8-16-2016

The Effects of Community-Building on Achievement, Motivation, and Engagement in Undergraduate Mathematics

Hannak Keith
Western Kentucky University, hannah.keith723@topper.wku.edu

THE EFFECTS OF COMMUNITY-BUILDING ON ACHIEVEMENT, MOTIVATION, AND ENGAGEMENT IN UNDERGRADUATE MATHEMATICS

A Capstone Experience/Thesis Project

Presented in Partial Fulfillment of the Requirements for

the Degree Bachelor of Arts with

Honors College Graduate Distinction at Western Kentucky University

By

Hannah B. Keith

*****

Western Kentucky University
2016

CE/T Committee:

Professor Lisa C. Duffin, Advisor

Dr. Jennifer D. Cribbs

Dr. Benjamin LaPoe

Approved by

____

Advisor
Department of Psychology

ABSTRACT

This 2 x 2 quasi-experimental study examined the effects of pedagogical method (i.e., direct instruction vs. 5E inquiry) and intentional community-building (i.e., absence or presence) on undergraduate student (*N* = 103) motivation, engagement, and achievement in mathematics. Conditions were randomly assigned to one of four different College Algebra classes with a one-time occurrence and taught by a trained expert teacher. Findings indicated that intentional community-building – regardless of pedagogical method – had the strongest effects on students' motivation, engagement, and achievement. Although no differing pedagogical effects were discovered (most likely due to the one-time implementation of the lesson formats), the findings provide evidence for the necessity of community-building efforts -- an aspect of education that is often overlooked in the undergraduate STEM classroom.

Keywords: college mathematics, pedagogy, community-building, motivation, Self-Determination Theory, engagement, achievement

ACKNOWLEDGEMENTS

First of all, I would like to thank Dr. Lisa Duffin, my advisor and mentor. She has invested an incredible amount of time into this project, including providing guidance throughout the study, taking the time to explain research procedures, and encouraging me to take new perspectives on education. She has not only given me a better understanding of the research process, but she has also made me a much better learner and teacher. I cannot thank her enough for the investment she has made in me during my time at WKU!

In addition to Dr. Duffin, I would like to thank the rest of the SKyTeach faculty and staff for providing lesson materials, constant encouragement, and wisdom.

Many individuals contributed to the success of this research project. Thank you to Dr. Bruce Kessler and the WKU Mathematics Department for allowing me the opportunity to perform experimental research in a realistic classroom setting. Thank you to Mrs. Amanda Nutt for allowing me to use your Math 116 classrooms for this study. Thank you to Mrs. Melissa Rudloff for assisting in lesson plan development and teaching the experimental lessons. Thank you to Dr. Martha Day for providing feedback on lesson plans and research presentations. Thank you to Mrs. Karen Long, who assisted in getting study materials printed and always made sure I had everything I needed. Thank you Dr. Lee Ann Smith for serving as a sounding board for project ideas. Finally, thank you to

Courtney Inabnitt, who wrote substantial portions of the experimental lessons and assisted in filming lessons. The Honors College at WKU provides students with the opportunity to make their learning experiences more meaningful through hands-on, mentored research. I would like to thank them for the time they invest in improving undergraduate education through the CE/T project and other endeavors.

This project was primarily funded by monies from the Faculty-Undergraduate Student Engagement (FUSE) Internal Grant. The FUSE Grant allowed for the purchase of lesson supplies and also allowed for travel to the American Educational Research Association (AERA) Annual Meeting in Washington, D.C. to present the study.

I would like to thank Dr. Jennifer Cribbs for serving as my second reader and providing encouragement throughout this project. I would also like to thank Dr. Benjamin LaPoe for serving as my third reader on the committee.

This project would not have been possible without God and my parents. God has blessed me with two passions— teaching and research— and has given me the strength to pursue them both throughout this project. In addition, this project would not have been successful without the values of hard work and perseverance that my parents instilled in me by their own example.

VITA

FIELDS OF STUDY

Mathematics; Science and Math Education

PRESENTATIONS

Keith, H.B. & Cribbs, J.D. (2016, April). *The relationship between secondary mathematics teachers' college experiences and their personal classroom practices*. Presented at the Western Kentucky University Student Research Conference, Bowling Green, KY.

Keith, H.B. & Duffin, L.C. (2016, April). *The effects of community-building on achievement, motivation, and engagement in undergraduate mathematics*. Presented at the American Educational Research Association Annual Meeting, Washington, D.C.

Keith, H.B., Duffin, L.C., & Cribbs, J.D. (2015, April). *How do pedagogy and rapport impact college mathematics?*. Presented at the Western Kentucky University Student Research Conference, Bowling Green, KY.

Keith, H.B., Duffin, L.C., & Day, M.M. (2015, May). *Whatever you like: The effects of 5E inquiry versus direct instruction on teacher rapport*. Presented at the University of Texas UTeach Institute Annual Conference, Austin, TX.

Keith, H.B. & Inabnitt, C.P. (2014, April). *The Pursuit of Effective Teaching*. Presented at the Western Kentucky University Student Research Conference, Bowling Green, KY.

GRANTS

Keith, H.B. & Duffin, L.C. (2014). WKU Faculty-Undergraduate Student Engagement (FUSE) Internal Grant. *Does Pedagogy Really Matter for Student Motivation and Achievement in College Mathematics?*. Funded: $4,500. (FUSE Award #: 15-SP201).

AWARDS

May 2016 .... Awarded the Outstanding SKyTeach Undergraduate Student, Secondary Award

April 2016 .... Awarded the Pauline Lowman Memorial Secondary Education Award for outstanding graduating secondary education mathematics major

August 2015 .... Awarded the Henry M. and Zula G. Yarbrough Scholarship in Mathematics, $775

May 2015 .... Session Winner, Undergraduate Poster Session: Research Category, UTeach Institute Annual Conference, Austin, TX

2014-2015 .... Awarded the Robert Noyce Teacher Scholarship, $7,000-$14,000

April 2014 .... Awarded the Pi Mu Epsilon Award for outstanding second-year mathematics major

April 2013 .... Awarded the Robert C. Bueker Mathematics Award for outstanding first-year mathematics major

2012-2016 .... Awarded the Award of Excellence Scholarship by Western Kentucky University, $9,982

2012...................... Valedictorian, South Haven Christian School, Springfield, Tennessee

TABLE OF CONTENTS

Page

Abstract ii

Acknowledgements iii

Vita v

List of Figures viii

Chapters:

1. Introduction 1

2. Literature Review 5

3. Methods 19

4. Results 29

5. Discussion 33

References 39

Appendices 53

LIST OF FIGURES

Table		Page
3.1	Experimental Conditions	20
3.2	Comparison of 5E and Direct Instruction Lesson Plans	21
4.1	Descriptive and Inferential Statistics for Pre-Experimental Variables	30
4.2	Descriptive and Inferential Statistics for Dependent Variables	32

CHAPTER 1

INTRODUCTION

As humans, we are forced to decide which achievements possess enough importance to be pursued. Often, the goals we select and our subsequent persistence in striving to achieve them are a direct result of the interactions between our inner motivations and the environment around us (Deci & Ryan, 2000). According to self-determination theory, the extent to which one finds that his environment allows him to act autonomously, build competence, and relate to others will determine the extent to which he internalizes the beliefs and practices of those around him, or becomes intrinsically motivated (Deci & Ryan, 2000). The extent and type of this motivation are then evidenced by an individual's decisions about whether or not to engage with his environment (Skinner, Kindermann, Connell, & Wellborn, 2009).

Nowhere are a person's inner motivations and engagement patterns more apparent than in the classroom. At the undergraduate level, where students are choosing a career path, these factors play an even more important role. However, it is in this realm of learning that a vast number of science, technology, engineering, and mathematics (STEM) students seem to find a lack of motivating and engaging environments. In fact, nearly 25% of undergraduates pursue a degree in a STEM field at the onset of higher education, but only half of these individuals will complete the major (Business-Higher Education Forum; BHEF, 2011b). The decision to change majors does not only impact the person who makes it. Recent reports by the President's Council of Advisors on Science and Technology (PCAST, 2012) indicate "a need for approximately 1 million more STEM professionals than the U.S will produce at the current rate over the next decade if the country is to retain its historical preeminence in science and technology" (p. i). If this problem is to be addressed at both the personal and national level, the factors underlying undergraduates' decisions not to pursue or complete STEM degrees must be examined. Unfortunately, less than half of high school seniors are considered to be math proficient. Of this group, 14% express a potential interest in pursuing a STEM career (BHEF, 2011a). Among those proficient undergraduates who enroll in mathematics courses, deficits in motivation and engagement can contribute to a loss of interest in STEM (PCAST, 2012). Specific documented complaints include an unwelcoming faculty, beginning-level courses that are uninspiring (PCAST, 2012), lack of faculty concern, the promotion of rivalry in the classroom, and a lack of emphasis on true understanding (Kardash & Wallace, 2001). To address these concerns, instructors must examine an educational aspect that they can control: the classroom environment.

According to the self-system model of motivational development (SSMMD), it is social interactions in an environment that initiate the processes that lead to motivation, engagement, and subsequent performance (Connell, 1990). In a classroom setting, the quality of these interactions are determined by the type of community that is present in the classroom. Accordingly, positive experiences of community in school settings have been associated with higher levels of student motivation and engagement (Goodenow, 1993; Osterman, 2000). Teachers determine the structure of the community as they make pedagogical and relational decisions. In the mathematics classroom, direct instruction (DI), a lecture-based model of pedagogy, is commonly utilized (Walczyk & Ramsey, 2003). The stages in the DI method are based upon the premise that individual students learn best when they are first given explicit content knowledge by the teacher (Kozioff, LaNunziata, Cowardin, & Bessellieu ,2001). However, in the last few decades, research has shown other, more student-centered models of instruction – like inquiry -- to be more effective in promoting student learning (Clarke, Breed, & Fraser, 2004; Kwon, Rasmussen, & Allen, 2005; Rasmussen, Kwon, Allen, Marrongelle, & Burtch, 2006) and achievement (Applebee, Langer, Nystrand, & Gamoran, 2003; White, Shimoda, & Frederiksen, 1999). Although changes in secondary institutions have occurred slowly, recent education reforms have shifted towards inquiry based-instruction in mathematics (National Council for Teachers of Mathematics; NCTM, 1991) and its constructivist, largely student-centered approach to teaching and learning ("Old Standards v. Common Core," n.d.; Walczyk & Ramsey, 2003). A specific inquiry-based pedagogy that has been validated in the STEM fields is the 5E model. The 5E model consists of five stages that are sequenced so that students work together to formulate unique solutions to problems while the teacher provides necessary support (Bybee et al., 2006). Because the DI and 5E models are so different in nature, it is reasonable to conclude that the types of community they facilitate will be just as different.

Regardless of pedagogical preference, a teacher has influence over the relationships that are built in her classroom. In a community of learners, two relationships are critical to success: the teacher-student relationship and the student-peer relationship. Teacher rapport serves as “a contextual variable that sets the stage for effective teaching” (Buskist & Saville, 2001, p. 12). As a result, studies show that undergraduate students place a higher emphasis on teacher rapport than do their professors (Buskist, Sikorski, Buckley, & Saville, 2002), and relatively few students report feelings of rapport with more than 50% of their professors (Benson, Cohen, & Buskist, 2005). Because the college mathematics classroom is predominantly teacher-centered, it is safe to say that students’ opportunities to build relationships with their peers are limited as well. Therefore, the goal of this study was to determine the effects of the absence or presence of intentional community-building in conjunction with two different models of instruction (DI vs. 5E) in the undergraduate mathematics classroom using a quasi-experimental design. Two questions were addressed in particular:

1. Is there a difference in student motivation, engagement, and achievement depending on the pedagogy (DI vs 5E) experienced in the college mathematics classroom?
2. Is there a difference in student motivation, engagement, and achievement in the two pedagogical types (DI vs 5E) when an educator intentionally facilitates a community environment?

CHAPTER 2

LITERATURE REVIEW

Self-System Model of Motivational Development

A theoretical framework of particular interest to this study is the self-system model of motivational development (SSMMD). The SSMMD framework examines the interactions between a person's internal motivational needs and significant others in the social environment. As a result of these exchanges, individuals form self-system processes, or judgments about oneself in relation to social interactions (Connell, 1990). In a classroom setting, it is the teacher's responsibility to create an environment that meets students' motivational needs and allows them to form positive self-system processes. According to self-determination theory, which is embedded within the SSMMD framework, each individual possesses three inherent needs that he seeks for his environment to meet: competence, autonomy, and relatedness (Deci & Ryan, 2000). Engagement then comes as a result of the satisfaction of these needs and disaffection as the result of the frustration of these needs (Connell, 1990). It is engaged students that reach the levels of learning and achievement that are quintessential to the success of our nation. In order to reach these levels of performance, it is important for educators to understand students' dynamic needs and the subsequent impact that they have on engagement.

