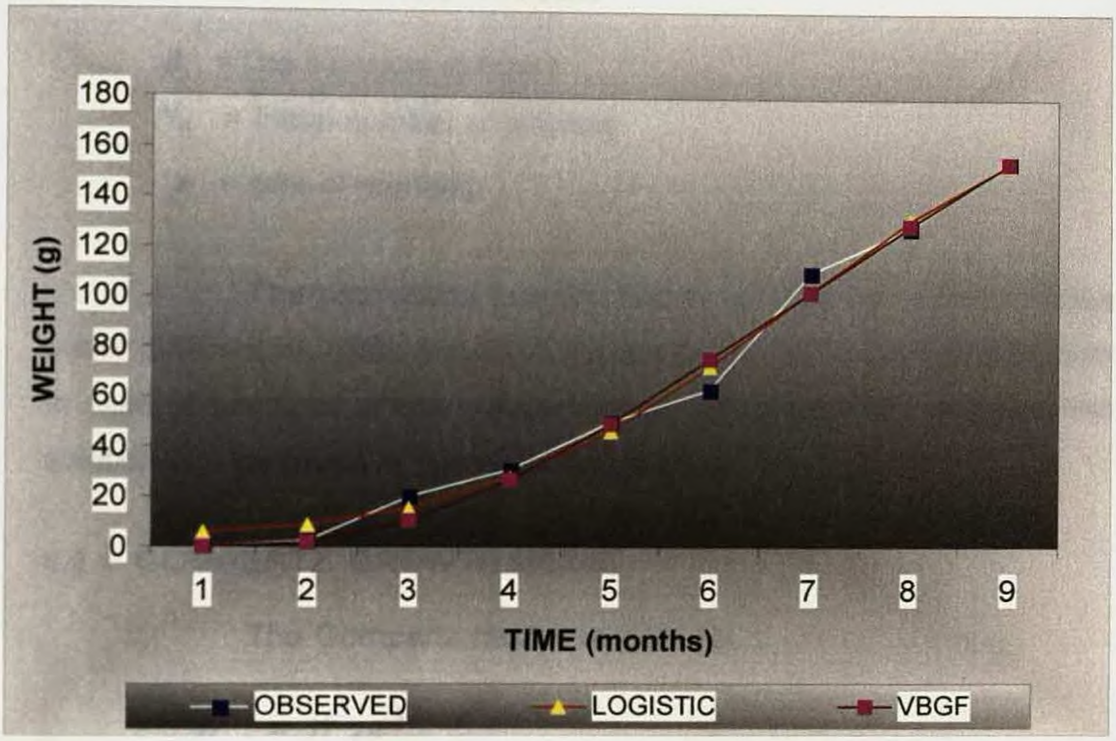

Fig.5. Different growth models fitted for the data-3.

