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Essays on Expected Equity Returns and Volatility: Modeling and Prediction

Autor:

DANIEL DE ALMEIDA

Director:

ESTHER RUIZ

Tutores:

LUIZ KOODI HOTTA

ANA-MARÍA FUERTES

DEPARTAMENTO DE ESTADÍSTICA

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Autor: *Daniel de Almeida*

Director: Esther Ruiz

Firma del Tribunal Calificador:

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Essays on Expected Equity Returns and Volatility: Modeling and Prediction

Author:

Daniel de Almeida

Advisor:

Esther Ruiz, Luiz Koodi Hotta & Ana-Mara Fuertes

DEPARTMENT OF STATISTICS

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To my mom Marilney Saipp!

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Abstract

This thesis is based on modeling and predicting expected equity returns and volatility. In the first step, it focus on multivariate conditional volatility models, where multivariate GARCH (MGARCH) models are the most traditional approach considered in literature. However, the traditional MGARCH models need to be restricted so that their estimation is feasible in large systems and covariance stationarity and positive definiteness of conditional covariance matrices are guaranteed. To overcome this gap, this thesis analyzes the limitations of some very popular restricted parametric MGARCH models often implemented to represent the dynamics observed in real systems of financial returns. These limitations are illustrated using simulated data and a five-dimensional system of exchange rate returns. We show that the restrictions imposed by the BEKK model are very unrealistic generating potentially missleading forecasts of condicional correlations. On the contrary, models based on the DCC specification provide appropriate forecasts. Alternative estimators of the parameters are important to simplify the computations but do not have implications on the estimates of conditional correlations.

In the second step, this thesis focus on predicting the mean of equity risk premium. In particular, we show that existing equity premium forecasts can be improved by combining parsimonious state-dependent regression models, where well-known macroeconomic predictors are interacted with an economic state variable based on technical indicators. The combining forecasts proposed deliver statistically and economically out-of-sample gains vis-a-vis the historical average, traditional univariate regressions and equal-weighted (EW) combination of macroeconomic forecasts. The EW combination is widely reported to be not worse than combining forecasts using estimated weights in equity-premium literature. However, given the relative large set of macroeconomic variables available as candidate predictors, we show that sparse combining method produces promising results for equity risk premium prediction.

Resumen

Esta tesis se basa en modelar y predecir el rendimiento esperado de las acciones y su volatilidad. En la primera etapa se analizan los modelos multivariantes de volatilidad condicional. Los modelos GARCH multivariantes (MGARCH) son los más utilizados en la literatura. Sin embargo, estos modelos necesitan ser restringidos para que su estimación sea factible en grandes sistemas y para que la estacionariedad de segundo orden así como la positividad de las matrices de covarianzas condicionales estén garantizadas. Para poder proponer una solución a este problema, la tesis analiza las limitaciones de algunos modelos MGARCH paramétricos restringidos, los cuales son muy utilizados para representar la dinámica observada en los sistemas reales de rentabilidad financiera. Estas limitaciones se ilustran usando datos simulados y un sistema de cinco series de rendimientos de tipos de cambio. Mostramos que las restricciones impuestas por el modelo BEKK son muy poco realistas, lo que puede generar una mala especificación de las previsiones de correlaciones condicionales. Por el contrario, los modelos DCC generan previsiones apropiadas. Los estimadores alternativos de parámetros son importantes para simplificar los cálculos, pero no tienen implicaciones en las estimaciones de las correlaciones condicionales.

En la segunda etapa, la tesis estudia la predicción de la media de la prima de riesgo. En particular, se muestra que las previsiones de la prima de riesgo se pueden mejorar mediante la combinación de modelos de regresión *state-dependent* parsimoniosos, donde los predictores macroeconómicos interactúan con una variable de estado econmico basada en indicadores técnicos. Las combinaciones de previsiones propuestas otorgan estadística y económicamente ganancias *out-of-sample* con relación a la media histórica, modelos de regresión univariantes y la media de la combinación de previsiones macroeconómicas utilizando iguales pesos (EW). La combinación EW es generalmente aceptada en la literatura de prima de riesgo por no ser peor que la combinación de pronósticos utilizando pesos estimados. Sin embargo, dado el gran conjunto de variables macroeconómicas disponibles como posibles predictores, demostramos que la combinación parsimoniosa del método produce resultados prometedores para la predicción de la prima de riesgo.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1. Motivation

Modeling and predicting the mean, volatilities and conditional covariances and correlations of financial time series have been attracting increasing interest of researchers and practitioners in areas such as asset pricing, asset allocation, hedging strategies, option pricing and risk management. In particular, investors and banks wish to measure the risk (volatility) and the risk premia (expected mean in excess of risk free rate) to decide whether or not invest in a risky asset. They have been addressing three important decisions: choosing the best model, selecting the most important predictors and using the most efficient estimator.

Forecasting the equity risk premia have attracted a great deal of attention specially in asset allocation, which requires real-time forecasts of stock returns. Although the crescent interest for academics and practitioners and the plethora of variables proposed as predictors, stock return forecasting can be frustrating as argued by Rapach and Zhou (2012). Indeed, the unpredictable component of stock returns are much larger than the predictable part, so any forecasting model can explain only a relatively small part of stock returns. Furthermore, some authors argue that the stock return predictability is restricted to in-sample evaluations and support its unstable out-of-sample performance; see, for example, Bossaerts and Hillion (1999), Goyal and Welch (2003) and Butler *et al.* (2005).

Regarding the predictability of multivariate conditional volatility, the multivariate generalized autoregressive conditional heteroscedastic GARCH (MGARCH) models are the more traditional approach considered in literature since the extension, proposed by Bollerslev *et al.* (1988), of univariate GARCH models of Engle (1982) and Bollerslev (1986) to a multivariate framework.

Nowadays, MGARCH models plays a fundamental role in several financial implementations that require estimates of conditional variances, covariances and correlations of multivariate time series. The largest number of implementations of MGARCH models appear in the context of systems of financial returns; see, for example, Bollerslev *et al.* (1988), Attanasio (1991), De Santis and Gerard (1997), Hansson and Hordahl (1998), Lien and Tse (2002), Engle and Colacito (2006), Andersen *et al.* (2007), McNeil *et al.* (2010), Santos *et al.* (2012) and Santos *et al.* (2013), for a few selected asset pricing, portfolio selection, risk management and future hedging applications. The majority of fund managers use volatility and conditional correlation forecast to construct equity portfolios; see Amenc *et al.* (2012) for a survey based on 139 North American Investment managers. It is important to note that, depending on the particular application, the number of returns in the system can vary from being rather small to extremely large; see, for example, Bollerslev *et al.* (1988), Kavussanos and Visvikis (2004), Kawakatsu (2006) and Beirne *et al.* (2013) for small systems and Cappiello *et al.* (2006), Diebold and Yilmaz (2009), Santos *et al.* (2012), Santos *et al.* (2013) and Rombouts *et al.* (2014) for large systems. Besides financial applications, MGARCH models have also been fitted to systems of macroeconomic and commodity related variables. For example, Conrad and Karanasos (2010) explain the inflation-growth interaction fitting a generalized version of the constant conditional correlation (CCC) model, while Baillie and Myers (1991) estimate the optimal hedge ratios of commodity futures, Kavussanos and Visvikis (2004) analyze the interaction between spot and forward returns and volatilities in the shipping freight markets and Bampinas and Pangiotis (2015) fit vector autoregressive (VAR) models with MGARCH errors to explain the relationship between oil and gold prices. MGARCH models have also been implemented in agricultural economics; see Gardebroek and Hernandez (2013) and Haixia and Shiping (2013) for some references. Finally, MGARCH models are also useful when modeling and forecasting non-economic time series. For example, Cripps and Dunsmuir (2003) and Jeon and Taylor (2012) fit bivariate vector autoregressive moving average (VARMA) models with MGARCH errors to model the wind speed and direction.

1.2. Multivariate GARCH models

The VECH model of Bollerslev *et al.* (1988) is a direct generalization from the GARCH model. In this model, each conditional variance and covariance are function of all lagged conditional variances and covariances as well as lagged cross product of returns and lagged square returns.

Nevertheless, in practice, the estimation of the VECH model is limited due to two main limitations. First, due to the need of estimating a large number of parameters, its implementation was restricted to systems of small dimensions. Second, their parameters need to be restricted to guarantee covariance stationarity and positive definiteness of conditional covariance matrices. Consequently, the majority of popular MGARCH models implemented to represent the dynamic evolution of volatilities, covariances and correlations of real systems are restricted so that parameter estimation is feasible and to guarantee covariance stationarity and positiveness. The corresponding restrictions are often based on assuming that volatilities depend on their own past without interrelations neither among them nor with covariances. However, if these restrictions are not satisfied, the estimated conditional variances, covariances and correlations may suffer from strong biases; see Kroner and Ng (1998), Ledoit *et al.* (2003), Caporin and McAleer (2008), Rossi and Spazzini (2010) and Amado and Teräsvirta (2014) for consequences of misspecifying conditional variances, covariances and correlations. Engle (2009) argues that, although it seems important to allow for square and cross-products of one asset to help forecasting variances and covariances of other assets, in fact, there are few striking examples of these interrelations in the literature. On the contrary, several works conclude that allowing for interrelations among conditional variances and correlations may be important. First, empirical evidence on volatility feedbacks is plentiful; see Granger *et al.* (1986), Engle *et al.* (1990), Comte and Lieberman (2000), Hafner and Herwartz (2006, 2008b), Bai and Chen (2008), Diebold and Yilmaz (2009), Nakatani and Teräsvirta (2009), Conrad and Karanasos (2010), Beirne *et al.* (2013) and Aboura and Chevallier (2015) for studies related with volatility feedbacks. Therefore, it seems important to allow for volatility interactions when specifying MGARCH models. Second, it has often been found that volatilities and cross-correlations across assets move together over time and, consequently, they cannot be estimated separately. For example, Ramchand and Susmel (1998), Longin and Solnik (2001) and Okimoto (2008) find that cross-correlations between markets are higher during unstable periods when the markets are more volatile. Kasch and Caporin (2013) also describe correlations between conditional correlations and variances, while Bauwens and Otranto (2013) have a very complete description of the literature on volatility as a determinant of correlations. Finally, we did not find empirical evidence in the related literature about the influence of correlations on volatilities.

The main goals are to survey the main developments of parametric MGARCH models, updating previous surveys by Bauwens *et al.* (2006), Engle (2009) and Silvennoinen and Teräsvirta (2009a) and to analyze the potential biases incurred when the restricted models are fitted to systems with rich dynamics as those usually encountered in real data.

We contribute to literature by simulating a very general model, where all conditional variances, covariances and correlations are related with each other, and fitting the most popular models and estimation methods available in the literature to evaluate how much we lose in restricting the model and/or simplifying the estimation procedure. Our main finding is that the full BEKK (after Baba-Engle-Kraft-Kroner) model in the version usually consider in literature (with just one factor) restricts strongly the dynamics of the series and performs poorly compared with the other models, even with their restricted versions. The O-GARCH and GO-GARCH models also have bad performances both empirically and to the simulated data. Furthermore, the performance in fitting and forecasting volatilities, conditional covariances and correlations are similar for a given model regardless of the particular procedure used to estimate the parameters. Finally, the dynamic conditional correlation models estimated in three-steps is the best alternative model to explain the general model.

1.3. Predicting the mean of equity risk premium

Welch and Goyal (2008) conclude that several macroeconomic predictors fail to beat the simple historical average benchmark in out-of-sample exercises and recommend researchers to explore alternative variables and/or more sophisticated models to forecast equity premia. Subsequently, various studies have put forward alternative forecasting methods that provide statistical and economic evidence of out-of-sample predictability. Here we focus in two kinds of methods.

First, there is a growing body of literature indicating that asset returns follow a more complex process with more than one regime; see, for example, Turner *et al.* (1989), Garcia and Perron (1996), Perez-Quiros and Timmermann (2000), Ang and Bekaert (2002, 2007), Ang and Chen (2002), Guidolin and Timmermann (2006a,b) and Pettenuzzo and Timmermann (2011). Moreover, numerous others, such as Henkel *et al.* (2011), Dangl and Halling (2012), Gargano and Timmermann (2012) and Neely *et al.* (2014), find that numerous macroeconomic predictors tend to give stronger signals during recessions than expansions. Jacobsen *et al.* (2014) find that industrial metal prices are positive (negatively) related with future equity premia in recessions (expansions). From a theoretical standpoint, the intertemporal capital asset pricing model of Merton (1973) indicates that time-varying risk aversion may imply a time-varying dependence between future stock returns and macroeconomic variables. More recently, Bali (2008) proves that relative risk aversion coefficients are unstable, suggesting that predictability could be time varying. Intuitively, it is not hard to think that the level of investors' risk aversion increases from economic expansions to recessions (and vice versa) which would suggest that the predictive power of a given macroeconomic indicator is regime-sensitive or state-dependent.

Second, combining forecasts from different models offers a shield against model uncertainty and structural breaks, and allows parsimoniously to incorporate information from many variables. Although the equity premium predictability literature is rather ample, the notion of forecast combination has been devoted scant attention. To our best knowledge there are two exceptions: Rapach *et al.* (2010) who combine forecasts from simple linear regression models based on numerous macroeconomic variables, and Elliott *et al.* (2013) who further extend this work by averaging forecasts across complete subset regressions with the same number of predictive variables. The combining approaches of Rapach *et al.* (2010) and Elliott *et al.* (2013) are based either on equal weights or estimated weights. They conclude that the equal-weight (EW) forecast combination is at least as good as the combination based on estimated weights. However, a parallel literature suggests that pre-selecting a subset of "best" predictors among a large set of candidates is fruitful before deploying forecast combination methods given the large degree of co-movement between them. For example, Bai and Ng (2008), Dobreva and Schaumburg (2013) and Fuentes *et al.* (2015) combine variable selection with factor modeling and argue that *less is more* or that pooling the information from less (appropriately selected) variables can yield better forecasts. Following this lead, it is interesting using a sparse method to combine forecasts of equity premium.

Our objectives are to improve existing equity premium forecasts by considering combining forecasts from regime-switching predictive regressions and to propose new combination approaches as alternative to the EW combination.

We advocate the use of parsimonious regime-switching predictive regressions and extend equity-premium forecast strategies by combining regime-switching regression models with observable state, where well-known macroeconomic predictors are interacted with an economic state variable based on technical indicators. We demonstrate that technical indicators are reasonably good predictors (vis-a-vis the NBER dating approach) as proxies for the current state of economy and formulate simple regressions where the macroeconomic variable at hand is interacted with a technical indicator. Using conventional statistical and economic measures, we conclude that combining parsimonious state-dependent predictive regressions deliver robust out-of-sample forecasting gains compared to the historical average (HA) benchmark, and traditional linear predictive regressions based on macroeconomic or technical variables. We also propose a novel sparse EW forecast combination (SPAR hereafter) approach for equity premium prediction. The robust significance test of Kostakis *et al.* (2015) is employed in-sample to pre-select significant models (defined here by models whose slope coefficients are jointly significant). The combined out-of-sample forecast is then based on forecasts from significant models. Our investigation demonstrates empirically that the SPAR method is a good alternative to the EW combining approach which endorses from the lens of equity risk premium prediction the *less is more* view in the extant macroeconomic literature.

1.4. Organization

The rest of this thesis is organized as follows. Chapter 2 is an extension of Almeida *et al.* (2015) and analyzes the most popular MGARCH specifications often implemented in empirical applications when estimating and forecasting conditional variances, covariances and correlations. First, it describes the models, parameter estimators and their properties. Second, it compares their finite sample performance by carrying out Monte Carlo experiments to analyze the effect of restrictions on the estimation of conditional covariance matrices. Third, it carries out an empirical application to a five-dimensional system of daily exchange rate returns of the Euro (EUR), British Pound (GBP), Swiss Franc (CHF), Australian Dollar (AUD) and Japanese Yen (JPY) against the US Dollar (USD). Finally, it summarizes the main finds and conclusions. Chapter 3 describes the traditional econometric methodology to forecast the equity premium and proposes a new approach to construct regime-switching models to forecast equity premia. The performance of the new forecasting strategy is measured by estimating predictive regressions for Standard and Poor's (S&P) 500 equity premium. Finally, Chapter 4 concludes and gives directions for future projects.

Chapter 2

MGARCH models: Trade-off between feasibility and flexibility

2.1. Introduction

The original MGARCH model, proposed by Bollerslev *et al.* (1988) and denoted as VECH, is a direct generalization from its univariate counterpart. The VECH model is rather flexible allowing all volatilities and conditional covariances to be related with each other. However, in practice, the VECH model is difficult to be estimated given that the number of parameters can become excessively large even in systems of moderate size. Consequently, the early implementation of VECH models was originally restricted to systems with very few series. An additional problem faced when dealing with VECH models is the determination of the conditions to guarantee positive definite conditional covariance matrices and stationarity. To overcome these problems, numerous popular MGARCH models implemented in practice to represent the dynamic evolution of volatilities, covariances and correlations are restricted in a such a way that parameter estimation is feasible and/or they guarantee covariance stationarity and positiveness. The early restricted MGARCH models are the diagonal VECH (DVECH), suggested by Bollerslev *et al.* (1988), and the BEKK model of Engle and Kroner (1995), which are still rather popular in empirical applications; see, for example, Ledoit *et al.* (2003) and Bauwens *et al.* (2007) for the DVECH model and Beirne *et al.* (2013) for the BEKK model. Alternatively, several specifications of restricted MGARCH models are based on the decomposition of the covariance matrix into the product of conditional variances and correlations. Among these models, the most popular are the constant conditional correlation (CCC) model of Bollerslev (1990) and the dynamic conditional correlation (DCC) model of Engle

(2002); see, for example, Laurent *et al.* (2013) and Amado and Teräsvirta (2014) for some recent empirical applications of the CCC model and Engle and Kelly (2012), Bauwens *et al.* (2013), Aielli and Caporin (2014) and Audrino (2014) for the DCC model. However, if the restrictions behind these specifications are not satisfied, the estimated conditional variances, covariances and correlations may suffer from biases. Then, the restricted models may fail to represent the rich dynamics of real systems of financial returns. Furthermore, given that different models are based on different restrictions, the choice of a particular restricted MGARCH model can lead to substantially different conclusions when forecasting dynamic covariance matrices. However, in empirical applications, the particular specification fitted to the data is often chosen in an ad hoc basis with the easy of estimation being a primary factor affecting the selection of the model; see the discussion by Caporin and McAleer (2012). Consequently, in order to choose an appropriate restricted specification, it is important to analyze how the restrictions affect the estimation of conditional variances, covariances and correlations. Kroner and Ng (1998) and Ledoit *et al.* (2003) compare the DVECH, BEKK and CCC models estimating the conditional variances, covariances and correlations of systems with 2 and 7 series of financial returns, respectively, and conclude that they may have different economic implications. An empirical comparison of the scalar versions of the BEKK and DCC models has been carried out by Caporin and McAleer (2008) who found that both models are very similar in forecasting conditional variances, covariances and correlations. Latter, Caporin and McAleer (2012) compare the BEKK and DCC models from a theoretical point of view and conclude that the BEKK model could be used to obtain consistent estimates of the DCC model with a direct link to the indirect DCC model suggested in Caporin and McAleer (2008).[1]

Our objective is to analyze the potential biases incurred when the restricted models are fitted to systems with rich dynamics as those usually encountered in real data. We extend previous research by carrying out Monte Carlo experiments, including new models and very recently proposed parameter estimators. In this way, we fit the restricted models and parameter estimators to systems simulated by specifications without restrictions and analyze the empirical implications on estimated conditional variances, covariances and correlations of the restrictions imposed on MGARCH models to reduce the number of parameters and/or to guarantee covariance stationarity and/or positiveness. Our results have important implications for the empirical implementation of MGARCH models.

[1]Rossi and Spazzini (2010) and Caporin and McAleer (2014) carry out a Monte Carlo analysis to compare MGARCH models focusing on the interaction between the specification of the conditional covariance matrix and the conditional distribution of returns and on the effect of the dimension of the system, respectively.

The rest of this Chapter is organized as follows. Section 2.2 describes several popular MGARCH specifications often implemented in empirical applications. Section 2.3 compares their finite sample performance by carrying out Monte Carlo experiments to analyze the effect of restrictions on the estimation of conditional covariance and correlation matrices. An empirical application to a five-dimensional system of daily exchange rate returns of the EUR, GBP, CHF, AUD and JPY against the USD is carried out in Section 2.4. Finally, Section 2.5 concludes.

2.2. Multivariate GARCH models

This section describes several popular MGARCH models often implemented to represent the dynamic evolution of conditional variances, covariances and correlations of multivariate conditionally heteroscedastic time series.

2.2.1. The VECH model

Assuming zero conditional mean, the VECH model is given by[2]

$$\mathbf{r}_t = \mathbf{H}_t^{1/2}\varepsilon_t, \tag{2.1}$$

$$\text{vech}(\mathbf{H}_t) = \mathbf{C} + \mathbf{A}\,\text{vech}(\mathbf{r}_{t-1}\mathbf{r}'_{t-1}) + \mathbf{B}\,\text{vech}(\mathbf{H}_{t-1}), \tag{2.2}$$

where $\mathbf{r}_t$ is the $N \times 1$ vector of returns[3] observed at time t, for $t = 1, \dots, T$, $\mathbf{H}_t$ is the $N \times N$ conditional covariance matrix of $\mathbf{r}_t$ given past information until time $t-1$, $\mathbf{C}$ is an $N(N+1)/2$ vector of constants, $\mathbf{A}$ and $\mathbf{B}$ are square $N(N+1)/2$ parameter matrices and ε_t is a serially independent multivariate white noise process with covariance matrix $\mathbf{I}_N$, the identity matrix of order N. The operator vech$(\cdot)$ stacks the columns of the lower triangular part of a square matrix. Finally, $\mathbf{H}_1 = \mathbf{\Sigma} = E(\mathbf{r}_t\mathbf{r}'_t)$ is the unconditional covariance matrix of returns. The VECH model is covariance stationary if the moduli of the eigenvalues of $\mathbf{A} + \mathbf{B}$ are less than one; see Engle and Kroner (1995). Furthermore, Hafner (2003) derives analytical expressions of the fourth order moments of returns when the distribution of ε_t is spherical and Hafner and Preminger (2009b) establish sufficient conditions for geometric ergodicity. Although there are not known necessary conditions for

[2]For the sake of simplicity, in this thesis, we focus on the simplest specification which only includes one lag of past returns, conditional variances and covariances.

[3]We refer to the observed vector of time series as vector of returns. Nevertheless, it should be understood that we may also refer to the residuals of a multivariate VARMA model fitted to represent the conditional means when dealing with non-financial data.

the positivity of $\mathbf{H}_t$, Gourieroux (1997), Francq and Zakoan (2010) and Chrétien and Ortega (2014) discuss sufficient conditions. It is important to point out that although the restrictions are usually imposed on the matrices of parameters that govern the dynamic evolution of the conditional covariances, $\mathbf{A}$ and $\mathbf{B}$, very recently, Caporin and Paruolo (2015) propose a model with restrictions on the unconditional covariance matrix.

The VECH model in equations (2.1) and (2.2) is very flexible to represent symmetric responses of conditional variances and covariances to past squared returns and cross-products of returns. The most popular estimator of the parameters of the VECH model is Gaussian quasi-maximum likelihood (G-QML) based on maximizing the Gaussian log-likelihood function. If the conditional covariance matrix is well specified, Bollerslev and Wooldridge (1992) show that the G-QML estimator is consistent. Hafner and Preminger (2009b) establish consistency and asymptotic normality of the G-QML estimator under the existence of sixth-order moments; see also Hafner and Herwartz (2008a) who provide analytical expressions of the score and the Hessian. It is important to note that the dimension of the covariance matrix of the parameter estimator is at least of order N^2. Consequently, the asymptotic covariance matrix of the parameter estimator, computed as the average of the T outer-products of the score, will not be full rank for large N. This is a feature inherent to all MGARCH models and all their estimators; see Palandri (2009).

Given that conditional financial returns often exhibit fat tails and are skewed, a natural alternative to the G-QML estimator is based on maximizing the Student-ν likelihood, where ν is the degree of freedom; see Fiorentini *et al.* (2003), Hafner and Herwartz (2006) and Bai and Chen (2008). The corresponding estimator will be denoted by S-QML.[4]

Both QML estimators are computationally demanding and could be unfeasible when the dimension of the system is relatively large. Indeed, the log-likelihood function is nonlinear on the parameters and, in each iteration of its maximization algorithm, the matrix $\mathbf{H}_t$ needs to be inverted T times. Another difficulty in estimating VECH models is that their parameters need to be subjected to nonlinear constraints to ensure the existence of covariance stationary solutions and the positive semidefinite character of the conditional covariance matrices. Chrétien and Ortega (2014) solve the estimation problem by incorporating these non-linear constraints in an efficient and natural way, using a Bregman-proximal trust-region method. They fit the VECH model for

[4]Bauwens and Laurent (2005) further propose a multivariate skew-Student distribution. Other distributions used in MGARCH models are the fat-tailed multivariate Laplace of Rombouts *et al.* (2014) and the multiple degrees of freedom t of Serban *et al.* (2007); see Rossi and Spazzini (2010) for a comparison of the performance of these distributions associated with different MGARCH specifications. The asymptotic distribution of these non-Gaussian estimators has not been derived yet.

real systems of stock returns for dimensions up to eight and, with considerable computational effort, find a superior performance of the estimated VECH model in relation to other traditional parsimonious models.

The parameters of the VECH model can also be estimated using the two-step covariance targeting (CVT) procedure originally proposed by Engle and Mezrich (1996) in the context of univariate time series; see Caporin and McAleer (2012) for a general description. The CVT estimator can be implemented after rewriting equation (2.2) in terms of the unconditional covariance matrix as follows

$$\text{vech}(\mathbf{H}_t) = (\mathbf{I}_{\frac{N(N+1)}{2}} - \mathbf{A} - \mathbf{B})\text{vech}(\mathbf{\Sigma}) + \mathbf{A}\ \text{vech}(\mathbf{r}_{t-1}\mathbf{r}'_{t-1}) + \mathbf{B}\ \text{vech}(\mathbf{H}_{t-1}). \tag{2.3}$$

The estimation procedure is then divided into two steps. First, the unconditional covariance matrix, $\mathbf{\Sigma}$, is estimated by the sample unconditional covariance matrix of $\mathbf{r}_t$ and substituted in (2.3). Then, the remaining parameters are estimated by G-QML, conditional on the sample estimates of the unconditional covariances.

Recently, Sbrana and Poloni (2013) and Poloni and Sbrana (2014) propose two alternative consistent estimators of the parameters of the VECH model that can be used as starting values for alternative efficient estimators. Sbrana and Poloni (2013) propose a closed-form estimator based on the VARMA representation of the VECH model while Poloni and Sbrana (2014) propose a feasible generalized least squares type estimator.