Motivation and Self-Determination Theory. Motivation is conceptualized by self-determination theory (SDT) as the result of interactions between an individual's three inherent needs (i.e., autonomy, competence, and relatedness) and his environment (Deci & Ryan, 2000). Autonomy is defined as the need to engage in behaviors that are "volitional and reflectively self-endorsed" (Niemiec & Ryan, 2009, p. 135), competence reflects the need to demonstrate adequacy in a social or physical context, (Niemiec, Lynch, Vansteenkiste, Bernstein, Deci, & Ryan, 2006), and relatedness is the need to belong (Deci & Ryan, 2000). As students experience the satisfaction of these needs, they are able to achieve a state of intrinsic motivation (Deci & Ryan, 2000), or the drive to participate in activities because of an inherent like for the activity (Ryan & Deci, 2000). In the classroom, motivation is valued not only for its own sake but for the fact that it can lead students to initiate their engagement in the classroom (Appleton, Christenson, & Furlong, 2008; Connell & Wellborn, 1991), which is the next step in the SSMMD framework (Connell, 1990).

Engagement. Engagement has been broadly conceptualized in a number of ways, leading to a disagreement among scholars about its specific definition (Appleton, Christenson, & Furlong, 2008; Fredricks et al., 2004; Jimerson, Campos, & Grief, 2003). Germane to this study and the SSMMD framework, engagement is conceptualized as a meta-construct composed of three sub-components: behavior, cognition, and emotion (Fredericks, Blumenfeld, & Paris, 2004; Fredricks & McColsky, 2012). Behavioral engagement represents active participation (Fredericks, Blumenfeld, & Paris, 2004), taking into account both the quality and duration of a student's efforts (Skinner et al., 2009). Cognitive engagement denotes one's investment in his own learning process -- i.e., the degree to which one actively processes information during the lesson (Kong, Wong, & Lam, 2003). Emotional engagement is defined as the extent to which an individual likes or enjoys an activity or environment (Fredericks, Blumenfeld, & Paris, 2004). Together in the classroom setting, these three sub-components of engagement are measurable indicators of students' underlying motivations (Skinner, Kindermann, Connell, & Wellborn, 2009).

In the last few decades, researchers have begun to focus on the concept of engagement as a way of improving achievement and keeping students in school (Appleton et al., 2008). Engagement as an educational variable is valued for two primary reasons: its outcomes and malleability. In respect to academic outcomes, engagement has been positively correlated with achievement (Connell, Spencer, & Aber, 1994) and negatively correlated with high school dropout rates (Cairns & Cairns, 1994). In his review of the National Survey of Student Engagement, an instrument designed to measure engagement at the postsecondary level, Kuh (2004) states that, "The voluminous research on college student development shows that the time and energy students devote to educationally purposeful activities is the single best predictor of their learning and personal development" (p. 1). In other words, engagement plays a significant role in a college student's educational achievement. Fortunately, a currently disengaged mathematics student is not a lost cause. Research indicates that a person's engagement is relative to the environment in which he is participating (Fredricks, Blumenfeld, & Paris, 2004; Russell, Ainsley, & Frydenberg, 2005). Therefore, it is a malleable construct which educators have the opportunity to impact by changing elements of the classroom environment – i.e., classroom community (Fredricks et al., 2004; Newmann, Wehlage, & Lamborn, 1992).

Classroom Community

In the classroom, community is a construct which describes both the academic and social interactions among people. (Boaler, 1999; Rovai, Wighting, & Lucking, 2004). Although all mathematics classrooms can be classified as a community in some sense, some environments emphasize mathematics as a practice of inquiry, while others emphasize the field as a practice of repetition (Goos, 2004). Social interactions in the classroom will differ according to the emphasis of the environment (Boaler, 1999; Goos, 2004). Therefore, some classrooms can be described as having a strong sense of community, and others as weak. McMillan (1996) identifies four social elements that compose a strong sense of community: an atmosphere of belonging, a trustworthy hierarchical structure, positive interdependence, and the bond that comes from group experiences. When such a community is formed in the classroom, many positive outcomes can be expected, including increased engagement and academic achievement (McKinney, McKinney, Franiuk, & Schweitzer, 2006; Patrick, Ryan, & Kaplan, 2007). Because the teacher serves as the trustworthy authority in the classroom community, she assumes the responsibility of facilitating a community-driven environment by supplying structure, autonomy support, and involvement for the learner (Kiefer, Alley, &

Ellerbrock, 2015). Structure is established as the teacher sets expectations and provides the feedback that determines socially appropriate behavior (Ryan & Patrick, 2001). The type of structure established will then determine if the environment is autonomy supportive, or places emphasis on students' "communication of choice, room for initiative, recognition of feelings, and a sense that activity is connected to personal goals and values" (Connell, 1990, p. 66). The interactions occurring in the classroom will reflect the structure and autonomy support of the environment. Involvement, which occurs as others in the classroom show expressed interest in or pleasurably interact with an individual (Connell, 1990), is experienced by students during these interactions.

A teacher's beliefs about the facilitation of a community environment are often reflected in the model of pedagogy she chooses. Instructivist pedagogies such as the direct instruction (DI) model flow from the assumption that it is the teacher's responsibility to supply the context by which knowledge will be understood (Kozioff et al., 2001). This assumption formulates mathematics as a practice of memorization, and as a result, focuses more on the individual's ability to replicate what he has seen, rather than on his ability to contribute to the overall learning process. Therefore, the DI model would appear to facilitate a weak sense of community. In contrast, constructivist theory that drives pedagogies such as the 5E model of inquiry dictates that knowledge is constructed through both individual critical thinking and social interactions (Powell & Kalina, 2009). These underlying principles treat mathematics as a practice of inquiry, which requires students to share their ideas with one another and with the teacher. Subsequently, community-building is viewed as a necessary component of the problem-solving process.

Therefore, the 5E model would seem to be conducive to a strong sense of community. However, regardless of the model of pedagogy she uses, the teacher can choose to personally relate to students and to encourage them to relate with one another. This intentional approach to community-building has the potential to compensate for the negative impacts or to compound the positive impacts of a particular model of pedagogy. The question then becomes: Is classroom community best facilitated by particular models of pedagogy, teacher intentionality, or some combination of the two?

Direct Instruction. In the college mathematics classroom, direct instruction (DI), a lecture-based model of pedagogy, is commonly utilized (Walczyk & Ramsey, 2003). The model teaches concepts "explicitly and systematically" (Kozioff, et al., 2001, p. 56) through five basic stages: launch, worked example, guided practice, independent practice, and evaluation. While some studies include additional stages or use different terminology to refer to them, these basic components are seen throughout the DI literature (Kozioff, et al., 2001; Moore, 2007; Watkins & Slocum, 2004). The teacher begins by explicitly stating objectives (Kozioff et al., 2001; Watkins & Slocum, 2004). She then models a mathematical concept, guides students through a subsequent example as they work the problem simultaneously, and gives students the opportunity to work similar problems on their own. Finally, student progress is evaluated (Kozioff et al., 2001). In other words, instruction centers mainly on the teacher's ability to communicate specific mathematical steps, procedures, and rules that combine to form fundamental ideas and students' subsequent ability to perform them accurately (Kozioff et al., 2001).

Overall, the DI model seems to exhibit a controlling environment. Its explicit structure means that students are aware of exactly what they need to do throughout the lesson. However, there is then no need for an emphasis on autonomy support. An autonomy-supportive environment is one in which the teacher provides the support necessary for success while encouraging students' unique problem-solving approaches and mastery of content (Reeve, 2006). In contrast, the DI model emphasizes that "strategies be taught to allow students to solve the greatest number of problems with the fewest possible number of steps" (Przychodzin, Marchand-Martella, Martella, & Azim, 2004, p. 58) using scripted content delivery (Przychodzin et al., 2004; Watkins & Slocum, 2004). While no studies appear to have specifically measured the levels of autonomy-support facilitated by the DI model, the characteristics of an autonomy-supportive environment that have been researched do not seem to appear naturally in the DI classroom. By this explanation, the DI model would seem to thwart, rather than to support, students' need for autonomy.

In keeping with the lack of emphasis on autonomy, as learners rehearse mathematical processes during guided and independent practice, feedback regarding progress and the correction of mistakes flows almost entirely from teacher to student (Kozioff et al., 2001). Schunk and Zimmerman (1997) state that imitative competence is obtained when a student can generally model the same process he has just seen demonstrated before him. Therefore, if the type of evaluation used to assess students' competence is comparable to the types of worked examples during the lesson, students receive accurate feedback of their capabilities in the DI classroom, and thus, have their need for competence met. However, since independent and peer problem-solving strategies are not emphasized in the DI model, its ability to increase competence and achievement in mathematical understanding and thinking skills is questioned.

Ultimately, the DI model's perspective on competence-building as a one-way flow of information negates an emphasis on relatedness in the community. Because accuracy and efficiency are emphasized over student choice and creative problem solving, there is no need for the teacher to express an interest in a student's individual ideas, or to encourage this same interest among peers. Therefore, in most college mathematics classrooms where the DI model is utilized, it is hypothesized that little importance is placed on cooperative learning, and subsequently, on the concept of positive interdependence. In a community, the currency an individual utilizes to get what he needs is self-disclosure (McMillan, 1996). Through discussion, learners discover both what they have in common (the beginning of the bonding process) and where they differ in academics and personal experiences (McMillan, 1996). The swapping of ideas and explanations with the educator and other peers allows students to see their own academic and social needs met in exchange for meeting someone else's needs. However, as knowledge is disseminated by the teacher through scripted content, this trade does not occur in the DI environment. Therefore, it is difficult for students to develop "alliances with trusted others" (Furrer & Skinner, 2003, p. 148) in the DI environment. Accordingly, it would appear that feelings of relatedness are not facilitated. While relatedness is not measured as an outcome of this study, it is an important component of the classroom community manipulation.

Together, this research suggests that the DI model as an approach to mathematical instruction, while very focused, seems to create an environment which places little emphasis on meeting students' motivational needs or inspiring critical mathematical thinking. When the classroom environment does not meet students' needs of competence, relatedness, and autonomy, they become disengaged (Skinner, et al., 2009). Since engagement is a predictor of achievement according to the SSMMD (Connell & Wellborn, 1991), it would seem plausible that students in a DI classroom would report low levels of engagement and would do poorly on assessments that measure mathematical thinking. In a longitudinal ethnographic study that followed students (N = 310) from year 9 (age 13) to year 11 (age 16), Boaler (1999) found that students (n = 200) subjected to classroom conditions typical of the DI model scored lower on two different mathematical project-based assessments than did students (n = 110) subjected to an inquiry environment. In addition, when given the General Certificate of Secondary Education- the national test taken by all graduating high school students in the UK- , these same students seemed to struggle with problems involving conceptual understanding, answering two times more procedural questions than conceptual ones. Therefore, the DI model appears to come up short in its facilitation of student motivation, engagement, and achievement. Accordingly, recent reforms in STEM education have shifted towards inquiry pedagogies.

5E Model of Inquiry. In the realm of STEM education, emphasis has recently been placed on the constructivist ideologies demonstrated in inquiry-based instruction (NCTM, 1991) – a stark contrast to the DI model's instructivist foundations. "Constructivists shift the focus from knowledge as a product to knowing as a process"

(Jones & Brader-Araje, 2002, p. 3). A specific model of inquiry-based pedagogy which has been validated in the STEM fields is the 5E model (Bybee et al., 2006). The 5E model consists of five stages: Engagement, Exploration, Explanation, Elaboration, and Evaluation. Instruction begins by sparking student interest and attention while connecting the lesson topic to students' previous experiences and knowledge. It then provides students with activities and contexts to actively explore and discover overriding themes of the lesson content through a variety of means. Together, the learners and educators discuss students' discoveries and make connections to the body of mathematical concepts they have previously examined. Finally, students engage with additional challenges that help them to transfer and apply the newly learned content to different or novel contexts, thereby deepening their conceptual understanding and application of skills. Evaluation occurs throughout the lesson and allows for the educator to determine whether or not students have met objectives and for learners to evaluate their own understanding (Bybee et al., 2006). Just as the stages of the DI model inherently impact the classroom environment, the 5E model brings its own unique contributions to the learning atmosphere when viewed from the SSMMD perspective.

Overall, the 5E model appears to inherently facilitate a strong sense of community in the classroom. Its structure dictates that students are expected to critically and collaboratively solve mathematical problems. In such an environment, autonomy support is necessary to help students experience success as they connect their personal explorations to the desired mathematical content. During this process, the teacher allows students to work in their own way, provides hands-on opportunities, facilitates student conversations, and actively listens to learners. All of these actions have been identified as autonomy-supportive (Deci, Spiegal, Ryan, Koestner, & Kauffman, 1982; Flink, Boggiano, & Barrett, 1990; Reeve & Jang, 2006; Reeve, Bolt, & Cai, 1999). While no studies have quantitatively measured the levels of perceived student autonomy support associated with a specific model of inquiry instruction, the autonomy-supportive characteristics of the 5E model would appear to support learners' need for autonomy.