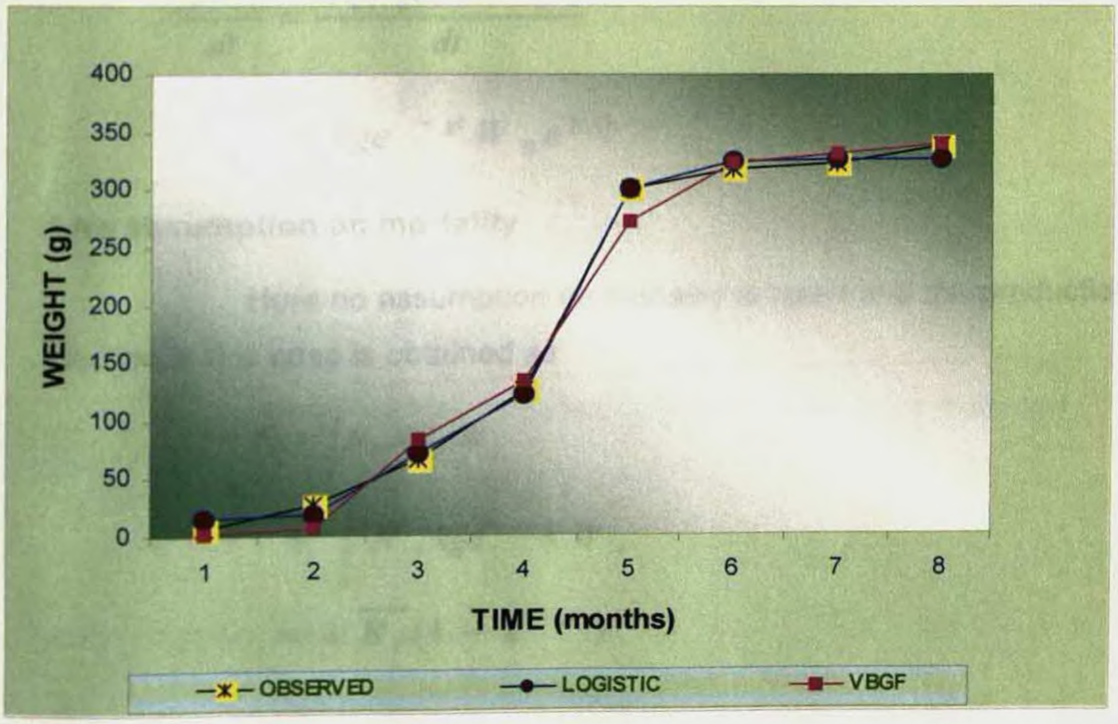

Fig.6. Different growth models fitted for data-4.

Other notations used in the results

B_t =The biomass at time t

N_0 = Initial number of animals

z = rate of mortality

The production function based on different growth models and under different mortality assumptions are described in the following sections. In case of functions whose integral cannot be obtained by analytic methods, solution can be found by numerical integration.

4.2.1 GOMPERTZ GROWTH MODEL

The Gompertz model is given by

$$W_t = W_0 e^{k(1-e^{(-gt)})}$$

$$\frac{dW_t}{dt} = \frac{d(W_0 e^{k(1-e^{(-gt)})})}{dt}$$

$$= kge^{-gt} W_0 e^{k(1-e^{(-gt)})}$$

1.No assumption on mortality

Here no assumption on mortality is taken and the production function in this case is obtained as

$$P_t = \int_0^t N_t.dW_t.dt$$

$$= \int_0^t N_t kge^{-gt} W_t$$

$$= k\overline{B_t}(1 - e^{-gt})$$

2. Linear assumption on mortality

When a linear relationship is assumed in mortality, we can write N_t as

$$N_t = a_1 + b_1 t$$

and production can be estimated as

$$P_t = \int_0^t N_t . dW_t . dt$$

$$= \overline{B_t} - \int_0^t bW_0 e^{k(1-e^{(-gt)})} . dt$$

3. Exponential assumption on mortality.

When the change in population size is exponential, we have

$$N_t = N_0 e^{-Zt}$$

and then production will be

$$P_t = \int_0^t N_t . dW_t . dt$$

$$= \overline{B_t} + N_0 zW_0 e^k \int_0^t e^{-zt - ke^{-gt}}$$

4.2.2 LOGISTIC GROWTH MODEL

Let us assume that growth is according to the logistic law then,

$$W_t = \frac{\alpha}{1 + \beta e^{-kt}}$$

which on differentiation gives

$$\frac{d(W_t)}{dt} = k\beta e^{-kt} \frac{\alpha}{(1+\beta e^{-kt})^2}$$

1.No assumption on mortality

The production function in this case can be obtained as

$$P_t = \int_0^t N_t.dW_t.dt$$

$$= \int_0^t \frac{N_t.W_t k\beta e^{-kt}}{(1+\beta e^{-kt})}.dt$$

$$= k\beta \overline{B_t} \int_0^t \frac{e^{-kt}}{(1+\beta e^{-kt})}.dt$$

$$= B_T((\log(1+\beta e^{-kt})) - (\log(1+\beta)))$$

2. Linear assumption on mortality

When there is a linear assumption on mortality,

$$N_t = a_1 + b_1 t$$

The corresponding production function is obtained as

$$P_t = \int_0^t N_t.dW_t.dt$$

$$= \overline{B_T} - \int_0^t \frac{b_1\alpha}{(1+\beta e^{-kt})}.dt$$

$$= \overline{B_t} + \frac{b_1\alpha}{k}\log(\frac{\beta(e^{-kt}-1)}{1+\beta(e^{-kt}-1)})$$