As an illustration, we generate a bivariate system of size $T = 1000$ by a VECH model with Gaussian errors and the values of parameters used to generate the simulated systems reported in the Appendix A. The parameters are chosen so as to represent the conditional variances and covariances usually estimated when dealing with real data and to guarantee covariance stationarity and positiveness of the covariance matrices; see, for example, Bai and Chen (2008) and Conrad and Karanasos (2010).[5]

The first two rows of Figure 2.1 plot the simulated conditional standard deviations for each of the two variables in the system, while the last two rows plot the conditional covariances and correlations, respectively. It is important to point out that, although we estimate the parameters by G-QML[6] and by VT without restricting them to ensure positivity and covariance stationarity,

[5]All the programming used in this thesis has been developed by the author using MATLAB codes, except when explicitly mentioned.

[6]Note that, as the data generating process has Gaussian errors, the QML is in fact ML.

the estimated parameters satisfy the corresponding conditions. The estimated conditional standard deviations, covariances and correlations are also plotted in the first column of Figure 2.1, while the second column plots the simulated values (x-axis) versus the estimated values (y-axis) corresponding to the estimates obtained with the two procedures. In the second column plots, the errors can be measured as the Euclidean distance between the points and the identity line. Note that, as the fitted model is the true data generating process (DGP), the errors plotted in Figure 2.1 can be attributed to parameter estimation. This figure illustrates that the errors have means close to zero and relatively small dispersion. Furthermore, there are not large differences when estimating the parameters using G-QML or VT, but both procedures tend to overestimate the conditional standard deviations when they are small.

2.2.2. Restricted models for conditional covariance matrices

Diagonal VECH model

The DVECH model assumes that the matrices $\mathbf{A}$ and $\mathbf{B}$ in equation (2.2) are diagonal. Consequently, each conditional variance and covariance in the system has a univariate GARCH-type specification without allowing for feedbacks among volatilities and between volatilities and covariances.

Sufficient conditions to guarantee the positivity of the covariance matrices of the DVECH models have been derived by Ding and Engle (2001) and Ledoit *et al*. (2003). Later, Gourieroux (2007) derives necessary and sufficient conditions for the bivariate DVECH model and shows that the sufficient condition by Ding and Engle (2001) is also a necessary one. The extension of the necessary positivity conditions to systems with 3 or more variables is an open question. Finally, Ledoit *et al.* (2003) derive necessary conditions to ensure covariance stationarity.

Although G-QML estimation of the parameters of the DVECH model is easier than in the complete VECH model, Ledoit *et al.* (2003) argue that it is not computationally feasible for systems of dimension $N > 5$. The DVECH model still has too many parameters that interact in a complex way and, as a consequence, it is difficult to obtain convergence using existing optimization algorithms. Moreover, the estimation of the DVECH model does not necessarily yield positive semidefinite conditional covariance matrices. To solve these issues, Ledoit *et al.* (2003) propose estimating the DVECH model using a flexible two-step procedure, hereafter denoted by LSW (after Ledoit-Santa Clara-Wolf). In the first step, the volatilities are estimated by fitting univariate GARCH models and the conditional covariances by fitting bivariate GARCH models. These estimates do not necessarily yield positive conditional covariance matrices. Consequently, in the second step, the estimated parameters are transformed in such a way that they guarantee positive semi-definite conditional covariance matrices with the transformation being the least disruptive. Although the asymptotic distribution of this estimator is unknown, standard errors of the parameters can be obtained by bootstrapping.

Next, we illustrate the biases incurred when estimating conditional variances, covariances and correlations after fitting the DVECH model to the bivariate system generated by the DGP described in Appendix A. The pseudo-parameters of the DVECH model are estimated by the G-QML, VT and LSW procedures. The first column of Figure 2.2 plots the simulated and estimated conditional standard deviations, covariances and correlations, while in the second column are the scatter plots of the simulated versus the estimated values. We observe that, first, the estimates of the conditional standard deviations, covariances and correlations obtained when implementing the three alternative estimators considered are very similar. Second, as expected, the errors in the conditional standard deviations and covariances are larger than when the true VECH model is fitted. Finally, the conditional covariances are underestimated when they are large. Therefore, in the particular DGP considered in this illustration and for the particular time series generated, the restrictions imposed by the DVECH model have greater influence on the estimation of the conditional standard deviations and covariances than on conditional correlation estimates.

BEKK models

Engle and Kroner (1995) propose the BEKK model which guarantees positivity of the conditional covariance matrices.[7] The BEKK(1,1,K) is given by

$$\mathbf{H}_t = \mathbf{W} + \sum_{k=1}^{K} \mathbf{A}_k' \mathbf{r}_{t-1} \mathbf{r}_{t-1}' \mathbf{A}_k + \sum_{k=1}^{K} \mathbf{B}_k' \mathbf{H}_{t-1} \mathbf{B}_k, \tag{2.4}$$

where K determines the generality of the process and $\mathbf{W}$, $\mathbf{A}_k$ and $\mathbf{B}_k$ are square $N \times N$ parameter matrices, with $\mathbf{W}$ being a positive definite symmetric matrix. Denoting by $\otimes$ the Kronecker product of two matrices, the BEKK model is covariance stationary if and only if the modulus of the eigenvalues of $\sum_{k=1}^{K} \mathbf{A}_k \otimes \mathbf{A}_k + \sum_{k=1}^{K} \mathbf{B}_k \otimes \mathbf{B}_k$ are smaller than one; see Engle and Kroner

[7] Kawakatsu (2006) proposes an alternative model based on $\exp\{\mathbf{H}_t\}$ that also guarantees positivity. Other models that guarantee the symmetry and positive definiteness of the conditional covariance matrices are given in Tsay (2002) and Bai and Chen (2008), who consider a Cholesky decomposition of $\mathbf{H}_t$, and Hendrych and Cipra (in press), who propose a formulation based on the LDL decomposition. These models are not considered in this thesis as they are not as popular as the BEKK model.

(1995). The conditions for strict stationarity and geometric ergodicity can be found in Boussama *et al.* (2011).

Engle and Kroner (1995) show that all BEKK models are representable as VECH models and that the BEKK parametrization eliminates very few if any interesting model allowed by the VECH representation. Indeed, Scherrer and Ribarits (2007) and Stelzer (2008) show that, when $N = 2$, both specifications are equivalent. Nonetheless, when $N > 3$, the VECH model is more flexible than the BEKK model; see, for example, Stelzer (2008) who presents a three-dimensional VECH model with no BEKK representation. It is important to note that the equivalence between the BEKK and VECH models can be established when $K > 1$. However, the version of the BEKK model predominantly fitted in practice restricts $K = 1$;[8] see, for instance, Kroner and Ng (1998), Ledoit *et al.* (2003), Hafner and Herwartz (2006), Silvennoinen and Teräsvirta (2009a), Rossi and Spazzini (2010), Caporin and McAleer (2012), Laurent *et al.* (2012), Beirne *et al.* (2013), Pedersen and Rahbek (2014) and Burda (2015). In this case, the VECH and BEKK models are not equivalent even if $N = 2$.

The parameters of the BEKK models can be estimated by QML and CVT with the same computational problems described in subsection 2.2.1. Comte and Lieberman (2003) and Hafner and Preminger (2009b) prove consistency and asymptotic normality of the G-QML estimator under eighth and sixth finite moments of the observed variables, respectively. Avarucci *et al.* (2013) argue that finite fourth order moment restrictions for the G-QML estimator cannot be relaxed, even in the simple ARCH form of the BEKK model.[9] Very recently, Pedersen and Rahbek (2014) study the asymptotic properties of the CVT estimator and establish its strong consistency under finite second-order moments and asymptotic normality under finite sixth-order moments. Note that these conditions are identical to those of the G-QML estimator; see Hafner and Preminger (2009b). However, as pointed out by Caporin and McAleer (2012), when estimating the BEKK model by CVT, it is extremely complicated to impose positive definiteness and covariance stationary restrictions. Recently, Burda (2015) deals with this shortcoming and suggests an approach based on Constrained Hamiltonian Monte Carlo that solves both the nonlinear constraints resulting from BEKK targeting and the complex form of the BEKK likelihood in relatively large dimensions.

Alternatively, Noureldin *et al.* (2014) propose a rotated version of the BEKK model, denoted by rotated BEKK (RBEKK), based on fitting a covariance-targeting BEKK-type specification to the

[8]Hereafter, the BEKK model refers to the BEKK(1,1,1) model.

[9]Alternatively, Boudt and Croux (2010) show the good robustness properties of an M-estimator with a fat-tail Student-ν loss function.

rotated returns, $\mathbf{e}_t = \mathbf{\Sigma}^{-1/2}\mathbf{r}_t$, as follows

$$\mathbf{G}_t = \mathbf{I}_N - \mathbf{A}\mathbf{A}' - \mathbf{B}\mathbf{B}' + \mathbf{A}'\mathbf{e}_{t-1}\mathbf{e}'_{t-1}\mathbf{A} + \mathbf{B}'\,\mathbf{G}_{t-1}\mathbf{B}, \tag{2.5}$$

where $\mathbf{G}_t$ is the conditional covariance matrix of $\mathbf{e}_t$ with $\mathbf{G}_1 = \mathbf{I}_N$ and $\mathbf{A}$ and $\mathbf{B}$ are square $N \times N$ parameter matrices. The conditions for convariance stationary are obtained by writing (2.5) in the vectorized form and using the analysis in Engle and Kroner (1995); see Noureldin *et al.* (2014). The RBEKK model can be easily estimated by CVT, ensuring positiveness and covariance stationary. In the first step, $\mathbf{\Sigma}$ is estimated by the sample unconditional covariance matrix of $\mathbf{r}_t$, denoted by $\hat{\mathbf{\Sigma}}$. Using the spectral decomposition of this estimate, $\hat{\mathbf{e}}_t$ is computed. The second step is based on estimating the parameters in (2.5) by G-QML, conditional on $\hat{\mathbf{e}}_t$.

As mentioned above, even if K is restricted to be 1, fully parametrized BEKK models are only feasible for small systems. Two restricted popular versions of the BEKK model that further reduce the number of parameters are the diagonal BEKK (DBEKK) and the scalar BEKK (SBEKK) models, which are obtained by restricting the $\mathbf{A}_k$ and $\mathbf{B}_k$ matrices in (2.4) to be diagonal and scalar, respectively; see Ding and Engle (2001). The DBEKK model is representable as a DVECH model; see Bauwens *et al.* (2006). Consequently, the variances only depend on their own lags and past squared returns and the covariances only depend on their own lags and past cross products of returns. Noureldin *et al.* (2014) also study restricted versions of the RBEKK model denoted as diagonal RBEKK (D-RBEKK) and scalar RBEKK (S-RBEKK) models.

Consider again the same simulated system described in Appendix A. Figure 2.3 plots the estimated conditional standard deviations, covariances and correlations obtained after estimating the parameters of the BEKK model by G-QML and VT, while Figure 2.4 plots the same quantities after estimating the parameters of the DBEKK and SBEKK models by G-QML and of the D-RBEKK and S-RBEKK models by VT. For the sake of comparison, we also plot the simulated conditional standard deviations, covariances and correlations in both figures. Figure 2.3 shows that the BEKK model estimated by VT has slight larger errors than the BEKK model estimated by G-QML. However, the errors in the BEKK model are much larger than in the VECH model (see Figure 2.1) and, surprisingly, larger than in the DBEKK, SBEKK, D-RBEKK and S-RBEKK models plotted in Figure 2.4, which are restricted versions of the BEKK model. In order to find a possible explanation for it, we write the equations of the volatilities and conditional correlations for the bivariate BEKK, i.e.,

$$h_{11,t} = w_{11} + a_{11}^2 r_{1,t-1}^2 + 2a_{11}a_{12}r_{1,t-1}r_{2,t-1} + a_{12}^2 r_{2,t-1}^2 + b_{11}^2 h_{11,t-1} + 2b_{11}b_{12}h_{12,t-1} + b_{12}^2 h_{22,t-1}, \quad (2.6)$$

$$h_{12,t} = w_{12} + a_{11}a_{21}r_{1,t-1}^2 + (a_{11}a_{22} + a_{12}a_{21})r_{1,t-1}r_{2,t-1} + a_{12}a_{22}r_{2,t-1}^2 + b_{11}b_{21}h_{11,t-1} \quad (2.7)$$

$$+(b_{11}b_{22} + b_{12}b_{21})h_{12,t-1} + b_{12}b_{22}h_{22,t-1},$$

$$h_{22,t} = w_{22} + a_{21}^2 r_{1,t-1}^2 + 2a_{21}a_{22}r_{1,t-1}r_{2,t-1} + a_{22}^2 r_{2,t-1}^2 + b_{21}^2 h_{11,t-1} + 2b_{21}b_{22}h_{12,t-1} + b_{22}^2 h_{22,t-1}, \quad (2.8)$$

where w_{ij}, a_{ij} and b_{ij}, for $i, j = 1, 2$, are the elements of matrices **W**, **A** and **B**, respectively, in equation (2.4). Note that the non-linear restrictions in the parameters imposed by the BEKK model do not hold in practice for volatilities and conditional correlations. For example, in equation (2.6), the term that multiplies $r_{1,t-1}r_{2,t-1}$ is imposed to be the double of the geometric mean of the terms that multiply $r_{1,t-1}^2$ and $r_{2,t-1}^2$, respectively. Suppose that the terms that multiply $r_{1,t-1}^2$ and $r_{2,t-1}^2$ were corrected estimated, i.e., 0.097 and 0.022, respectively. It would imply, by the definition of the model, that the term that multiplies $r_{1,t-1}r_{2,t-1}$ is 0.093. However, in real data, we expect that the influence of $r_{1,t-1}r_{2,t-1}$ in $h_{11,t}$ is much smaller than the influence of $r_{1,t-1}^2$. Analogous arguments apply to the volatilities of the second series and conditional covariances. The restrictions imposed in BEKK model could be even worse than considering that the matrices in (2.4) are diagonal or scalar. Note that Rossi and Spazzini (2010) also find the counterintuitive result that the SBEKK model has a better performance than the less restrictive DBEKK model. Figure 2.4 shows that the errors of the estimated conditional correlations are larger than when estimating the true DGP, besides the fact that, when the conditional correlation are small, they are underestimated. However, the errors obtained when estimating conditional volatilities and covariances are similar in magnitude to those obtained when the true DGP is fitted. Furthermore, Figure 2.4 shows that the estimates of conditional standard deviations, covariances and correlations are very similar regardless of whether the DBEKK, SBEKK, D-RBEKK or S-RBEKK models are fitted.

2.2.3. Conditional correlation models

Instead of modeling directly the conditional covariance matrix, $\mathbf{H}_t$, several authors propose specifying it as the following product of conditional variances and correlations

$$\mathbf{H}_t = \text{diag}(\mathbf{H}_t)^{1/2}\mathbf{R}_t\text{diag}(\mathbf{H}_t)^{1/2}, \quad (2.9)$$

where $\text{diag}(\mathbf{H}_t) = \text{diag}(h_{11,t}, \cdots, h_{NN,t})$ is a diagonal matrix whose elements are the conditional variances of each series and $\mathbf{R}_t$ contains the conditional correlations.[10] In this way, assuming that conditional variances and correlations are not related, it is possible to simplify the estimation, estimating first the conditional variances and second the conditional correlations. This two-step (2s) estimation procedure is relatively simple allowing working with high dimensional systems. Carnero and Eratalay (2014) show that, if the innovations are Gaussian, estimating the parameters in multiple steps has a very similar performance to that of the QML estimator.

CCC models

Bollerslev (1990) introduces the CCC model, assuming that the conditional correlation matrix is constant over time, that is, $\mathbf{R}_t = \mathbf{R}$, where, $\mathbf{R}$ is a symmetric positive definite matrix. The matrices $\mathbf{H}_t$ are definite positive if and only if all the conditional variances $h_{ii,t}$, $i = 1, \cdots, N$, are positive and $\mathbf{R}$ is a positive definite matrix.

Allowing for feedback among volatilities, Jeantheau (1998) proposes an extension of the original CCC model, the extended CCC (ECCC) model, with volatilities fitted by a vector GARCH(1,1) model, as follows

$$\mathbf{h}_t = \mathbf{c} + \mathbf{A}\,\mathbf{r}_{t-1}^2 + \mathbf{B}\,\mathbf{h}_{t-1}, \tag{2.10}$$

where $\mathbf{c}$ is an N-dimensional vector, $\mathbf{A}$ and $\mathbf{B}$ are $N \times N$ parameter matrices, $\mathbf{r}_t^2 = (r_{1,t}^2 \dots r_{N,t}^2)'$ and $\mathbf{h}_t = (h_{11,t} \dots h_{NN,t})'$; see also Caporin (2007), Conrad and Karanasos (2010) and Francq and Zakian (in press). He and Teräsvirta (2004) give sufficient conditions for covariance and strictly stationarity, whereas Aue *et al.* (2009) establish a sufficient condition for strict stationarity and the existence of fourth-order moments. On the other hand, Nakatani and Teräsvirta (2008) establish sufficient positivity conditions, while Conrad and Karanasos (2010) provide less restrictive assumptions to ensure the positive definiteness of $\mathbf{H}_t$ and also show that there is a representation of the form (2.10) in which $\mathbf{B}$ is diagonal; Nakatani and Teräsvirta (2009) suggest a procedure for testing the hypothesis of a diagonal structure against the hypothesis of volatility feedbacks.

With respect to estimation of the ECCC parameters, Jeantheau (1998) proves the strong consistency of the G-QML estimator and Ling and McAleer (2003) prove its asymptotic normality. More recently and under mild conditions that coincide with the minimal ones in the univariate case, Francq and Zakoan (2012) establish the strong consistency and asymptotic normality of the

[10] As the original CCC and DCC models, this thesis assumes that each conditional variance follows a GARCH(1,1) model; see, for example, Audrino (2006) and Laurent *et al.* (2012) for a comparison of different specifications for individual volatilities, focusing on their impact on the accuracy of the conditional covariance estimates.

G-QML estimator. Specifically, although it is required strict stationarity, no moment assumption is made concerning the existence of moments of the observed process. Francq *et al.* (in press) establish the strong consistency and the asymptotic normality of the CVT estimator under finite fourth moments of the data generating process, whereas Pedersen (in press) derive the large-sample properties of this estimator when the distribution of the data generating process has infinite fourth moments. Finally, Francq and Zakoan (in press) estimate (2.10) imposing $\mathbf{B}$ to be diagonal so that volatilities can be estimated separately, i.e., using an equation-by-equation (EBE) estimator. They show strong consistency and asymptotic normality of the EBE estimator in a very general framework and prove consistency and joint asymptotic normality of the EBE volatility and conditional correlation matrix estimator of the ECCC model, which includes the original CCC model.[11]

The first column of Figure 2.5 plots the estimated conditional standard deviations, covariances and correlations obtained after fitting the CCC and ECCC models estimated EBE to the same simulated system described in Appendix A, while the second column plots the real values versus the estimated values considering alternative procedures. The errors in the conditional correlation estimates of the CCC and ECCC models are notably greater than the errors of the full model. However, when compared with the errors plotted in Figure 2.3, we can observe that the errors are smaller than when the BEKK model is fitted and similar to those plotted in Figure 2.4 for the DBEKK and SBEKK models. Furthermore, Figure 2.5 illustrates that the errors when fitting the CCC and ECCC models are similar when looking at conditional correlations, but the ECCC model have smaller errors than the CCC model for the conditional standard deviations of the second series and for the conditional covariances.

Conrad and Karanasos (2010) propose a further extension that also allows for negative feedback among volatilities and derive necessary and sufficient conditions for the positive definiteness of the covariance matrix. It is important to point out that Conrad and Karanasos (2010) claim that their results are also valid for models in which the correlations are time-varying, which will be considered latter in this thesis. Most of the results obtained by them are referred to bivariate models. Whether they can be generalized for large systems is an open question.

DCC models

Assuming constant conditional correlations is not reasonable in many practical situations. Consequently, several authors suggest models with time-varying conditional correlations. Be-

[11] Very recently, Chen (2015) propose an efficient method-of-moments based estimator of CCC models which is robust to the unknown conditional distribution of financial returns.

cause of its popularity, we focus on the (scalar) DCC model of Engle (2002) and its consistent correction by Aielli (2013), known as cDCC, which is given by

$$\mathbf{R}_t = \text{diag}(\mathbf{Q}_t)^{-1/2}\mathbf{Q}_t \, \text{diag}(\mathbf{Q}_t)^{-1/2}, \tag{2.11}$$

$$\mathbf{Q}_t = (1-a-b)\mathbf{S} + a \, \text{diag}(\mathbf{Q}_{t-1})^{1/2}\mathbf{u}_{t-1}\mathbf{u}'_{t-1}\text{diag}(\mathbf{Q}_{t-1})^{1/2} + b\mathbf{Q}_{t-1}, \tag{2.12}$$

where $\mathbf{Q}_t$ is an $N \times N$ positive definite matrix, $\mathbf{u}_t = (u_{1t}, \cdots, u_{Nt})'$ with $u_{it} = r_{it}/\sqrt{h_{iit}}$ being the standardized correlated returns, $\mathbf{S}$ is the unconditional covariance matrix of $\text{diag}(\mathbf{Q}_{t-1})^{1/2}\mathbf{u}_t$ and a and b are scalars. A sufficient condition for the positivity of $\mathbf{H}_t$ is that all conditional variances are positive, $a, b > 0$ and $a + b < 1$. Aielli (2013) also proves that if $\mathbf{S}$ is positive definite and $a + b < 1$, the correlated and the standardized return processes are strictly and covariance stationary.

The parameters of the cDCC model can be estimated by the three-step (3s) estimator described by Aielli (2013); see also Caporin and McAleer (2012) for a description of available asymptotic results on the estimation of the parameters of DCC models. As argued by Franq and Zakoan (in press), the asymptotic properties of the three-step estimator are an open issue. Although the three-step estimator of the cDCC model overcomes the bias problem in the parameter $\mathbf{S}$, this estimator is still downward biased in high dimensions.[12] Noureldin *et al.* (2014) also apply the rotation technique to the DCC model of Engle (2002), resulting in the rotated DCC (RDCC) model.

We also consider the extended DCC (EDCC) model, where volatilities are given by equation (2.10), assuming diagonal $\mathbf{B}$ and the conditional correlations are given by equations (2.11) and (2.12). This model can also be estimated by the 3s estimator, with volatilities estimated by the EBE estimator; see Franq and Zakoan (in press).

Next, we illustrate the performance of the cDCC and RDCC models when fitted to the simulated system described in the Appendix A. The first column of Figure 2.5 plots, together with the simulated conditional standard deviations, covariances and correlations and the estimates of the CCC and ECCC models, the corresponding estimates obtained after fitting the cDCC and RDCC models, while the second column plots the real values versus the estimated values. Note that the conditional standard deviations of the CCC, cDCC and RDCC models are exactly the same given that they are estimated separately in the first step. The cDCC and RDCC models seem to have slightly larger errors than the VECH model, but lower errors than the CCC and ECCC mod-

[12] Pakel *et al.* (2014) propose the composite likelihood estimator, allowing to estimate models even when the cross-sectional dimension is larger than the sample size. Alternatively, Hafner and Reznikova (2012) suggest a reduction of the bias by using shrinkage techniques applied to the sample covariance matrix of the standardized residuals.

els. Also, the RDCC and cDCC models tend to underestimate the correlations when they are large and underestimate when they are small. On the other hand, the estimated conditional covariances of the cDCC and RDCC models seem to be robust to the misspecification.

Equation (2.12) imposes a common dynamic structure for all conditional correlations, governed by the parameters a and b. This might not be realistic when pairwise correlations between different returns have different behaviors. To avoid this constraint, several generalizations of the DCC model of Engle (2002) have been proposed. For example, Billio *et al.* (2006) introduce a block-diagonal structure, where the dynamics are restricted to be equal only among certain groups of variables, and a BEKK structure on the conditional correlations is proposed. Bauwens *el al.* (in press) estimate the cDCC model in (2.12) when a and b are replaced by matrices using a Bregman-proximal trust-region method and conclude empirically that the use of richly parametrized models have better performance than the scalar case. Hafner and Franses (2009) also extend the DCC model by allowing the parameters to vary across assets and Otranto (2010) proposes a clustering algorithm to identify similar structures of correlation dynamics in the DCC models. Finally, several authors propose different short- and long-run sources that affect correlations; see, for example, Colacito *et al.* (2011), Rangel and Engle (2012), Audrino and Trojani (2011) and Audrino (2014). Also, Silvennoinen and Teräsvirta (2009b, 2015) propose the smooth transition conditional correlation (STCC) model allowing the correlations to vary smoothly between two different states.

The cDCC model has been extended by Bauwens and Otranto (2013) to include volatility as determinant of correlations by introducing measures of volatility as exogenous variables. On the other hand, Palandri (2009) proposes breaking the conditional correlation matrices into the product of a sequence of matrices in such a way that they preserve positive definiteness without imposing constraints on the parameters. The sequential conditional correlations (SCC) model separates the correlations and partial correlations, allowing for a multi-step estimation procedure. Consequently, very complex optimization problems are converted into a series of mere univariate and bivariate estimations, which enables working with very high dimensional systems and at the same time complex functional forms for the conditional correlation process. However, the SCC model still assumes that variances and correlations are not related between them. Finally, Boudt *et al.* (2013) propose a robust extension of the model, known as BIP-cDCC model, for forecasting correlations in the presence of one-off events which cause large changes in prices whilst not affecting the volatility dynamics. They apply the new model to daily exchange rate returns of the EUR and JPY against the USD and conclude that the BIP-cDCC model is always better or have similar performance in relation to the cDCC model when forecasting future covariance matrices at different forecast horizons.

2.2.4. Factor models

There is a growing literature on multivariate conditionally heteroscedastic factor models; see Engle *et al.* (1990), Alexander and Chibumba (1997) and van der Weide (2002) for early references. In the (G)O-GARCH model, each series in the system is generated by an orthogonal transformation of m ($m \leq N$) univariate GARCH-type processes, the unobserved factors. The O-GARCH(1,1,m) model of Alexander (2001) is given by

$$\mathbf{\Sigma}^{-1/2}\mathbf{r}_t = \mathbf{\Lambda}_m \mathbf{f}_t, \tag{2.13}$$

where the $\mathbf{\Lambda}_m$ is a $N \times m$ matrix given by $\mathbf{\Lambda}_m = \mathbf{P}_m \mathbf{L}_m^{1/2}$, with $\mathbf{L}_m = \text{diag}(l_1 \cdots, l_m)$ such that $l_1 \geq \cdots \geq l_m > 0$ are the m largest eigenvalues of the unconditional correlation matrix of $\mathbf{e}_t = \mathbf{\Sigma}^{-1/2}\mathbf{r}_t$ and $\mathbf{P}_m$ is an orthogonal matrix of their associated eigenvectors. The vector $\mathbf{f}_t = (f_{1t}, \cdots, f_{mt})'$ is a random process with **0** conditional mean and diagonal conditional variance matrix, $\mathbf{G}_t$, with each element satisfying

$$g_{ii,t}^2 = (1 - \alpha_i - \beta_i) + \alpha_i f_{i,t-1}^2 + \beta_i g_{ii,t-1}^2, \tag{2.14}$$

for $i = 1, \cdots, m$. Hereafter, we denote O-GARCH as the O-GARCH(1,1,N) model as it is the most popular specification; see, for example, Noureldin *et al.* (2014).