As learners participate in an autonomy-supportive environment, they are able to develop competence for mathematical thinking skills. Halpern and Hakel (2003) state that "What learners *do* determines what and how much is learned, how well it will be remembered, and the conditions under which it will be recalled" (p. 41). In the 5E model, in-depth understanding is emphasized as students explore, explain, and apply their knowledge to new contexts (Bybee et al., 2006). Therefore, the competence students develop involves more than the ability to imitate what they have seen. Critical mathematical thinking abilities are improved; however, inquiry models of instruction do not produce results overnight. Many students have become accustomed to lecture-based classrooms in the realm of college mathematics (Walczyk & Ramsey, 2003). Throughout the literature on inquiry-based instruction, numerous authors note that students are likely to experience some form of the grieving process when faced with a major change in their typical learning environment (Felder & Brent, 1996; Spector, Burkett, & Leard, 2007; Woods, 1994). In his discussion on problem-based learning (PBL) environments, Woods (1994) suggests eight progressive stages of the grieving process that students may experience: shock, denial, strong emotion, resistance and withdrawal, surrender and acceptance, struggle and exploration, sense of direction, and integration and success. Therefore, it is expected that the full positive effects of the 5E model would take some time to surface. Over time, when achievement measures assess true understanding, it is believed that students would demonstrate increased performance in the 5E classroom (DeHaan, 2005; Rasmussen & Kwon, 2007; Songer, Lee, & McDonald, 2003).

As students participate in the 5E environment, cooperative learning is a key tool in the process of developing critical thinking skills. In a study of middle school students (n = 91), Johnson, Johnson, Buckman, and Richards (1985) found that as students' perceptions of interdependence with their peers increased, so did their perceptions that they were supported academically and socially. As students participate in the social construction of knowledge, their most important tool is language (Powell & Kalina, 2009). In addition to the dialogue that occurs during group work, the 5E model allows students to share their ideas. At the elementary level, the facilitation of cooperation among students and the extraction of student ideas have been associated with classroom community (Solomon, Battistich, Kim, & Watson, 1997). Community research (McMillan, 1996) dictates that group cohesion is greater when authorities and "citizens" influence one another simultaneously. In the 5E model of inquiry, the teacher guides students toward a common endpoint, but it is student responses that determine the flow of conversation. In other words, students and teachers influence one another throughout the course of the lesson. Based upon the elements of cooperative learning and group cohesion that are inherent in the 5E model, it would appear that such an environment is conducive to feelings of relatedness and community.

Taken together, research on inquiry models of pedagogy seems to indicate that the 5E model inherently facilitates a strong sense of community and therefore nurtures student motivation and mathematical thinking. As student motivation is the driving factor behind the choice to engage in the SSMMD framework (Connell, 1990), it is predicted that students in the 5E environment would report higher levels of engagement than individuals participating in a lecture-based classroom. In the long run, engagement and the development of critical thinking skills are projected to be manifested in terms of improved student achievement. It remains to be seen whether these results would be even more pronounced if the 5E model's inherent community-building were to be combined with a teacher's intentional choices to build community.

Current State of Research

While a significant amount of research has been done on mathematics education as a whole, the majority of reports and initiatives focus on impacting the quality of mathematics at the K-12 level (Mathematics Learning Study Committee, 2001; National Mathematics Advisory Panel, 2008; NCTM, 2000). Indeed, the ideal educational experience for any student would consist of a firm, engaging mathematics background extending throughout his elementary, secondary, and post-secondary career. However, the reality of early educational experiences for many current and future college students is reflected in the depressing statistics of high school math proficiency (less than 50% of seniors; BHEF, 2011a) and STEM interest among those seniors who are proficient (39%; BHEF, 2012). Based on these statistics, a fair amount of research is needed to target mathematics students at the college level if America is to see an increase in STEM graduates.

While the concepts of community and relationships in the mathematics classroom have been examined (Boaler, 1999; Goos, 2004; Ryan & Patrick, 2001), the focus of these studies is not on the impacts of a specific model of pedagogy on student outcomes. If the DI model is to be present in a positive classroom environment, it appears that some other environmental factor must also be present to facilitate students' motivation, engagement, and critical thinking. On the other hand, although the 5E model appears to establish its own classroom community and subsequently produces positive results, the concept of intentional community-building is not moot. Looking at the resources teachers have at their disposal, it would appear that intentional community-building could be the most effective way to both address the concerns associated with the DI model and to compound the positive effects of the 5E model. To confirm these hypotheses, research is needed to examine the impacts of both common models of pedagogy and teacher intentionality in the college mathematics classroom. The purpose of the current study, then, was to utilize a 2 x 2 quasi-experimental design to examine the effects of intentional community-building efforts and pedagogical style within college mathematics on students' perceived autonomy-support, competence, engagement (i.e., behavioral, cognitive, and emotional), and achievement.

CHAPTER 3

METHODS

Participants and Experimental Design

For this study, participants were 103 students enrolled in a college algebra course at a large comprehensive university in the Mid-South of the United States. Participants were 59.2% female, with a mean age of 19.94 years. 70.9% self-identified as White, 7.8% as African American, 3.9% as Hispanic, 1.9% as Asian, 8.7% as mixed, and 2.9% as other. Students' intended majors were: 33.2% STEM, 55.3% Non-STEM, and 11.1% either undecided or exploratory; 69.9% of the students' majors required the college algebra course.

The study utilized a 2 x 2 quasi-experimental design where pedagogical style (DI or 5E model) and intentionality of the teacher to build a classroom community (absent or present) were manipulated. Four sections of college algebra taught by the same instructor were used in this study. Each class was randomly assigned to one of the four experimental conditions (i.e., DI+, DI-, 5E+, 5E-) identified in Table 3.1 below.

Table 3.1

Experimental Conditions

	-	+
5E	Scripted inquiry lesson on repeatable permutations	Scripted inquiry lesson on repeatable permutations Intentional actions designed to enhance sense of community
DI	Scripted direct instruction lesson on repeatable permutations	Scripted direct instruction lesson on repeatable permutations Intentional actions designed to enhance sense of community

Experimental Conditions

Before the study took place, two lessons exploring the concept of repeatable permutations were carefully developed. One lesson followed the DI model and the other followed the 5E model. Both lessons covered the same content and were designed to meet content standard CCSS.MATH.CONTENT.HSS.CP.B.9 (+), "Use permutations and combinations to compute probabilities of compound events and solve problems." In each lesson, students were expected to meet the same two objectives:

1. Compute the number of permutations in a situation, given a limited set of options using both logic and a mathematical relationship. (applying)

2. Explain in detail the steps taken in calculating a repeatable permutation and the logic behind them (understanding)

To ensure consistency of lesson delivery across conditions, a carefully trained and qualified master teacher with 20 plus years of teaching experience taught each of the lessons with fidelity to each condition. A basic outline of the five stages of each lesson plan is given in Table 3.2 below.

Table 3.2

Comparison of 5E and Direct Instruction Lesson Plans

	5E Model of Inquiry	Direct Instruction
Stage 1	Engagement - Video clip of improvisation to engage students - Discussion of improvisation - Short music lesson to familiarize students with context of future problems - Introduce musical permutations problem	Launch - Objectives stated - Definition (including formula) and examples of permutations given
Stage 2	Exploration - Introduce same basic musical problem on a smaller scale - In pairs, students list musical permutations - Class list of permutations created	Worked Example - Introduce musical permutations problem - Short music lesson to familiarize students with context of future problems - Introduce same basic musical problem on a smaller scale - Students list permutations individually - Teacher explains logic and solves both simple and original complex problem
Stage 3	Explanation	Guided Practice - Students solve password problem individually

	- Students and teacher observe list to answer simpler problem - Teacher and students discuss patterns in the permutations - Teacher introduces and generalizes formula - In pairs, students use general formula to solve original problem	- Teacher explanation
Stage 4	Elaboration - In pairs, students solve ice cream problem using m&m's and check using formula to build understanding - Students explain how they reached answer - Non-repeatable permutation example with ice cream - Small group and whole class discussion: Could you use the formula n^r in this situation?	Independent Practice - Students solve ice cream problems individually using logic and formula to build understanding - Teacher explanation
Stage 5	Evaluation - Summative assessment: students identify, solve, and justify repeatable permutations - Student self-evaluation	Evaluation - Summative assessment: students identify, solve, and justify repeatable permutations - Student self-evaluation

In the two community-building conditions, the master teacher exercised both scripted verbal interactions and natural, non-verbal interactions with students. A 5-minute ice-breaker activity was created and utilized at the beginning of the lesson to intentionally build rapport between teacher and student, and among students. In the activity, pairs of students drew at random from a deck of "getting-to-know-you" cards a question that both individuals had to answer (e.g., What is your favorite ice cream flavor and why?). Each person in the pair took turns drawing a card and asking/answering the question on the card. At the end of the activity, the instructor called on pairs of students to share some of their newly found commonalities. In this community-building condition, the instructor was able to use participants' names throughout the lesson because each student also had a pre-made name tent in front of them during the lesson. In addition, the instructor intentionally displayed vocal enthusiasm about the topic, smiled at students frequently, and conversed with them as they worked independently. All manipulations chosen for this study are validated ways of building a relationship with students found in the teacher immediacy literature (Gorham, 1988; Richmond, Gorham, & McCroskey, 1987) or components of community building (McMillan, 1996). In the two non-community-building conditions, no ice-breaker activity or name tents were used. The teacher followed the instructional model that was carefully scripted in the lesson plans, only engaging in non-scripted interactions that were necessary to uphold the integrity of the classroom experience.

Instrumentation

To confirm that experimental conditions contained students with similar beliefs in their abilities prior to the experimental manipulations, students' self-efficacy for real-world mathematics was measured using the Tasks subscale of the Mathematics Self-Efficacy Scale- Revised (MSES-R; Betz & Hackett, 1983). The scale consisted of 18 items (α=0.91) which were each assessed on a 6-point Likert scale, ranging from 1 (Not confident at all) to 6 (Completely confident). Questions targeted learners' beliefs about their mathematical abilities in the real world, such as *"How much confidence do you have that you are able to determine the amount of sales tax on a clothing purchase?"*. For additional reliability and validity information on the MSES-R Tasks subscale, please refer to Kranzler and Pajares (1997).

In addition, students' basic computational and algebraic abilities before the lesson were measured and statistically compared by three questions taken from the MSES-R (Betz & Hackett, 1983) Math Problems subscale. Specifically, each participant completed the following items: (1) *In a certain triangle, the shortest side is 6 inches. The longest side is twice as long as the shortest side, and the third side is 3.4 inches shorter than the longest side. What is the sum of the three sides in inches?*, (2) *If* $y = 9 + x/5$, *find* x *when* $y = 10$, and (3) $3\ 4/5 - 1/2 =$ ______ *Please write your answer as a mixed number.* A content rubric evaluating work shown and correctness was created to evaluate each participant's responses (see Appendix A). Performance scores were computed as a percentage of correctness out of a total score of 14.

Student achievement for the lesson content (i.e., repeatable permutations) was measured using a researcher-designed assessment (see Appendix B) that aligned with the learning objectives and specific content covered in the lessons. Learners were asked to solve problems involving repeatable permutations using both logic and the n^r formula, which was taught in the lessons. In addition, students were required to assess three real-world scenarios and determine whether or not the scenarios constituted a repeatable permutation, explain how they made this decision (the understanding component), and find a solution to the problem if they determined it to be a repeatable permutation. Again, a researcher-created rubric (see Appendix C) assessed each item for work shown, explanation, and correctness. Achievement scores were computed as a percentage out of a total score of 17.

Perceived competence was measured using a modified version of the Perceived Competence subscale of the Intrinsic Motivation Inventory (IMI; Ryan, 1982), which included 6 items (α=0.90) —e.g., *"I think I am pretty good at the math activity we did today."* Participants were asked to respond using a 7-point Likert scale, ranging from 1 (Strongly Disagree) to 7 (Strongly Agree). Perceived autonomy support was evaluated using a modified version of the Learning Climate Questionnaire (LCQ; Williams & Deci, 1996). The LCQ contained six items (α=0.90) – e.g., *"I feel that today's math instructor provided me choices and options,"* and participants rated each item using a 7-point Likert scale, ranging from 1 (Strongly Disagree) to 7 (Strongly Agree). All statements in both measures were slightly altered to refer to the "math activity" of the one-day experimental lesson, rather than to a general activity over a longer period of time.

