UNIVERSIDAD CARLOS III DE MADRID
working papers

UC3M Working Papers
Statistics and Econometrics
16-14
ISSN 2387-0303
December 2016

Departamento de Estadística
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 91 624-98-48

Measuring the uncertainty of Principal Components in Dynamic Factor Models.

Javier de Vicente[a] and Esther Ruiz[a]

Abstract

In the context of Dynamic Factor Models, factors are unobserved latent variables of interest. One of the most popular procedures for the factor extraction is Principal Components (PC). Measuring the uncertainty associated to factor estimates should be part of interpreting these estimates. Several procedures have been proposed in the context of PC factor extraction to estimate this uncertainty. In this paper, we show that these methods are not adequate when implemented to measure the uncertainty associated to the factor estimation. We propose an alternative procedure and analyze its finite sample properties. The results are illustrated in the context of extracting the common factors of a large system of macroeconomic variables.

Keywords: Dynamic Factor Models, Principal Components, Extraction uncertainty, Bootstrap.

[a] Department of Statistics, Universidad Carlos III de Madrid.

Acknowledgements: both authors acknowledge financial support from the Spanish Ministry of Education and Science, research project ECO2015-70331-C2-2-R.

Measuring the uncertainty of Principal Components in Dynamic Factor Models

Javier de Vicente[1] and Esther Ruiz *[1]

[1]Department of Statistics, Universidad Carlos III de Madrid

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Abstract

In the context of Dynamic Factor Models, factors are unobserved latent variables of interest. One of the most popular procedures for the factor extraction is Principal Components (PC). Measuring the uncertainty associated to factor estimates should be part of interpreting these estimates. Several procedures have been proposed in the context of PC factor extraction to estimate this uncertainty. In this paper, we show that these methods are not adequate when implemented to measure the uncertainty associated to the factor estimation. We propose an alternative procedure and analyze its finite sample properties. The results are illustrated in the context of extracting the common factors of a large system of macroeconomic variables.

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1 Introduction

Currently, large systems of macroeconomic variables are easily accessible. Thus, the reduction of the dimension and the consequent extraction of the underlying factors are important issues for econometricians and policy decision makers. In this context, the Dynamic Factor Models (DFMs), originally introduced by Geweke (1977) and Sargent and Sims (1977), have received a lot of attention; see Breitung and Eickmeier (2006), Bai and Ng (2008), Stock and Watson (2011), Breitung and Choi (2013) and Bai and Wang (2016) for excellent surveys on DFMs. The main goal of DFMs is to explain the dynamics of the system using a reduced number of unobservable common factors, which determine the dynamics of the macroeconomic variables. The estimated latent factors are useful

*We gratefully acknowledge the financial support from the Spanish Government, contract grant ECO2015-70331-C2-2-R.

instruments for a wide range of applications: i) to represent economic cycles, trends and structural shocks; see Diebold and Rudebush (1996), Forni and Reichlin (1998), Kose et al. (2008), Arouba et al. (2012), Camacho et al. (2012) and Breitung and Eickmeier (2016) among others; ii) to serve as instrumental variables; see Favero et al. (2005), Bai and Ng (2010) and Kapetanios and Marcellino (2010); iii) as regressors for the construction of Factor-Augmented Vector Autorregresive models (FAVAR) or Factor-augmented Error Correction models (FECM); see, for example, Bernanke et al. (2005), Banerjee et al. (2014), Abbate et al. (2016) and Bai et al. (2016) or *iv*) in the context of factor-augmented predictive regressions, to improve the forecasting of the objective variable; see, for example, Stock and Watson (1999, 2002a, 2002b and 2006), Marcellino et al. (2003), Bernanke and Boivin (2003), Boivin and Ng (2005), Banerjee et al. (2008) and, more recently, Ando and Tsay (2014) and Bräuning and Koopman (2014). Several methods have been proposed in the literature for factor extraction. The most popular procedures for large data sets are still based on Principal Components (PC) techniques; see, for example, Ludvigson and Ng (2007, 2009, 2010), Ando and Tsay (2014), Gonçalves and Perron (2014) and Djogbenov et al (2015) for recent references. The factors correspond to the first few principal components (arranged in decreasing order by their eigenvalues) of the entire system of variables; see, for example, Stock and Watson (2002a) for an excellent discussion on PC factor extraction. Consequently, PC factor extraction is computationally simple and allows dealing with very large systems. However, it is crucial to obtain not only accurate point estimates of the latent factors, but also of their associated uncertainty. Bai (2003) remarks the importance of constructing confidence intervals of the extracted factors in empirical applications in which these represent economic indices. While Bai and Ng (2006) argue about the importance of measuring correctly the uncertainty of the factor extraction in FAVAR models. More recently, Jackson et al. (2016) argue that measures of factor uncertainty should always accompany applied work in order to establish the statistical legitimacy of the results.

The asymptotic distribution of the factors extracted using PC is derived by Bai (2003) assuming weak dependence in the idiosyncratic term while Bai and Ng (2006) deal with the asymptotic distribution of the OLS estimator of a factor-augmented predictive regression model and propose three different estimators of the asymptotic covariance matrix of the factors depending on the structure of the errors. More recently, Bai and Ng (2013) derive the limiting distribution of the factors and its corresponding covariance matrix estimation for different identification restrictions. However, results on the performance of the asymptotic distribution to approximate the finite sample distribution of the estimated factors are scarce. As far as we are concerned, only Poncela and Ruiz (2016) show that PC intervals based on the asymptotic distribution could underestimate the uncertainty of the extracted factors [1]. Alternatively, the finite sample distribution of the estimated factors can be obtained using resampling procedures that incorporate the parameter uncertainty. Several authors propose using bootstrap in the context of DFMs with other objectives than obtaining the distribution of the underlying factors. For example, Yamamoto (2012) obtains bootstrap bands for impulse response functions in the context of FAVAR models. Ludvigson and Ng (2007,2009 and 2010), Gospodinov and Ng (2013), Gonçalves and Perron (2014), Djogbenou et al. (2015) and Jackson et al. (2016) implement bootstrap procedures to carry out inference on the OLS estimator of the parameters of factor-augemented predictive regression models [2]. More recently, Shintani and Guo (2015) also propose using bootstrap to test about the autoregressive parameter governing the dependence of the latent factor.

This paper has three main contributions. First, we provide extensive Monte Carlo experiments in order to asses the conditions under which the asymptotic distribution of the factors extracted using PC is a good approximation of the finite sample distribution. In concordance with the results in Poncela and Ruiz (2016), we show that in a wide range of scenarios, the asymptotic confidence intervals of the estimated factors are unrealistically tiny. The second contribution is to analyze the performance of the available bootstrap methods when implemented to obtain confidence bands of the PC extracted factors. We show that these methods either obtain the marginal distribution of the factors and, consequently, the corresponding intervals are too wide as to be informative or they are based on independent extractions and, therefore, they are not appropriate to represent the dynamic of the factors. The third contribution of this paper is to propose a new bootstrap procedure designed to construct conditional confidence bands for the estimated factors that take into account the dynamic dependence in the system. The finite sample performance of the proposed procedure is analyzed.

The rest of the paper is organized as follow. Section 2 describes the PC factor extraction procedure and its asymptotic distribution. Monte Carlo experiments are carried out to asses the

[1] In the context of inference for the OLS estimator of the parameters of factor-augmented predictive regression models, Gonçalves and Perron (2014) show that the finite sample properties of the asymptotic approach of Bai and Ng (2006) can be poor, especially if N is not sufficiently large relative to T

[2] The procedure proposed by Corradi and Swanson (2014) does not allow to obtain bootstrap intervals for the estimated factors and is not considered further in this paper

adequacy of the asymptotic distribution to approximate the finite sample distribution of the factors. Section 3 describes available bootstrap procedures proposed for DFM and analyzes their finite sample performance. In Section 4, the new resampling procedure is proposed and its finite sample performance analyzed. Section 5 illustrates the results with an empirical application. Section 6 concludes.