. 3. Exponential assumption on mortality

When the mortality is assumed to follow the exponential law, we have

$$N_t = N_0 e^{-Zt}$$

In this production function can be derived as

$$P_t = \int_0^t N_t . dW_t . dt$$

$$= \overline{B_t} + N_0 \alpha z \int_0^t \frac{1}{e^{Zt}(1+\beta e^{-kt})} . dt$$

4.2.3 RICHARDS GROWTH MODEL

According to Richards model we have

$$W_t = A(1 + e^{-kt})^{-M}$$

and

$$\frac{d(W_t)}{dt} = AMk\,(1 + e^{-kt})^{(M+1)} e^{-kt}$$

1.No assumption on mortality

In the absence of pre assumed relation on mortality we have

$$P_t = \int_0^t N_t . dW_t . dt$$

$$= \int_0^t N_t AMk\,(1 + e^{-kt})^{-(M+1)} e^{-kt} . dt$$

$$= M\overline{B_t} \log(2 - e^{-kt})$$

2. Linear assumption on mortality

Here $N_t = a_1 + b_1 t$

The production function in this situation is

$$P_t = \int_0^t N_t . dW_t . dt$$

$$= \overline{B_T} - \int_0^t \frac{b_1 A (1 + e^{-kt})^{-M}}{K(e^{-kt})}.dt$$

$$= \overline{B_T} - \frac{b_1 A}{K} \int_0^t \frac{(1 + e^{-kt})^{-M}}{(e^{-kt})}.dt$$

3. Exponential assumption on mortality.

With the exponential assumption on mortality we have

$$N_t = N_0 e^{-Zt}$$

$$P_t = \int_0^t N_t.dW_t.dt$$

$$= \overline{B_t} + N_0 Az \int_0^t e^{-Zt} (1 + e^{-kt})^{-M}$$

4.2.4 MODIFIED JANOSCHEK GROWTH MODEL

This growth model is given by

$$W_t = (W_0) e^{-kt^p}$$

$$\frac{dW_t}{dt} = -W_0 kpe^{-kt^p} t^{p-1}$$

1.No assumption on mortality

In this case the evaluated production function is

$$P_t = \int_0^t N_t.dW_t.dt$$

$$= \int_0^t - W_0 kpe^{-kt^p} t^{p-1} N_t.dt$$

$$= -k\overline{B_t}t^p$$

2. Linear assumption on mortality

When $N_t = a_1 + b_1 t$ i.e., under the linear assumption on mortality ,the production function can be obtained as

$$P_t = \int_0^t N_t.dW_t.dt$$

$$= \overline{B_T} - \int_0^t b_1 W_0 e^{-kt^p}.dt$$

$$= B_T - b_1 W_0 \int_0^t e^{-kt^p}.dt$$

3. Exponential assumption on mortality .

When mortality is according to the exponential law we have $N_t = N_0 e^{-Zt}$ and consequently the production function becomes

$$P_t = \int_0^t N_t.dW_t.dt$$

$$= \overline{B_t} + N_0 z W_0 \int_0^t e^{-Zt} e^{-kt^p}.dt$$

4.2.5 POWER FUNCTION

The following power function was used to evaluate the production function under different assumptions on mortality.