In the GO-GARCH of Boswijk and van der Weide (2011), the singular value decomposition of the matrix $\mathbf{\Lambda}_N$ is used as a parametrization, i.e. $\mathbf{\Lambda}_N = \mathbf{P}_N \mathbf{L}_N^{1/2}\mathbf{U}(\delta)$, where $\mathbf{U}(\delta)$ is parameterized by a $N(N-1)/2$-dimensional vector, δ, whose elements are rotation angles such that $-180 \leq \delta_j \leq 180$. The O-GARCH model corresponds to the particular case $\mathbf{U}(\delta) = \mathbf{I}_N$.

The estimation and asymptotic theory of the O-GARCH and GO-GARCH models is described in Boswijk and van der Weide (2011) and Hafner and Preminger (2009a), respectively.

2.3. Monte Carlo simulation

In this section, we carry out Monte Carlo experiments to analyze the finite sample properties of the estimated conditional variances, covariances and correlations obtained after fitting restricted specifications to systems in which all variances and covariances are interrelated with each other.

We also compare alternative methods to estimate the pseudo-parameters in the restricted specifications considered.

We simulate 500 replicates of sizes $T = 1000$ and 2000 by the VECH model in equations (2.1) and (2.2) with $N = 2$ and 5 and Gaussian and Student-7 errors. The values of the parameters used to generate the simulated systems are reported in the Appendix A. When dealing with the bivariate system, we fit all the restricted models described in section 2 plus the VECH model which is the true DGP. The parameters of each model are estimated using the alternative available estimators considered. In this way, we can conclude about how the estimates of the conditional variances, covariances and correlations are affected by the restrictions imposed by each model, as well as the differences among alternative parameter estimators implemented to the same model. Table 2.1 summarizes the models fitted, the estimators considered and the parameter restrictions imposed in the bivariate case to ensure covariance stationarity and positivity. When dealing with the system with $N = 5$, the parameters of the VECH model are difficult to be estimated. In this case, we only fit the restricted models and estimate the parameters using those estimators which are feasible.

Table 2.1: Restrictions imposed in the parameters

Model	Estimators	Stationarity	Positivity
VECH	G(S)-QML, CVT	—	—
DVECH	G(S)-QML, CVT	—	—
	LSW	LSW estimation	LSW estimation
BEKK	G-QML, S-QML	—	model
	CVT	—	—
DBEKK	G-QML, S-QML	$a_i^2 + b_i^2 < 1, 0 < a_i, b_i < 1, \mathbf{C} > 0$ in (2.4)[1]	model
	CVT	—	—
SBEKK	G-QML, S-QML	$a^2 + b^2 < 1, 0 < a, b < 1, \mathbf{C} > 0$ in (2.4)[2]	model
	CVT	—	—
D-RBEKK	CVT, S-CVT	$a_i^2 + b_i^2 < 1, 0 < a_i, b_i < 1$ in (2.5)[1]	$a_i^2 + b_i^2 < 1, 0 < a_i, b_i < 1$ in (2.5)[1]
S-RBEKK	CVT, S-CVT	$a^2 + b^2 < 1, 0 < a, b < 1$ in (2.5)[2]	$a^2 + b^2 < 1, 0 < a, b < 1$ in (2.5)[2]
O-GARCH	G-QML	$\alpha_i + \beta_i < 1, 0 < \alpha_i, \beta_i < 1$ in (2.14)	$\alpha_i + \beta_i < 1, 0 < \alpha_i, \beta_i < 1$ in (2.14)
GO-GARGH	G-QML	$\alpha_i + \beta_i < 1, 0 < \alpha_i, \beta_i < 1$ in (2.14)	$\alpha_i + \beta_i < 1, 0 < \alpha_i, \beta_i < 1$ in (2.14)
CCC	G-2s, S-2s	usual restrictions to fit GARCH	usual restrictions to fit GARCH
ECCC	G-2s, S-2s	usual restrictions to fit aug-GARCH	usual restrictions to fit aug-GARCH
cDCC	G-3s, S-3s	$a + b < 1, 0 < a, b < 1$ in (2.12) and [3]	$a + b < 1, 0 < a, b < 1$ in (2.12) and [3]
RDCC	G-3s, S-3s	$a + b < 1, 0 < a, b < 1$ in (2.12) and [3]	$a + b < 1, 0 < a, b < 1$ in (2.12) and [3]
EDCC	G-3s, S-3s	$a + b < 1, 0 < a, b < 1$ in (2.12) and [4]	$a + b < 1, 0 < a, b < 1$ in (2.12) and [4]

Notes: Summary of restrictions imposed when maximizing the bivariate likelihood to ensure covariance stationarity (second row) and positivity (third row). *model* means that the positivity is ensured by the model parametrization and — means that we do not restrict the parameters.
[1] where a_i and b_i, $i = 1, 2$, are the diagonal elements of the $\mathbf{A}$ and $\mathbf{B}$.
[2] where a and b are given when the $\mathbf{A}$ and $\mathbf{B}$ matrices are replaced by these scalars.
[3] the usual restrictions to estimate the GARCH(1,1) model in the first step.
[4] the usual restrictions to estimate the augmented GARCH(1,1) model in the first step.

For each replicate and estimator considered, the performance of the estimated conditional covariance and correlation matrices is measured by the following Frobenius norms

$$LF_1 = \frac{\sum_{t=1}^{T} \mathrm{Tr}[(\hat{\mathbf{H}}_t - \mathbf{H}_t)'(\hat{\mathbf{H}}_t - \mathbf{H}_t)]}{T}, \quad LF_2 = \frac{\sum_{t=1}^{T} \mathrm{Tr}[(\hat{\mathbf{R}}_t - \mathbf{R}_t)'(\hat{\mathbf{R}}_t - \mathbf{R}_t)]}{T}, \tag{2.15}$$

where $\hat{\mathbf{H}}_t$ and $\hat{\mathbf{R}}_t$ are the estimated conditional covariance and correlation matrices at time t; see Laurent *et al.* (2013) for a comprehensive list of different loss functions and their impacts on ranking forecasting performances of MGARCH models.

2.3.1. Bivariate case

Table 2.2 reports the number of replicates of the bivariate system in which stationarity and positivity of covariances matrices are not empirically satisfied. We consider that a fitted model is not positive for some replicate if at least one of the conditional covariance matrices is not positive defined. On the other hand, we verify whether the parameter estimates satisfy the sufficient restrictions to ensure covariance stationarity. Note that, the estimated parameters of all rotated models, the (G)O-GARCH model and the models based on representing conditional correlations always satisfy the stationarity and positivity restrictions, as they are imposed in the estimation process. Moving on to the results of the VECH and BEKK models, the stationarity conditions are violated in a relatively large number of replicates when the parameters are estimated by QML. The number of violations of covariance stationarity increases when the DGP is the Student-7 VECH model, regardless of whether the Gaussian or the Student likelihoods are maximized. Still, the number of covariance stationarity violations decreases with the sample size. On the other hand, when the parameters of the VECH and BEKK models are estimated by CVT, the estimates always satisfy the covariance stationarity. However, the covariance matrix estimates by CVT are not positive in a relative large number of replicates, while QML estimates are positive in the majority of the cases. Finally, when the DVECH, DBEKK or SBEKK models are fitted, we can observe that, regardless of the estimator, they always satisfy covariance stationarity and positivity restrictions, except in some cases when the DGP is the Student-7 VECH model and the DVECH model is estimated by G-QML or S-QML. It is important to emphasize that although the conditional covariances of the BEKK, DBEKK and SBEKK models are positive by definition, when they are estimated by CVT without restrictions, positivity is not ensured; see Caporin and McAleer (2012).

Table 2.2 also reports the average computer time involved in the estimation. In each estimation that does not converge for an initial value, we try alternative initial values until it converges; see

Table 2.2: Summary of bivariate Monte Carlo simulations

		Stationarity				Positivity				Computer time			
Model	Estimator	Gaussian		Student-7		Gaussian		Student-7		Gaussian		Student-7	
		T_1	T_2	T_1	T_2	T_1	T_2	T_1	T_2	T_1	T_2	T_1	T_2
VECH	G-QML	15	5	65	30	2	0	5	0	28.8	57.2	31.4	58.4
	S-QML	19	6	50	18	2	0	6	0	25.6	49.1	55.2	104.7
	CVT	0	0	0	0	41	31	85	67	20.5	40.7	22.7	41.8
DVECH	G-QML	0	0	21	3	0	0	0	0	9.4	20.3	11.8	21.6
	S-QML	0	0	13	2	0	0	0	0	20.8	41.8	25.2	48.3
	CVT	0	0	0	0	0	0	0	0	7.8	17.2	7.5	18.4
	LSW		—					—		0.27	0.43	0.26	0.39
BEKK	G-QML	15	9	96	57			—		17.2	34.7	17.0	30.5
	S-QML	13	8	60	37			—		24.3	51.5	28.3	54.5
	CVT	0	0	0	0	19	12	47	32	11.5	22.7	13.2	23.5
DBEKK	G-QML		—					—		3.7	7.7	4.3	8.4
	S-QML		—					—		6.2	12.5	9.5	17.2
	CVT	0	0	0	0	0	0	0	0	3.6	4.8	3.0	5.3
SBEKK	G-QML		—					—		2.7	6.2	2.7	5.8
	S-QML		—					—		3.4	7.3	4.7	8.9
	CVT	0	0	0	0	0	0	0	0	1.3	2.7	1.6	3.1
D-RBEKK	CVT		—					—		1.6	3.1	1.6	3.0
	S-CVT		—					—		3.2	6.6	4.2	7.8
S-RBEKK	CVT		—					—		0.80	1.6	0.82	1.5
	S-CVT		—					—		1.6	3.3	1.7	3.1
O-GARCH	G-QML		—					—		1.3	2.3	1.3	2.4
GO-GARGH	G-QML		—					—		3.1	5.9	3.1	5.9
CCC	G-2s		—					—		0.23	0.33	0.23	0.34
	S-2s		—					—		1.1	1.3	0.74	1.3
ECCC	G-2s		—					—		1.2	2.1	1.3	2.3
	S-2s		—					—		6.1	11.0	7.3	12.1
cDCC	G-3s		—					—		1.1	1.7	1.2	1.8
	S-3s		—					—		3.2	4.9	3.0	5.3
RDCC	G-3s		—					—		1.2	2.0	1.3	2.1
	S-3s		—					—		3.9	5.7	3.7	6.4
EDCC	G-3s		—					—		2.1	3.3	2.2	3.9
	S-3s		—					—		8.6	13.9	16.3	16.5

Notes: The Monte Carlo simulations are carried out considering four different DGP: Gaussian VECH and Student-7 VECH models and $T_1 = 1000$ and $T_2 = 2000$. It is considered 500 replications for each DGP. From the third to sixth and from the seventh to tenth columns are the number of cases where the fitted model is not stationary and not positive, respectively. *model* means that the positivity is ensured by the model parametrization. From the eleventh to fourteenth columns are the mean time in seconds required to estimate the model.

Asai (2013) and Chrétien and Ortega (2014) for methods to choose initial values for the BEKK and VECH models, respectively. It is obvious that CVT is faster than G-QML and S-QML with G-QML being faster than S-QML. In the DVECH model, the LSW estimator is even faster. When comparing the DBEKK and SBEKK models with their rotated versions, we can observe that the computer time involved in the estimation of the rotated cases is lower and, in addition, they ensure positivity and stationarity by imposing linear constraints in the estimation. Therefore, it seems that it is worth using the rotated BEKK models rather than non-rotated BEKK models.

Table 2.3 reports the Monte Carlo averages of LF_1 and LF_2 of the replicates that satisfy positivity and covariance stationarity. We can see that the errors have similar values when the true DGP is fitted by QML and CVT, with CVT being, in general, slightly worse estimating correlations and better estimating covariances. When the DGP is the Student-7 VECH model, the VECH estimated by S-QML is slightly better than the VECH estimated by G-QML or by CVT. Moving on to the restricted misspecified models, we observe that, for a given model, different estimators lead to similar results. Therefore, there are not large differences when alternative parameter estimators are implemented to the same model specification.

Table 2.3: Summary of the results of bivariate Monte Carlo simulations based on the LF statistics

Model	Estimator	Gaussian				Student-7			
		$T = 1000$		$T = 2000$		$T = 1000$		$T = 2000$	
		LF_1	LF_2	LF_1	LF_2	LF_1	LF_2	LF_1	LF_2
VECH	G-QML	3.00	**0.72**	2.53	**0.32**	3.24	0.66	3.07	0.32
	S-QML	3.03	0.72	2.55	0.32	**2.73**	**0.61**	**2.62**	**0.30**
	CVT	**2.75**	0.79	**2.47**	0.38	2.81	0.75	3.04	0.56
DVECH	G-QML	5.24	0.91	6.70	0.76	4.97	0.89	6.97	0.75
	S-QML	5.25	0.91	6.73	0.76	4.77	0.80	6.91	0.68
	CVT	5.50	1.08	6.40	0.95	5.00	1.05	6.43	0.92
	LSW	5.82	2.34	6.45	2.15	5.12	2.20	7.06	2.42
BEKK	G-QML	8.78	3.13	8.93	3.10	8.28	3.29	10.0	3.20
	S-QML	9.17	3.07	8.88	3.01	7.77	2.92	9.21	2.74
	CVT	8.20	3.16	8.89	3.03	7.82	3.37	10.4	3.19
DBEKK	G-QML	5.75	2.27	7.10	2.50	5.34	2.20	7.09	2.39
	S-QML	5.70	2.19	7.01	2.38	4.75	1.95	6.29	2.09
	CVT	5.21	2.31	6.67	2.51	4.67	2.31	6.44	2.55
SBEKK	G-QML	5.22	2.52	6.66	2.66	4.32	2.38	6.37	2.52
	S-QML	5.16	2.42	6.56	2.55	4.04	2.08	5.82	2.19
	CVT	4.75	2.69	6.29	2.83	3.81	2.72	5.67	2.88
D-RBEKK	G-CVT	5.06	2.57	6.56	2.75	4.35	2.61	6.36	2.82
	S-CVT	5.06	2.55	6.55	2.74	4.22	2.55	6.03	2.75
S-RBEKK	G-CVT	4.75	2.69	6.29	2.83	3.81	2.72	5.67	2.88
	S-CVT	4.77	2.67	6.29	2.81	3.80	2.65	5.62	2.81
O-GARCH	G-QML	7.52	3.13	9.86	3.22	6.74	3.06	9.83	3.17
GO-GARCH	G-QML	8.76	2.34	11.0	2.32	6.87	2.68	9.97	2.89
CCC	G-2s	5.00	1.74	5.98	1.76	4.56	1.86	6.58	1.92
	S-2s	5.09	1.74	6.06	1.76	4.28	1.85	6.08	1.91
ECCC	G-2s	5.44	1.74	4.08	1.76	5.05	1.85	4.85	1.91
	S-2s	4.77	1.74	3.86	1.76	3.83	1.85	3.94	1.91
cDCC	G-3s	4.05	1.01	4.69	0.88	3.99	1.22	5.47	1.15
	S-3s	4.13	1.01	4.75	0.87	3.69	1.20	4.85	1.11
RDCC	G-3s	4.04	1.00	4.69	0.87	3.99	1.20	5.46	1.13
	S-3s	4.10	0.99	4.76	0.86	3.67	1.17	4.88	1.10
EDCC	G-3s	3.79	1.00	2.90	0.85	4.50	1.22	3.99	1.14
	S-3s	3.93	0.99	2.68	0.84	3.26	1.18	2.92	1.10

Notes: This Table reports the means of the LF statistics for the conditional covariances (1) and correlations (2) matrices through the Monte Carlo replicates whose all of the models give only positive volatilities and ensure stationarity. These statistics are evaluated after fitting alternative bivariate MGARCH models by different estimation procedures when the DGP is the Gaussian VECH and Student-7 VECH models with paramaters given in subsection 2.2.1, considering sample sizes $T = 1000$ and 2000. After the exclusions of non-positive and non-stationary models, it remains 459, 475, 365 and 422 replicates for Gaussian with $T = 1000$, Gaussian with $T = 2000$, Student-7 with $T = 1000$ and Student-7 with $T = 2000$, respectively. The LF_1 and LF_2 statistics are in the scale $\times 10^{-9}$ and $\times 10^{-2}$, respectively. In bold, it is the best model.

Hereafter, we focus on comparing different restricted models. First, we observe that although the DVECH model has a LF_1 statistic much larger than when the true VECH model is fitted, this difference is not so large when estimating correlations. We also note that increasing the sample size from $T = 1000$ to 2000, reduces the LF_2 when the true model is fitted but not necessarily when the DVECH model if fitted. Second, the errors corresponding to the BEKK model are much larger compared to the DVECH model. However, when fitting the more restrictive DBEKK model, the results are improved with respect to its full version. Surprisingly, when the more restrictive SBEKK model is fitted, the results are even better for conditional covariance matrices. Note that Rossi and Spazzini (2010) also find the counterintuitive result that the SBEKK model has a better performance than the less restrictive DBEKK model. We also note that the performance of the DVECH model is similar to the restricted BEKK-type models when estimating covariances and better when estimating correlations. Third, the rotation of BEKK-type models only improves marginally the estimated covariance matrices of the DBEKK and SBEKK models. However, the computer time involved in the estimation of the rotated cases is lower and the rotated models ensure positivity of covariance matrices and covariance stationarity; see Table 2.1. Finally, when the O-GARCH and GO-GARCH models are fitted, the restrictions imposed in these models generate estimates of conditional variances, covariances and correlations with larger distances than those of the DVECH or restricted BEKK-type models.

Turning now to the results of the models that represent the dynamics of conditional correlations, we can see that the errors corresponding to the cDCC and RDCC models are very similar between them and similar, in terms of correlations, to the DVECH model. On the other hand, the dynamic correlation models are much better than the DVECH model and BEKK-type models in terms of covariances. When estimating covariances, the ECCC and cDCC models are comparable and the difference with respect to the true DGP is 50%. When estimating correlations, the EDCC, cDCC and RDCC models remarkably outperform the constant correlation models. Finally, the EDCC is better than the cDCC and RDCC in relation to LF_1 statistic.

To summarize the results, Figure 2.6 presents 95% non-parametric confidence intervals of the rank average of the each fitted model, where the rank for each replicate is equal to 1 for the best fitted model according to LF_1 (or LF_2) and equal to 32 for the worst one. Once again, we see that the ranks are very similar when the parameters of a particular model are estimated by different estimators and that the ranks of the BEKK, CCC and O-GARCH models are always worse than those of the DCC models.

As conclusion, for the set of parameters considered in the simulation, the restrictions imposed by a misspecified model are much more relevant than the choice of the estimation method. Moreover, all alternative models have notably inferior performances in relation to the VECH model, specially when $T = 2000$, and among the alternative models, the cDCC, RDCC and EDCC models have, in general, a superior performance. Indeed, Laurent *et al.* (2012) conclude empirically that it is very difficult to outperform the DCC model after comparing 125 models fitted to forecast the correlations of a system of 10 assets from the New York Stock Exchange. Caporin and McAleer (2014) also support empirically the preference for correlation models over covariance models when analyzing different cross-sectional dimensions from 5 up to 89 assets.

2.3.2. Five-dimensional case

Following the same procedure of the previous subsection, we simulate a five-dimensional VECH model, where all volatilities, conditional covariances and correlations are related with other. The VECH and BEKK models and the DVECH and DBEKK models estimated by QML are not considered as their estimation is unfeasible when $N = 5$. All the other fitted models are covariance stationary and positive. Table 2.4 reports the Monte Carlo averages of LF_1 and LF_2. The conclusions are very similar to those of bivariate case. In particular, the O-GARCH and GO-GARCH models are not good and the dynamic conditional correlation models outperform the rest of the models. Regardless of the sample size and error distribution, the LF_1 and LF_2 distances are minimum when the EDCC model if fitted. Figure 2.7 presents the 95% confidence intervals of the ranks of the alternative models and estimators through the Monte Carlo replicates. Once more, the models based on correlations are ranked best in relation the those based in covariances. In particular, the DCC models have better performance than the CCC models, with the EDCC model being the winner.

2.4. Empirical application

In this section, we compare empirically the performance of several MGARCH models fitted to a system of five exchange rate returns, namely EUR, GBP, CHF, AUD and JPY against the USD. The data are daily closing exchange rates observed at 12:00 AM (New York time) from January 2, 2004 to December 31, 2013, with a total of 2582 daily observations. We define the exchange rate returns as usual by $r_{i,t} = 100 \times \log(y_{i,t}/y_{i,t-1})$, $i = 1, 2, \cdots, 5$, where $y_{i,t}$ is the daily exchange rate

Table 2.4: Summary of the results of five-dimensional Monte Carlo simulations based on the LF statistics

Model	Estimator	Gaussian				Student-7			
		$T = 1000$		$T = 2000$		$T = 1000$		$T = 2000$	
		LF_1	LF_2	LF_1	LF_2	LF_1	LF_2	LF_1	LF_2
DVECH	LSW	1.03	2.15	1.29	2.04	2.18	2.54	2.59	2.60
DBEKK	CVT	1.16	1.93	1.47	2.05	2.39	2.21	2.97	2.38
SBEKK	G-QML	1.49	2.16	1.63	2.26	3.50	2.42	3.53	2.56
	S-QML	1.35	1.93	1.55	2.02	3.21	1.94	3.02	2.04
	CVT	1.17	2.10	1.50	2.21	2.38	2.38	3.01	2.53
D-RBEKK	G-CVT	1.16	2.03	1.46	2.14	2.40	2.33	2.98	2.48
	S-CVT	1.15	1.93	1.46	2.03	2.38	2.14	2.95	2.26
S-RBEKK	G-CVT	1.17	2.10	1.50	2.21	2.38	2.38	3.01	2.53
	S-CVT	1.17	2.00	1.51	2.10	2.42	2.19	3.05	2.31
O-GARCH	G-QML	3.56	15.8	4.25	15.33	6.46	17.5	7.94	17.4
GO-GARCH	G-QML	2.04	7.04	2.68	6.43	3.85	7.71	5.07	7.66
CCC	G-2s	0.78	1.81	0.98	1.75	1.78	1.97	2.31	1.95
	S-2s	0.79	1.80	0.98	1.74	1.67	1.93	2.28	1.92
ECCC	G-2s	0.68	1.79	0.66	1.73	1.34	1.92	1.36	1.91
	S-2s	0.59	1.78	0.62	1.73	1.15	1.91	1.17	1.90
cDCC	G-3s	0.55	0.77	0.67	0.67	1.46	1.03	1.87	0.96
	S-3s	0.55	0.77	0.68	0.67	1.35	1.02	1.83	0.96
RDCC	G-3s	0.54	0.76	0.67	0.67	1.45	1.02	1.86	0.96
	S-3s	0.54	0.76	0.67	0.67	1.34	1.01	1.82	0.95
EDCC	G-3s	0.45	0.74	0.36	0.65	1.06	0.98	0.89	0.92
	S-3s	**0.38**	**0.73**	**0.31**	**0.64**	**0.87**	**0.97**	**0.69**	**0.91**

Notes: This Table reports the means through the Monte Carlo replicates of the LF statistics for the conditional covariances (1) and correlations (2) matrices after fitting alternative MGARCH models by different estimation procedures when the DGP is the five-dimensional Gaussian VECH and Student-7 VECH models, considering sample sizes $T = 1000$ and 2000 and 500 replications. The LF_1 and LF_2 statistics are in the scale $\times 10^{-7}$ and $\times 10^{-1}$, respectively. In bold, it is the best model.

of the i-th series at time t. Figure 2.8 plots the returns for the full sample period, which is split into estimation in-sample and forecast out-of-sample periods. The out-of-sample period spans from January 2, 2013 to December 31, 2013 with $H = 258$ forecasts. Table 2.5 reports descriptive statistics and the 20-lag Ljung-Box statistics for returns ($Q(20)$) and squared returns ($Q_2(20)$) for the overall, in-sample and out-of-sample periods. The traditional features of returns like approximately zero mean, skewness and excess of kurtosis are present in all the currencies and for most periods. According to the Ljung-Box statistics, the returns and squared returns are significantly autocorrelated for the overall and in-sample periods. Although the Ljung-Box statistic for serial correlation of returns is significant, an analysis of the sample autocorrelation functions shows that the magnitudes of the correlations are very small, and generally not significant in the first two lags and multiple of five lags. Consequently, we fit MGARCH models without any dependence in the conditional mean. As an illustration, Figure 2.9 plots the estimated pairwise conditional correlation between GBP and CHF and between AUD and JPY after fitting the R-DBEKK and cDCC models. We can observe that the correlations estimated by these models could be rather different in some particular periods.

Table 2.5: Descriptive statistics of daily returns of the series considered in the application

Currency	Mean	Min	Max	Var	Skew	Kurt	$Q(20)$	$Q_2(20)$
Panel A: Full sample, January 2, 2004 – December 31, 2013								
EUR	0.0034	-3.84	4.62	0.41	0.123*	3.054***	16.9	39.2***
GBP	-0.0030	-3.92	4.47	0.40	-0.079	4.181***	77.0***	1359.6***
CHF	0.0128	-8.48	5.45	0.50	-0.514***	11.11***	36.5**	45.3***
AUD	0.0067	-8.83	6.70	0.86	-0.950***	12.35***	46.6***	2610.5***
JPY	0.0004	-5.17	3.81	0.46	0.072	4.270***	42.7***	738.1***
Panel B: In-sample, January 2, 2004 – December 31, 2012								
EUR	0.0019	-3.84	4.62	0.43	0.139*	2.987***	19.8	37.5**
GBP	-0.0042	-3.92	4.47	0.42	-0.072	4.103***	75.6***	1194.3***
CHF	0.0130	-8.48	5.45	0.52	-0.541***	11.28***	34.3**	46.4***
AUD	0.0138	-8.83	6.70	0.91	-0.955***	12.07***	49.5***	2353.6***
JPY	0.0089	-5.17	3.81	0.44	0.098	4.698***	54.3***	778.4***
Panel C: Out-of-sample, January 2, 2013 – December 31, 2013								
EUR	0.0171	-1.72	1.45	0.23	-0.187	0.963**	40.7***	26.9
GBP	0.0073	-1.27	1.36	0.21	-0.180	0.301	17.6	19.7
CHF	0.0112	-1.74	1.73	0.31	0.097	0.609*	33.1**	12.2
AUD	-0.0577	-3.77	1.52	0.40	-0.802***	4.118***	24.6	19.4
JPY	-0.0756	-3.42	2.81	0.59	-0.008	1.717***	13.3	15.5

Notes: This Table reports daily summarizing statistics for the returns of Euro (EUR), British Pound (GBP), Swiss Franc (CHF), Australian Dollar (AUD) and Japanese Yen (JPY) against the US Dollar (USD): mean, minimum (Min), maximum (Max), variance (Var), skewness (Skew), excess of kurtosis (Kurt) and Ljung-Box for returns ($Q(20)$) and squared returns ($Q_2(20)$). The top panel corresponds to the full sample period; the second panel to the in-sample period; and the third panel to the out-of-sample period. *, ** and *** mean significant at 10%, 5% and 1% levels for the skewness, excess of kurtosis and Ljung-Box asymptotic tests.