2 Factor extraction

In this section, we describe the DFM considered in this paper and introduce notation. We also describe the asymptotic properties of the PC factor extraction procedure. Finally, we carry out Monte Carlo experiments to asses the adequacy of the asymptotic distribution to approximate the finite sample distribution of the extracted factors.

2.1 The Dynamic Factor Model

We consider the following DFM in which the latent factors and the idiosyncratic components are VAR(1) processes

$$Y_t = PF_t + \varepsilon_t, \tag{1}$$

$$F_t = \Phi F_{t-1} + \eta_t, \tag{2}$$

$$\varepsilon_t = \Gamma \varepsilon_{t-1} + a_t \tag{3}$$

where $Y_t = (y_{1t}, ..., y_{Nt})'$ is the $N \times 1$ vector of observed variables at time t for $t = 1, ..., T$, P is the $N \times r$ matrix of factor loadings, $F_t = (f_{1t}, ..., f_{rt})'$ is the $r \times 1$ matrix of unobservable factors and $\varepsilon_t = (\varepsilon_{1t}, ..., \varepsilon_{Nt})'$ is the $N \times 1$ vector of idiosyncratic noises. The disturbances $\eta_t = (\eta_{1t}, ..., \eta_{rt})'$ and $a_t = (a_{1t}, ..., a_{Nt})'$ are mutually independent Gaussian white noise vectors with finite covariance matrices Σ_η and Σ_a respectively. The matrices Φ and Γ are diagonal with their parameters restricted so that Y_t is stationary. The number of factors, r, is assumed to be known and fixed as the cross-sectional and temporal dimensions, N and T, respectively, grow.

The DFM in equations (1) to (3) has been frequently used in the related literature; see, for example, Jungbacker and Koopman (2015) and Alvarez et al. (2016) for recent references.

Next, we describe the PC procedure to extract the factors in DFMs.

2.2 Principal Components Factor Extraction

The factor extraction PC procedure is very popular because of its computational simplicity even in large data sets. The $r \times T$ matrix of extracted factors $\hat{F} = (\hat{F}_1, ..., \hat{F}_t)$ is given by $\sqrt{T}$ times the eigenvectors corresponding to the largest eigenvalues of the $T \times T$ matrix $Y'Y$ where $Y = (Y_1, ..., Y_T)$. The estimated factor loadings matrix, $\hat{P}$, is estimated by $\hat{P} = \frac{Y\hat{F}'}{T}$. This method only identifies the rotation of the factors and their loadings. For a unique identification of the factors, a normalization as, for example, $\frac{F'F}{T} = I_r$ is imposed; see Bai and Ng (2013) for a extensive discussion on identification issues. Connor and Korajczyk (1986) prove consistency for the PC factors when N goes to infinity and T is fixed. Stock and Watson (2002a) show that the space spanned by the estimated factors is consistent when both N and T tend simultaneously to infinity if the serial and cross-sectional correlations of the idiosyncratic noises are weak and the factors are pervasive. Bai (2003) shows that, for a consistent extraction of the factors in the case of large N but fixed T, it is necessary to assume asymptotic orthogonality and homoscedasticity of the idiosyncratic term. Only under large N and T, Bai (2003) establishes consistency in the presence of serial correlation and heteroscedasticity. Furthermore, if $\frac{\sqrt{N}}{T} \to 0$, Bai (2003) derives the following limiting distribution assuming that the eigenvalues of the covariance matrix of the factors and loadings are distinct:

$$\sqrt{N}(\hat{F}_t - H'F_t) \xrightarrow{d} N(0, \Pi_t) \tag{4}$$

where H is an invertible matrix such that $\hat{F}_t$ is an estimator of $H'F_t$. It is important to note that $\hat{F}_t$ is estimating a rotation of F_t. The asymptotic covariance matrix and therefore, the confidence bands, are constructed for this rotation and not for F_t. Bai and Ng (2006) propose the three following estimators of the covariance matrix, Π_t, depending on the underlying assumptions regarding the idiosyncratic noises:

1. Cross-sectionally uncorrelated but heteroscedastic noises:

$$\hat{\Pi}_t = \hat{V}^{-1} \left(\frac{1}{N} \sum_{i=1}^{N} \hat{p}_{i.} \hat{p}_{i.}^{'} \hat{\epsilon}_{it}^2 \right) \hat{V}^{-1}, \tag{5}$$

where $\hat{V}$ is the r x r diagonal matrix of the first r eigenvalues of $YY'/(TN)$ arranged in decreasing order, $\hat{p}_{i}.$ is the $i-th$ row of the factor loading matrix $\hat{P}$ and the residuals are given by $\hat{\epsilon}_{it} = Y_{it} - \hat{p}_{i.}^{'} \hat{F}_t$.

2. Cross-sectionally uncorrelated and homoscedastic noises:

$$\hat{\Pi}_t = \hat{V}^{-1}\left(\hat{\sigma}_\epsilon^2 \frac{1}{N}\sum_{i=1}^{N}\hat{p}_{i.}^{'}\hat{p}_{i.}\right)\hat{V}^{-1}, \tag{6}$$

where, according to Bai and Ng (2008), $\hat{\sigma}_\epsilon^2 = \frac{1}{NT-r(T+N-r)}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{\epsilon}_{it}^2$.

3. Cross-sectionally correlated noises:

$$\hat{\Pi}_t = \hat{V}^{-1}\left(\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\hat{p}_{i.}^{'}\hat{p}_{j.}\frac{1}{T}\sum_{t=1}^{T}\hat{\epsilon}_{it}\hat{\epsilon}_{jt}\right)\hat{V}^{-1}. \tag{7}$$

Bai and Ng (2006) propose estimating the asymptotic covariance matrix of the factors using expression (5) regardless of the properties of the idiosyncratic noises. They argue that, if the cross correlation in the errors is small, assuming that they are zero could be convenient because the sampling variability from their estimation could cause an efficiency loss.

2.3 Finite sample distribution of factors

We carry out Monte Carlo experiments to analyze the adequacy of the asymptotic distribution of the PC factors to approximate their finite sample distribution and, consequently, to construct confidence bands. The Monte Carlo experiments are performed using DFM of increasing complexity. The first model is the ubiquitous single factor model with temporal and cross-sectionally independent idiosyncratic errors. Then, we consider the same model in which the idiosyncratic term is either cross-correlated, temporally dependent or heteroscedastic. The data generating process (DGP) is given by the DFM in equations (1)-(3) with $N = 50, 100, 200$ and 1000, and $T = 50, 100, 200$ and 1000. The number of the Monte Carlo replications is $R = 500$. Consider the first DGP given by the DFM in equations (1) to (3) in which $r = 1$ and the idiosyncratic noises are homoscedastic and cross-sectionally uncorrelated white noises. The matrix of factor loadings, P, is generated once from a uniform distribution in [0,1] with $\sum_{i=1}^{50} p_i^2 = 15.85$, $\sum_{i=1}^{100} p_i^2 = 32.12$, $\sum_{i=1}^{200} p_i^2 = 65.56$ and $\sum_{i=1}^{1000} p_i^2 = 331.04$ for $N = 50, 100, 200$ and 1000 respectively. In order to analyze the effect of the temporal dependence of the factor on its accuracy, we consider several values of the autorregresive parameter of the factor, $\Phi = 0.2, 0.5$ and 0.9. In each case, the noise in equation (2), η_t, has variance such that $Var\left(F_t\right) = 1$. The covariance of the idiosyncratic noises is given by $\Sigma_a = q^{-1}I$. Given that $Var\left(F_t\right) = 1$, the signal to noise ratio is given by $q = \frac{Var(F_t)}{Var(\varepsilon_t)}$. Breitung and Eickmeier (2016) point out that the accuracy of factor estimates can depend on the signal to noise ratio. Consequently, in our Monte Carlo experiments, we consider several values of q, namely $q = 5, 3, 2, 1, 0.5, 0.3$ and 0.2.