$$W_T = at + bt^c$$

$$\frac{dW_t}{dt} = a + cbt^{c-1}$$

1.No assumption on mortality

In this case the production function can be derived as

$$P_t = \int_0^t N_t . dW_t . dt$$

$$= \int_0^t N_t (cbt^{c-1} + a)$$

$$= N_t at + \int_0^t N_t (cbt^{c-1})$$

$$= N_t (at + bt^c)$$

2. Linear assumption on mortality

The production function evaluated under the linear assumption on mortality, i.e., with $N_t = a_1 + b_1 t$ is

$$P_t = \int_0^t N_t . dW_t . dt$$

$$= N_t (at + bt^c) - \left[\frac{b_1 at^2}{2} + \frac{b_1 bt^c}{c+1} \right]$$

3. Exponential assumption on mortality.

Under this assumption we have $N_t = N_0 e^{-Zt}$ and the derived production function is

$$P_t = \int_0^t N_t.dW_t.dt$$

$$= N_t(at + bt^c) - N_0 a(e^{-zt} - 1) + bz\int_0^t e^{-zt}t^c$$

4.2.6 POLYNOMIAL GROWTH MODELS

The quadratic model $W_T = A + Bt + Ct^2$ is considered here for the derivation of production functions. On differentiating this function with respect to time we get

$$\frac{dW_t}{dt} = B + 2Ct$$

1.No assumption on mortality

The production function under the quadratic model with no assumption on mortality is

$$P_t = \int_0^t N_t.dW_t.dt$$

$$= \int_0^t N_t(B + 2Ct)$$

$$= N_t(Bt + Ct^2)$$

2. **Linear assumption on mortality**

In this case $N_t = a_1 + b_1 t$, i.e, the change in population size is linear with respect to time and the production function can be evaluated as

$$P_t = \int_0^t N_t . dW_t . dt$$

$$= N_t \left[(Bt + Ct^2)\right]_0^t - b_1 \left[(\frac{Bt^2}{2} + \frac{ct^3}{3}) \right]_0^t$$

$$= N_t (Bt + Ct^2) \; - b_1 (\frac{Bt^2}{2} + \frac{ct^3}{3})$$

3. Exponential assumption on mortality

The production function under the exponential assumption of mortality is

$$P_t = \int_0^t N_t . dW_t . dt$$

$$= \int_0^t N_0 e^{-Zt} (b + 2ct)$$

$$= \int_0^t N_0 e^{-Zt} b + \int_0^t N_0 e^{-Zt} 2ct$$

$$= N_0 b \int_0^t e^{-Zt} + N_0 2c \int_0^t e^{-Zt} t$$

$$= \frac{N_0 b}{z} \left[1 - e^{-zt}\right] \; - N_0 2c[\frac{1}{z^2} (e^{-zt} - 1)(zt - 1)]$$

4.3 Evaluation of production function

The derived production function was fitted to the data. Production of Oysters cultured in the Ashtamudi Lake of Quilon district was used for analysis. All the models derived were fitted and since good results were given by Gompertz and logistic growth models with linear and exponential assumptions on mortality. The results are given in Table-8 and the fitted models are graphically described in fig.7, 8,9 respectively

Table. 8 The production function fitted along with the assumptions taken and the index of fit.

Sl.No	Growth model	Mortality	Index of fit
1	Gompertz	No assumption	0.61
		linear	0.71
		exponential	0.71
2	Logistic	No assumption	0.68
		linear	0.66
		exponential	0.71