The final set of scales measured engagement along three dimensions: emotional, cognitive, and behavioral. Emotional engagement was measured using a modified version of the Interest/Enjoyment Subscale of the IMI (Ryan, 1982) because the operational definition -- the extent to which an individual likes or enjoys an activity or environment (Fredericks, Blumenfeld, & Paris, 2004) -- is analogous to intrinsic motivation. The subscale consisted of seven items (α=0.91) – e.g., *"I enjoyed doing today's math activity very much."* Both behavioral and cognitive engagement were measured using modified versions of the respective subscales of the Student Engagement in the Mathematics

Classroom Scale (Kong, Wong, & Lam, 2003). The Behavioral Engagement subscale was comprised of 9 items (α=0.89) selected to measure learners' attentiveness (4 items; e.g., *"I listened attentively to the teacher's instruction in today's math lesson."*), diligence (3 items; e.g., *"When I faced a difficult problem in today's math lesson, I kept working until I finished it."*), and allocation of time during the lesson (2 items; e.g., *"During today's class period, I spent most of my time focusing and working on the math tasks in the lesson."*). Modifications to scale items focused on making the items pertinent to the one-day lesson instead of time spent doing mathematics homework outside of class, for example. The nine items of the Behavioral Engagement subscale were subjected to an exploratory factor analysis (EFA) using IBM SPSS version 23 and revealed one component explaining 51.84% of the variance (loadings: .430-.885). Results from a Parallel Analysis confirmed this one-factor solution for randomly generated data.

The Cognitive Engagement subscale was composed of 6 items (α=0.77) that focused on deep strategy use (3 items; e.g., *"I would try to connect what I learned in today's math lesson with what I encounter in real life or in other subjects."*) and surface strategy use (3 items; e.g., *"I found memorizing formulas is the best way to learn the math in today's lesson."*). EFA was also used to examine the six items on the Cognitive Engagement subscale and revealed a two-component solution explaining 54.94% of the variance. Component 1 (deep strategy use) contributed 40.76% to the total variance explained while Component 2 (surface strategy use) contributing 14.18% (loadings: .520-.832). A Parallel Analysis confirmed this two-factor solution for randomly generated data. The two components, while separate yet significantly correlated, can be used combinatorially to measure cognitive engagement as suggested by the scale authors (Kong et al., 2003).

Procedures

A week prior to the experimental lessons, students' self-efficacy for and basic abilities in mathematics were assessed using the MSES-R to ensure the comparison of similar conditions. The following week, the four experimental lessons were conducted and recorded. At the conclusion of each lesson, students were invited to self-assess their performance on the summative evaluation. This instructional strategy was designed for two purposes: 1) to help students reflect upon their own learning, and 2) to prime their competence-related thinking for the post-experimental measures. Post-experimental measures, which evaluated perceived competence, autonomy support, and engagement, were administered in the last few minutes of the class.

Analyses

In order to determine if statistically significant differences occurred between the four conditions on the six dependent variables ($p < .05$), a one-way between-groups analysis of variance (ANOVA) was conducted using the IBM SPSS 23 statistical program. To determine where mean-level differences occurred between groups on each statistically significant dependent variable, a Tukey post hoc test was used. Eta squared (η^2) statistics were calculated on significant dependent variables to determine the relative magnitude of the differences between the means (Sun, Pan, & Wang, 2010; Tabachinick

& Fidell, 2013). To interpret the strength of the effect sizes detected in this study, we used the guidelines proposed by Ferguson (2009): small = .04, medium = .25, and large = .64 while evaluating the effects in the context of the study and supporting literature (Trusty, Thompson, & Petrocelli, 2004).

CHAPTER 4

RESULTS

Randomization Check

To determine whether the groups differed prior to the experimental manipulation, we examined two variables: real-world mathematics self-efficacy and basic computational abilities. Table 4.1 summarizes the descriptive and inferential statistics for the pre-study measures. There were no statistically significant differences between group means as determined by a one-way ANOVA in both self-efficacy for real-world mathematics ($F(3,86) = 0.268$, $p = .85$) and basic computational abilities ($F(3,86) = 0.262$, $p = .85$). Therefore, there were no mean-level differences between the participants in the four conditions for self-efficacy ($M_{DI+} = 4.12$, $SD = 0.85$; $M_{DI-} = 3.96$, $SD = 0.91$; $M_{\mathrm{5E+}} = 4.07$, $SD = 1.11$; $M_{\mathrm{5E-}} = 3.89$, $SD = 0.63$) or for basic computational skills ($M_{DI+} = 51.95$, $SD = 21.05$; $M_{DI-} = 53.30$, $SD = 22.24$; $M_{\mathrm{5E+}} = 56.80$, $SD = 20.64$; $M_{\mathrm{5E-}} = 19.15$, $SD = 19.15$).

Table 4.1

Descriptive and Inferential Statistics for Pre-Experimental Variables

	DI+		DI-		5E+		5E-			
	Mean	(SD)	Mean	(SD)	Mean	(SD)	Mean	(SD)	*F*	*p*
SE	4.12	(0.86)	3.97	(0.91)	4.07	(1.11)	3.89	(0.63)	0.26	0.85
Math	51.95	(21.05)	53.30	(22.24)	56.80	(20.64)	51.70	(19.15)	0.27	0.85

Note. SE = self-efficacy for real-world mathematics and Math = mathematics achievement.

Differences amongst Dependent Variables

Table 4.2 summarizes the descriptive and inferential statistics for all six dependent variables: perceived autonomy support, perceived competence, emotional engagement, behavioral engagement, cognitive engagement, and achievement. A one-way between groups analysis of variance (ANOVA) revealed statistically significant differences among groups for autonomy ($F(3,94) = 5.159$, $p = .002$), competence ($F(3,96) = 4.479$, $p = .005$), emotional engagement ($F(3,92) = 4.231$, $p = .008$), behavioral engagement ($F(3,93) = 3.226$, $p = .026$), and achievement ($F(3,99) = 4.489$, $p = .005$). Marginally significant differences were noted among groups for cognitive engagement ($F(3,91) = 2.390$, $p = .074$). Although statistically significant differences in the six dependent variables were detected, the actual differences in the mean scores between groups were relatively small based on the calculated effect sizes: autonomy ($\eta^2 = 0.14$), competence ($\eta^2 = .12$), emotional engagement ($\eta^2 = .12$), behavioral engagement ($\eta^2 = .09$), cognitive engagement ($\eta^2 = .07$), and achievement ($\eta^2 = .12$).

Using a Tukey's post-hoc test, statistically significant differences between groups were found for every variable, although differences in cognitive engagement were only marginally significant. For perceived autonomy support, statistically significant differences were noted between both community-building conditions (M_{D+} = 5.45 and M_{5+} = 5.45) and the non-community building direct instruction condition (M_{D-} = 4.43). The same differences were noticed in students' perceived competence, with the highest levels being noted in the community-building conditions (M_{D+} = 4.82 and M_{5+} = 5.08) and the lowest levels reported in the non-community building direct instruction condition (M_{D-} = 3.83). Reported emotional engagement was statistically higher in the 5E community-building classroom (M_{5+} = 4.84) than in the non-community building direct instruction classroom (M_{D-} = 3.48). Behavioral engagement demonstrated statistically significant differences between only the 5E community-building (M_{5+} = 5.43) and non-community building (M_{5-} = 4.37) conditions. For cognitive engagement, marginally significant differences were noted between the two community-building conditions (M_{D+} = 4.62 and M_{5+} = 4.72) and the two non-community building (M_{D-} = 4.00 and M_{5-} = 4.04) conditions. Finally, for student achievement, differences were noted between both community-building conditions (M_{D+} = 56.30 and M_{5+} = 58.31) and the non-community building 5E condition (M_{5-} = 38.11).

Table 4.2

Descriptive and Inferential Statistics for Dependent Variables

	DI+		DI-		5E+		5E-			
	Mean	(SD)	Mean	(SD)	Mean	(SD)	Mean	(SD)	*F*	η^2
AU	5.45_a	(0.96)	4.43_b	(1.18)	5.45_a	(1.10)	4.72_{ab}	(1.31)	5.16^*	0.14
CO	4.82_a	(1.31)	3.83_b	(1.41)	5.08_a	(1.46)	4.19_{ab}	(1.26)	4.48^*	0.12
EE	4.08_{ab}	(1.44)	3.48_b	(1.15)	4.84_a	(1.15)	4.20_{ab}	(1.48)	4.23^*	0.12
BE	5.12_{ab}	(1.11)	4.97_{ab}	(1.17)	5.43_a	(1.12)	4.37_b	(1.28)	3.23^*	0.09
CE	4.62_a	(1.15)	4.00_b	(1.27)	4.72_a	(1.19)	4.04_b	(1.12)	2.39^{**}	0.07
AC	56.30_a	(20.34)	47.67_{ab}	(22.86)	58.31_a	(22.91)	38.11_b	(18.51)	4.49^*	0.12

Note. AU = perceived autonomy support, CO = perceived competence, EE = emotional engagement, BE = behavioral engagement, CE = cognitive engagement, AC = achievement. *=$p < .05$. **$p = .07$. Means in the same row that do not share subscripts differ at the $p < .05$ level.

CHAPTER 5

DISCUSSION

Findings from this study indicate that undergraduate college algebra students who experienced intentionally created community-building mathematics classrooms demonstrated many educational benefits. Positive impacts were seen in students' perceived autonomy-support, competence, engagement (emotional, behavioral, and cognitive) and achievement in a college mathematics classroom. These results are consistent with prior research that indicates associations between positive community experiences, achievement, motivation, and engagement (Black & Deci, 2000; McKinney et al., 2006; Patrick, Ryan, & Kaplan, 2007). Contrary to predictions, neither model of instruction appeared to significantly impact dependent variables in and of itself. Prior research has demonstrated that students require time to adjust to new methods of instruction, and it is believed that the short time period in which the experiment occurred influenced this portion of the results (Felder & Brent, 1996; Spector et al., 2007; Woods, 1994). To truly understand the implications of our study regarding classroom practice, a more in-depth comparison of student outcomes is needed.

Findings from this study support prior research, which suggests that students perceive an environment to be autonomy-supportive if teachers demonstrate supportiveness and intentionally acknowledge their needs and interests (Reeve, 2006; Reeve & Jang, 2006). The statistically significant difference noted between the two community-building conditions and the DI- condition indicate that direct instruction with no intentional community-building efforts was perceived to be the least autonomy-supportive environment. However, statistical analyses revealed identical scores between the 5E+ and DI+ conditions, indicating that intentional community-building efforts superseded any differences in autonomy support inherent within the two pedagogical models. Research has shown that an autonomy-supportive environment results in higher levels of perceived competence (Deci, Nezlek, & Sheinman, 1981). Therefore, the fact that the community-building conditions also demonstrated the highest levels of perceived student competence is not surprising. In the instances of both perceived autonomy support and competence, the DI- condition produced the lowest scores. The addition of intentional community-building efforts could have led to an increased perception of autonomy support which then impacted student competence, or the presence of a positive community could have directly impacted student competence. While the pathway of this influence is unknown, the fact remains that the intentional community-building efforts of the teacher displaced any differences between the pedagogical models in their impacts on student autonomy and competence.

In the categories of emotional and behavioral engagement, the effects of community-building efforts are present but not as conclusive as other dependent variables. Findings suggest that both a more student-centered model of pedagogy and intentional community-building efforts contributed to the differences noted in student responses to the environment. In general, autonomy-supportive behaviors have been associated with increased student engagement (Reeve, Jang, Carrell, Jeon, and Barch, 2004). More specifically, studies have shown that the presence of an autonomy-supportive teacher is correlated with higher levels of positive emotionality and intrinsic motivation, constructs similar to emotional engagement (Deci et al., 1981; Patrick, Skinner, & Connell, 1993). Logically, then, the differences between the 5E+ and 5E- conditions make sense. As was noted previously, students generally react adversely when they are first faced with taking responsibility for their own learning (Felder & Brent, 1996; Spector, Burkett, & Leard, 2007; Woods, 1994). In the two 5E environments, students were faced with a new teacher, a new learning environment and new peer interactions. In the 5E- and DI- conditions, students faced these new experiences without any teacher support above what was necessary for the validity of the study. Why would students choose to engage behaviorally and emotionally in a new, potentially frightening environment when the teacher does not seem to enjoy their presence or intentionally make them feel safe and welcome among their peers? The 5E+ condition provided learners with new, more hands-on experiences that were made welcoming and less frightening by the presence of a trusted, warm authority. It is believed that this is one reason why students demonstrated higher levels of emotional and behavioral engagement in the 5E+ condition than in the 5E- and DI- conditions.

While this explanation accounts for the differences in behavioral and emotional engagement in the two 5E conditions, it does not address the difference between the 5E+ and the DI- condition. Predictions before the study were that, based on prior research, the cooperative learning and learner involvement of the 5E model would facilitate a greater sense of community. This would lead to higher student motivation, and subsequently to higher student engagement. Therefore, it is believed that the inherent sense of community and student-driven activities built into the 5E model were the inherent differences that led to the higher levels of emotional and behavioral engagement. Differences were also noted among classroom conditions for cognitive engagement, but these differences were only marginally significant. However, as the two community-building conditions produced higher levels of cognitive engagement than did the two non-community building conditions, the importance of intentional community-building efforts was still highlighted.