For each Monte Carlo replicate, $i = 1, ..., R$, we compute i) the Root Mean Squared Error of the factor computed as $\text{RMSE}^{(i)} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(F_t^{(i)} - \hat{F}_t^{(i)}\right)^2}$, ii) the Asymptotic Root Mean Squared Error computed as $\text{RMSEA}^{(i)} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\hat{\Pi}_t^{(i)}/N}$, and iii) the empirical coverages computed as $C^{(i)} = \frac{1}{T}\sum_{t=1}^{T} I\left(F_t^{(i)} \in CI_t^{(i)}\right)$ where $I(\cdot)$ is the indicator function and $CI_t^{(i)} = \left[\hat{F}_t^{(i)} \pm z_{\alpha/2}\sqrt{\hat{\Pi}_t^{(i)}/N}\right]$ with $z_{\alpha/2}$ being the $\alpha/2$ quantile of a standard normal distribution. Table 1 reports the Monte Carlo averages of these measures for different temporal and cross-sectional dimensions. Observe that, regardless the sample dimensions, N and T, the signal to noise ratio, q, and the serial dependence of the factor, Φ, the RMSEA underestimates the sample RMSE producing coverages well below the nominal. Furthermore, the sample RMSE increases with Φ while the asymptotic RMSEA is stable, leading to larger undercoverages as the persistence of the factor increases. For example, when $q = 5$, $\Phi = 0.8$, $T = 100$ and $N = 1000$, the sample coverage is 44% when the nominal is 95%.

In addition, as N increases with a fixed T, both the RMSE and RMSEA decrease. However, the RMSEA does it at a much faster rate, leading to tiny coverages when $T = 100$ and $N = 1000$. On the other hand, when N remains fixed as T increases, the RMSE reduces again but the RMSEA remains stable. This causes an improvement in the coverages, as expected since $\frac{\sqrt{N}}{T} \to 0$ as assumed by Bai (2003) when deriving the asymptotic distribution.

In order to have a better understanding of the finite sample properties of the PC estimator when data has a more realistic structure, we also generate the idiosyncratic errors by equation (3) with $\Gamma = \gamma I_N$ and $\gamma = 0.2, 0.5$ and 0.8. When $\gamma = 0$, cross-sectionally correlated errors and cross-section heteroscedastic errors are also generated such that Σ_e is a Toeplitz matrix with parameter 0.5 and $\Sigma_a = diag\left[\sigma_a^2 U(a, b)\right]$, respectively. When the errors are heteroscedastic, $\sigma_a^2 = 0.1, 1, 2$ and 10, $a = 0.1, 0.5$ and 0.9 and $b = 2, 1.5$ and 1.1.

Table 2 provides the Monte Carlo averages when there is serial or cross-sectional dependence in the idiosyncratic term ($\gamma = 0.5$) and also in the presence of cross-sectional heteroscedasticity. When the factors are dominant, the signal to noise ratio is large (q=5), introducing temporal dependence in the idiosyncratic noise does not affect the RMSE and only has a marginal effect on the coverages. However, in all other cases, the RMSE increase considerably while the RMSEA remains almost stable. Consequently, the effect of introducing serial correlation in the idiosyncratic term will be dramatic with extremely low sample coverages. Furthermore, as N increases, the coverages decrease. This is consistent with Bai (2003) who point out that for a fixed T, it is not possible to obtain a consistent estimation in presence of serial correlation.

Something similar occurs when there is cross-correlation. If q is not large, $q = 0.2$, the effect of introducing correlation is remarkable. This is also consistent with Boivin and Ng (2006) who argue that more data is not always desirable when q is not large and the errors are cross-correlated. Results when the idiosyncratic terms are heteroscedastic are quite similar to the cross-correlation scenario.[3]

3 Extant bootstrap procedures for PC factors

Several authors propose implementing resampling techniques in order to construct confidence intervals in the context of PC; see, for example, Beran and Srivastra (1985), Stauffer et al. (1985), Timmerman et al. (2007), Babamoradi et al. (2013), Van Aelst et al. (2013) and Fisher et al. (2015). However, these procedures assume iid observations and, consequently, they are not appropiate for DFM. In recent years, several resampling methods have been proposed in the context of DFMs with other objectives than constructing confidence bands for the extracted factors. These procedures allow the construction of bootstrap bands of the factors as a subproduct. In this section, we describe these extant resampling algorithms and carry out Monte Carlo experiments to assess their adequacy to construct confidence bands for the extracted PC factors. The extant algorithms can be classified into two main classes: i) Block bootstrap algorithms and ii) residual algorithms.

3.1 Block bootstrap algorithms

Gospodinov and Ng (2013) propose a moving block bootstrap of the original vector of observations. The algorithm adapted to obtain the bootstrap distribution of the factors is as follows:

1. Denote by $B_{t,m} = (Y_t, Y_{t+1}, ..., Y_{t+m-1})$ a block of m $(1 \leq m < T)$ consecutive observations of Y_t. Obtain bootstrap replicates of $Y_t^{*(b)}$ drawing with replacement $K = T/m$ blocks from $(B_{1,m}, B_{2,m}, ..., B_{T-m+1,m})$.

[3] Results for different sample sizes and different idiosyncratic structures are available from the authors from request.

2. Using $Y_t^{*(b)}$, obtain PC estimates $\hat{F_t^*}^{(b)}$.

3. Repeat steps 1 and 2 for $b = 1, ..., B$.

The block size m is allow to grow but in a slower rate than T.[4] Denote by $\hat{G}^*_{F_t}(x)$ the empirical distribution of $\hat{F}_t^{(b)}$ given by

$$\hat{G}^*_{F_t}(x) = \#\left(\hat{F_t^*}^{(b)} \leq x\right)/B. \tag{8}$$

Then, $(1-\alpha)\%$ confidence bands for the extracted factors can be constructed as

$$\left(q^*_{\alpha/2}, q^*_{1-\alpha/2}\right) \tag{9}$$

where $q^*_{\alpha/2}$ and $q^*_{1-\alpha/2}$ are the $\alpha/2$ and $1-\alpha/2$ empirical quantiles of $\hat{G}^*_{F_t}(x)$ respectively. Alternatively, it is possible to compute the bootstrap RMSE, as follows

$$RMSEB = \sqrt{\frac{1}{B}\sum_{b=1}^{B}\left(\hat{F_t^*}^{(b)} - \frac{1}{B}\sum_{b=1}^{B}\hat{F_t^*}^{(b)}\right)^2} \tag{10}$$

Then assuming normality of the factors the $(1-\alpha)\%$ confidence intervals are given by

$$\hat{F}_t \pm Z_{\alpha/2} RMSEB \tag{11}$$

It is important to note that the confidence bands constructed as in (9) are marginal. Obtaining the marginal distribution of the factors could be appropriate in the context of carrying out inference on the parameter estimates of the factor-augmented regression model as it is the objective of Gospodinov and Ng(2013). However, they are not appropriate for the extracted factors because the marginal bands are not informative. As a illustration, the second row of Figure 1 plots a factor generated by the DFM described in the previous section with $r = 1$ and white noise and cross-sectionally uncorrelated and homoscedastic idiosyncratic errors. The bands are constructed as in expressions (9) and (11) with $B = 1000$ bootstrap replicates; see Gonçalves and Perron (2014) who consider $B = 399$ while Ludvigson and Ng (2007, 2009, 2010) consider $B = 1000$. Therefore, we expect the corresponding bands to be too wide having coverages well above the nominal.

[4] Gospodinov and Ng (2013) consider $m = 4$ for forecasting purposes. The authors obtain similar results for other block sizes while $m \in [4, 24]$.

3.2 Residual bootstrap algorithms

There are two main algorithms proposed in the literature. First, Gonçalves and Perron (2014) propose a residual-based bootstrap for inference about the OLS estimator in a factor-augmented predictive regression model. The algorithm to obtain bootstrap replicates of the factors is as follows: [5]

1. Estimate $\hat{P}$ and $\hat{F}_t$ using PC. Obtain the residuals $\hat{\varepsilon}_t = Y_t - \hat{P}\hat{F}_t$ and its empirical distribution $\hat{G}_\varepsilon$.