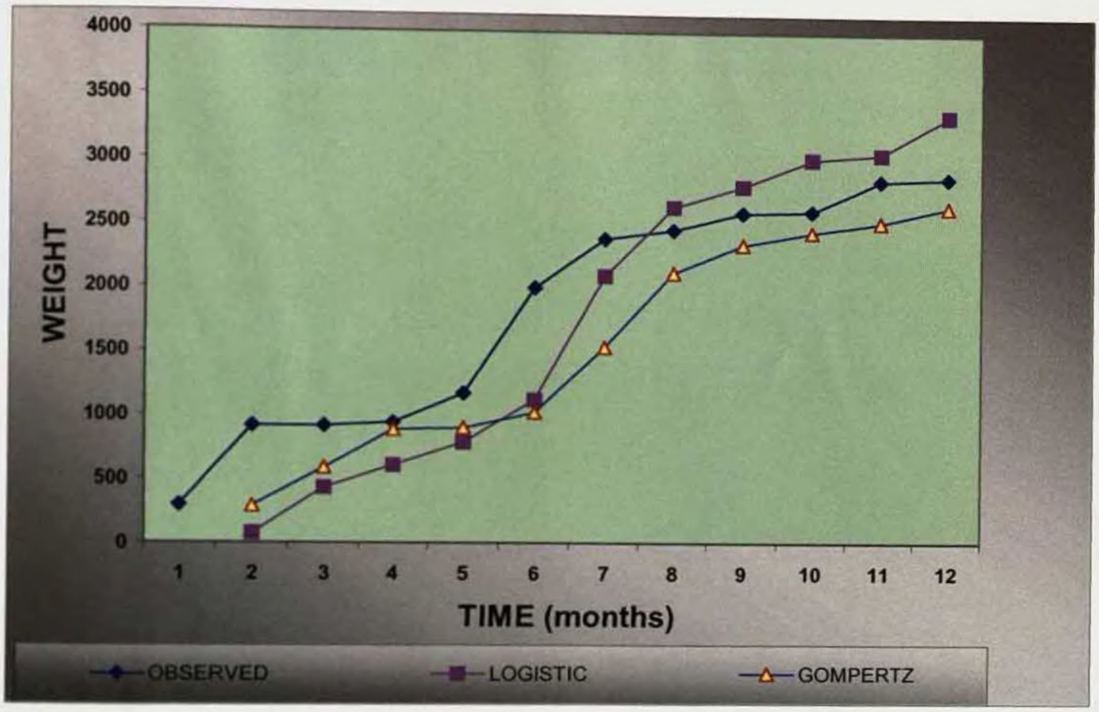

Fig.7. Production function when there is no assumption on mortality.

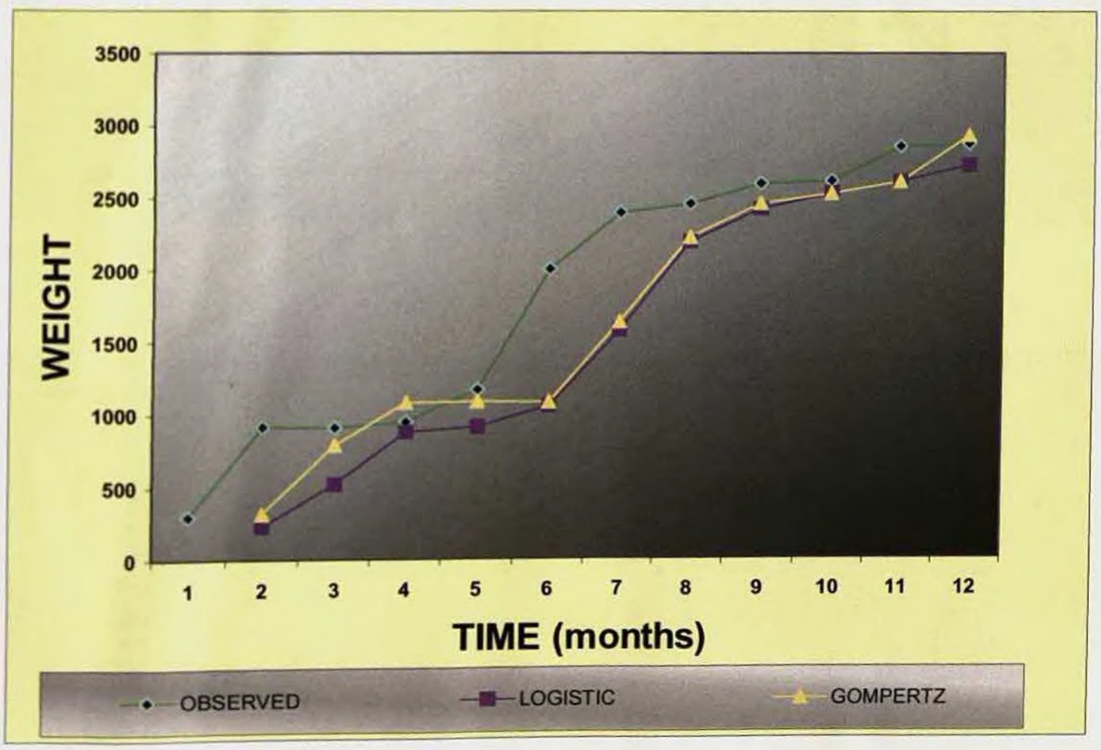

Fig.8. Production function when assumption on mortality is linear.