The out-of-sample forecasts are evaluated using the rolling scheme procedure of Giacomini and White (2006) and described as follows. For each model and estimation method, we denote by W_1 the first window which incorporates the first T_1 observations and by $\hat{\theta}_1$ the corresponding parameter estimates, and compute h-step-ahead forecasts for $h = 1, 5, 20$. For the next four windows, W_2 to W_5, we add a new observation, one by one, and use the estimate $\hat{\theta}_1$ from the previous window, i.e., $\hat{\theta}_5 = \cdots = \hat{\theta}_2 = \hat{\theta}_1$ to compute the forecasts. For window W_6, we add the $(T_1+5)-th$ observation and drop the first five observations to obtain the new parameter estimate, $\hat{\theta}_6$. For the next four windows, we repeat the same procedure used for windows W_2 to W_5. Then, we repeat this whole procedure until the last window W_{258}.

In order to compare the out-of-sample forecasts, we evaluate the out-of-sample negative log-likelihood (NL), used by Audrino (2014), which is given by

$$NL(h) = -\sum_{i=0}^{H-h} \log[L(\mathbf{r}_{T+i+h}; \hat{\theta}_{i+1}, \hat{\mathbf{H}}_{T+i}(h))], \tag{2.16}$$

where $L(.;.)$ is the conditional likelihood, which can be Gaussian or Student, and $\hat{\mathbf{H}}_{T+i}(h)$ is the h-step-ahead forecast of $\mathbf{H}_{T+i+h}$ for $h = 1, 5, 20$. The prediction $\hat{\mathbf{H}}_{T+i}(h)$ is obtained using observations up to $T+i$ and $\hat{\theta}_{i+1}$. This method is reasonable to compare the predictive accuracy of models since the parameters of the models are estimated in-sample using the same functions. Furthermore, the NL has the strength of not considering any proxies for the unobservable covariance matrices.

The comparison of models is also carried out using the following Frobenius loss functions

$$LF_1(h) = \frac{\sum_{i=0}^{H-h} \mathrm{Tr}[(\mathbf{H}_{T+i+h} - \hat{\mathbf{H}}_{T+i}(h))'(\mathbf{H}_{T+i+h} - \hat{\mathbf{H}}_{T+i}(h))]}{H-h+1}, \tag{2.17}$$

$$LF_2(h) = \frac{\sum_{i=0}^{H-h} \mathrm{Tr}[(\mathbf{R}_{T+i+h} - \hat{\mathbf{R}}_{T+i}(h))'(\mathbf{R}_{T+i+h} - \hat{\mathbf{R}}_{T+i}(h))]}{H-h+1}. \tag{2.18}$$

As the true covariance and correlation matrices, $\mathbf{H}_t$ and $\mathbf{R}_t$, respectively, are unobservable, we use the realized variances, covariances and correlations as a proxy; see Patton and Sheppard (2009) for the value of high-precision proxies for the evaluation of volatilities and conditional correlation forecasts. It is widely recognized that the estimation of realized covariances and correlations suffers from asynchronous trading and market microstructure noise, causing the covariance and correlation estimators to be biased and inconsistent; see, for instance, McAleer and Medeiros (2008), Patton (2011) and Corsi and Audrino (2012). We sample the intraday returns into 288 five-minute intervals[13] to avoid the asynchronous effect and compute the realized covariance by the realized outlyingness weighted covariance (rOWCov) of Boudt *et al.* (2011).[14]

Finally, we perform the superior predictive ability (SPA) test of Hansen (2005) and the model confidence set (MCS) of Hansen *et al.* (2011) to verify whether the performance of alternative models and methods are significantly different according to the NL and LF criteria.[15] The first test allows for multiple comparison against a pre-specified benchmark model, with the null hypothesis being that each model is not outperformed by at least one of the other competing models.

[13] We also compute the realized covariances using ten-minute and thirty-minute intervals with similar results.
[14] The proxies are estimated in R by the package *highfrequency*.
[15] We compute the SPA test and MSC using the Sheppard's MFE Toolbox package, written in MATLAB, by considering 10,000 bootstrap replications and block length of 6. The results do not change when we change the block length to 3 or 9. The MSC is constructed using the R method within the MFE Toolbox package.

The second one chooses from an initial set, a subset of forecasts that outperforms all the other alternatives.

Table 2.6 reports the observed NL, LF_1 and LF_2 loss functions, the corresponding p-values of the SPA tests and the MCS with $\alpha = 10\%$ significance level for the different models and estimation methods considered. According to the NL criteria, the models estimated by maximizing the Student likelihood have notably better performance than models estimated by maximizing the Gaussian likelihood for the three forecasting horizons. Indeed, with the exception of the CCC model, all the models estimated by Student likelihood are not significantly outperformed in the SPA test at 5% confidence level, while all the models estimated by Gaussian likelihood are outperformed for one- and twenty-step-ahead. According to LF_1 and LF_2 loss functions, the models estimated by Student likelihood have in general a better performance in relation to the models estimated by Gaussian likelihood. However, the difference between the estimation methods are not as large as in the case of NL.

The full-sample period is also split into two other in-sample and out-of-sample periods, such that the resulting second and third out-of-sample periods are from January 2, 2008 to December 31, 2013, with $H = 1550$; and from January 2, 2009 to December 31, 2013, with $H = 1291$, respesctively. In the second split, the estimation period is a low volatility period and has low heteroscedasticity, whereas the forecasting period has extreme market conditions and large heteroscedasticity. In the last split, both the in-sample and out-of-sample periods have periods of low and high volatilities. Table A.1 reports the performance of the alternative models and estimators considered when implemented to forecast volatilities, covariances and correlations during the out-of-sample periods corresponding to the second and third splits. Given that we do not have high frequency data for these periods, Table A.1 only reports the results corresponding to the out-of-sample negative log-likelihood. Once again, the SBEKK model and the DCC models estimated by maximizing the Student likelihood are the best models.

When looking at the results for NL, we can observe that the SBEKK and the three DCC models estimated by maximizing the Student likelihood are included in the MCS for all horizons. However, when looking at the LF_1 loss for covariances and LF_2 loss for correlations, the BEKK-type models are not included in the MCS and only the DCC, RDCC and EDCC models are selected. Noureldin *et al.* (2014) also conclude that the RDCC is better than the RBEKK, O-GARCH and GO-GARCH models. Finally, Francq and Zakoan (in press) also reject empirically the BEKK model.

2.5. Concluding remarks

In this Chapter we discuss the main strengths and limitations of the most popular symmetric multivariate GARCH models available in the literature. A simulation study is carried out to compare different restricted specifications and estimators, when the series are generated by the general VECH model. The DCC models explain adequately the evolution of volatilities, conditional covariances and correlations generated by the general model. However, the BEKK, O-GARCH and GO-GARCH fail to estimate conditional covariances and correlations. We emphasize that although the BEKK(1,1,K) is as general as the VECH model in the bivariate case, it is strongly restricted when $K = 1$, as usually considered in empirical applications. In this way, it is advisable to fit the restricted DBEKK or SBEKK models, or their rotated versions. We conclude that the performance of the forecasts of volatilities, conditional covariances and correlations are similar for a given model regardless of the particular procedure used to estimate the parameters. Therefore, we advise choosing the estimation method based on computational advantages. An empirical application to a five-dimensional system of exchange rates returns is carried out. We conclude that the SBEKK model estimated by S-QML and the DCC models estimated by S-QML have a reasonable forecast performance according to the negative log-likelihood criterion, while the three DCC models outperform all the other models according to the LF criteria.

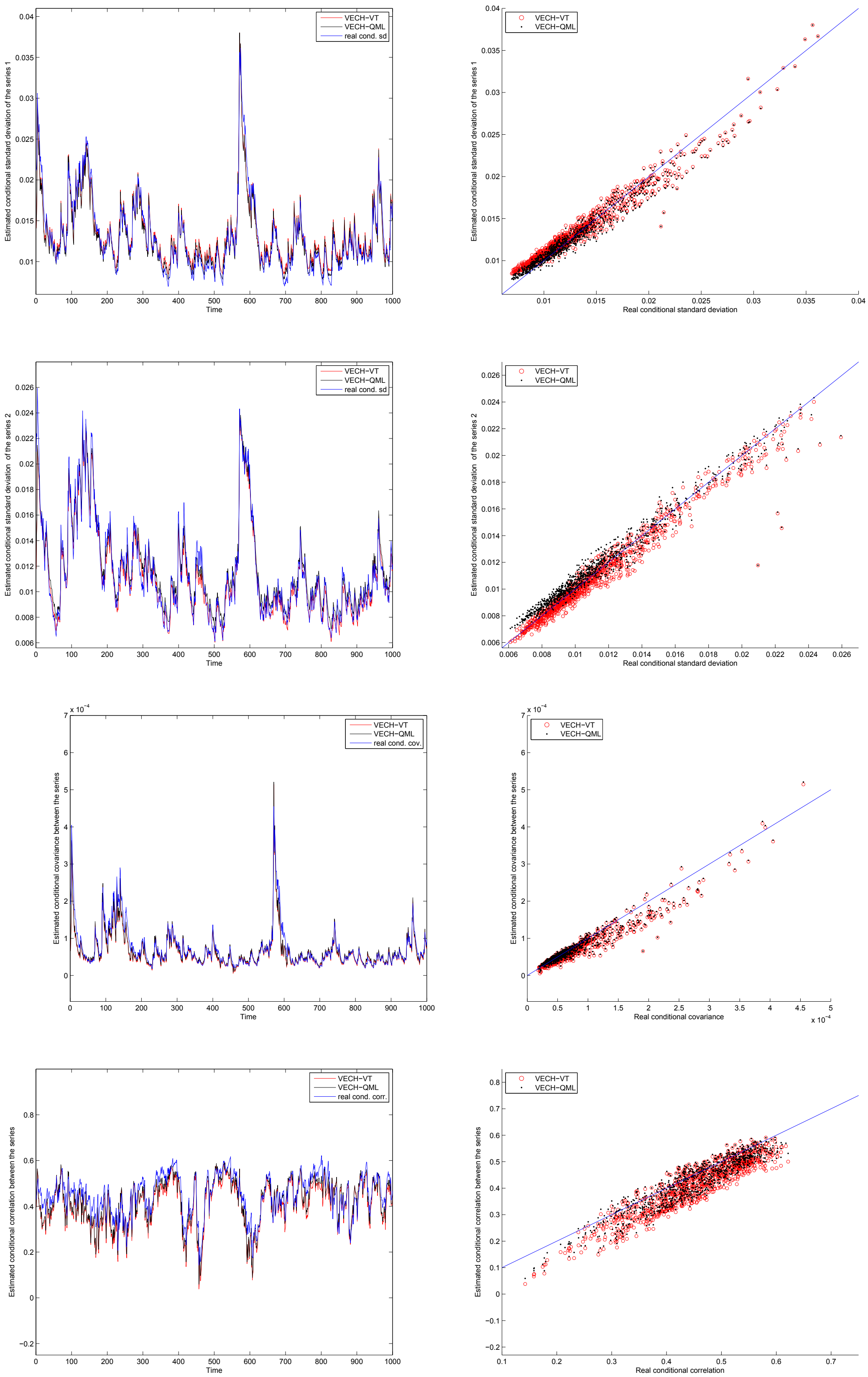


Figure 2.1: Plot of estimated conditional covariances and correlations of the VECH model

First column: simulated conditional standard deviations (first two rows), covariances (third row) and correlations (fourth row) of the VECH series together with the corresponding estimates obtained after fitting the VECH model by QML and VT. Second column: corresponding simulated vs fitted values.

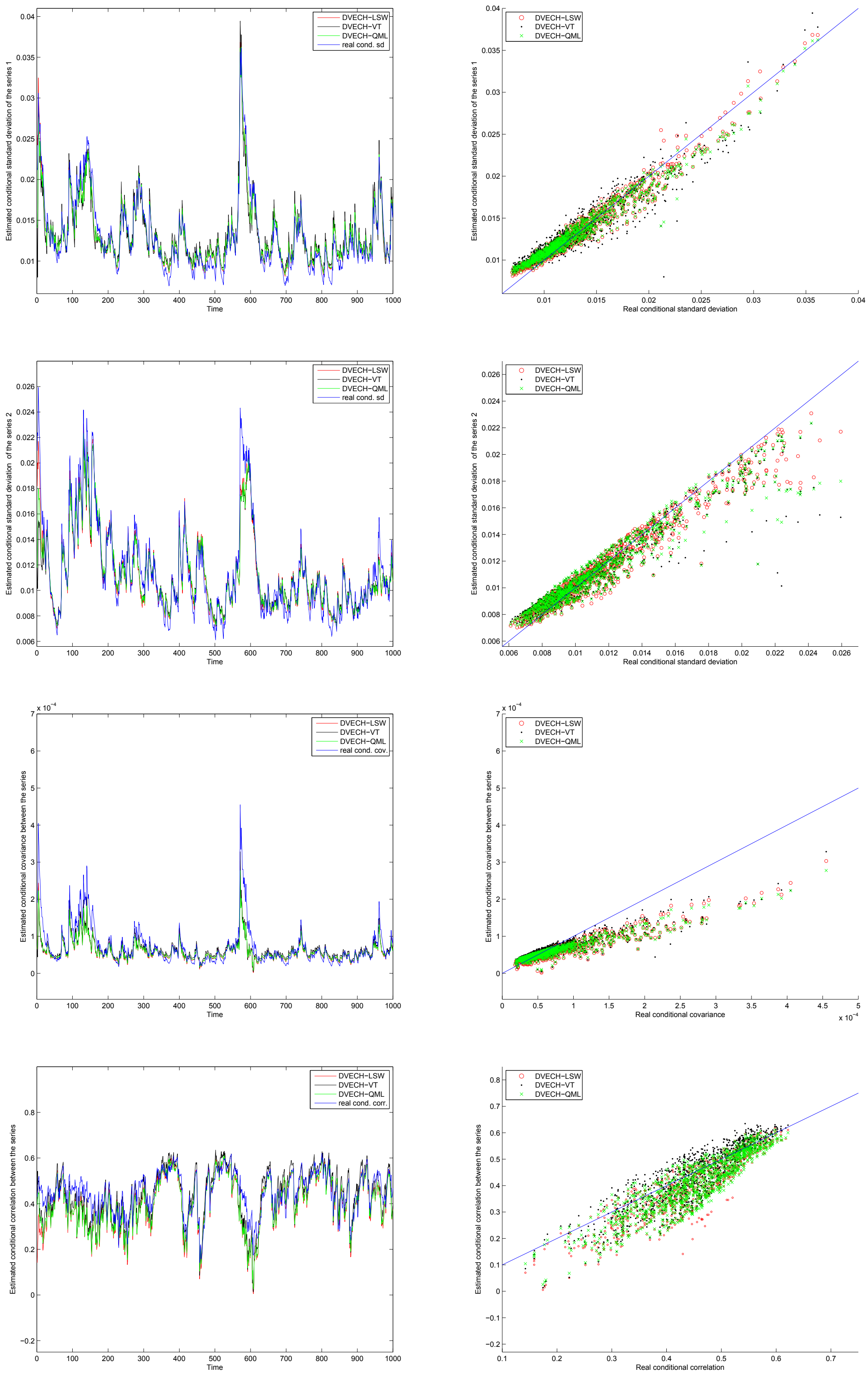


Figure 2.2: Plot of estimated conditional covariances and correlations of the DVECH model

First column: simulated conditional standard deviations (first two rows), covariances (third row) and correlations (fourth row) of the VECH series together with the corresponding estimates obtained after fitting the DVECH model by QML, VT and LSW. Second column: corresponding simulated vs fitted values.

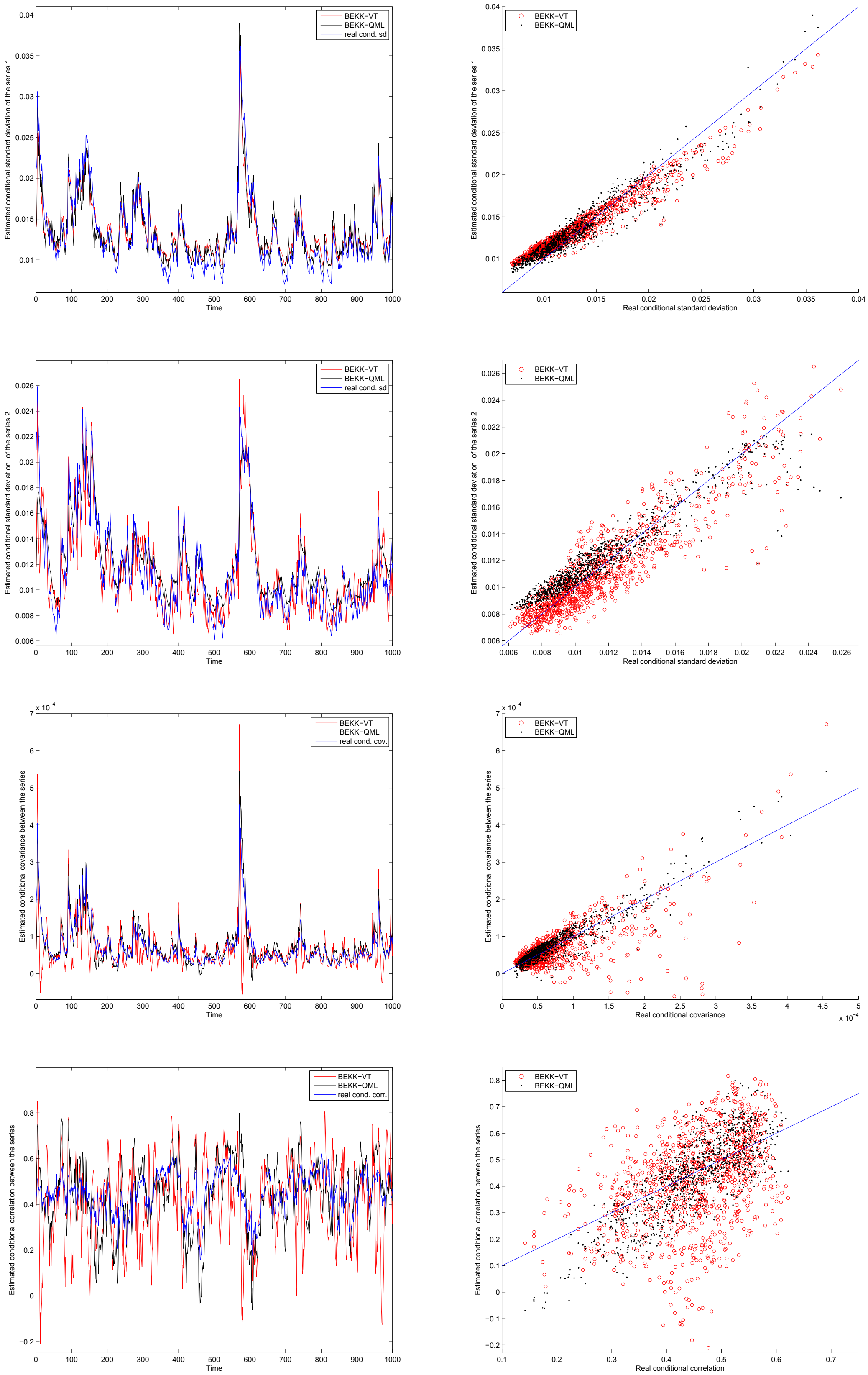


Figure 2.3: Plot of estimated conditional covariances and correlations of the BEKK model

First column: simulated conditional standard deviations (first two rows), covariances (third row) and correlations (fourth row) of the VECH series together with the corresponding estimates obtained after fitting the BEKK by QML and VT. Second column: corresponding simulated vs fitted values.

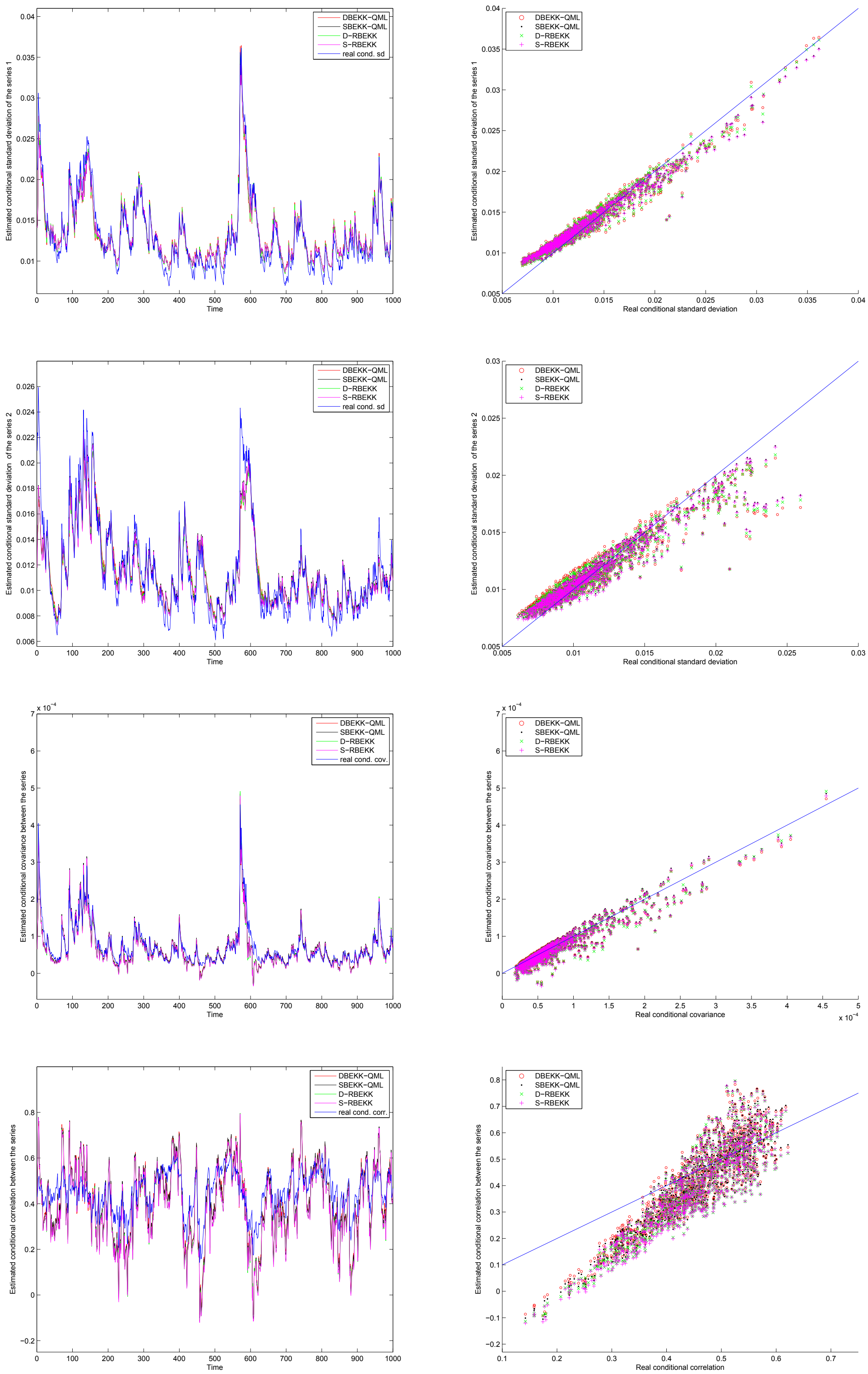


Figure 2.4: Plot of estimated conditional covariances and correlations of the DBEKK and SBEKK models

First column: simulated conditional standard deviations (first two rows), covariances (third row) and correlations (fourth row) of the VECH series together with the corresponding estimates obtained after fitting the DBEKK, SBEKK and D-RBEKK and S-RBEKK models. Second column: corresponding simulated vs fitted values.

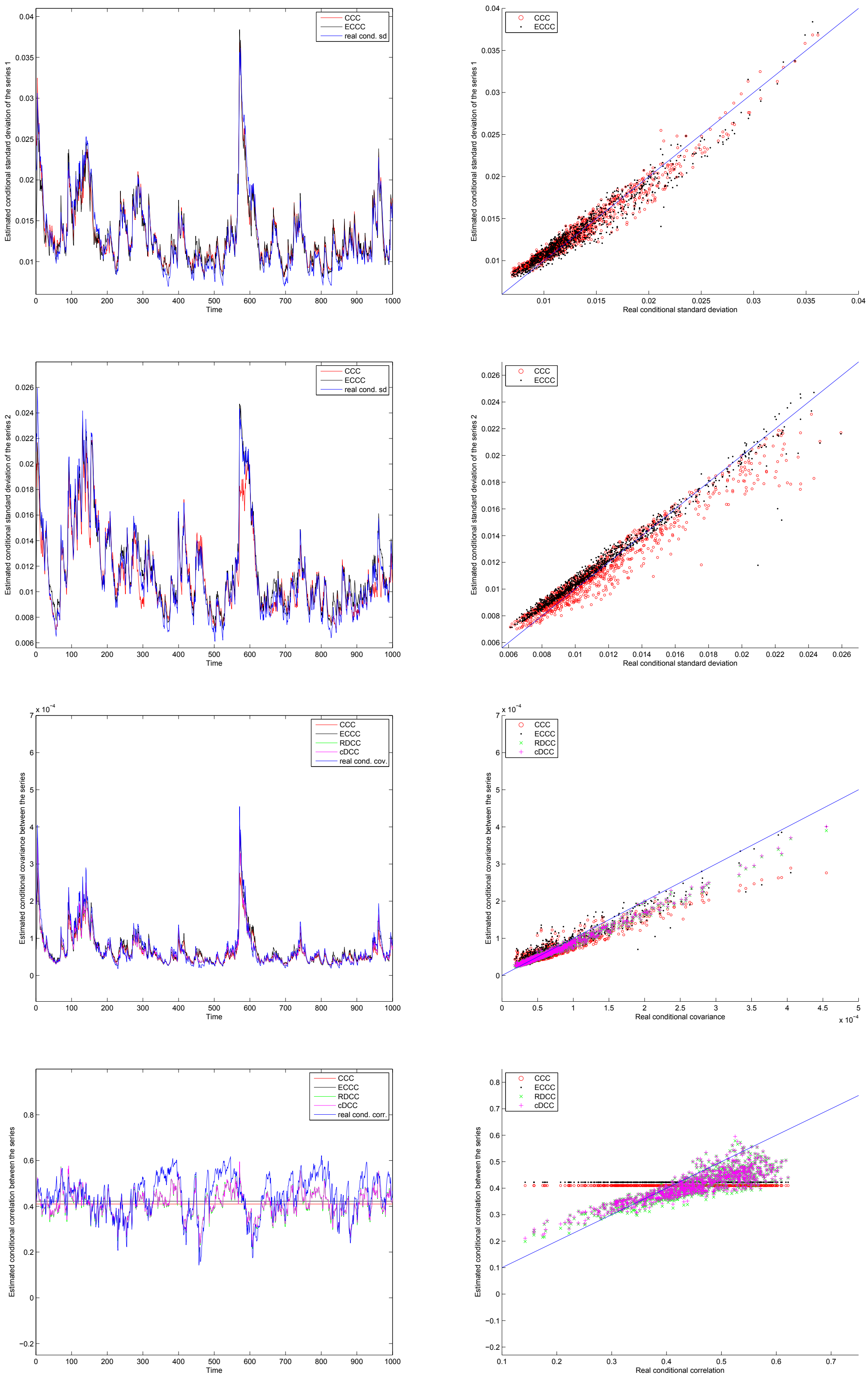


Figure 2.5: Plot of estimated conditional covariances and correlations of the CCC and DCC models

First column: simulated conditional standard deviations (first two rows), covariances (third row) and correlations (fourth row) of the VECH series together with the corresponding estimates obtained after fitting the CCC, ECCC, cDCC and RDCC models. Second column: corresponding simulated vs fitted values.