2. Obtain the bootstrap replicates of Y_t as follows

$$Y_t^{*(b)} = \hat{P}\hat{F}_t + \varepsilon_t^{*(b)} \tag{12}$$

   where ε_t^* are random extractions from $\hat{G}_\varepsilon$.

3. Using $Y_t^{*(b)}$, obtain PC estimates of the factors, $\hat{F}_t^{*(b)}$.

4. Repeat steps 2 and 3 for $b = 1, ..., B$.

The bootstrap replicates of Y_t obtained as in equation (12) are centered in the estimated common factor $\hat{P}\hat{F}_t$ and incorporate uncertainty about the idiosyncratic noise. However, they do not add the uncertainty associated with the estimation of the common factor. As a consequence, the confidence bans are conditional but narrower than they should be. This procedure has also been proposed by Ludvigson and Ng (2007, 2009, 2010). The third row of Figure 1 illustrates this procedure with the same factor as used for the Gospodinov and Ng (2013) algorithm.

The second residual bootstrap in the literature is due to Yamamoto (2002). This procedure is a residual-based bootstrap intended to carry out inference for impulse response functions in structural FAVARs models. The algorithm adapted to measure the uncertainty of the factor extraction is as follow:

1. Estimate $\hat{P}$ and $\hat{F}_t$ using PC. Regress $\hat{F}_t$ on $\hat{F}_{t-1}$ and estimate $\hat{\Phi}$ by Least Squares (LS). Obtain the corresponding residuals $\hat{u}_t = \hat{F}_t - \hat{\Phi}\hat{F}_{t-1}$ and $\hat{\varepsilon}_t = Y_t - \hat{P}\hat{F}_t$ [6]. Obtain the empirical distribution functions $\hat{G}_u$ and $\hat{G}_\varepsilon$ of the centered residuals $\hat{u}_t$ and $\hat{\varepsilon}_t$, respectively.

[5] Gonçalves and Perron (2014) propose a wild bootstrap algorithm to obtain replicates of $\varepsilon_t^{*(b)}$

[6] Note that this procedure is similar to the first step of the factor extraction procedure proposed by Doz et al. (2011).

2. For $t = 2, ..., T$, obtain bootstrap replicates of Y_t that mimic the dynamic characteristics of the original system as follows

$$F_t^{*(b)} = \hat{\Phi} F_{t-1}^{*\ (b)} + u_t^{*(b)} \tag{13}$$

$$Y_t^{*(b)} = \hat{P} F_t^{*(b)} + \varepsilon_t^{*(b)} \tag{14}$$

where $F_1^{*(b)} = \hat{F}_1$ and η_t^* and ε_t^* are random extractions from $\hat{G}_\eta$ and $\hat{G}_\varepsilon$ respectively.

3. Using $Y_t^{*(b)}$, obtain PC estimates of the factors, $\hat{F}_t^{*(b)}$.

4. Repeat steps 2 and 3 for $b = 1, ..., B$.

The bands constructed using this procedure are either marginal or based on the marginal RMSE as, at each moment of time, they are based on bootstrap replicates of the factors which are not based on the available information set. Therefore, we expect a similar behaviour as that of the bands constructed using Gospodinov and Ng (2013) algorithm; see the fourth row of Figure 1.

Shintani and Guo (2015) propose a slight modification of the above algorithm in order to approximate the distribution of $\hat{\Phi}$. Next we describe the procedure when implemented to obtain bootstrap replicates of the factors[7]:

1. Estimate $\hat{P}$ and $\hat{F}_t$ using PC. Regress $\hat{F}_t$ on $\hat{F}_{t-1}$ and estimate $\hat{\Phi}$ by Least Squares (LS). Obtain the corresponding residuals $\hat{\eta}_t = \hat{F}_t - \hat{\Phi}\hat{F}_{t-1}$ and $\hat{\varepsilon}_t = Y_t - \hat{P}\hat{F}_t$. Denote $\hat{G}_\eta$ the empirical bootstrap distribution of the centered residuals $\hat{\eta}_t$.

2. Obtain bootstrap replicates of the factor $F_t^{*(b)} = \hat{\Phi} F_{t-1}^{*(b)} + \eta_t^{*(b)}$ where η_t^* are random extractions with replacement from $\hat{G}_\eta$.

3. Obtain bootstrap replicates of Y_t with $y_{1t}^{*(b)} = \hat{p}_{1r}^{*(b)} \hat{f}_{tr}^{\prime *(b)} + \epsilon_{1t}^{*(b)}$ by drawing pairs $\left(\hat{p}_{1r}^{*(b)}, \epsilon_{1t}^{*(b)}\right)$ from $\left(\hat{p}_{ir}^{*(b)}, \epsilon_{it}^{*(b)}\right)$. Repeat the same procedure N times to generate all $y_{it}^{*(b)}$ for $i = 1, ..., N$.

4. Apply PC to $Y_t^{*(b)}$ and extract the factors $\hat{F}_t^{*(b)}$.

5. Repeat steps 2 and 3 for $b = 1, ..., B$.

[7]The authors describe the procedure for $r = 1$ factors.

As explained before, the confidence bands of the extracted factors can also be constructed using the corresponding percentiles of $\hat{G}^*_{F_t}(x)$ or with Gaussian approximations. It is important to point out that the confidence bands are also marginal as they are based on bootstrap replicates of the factors generated as in (13). As a consequence, we expect these bands to be wider than if the conditional bands were obtained. [8]

3.3 Finite sample performance

We carry out Monte Carlo experiments to analyze the finite sample performance of the resampling methods described in the previous subsection. The design is the same as that considered in subsection 2.3. The reported coverages are the corresponding ones to the confidence bands constructed using the appropriate percentile of the bootstrap distribution of the extracted factors or through the Gaussian approximation.

Tables 3, 4 and 5 report Monte Carlo averages of the resampling procedures based on Gospodinov and Ng(2013), Yamamoto (2012) and of the second procedure proposed by Shintani and Guo (2015) respectively. As already pointed out, with these procedures the marginal distributions of the extracted factors are obtained. Consequently, in these three tables and for all models considered, the RMSEB are very close to one, the true value of the marginal variance. Thus, the coverages when the confidence bands are computed with Gaussian approximations are almost always 100%. However, since both procedures are designed to obtain the uncertainty associated with the marginal distribution, the quantile confidence bands are parallel and uninformative lines that do not reflect any kind of temporal dynamic. This leads to coverages below 100% in both cases.

Table 6 reports the Monte Carlo averages corresponding to the Gonçalves and Perron (2014) bootstrap procedure. Comparing the results in table 7 with those reported in table 2 corresponding to the asymptotic intervals, we can observe that, if the signal to noise ratio, q, is large, the average coverages obtained using the asymptotic approximation and the bootstrap procedure are pretty similar and bellow the nominal coverage. When q is small, the bootstrap gets a considerable underestimation of the uncertainty, leading to tiny coverages. Recall that the procedure proposed by Gonçalves and Perron (2014) only consider the idiosyncratic uncertainty and does not incorporate the estimation one and, therefore, the RMSEB is smaller than the empirical RMSE specially when

[8] The first algorithm proposed by Shintani and Guo (2015) is valid under the assumption of independence of Y_t given that the cross-sectional dimension is being bootstrapped.

the idiosyncratic term is not relevant.

Therefore, we can conclude that none of the bootstrap procedures already proposed in the context of PC factor extraction in DFM, are valid to measure the uncertainty of the factor, However, it is important to mention that none of these methods was designed with this purpose.

4 New Procedure

In this section, we propose a new resampling strategy to measure uncertainty and to construct confidence bands for PC factors extraction which takes into account the specific problem at hand.