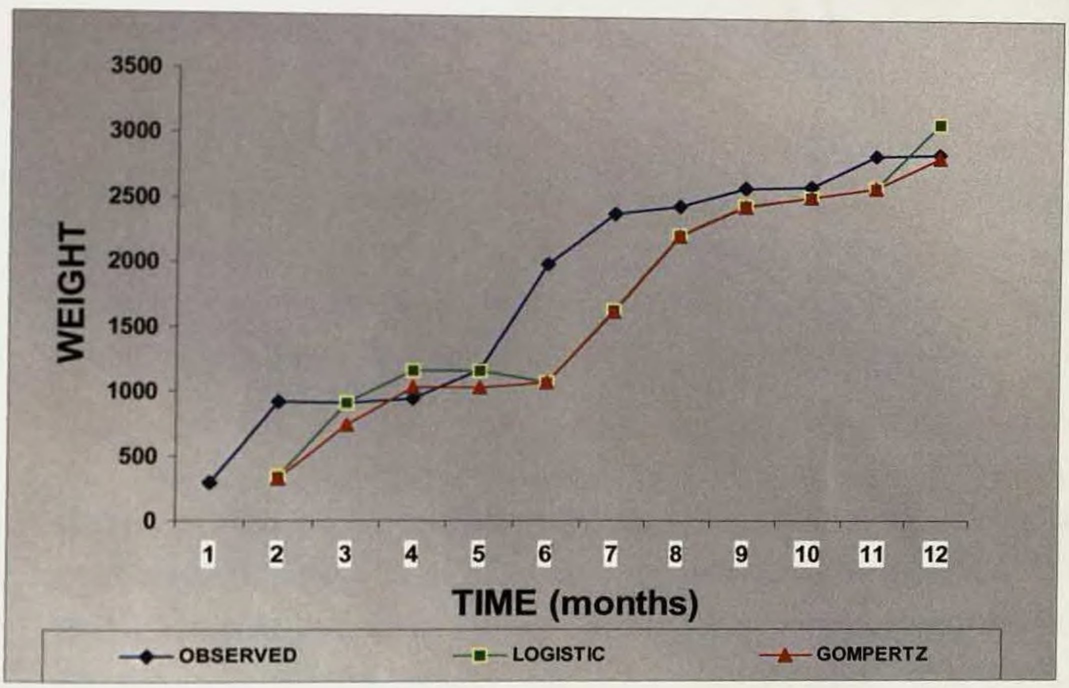

Fig.9. Production function when assumption on mortality is exponential.

5. DISCUSSION

5. DISCUSSION

As man is on the lookout of new sources of protein, aquaculture especially mariculture is gaining momentum globally. Growth is generally defined as the three-dimensional increase in the size of the organism over time. It represents the net outcome of a series of behavioural and physiological processes. Growth is the most important aspect of biology that is important for the farmer for determining the success of the culture.

Immense work has been done in modeling the growth of animals in relation to age, ecological factors, nutritional factors etc. In the present work an attempt has been made to model the growth of organism of mariculture importance in relation to the time of stocking. Molluscs are one of the most important groups that are gaining importance in culture worldwide.

In the present contest we had tried to model the growth of two species of mollusc *Crassostrea madrasensis* and *Perna virdis*. All the growth models earlier described in materials and methods were fitted to both average shell on weight and meat weight. In the case of oyster best results for the average shell on weight were obtained for Logistic model. R^2 was found to be as high as 0.98 but for meat weight Gompertz fitted best. When the mussels cultured in Dalavapuram cultured were modeled Gompertz was found to give a better fit in both shell on and meat weight.