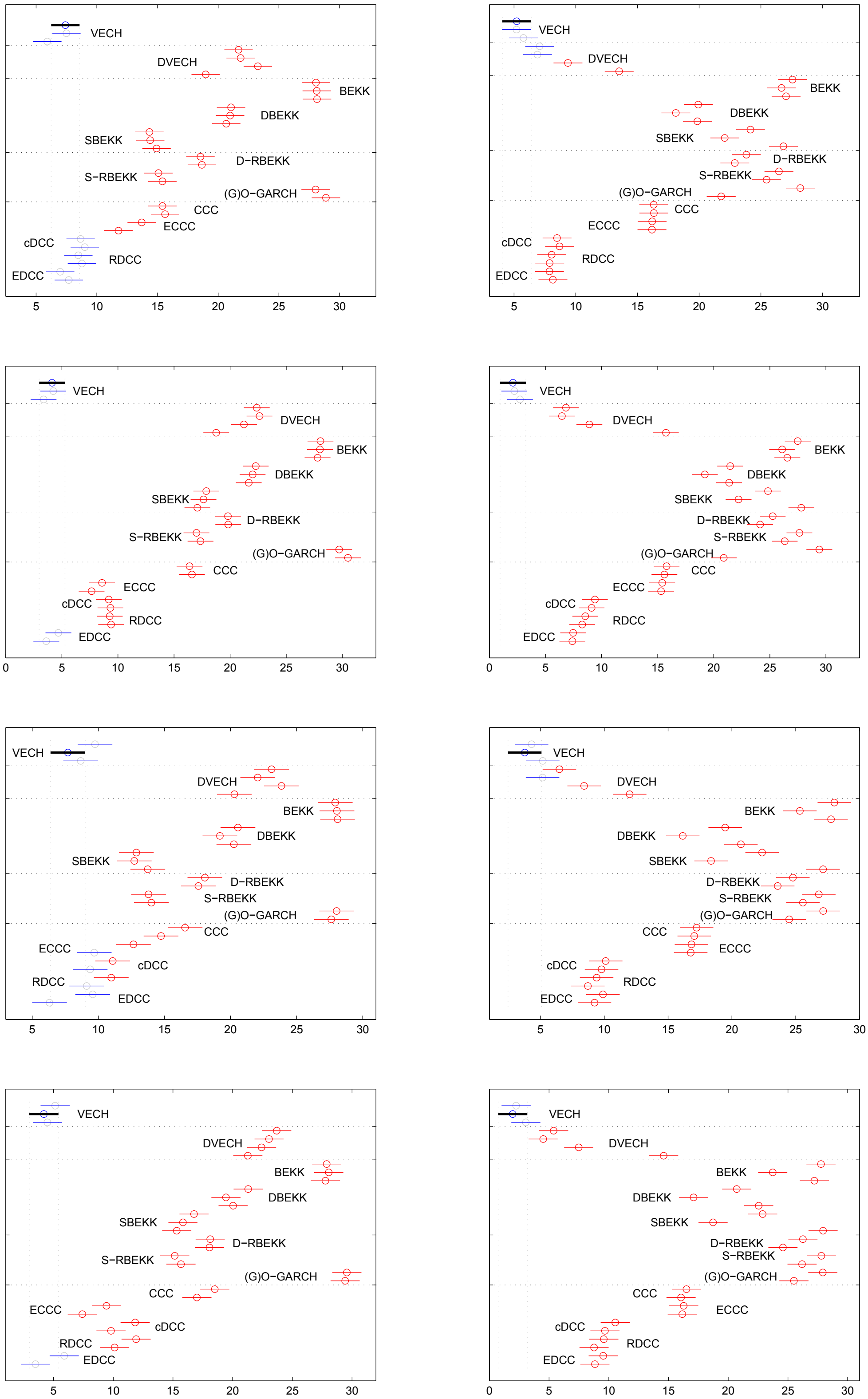


Figure 2.6: Multiple comparison based on Friedman test for bivariate Monte Carlo simulations

Multiple comparison based on Friedman test for the simulation described in Table 2.2. Results for LF_1 and LF_2 on the left and right columns, respectively. The first two rows are for Gaussian and the last two for Student-7 error distributions. Results for sample size $T = 1000$ (2000) are presented in the first and third (second and fourth) rows. The 95% confidence interval are for the average rank of the estimation method. In black is the confidence interval for the reference method. Methods which are statistically different are in red, while the others are in blue.

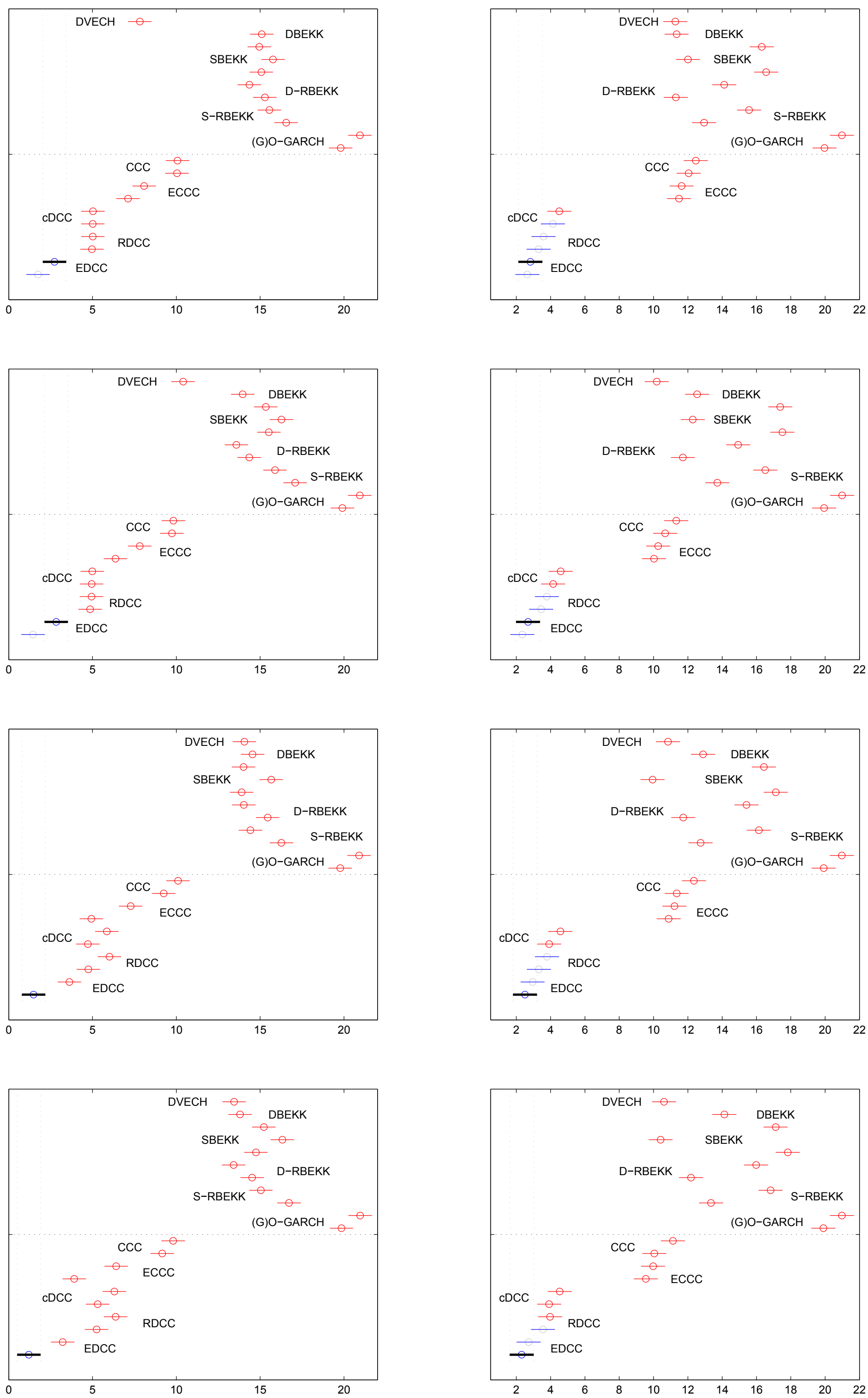


Figure 2.7: Multiple comparison based on Friedman test for five-dimensional Monte Carlo simulations

Multiple comparison based on Friedman test for the simulation described in Table 3. Results for LF_1 and LF_2 on the left and right columns, respectively. The first two rows are for Gaussian and the last two for Student-7 error distributions. Results for sample size $T = 1000$ (2000) are presented in the first and third (second and fourth) rows. The 95% confidence interval are for the average rank of the estimation method. In black is the confidence interval for the best model (EDCC). Methods which are statistically different are in red, while the others are in blue.

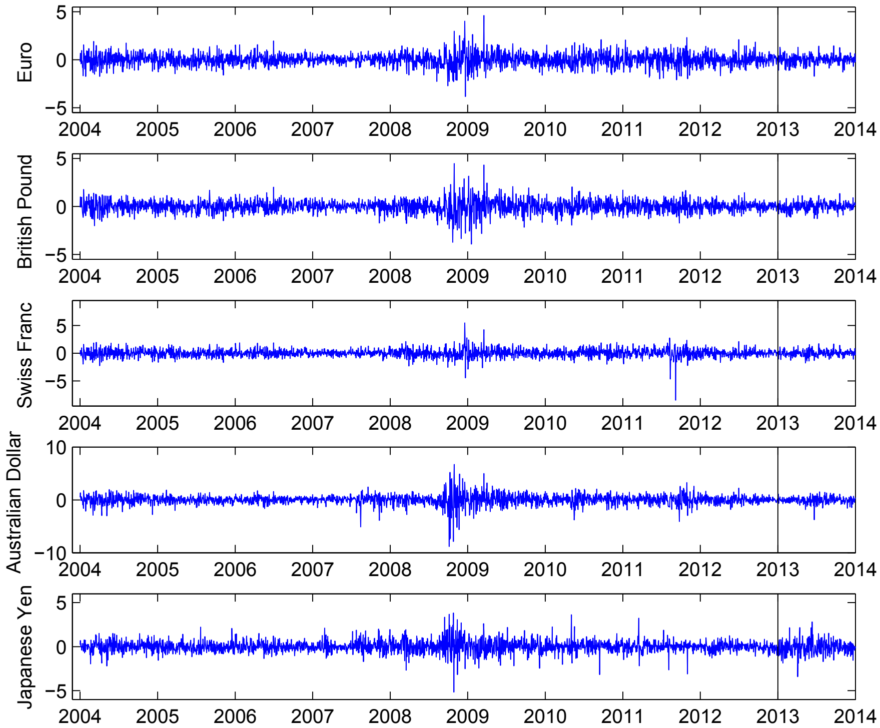


Figure 2.8: Plot of return series of the currencies considered in application

Return series of the Euro (first row), British Pound (second row), Swiss Franc (third row), Australian Dollar (fourth row) and Japanese Yen currencies (fifth row) against the US Dollar currency in the period starting on January 2, 2004 and ending on December 31, 2013, for a total of 2581 daily observations.

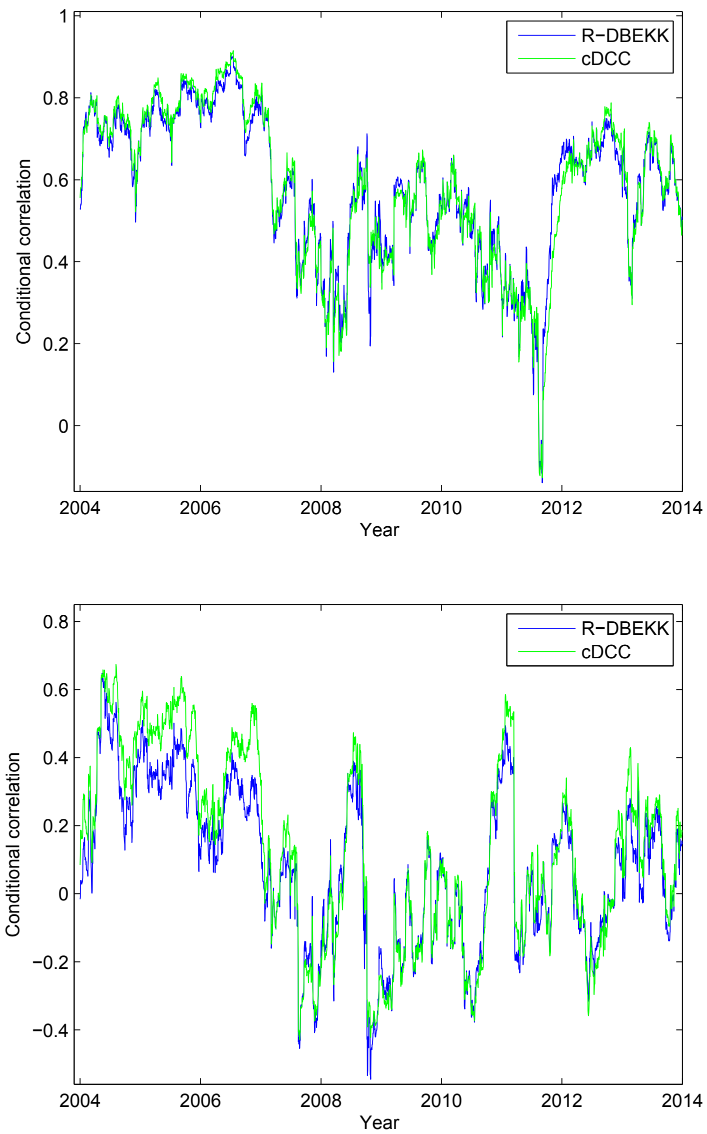


Figure 2.9: Plot of estimated conditional correlations between currencies

Pairwise conditional correlations between GBP and CHF (top panel) and AUD and JPY (bottom panel) series after being estimated by the R-DBEKK and cDCC models.

Table 2.6: Results of the empirical application to a five-dimensional system of daily exchange rate returns

Model-Estimator	NL						LF_1						LF_2					
	1-step-ahead		5-step-ahead		20-step-ahead		1-step ahead		5-step ahead		20-step ahead		1-step ahead		5-step ahead		20-step ahead	
	NL	SPA	NL	SPA	NL	SPA	LF_1	SPA	LF_1	SPA	LF_1	SPA	LF_2	SPA	LF_2	SPA	LF_2	SPA
DVECH-LSW	822.0	0.00	822.7	0.00	800.8	0.00	0.319	0.00	0.372	0.00	0.407	0.00	0.485	0.00	0.499	0.00	0.502	0.00
DBEKK-CVT	814.5	0.02	834.4	0.06	790.4	0.00	0.304	0.00	0.369	0.05	0.400	0.00	0.456	0.00	0.452	0.00	0.482	0.03
SBEKK-G-QML	804.9	0.02	**823.1**	0.06	789.2	0.01	0.295	0.03	0.366	0.04	0.396	0.00	0.461	0.00	0.445	0.00	0.499	0.00
-S-QML	**771.7**	1.00	**773.9**	1.00	**744.3**	0.58	0.304	0.00	0.367	0.00	0.396	0.00	0.470	0.00	0.435	0.00	0.489	0.04
-CVT	815.2	0.00	820.8	0.00	792.9	0.00	0.299	0.00	0.381	0.00	0.405	0.00	0.449	0.00	0.451	0.00	0.495	0.00
D-RBEKK-CVT	818.6	0.01	**840.5**	0.05	797.2	0.00	0.295	0.12	**0.364**	0.21	0.392	0.02	0.426	0.00	0.439	0.00	0.493	0.00
S-CVT	**779.0**	0.14	**781.4**	0.25	752.1	0.20	**0.285**	1.00	**0.352**	1.00	0.387	0.09	0.435	0.00	0.430	0.02	0.494	0.00
S-RBEKK-CVT	815.2	0.00	820.8	0.00	792.9	0.00	0.301	0.00	0.370	0.01	0.399	0.00	0.444	0.00	0.441	0.00	0.495	0.00
S-CVT	**777.9**	0.29	**774.1**	0.92	747.6	0.13	0.299	0.01	0.370	0.02	0.398	0.00	0.441	0.00	0.449	0.00	0.492	0.00
O-GARCH	1131.5	0.00	1115.1	0.00	1060.8	0.00	0.331	0.00	0.422	0.00	0.433	0.00	0.510	0.00	0.530	0.00	0.542	0.00
GO-GARCH	962.9	0.00	954.3	0.00	911.3	0.00	0.322	0.00	0.419	0.00	0.421	0.00	0.506	0.00	0.525	0.00	0.529	0.00
CCC-G-2s	876.7	0.00	861.0	0.00	818.5	0.00	0.316	0.00	0.413	0.00	0.409	0.00	0.492	0.00	0.511	0.00	0.512	0.00
S-2s	811.9	0.03	**799.2**	0.07	755.1	0.03	0.318	0.00	0.421	0.00	0.412	0.00	0.493	0.00	0.504	0.00	0.516	0.00
ECCC-G-2s	888.2	0.00	873.7	0.00	828.1	0.00	0.314	0.00	0.380	0.00	0.402	0.01	0.424	0.00	0.423	0.10	0.490	0.00
S-2s	808.5	0.07	**798.2**	0.16	756.4	0.24	0.309	0.00	0.368	0.08	0.409	0.00	0.412	0.03	0.432	0.00	0.484	0.01
cDCC-G-3s	819.4	0.01	828.9	0.02	789.0	0.00	0.304	0.00	0.372	0.03	0.392	0.00	0.410	0.05	**0.423**	0.08	**0.481**	0.11
S-3s	**779.2**	0.21	**776.6**	0.49	**742.8**	0.17	0.297	0.02	**0.367**	0.15	0.398	0.00	0.415	0.01	**0.412**	0.36	**0.470**	0.68
RDCC-G-3s	819.2	0.01	828.0	0.03	784.3	0.00	**0.288**	0.65	**0.363**	0.22	0.392	0.02	**0.402**	0.12	**0.416**	0.29	**0.478**	0.52
S-3s	**778.4**	0.25	**774.9**	0.89	**739.0**	1.00	**0.286**	0.89	**0.354**	0.91	**0.381**	1.00	**0.376**	1.00	**0.408**	1.00	**0.469**	1.00
EDCC-G-3s	845.6	0.00	852.5	0.00	798.5	0.00	**0.289**	0.37	0.369	0.15	0.389	0.05	**0.378**	0.59	**0.420**	0.44	0.475	0.21
S-3s	**785.8**	0.15	**784.9**	0.20	**740.4**	1.00	**0.281**	0.98	**0.352**	0.93	**0.380**	1.00	**0.369**	1.00	**0.408**	1.00	**0.461**	1.00

Notes: The results are based on the out-of-sample negative log-likelihood (NL) and Frobenius loss function (LF) for $h = 1$, 5 and 20 horizon forecasting of the system of the Euro, British Pound, Swiss Franc, Australian Dollar and Japanese Yen currency returns against the US Dollar currency. We evaluate the realized covariance using a five-minute frequency data and rOWCov method. In the top left panel is the NL values; in the top right panel is the LF_1; and in the bottom panel is the LF_2. In all the panel are the p-value for the superior predictive ability (SPA) tests for the null hypothesis that each model is not outperformed by any of the other models according with the corresponding criterion. In bold, the models included in the confidence sets (MCS) at 10% significance level. The out-of-sample periods goes from January 2, 2012 to December 31, 2013, for a total of 258 daily observations.

Chapter 3

Combining State-Dependent Forecasts of Equity Risk Premium

3.1. Introduction

The predictability of the equity market return in excess of the risk-free interest rate plays an important role in several financial applications such as asset pricing, asset allocation and risk management. There is convincing evidence to date that a host of macroeconomic variables have in-sample predictive power for the equity premia; see, for example, Fama and French (1988), Lettau and Ludvigsson (2001) and Ang and Bekaert (2002, 2007). However, a degree of skepticism remains as regards the out-of-sample predictability of the equity premium; see, for instance, Bossaerts and Hillion (1999), Goyal and Welch (2003) and Butler *et al.* (2005). In particular, Welch and Goyal (2008) document that most macroeconomic predictors fail to beat the simple historical average benchmark in out-of-sample forecasting exercises and recommend researchers to explore alternative predictors and/or more sophisticated forecasting methods. Consequently, recent studies have tried to find alternative forecasting methods that provide statistical and economic evidence of out-of-sample predictability; see Campbell and Thompson (2008), Rapach *et al.* (2010), Ferreira and Santa-Clara (2011) and Pettenuzzo *et al.* (2014). In a recent study, Neely *et al.* (2014) show that technical indicators contain predictive information for the equity risk premium and that they are not encompassed by macroeconomic variables.

This thesis contributes to the stock return forecasting literature by constructing parsimonious regime-switching predictive regressions where the equity premium can have two different states. Although the traditional works assume that equity premia are generated by a linear process with stable coefficients on macroeconomic variables, recent literature indicates that asset returns follow a complex process with more than one regime; see, for example, Turner *et al.* (1989), Garcia and Perron (1996), Perez-Quiros and Timmermann (2000), Ang and Bekaert (2002, 2007), Ang and Chen (2002), Guidolin and Timmermann (2006a,b) and Pettenuzzo and Timmermann (2011). In particular, Tu (2010) concludes that the certainty-equivalent losses associated with ignoring regime switching are generally above 2% per year. Furthermore, Henkel *et al.* (2011), Dangl and Halling (2012), Gargano and Timmermann (2012), Neely *et al.* (2014), among many others find that numerous predictors tend to give stronger signals during recessions than expansion. Jacobsen *et al.* (2014) find an even more strong evidence when considering industrial metals prices as predictors, which are strongly positive related with future equity premia in recessions and strongly negative related in expansions. From a theoretical point of view, the intertemporal asset pricing model of Merton (1973) indicates that time-varying risk aversion may imply a time-varying dependence between stock returns and macroeconomic predictors. More recently, Bali (2008) proves that relative risk aversion coefficients are unstable, suggesting that predictability could be time-varying. On the basis that predictability changes over time, some authors (see, for example, Guidolin and Timmermann (2007), Henkel *et al.* (2011), Dangl and Halling (2012) and Zhu and Zhu (2013)) provide equity premium forecasting strategies using regime-switching, where each regime is unobservable. We propose a more parsimonious approach to construct regime-switching models to forecast equity premia. We rely on the technical variables of Neely *et al.* (2014) as proxy of the current state of economy and allow each macroeconomic variable at hand to be interacted with this state variable; thus, the equity premium predictability becomes state-dependent. Our methodology is very simple and easy to be interpreted. We analyze the performance of our forecasting strategy by estimating predictive regressions for S&P 500 equity premium. Using conventional statistical and economic measures, we conclude that the combining regime-switching predictive regressions deliver stable out-of-sample forecasting gains compared to the historical average, traditional regressions based on macroeconomic or technical variables and combining single-state models.

Our second contribution of this Chapter is to propose a novel sparse forecast combination method. Since the seminal paper of Bates and Granger (1969), the idea of forecast combination has received ample support from successful economics and finance applications; see Stock and Watson (1999, 2003, 2004) who combine forecasts of inflation and real output growth with implications for macroeconomic policy-making, Fuertes and Olmo (2013) who combine intra-day and inter-day forecasts of Value-at-Risk for risk management and Caldeira *et al.* (2015) who combine high dimensional multivariate volatility forecasts using economic criteria based on portfolio selection. This method accounts for model uncertainty and incorporates information from several variables, which reduces forecast volatility. In the equity risk premium context, the notion of forecasting combination is first explored by Rapach *et al.* (2010) who combine forecasts from assorted univariate macroeconomic regression models. Elliott *et al.* (2013) further extend this work by averaging forecasts across complete subset regressions with the same number of predictive variables. The combining approaches of Rapach *et al.* (2010) and Elliott *et al.* (2013) are based either on equal weights or estimated weights over an estimate period. They conclude that the EW combining forecasts are not worse than combining forecasts using estimated weights. However, numerous authors suggest that a selection of relevant variables is suitable before using combining methods, given the relative large collection of macroeconomic variables typically employed as candidate predictors and the large degree of co-movement between them. For example, Bai and Ng (2008), Dobreva and Schaumburg (2013) and Fuentes *et al.* (2015) combine a variable selection process with factor models and argue that more variables do not necessarily yield better forecasts. Based on the last three references, we extend their sparse combining approach to equity premium forecast. A in-sample significance test of (joint) predictability is employed as a threshold rule to select which of the individual models will be considered or excluded in the combining forecasts. Then, the one-step-ahead forecast is given by the mean average of forecasts from models where the null hypothesis of zero slope coefficient(s) is not rejected statistically under a selected level. Our results show that our SPAR method is a good alternative to the EW combining approach.

The rest of this Chapter is organized as follows. Section 3.2 presents the econometric methodology and describes our macroeconomic and technical data set, while Section 3.3 outlines statistical and economic measures of forecasting performance. Section 3.4 carries out an empirical analysis of the US equity premium. Finally, Section 3.5 concludes.