4.1 Bootstrap Procedure

The proposed procedure builds on ideas of Pascual et al.(2004) to obtain conditional densities of the unobserved factors that incorporates the estimation uncertainty. The algorithm is as follows:

1. Estimate $\hat{P}$ and $\hat{F}_t$ using PC. Regress $\hat{F}_t$ on $\hat{F}_{t-1}$ and estimate $\hat{\Phi}$ by Least Squares (LS). Obtain the corresponding residuals $\hat{\eta}_t = \hat{F}_t - \hat{\Phi}\hat{F}_{t-1}$ and $\hat{\varepsilon}_{it} = Y_{it} - \hat{P}_{ir}\hat{F}_{rt}$ for $i = 1$ to N. Denote $\hat{G}_\eta$ and $\hat{G}_\varepsilon$ the empirical distribution functions of $\hat{\eta}_t$ and $\hat{\varepsilon}_{it}$, respectively.

2. For $t = 1, ..., T$ obtain bootstrap replicates $F_t^* = \hat{\Phi}F_{t-1}^* + \hat{u}_t^*$ and $Y_t^* = \hat{P}F_t^* + \varepsilon_t^*$, being $\hat{u}_t^*$ random extractions with replacement from the empirical distribution functions of $\hat{u}_t$ and ε_t^* are random extractions with replacement from $\hat{G}_\varepsilon$. Based on Pascual et al. (2004) obtain $\hat{\Phi}^*$ and $\hat{P}^*$.

3. Obtain bootstrap replicates of Y_t that mimic the dynamic of the original system as follows:

$$F_t^{*(b)} = \hat{\Phi}^{*(b)}\hat{F}_{t-1} + \hat{\eta}_t \tag{15}$$

$$Y_t^{*(b)} = \hat{P}^{*(b)}F_t^{*(b)} + \varepsilon_t^{*(b)} \tag{16}$$

4. Using $Y_t^{*(b)}$, obtain PC estimates $\hat{F_t^*}^{(b)}$.

5. Repeat steps 2,3 and 4 for $b = 1, ..., B$.

It is important to note that the pseudo-factors $\hat{F}_t^*$ have to be normalized for identification purposes. The confidence bands are constructed as in equations (9) and (11) for $r = 1$ extracted factors.

4.2 Finite Sample Performance

We also carry out extensive Monte Carlo experiments to analyze the finite sample performance of the new bootstrap procedure introduced in the previous section. The structure of the experiment is the same as the one considered in sections 2.3 and 3.3. The RMSE, the RMSEB computed as in equation (10) are reported. In the same way, the coverages related to the confidence bands constructed using the appropriate percentile of the bootstrap distribution of the extracted factors or through Gaussian approximation are also presented.

In order to have a broad view of the finite sample performance of this new procedure, we present its behavior in scenarios of different nature. Table 7 shows the results for a wide range of signal to noise ratios. Table 8 reports the performance of the proposed procedure for different sample sizes and, finally, the outcomes for different structures in the idiosyncratic term are presented in Table 9.

It is noticeable that, for all the signal to noise ratios, the new algorithm measures considerably well the uncertainty associated with the factor extraction. The RMSEB and RMSE are almost identical in almost all cases and the coverages are very close to the nominal ones. The averages of the RMSE and the RMSEB of the new procedure are equal; 0.18, with a mean coverage of 0.92. Although it is very close to the nominal, this small difference could be a sign of non-gaussianity in the behavior of the factors extracted by PC or of correlation between residuals and the idiosyncratic term. If the results of the proposed procedure are compared to the asymptotic approach for the same signal to noise ratios, it can also be seen that the average of the RMSEA is 0.14 instead of 0.18, well bellow the real one, and the mean coverage is 0.83 instead of 0.92.

Moreover, looking the results reported in Table 7, the bootstrap algorithm leads to almost the same results no matter the sample size. The averages of the RMSE and the RMSEB of the new procedure for the different sample sizes that appear in Table 7 are equal; 0.21, with a mean coverage of 0.91. However, the average of the RMSEA for the same sample sizes is 0.13 with a mean coverage of 0.77. Once again the new procedure outperform the asymptotic approach.

In addition, regardless the idiosyncratic structures associated with the data, the resampling procedure behaves also properly. Only when the signal to noise ratio is small and the there is serial dependence in the idiosyncratic term, the coverages are considerably bellow the nominal. The averages of the RMSE and the RMSEB of the new procedure for the different idiosyncratic structures are also equal; 0.18, with a mean coverage of 0.91. However, the average of the RMSEA for the same structures is 0.15 with a mean coverage of 0.82. Again, the finite sample performance of the algorithm proposed is better than the asymptotic one.

Therefore, the new procedure outperform considerably the asymptotic approach, measuring better the uncertainty in all the cases. Only when there is a strong serial dependence in the factors or in the idiosyncratic term, the results of the new procedure worsen. Nevertheless, the results are still considerably better than those obtained for the asymptotic covariance matrix. This small inconvenience is probably due to the correlation between residual and the idiosyncratic components when the scenarios are close to non-stationarity.

5 Empirical Analysis

In this section, we exemplify the importance of a proper measurement of the uncertainty associated with the factors extracted by PC. For this purpose we analyze the quarterly series belonging to the database of the Treasury Department of Spain, which consists of a panel of 75 Spanish macroeconomic variables observed quarterly from the first quarter of 1980 to the last of 2014. The variables have been seasonally adjusted and converted to stationary. Moreover, they have been standardized to have zero mean and unite variance. Therefore, the total panel of data consists of N=75 variables and T=140 observations. We start the analysis estimating the number of factors to extract. More specially, we consider the information criteria of Bai and Ng (2002), the edge distribution of Onatski (2010) and the ratios of eigenvalues proposed by Ahn and Horenstein (2013). The number of factors to compute is one. The factors are extracted by PC and the confidence intervals are constructed following the procedures described above. The sum of the weights is $\sum_{i=1}^{N} p_i = 12.21$ with estimated weights larger than 0.8 in absolute value corresponding to: Gross capital formation, capital stock, imports, unemployment rate, rest of the word clients' GDP, total resources of public administrations. $\hat{\Phi} = 0.7$, $\hat{\sigma}_a^2 = [-0.93, 0.86]$ with the mode around 0, and serial dependence $\hat{\gamma} = [-0.74, 0.97]$ distributed uniformly in this interval. Figure 2 plots the results of this analysis. It can be observed that the asymptotic confidence intervals are considerably narrower than the ones constructed following the procedure proposed in the previous section. The amplitude of the asymptotic confidence intervals correspond only to a 62% of the proposed amplitude of the intervals. Moreover, the 95% asymptotic confidence intervals are almost equivalent to the 75% confidence intervals constructed using our procedure which, through an intense Monte Carlo experiment, has proven to be a better approach for measuring the uncertainty of factor extraction with PC in this kind of scenario. In other words, it is important to point out the importance of measuring the uncertainty of the factors correctly. If practitioners and policy decision makers use the asymptotic covariance matrix for measuring the uncertainty or for constructing confidence bands for the latent factors, could lead to a wrong interpretation of the economic reality -cycles and recessions- and to an incorrect density forecast in the context of diffusion indexes.

6 Conclusions

This paper explores different methods for improving the computation of the uncertainty associated to factor extraction using PC in DFM. By means of an intense Monte Carlo exercise, the finite sample performance of the asymptotic covariance matrix proposed by Bai and Ng (2006) is investigated, we see that this estimation underestimates the uncertainty of the PC factors, causing narrower gaussian confidence bands than desired. Moreover, it has been shown that the existing resampling procedures in the context of PC in DFM are not capable of measuring the uncertainty associated to the factor extraction correctly. Partly because some of them compute the marginal variance instead of the conditional one, or because they do not take into account the parameter uncertainty of the DFM. Finally, a bootstrap algorithm to compute the uncertainty of PC factors and to construct confidence intervals is presented. This algorithm has proven to have a better finite sample performance than the existing methods for a wide range of scenarios of very different nature. Many topics remain to be developed. First of all, it is desirable to expand the algorithm for the cases in which more than one factor is extracted and also, it is necessary to improve the performance of the procedure when strong serial dependence exists both in the factors and in the idiosyncratic term. A second interesting area of research would be to study the importance of a correct measurement of the factor extraction uncertainty in density forecast using diffusion indexes and, moreover, to study the effect of a non-Gaussian idiosyncratic term. Another important extension would be to apply the procedure to empirical cases with the objective of constructing a stress indicator and warning signals for economic recessions.