In the case of student achievement, significant differences were noted between the 5E- condition and the two community-building conditions. Similar to the results of autonomy and competence, differences in student achievement seem to hinge upon intentional community-building efforts. This finding is corroborated by prior research (Black & Deci, 2000; McKinney, McKinney, Franiuk, & Schweitzer, 2006). As was noted earlier, it is believed that over time, an inquiry-based environment would lead to higher levels of achievement defined as in-depth understanding.

Implications

In the college classroom, these findings reveal the importance of a component of the classroom environment that is often overlooked by instructors: the building of a community structure. Studies have shown that many educators at the post-secondary level are resistant to changing their methods of instruction for many reasons, including the following: the change does not seem to be a valuable use of professional time, funds are inadequate, innovation is not always supported by faculty leadership, and students may reject the change to traditional instruction (Harwood, 2003; Marsh & Hattie, 2002; Wright & Sunal, 2004). While instructional innovations are important to student success, they do, in fact, require a long-term investment of time (and sometimes money). However, choosing to build a community environment in the classroom costs little time or money. It simply requires intentionality. By determining to build relationships with students and to allow students to build relationships among themselves, educators have the potential to impact student achievement, motivation, and engagement in a relatively short period of time.

Limitations

The current study does possess limitations, particularly in the form of time. Because the experiment involved conducting each lesson only once, only the short-term results of differing instructional methods and community-building efforts could be examined. Therefore, future research should examine the long-term effects of inquiry-based and direct instruction pedagogies in combination with community-building efforts to determine their effects on achievement, motivation, and engagement in the undergraduate mathematics classroom. Studies which oppose inquiry-based approaches often question students' ability to discover overarching concepts on their own (Kozioff et al., 2001). However, many studies consider inquiry methods that provide learners with little support and feedback (Kirschner, Sweller, & Clark, 2006; Klahr & Nigam, 2004). These practices are not consistent with the characteristics of the 5E environment (Bybee et al., 2006) or with the expectations associated with most current explanations of inquiry instruction (Bell, Smetana, & Binns, 2005; Marshall & Horton, 2011). Therefore, future studies examining instructional methods should focus on comparing traditional instructional methods like direct instruction to an inquiry-based approach with integrity. Only then can valid inferences be made to inform instructional decisions in the classroom.

REFERENCES

Applebee, A. N., Langer, J.A., Nystrand, M. & Gamoran, A. (2003). Discussion-Based approaches to developing understanding: Classroom instruction and student performance in middle and high school English. *American Educational Research Journal*, *40*, 685-730. doi: 10.3102/00028312040003685

Appleton, J.J., Christenson, S.L., Furlong, M.J. (2008). Student engagement with school: Critical conceptual and methodological issues of the construct. *Psychology in the Schools, 45*, 369-386. doi: 10.1002/pits.20303

Bell, R. L., Smetana, L., & Binns, I. (2005, October). Simplifying inquiry instruction. *The Science Teacher*, *72*(7), 30-33.

Benson, T. A., Cohen, A.L., & Buskist, W. (2005). Rapport: Its relation to student attitudes and behaviors toward teachers and classes. *Teaching of Psychology*, 32, 237–39.

Betz, N. E., & Hackett, G. (1983). The relationship of mathematics self-efficacy expectations to the selection of science-based college majors. *Journal of Vocational Behavior*, *23*, 329-345. doi: 10.1016/0001-8791(83)90046-5

Black, A. E., & Deci, E. L. (2000). The effects of instructors' autonomy support and students' autonomous motivation on learning organic chemistry: A self-determination theory perspective. *Science Education*, *84*, 740-756. doi:

10.1002/1098-237X(200011)84:6<740::AID-SCE4>3.0.CO;2-3

Boaler, J. (1999). Participation, knowledge and beliefs: A community perspective on mathematics learning. *Educational studies in mathematics, 40*, 259-281. doi: 10.1023/A:1003880012282

Business-Higher Education Forum (2011a). *Meeting the STEM workforce demand: Accelerating math learning among students interested in STEM*. Washington, DC: Author.

Business-Higher Education Forum (2011b). *Meeting the STEM workforce challenge: Leveraging higher education's untapped potential to prepare tomorrow's STEM workforce*. Washington, DC: Author.

Business-Higher Education Forum (2012). *STEM interest among college students: Where they enroll*. Washington, DC: Author.

Buskist, W., & Saville, B. K. (2001). Creating positive emotional contexts for enhancing teaching and learning. *APS Observer, 19*, 12-13.

Buskist, W., Sikorski, J., Buckley, T., & Saville, B.K. (2002). Elements of master teaching. In S. F. Davis & W. Buskist (Eds.), *The teaching of psychology: Essays in honor of Wilbert J. McKeachie and Charles L. Brewer,* (pp. 27–39). Mahwah, N.J.: Erlbaum.

Bybee, R. W., Taylor, J. A., Gardner, A., Van Scotter, P., Powell, J. C., Westbrook, A., & Landes, N. (2006). *The BSCS 5E instructional model: Origins and effectiveness*. Colorado Springs, CO: BSCS.

Cairns, R.B. & Cairns, B.D. (1994). *Lifelines and risks: Pathways of youth in our time*. New York: Cambridge University Press.

Clarke, D., Breed, M., & Fraser, S. (2004). The consequences of a problem-based mathematics curriculum. *Mathematics Educator*, *14*(2), 7-16.

Cohen, J.W. (1988). *Statistical power analysis for the behavioral sciences* (2nd edn). Hillsdale, NJ: Lawrence Erlbaum Associates.

Connell, J. P. (1990). Context, self, and action: A motivational analysis of self-system processes across the life span. In D. Cicchetti & M. Beeghly (Eds.), *The self in transition: Infancy to childhood* (pp. 61-97). Chicago, IL: University of Chicago Press.

Connell, J. P., Spencer, M. B., & Aber, J. L. (1994). Educational risk and resilience in African-American youth: Context, self, action, and outcomes in school. *Child Development*, *65*, 493-506. doi: 10.1111/j.1467-8624.1994.tb00765.x

Connell, J. P., & Wellborn, J. G. (1991). Competence, autonomy, and relatedness: A motivational analysis of self-system processes. *Self-Processes and Development: The Minnesota Symposia on Child Psychology*, *23*, 43-77.

Deci, E. L., & Ryan, R. M. (2000). The" what" and" why" of goal pursuits: Human needs and the self-determination of behavior. *Psychological Inquiry*, *11*, 227-268. doi: 10.1207/S15327965PLI1104_01

Deci, E. L., Nezlek, J., & Sheinman, L. (1981). Characteristics of the rewarder and intrinsic motivation of the rewardee. *Journal of Personality and Social Psychology*, *40*, 1-10. http://dx.doi.org/10.1037/0022-3514.40.1.1

Deci, E. L., Spiegel, N. H., Ryan, R. M., Koestner, R., & Kauffman, M. (1982). Effects of performance standards on teaching styles: Behavior of controlling teachers. *Journal of Educational Psychology*, *74*, 852-859. doi: 10.1037/0022-0663.74.6.852

DeHaan, R. L. (2005). The impending revolution in undergraduate science education. *Journal of Science Education and Technology*, *14*, 253-269. doi: 10.1007/s10956-005-4425-3

Felder, R. M., & Brent, R. (1996). Navigating the bumpy road to student-centered instruction. *College Teaching*, *44*, 43-47. doi: 10.1080/87567555.1996.9933425

Ferguson, C.J. (2009). An effect size primer: A guide for clinicians and researchers. *Professional Psychology: Research and Practice, 40*, 532-538. doi: 10.1037/a0015808

Flink, C., Boggiano, A. K., & Barrett, M. (1990). Controlling teaching strategies: Undermining children's self-determination and performance. *Journal of Personality and Social Psychology*, *59*, 916-924. http://dx.doi.org/10.1037/0022-3514.59.5.916

Fredricks, J. A., Blumenfeld, P. C., & Paris, A. H. (2004). School engagement: Potential of the concept, state of the evidence. *Review of Educational Research*, *74*, 59-109. doi: 10.3102/00346543074001059

Fredricks, J. A., & McColskey, W. (2012). The measurement of student engagement: A comparative analysis of various methods and student self-report instruments. In S.L. Christenson, A.L. Reschly, & C. Wylie (Eds.), *Handbook of research on* *student engagement* (pp. 763-782). New York, NY: Springer US. doi: 10.1007/978-1-4614-2018-7_37

Furrer, C., & Skinner, E. (2003). Sense of relatedness as a factor in children's academic engagement and performance. *Journal of Educational Psychology*, *95,* 148-162. http://dx.doi.org/10.1037/0022-0663.95.1.148

Goodenow, C. (1993). The psychological sense of school membership among adolescents: Scale development and educational correlates. *Psychology in the Schools*, *30*, 79-90. doi: 10.1002/1520-6807(199301)30:1<79::AID-PITS2310300113>3.0.CO;2-X

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education*, 35, 258-291. doi: 10.2307/30034810

Gorham, J. (1988). The relationship between verbal teacher immediacy behaviors and student learning. *Communication Education, 37,* 40-53. doi: 10.1080/03634528809378702

Halpern, D. F., & Hakel, M. D. (2003). Applying the science of learning to the university and beyond: Teaching for long-term retention and transfer. *Change: The Magazine of Higher Learning*, *35*(4), 36-41. doi: 10.1080/00091380309604109

Harwood, W. S. (2003). Course enhancement: A road map for devising active-learning and inquiry-based science courses. *International Journal of Developmental Biology*, *47*, 213-222.

Jimerson, S. R., Campos, E., & Greif, J. L. (2003). Toward an understanding of definitions and measures of school engagement and related terms. *The California School Psychologist*, *8*, 7-27. doi: 10.1007/BF03340893

Johnson, D. W., Johnson, R. T., Buckman, L. A., & Richards, P. S. (1985). The effect of prolonged implementation of cooperative learning on social support within the classroom. *The Journal of Psychology*, *119*, 405-411. doi: 10.1080/00223980.1985.10542911

Jones, M. G., & Brader-Araje, L. (2002). The impact of constructivism on education: Language, discourse, and meaning. *American Communication Journal*, *5*(3), 1-10.

Kardash, C. M., & Wallace, M. L. (2001). The Perceptions of Science Classes Survey: What undergraduate science reform efforts really need to address. *Journal of Educational Psychology, 93*, 199-210. http://dx.doi.org/10.1037/0022-0663.93.1.199

Kiefer, S. M., Alley, K. M., & Ellerbrock, C. R. (2015). Teacher and peer support for young adolescents' motivation, engagement, and school belonging. *RMLE Online*, *38*(8), 1-18. doi: 10.1080/19404476.2015.11641184

Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist*, *41*, 75-86. doi: 10.1080/00461520701263426

Klahr, D., & Nigam, M. (2004). The equivalence of learning paths in early science instruction effects of direct instruction and discovery learning. *Psychological Science, 15*, 661-667. doi: 10.1111/j.0956-7976.2004.00737.x

Kong, Q. P., Wong, N. Y., & Lam, C. C. (2003). Student engagement in mathematics: Development of instrument and validation of construct. *Mathematics Education Research Journal*, *15,* 4-21. doi: 10.1007/BF03217366

Kozioff, M. A., LaNunziata, L., Cowardin, J., & Bessellieu, F. B. (2001) Direct instruction: Its contributions to high school achievement. *The High School Journal*, 84(2), 54-71.

Kranzler, J. H., & Pajares, F. (1997). An Exploratory Factor Analysis of the Mathematics Self-Efficacy Scale Revised (MSES-R). *Measurement and Evaluation in Counseling and Development*, *29*, 215-228.

Kuh, G. D. (2004). *The national survey of student engagement: Conceptual framework and overview of psychometric properties*. Center for Postsecondary Research and Planning, Indiana University, Bloomington, IN. Retrieved on April 3, 2016 from: http://nsse.indiana.edu/2004_annual_report/pdf/2004_conceptual_framework.pdf

Kwon, O. N., Rasmussen, C., & Allen, K. (2005). Students' retention of mathematical knowledge and skills in differential equations. *School Science and Mathematics*, *105*, 227-239. doi: 10.1111/j.1949-8594.2005.tb18163.x

Marsh, H. W., & Hattie, J. (2002). The relation between research productivity and teaching effectiveness: Complementary, antagonistic, or independent constructs?. *Journal of Higher Education*, 73, 603-641.