3.2. Forecasting methodology

This section describes the forecasting strategies and variables considered in this thesis.

3.2.1. Predictive variables: Macroeconomic and technical indicators

The most widely-used predictive regression for the equity risk premium can be formalized as

$$r_{t+1} = \alpha_i + \beta_i x_{i,t} + \varepsilon_{t+1}, \tag{3.1}$$

where r_{t+1} is the continuously compounded return (including dividends) in excess of the risk-free interest rate, $x_{i,t}$ is the i-th macroeconomic variable at time t, ε_{t+1} is a zero-mean error, and $t = 1, 2, \cdots, T$ are the sample months. Denoting by $E_t(\cdot)$ the conditional expectation based on the information until time t, one forecast of the one-month-ahead equity risk premium is $\hat{r}_{t+1} = E_t(r_{t+1}) = E_t(\alpha_i + \beta_i x_{it} + \epsilon_{t+1}) = \alpha_i + \beta_i x_{it}$. A simple no-predictive benchmark is obtained by imposing $\beta_i = 0$ in Equation (3.1), which results in the constant expected equity risk premium model, with the one-month-ahead forecast given by the historical average (HA) of excess returns from months 1 to t, i.e., $\bar{r}_{t+1} = (\sum_{i=1}^{t} r_i)/t$. If $x_{i,t}$ contains predictive information for the equity risk premium, then $\hat{r}_{t+1}$ should outperform $\bar{r}_{t+1}$. The reason on why assorted macroeconomic variables, such as the dividend yield, default spread and term structure, have been employed in the equity risk premium literature is well described by Cochrane (2011). In particular, according to asset price theory, time-varying expected stock returns are basically determined by future macroeconomic conditions. Once macroeconomic variables can predict changing in economy state, they should also have predictive ability for equity risk premium tendencies, which is consistent with rational asset pricing.

We base our analysis on the updated 12 macroeconomic variables studied by Welch and Goyal (2008) inter alios with observations over the period from December 1950 to December 2014. They comprise

- Dividend yield (D/Y): difference between the log of 12-month moving sums of dividends paid on the S&P 500 Index and the log of lagged prices.

- Earnings-price ratio (E/P): difference between the log of 12-month moving sums of earnings on the S&P500 Index and the log of prices.

- Dividend payout ratio (D/E): difference between the log of 12-month moving sums of dividends paid on the S&P 500 Index and the log of 12-month moving sums of earnings on the S&P500 Index.

- Equity premium volatility (RVOL): based on a 12-month moving standard deviation estimator of Mele (2007).
- Book-to-market ratio (B/M): ratio of book value to market value for the Dow Jones Industrial Average.
- Net equity expansion (NTIS): ratio of 12-month moving sums of net issues by New York Stock Exchange (NYSE) listed stocks to the total end-of-year market capitalization of NYSE stocks.
- Treasury bill (TBill): interest rate on a 3-month Treasury bill.
- Long-term rate of returns (LTR): return on long-term government bonds.
- Term Spread (TMS): difference between the long-term yield on government bonds and the Treasury bill rate.
- Default yield spread (DFY): difference between BAA- and AAA-rated corporate bond yields from FRED.
- Default return spread (DFR): difference between long-term corporate bond and long-term government bond returns.
- Inflation (INFL): Consumer Price Index (all urban consumers) from the Bureau of Labor Statistics. As inflation information is released only with a month of delay, we use $x_{i,t-1}$ in Equation (3.1) for inflation to forecast the equity premium at time $t+1$.[1]

Equity risk premium forecasts can also be predicted by technical variables as follows

$$r_{t+1} = \alpha_i + \beta_i \mathrm{TECH}_{i,t} + \varepsilon_{t+1}, \tag{3.2}$$

where $\mathrm{TECH}_{i,t}$ is the i-th technical indicator at time t, which is equal to 1 (buy signal) or 0 (sell signal). Typical technical indicators are trend-chasing rules based on moving average of past price and volume patterns. Although the economic intuition on why technical indicators have predictive power in the equity risk premium is not well established, Neely *et al.* (2014) provide four possible theoretical models to explain the influence of technical indicators in future equity

[1] We exclude two variables, the dividend price ratio (D/P) and the long term yield (LTY) since they are by construction a combination of other variables in the set; namely, D/P=E/P+D/E and LTY=TBL+TMS.

premium trends. Furthermore, they find empirically statistical and economic evidence on US equity predictability by considering numerous technical indicators.

As candidates for $\text{TECH}_{i,t}$ in the predictive regression (3.2) we consider the 14 technical indicators adopted by Neely et al. (2014). The moving average rule with lengths a and b, MA(a,b), is given by

$$\text{MA}_t(a,b) = \begin{cases} 1 & \text{if} \quad (\sum_{j=0}^{a-1} P_{t-j})/a \leq (\sum_{i=0}^{b-1} P_{t-j})/b, \\ 0 & \text{if} \quad (\sum_{j=0}^{a-1} P_{t-j})/a > (\sum_{i=0}^{b-1} P_{t-j})/b, \end{cases} \tag{3.3}$$

where P_t denotes the asset price at time t. We generate moving average indicators for $a = 1, 2, 3$ and $b = 9, 12$.

The moment rule with level l, denoted by MOM(l), is defined by

$$\text{MOM}_t(l) = \begin{cases} 1 & \text{if} \quad P_t \leq P_{t-l}, \\ 0 & \text{if} \quad P_t > P_{t-l}. \end{cases} \tag{3.4}$$

We analyze moment rules for $l = 9, 12$.

The on-balance volume is defined as

$$OBV_t = \sum_{j=0}^{t} Vol_j D_j, \tag{3.5}$$

where Vol_j is the volume traded in the S&P 500 Index in the period j and D_j is a variable such that $D_j = 1$ if $P_j - P_{j-1} \geq 0$ and $D_j = -1$ if $P_j - P_{j-1} < 0$. The third trend-following strategy is given as function of the on-balance volume as follows

$$\text{VOL}_t(a,b) = \begin{cases} 1 & \text{if} \quad (\sum_{j=0}^{a-1} OBV_{t-j})/a \leq (\sum_{j=0}^{b-1} OBV_{t-j})/b, \\ 0 & \text{if} \quad (\sum_{j=0}^{a-1} OBV_{t-j})/a > (\sum_{j=0}^{b-1} OBV_{t-j})/b. \end{cases} \tag{3.6}$$

We compute monthly signals for $a = 1, 2, 3$ and $b = 9, 12$ and denote it as VOL(a, b).

We further introduce an agreement variable to summarize the information from the 14 technical variables as follows

$$\text{A}_k = \begin{cases} 1 & \text{if} \quad \sum_{i=1}^{14} \text{TECH}_{i,t} \geq k, \\ 0 & \text{if} \quad \sum_{i=1}^{14} \text{TECH}_{i,t} < k, \end{cases} \tag{3.7}$$

where $i = 1, \cdots, 14$ and k is the minimum number of technical variables that have to be equal to 1 to A_k take value 1. Hence the agreement variable is equal to 1 if at least k of the 14 technical variables are equal to 1, and 0 otherwise.

3.2.2. Two-state predictive model

Our forecasting strategy consists on modeling the equity premium as having two different states as follows

$$r_{t+1} = \alpha_i + \beta_{1i} x_{i,t} + \beta_{2i} S_t x_{i,t} + \varepsilon_{t+1}, \tag{3.8}$$

where S_t is a dummy state-variable, which takes values 0 or 1. Consequently, the mean of equity premium at time t is equal to $\alpha_i + \beta_{1i} x_{i,t}$ if $S_t = 0$ and to $\alpha_i + (\beta_{1i} + \beta_{2i}) x_{i,t}$ if $S_t = 1$, so the equity premium is state-dependent.[2] We can test the state effect in the equity premium forecasts by doing H$_0$: $\beta_{2i} = 0$ against H$_A$: $\beta_{2i} \neq 0$.

The first challenge is to find an appropriate state variable. As the NBER business cycle data is released with a lag, we have to look for a proxy of the current state of the economy that is based on historical data up to the time when the forecast is made. Jacobsen *et al.* (2014) suggest three alternative proxies for S_t: the CFNAI (Chicago Fed National Activity Index) which is available on a real time basis[3], the recession probabilities from Chauvet and Piger (2008) using a regime switching model and recession probabilities based on the four macroeconomic variables of Henkel *et al.* (2011) (term spread, default spread, dividend yield, and the short rate) also using a regime switching model. We consider information from technical indicators as plausible candidates for S_t, which are very simple functions of past return prices and can identify price trends by relying that these trends persist into the future. Moreover, technical variables are strongly positive correlated with the NBER-cycle, as we can see in Table 3.1 the percentage of agreement between each technical variable and the business cycle. We report two alternatives for S_t, the moving average MA(2,12) technical indicator and the agreement variable A_{10}, both with more than 80% of agreement the NBER business cycle and less then 7.5% of transitions (therefore persistent).[4] Figure 3.1 plots the MA(2,12) and A_{10} technical trading indicators over the sample period from December 1950 to December 2014. We observe that these state variables capture most of the actual NBER-dated recession months, albeit at the expense of triggering some false recession signals. Therefore, technical indicators can be thought as a proxy of the present state of economy, such that a value of

[2] There is no empirical work in literature that justifies considering a predictive model for the equity premium with a switching intercept α_i. Nevertheless, we considered a generalization of model (3.8) with intercept term $\alpha_{1i} + \alpha_{2i} S_t$, using various proxies for S_t. The resulting forecasts did not offer any improvement neither according to out-of-sample R^2_{OOS} nor according to the certainty equivalent return gains.

[3] The CFNAI is a weighted average of 85 existing monthly indicators released at the end of each month; see Stock and Watson (1999). We do not use it because the CFNAI was first released in 2001 and our sample starts in 1950.

[4] The percentage of transitions of NBER data is 2.61%, so we look for a persistent variable S_t.

1 indicates current recession and a value of 0 indicates expansion.

Table 3.1: Agreement with NBER-data

Technical variables					Agreement variables				
Predictor	Agreement (%)			Transit-	Pred-	Agreement (%)			Transit-
	REC	EXP	Full sample	ions(%)	ictor	REC	EXP	Full sample	ions(%)
MA(1,9)	74.38	77.13	76.69	13.04	I_1	**90.91**	54.40	60.16	12.39
MA(1,12)	80.17	81.30	81.12	8.87	I_2	87.60	62.29	66.28	11.34
MA(2,9)	77.69	78.36	78.26	9.91	I_3	85.95	66.92	69.92	10.56
MA(2,12)	80.17	81.14	80.99	**7.04**	I_4	85.12	70.02	72.40	10.95
MA(3,9)	79.34	79.29	79.30	8.60	I_5	83.47	72.80	74.48	9.91
MA(3,12)	80.17	81.61	81.38	7.04	I_6	80.17	75.27	76.04	9.91
MOM(9)	79.34	80.83	80.60	9.65	I_7	76.86	80.53	79.95	8.34
MOM(12)	**80.99**	**83.31**	**82.94**	7.56	I_8	76.03	83.31	82.16	7.30
VOL(1,9)	67.77	73.42	72.53	17.47	I_9	72.73	85.47	83.46	6.00
VOL(1,12)	71.90	76.97	76.17	12.39	I_{10}	69.42	86.09	83.46	5.74
VOL(2,9)	70.25	73.42	72.92	10.95	I_{11}	67.77	87.64	84.51	6.26
VOL(2,12)	71.90	76.35	75.65	7.69	I_{12}	64.46	88.41	84.64	**5.48**
VOL(3,9)	69.42	75.27	74.35	10.43	I_{13}	61.16	89.64	85.16	6.26
VOL(3,12)	71.90	76.51	75.78	7.17	I_{14}	53.72	**92.12**	**86.07**	6.78

Notes: Agreement of each indicator variable described in Section 3.2.1 at time t with the present state of economy, considering the NBER data from January 1951 to December 2014. It is also reported the percentage of true recessions (REC) and true expansions (EXP) and the percentage that each technical variable transits from one state to the other. In the NBER data, the percentage of transition is 2.61%. In bold, it is the best model of each panel.

3.2.3. Equal-weighted combination of forecasts

Let N the number of individual forecasts of r_{t+1} at hand, all of them evaluated from information up to time t. The combining forecasts is a weighted average of them

$$\hat{r}^C_{t+1} = \sum_{i=1}^{N} \omega_{i,t} \hat{r}_{i,t+1}, \tag{3.9}$$

where $\hat{r}_{i,t+1}$ is the i-th individual forecast with the corresponding loading or weight $\omega_{i,t}$. Different versions of combining methods are given by different choices of the weights $\omega_{i,t}$. The more popular version is the EW combining forecasts, also known as mean combining forecasts, which set $\omega_{i,t} = 1/N$ on each individual predictive regression forecast in Equation (3.9). Other two direct alternatives are the median combination, where the forecast is the median of $\hat{r}_{i,t+1}$, $i = 1, \cdots, N$, and the trimmed mean combination, which imposes $\omega_{i,t} = 0$ for the individual forecasts with the smallest and largest values and sets $\omega_{i,t} = 1/(N-2)$ for the remaining individual forecasts in Equation (3.9).

Elliot *et al.* (2013) extend the combining forecasts by averaging forecasts across complete subset regressions with the same number of predictive variables, i.e., they combine forecasts from all possible linear regression models considering a fixed number of predictors, say k. For $k = 2$, for example, there is a total of $\binom{N}{2}$ models with two explaining variables

$$r_{t+1} = \alpha_i + \beta_i x_{i,t} + \beta_j x_{j,t} + \varepsilon_{t+1}, \tag{3.10}$$

such that $i \neq j$. After estimating all the α_i, β_i and β_j by OLS, it is possible to compute $\binom{N}{2}$ different one-step-ahead forecasts. The combining one-step ahead forecast is obtained by averaging all these $\binom{N}{2}$ forecasts. We analyze complete subset regressions on macroeconomic predictive variables for $k = 1, 2, 3$ to analyze which cases produces better forecasts; see Elliott *et al.* (2013) for a comparison of different values of k. They argue that as k grows larger, the out-of-sample forecasting performance quickly deteriorates. The combining approach of Rapach *et al.* (2010) is a special case of the complete subset regression, when k is equals to 1. For for $k = 1, 2, 3$, we compare traditional combining forecasts to combining forecasts applied to regime-switching models.

3.2.4. Sparse combination of model forecasts

Rapach *et al.* (2010) and Elliott *et al.* (2013) conclude that the EW combining forecasts are not outperformed by the median and trimmed mean combinations and any other estimated-weighted methods considered. As an alternative to the EW combination, we introduce the sparse EW combining forecasts, where the one-step-ahead prediction of the equity risk premium is the mean average of forecasts from selected models through in-sample predictability tests, as will be explained in the following. As argued by Campbell and Yogo (2006), traditional in-sample predictability tests can be severely distorted when the degree of persistence of the predictive variables is not well specified. However, it is widely known that most of the macroeconomics variables used in predictive regressions are highly persistent with autoregressive roots close to unity, which generating uncertainty whether these variables are stationary or not. For this reason, we evaluate the (joint) significance of parameters by the Kostakis *et al.* (2015) test which is robust to regressors' degree of persistence and accommodates testing the joint predictive ability in multiple regression; see also Polk *et al.* (2006) and Amihud *et al.* (2009) for other hypothesis-testing methods for multiple regressors. For each individual forecast, we first test if the predictive variables are jointly significant according to the Kostakis *et al.* (2015). For example, in the case of combining forecasts from simple predictive regressions, we test whether each $\beta_i = 0$ in equation (3.1) against $\beta_i \neq 0$, whereas in case of combining forecasts from two-variable regressions, $k = 2$, we test $\beta_i = \beta_j = 0$ in equation (3.10) for each par i, j with $i \neq j$. Then, we construct the one-step-ahead prediction of the equity risk premium as the mean average of forecasts from models whose coefficients are jointly significant. If at any given rolling estimation window (ending at month t) none of the individual forecasts is significant, the corresponding prediction at time $t + 1$ is the historical average. Formally, from a total of $\binom{N}{k}$ forecasts at hand, the sparse EW forecast combination is given by

$$\hat{r}_{t+1}^{SPAR} = \begin{cases} \dfrac{\sum_{i \in E} \hat{r}_{i,t+1}}{M}, & \text{if} \quad E \neq \emptyset, \\ \dfrac{\sum_{t=1}^{T} r_t}{T}, & \text{if} \quad E = \emptyset, \end{cases} \tag{3.11}$$

where $E = \{i$; predictive variable(s) (is)are (jointly) significant at 10% level; $i = 1, \cdots, \binom{N}{k}\}$ and M is the cardinality of E.

3.3. Out-of-sample forecast evaluation

Inoue and Kilian (2005) and Diebold (2012) compare out-of-sample versus in-sample tests of predictability and conclude that in-sample tests of predictability have higher power than out-of-sample tests. Nevertheless, the objective of our empirical application is to verify whether combining state-dependent forecasts can be employed for investors on a real-time basis who want to predict the one-step-ahead equity premium, implying in better economic decisions, i.e., portfolio holdings. Therefore, an out-of-sample analysis is more suitable.

The full-sample is split into the in-sample estimate and out-of-sample forecast periods with T and H observations, respectively. Following Welch and Goyal (2008), out-of-sample forecasts of the equity premium are obtained sequentially in a recursive approach (expanding estimation windows). We describe next the statistical and economic measures of out-of-sample predictability employed in the thesis.

3.3.1. Statistical performance

We compare the statistic performance of different forecasts by three measures. First, the popular out-of-sample R^2, R^2_{OOS}, of Campbell and Thompson (2008) is computed to measure the reduction in the mean squared predictive error (MSE) of a specific forecast in relation to the HA benchmark as follows

$$R^2_{OOS} = 1 - \frac{\sum_{h=1}^{H}(\hat{r}_{T+h} - r_{T+h})^2}{\sum_{h=1}^{H}(\bar{r}_{T+h} - r_{T+h})^2}, \tag{3.12}$$

where $h = 1, \cdots, H$ are the out-of-sample months, $\hat{r}_{T+h}$ is a one-ahead forecast based on information until time $T + h - 1$ and $\overline{r}_{T+h}$ is the mean of the first $(T + h - 1)$ observations. If $R^2_{OOS} > 0$, it means that the forecast $\hat{r}_{T+h}$ performs better, in mean, than the historical average; whereas, if $R^2_{OOS} \leq 0$, the historical average is not outperformed by the alternative approach. We report the R^2_{OOS} statistic separately for expansionary (EXP) and recessionary (REC) months according to the NBER business cycle dating. We also report an alternative version of the conventional R^2_{OOS} that compares the square forecast error of the predictive approach at hand versus the EW combination of forecasts from conventional one-state predictive models based on individual macroeconomic variables ($k = 1$), which corresponds to the traditional combining forecasts of Rapach *et al.* (2010).[5]

Second, the MSE-*adjusted* statistic of the Clark and West (2007) is performed to test the null hypothesis that the historical average performs better or equal to the alternative forecast, which correspond to the hypothesis test H_0: $R^2_{OOS} \leq 0$ against $R^2_{OOS} > 0$. This test is an extension of the Diebold and Mariano (1995) and West (1996) tests and allows to compare forecast of nested models.

The R^2_{OOS} (like MSE) is an overall measure of forecast performance computed from forecast sequences and hence, as point statistics, they can mask important instability in forecast performance. To gauge the dynamics of the forecast performance over the entire out-of-sample period we graph the variation in the cumulative square error (CSE) given by

$$\Delta\text{CSE}_t = \sum_{h=1}^{t}(\bar{r}_{T+h} - r_{T+h})^2 - (\hat{r}_{T+h} - r_{T+h})^2, \tag{3.13}$$

such that a positively-sloped ΔCSE_t graph indicates that the forecasting model at hand consistently outperforms the HA benchmark, while a negatively-sloped curve indicates the opposite. A switch from a positive to a negative slope or vice versa indicates unstable forecast performance. We also graph the variation in the CSE when the benchmark is the model of Rapach *et al.* (2010) to compare the gains of considering combinations of two-state models rather than combinations of one-state models.

[5]Hereafter, unless specified, the benchmark of the R^2_{OOS} is the historical average forecast.

3.3.2. Economic performance

As reported by Cenesizoglu and Timmermann (2012), for the R^2_{OOS} statistic being a point forecast, it does not reflect the accuracy of predictive movements in the entire distribution of equity premium. To overcome this, we carry out an asset allocation exercise to evaluate the economic performance of the equity premium forecasts. We consider investors who at time t allocate ω_t% of his total wealth to stocks and the reminding (1-ω_t)% to risk-free bills. In this way, the total wealth at month $t+1$ is

$$W_{t+1} = [(1-\omega_t)\text{exp}(r^f_{t+1}) + \omega_t\text{exp}(r^f_{t+1} + r_{t+1})]W_t, \tag{3.14}$$

where the equity premium, r_t, and the risk-free interest rate, r^f_{t+1}, are continuously compounded. We assume that investors maximize the expected one-month ahead wealth, which excludes any intertemporal hedging component in the choice of the portfolio weights. Hence, portfolio weights for the period t are the solution to the following optimizing problem

$$\omega_t^* = \arg\max_{\omega_t} E_t[U(W_{t+1})], \tag{3.15}$$

where the utility function $U(W_{t+1})$ is defined according to the investor's preference. For a mean-variance investor, the corresponding utility is $U(W_{t+1}) = E_t[W_{t+1}] - \frac{\gamma}{2}Var_t[W_{t+1}]$, where γ is the relative risk aversion parameter. This investor's preference implies that the optimal proportion of wealth allocated to equities on month $t+1$ is

$$\omega_t^* = \frac{\text{exp}(\hat{r}_{t+1} + \bar{\sigma}^2_{t+1}/2) - 1}{\gamma\text{exp}(r^f_{t+1})\text{exp}(\bar{\sigma}^2_{t+1} - 1)\text{exp}(2\hat{r}_{t+1} + \bar{\sigma}^2_{t+1})}, \tag{3.16}$$

where $\hat{r}_{t+1}$ is the equity premium forecast relying on a forecast strategy and $\bar{\sigma}^2_{t+1}$ is the predicted conditional variance of the excess return. As in Neely *et al.* (2014), we evaluate the predictive conditional variance as the mean squared excess returns over five-years rolling windows, regardless of the forecast method considered. We follow Campbell and Thompson (2008) and impose the constrain $0 \leq \omega_t \leq 1.5$ to rule out short sales and leverage above 50%.

We also consider constant relative risk aversion (CRRA) investors, where $U(W_{t+1}) = W^{1-\gamma}_{t+1}/(1-\gamma)$, so that the optimal portfolio weight can be approximated by

$$\omega_t^* = \frac{\hat{r}_{t+1} + \bar{\sigma}^2_{t+1}/2}{\gamma\bar{\sigma}^2_{t+1}}. \tag{3.17}$$

We compare the asset allocation performance of different models by the certainty equivalent return (CER) measure, which is the risk-free rate of return that an investor is willing to accept instead of adopting the given risky portfolio. The CER corresponding to a mean-variance investor is given by

$$\text{CER} = \hat{\mu}_p - \frac{\gamma\hat{\sigma}_p^2}{2}, \tag{3.18}$$

where $\hat{\mu}_p$ and $\hat{\sigma}_p^2$ are, respectively, the mean and variance of the portfolio returns over the out-of-sample period. On other other hand, the CER corresponding to the a CRRA investor is given by

$$\text{CER} = \left[\frac{(1-\gamma)}{H}\sum_{h=1}^{H}\frac{W_{T+h}^{1-\gamma}}{1-\gamma}\right]^{1/(1-\gamma)} - 1. \tag{3.19}$$

Our forecast evaluation metric is the CER gain, Δ, defined as the difference between the CER of a investor who employs a forecasting model to predict the risk premium and the CER of a investor that assumes no predictability and, accordingly, relies on the historical average. We report monthly CER gains in annualized form by multiplying it by 1200. Positive CER gains represent the annualized fee that an investor would be willing to pay in order to have access to the forecasting model. We also report the CER gains for expansionary (EXP) and recessionary (REC) months according to the NBER business cycle dating.

Additionally, we compare the investor's performance through the Sharpe ratio, defined by the quotient between the mean portfolio return in excess of the risk-free rate and the standard deviation of the excess of portfolio return, both over the forecast period.

3.4. Empirical analysis

In this section we carry out an empirical application using the monthly data on US equity premium along the 12 macroeconomic and 14 technical variables described in section 3.2.1 in the period from December 1950 to December 2014. The interest lies in illustrating combining state-dependent forecasts and comparing its performance to other approaches. The equity risk premium is given by the difference between the log return on the S&P 500 (including dividends) and the log return on the Treasury-bill rate. After accounting for lag in the predictive regressions, the sample is split into in-sample estimate and out-of-sample forecast periods. The in-sample spans from December 1950 to December 1965 with a total of $T = 180$ observations, and the forecast evaluation period is from January 1966 to December 2014 with a total of $H = 588$ one-step-ahead predictions.

3.4.1. Results from statistical evaluation

Table 3.2 reports the out-of-sample forecasting performance of the individual regression models. Consistent with the findings of Welch and Goyal (2008), the majority of the macroeconomic variables do not add extra information in relation to the HA benchmark, as only two of the corresponding R^2_{OOS} are positive. Also these two positive R^2_{OOS} are just 0.06% and 0.26%. On the other hand, all the indicator variables have MSE lower than the benchmark forecast, with seven of them also beating statistically the HA, based on the MSE-adjusted statistics with 10% of significance. The highest values of the R^2_{OOS} is 0.864% for the MA(2,12) variable. Figure 3.2 plots the cumulative square forecast errors for the historical average forecast minus the cumulative square forecast error for the macroeconomic and technical variables. In the case of technical indicators, we just plot one of each group, MA, MOM and VOL, as the other ones have similar pattern. These figures show that none of the individual macroeconomic or technical variables can beat the historical average. Although some of the curves are crescent during certain periods, all individual models exhibit relatively long periods with negatively sloped curves.