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Table 1: Monte Carlo results of the asymptotic approximation when the idiosyncratic component is homoscedastic and serial and cross-sectionally uncorrelated.

		T=50; N=100			T=50; N=200		
q	Φ	RMSE	RMSEA	95%	RMSE	RMSEA	95%
5	0.2	0.13	0.08	0.79	0.11	0.05	0.73
5	0.5	0.15	0.08	0.77	0.13	0.05	0.68
5	0.8	0.19	0.08	0.65	0.18	0.06	0.58
1	0.2	0.19	0.16	0.88	0.18	0.12	0.85
1	0.5	0.23	0.16	0.86	0.18	0.12	0.83
1	0.8	0.26	0.18	0.83	0.22	0.12	0.75
0.2	0.2	0.44	0.32	0.82	0.33	0.24	0.83
0.2	0.5	0.46	0.32	0.82	0.37	0.25	0.81
0.2	0.8	0.49	0.33	0.79	0.36	0.23	0.79
		T=100; N=200			T=100; N=1000		
q	Φ	RMSE	RMSEA	95%	RMSE	RMSEA	95%
5	0.2	0.08	0.05	0.79	0.06	0.02	0.63
5	0.5	0.10	0.05	0.76	0.08	0.03	0.58
5	0.8	0.13	0.05	0.63	0.12	0.03	0.44
1	0.2	0.13	0.11	0.89	0.09	0.05	0.78
1	0.5	0.15	0.12	0.88	0.10	0.05	0.75
1	0.8	0.18	0.12	0.82	0.13	0.05	0.66
0.2	0.2	0.30	0.25	0.89	0.15	0.11	0.87
0.2	0.5	0.29	0.24	0.89	0.15	0.11	0.85
0.2	0.8	0.33	0.25	0.87	0.19	0.12	0.78

Table 2: Monte Carlo results of the asymptotic approximation for different idiosyncratic structures with T=100 and N=200.

		Independecy			Serial Dependency		
q	Φ	RMSE	RMSEA	95%	RMSE	RMSEA	95%
5	0.2	0.08	0.05	0.80	0.10	0.05	0.78
5	0.5	0.10	0.05	0.75	0.10	0.06	0.76
5	0.8	0.14	0.06	0.65	0.14	0.06	0.65
1	0.2	0.14	0.12	0.90	0.15	0.12	0.88
1	0.5	0.15	0.12	0.87	0.15	0.12	0.86
1	0.8	0.18	0.13	0.84	0.18	0.12	0.80
0.2	0.2	0.29	0.25	0.89	0.38	0.25	0.79
0.2	0.5	0.30	0.25	0.89	0.34	0.23	0.80
0.2	0.8	0.32	0.25	0.87	0.36	0.23	0.80
		Heteroscedasticity			Cross-section Dependency		
q	Φ	RMSE	RMSEA	95%	RMSE	RMSEA	95%
5	0.2	0.10	0.06	0.80	0.08	0.05	0.79
5	0.5	0.10	0.06	0.75	0.10	0.05	0.75
5	0.8	0.14	0.05	0.62	0.14	0.06	0.65
1	0.2	0.15	0.13	0.89	0.15	0.12	0.87
1	0.5	0.15	0.12	0.87	0.16	0.12	0.84
1	0.8	0.19	0.13	0.82	0.19	0.13	0.82
0.2	0.2	0.32	0.25	0.87	-	-	-
0.2	0.5	0.33	0.26	0.86	-	-	-
0.2	0.8	0.35	0.26	0.84	-	-	-

Note: When the signal to noise ratio is small (q=0.2), the covariance matrix of the data is not positive definite.

Table 3: Finite sample performance of the marginal block bootstrap in a DFM with T=100 and N=200 and idiosyncratic noises with different properties; a) homoscedastic and serial and cross-sectionally uncorrelated; b) serially dependent; c) heteroscedastic amd d)cross-sectionally dependent.

		Independency				Serial Dependency			
q	Φ	RMSE	RMSEB	Gaussian 95%	Quantile 95%	RMSE	RMSEB	Gaussian 95%	Quantile 95%
5	0.2	0.08	1.00	1.00	0.95	0.08	0.99	1.00	0.95
5	0.5	0.10	0.99	1.00	0.96	0.10	0.99	1.00	0.96
5	0.8	0.14	1.00	1.00	0.96	0.14	0.99	1.00	0.96
1	0.2	0.15	1.00	1.00	0.96	0.16	0.99	1.00	0.96
1	0.5	0.15	1.00	1.00	0.96	0.16	0.99	1.00	0.96
1	0.8	0.19	0.99	1.00	0.96	0.19	0.99	1.00	0.96
0.2	0.2	0.29	1.00	1.00	0.95	0.34	0.99	1.00	0.95
0.2	0.5	0.32	1.00	1.00	0.96	0.38	0.99	1.00	0.96
0.2	0.8	0.32	1.00	1.00	0.96	0.34	0.99	1.00	0.96
		Heteroscedasticity				Cross-section Dependency			
q	Φ	RMSE	RMSEB	Gaussian 95%	Quantile 95%	RMSE	RMSEB	Gaussian 95%	Quantile 95%
5	0.2	0.09	1.00	1.00	0.96	0.09	1.00	1.00	0.96
5	0.5	0.10	1.00	1.00	0.96	0.10	1.00	1.00	0.95
5	0.8	0.14	1.00	1.00	0.97	0.14	1.00	1.00	0.96
1	0.2	0.15	1.00	1.00	0.96	0.16	1.00	1.00	0.96
1	0.5	0.16	1.00	1.00	0.96	0.16	0.99	1.00	0.95
1	0.8	0.18	0.99	1.00	0.96	0.18	0.99	1.00	0.96
0.2	0.2	0.30	1.00	1.00	0.97	-	-	-	-
0.2	0.5	0.34	1.00	1.00	0.96	-	-	-	-
0.2	0.8	0.35	0.99	1.00	0.97	-	-	-	-

Table 4: Finite sample performance of the marginal residual bootstrap proposed by Yamamoto (2012) in a DFM with T=100 and N=200 and idiosyncratic noises with different properties; a) homoscedastic and serial and cross-sectionally uncorrelated; b) serially dependent; c) heteroscedastic and d) cross-sectionally dependent.

		Independency				Serial Dependency			
q	Φ	RMSE	RMSEB	Gaussian 95%	Quantile 95%	RMSE	RMSEB	Gaussian 95%	Quantile 95%
5	0.2	0.09	0.99	1.00	0.95	0.09	0.99	1.00	0.95
5	0.5	0.10	0.99	1.00	0.95	0.10	0.99	1.00	0.95
5	0.8	0.13	0.97	1.00	0.96	0.13	0.97	1.00	0.96
1	0.2	0.14	1.00	1.00	0.95	0.15	1.00	1.00	0.95
1	0.5	0.15	0.99	1.00	0.96	0.16	0.99	1.00	0.96
1	0.8	0.18	0.97	1.00	0.96	0.19	0.97	1.00	0.96
0.2	0.2	0.30	1.00	1.00	0.95	0.36	0.99	1.00	0.96
0.2	0.5	0.30	0.99	1.00	0.96	0.35	0.99	1.00	0.96
0.2	0.8	0.33	0.97	1.00	0.96	0.35	0.96	1.00	0.96
		Heteroscedasticity				Cross-Section Dependency			
q	Φ	RMSE	RMSEB	Gaussian 95%	Quantile 95%	RMSE	RMSEB	Gaussian 95%	Quantile 95%
5	0.2	0.09	1.00	1.00	0.95	0.09	1.00	1.00	0.95
5	0.5	0.10	0.99	1.00	0.95	0.10	0.99	1.00	0.95
5	0.8	0.13	0.97	1.00	0.95	0.13	0.97	1.00	0.95
1	0.2	0.15	1.00	1.00	0.95	0.15	1.00	1.00	0.95
1	0.5	0.16	0.99	1.00	0.95	0.16	0.99	1.00	0.95
1	0.8	0.20	0.97	1.00	0.96	0.20	0.97	1.00	0.96
0.2	0.2	0.28	1.00	1.00	0.95	-	-	-	-
0.2	0.5	0.34	0.99	1.00	0.96	-	-	-	-
0.2	0.8	0.34	0.98	1.00	0.97	-	-	-	-

Table 5: Finite sample performance of the marginal second residual bootstrap proposed by Shintani and Guo (2015) in a DFM with T=100 and N=200 and idiosyncratic noises with different properties; a) homoscedastic and serial and cross-sectionally uncorrelated; b) serially dependent; c) heteroscedastic and d) cross-sectionally dependent.