Shin *et al.* (1995) found that the growth in shell length fitted to the von Bertalanffy model and the flesh dry weight to the Gompertz model. When the northern quahogs *Mercenaria mercenaria* growth was modeled the Richards model gave the most accurate prediction. The modified Gompertz function provided a better fit to the Chilean loco *Concholepas concholepas* growth rate data. (Rodriguez *et al.*, 2001). All this helps us to conclude that the growth of mollusc is commonly sigmoid. The fact that in natural population seasonal changes in growth rates can give a sigmoid curve for each year, (Wilbur and Yonge, 1968) also substantiate our findings.

Growth of an organism depends directly on the suitability of the habitat for the organism concerned in terms of environment, food and space availability. In both cases the growth rate of the organism was found to increase during the first six months and later decrease tremendously. It might be due to the fact the prevailing ecological conditions as ecological condition influence the growth rate of the mollusc (Wilbur and Yonge, 1968). It may also due to the fact as Walne (1958) has stated that larger individuals grew less rapidly i.e. individuals of the same age exhibited a growth deficiency as their size increased. Perna has generally better growth rate than its cold-water relative Mytilus because of the tropical distribution where elevated water temperature and less prominent seasonality in food supply favour continuous growth throughout the year. Further works in tune with the variation of growth rate and ecological and physical conditions should be studied and growth be modeled using the corresponding parameters.

One of the objectives of aquaculture is to obtain a maximum economic return within a minimum time. In aquaculture experiments even of a short duration, organisms may reach their asymptotic size and growth rates may vary greatly (Springborn, 1994). In the current experiment-involving oyster at the point of harvest the weight was found to be 41g while the asymptotic weight was found to be only 39. During the last phase of growth we can easily find that there is slower growth and the farmer is not profited by this extra month and hence an earlier harvest by one month could be recommended.

Fish growth data are usually fitted through production models appropriate to predict and compare growth patterns between population and within species (Chen *et al.*, 1992). Various growth models were used to model the growth of grey mullet *Mugil cephalus* that were reared in two different culture conditions viz. polyculture and monoculture. In both the cases, Logistic growth models fitted the data significantly better with relatively higher R^2. The growth rate was found to be higher in the monoculture condition. VBGF is a simple growth model in which fish grows to an asymptotic size is described in terms of biologically meaningful parameters of asymptotic size and a growth coefficient. The VBGF model was able to explain about 96%variation in the data. It gave an asymptotic weight nearer to 330,eventhough the growth rate between the two culture systems showed a three-time difference.

In open waters, environmental conditions (e.g., temperature and presence of predators), cause fish to either grow rapidly toward a small size (high k, low W_∞), or leisurely toward a large size (low k, high W_∞). This leads to their growth performance index (ø' = ln k +.67 ln W_∞) remaining nearly constant among different populations of the same species (Pauly, 1994). In our case the performance index was around 5 in both the cases.

The reasons for this near constancy of ø', which is ultimately due to the way fish allocate the scarce oxygen diffusing through their gills, are discussed in Pauly (1981, 1994). For most captive fish, the absence of predators and sexual competitors allows the allocation of more oxygen to feeding and growth, and away from behaviors that are costly in terms of oxygen demand, such as darting about to evade predators, or fighting against sexual competitors. This results in captive fish usually having ø' values higher than those predicted from the growth performance of free-living populations. Moreover, the strength of this effect increases with the sophistication of the culture system (Pauly *et al.*, 1988). Obviously, this effect will be strengthened by genetic enhancement for fast growth, e.g., in Nile tilapia (Pullin, 1988) or Atlantic salmon (Gjedrem, 1985), which, if often unwittingly, selects for the calm behavior that allows optimal allocation of oxygen to growth (Jones, 1996; Bozynski, 1998). Combined, these effects cause the ø' values of fish in intensive culture systems to be much higher than for their conspecifics in open waters.