Table 3.2: Results from statistical evaluation of individual predictive variables

Macroeconomic variables				Technical variables			
Predictor	R^2_{OOS}(%)			Predictor	R^2_{OOS}(%)		
	ALL	EXP	REC		ALL	EXP	REC
D/Y	-0.168	-1.366	2.382	MA(1,9)	0.302	-0.676	2.384
E/P	-0.579	-0.300	-1.173	MA(1,12)	0.697*	-0.521	3.289
D/E	-0.883	-1.720	0.898	MA(2,9)	0.392*	-0.611	2.527
RVOL	0.064*	-0.156	0.531	MA(2,12)	**0.846****	-0.409	**3.518**
B/M	-1.327	-0.399	-3.303	MA(3,9)	0.480*	-0.674	2.937
NTIS	-0.922	-0.129	-2.611	MA(3,12)	0.088	-0.426	1.181
TBL	-0.837**	-1.907	1.441	MOM(9)	0.122	-0.447	1.333
LTR	**0.260****	-1.931	**4.922**	MOM(12)	0.161	-0.414	1.383
TMS	-0.834**	-3.136	4.064	VOL(1,9)	0.476*	-0.527	2.611
DFY	-0.634	-0.542	-0.830	VOL(1,12)	0.803**	-0.201	2.940
DFR	-0.420	**0.349**	-2.055	VOL(2,9)	0.467*	0.042	1.371
INFL	-0.272	0.159	-1.190	VOL(2,12)	0.348	**0.189**	0.688
				VOL(3,9)	0.034	-0.368	0.890
				VOL(3,12)	0.673**	0.095	1.903

Notes: The forecasts are based on individual macroeconomic (on the left) and technical (on the right) predictive variables, both described in Section 3.2.1, over the out-of-sample period from January 1966 to December 2014. R^2_{OOS} is the out-of-sample R^2 given in Equation (3.12). The R^2_{OOS} for the full-sample (ALL) is computed separately for NBER-dated expansion (EXP) and recession (REC) months. The hypothesis test H_0: $R^2_{OOS} \leq 0$ against $R^2_{OOS} > 0$ is performed by the Clark and West (2007) out-of-sample MSE-adjusted statistic, where *, ** and *** indicate significance at the 10%, 5%, and 1% levels. In bold, it is the best model of each panel.

Panel A of Table 3.3 summarizes the statistical performance of combining forecasts from traditional one-state predictive models. In this panel, all the R^2_{OOS} values are larger than the R^2_{OOS} values of the 26 forecasts from individual macroeconomic predictors. Also combining subset regressions with two or three predictive variables produce better point forecasts than the original combining forecasts from individual macroeconomic predictive variables, with the R^2_{OOS} for the $k = 3$ being the largest one. Furthermore, combining methods outperform statistically the HA considering the MSE-adjusted statistics with significance level of 1%. Rapach *et al.* (2010) and Elliott *et al.* (2013) also conclude that combinations of macroeconomic predictive models beat the HA forecast and forecasts from individual predictive variables. The R^2_{OOS} of the sparse EW combining forecasts increases in relation to the corresponding EW combination when considering $k = 1$ or $k = 2$, but this is not true when $k = 3$, where the R^2_{OOS} for the EW combination is slightly larger.

The performance of combining forecasts from two-state models, with states defined by MA(2, 12) and I_{10} variables (Panels B and C of Table 3.3, respectively) is improved the in relation to the nested combining forecasts based on one-state models. Furthermore, the R^2_{OOS} of combining two-state models are significantly larger than the traditional combining forecasts of Rapach *et al.* (2010), denoted by EW($k = 1$), at 10% level. The results are more evident when using I_{10} as a proxy, once the R^2_{OOS} values increase from 1.034, 1.611 and 1.792% to 1.763, 2.23 and 2.145% for the EW combination with $k = 1$, $k = 2$ and $k = 3$ forecast strategies, respectively. When the proxy is the MA(2,12) variable, the R^2_{OOS} of the SPAR combining forecasts increases in relation to the corresponding EW combining method for $k = 1$ and $k = 2$ and it is almost the same when $k = 3$.

We also compare our combining state-dependent forecasts with the principal component (PC) combining method of Neely *et al.* (2014). Panel D of Table 3.3 displays the results of the three PC combining cases: of the 12 macroeconomic variables (PC-ECON), of the 14 technical indicators (PC-TECH) and of all the macroeconomic and indicators together (PC-ALL). We can see that PC-ECON and PC-TECH do not produce more accurate point forecasts than the simple regression based on the MA(2,12) variable. On the other hand the PC-ALL combination improves the predictability of baseline bivariate regressions, with an R^2_{OOS} of 1.48%. Still, the R^2_{OOS} are larger for combining forecasts from two-state model compared to PC-ALL forecasts.

From Tables 3.2 and 3.3, it is evident that, for almost all forecast strategies, the predictability is substantially larger for recessions vis--vis expansions, which matches with several works in

Table 3.3: Results from statistical evaluation of combining forecast methods

Predictor	Equal-weighted R^2_{OOS} (%)				Sparse R^2_{OOS} (%)			
	ALL	EXP	REC	EW($k=1$)	ALL	EXP	REC	EW($k=1$)
Panel A: One-state regressive models								
$k=1$	1.034***	0.722	1.696	0	1.507***	0.736	3.149	0.479*
$k=2$	1.611***	**1.005**	2.901	0.584**	1.709***	**0.961**	3.300	0.683**
$k=3$	**1.792*****	0.947	**3.592**	**0.767****	**1.779*****	0.885	**3.682**	**0.753****
Panel B: Two-state regressive models - state variable MA(2,12)								
$k=1$	1.451***	**0.523**	3.424	0.421*	**2.041*****	**0.657**	4.984	**1.017****
$k=2$	**1.829*****	0.442	4.782	**0.804****	1.908***	0.398	5.121	0.883**
$k=3$	1.769***	0.065	**5.394**	0.743**	1.759***	0.009	**5.482**	0.733**
Panel C: Two-state regressive models - state variable A_{10}								
$k=1$	1.763***	**0.868**	3.667	0.737**	1.894***	**0.539**	4.778	0.869**
$k=2$	**2.230*****	0.848	5.171	**1.209****	1.878***	0.380	5.065	0.853**
$k=3$	2.145***	0.552	**5.535**	1.123**	**2.090*****	0.459	**5.560**	**1.067****
Panel D: Principal component analysis								
ECON	-0.224***	-2.672	4.984					
TECH	0.689*	**-0.321**	2.838					
ALL	**1.489*****	-2.678	**10.356**					

Notes: The forecasts over the out-of sample-period from January 1966 to December 2014 are evaluated by the equal-weighted (EW) combination on the right and by the sparse EW (SPAR) combination on the left. The combining forecasts based on conventional one-state predictive regressions (Panel A), two-state predictive regressions (with state variable proxied by the MA(2,12), Panel B, or A_{10}, Panel C, technical indicators) and principal component (PC) combination (Panel D). In Panels A, B and C, the candidate predictors are the set of 12 macroeconomic variables considered either in single-, two- or three-variable regressions ($k=1,2,3$). The PC analysis is conducted separately for macroeconomic predictive variables (ECON), technical indicators (TECH) and all of them together (ALL). R^2_{OOS} is the out-of-sample R^2 given in Equation (3.12). Besides the traditional historical average (HA) benchmark, the R^2_{OOS} is also evaluated by considering the EW combining forecasts from individual regressive variables ($k=1$) as a benchmark. When the HA is the benchmark, the R^2_{OOS} for the full-sample (ALL) is also computed separately for NBER-dated expansion (EXP) and recession (REC) months. The hypothesis test H_0: $R^2_{OOS} \leq 0$ against $R^2_{OOS} > 0$ is performed by the Clark and West (2007) out-of-sample MSE-adjusted statistic, where *, ** and *** indicate significance at the 10%, 5%, and 1% levels. In bold, it is the best model of each panel.

literature; see, for example, Henkel *et al.* (2011), Gargano and Timmermann (2012), Dangl and Halling (2012) and Neely *et al.* (2014). For example, the highest R^2_{OOS} for expansions is 1.005% when using the $k=2$ as predictor, while the R^2_{OOS} in recession months for combining methods is always larger than 1.5%.

Figure 3.3 plots the two components of Theil (1971) MSE decomposition for the individual predictive regression models and the combining methods over the out-of-sample period. These components are the squared forecast bias and a remainder term that depends basically on the forecast volatility. To avoid cluttering the diagram, we just plot one technical variable of each group, MA(2,12), MOM(12) and VOL(2,12), as the other variables lie close to the others in the same group. Analogously, we just display the results for combining forecasts from single predictive variables, $k=1$, but considering the EW and SPAR combinations and one-state and two-state models. All combination of forecasts has a lower forecast variance than all of the individual predictive regression models. Furthermore, the SPAR combining forecasts have lower bias than the EW combining forecasts. Finally, two-state models have lower forecast variance than one-state models.

To check the stability of our results over time, Figure 3.4 plots the CSE for the historical average benchmark model minus the CSE for combining forecasting methods. The bottom figures have positively sloped curves during certain periods, but also present relatively long periods where the curves are decreasing, which indicates that PC models do not deliver stable forecasts. We also observe that the predictive ability of combining one-state models deteriorates after the second half of 1990's, as the lines for the EW (or SPAR) combining forecasts from one-state models are predominantly non-crescent. On the other hand, the slopes corresponding to combining two-state forecasts are predominantly crescent. Therefore, combining forecasts from two-state models consistently outperform the benchmark model. Finally, Figure 3.5 plots the CSE for the Rapach *et al.* (2010) model minus the CSE for alternative combining forecasts. We can see that the improvement of introducing two-state models in the combining methods is more evident during recessions, where combining two-state models consistently outperforms combining one-state models.

3.4.2. Results from economic evaluation

The results for asset allocation exercise for the 26 individual regression models are in Table 3.4. Once again, we observe that the indicators variables can explain better future equity premia than macroeconomic predictors. The CER gains are always positive for the technical indicators and ranges from 73 to 315 basis points for a mean-variance investor and from 91 to 298 basis points for a CRRA investor. The largest CER gains and Sharpe ratios is reached for the MA(2,12) variable, for any investor preference. The CER gains are also computed separately for NBER-dated expansion and recession months, but we do not report them here for brevity. As the statistic measure, the economic gains are more evident for recessions in comparison to expansions for all the technical and the majority of macroeconomic variables.

Panel A of Table 3.5 reports the economic results for combining macroeconomic models. Combining forecasts based on one-state models do not outperform the MA(2,12) indicator neither for mean-variance nor for CRRA preferences. In particular, the EW combining forecasts of Rapach *et al.* (2010) have CER gains lower than at least six of individual technical indicators. Although the SPAR combining method increases the CER gains and Sharpe rations in relation to the EW

Table 3.4: Results from asset allocation exercise for individual predictive variables

Macroeconomic variables					Technical variables				
	Mean-variance		CRRA			Mean-variance		CRRA	
Predictor	Δ(%)	Sharpe	Δ(%)	Sharpe	Predictor	Δ(%)	Sharpe	Δ(%)	Sharpe
HA	3.88	0.065	3.65	0.048	MA(1,9)	1.71	0.096	1.84	0.084
					MA(1,12)	2.89	0.126	2.76	0.108
D/Y	-0.03	0.045	0.52	0.036	MA(2,9)	2.09	0.106	2.10	0.092
E/P	0.37	0.064	0.61	0.049	MA(2,12)	**3.15**	**0.133**	**2.98**	**0.113**
D/E	-0.34	0.046	-0.72	0.025	MA(3,9)	2.53	0.119	2.41	0.103
RVOL	-0.65	0.081	-0.71	0.060	MA(3,12)	1.37	0.089	1.48	0.076
B/M	-1.34	0.042	-1.27	0.025	MOM(9)	1.35	0.088	1.50	0.075
NTIS	0.15	0.085	-0.36	0.062	MOM(12)	1.30	0.088	1.46	0.075
TBL	1.94	0.101	**1.66**	0.082	VOL(1,9)	1.66	0.098	1.73	0.084
LTR	1.01	0.102	1.24	0.083	VOL(1,12)	2.58	0.119	2.59	0.104
TMS	**1.97**	**0.127**	1.17	**0.101**	VOL(2,9)	1.50	0.096	1.62	0.083
DFY	-0.79	0.062	-1.02	0.040	VOL(2,12)	1.33	0.091	1.46	0.078
DFR	0.24	0.069	0.31	0.053	VOL(3,9)	0.73	0.077	0.91	0.064
INFL	0.81	0.081	0.60	0.064	VOL(3,12)	2.16	0.109	2.20	0.094

Notes: The forecasts are based on individual macroeconomic (on the left) and technical (on the right) predictive variables, both described in Section 3.2.1, over the out-of sample-period from January 1966 to December 2014. The investor who allocates his wealth between stocks and risk-free bills at the end of each out-of-sample month is assumed to have a mean-variance or constant risk relative risk aversion (CRRA) preferences and a relative risk aversion parameter $\gamma = 5$. $\Delta(\%)$ is the monthly certainty equivalent return (CER) gains, in the annualized form, for a investor who employs a forecasting model to predict the risk premium rather than assuming historical average (HA) forecast. Sharpe ratio is the mean portfolio return in excess of the risk-free rate divided by the standard deviation of the excess of portfolio return. In bold, it is the best model of each panel.

combination, the SPAR combining forecast from one-state models can beat the MA(2,12) model.

We can see in Panels B and C of Table 3.5 that combining forecasts from two-state models improve the economic performance in relation to combinations from one-state models, considering any utility gain. The improvement is notable for either MA(2,12) or A_{10} as state variable. Furthermore, for mean-variance or a CRRA investor, any combining two-state models considered in the analysis have larger CER gains and Sharpe ratios than any forecasts from individual model or combinations from one-state models. The results are even better when the SPAR method is applied to two-state models. Therefore, the SPAR combining forecasts from state-dependent models are economic better than their nested forecasting strategies.

Panel D of Table 3.5 shows that the PC-ALL forecast has CER gains of 412 and 400 basis points for the mean-variance and CRRA preferences, respectively, which is larger than all the 26 individual regression models and combining forecasts from one-state models. However, the CER gains and Sharpe ratios of the SPAR combining forecasts from two-state models are larger than the CER gains and Sharpe ratios of the PC-ALL model for any investor's preference.

To summarize, the combining forecasts from regime-switching predictive models deliver larg-

Table 3.5: Asset allocation exercise for combining forecasts

	Equal-weighted						Sparse					
	Mean-variance			CRRA			Mean-variance			CRRA		
Pred-ictor	$\Delta(\%)$ HA	$\Delta(\%)$ EW	Sharpe ratio	$\Delta(\%)$ HA	$\Delta(\%)$ EW	Sharpe ratio	$\Delta(\%)$ HA	$\Delta(\%)$ EW	Sharpe ratio	$\Delta(\%)$ HA	$\Delta(\%)$ EW	Sharpe ratio
Panel A: One-state regressive models												
$k=1$	1.65	0	0.097	1.67	0	0.080	2.75	1.11	0.127	2.45	0.79	0.106
$k=2$	2.70	1.05	0.121	2.66	1.00	0.103	**3.02**	**1.38**	**0.131**	2.63	0.97	**0.106**
$k=3$	**2.91**	**1.26**	**0.127**	**2.89**	**1.22**	**0.109**	2.85	1.20	0.126	**2.66**	**0.99**	0.105
Panel B: Two-state regressive models - state variable MA(2,12)												
$k=1$	3.30	1.66	0.136	3.53	1.86	0.123	4.36	2.72	0.163	4.38	2.71	0.146
$k=2$	**4.36**	**2.71**	**0.163**	4.46	2.80	**0.146**	**4.50**	**2.86**	**0.166**	4.51	2.84	**0.148**
$k=3$	4.31	2.66	0.162	**4.50**	**2.84**	0.146	4.36	2.71	0.163	**4.55**	**2.88**	0.147
Panel C: Two-state regressive models - state variable A_{10}												
$k=1$	3.62	1.97	0.144	3.84	2.17	0.130	4.12	2.47	0.157	4.31	2.64	0.144
$k=2$	**4.38**	**2.73**	**0.163**	**4.50**	**2.83**	**0.147**	**4.54**	**2.90**	**0.168**	**4.63**	**2.97**	**0.151**
$k=3$	4.25	2.60	0.160	4.38	2.72	0.143	4.38	2.73	0.163	4.49	2.83	0.146
Panel D: Principal component analysis												
ECON	1.60	-0.04	0.113	0.81	-0.83	0.088						
TECH	2.49	0.84	0.117	2.35	0.70	0.100						
ALL	**4.12**	**2.47**	**0.158**	**4.00**	**2.35**	**0.143**						

Notes: The table summarizes the out-of-sample economic performance of equity premium forecasts of equal-weighted (on the left) and sparse EW (on the right) combination of conventional one-state predictive regressions (Panel A), two-state predictive regressions (with state variable proxied by the MA(2,12), Panel B, or A_{10}, Panel C, technical indicators) and principal component (PC) combination (Panel D). The out-of sample-period spans from January 1966 to December 2014. In Panels A, B and C, the candidate predictors are the set of 12 macroeconomic variables considered either in single-, two- or three-variable regressions ($k = 1, 2, 3$). The PC analysis is conducted separately for macroeconomic predictive variables (ECON), technical indicators (TECH) and all of them together (ALL). The investor who allocates his wealth between stocks and risk-free bills at the end of each month is assumed to have a mean-variance or constant relative risk aversion (CRRA) preferences and a relative risk aversion parameter $\gamma = 5$. $\Delta(\%)$ HA is the monthly certainty equivalent return (CER) gains, in the annualized form, for a investor who uses as risk premium forecast a combined forecasts from competing models instead of the historical average (HA) excess return. $\Delta(\%)$ EW is similarly defined with reference to equal-weighted combining forecast from individual regressive variables ($k = 1$). Sharpe ratio is the mean portfolio return in excess of the risk-free rate divided by the standard deviation of the excess of portfolio return. In bold, it is the best model of each panel.

er CER gains and Sharpe ratios compared to forecasts from historical average, simple regressive variables and combining single-state models. Furthermore, the results considering the SPAR combination are better in relation to the EW combination.

3.4.3. Robustness checks

We employ robustness checks to verify whether our results are still valid in other sub-sample periods and for other risk aversion parameter γ . Following Rapach *et al.* (2010), we consider an out-of-sample period starting on January of 1976 and other starting on January 2000. The first case is motivated for the results of Welsh and Goyal (2003) that points that the out-of-sample predictive ability of several economic variables deteriorates notably after the Oil Shock in the period between 1973 and 1975. The second one corresponding to the last 15 years of the full sample. In both forecast-periods, predictability is more unstable and even argued to be spurious by some authors.

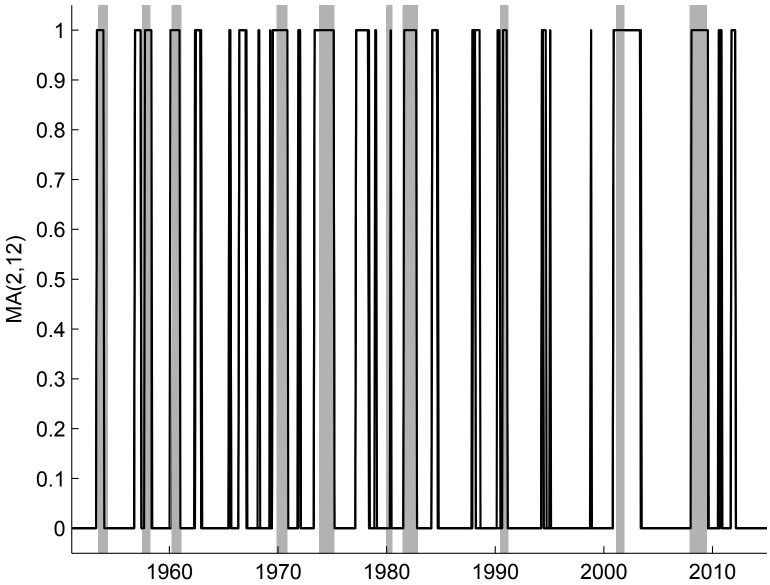
The statistical and economic results in Tables B.1 and B.2 confirm that combining regime-switching model outperforms the other methods for the two alternative out-sample periods. In these sub-samples, the R^2_{OOS} of combining forecast of Rapach *et al.* (2010) and its extension of Elliott *et al.* (2013) are not significantly greater than zero at the 10% level, according to the Clark and West (2007) test, whereas the combining two-state model are in all the cases. Moreover, the CER gains when considering combining two-state models rather than combining one-state models are, in average, 250 and 650 basis points for the out-of samples starting in January 1976 and January 2000, respectively.

Following Cenesizoglu and Timmermann (2012), we also report the asset allocation exercise by considering relative risk aversion parameter equal to 3 and 10 . Tables B.3 and B.4 confirm the better performance of combining two-state models in relation to the other competitors. Also the CER gains of the SPAR combination are larger than the corresponding EW combination. Therefore, our methods are robust to the out-of-sample periods considered and to the relative risk aversion parameter.

3.5. Conclusions

In this Chapter, we extend equity-premium forecast strategies by combining regime-switching models, where, in each individual model, macroeconomic variables are allowed to be interacted with a state variable. We construct proxies of state variables by using technical variables of Neely *et al.* (2014), which are associated with the business cycle and and can identify stock's price trends. In the empirical out-of-sample analysis to the monthly US equity premium series, our new method deliver statistically and economically out-of-sample gains vis-a-vis the historical average, traditional univariate regressions and equal-weighted combination of macroeconomic forecasts. The results are robust to different out-of-sample periods, alternative investor's preferences and different relative risk aversion parameters.

We also propose a sparse EW combining methods as an alternative to the EW combination. In this approach only models with slope coefficients jointly significant are considered in the combining forecasts. Although we observe statistically a mild improvement of the SPAR method in relation to the EW combination, the asset allocation results show evident better performance of the SPAR strategy compared to the EW combining forecasts.



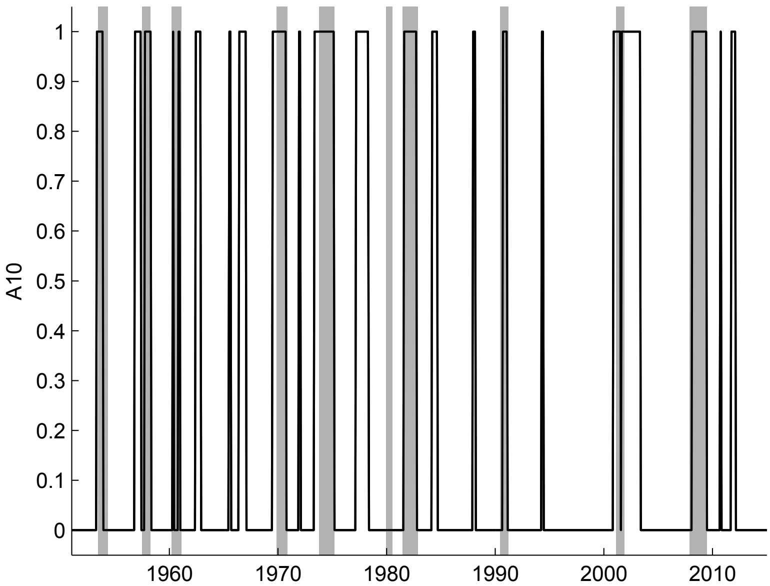


Figure 3.1: Plot of MA(2,12) and A_{10} technical indicators

This figure plots the recession months as signaled by the moving average MA(2,12), top graph, and the agreement (of technical indicators) variable A_{10}, bottom graph. Shaded areas indicate recession months according to NBER business cycle dating. The sample period is from December 1950 to December 2014.

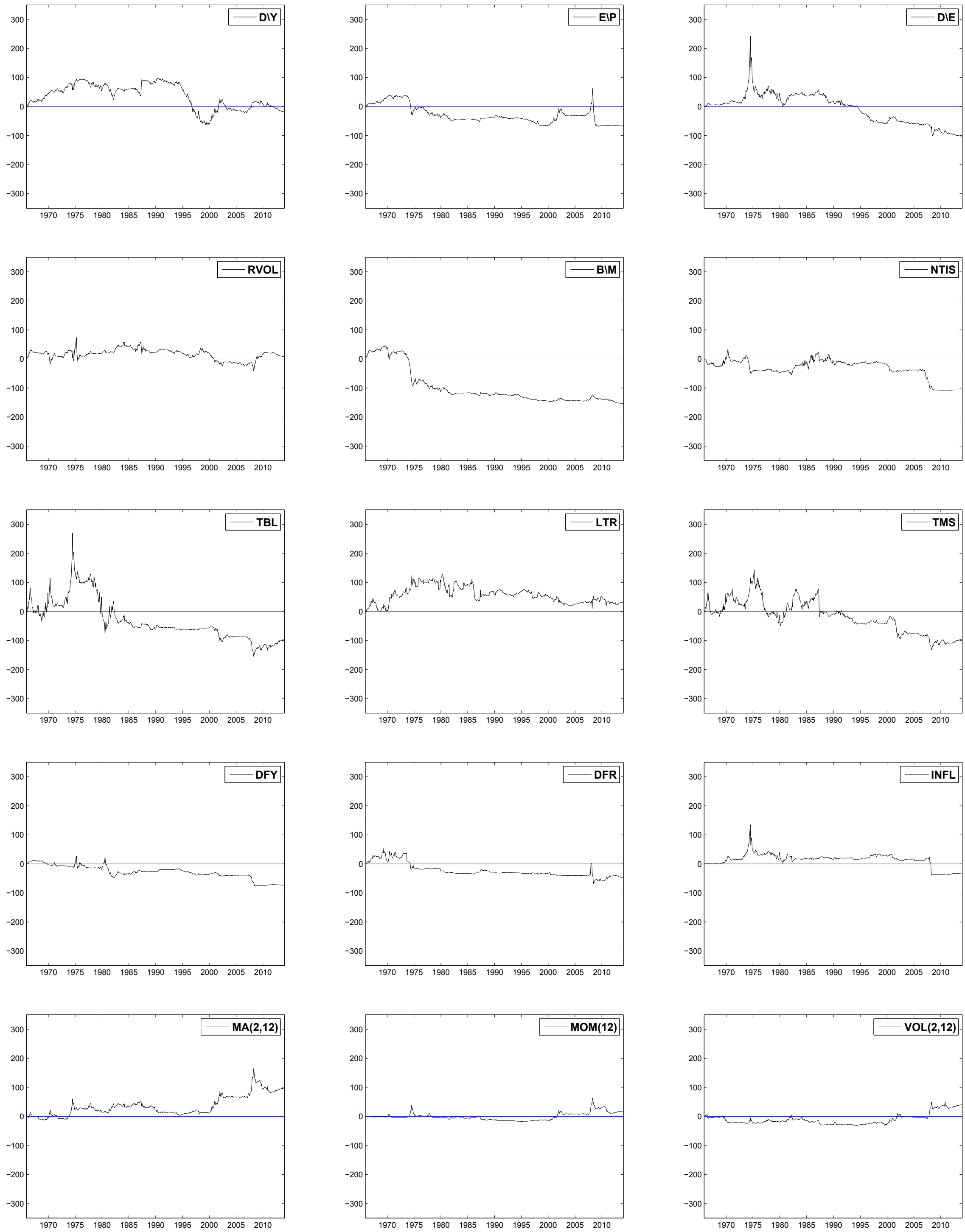


Figure 3.2: Cumulative squared forecast errors from individual models

This figure plots the ΔCSE_t defined as the cumulative difference in the square forecast errors between the historical average (HA) benchmark model and each simple predictive regression model. The individual predictive variables, as labeled in each graph, are described in Section 3.2.1. The out-of-sample period spans from January 1966 to December 2014.