Marshall, J. C., & Horton, R. M. (2011). The relationship of teacher-facilitated, inquiry-based instruction to student higher-order thinking. *School Science and Mathematics*, *111*, 93-101. doi: 10.1111/j.1949-8594.2010.00066.x

Mathematics Learning Study Committee. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academies Press.

McKinney, J. P., McKinney, K. G., Franiuk, R., & Schweitzer, J. (2006). The college classroom as a community: Impact on student attitudes and learning. *College Teaching*, *54*, 281-284. doi: 10.3200/CTCH.54.3.281-284

McMillan, D. W. (1996). Sense of community. *Journal of Community Psychology*, *24*, 315-325.

Moore, D. W. (2007). Direct instruction: Targeted strategies for student success. *National Geographic*. Retrieved from http://insideng.com/profdev/guides/Moore_ Instruction.Pdf

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics, Commission on Teaching Standards for School Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: National Council of Teachers of Mathematics, Inc.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. U.S. Department of Education.

Newmann, F., Wehlage, G. G., & Lamborn, S. D. (1992). The significance and sources of student engagement. In F. Newmann (Ed.), *Student engagement and achievement in American secondary schools* (pp. 11–39). New York: Teachers College Press.

Niemiec, C. P., Lynch, M. F., Vansteenkiste, M., Bernstein, J., Deci, E. L., & Ryan, R. M. (2006). The antecedents and consequences of autonomous self-regulation for college: A self-determination theory perspective on socialization. *Journal of Adolescence, 29*, 761-775. doi: 10.1016/j.adolescence.2005.11.009

Niemiec, C.P. & Ryan, R.M. (2009). Autonomy, competence, and relatedness in the classroom: Applying self-determination theory to educational practice. *Theory and Research in Education, 7*, 133-144. doi: 10.1177/1477878509104318

Old Standards v. Common Core: A Side-by-Side Comparison of Math Expectations - Foundation for Excellence in Education. (n.d.). Retrieved December 3, 2014, from http://excelined.org/common-core-toolkit/old-standards-v-common-core-a-side-by-side-comparison-of-math-expectations/

Osterman, K. F. (2000). Students' need for belonging in the school community. *Review of Educational Research*, *70*, 323-367. doi: 10.3102/00346543070003323

Patrick, H., Ryan, A., & Kaplan, A. (2007). Early adolescents' perceptions of the classroom social environment, motivational beliefs, and engagement. *Journal of Educational Psychology, 99*, 83-98. doi: 10.1037/0022-0663.99.1.8

Patrick, B. C., Skinner, E. A., & Connell, J. P. (1993). What motivates children's behavior and emotion? Joint effects of perceived control and autonomy in the academic domain. *Journal of Personality and Social Psychology*, *65*, 781-791.

http://dx.doi.org/10.1037/0022-3514.65.4.781

Powell, K. C., & Kalina, C. J. (2009). Cognitive and social constructivism: Developing tools for an effective classroom. *Education*, *130*, 241-250.

President's Council of Advisors on Science and Technology (PCAST; 2012, February). *Engage to excel: Producing one million additional college graduates with degrees in science, technology, engineering, and mathematics* (Executive Report). Retrieved from The White House Office of Science and Technology Policy website: http://www.whitehouse.gov/sites/default/files/microsites/ostp/pcast-executive-report-final_2-13-12.pdf

Przychodzin, A. M., Marchand-Martella, N. E., Martella, R. C., & Azim, D. (2004). Direct instruction mathematics programs: An overview and research summary. *Journal of Direct Instruction*, *4*, 53-84.

Rasmussen, C., & Kwon, O. N. (2007). An inquiry-oriented approach to undergraduate mathematics. *The Journal of Mathematical Behavior*, *26*, 189-194. doi: 10.1016/j.jmathb.2007.10.001

Rasmussen, C., Kwon, O. N., Allen, K., Marrongelle, K., & Burtch, M. (2006). Capitalizing on advances in mathematics and K-12 mathematics education in undergraduate mathematics: An inquiry-oriented approach to differential equations. *Asia Pacific Education Review*, *7*, 85-93. doi: 10.1007/BF03036787

Reeve, J. (2006). Teachers as facilitators: What autonomy-supportive teachers do and why their students benefit. *The Elementary School Journal*, *106*, 225-236. doi: 10.1086/501484

Reeve, J., Bolt, E., & Cai, Y. (1999). Autonomy-supportive teachers: How they teach and motivate students. *Journal of Educational Psychology, 91*, 537-548. http://dx.doi.org/10.1037/0022-0663.91.3.537

Reeve, J., & Jang, H. (2006). What teachers say and do to support students' autonomy during a learning activity. *Journal of Educational Psychology*, *98*, 209-218. doi: 10.1037/0022-0663.98.1.209 Reeve, J., Jang, H., Carrell, D., Jeon, S., & Barch, J. (2004). Enhancing students' engagement by increasing teachers' autonomy support. *Motivation and Emotion*, *28*, 147-169. doi: 10.1023/B:MOEM.0000032312.95499.6f

Richmond, V.P., Gorham, J.S., McCroskey, J.C. (1987). The relationship between selected immediacy behaviors and cognitive learning. *Communication Yearbook, 10*, 574-590.

Rovai, A. P., Wighting, M. J., & Lucking, R. (2004). The classroom and school community inventory: Development, refinement, and validation of a self-report measure for educational research. *The Internet and Higher Education*, *7*, 263-280. doi: 10.1016/j.iheduc.2004.09.001

Ryan, R. M. (1982). Control and information in the intrapersonal sphere: An extension of cognitive evaluation theory. *Journal of Personality and Social Psychology*, *43*, 450-461. doi: 10.1037/0022-3514.43.3.450

Ryan, A. M., & Patrick, H. (2001). The classroom social environment and changes in adolescents' motivation and engagement during middle school. *American Educational Research Journal*, *38*, 437-460. doi: 10.3102/00028312038002437

Ryan, R.M. & Deci, E.L. (2000). Intrinsic and extrinsic motivations: Classic definitions and new directions. *Contemporary Educational Psychology*, *25*, 54-67. doi: 10.1006/ceps.1999.1020

Russell, J., Ainley, M., & Frydenberg, E. (2005). *Schooling issues digest: Student motivation and engagement.* Canberra, Australia: Department of Education, Science and Training, Australian Government

Schunk, D. H., & Zimmerman, B. J. (1997). Social origins of self-regulatory competence. *Educational Psychologist*, *32*, 195-208. doi: 10.1207/s15326985ep3204_1

Skinner, E. A., Kindermann, T. A., Connell, J. P., & Wellborn, J. G. (2009). Engagement and disaffection as organizational constructs in the dynamics of motivational development. In K.R. Wentzel & A. Wigfield (Eds.), *Handbook of motivation at school* (pp. 223-245). New York, NY: Taylor & Francis.

Solomon, D., Battistich, V., Kim, D. I., & Watson, M. (1996). Teacher practices associated with students' sense of the classroom as a community. *Social Psychology of Education*, *1*, 235-267. doi: 10.1007/BF02339892

Songer, N. B., Lee, H. S., & McDonald, S. (2003). Research towards an expanded understanding of inquiry science beyond one idealized standard. *Science Education*, *87*, 490-516. doi: 10.1002/sce.10085

Spector, B., Burkett, R. S., & Leard, C. (2007). Mitigating resistance to teaching science through inquiry: Studying self. *Journal of Science Teacher Education*, *18*, 185-208. doi: 10.1007/s10972-006-9035-2

Sun, S., Pan, W., & Wang, L.L. (2010). A comprehensive review of effect size reporting and interpreting practices in academic journals in education and psychology. *Journal of Educational Psychology, 102,* 989-1004. doi: 10.1037/a0019507

Tabachnick, B.G., & Fidell, L.S. (2013). *Using multivariate statistics* (6th edn). Boston: Pearson Education.

Trusty, J., Thompson, B., & Petrocelli, J.V. (2004). Practical guide for reporting effect size in quantitative research in the "Journal of Counseling & Development". *Journal of Counseling & Development, 82*, 107-110.

Walczyk, J.J., & Ramsey, L.L. (2003). Use of learner-centered instruction in college science and mathematics classrooms. *Journal of Research in Science Teaching, 40*, 566-584. doi: 10.1002/tea.10098

Watkins, C. L., & Slocum, T. A. (2004). The components of direct instruction. *Journal of Direct Instruction, 3,* 75-110.

White, B., Shimoda, T.A., & Frederiksen, J.R. (1999). Enabling students to construct theories of collaborative inquiry and reflective learning: Computer support for metacognitive development. *International Journal of Artificial Intelligence in Education*, *10*, 151-182.

Williams, G. C., & Deci, E. L. (1996). Internalization of biopsychosocial values by medical students: a test of self-determination theory. *Journal of Personality and Social Psychology*, *70*, 767-779. doi: 10.1037/0022-3514.70.4.767

Woods, D. R. (1994). *Problem-based learning: How to gain the most from PBL*. Waterdown, Ontario: DR Woods

Wright, E. L., Sunal, D. W., & Day, J. B. (2004). Reform in undergraduate science classrooms. In D.W. Sunal, E.L. Wright, & J.B. Day (Eds.), *Reform in undergraduate science teaching for the 21st century*, (pp. 137-152). Greenwich, CT: Information Age Publishing.

APPENDIX A

DIRECT INSTRUCTION LESSON PLAN

Launch

a. [Teacher will welcome students to classroom. Objectives will be explicitly stated.]

Today we are going to learn about repeatable permutations. [Display PowerPoint slide with objectives.] By the end of this class period, you should be able to both solve problems involving repeatable permutations and explain the logic behind the process. To start off, let's look at what a permutation is.

b. [Display PowerPoint slide with mathematical definition of a repeatable permutation.]

Given a set of n elements, the permutations with repetition are different groups formed by the r elements of a subset such that the order of the elements matters and the elements are repeated. Is anyone confused by this definition? [Pause for hands] Let's look at an example of a permutation. [Display PowerPoint slide with example of three friends.] Here I have my three friends Riley, Jack, and Priscilla standing in a group. However, they aren't just a group. They are a permutation because they have an order. Personally, Riley is my favorite, so I consider her my best friend. I've known Jack for a while and he's fun to hang out with, so I consider him my second best friend. Priscilla just hangs around with us because she likes Jack. So, in a permutation, the order matters. If I get mad at Riley and Jack becomes my first best friend, I have a whole new permutation. But, we said that we would be learning about repeatable permutations today. Repeatable simply means that an object can occupy more than one spot in the group. So, the example of my three friends is not a repeatable permutation because if Riley is my best friend, she can't also be my second best friend.

Transition: Now, let's work through some examples of repeatable permutations and figure out how to solve them.

Worked Example

a. [Display PowerPoint slide with music problem and read problem from slide.]

   Here is our first problem: "A jazz musician must improvise during his solo. For this particular song he can only work with four different components: quarter notes, eighth notes, sixteenth notes, and quarter rests. How many different ways could he write a four-count measure?". Before we try to solve this problem, let's take a quick timeout for a music lesson.

b. [Display PowerPoint slide with the definition of a measure.]

   A measure is a segment of time corresponding to a specific number of beats in which each beat is represented by a particular note value and the boundaries of the measure are indicated by vertical bar lines. So, the measure we are looking at will include different notes and rests that are combined to form a total of four beats. Now let's take a look at the notes and rests that our musician has to work with. [Display PowerPoint slide with the combinations of notes and rests used to create one beat.] The quarter note and the quarter rest are similar because they both are worth one beat. An eighth note is played a little bit faster. Each one is worth half a beat, so to make a full beat, you would need two eighth notes. Finally we have sixteenth notes, which are a worth a quarter of a beat each. So, if the jazz musician wanted to create one beat using sixteenth notes, he would have to hit the note four times.

c. [Give students instructions for first task of listing ways to write a two-count measure.]

   When we think about all the different ways that the jazz musician could possibly improvise this four-count measure, it seems overwhelming. So first, let's see how many different ways a two-count measure could be created. In just a minute, I am going to give you each a sheet of paper with some blanks on it that looks like this. [Hold up paper they are about to receive.] Each blank represents one beat. Your task is to come up with as many different ways to write these two beats as you can using the four types of notes and rests that we just talked about. [Display PowerPoint slide with note and rest options listed and examples of what students are being asked to do.] So for example, one way I could fill in these two beats would be to put a quarter rest in the first blank and four sixteenth notes in the second blank. Another way I could write two beats would be to swap and write the four sixteenth notes as the first beat and a quarter rest as the second beat. Also, don't forget that you can use the same note or rest for both beats. I'll go ahead and pass out your papers, and once everyone has one, I'll start the timer and you will have 90 seconds to write down as many permutations as you can think of.

d. [Give a paper to each student. Once every student has one, display the 90 second timer on the board, say "On your mark, get set, go!", and start the timer. Stand at the front of the classroom until students finish. Once time expires, begin explanation of logic behind formula.]