In general it can be concluded that among the different models fitted the Richards and the modified Janoschek provided the least fit, with highly varying values and overestimation of the adult values for the data. The Gompertz and Logistic models were found to be the best fit growth models.

Production is defined as the increase in biomass over time Ricker (1945), Chapman (1968) and Alagaraja (1986) have derived production functions based on different assumption of growth and mortality. But the models used for growth were mainly exponential or linear. From the earlier part of work it can be concluded that growth need not always be exponential and new production functions were derived using different popular growth curves and different assumptions on mortality.

Earlier works were mainly based on simple growth models since when on integrating with assumptions of mortality the function tend to become complex. With the advent of computers and efficient software these functions could be evaluated.

When the growth of oysters was modeled we found out that Gompertz and Logistic models gave the best fits. So production functions using these growth models were fitted to the data. The goodness of fit was estimated using the index of fit.

When no assumption on mortality was taken, both the models fitted equally well, with Logistic equation giving a slightly better index of fit. The best fit was obtained for Gompertz equation when linear assumption on mortality was taken into consideration. Equally good fits were obtained for

Gompertz and Logistic when exponential model was used. All the models estimated less during the initial period of production.

Day by day aquaculture is diversifying with different species and technologies are expanding rapidly with various types of culture. Any model useful for the farmer should be based on the environmental climatic, biological and economic assumptions. Even though the systems are complex, important factors can be identified and models can be developed. Growth is one of the most important aspects of fish physiology, which responds sensitively and rapidly to the changes. The growth of fish depends on number of factors both exogenous (environmental) and endogenous to the management of the culture. These factors, which govern the rate, must lie at the heart of any investigation of production. Since a commercial aqua culturist is profit oriented, his objective would be profit maximization. Then the model should be linked with economic factors like feed cost, operational cost, harvesting cost etc. along with biological constraints. This will help us to test the economic efficiency of the technology and to show how to operate the facility most efficiently. Also further research using new technologies like artificial neural network should also be undertaken to assess production.

SUMMARY

SUMMARY

Aquaculture, the farming and husbandry of aquatic organisms are important in the food and economy of many nations. Although, much practical knowledge and experience has accumulated in this field, mathematical modeling in aquaculture is relatively new and undeveloped. Models include a particular group of components and their relationship, which are deliberately chosen to answer a particular problem, question or intended use. An important principle of modeling is that mathematical equation serves as a model of a biological process.

In the present study, an attempt has been made to evaluate the production of mariculture systems. Production can be defined as an increase in biomass over a given period of time and so production necessarily involves both growth and mortality. Growth is a factor of utmost importance in aquaculture. It is influenced by both endogenous and exogenous factors. The endogenous factors influencing growth are age and feeding rate. Since feeding rate is constant in aquaculture farm and exogenous factors can be controlled by the farmer, growth can be expressed in terms of age i.e., weight as a function of age.

Earlier production functions were derived on the linear and exponential model of growth but the pattern of growth need not always be so. So different growth models like Gompertz, logistic, Richards, VBGF, modified janoschek and polynomial growth equations were fitted to the age-weight data of mussels, oysters and fish. The nonlinear algorithm of SYSTAT was used for analysis. Most of the equations accounted for the variation in the data set, but it was found that Gompertz and the logistic growth equations gave the best fit.

Production functions were then derived using these growth models and different assumptions on mortality, mainly three assumptions were taken. In the first case, no mortality assumptions were taken and for the next two it was assumed to be linear or exponential. When the integral functions became complex these were solved through numerical integration. These derived production functions were later fitted to data sets of oysters. It was found that linear and exponential mortality assumptions of Gompertz growth model gave good results. Aquaculture is a dynamic enterprise with many variables influencing its success. Since growth is the major factor, the growth of the organisms should be further modeled using climatic and environmental factors. Along with these, the production models should also be linked with economic factors like harvesting cost, operational cost, optimum harvesting time etc. to get good results. Further research including these factors should be undertaken to ensure success in this field.

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