		Independency				Serial Dependency			
q	Φ	RMSE	RMSEB	Gaussian 95%	Quantile 95%	RMSE	RMSEB	Gaussian 95%	Quantile 95%
5.00	0.20	0.09	1.00	1.00	0.95	0.09	1.00	1.00	0.95
5.00	0.50	0.10	0.99	1.00	0.95	0.10	0.99	1.00	0.95
5.00	0.80	0.14	0.97	1.00	0.96	0.14	0.97	1.00	0.96
1.00	0.20	0.15	1.00	1.00	0.95	0.15	1.00	1.00	0.95
1.00	0.50	0.15	0.99	1.00	0.95	0.16	0.99	1.00	0.95
1.00	0.80	0.18	0.97	1.00	0.96	0.18	0.97	1.00	0.96
0.20	0.20	0.29	1.00	1.00	0.95	0.35	0.99	1.00	0.94
0.20	0.50	0.29	0.99	1.00	0.95	0.34	0.99	1.00	0.95
0.20	0.80	0.32	0.98	1.00	0.96	0.35	0.97	1.00	0.95
		Heteroscedasticity				Cross-Section Dependency			
q	Φ	RMSE	RMSEB	Gaussian 95%	Quantile 95%	RMSE	RMSEB	Gaussian 95%	Quantile 95%
5.00	0.20	0.09	1.00	1.00	0.95	0.08	0.99	1.00	0.95
5.00	0.50	0.10	0.99	1.00	0.95	0.10	0.99	1.00	0.95
5.00	0.80	0.14	0.97	1.00	0.95	0.15	0.96	1.00	0.96
1.00	0.20	0.14	0.99	1.00	0.95	0.15	1.00	1.00	0.95
1.00	0.50	0.15	0.99	1.00	0.95	0.15	0.99	1.00	0.95
1.00	0.80	0.18	0.96	1.00	0.96	0.19	0.97	1.00	0.96
0.20	0.20	0.31	1.00	1.00	0.94	-	-	-	-
0.20	0.50	0.33	0.99	1.00	0.95	-	-	-	-
0.20	0.80	0.35	0.98	1.00	0.95	-	-	-	-

Table 6: Finite sample performance of the idiosyncratic bootstrap in a DFM with T=100 and N=200 and idiosyncratic noises with different properties; a) homoscedastic and serial and cross-sectionally uncorrelated; b) serially dependent; c) heteroscedastic and d)cross-sectionally dependent.

		Independency				Serial Dependency			
q	Φ	RMSE	RMSEB	Gaussian 95%	Quantile 95%	RMSE	RMSEB	Gaussian 95%	Quantile 95%
5	0.2	0.08	0.01	0.17	0.17	0.08	0.01	0.19	0.19
5	0.5	0.10	0.01	0.15	0.15	0.10	0.01	0.17	0.17
5	0.8	0.13	0.01	0.12	0.11	0.13	0.01	0.13	0.13
1	0.2	0.14	0.04	0.39	0.40	0.15	0.04	0.45	0.45
1	0.5	0.15	0.04	0.38	0.39	0.15	0.04	0.44	0.43
1	0.8	0.18	0.04	0.33	0.34	0.18	0.04	0.38	0.38
0.2	0.2	0.28	0.18	0.78	0.78	0.33	0.24	0.83	0.80
0.2	0.5	0.30	0.21	0.80	0.81	0.35	0.26	0.83	0.80
0.2	0.8	0.32	0.23	0.80	0.80	0.34	0.25	0.82	0.79
		Heteroscedasticity				Cross-Section Dependency			
q	Φ	RMSE	RMSEB	Gaussian 95%	Quantile 95%	RMSE	RMSEB	Gaussian 95%	Quantile 95%
5	0.2	0.09	0.01	0.18	0.18	0.09	0.01	0.15	0.16
5	0.5	0.10	0.01	0.17	0.17	0.10	0.01	0.14	0.15
5	0.8	0.14	0.01	0.14	0.14	0.14	0.01	0.12	0.12
1	0.2	0.14	0.04	0.43	0.44	0.14	0.03	0.36	0.37
1	0.5	0.15	0.04	0.46	0.46	0.16	0.04	0.38	0.38
1	0.8	0.18	0.05	0.40	0.40	0.19	0.04	0.35	0.35
0.2	0.2	0.32	0.34	0.93	0.91	-	-	-	-
0.2	0.5	0.30	0.27	0.88	0.87	-	-	-	-
0.2	0.8	0.34	0.32	0.87	0.86	-	-	-	-

Table 7: Finite sample performance of the new bootstrap procedure in a DFM with T=100 and N=200 and different signal to noise ratios.

q	Φ	RMSE	RMSEB	Gaussian 95%	Quantile 95%
5.00	0.2	0.09	0.11	0.92	0.91
5.00	0.5	0.10	0.11	0.90	0.87
5.00	0.8	0.13	0.12	0.86	0.85
4.00	0.2	0.09	0.11	0.93	0.92
4.00	0.5	0.10	0.12	0.90	0.89
4.00	0.8	0.14	0.12	0.85	0.85
3.00	0.2	0.09	0.12	0.94	0.93
3.00	0.5	0.11	0.12	0.91	0.89
3.00	0.8	0.15	0.13	0.87	0.85
2.00	0.2	0.11	0.12	0.95	0.93
2.00	0.5	0.12	0.13	0.92	0.91
2.00	0.8	0.16	0.15	0.87	0.87
1.00	0.2	0.14	0.14	0.94	0.93
1.00	0.5	0.15	0.16	0.94	0.92
1.00	0.8	0.19	0.18	0.90	0.89
0.50	0.2	0.19	0.20	0.95	0.94
0.50	0.5	0.20	0.20	0.94	0.93
0.50	0.8	0.21	0.21	0.92	0.90
0.30	0.2	0.23	0.24	0.95	0.94
0.30	0.5	0.24	0.24	0.94	0.92
0.30	0.8	0.27	0.26	0.93	0.90
0.25	0.2	0.27	0.27	0.94	0.94
0.25	0.5	0.28	0.28	0.94	0.93
0.25	0.8	0.30	0.29	0.93	0.90
0.20	0.2	0.28	0.28	0.94	0.94
0.20	0.5	0.29	0.28	0.93	0.92
0.20	0.8	0.33	0.32	0.92	0.90

Table 8: Monte Carlo results of the new procedure for different sample sizes when the idiosyncratic component is homoscedastic and serial and cross-sectionally uncorrelated.