   Ok, time's up! How many combinations did you get? [Wait for response.] Here is the list of all the different combinations you could have come up with. [Display PowerPoint slide with list of all repeatable permutations.] There should be a total of 16. Now that we know how many repeatable permutations there are, let's see if we can find the answer mathematically. [Display PowerPoint slide with breakdown explanation of where "4 times 4" comes from.] Let's say we select a quarter note for our first beat. Then, we could select any of the four notes or rests for our second beat. So, whenever a quarter note makes up the first beat, there are four possible pairs. The same thing holds true if I select any of the other notes or rests to fill in my first beat. I can create four pairs with each one. So, I have four different options to fill in the first beat, and I am multiplying each of these four options by the four different notes or rests they could be paired with in the second beat. Four times four gives me 16, which is the total number of repeatable permutations that we were able to list for a 2-count measure. [Flip back to PowerPoint slide with list of all repeatable permutations.]

e. [Display PowerPoint slide explaining the formula n^r.]

   Now, another way I could write "4 times 4" is "4^2." If we take a closer look, we were given 4 different types of notes and rests to work with, and we needed to fill 2 beats. If we were to write this as a general formula, we could write it as n^r, where n represents the number of objects we are given to work with and r represents the number of places these objects will fill. This formula will tell you how many different ways objects can be

combined for any repeatable permutation. A good question to ask yourself is "Why?".

Think about what a repeatable permutation is. First of all, if something is a permutation, then the order of the objects matters. For the example we just did, that means that if I fill two beats by playing a quarter note first and then two eighth notes, it will sound different than if I play two eighth notes and then a quarter note. Secondly, these permutations are repeatable. That means that if I play a quarter note for my first beat, I can play it again for my second beat. So, for every beat I play, I have four different note and rest options to choose from. So if you want to know if you can use the formula n^r to solve a problem, you need to first think about 1) "Does the order of the objects matter?" and 2) "Are the objects repeatable?". [Display these questions on a PowerPoint slide as you say them.]

f. [Display PowerPoint slide with original question. Click to bring up both questions students are asking themselves.]

   So let's look back at our original question: "A jazz musician must improvise during his solo. For this particular song he can only work with four different things: quarter notes, eighth notes, sixteenth notes, and quarter rests. How many different ways could he write a four-count measure?". For our first step, we say "Is the order of the objects (which are notes in this case) important?," and the answer is yes. If the musician plays a quarter note first and then has a quarter rest, that is different than resting first and then playing a quarter note. The second thing we need to think about is "Are the objects repeatable?". Again, the answer is yes. The jazz musician could play the quarter note four times in a row if he wanted to. So, since the order matters and notes and rests can be repeated, the jazz musician has four different options to choose from for each beat. [Display breakdown of 4^4 on current PowerPoint slide.] We can write this as 4 x 4 x 4 x 4 to represent the 4 different note options for 4 beats. Since we determined that the order of the notes mattered and they were repeatable, we can use the formula n^r. [Click to display formula breakdown on current PowerPoint slide.] n represents the number of notes and rests the musician has to choose from, which is 4. r represents the numbers of beats the musician needs to fill, which is also 4. So, we have 4^4. Do you agree with me that the formula 4^4 is the same as multiplying the number 4 four times? [Wait for response.] Can I get someone to type in 4^4 in your calculator and tell me what you get? [Wait for response.] That is correct. So, the jazz musician could play the 4-count measure in 256 different ways. [Click to display total on current PowerPoint slide.]

Transition: Now that you've seen an example of a repeatable permutation, let's work through an example together.

Guided Practice

a. [Display PowerPoint slide with password problem].

   For your computer log-in, you are required to create a 6-character password using only ten symbols: ! @ # $ % ^ & * - +. Symbols can be repeated. How many possible passwords can be created? I am going to give each of you a paper that looks like this. [Display handout.] On the front of this sheet, this question is written, and there is space for you to work out the problem. When you get the sheet, go ahead and try to solve only this problem. Then, I will go over it step-by-step with you. [Pass out sheet to each student.]

b. [Give the class approximately 2 minutes to work. Then, discuss the problem step-by-step with them.]

   Ok, let me have everyone's attention right up here. This problem tells you to find the number of computer log-in passwords you could create. The first thing you should have thought about was "Does the order of the objects in this problem matter?". [Display question on current PowerPoint slide.] In this case, the objects are symbols. The answer is "yes" because "!@#$%^" is a completely different password than "^%$#@!." Next, you should have thought, "Are the letters repeatable?". [Display question on current PowerPoint slide.] We know the answer is "yes" because the problem tells us that symbols can be repeated. This means the problem is a repeatable permutation, and you can use the formula nr. n represents the number of objects. In this case, we have 10 because there are 10 symbols. r represents the number of spaces these symbols are being used to fill. So, r would be 6 because the password is 6 characters long. When you put 10^6

into your calculator, what did you get? [Wait for student response.] That is correct! [Display answer on current PowerPoint slide.]

Transition: Ok, now that you've worked through some examples, I'm going to give you the chance to do a couple of problems on your own.

Independent Practice (10 minutes): (18:10)

a. [Instruct students to flip over the sheet they used for the password problem and give them time to work alone.]

   Go ahead and flip over your worksheet to the back side. There are two problems involving different flavors of ice cream. Do your best to figure them out on your own and then we'll go over the answers. [Wait approximately 3-5 minutes.]

b. [Go over problems with students.]

   Ok, it looks like most people are done. Let's go over the answers and see how you did. [Display PowerPoint slide with main question.] The problem tells us, "You and some friends decide to go get ice cream after a concert one night. Assume that getting a stack of chocolate, chocolate, and vanilla dips is different than getting a stack of chocolate, vanilla, and chocolate dips. [Display PowerPoint slide with questions.] 1a) Without using a formula, use your own logic to answer the following question: How many possible combinations of ice cream could you order if you have 2 different flavors to choose from and you will be ordering 3 scoops?". [Display example on current PowerPoint slide.] This is one example of how you could have logically solved this problem. You could draw out the different permutations of flavors, just like you did with the music notes. You also could have listed the flavors instead of drawing them, or created a completely different diagram. You should have gotten 8 different permutations. Part b asks us to solve the problem using the formula we have gone over in class. We know this problem is a repeatable permutation because the original problem tells us that the order in which we get the dips matters, and because we can repeat a flavor as much as we want to. So, using the formula n^r, n is our 2 different flavors, and r represents the number of places these flavors will fill, which is our 3 dips. So, 2^3 gives us 8 different ways to stack the ice cream. [Display answer on current

PowerPoint slide.] This answer should match the one you got when you used your own logic. Number 2 is done in exactly the same way, except this time you have 4 flavors to choose from. So, when we look at the formula n^r, r is still 3 because we are still looking to fill three scoops. However, n is now 4 because we had 4 different options to choose from. When you worked out 4^3, you should have gotten 64. [Display answer on current PowerPoint slide.] Are there any questions?

Transition: Go ahead and make sure you write your name on your Jazz It Up sheet and on your real world problems and pass them to the end of your row. [Allow a few seconds for students pass their papers. As they are doing this, display "Do your best!" PowerPoint slide.]

Evaluation (10 minutes): (24:50)

a. [Give instructions for evaluation.]

 The paper I am about to give you has a few questions about repeatable permutations. Make sure you work on your own, and do your best to answer every question. If you have a question, just raise your hand. Once you are finished, hold onto your paper, and we will go over the answers. You may begin as soon as you get your paper. [Pass out the evaluation sheet and collect Jazz It Up! sheet and real world problems. Allow approximately 3-5 minutes, or until most students seem to be done.]

b. [Go over answers and grading with students.]

 Ok, it looks like everyone is done. Before we look at the answers, I'm going to pass out a rubric so that you can grade yourself as we go. As soon as you get it, go ahead and put your name at the top. [Pass out rubric.] [Display PowerPoint slide with evaluation rubric.] In just a moment, I am going to display the answers in red. Every answer is worth one point. So for example, if you got the answer 8 for question number 1 without using the formula n^r, you would give yourself one point. But questions 3a-c asked you to do a number of different tasks. You will give yourself one point for each different task that you got right. Write down the total number of points you got for each question in the "Score" box for that question. Then at the end, add up the total number of points you got and write it in the "Total" box at the bottom. You can get up to 16 points. If you have any questions, just raise your hand. [Display PowerPoint slide with evaluation and answers.] [Give students 2 or 3 minutes to calculate their scores.] Is everyone done? I went ahead and calculated the percentages for the number of correct questions, so here is a list of those. Once you have taken a look at your grade, go ahead and flip your papers over and sit them in front of you.

c. [Give post-tests to students.]

Before you go, there are some questions that I would like to ask you about this lesson. Once you have answered them, go ahead and flip them over and sit them in a stack in front of you. [Allow time for students to fill out post-tests.][Ask students to fill in consent form.]

Transition into Questionnaires:

a. [Introduce Post-Tests]

Thinking about how you did on the evaluation and about the lesson you just had, go ahead and flip over to the next few sheets, and answer the questions. Please answer honestly.

b. [Introduce Informed Consent]

Thank you for allowing me to come to your classroom today. I am a math education student, and I would like to use the data from the questions we asked you and from your evaluations for a study I am doing, but I can't do that without your permission. This paper explains all about the study and tells you how we will make sure that your answers to these questions are not released to anyone else. If you agree to let us use your data, please sign your name at the bottom of this sheet. If you choose not to sign, there are no consequences. It is your personal choice. I'll pass out the consent forms to you, and once you are done you are free to leave. Please stack your informed consent document on top of your question packet, and stack them on the table on your way out the door.

APPENDIX B

5E LESSON PLAN

Lesson Plan Template

	Name of Student teaching: Hannah Keith ; Courtney Inabnitt
Teach Date: ___________ **Length of lesson: __________**	Title of Lesson: **Catch the Beat… Discovering Permutations**

Standard for the lesson:
CCSS.MATH.CONTENT.HSS.CP.B.9
(+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Unit goal that lesson addresses:

Objective/s- Write objective/s in ABCD format.	**Evaluation** In the space below, write at least one question to match the objective you listed or describe what you will look at to be sure that students can do this.
1. Compute the number of permutations in a situation, given a limited set of options using both logic and a mathematical relationship. (application) 2. Explain in detail the steps taken in calculating a repeatable permutation and the logic behind them, earning 6 out of 6 total points. (comprehension)	

1

Engagement: **Estimated Time:** 7 minutes **Description of Activity:**	
What the teacher does:	**What the student does:**
"Welcome to class today! We're going to start by watching a short video clip. Has anyone seen Who's Line? Did you like it?" **(PowerPoint)** Discuss how improvisation is used in our daily lives. "Scenes from a hat is all about improvisation." 1. What is improvisation? 2. What does the clip have to do with improvisation? 3. Where else would you see improvisation? *What about music? 4. How does improvisation show up in music? "With a show of hands, how many of you all know some musical knowledge? (Allow students to raise their hands.) Well I see that some of you do, but just so we're all on the same page let's take a few moments to review. **(PowerPoint)** A measure is a segment of time corresponding to a specific number of beats in which each beat is represented by a particular note value and the boundaries of the measure are indicated by vertical bar lines. So, the measure we are looking at will include different notes and rests that are combined to	1. Poll students for responses. 2. Answers will vary 3. Possible student answers include: conversations with others, dancing, music…. 4. Making slight changes to a song they know, doing what feels right, experimenting *Notes, what they have memorized, what they know about music

form a total of four beats. Now let's take a look at the notes and rests that our musician has to work with. **(PowerPoint)** The quarter note and the quarter rest are similar because they both are worth one beat. An eighth note is played a little bit faster. Each one is worth half a beat, so to make a full beat, you would need two eighth notes. Finally we have sixteenth notes, which are a worth a quarter of a beat each. So, if the jazz musician wanted to create one beat using sixteenth notes, he would have to play the note four times." "Do we all understand this? Now that we have reviewed a little about music, let's look at our problem." **(PowerPoint)** 5. Say a musician decides he wants to change up one of the measures in his song, and he can use 4 different musical symbols: quarter notes, eighth notes, sixteenth notes, and quarter rests. How many different ways do you think he could create this measure if it has four counts?	5. Answers will vary.

Resources Needed:

Safety Considerations:

Exploration: **Estimated Time:** 7.5 minutes **Overview of Activity:**	
What the teacher does:	**What the student does:**
"So if we were to try and figure it out by drawing every possible combination of the three notes and one rest, it would take a very long time. Instead of starting with a 4-count measure, why don't we scale it back? Let's try starting with a two-count measure. If we can find a pattern, we can use it to find the answer for a larger measure." Split them into groups of two based on where they are sitting. *Note: Students will have already been randomly grouped using name tents "In a moment, you and your partner will receive a stack of sticky notes, and each one has two blanks drawn on it. Each blank represents one beat, and your task is to draw as many as many different combinations of two beats as you can think of using the 4 different notes and rests that we have to work with. **(PowerPoint)** So for example, on my first note card I might draw a quarter rest in the first beat and four sixteenth notes in the second beat. Then on my second card, I might swap and draw the sixteenth notes in the first beat and the quarter rest in the second beat. Also, the notes can be played twice in a row, so you could draw a quarter rest in the first beat and a quarter rest in the second beat. Are there any questions?" Hand out baggies with 2 markers and 20 sticky notes per group with two blanks (one per beat) already drawn. As you are passing out sticky notes: "Please wait until I say go to begin drawing on the notecards."	Students will work together to list as many possible ways to create two beats as they can. After a few minutes, they will create a class list.