		T=50; N=100				T=50; N=200			
q	Φ	RMSE	RMSEB	Gaussian 95%	Quantile 95%	RMSE	RMSEB	Gaussian 95%	Quantile 95%
5	0.2	0.13	0.16	0.93	0.92	0.11	0.14	0.92	0.91
5	0.5	0.15	0.17	0.91	0.89	0.13	0.16	0.87	0.86
5	0.8	0.19	0.19	0.87	0.87	0.18	0.17	0.82	0.81
1	0.2	0.20	0.22	0.95	0.94	0.18	0.20	0.95	0.94
1	0.5	0.23	0.24	0.94	0.92	0.18	0.20	0.93	0.92
1	0.8	0.26	0.26	0.91	0.90	0.22	0.22	0.89	0.88
0.2	0.2	0.41	0.39	0.92	0.93	0.31	0.31	0.93	0.94
0.2	0.5	0.45	0.42	0.90	0.91	0.32	0.32	0.93	0.93
0.2	0.8	0.46	0.43	0.91	0.89	0.38	0.36	0.91	0.90
		T=100; N=200				T=100; N=1000			
q	Φ	RMSE	RMSEB	Gaussian 95%	Quantile 95%	RMSE	RMSEB	Gaussian 95%	Quantile 95%
5	0.2	0.09	0.11	0.92	0.91	0.06	0.10	0.96	0.99
5	0.5	0.10	0.11	0.90	0.87	0.08	0.10	0.95	0.97
5	0.8	0.13	0.12	0.86	0.85	0.17	0.16	0.85	0.84
1	0.2	0.14	0.14	0.94	0.93	0.10	0.12	0.89	0.84
1	0.5	0.15	0.16	0.94	0.92	0.11	0.12	0.89	0.89
1	0.8	0.19	0.18	0.90	0.89	0.13	0.13	0.91	0.91
0.2	0.2	0.28	0.28	0.94	0.94	0.16	0.16	0.90	0.88
0.2	0.5	0.29	0.28	0.93	0.92	0.16	0.16	0.93	0.90
0.2	0.8	0.33	0.32	0.92	0.90	0.26	0.23	0.84	0.84

Table 9: Finite sample performance of the new procedure in a DFM with T=100 and N=200 and idiosyncratic noises with different properties; a) homoscedastic and serial and cross-sectionally uncorrelated; b) serially dependent; c) heteroscedastic and d)cross-sectionally dependent.

		Independency				Serial Dependency			
q	Φ	RMSE	RMSEB	Gaussian 95%	Quantile 95%	RMSE	RMSEB	Gaussian 95%	Quantile 95%
5	0.2	0.09	0.11	0.92	0.91	0.1	0.11	0.93	0.92
5	0.5	0.10	0.11	0.9	0.87	0.1	0.11	0.89	0.90
5	0.8	0.13	0.12	0.86	0.85	0.13	0.12	0.89	0.89
1	0.2	0.14	0.14	0.94	0.93	0.14	0.16	0.94	0.93
1	0.5	0.15	0.16	0.94	0.92	0.16	0.16	0.92	0.91
1	0.8	0.19	0.18	0.90	0.89	0.19	0.17	0.89	0.89
0.2	0.2	0.28	0.28	0.94	0.94	0.36	0.29	0.88	0.88
0.2	0.5	0.29	0.28	0.93	0.92	0.35	0.28	0.89	0.90
0.2	0.8	0.33	0.32	0.92	0.9	0.34	0.28	0.89	0.90
		Heteroscedasticity				Cross-Section Dependency			
q	Φ	RMSE	RMSEB	Gaussian 95%	Quantile 95%	RMSE	RMSEB	Gaussian 95%	Quantile 95%
5	0.2	0.1	0.11	0.92	0.90	0.09	0.11	0.92	0.90
5	0.5	0.1	0.12	0.9	0.89	0.10	0.12	0.89	0.88
5	0.8	0.13	0.13	0.85	0.84	0.13	0.12	0.85	0.85
1	0.2	0.14	0.16	0.94	0.93	0.15	0.16	0.93	0.91
1	0.5	0.16	0.16	0.93	0.92	0.16	0.16	0.92	0.90
1	0.8	0.19	0.18	0.90	0.90	0.19	0.18	0.9	0.89
0.2	0.2	0.30	0.29	0.93	0.93	-	-	-	-
0.2	0.5	0.32	0.31	0.92	0.91	-	-	-	-
0.2	0.8	0.36	0.33	0.92	0.89	-	-	-	-

Figure 1: Illustration of 95% confidence bands constructed using different methods. asymptotic approximation(first row), block bootstrap (second row), idiosyncratic residual bootstrap (third row) and the marginal distribution bootstrap (fourth row). The first column is based on Gaussian bands with bootstrap RMSEs while the second column plots the bands constructed from the bootstrap densities

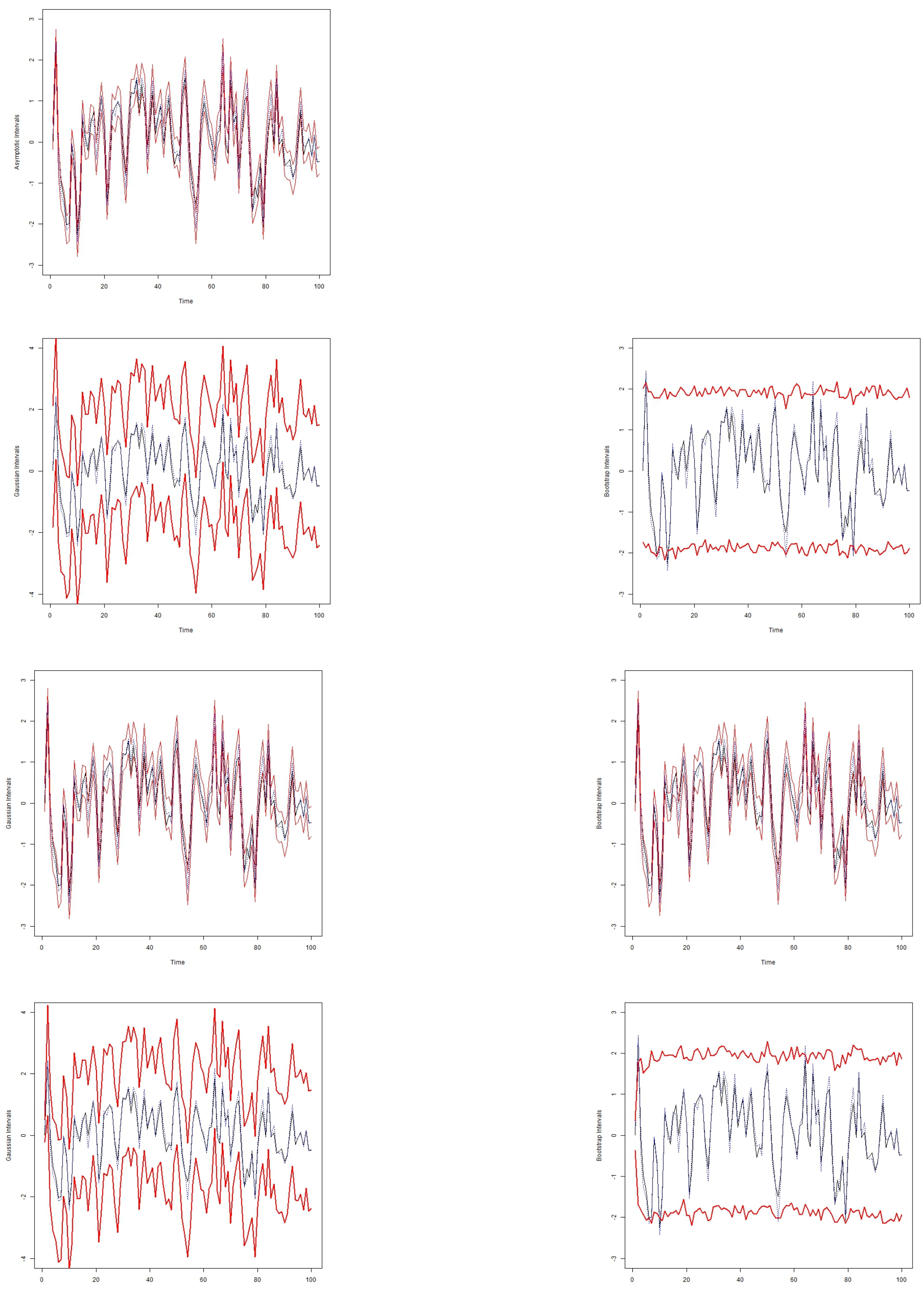

Figure 2: Asymptotic (blue lines) and Bootstrap intervals (red lines) for the economic cycle in Spain (black line).