Use a timer either on the computer or on a phone for the students. Make sure that the students can hear when the timer goes off. Inform students: "You have 2 minutes. Ready, set, go!" As students work, place the 4 flashcards around the room. After the 2 minutes have expired, have students arrange their sticky notes beneath 4 flash cards that are arranged around the room: Time's up! If you look around the room, you will notice that the four musical symbols you had to work with are posted on the wall. In just a moment, I want each partner to take half of the sticky notes. If you have any combinations that have a quarter note in the first blank, stick them beneath the quarter note sign. Do the same for the other symbols. If you get to a sign and the combination you have is already on the wall, stick yours right on top of it. Are there any questions? Before you do this, have one partner pack up all your materials in your bag and drop them in this box. You have 2 minutes to complete this task. On your mark, get set, go! As students are moving around the room, navigate around and make sure that sticky notes with the same options are placed on top of each other. lay down one note card per student (used later to define *n* and *r*). Once the class charts have been assembled, **ask:** 3. Have we listed every possible option?	3. yes or no

Resources Needed:

Safety Considerations:

another way I could write 4 times 4?	
7. **(PowerPoint)** How many different notes and rests did we have to work with? How many beats did we need to fill?	7. 4, 2
8. So, if I wanted to find a quicker way to count all the ways I could write a measure of music instead of drawing them all out, what 2 pieces of information do I need to know?	8. How many notes and rests we have to work with, how many beats are in the measure
"That is correct! The shortcut you just discovered is actually a tool that we can use to solve a number of different problems. You can write this tool as n^r." Give students a notecard to write down the formula and take notes.	
9. In the problem we just solved, what do you think that n was? Right! So if we wanted to generalize that, we could say that n is the number of objects we have to choose from. What was r in the problem with the music notes? Exactly! A way we could say this is that r represents the number of places that our objects will fill. On your notecard, go ahead and write down what "n" and "r" are in your own words.	9. The number of notes and rests we had, the number of beats
10. When I use the tool n^r, I am finding the number of ways that I can do what? *Refer back to note example if necessary	10. Rearrange or group objects
Right! Instead of saying a "group of objects," we're going to use the word "permutation." Write the word "permutation" on the board.	
11. Let's look back at our music problem. Does playing a quarter note and then resting sound exactly the same as resting and then playing a quarter note? So, is the order of the notes important? Right! So, when we are dealing with these permutations, the order of our objects matters.	11. No, Yes
12. Once you use a note for your first beat, can you repeat it again for your second beat?	12. Yes

Right! So, we can say that n^r helps us to find the number of REPEATABLE permutations. "What we have just discovered is the tool for finding the number of repeatable permutations. The formula only works for repeatable permutations. Let's look at a definition. **(PowerPoint)** "When given a set of n elements, the permutations with repetition are different groups formed by the r elements of a subset such that: the ORDER of the elements DOES MATTER and the elements are REPEATED." 13. Let's look back at our original question: **(PowerPoint)** Say a musician decides he wants to change up one of the measures in his song, and he can use 4 different musical symbols: quarter notes, eighth notes, sixteenth notes, and quarter rests. How many different ways do you think he could create this measure if it has four counts? Can we use the tool n^r to determine how many different ways a four count measure could be written with these same four notes and rests? Why? With your partner, go ahead and try to figure out the answer to this problem. If you need to write anything down, write it on the back of your notecard. Remember your notecard has a tool on it and if you need to you can use a calculator. Ask a few groups: 14. What was your answer? Where did the base (large) 4 come from? What about the exponent (small) 4? So when you typed in 4^4 in your calculator, what did you get? So, the musician can write the measure 256 different ways! Are there any questions? Now, I need three volunteers to help me give an example! Select three students. Here I have my three friends Riley, Jack, and Priscilla standing in a group. However, they aren't just a group. They have an order. I've known Riley the longest, so I consider her my best friend. Jack is pretty fun to hang out with, so I consider him my second best friend. Priscilla	13. Yes; the order of the notes/rests matters and notes/rests can be repeated 14. 4^4; the number of notes and rests, the number of beats

doesn't really like Riley or me. She just hangs around with us because she likes Jack. But let's say that I have an argument with Riley, and suddenly Jack is my new best friend (swap students). Now the same people are still in the group, but I have a completely different order. 15. So since the order of the people matters, what word can I use to describe my group of friends? 16. Are my friends an example of a repeatable permutation? Why or Why not?	15. Permutation 16. No; Riley can't be my first best friend and my second best friend

Resources Needed:

Safety Considerations:

Elaboration: **Estimated Time:** 15 minutes **Overview of Activity:**	
What the teacher does:	**What the student does:**
"Just as permutations are found in music, they are also found in other aspects of life." "Let's see if we can figure out another problem." **Inform the students of the task at hand.** **(PowerPoint)** "You and a friend work at an ice cream shop. Currently, the shop has a special on triple-dip cones, but there are only 2 flavors available" Present them with the question, "How many unique 3-scoop cones could you make?" You and your partner will be given a "What's the Scoop?" sheet to arrange your ice cream flavors. One partner will be responsible for scooping out the strawberry ice cream (pink m&m's), and the other will be responsible for scooping out the chocolate ice cream (brown m&m's). Your task is to lay out all the possible combinations of triple dip cones that you can think of. 1. Are the ice cream scoops an example of a repeatable permutation? Why or why not? *Is a stack of chocolate, chocolate, and strawberry scoops the same as a stack of chocolate, strawberry, and chocolate scoops? Since the ice cream is a repeatable permutation, after you and your partner think you have created all possible options of triple-dip ice cream	In groups of two students will model with m & m's the different possible orders they could make. Each group should then check their answer using the formula learned earlier in the lesson. 1. Yes, flavors are repeatable and the order of the scoops matters *No

10

cones using the two flavors, use the n^r tool to make sure that you and your partner found all the possible options. Pass out "What's the scoop?" sheet and m&m's. **After about 5 minutes call class back together:** 2. How many different 3-scoop cones could you possibly make? Explain how you arrived at your answer. (Make sure to ask what "n" and "r" were) 3. Our n^r relationship works for this scenario, but why? *Prompt until you get the answers of repeatability and order matters. "Let's say you get hired at the ice cream store down the street. This ice cream store has 4 different ice cream flavors, and also has a deal on triple-dip cones." 4. What would be the number of permutations for the 4 flavors and 3 scoops? How did you arrive at this answer? What does n mean? What does r mean? (Emphasize what n and r are) **(PowerPoint)** **Think, Pair, Share:** "Now, I want you to take a moment and consider this: **(PowerPoint)** 5. What if you were told you could only have one scoop of each flavor. Could you still use the formula n^r? Why or why not? After you have thought about your answer, take a few moments and discuss this with your partner. Then we will discuss our answers as a class." After giving the class time to discuss their answers call the class back together. Repeat the question and call on several groups to answer: 6. So, what do you think? Is this a repeatable permutation? Why or why	 2. Students should say that there are 8 possible orders that could be made. Students will describe how they were able to find the answer both with the m&m's and through using the formula. 3. Options can be used multiple times (repeatability) and the order of the scoops matters. 4. 4 raised to the third power is 64. 5. No; You only have 3 options for the second scoop once you select one for the first scoop 6. No; flavors cannot be repeated

11

not? Right! This is not a repeatable permutation because the flavors cannot be repeated. Once I get a scoop of chocolate ice cream I can't get another scoop of chocolate. This particular problem is an example of a non-repeatable permutation, which will be a lesson for another day.	

Resources Needed:

Safety Considerations:

Evaluation: **Estimated Time:** 10 minutes **Description of Activity:**	
Questions:	**Possible student answers:**
"Now that you have had a chance to work on some problems with a partner and discuss your ideas with the class, you're going to have the opportunity to show what you know. On your own, I would like for you to answer the questions on this first page (front and back). Once everyone has finished we will go over answers so you can see how well you did." Instruct students to pass their "What's the Scoop" sheet and notecards to the end of the row. Pass out evaluation sheets and pick up other materials as they take evaluation. 1. **Without using a formula**, use your own logic to answer the following question: You and your friend are arguing about where you should go have dinner. In order to make this decision, you decide to flip a coin three times. The coin has two sides: heads and tails. How many different permutations of heads and tails are possible? **Please show your work and/or drawings.** 2. Every time you unlock your cell phone, you are asked to enter a passcode. This passcode is four numbers long, and you may use the numbers 0-9. How many different four-digit passcodes could you create? 3. Look at the examples below. 1) Determine which ones are repeatable	

13

permutations and which ones are not and explain how you know this. 2) If it is a repeatable permutation, solve the problem and identify *n* and *r*. a) You and some friends are arranging 5 chairs in a row. How many different ways could you arrange the chairs b) You are playing a game in which you roll a die three times in a row. The sides of the die are labeled with the numbers 1-6. Assume that rolling the numbers 6, 4, and then 6 is different from rolling 6, 6, and then 4. How many possible results could you get? c) Social security numbers are made up of the numbers 0-9 and are 9 items long. How many different social security numbers are possible? Pass out rubric. Use script written on DI lesson.	

Resources Needed:

Safety Considerations:

APPENDIX C

PRE-TEST RUBRIC

Scale for grading student work shown			
0	**1**	**2**	**3**
No work shown.	Teacher could not replicate student answer based on work shown. One or more steps were missing.	Teacher could replicate student answer based on work shown, but the answer was incorrect.	Teacher could replicate student answer based on work shown, and the answer was correct.

Question	**Description**	**Score**
Q1	Circle the appropriate score for student work shown: 0 1 2 3 Give one point for each of the following: - Student determines the sum of the side lengths to be 26.6 - Student gives the unit “inches” as part of the answer	/5
Q2	Circle the appropriate score for student work shown: 0 1 2 3 Give one point for: - Student determines the answer to be x = 5	/4
Q3	Circle the appropriate score for student work shown: 0 1 2 3 Give one point for each of the following: - Student determines the answer to be 3 3/10, 33/10, or 3.3 - Student correctly writes the answer as the mixed number 3 3/10	/5
Total		**/14**

APPENDIX D

LESSON EVALUATION

1. **Without using a formula**, use your own logic to answer the following question:

   You and your friend are arguing about where you should go have dinner. In order to make this decision, you decide to flip a coin three times. The coin has two sides: heads and tails. How many different permutations of heads and tails are possible? **Please show your work and/or drawings.**

2. Every time you unlock your cell phone, you are asked to enter a passcode. This passcode is four numbers long, and you may use the numbers 0-9. How many different four-digit passcodes could you create?

CONTINUE TO THE BACK ➡

3. Look at the examples below. 1) Determine which ones are repeatable permutations and which ones are not and explain how you know this. 2) If it is a repeatable permutation, identify *n* and *r* and solve the problem.

Scenario	Repeatable Permutation?	Why?	n	r	Solution
a) You and some friends are arranging 5 different types of chairs in a row. How many different ways could you arrange the chairs?					
b) You are playing a game in which you roll a die three times in a row. The sides of the die are labeled with the numbers 1-6. Assume that rolling the numbers 6, 4, and then 6 is different from rolling 6, 6, and then 4. How many possible results could you get?					
c) Social security numbers are made up of the numbers 0-9 and are 9 items long. How many different social security numbers are possible?					

APPENDIX E

STUDENT EVALUATION RUBRIC

Question	**Description**	**Score**
Q1	Give one point for each of the following: - Students uses logic (no formula) to try to find solution - Student finds correct number of permutations	/2
Q2	Give one point for: -Student finds correct number of permutations	/1
Q3a	Give one point for each of the following: -Student states problem is not a repeatable permutation -Student states that chairs are not repeatable	/2
Q3b	Give one point for each of the following: -Student states problem is a repeatable permutation -Student states that order of the numbers matters -Student states that numbers are repeatable -Student identifies n as 6 (or the number of different options on the die) -Student identifies r as 3 (or the number of times the die was rolled) -Student states that solution is 216, or 6^3	/6
Q3c	Give one point for each of the following: -Student states problem is a repeatable permutation -Student states that order of the numbers matters -Student states that numbers are repeatable -Student identifies n as 10 (or the numbers to choose from) -Student identifies r as 9 (or the number of spaces in the password) -Student states that solution is 1,000,000,000 or 10^9	/6
Total		**/17**