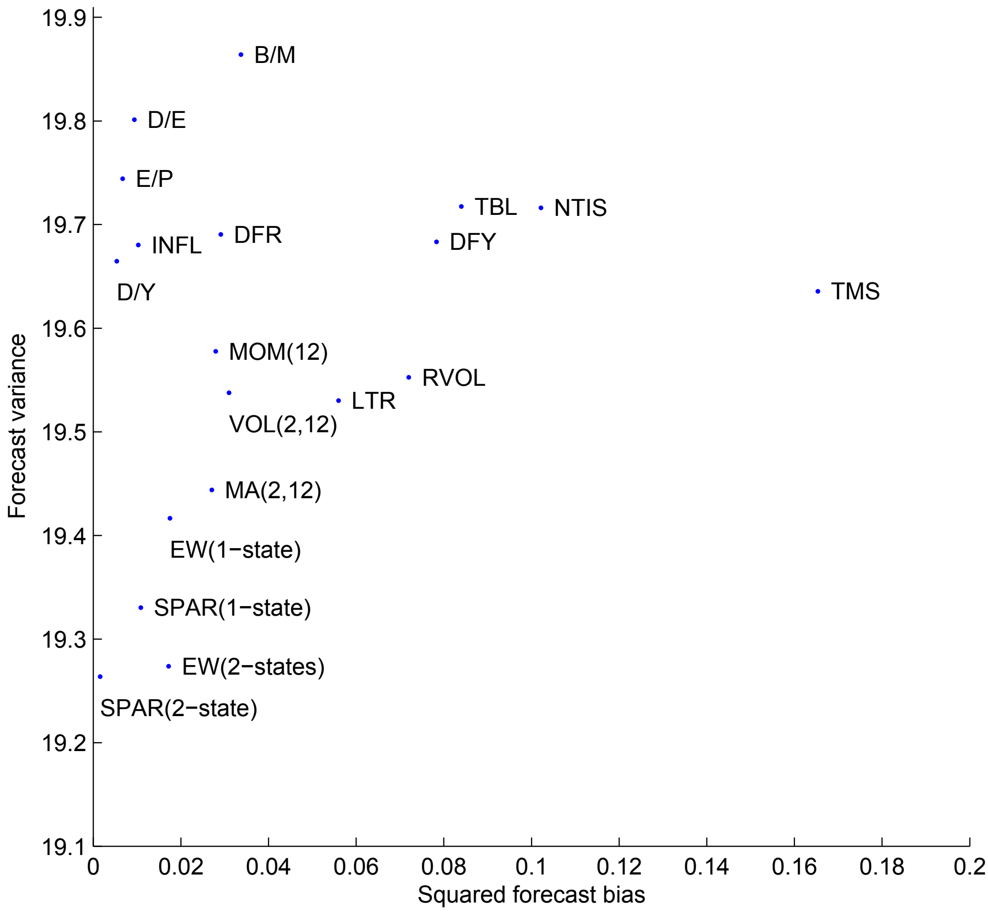


Figure 3.3: Scatterplot of the out-of-sample forecast variance versus the squared forecast bias

The figure is a scatterplot of the out-of-sample forecast variance (Y-axis) versus the squared forecast bias (X-axis) for each of the one-state simple predictive regressions (based on macroeconomic predictors or technical indicators), and equal-weighted (EW) and sparse EW (SPAR) combinations of one-state and two-state models. The macroeconomic predictors, D/Y, E/P, D/E, RVOL, B/M, NTIS, TBill, LTR, TMS, DFY, DFR and INFL, and the technical indicators MA(2,12), MOM(12), VOL(2,12) are as described in Section 3.2.1. The combined forecasts are of the 12 macroeconomic variables considered in one-variable regressions ($k = 1$). The out-of-sample period spans from January 1966 to December 2014.

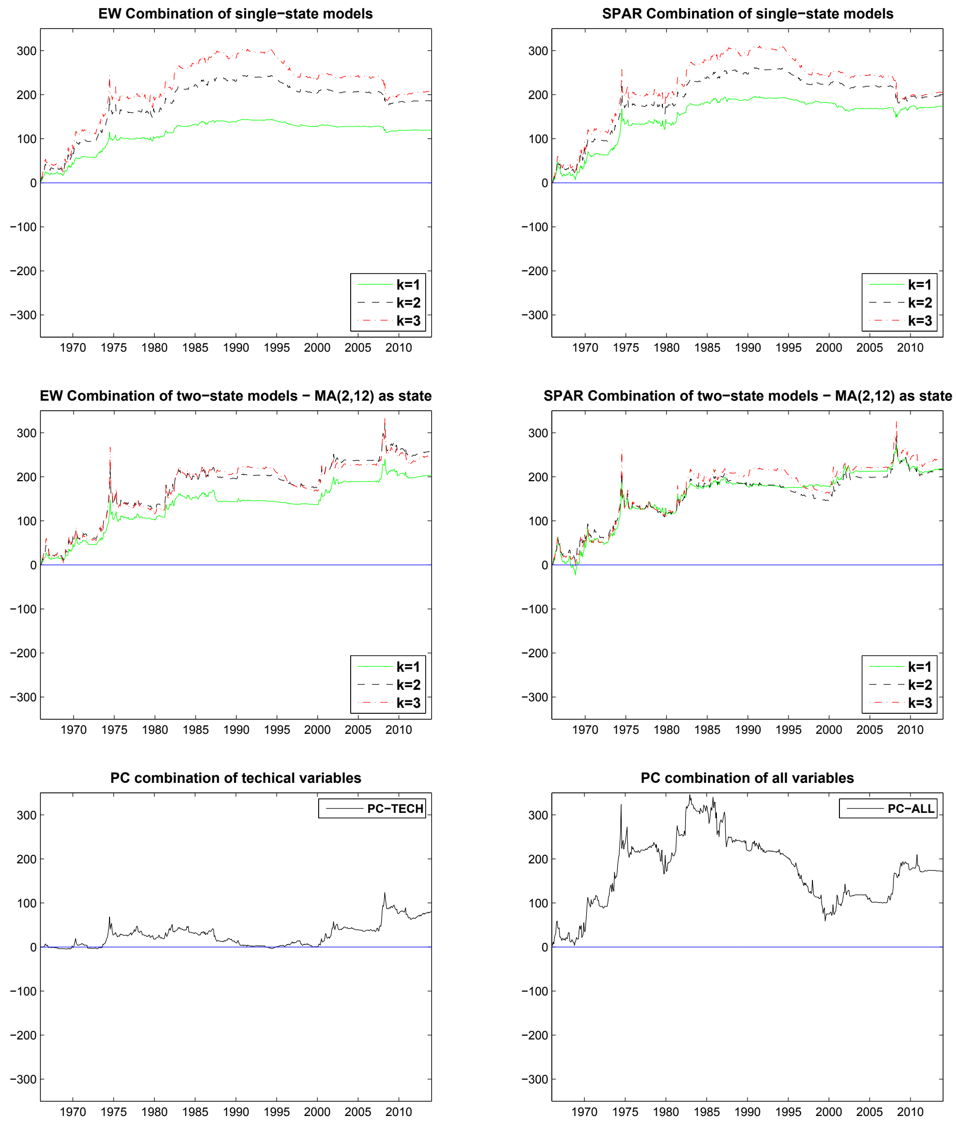


Figure 3.4: Cumulative squared forecast errors from combining forecasts - HA benchmark

This figure plots the ΔCSE_t defined as the cumulative squared forecast errors of the historical average (HA) benchmark model minus the cumulative squared forecast errors from combination of forecasting methods over the out-of-sample period from January 1966 to December 2014. We consider the equal-weighted (EW) and sparse EW (SPAR) combinations of conventional one-state predictive regressions and two-state predictive regressions, with state variable proxied by the MA(2,12), and principal component (PC) combination. In the EW and SPAR combinations, the candidate predictors are the set of 12 macroeconomic variables considered either in single-, two- or three-variable regressions ($k = 1, 2, 3$). The PC analysis is conducted for technical indicators (TECH) and macroeconomic predictive and technical variables together (ALL).

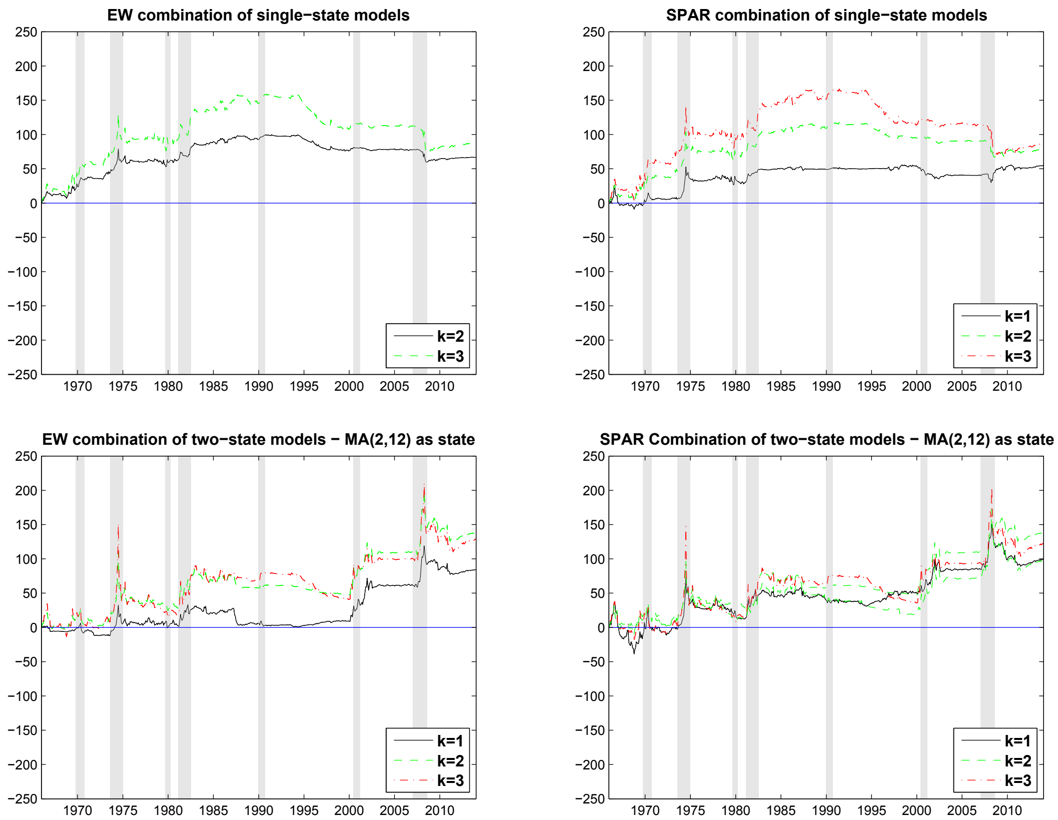


Figure 3.5: Cumulative squared forecast errors from combining forecasts - Rapach *et al.* (2010) benchmark

This figure plots the ΔCSE_t (EW) defined as the cumulative squared forecast errors of equal-weighted combining forecast from individual regressive variables ($k = 1$) minus the cumulative squared forecast errors from combination of forecasting methods over the out-of-sample period from January 1966 to December 2014. We consider the equal-weighted (EW) and sparse EW (SPAR) combinations of conventional one-state predictive regressions and two-state predictive regressions, with state variable proxied by the MA(2,12). In the EW and SPAR combinations, the candidate predictors are the set of 12 macroeconomic variables considered either in single-, two- or three-variable regressions ($k = 1, 2, 3$). Shaded areas indicate recession months according to NBER business cycle dating.

Chapter 4

Summary and Future Research

This thesis studies topics about modeling and predicting expected equity returns and conditional covariances and correlaions. In Chapter 2, we survey the main developments of parametric MGARCH models, updating previous surveys by Bauwens *et al.* (2006), Engle (2009) and Silvennoinen and Teräsvirta (2009a). Following, we carry out Monte Carlo simulations to analyze the potential biases incurred when the restricted models are fitted to systems with rich dynamics as those usually encountered in real data. We show that the restrictions imposed by the BEKK, O-GARCH and GO-GARCH models are very unrealistic generating potentially missleading forecasts of condicional correlations. We emphasize that although the BEKK(1,1,K) is as general as the VECH model in the bivariate case, it is strongly restricted when $K = 1$, as usually considered in empirical applications. On the contrary, models based on the dynamic conditional correlation specification provide appropriate estimates to explain adequately the evolution of volatilities, conditional covariances and correlations generated by the general model. Alternative estimators of the parameters are important to simplify the computations but do not have implications on the estimates of conditional correlations. Hence, we recommend choosing the estimation method based on computational advantages. We fitted the models considered to a five-dimensional system of exchange rate returns. We find that the SBEKK model estimated by S-QML and the DCC models estimated by S-QML have a reasonable forecast performance according to the negative log-likelihood criterion, while the three DCC models outperform all the other models according to the LF criteria.

In Chapter 3, we focus on predictability of the equity market return in excess of the risk-free interest rate. The combining forecasts from macroeconomic predictive models of Rapach *et al.* (2010) and its extension by Elliott *et al.* (2013) outperform statistically and economically the historical average benchmark to explain future equity market returns in excess of the riskless rate. We further extend equity-premium forecast strategies by combining regime-switching models, where, in each individual model, macroeconomic variables are allowed to be interacted with a state variable. We define state variables as simple functions of technical variables of Neely *et al.* (2014), which are associated with the business cycle and can identify stock's price trends. Our methodology is very parsimonious and easy to be interpreted. We carry out an empirical analysis to the monthly US equity premium series in the period from December 1950 to December 2014 and conclude that the combining regime-switching models deliver stable out-of sample gains in relation to the existing methods such as univariate regressions and combinations from one-state regressions widely considered in literature. Our results are confirmed by using three different out-of-sample periods and by an asset allocation exercise with two different investor's preferences and three different relative risk aversion parameters.

In the equal-weighted combining forecasts, each individual predictive regression model forecast have the same combining weight to explain future returns. Rapach *et al.* (2010), Elliott *et al.* (2013), among many other recent works, conclude that the EW combining forecasts are not worse than combining forecasts using estimated weights. Given the relative large set of macroeconomic variables available as candidate predictors, we propose a novel sparse forecast combination method and demonstrate through a comparison exercise with alternative forecast combination methods that it is effective for equity premium prediction. In this approach only models with slope coefficients jointly significant are considering in the combining forecast. Although we observe statistically a mild improvement of SPAR methods in relation to EW combinations, the asset allocation results show evident better performance of SPAR strategy compared to the EW combining forecasts.

It is also of interest to compare the models considered in Chapter 2 with those based on copulas; see, for example, Patton (2006), Lee and Long (2009), So and Yeung (2014) and Creal and Tsay (2015). Furthermore, we focus on symmetric models. However, there is a strong empirical evidence of asymmetries in the responses of conditional variances and covariances to positive versus negative past returns; see, for example, Bollerslev *et al.* (2006) for a comprehensive list of references with empirical evidence about the asymmetric response of volatility to past returns and Kroner and Ng (1998), Cappiello *et al.* (2006) and Caporin and McAleer (2011) for asymmetric response to simultaneous negative returns and simultaneous positive ones. Further research should focus on studying the economic implications of the restrictions imposed on asymmetric MGARCH models to reduce the number of parameters and/or to guarantee covariance stationarity and/or positiveness. The next step of the project will be propose a very general model dynamic conditional correlation model, where all conditional variances, covariances and correlations are related with each other and, at the same time, the model is feasible for systems with very large dimensions, like 50 or 100 or 200.

Numerous works seek to conduct an economic evaluation of forecasts by computing the utility gains (or CER gains). These CER calculations are based on mean-variance or CRRA preferences. However there is a gap in the literature that no paper has consider intertemporal hedging in the investor's decisions together with Epstein-Zin recursive preferences. In future works, we will develop an economic framework for evaluation of forecasts, i.e. the utility gains, to an investor that has recursive preferences, specifically the Epstein-Zin preference as it is assumed by most of the macro-finance literature, and intertemporal hedging choices.

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Appendix A

Appendix to Chapter 2

The DGP is given by the VECH model in equations (2.1) and (2.2) with

a) $\mathbf{\Sigma} = 10^{-4} \times ((2.217\ 0.887)'\ (0.887\ 1.763)')$, $\mathbf{A} = ((0.097\ 0.014\ 0.022)'\ (0.016\ 0.069\ 0.011)'\ (0.025\ 0.01\ 0.105)')$ and $\mathbf{B} = \mathrm{diag}(0.8695, 0.857, 0.85)$, where $\mathrm{vech}(\mathbf{\Sigma}) = (\mathbf{I}_{\frac{N(N+1)}{2}} - \mathbf{A} - \mathbf{B})^{-1}\mathbf{C}$.

b) $\mathbf{\Sigma} = 10^{-4} \times ((1.745\ 0.626\ 0.617\ 0.430\ 0.523)'(0.626\ 2.463\ 0.942\ 0.583\ 0.520)'\ (0.617\ 0.942\ 3.311\ 0.806\ 1.002)'\ (0.430\ 0.583\ 0.806\ 2.045\ 0.704)'\ (0.523\ 0.520\ 1.002\ 0.704\ 2.257)')$,

$$\mathbf{A} =$$

$$\begin{pmatrix}
0.081 & 0.02 & -0.01 & -0.012 & 0.015 & 0.02 & 0 & 0 & 0 & 0.005 & 0 & 0 & 0.011 & 0 & 0.01 \\
0.02 & 0.053 & 0 & 0 & 0 & 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.019 & 0 & 0.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0 & 0 & 0 & 0 & 0 \\
0.017 & 0 & 0 & 0.037 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0 & 0 \\
0.018 & 0 & 0 & 0 & 0.044 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 \\
0.021 & 0.008 & 0 & 0 & 0 & 0.098 & -0.01 & -0.011 & 0.017 & 0.02 & 0 & 0 & 0.012 & 0 & 0.015 \\
0 & 0 & 0 & 0 & 0 & 0.014 & 0.049 & 0 & 0 & 0.02 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.015 & 0 & 0.04 & 0 & 0 & 0 & 0 & 0.01 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.01 & 0 & 0 & 0.045 & 0 & 0 & 0 & 0 & 0 & 0.012 \\
0.018 & 0 & 0.01 & 0 & 0 & 0.017 & -0.009 & 0 & 0 & 0.1 & 0.01 & 0.011 & 0.015 & 0 & 0.019 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.015 & 0.039 & 0 & 0.023 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.02 & 0 & 0.045 & 0 & 0 & 0.019 \\
0.02 & 0 & 0 & -0.01 & 0 & 0.024 & 0 & -0.01 & 0 & 0.014 & -0.01 & 0 & 0.074 & -0.011 & 0.022 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.019 & 0.039 & 0.014 \\
0.017 & 0 & 0 & 0 & 0.007 & 0.02 & 0 & 0 & 0.009 & 0.015 & 0 & -0.01 & 0.02 & 0.01 & 0.07
\end{pmatrix},$$

and $\mathbf{B} = \mathrm{diag}(0.84\ 0.825\ 0.815\ 0.805\ 0.82\ 0.83\ 0.825\ 0.829\ 0.82\ 0.845\ 0.82\ 0.83\ 0.837\ 0.835\ 0.845)$.

Table A.1: Results of the empirical application to a five-dimensional system of daily exchange rate returns for alternative out-of-sample periods

Model-	Second split						Third split					
	1-step-ahead		5-step-ahead		20-step-ahead		1-step-ahead		5-step-ahead		20-step-ahead	
Estimator	NL	SPA	NL	SPA	NL	SPA	NL	SPA	NL	SPA	NL	SPA
DVEC-LSW	6948.1	0.00	7188.6	0.00	7755.3	0.00	5495.2	0.00	5609.2	0.00	6137.1	0.00
DBEKK-CVT	6316.9	0.00	6609.8	0.00	7220.6	0.00	4840.1	0.00	4995.6	0.00	5303.3	0.00
SBEKK-G-QML	6268.3	0.00	6587.8	0.00	7143.2	0.00	4805.6	0.00	5008.1	0.00	5245.5	0.00
-S-QML	**5891.5**	1.00	**6019.5**	1.00	**6242.3**	1.00	**4514.6**	1.00	**4585.8**	1.00	**4849.5**	1.00
-CVT	6366.2	0.00	6678.6	0.00	7080.1	0.00	4849.2	0.00	4942.7	0.00	5334.2	0.00
D-RBEKK-CVT	6284.4	0.00	6543.0	0.00	7022.2	0.00	4816.6	0.00	4940.7	0.00	5311.8	0.00
S-CVT	5924.2	0.02	**6033.6**	0.28	**6242.9**	0.54	4563.2	0.00	4655.3	0.02	4934.8	0.01
S-RBEKK-CVT	6352.7	0.00	6633.2	0.00	7052.4	0.00	4887.2	0.00	4953.7	0.00	5334.2	0.00
S-CVT	5957.2	0.00	6071.9	0.05	6320.1	0.03	4590.7	0.00	4649.9	0.02	4955.8	0.00
O-GARCH	8422.7	0.00	7969.4	0.00	8765.2	0.00	6604.3	0.00	5782.3	0.00	6635.9	0.00
GO-GARCH	7287.4	0.00	7029.2	0.00	7641.9	0.00	5875.6	0.00	5201.8	0.00	5891.9	0.00
CCC-G-2s	6932.1	0.00	7101.2	0.00	7564.7	0.00	5389.6	0.00	5445.3	0.00	5947.0	0.00
S-2s	6473.0	0.00	6526.6	0.00	6661.8	0.00	5049.5	0.00	5049.9	0.00	5079.4	0.00
ECCC-G-2s	7062.3	0.00	7140.4	0.00	7821.3	0.00	5512.2	0.00	5465.5	0.00	6018.1	0.00
S-2s	6547.1	0.00	6414.0	0.00	6873.5	0.00	5090.3	0.00	5052.4	0.00	5247.2	0.00
cDCC-G-3s	6307.6	0.00	6546.3	0.00	7088.9	0.00	4873.0	0.00	4997.3	0.00	5434.5	0.00
S-3s	5995.8	0.00	6105.6	0.01	6336.9	0.03	4628.6	0.00	4683.0	0.00	5026.2	0.00
RDCC-G-3s	6294.5	0.00	6529.7	0.00	7035.4	0.00	4862.9	0.00	4990.5	0.00	5428.4	0.00
S-3s	5987.4	0.00	6091.3	0.01	6307.4	0.02	4619.1	0.00	4669.9	0.00	5019.7	0.00
EDCC-G-3s	6416.8	0.00	6640.1	0.00	7186.0	0.00	4940.7	0.00	4977.4	0.00	5649.7	0.00
S-3s	6074.8	0.00	6097.0	0.00	6388.9	0.01	4658.9	0.00	4661.1	0.00	4926.8	0.00

Notes: Results based on the out-of-sample negative log-likelihood (NL) for $h = 1, 5$ and 20 horizon forecasting of the system of the Euro, British Pound, Swiss Franc, Australian Dollar and Japanese Yen currency returns against the US Dollar currency. On the right of each observed NL is the corresponding p-value for the superior predictive ability (SPA) tests for the null hypothesis that each model is not outperformed by any of the other models. The left panel correspond to the out-of-sample period from January 2, 2008 to December 31, 2013, with for a total of 1550 daily observations (second split) and the right panel to the out-of-sample period from January 2, 2009 to December 31, 2013, for a total of 1291 daily observations (third split). In the bold, the models included in the model confidence set (MCS) at $\alpha = 10\%$.

Appendix B

Appendix to Chapter 3

Table B.1: Results for the out-of sample-period starting on January 1976

Pred-ictor	Equal-weighted R^2_{OOS} HA	R^2_{OOS} EW	$\Delta(\%)$-MV HA	$\Delta(\%)$-MV EW	$\Delta(\%)$-CRRA HA	$\Delta(\%)$-CRRA EW	Sparse R^2_{OOS} HA	R^2_{OOS} EW	$\Delta(\%)$-MV HA	$\Delta(\%)$-MV EW	$\Delta(\%)$-CRRA HA	$\Delta(\%)$-CRRA EW
Panel A: One-state regressive models												
$k=1$	0.24	0	0.19	0	**0.15**	**0**	**0.48***	**0.24**	**0.73**	**0.54**	**0.34**	**0.19**
$k=2$	**0.33**	**0.09**	**0.23**	**0.04**	0.14	0.00	0.30	0.06	0.58	0.40	0.05	-0.10
$k=3$	0.23	-0.02	0.17	-0.02	0.12	-0.03	0.15	-0.10	0.24	0.06	-0.03	-0.18
Panel B: Two-state regressive models - state variable MA(2,12)												
$k=1$	0.79**	0.54	1.94	1.75	2.16	2.01	**1.10****	**0.85****	2.78	2.60	2.72	2.57
$k=2$	**0.99****	**0.75****	2.86	2.67	2.91	2.76	1.09**	0.85**	**3.05**	**2.86**	2.98	2.83
$k=3$	0.92**	0.67**	**2.95**	**2.77**	**3.09**	**2.94**	0.92**	0.68**	3.01	2.82	**3.13**	**2.98**
Panel C: Two-state regressive models - state variable A_{10}												
$k=1$	1.13**	0.89**	2.37	2.19	2.56	2.41	1.03**	0.78**	2.36	2.17	2.54	2.39
$k=2$	**1.37****	**1.13****	**2.61**	**2.42**	2.62	2.47	1.00**	0.75**	**2.94**	**2.75**	**2.93**	**2.78**
$k=3$	1.27**	1.03**	2.59	2.40	**2.65**	**2.50**	**1.25****	**1.01****	2.58	2.40	2.61	2.46
Panel D: Principal component analysis												
ECON	-2.64	-2.89	-0.84	-1.02	-1.86	-2.01						
TECH	**0.61***	**0.37**	**2.28**	**2.09**	**2.03**	**1.88**						
ALL	-0.42**	-0.66*	1.98	1.79	1.76	1.61						

Notes: Statistical and economic results of equity premium forecasts of equal-weighted (on the left) and sparse (on the right) combination of conventional one-state predictive regressions (Panel A), two-state predictive regressions (with state variable proxied by the MA(2,12), Panel B, or A_{10}, Panel C, technical indicators) and principal component (PC) combination (Panel D). In Panels A, B and C, the candidate predictors are the set of 12 macroeconomic variables considered either in single-, two- or three-variable regressions ($k = 1, 2, 3$). The PC analysis is conducted separately for macroeconomic predictive variables (ECON), technical indicators (TECH) and all of them together (ALL). The out-of sample-period spans from January 1976 to December 2014. R^2_{OOS} is the out-of-sample R^2 given in Equation (3.12). Besides the traditional historical average (HA), the R^2_{OOS} is also evaluated by considering the equal-weighted (EW) combining forecast from individual regressive variables ($k = 1$) as benchmark. The hypothesis test H_0: $R^2_{OOS} \leq 0$ against $R^2_{OOS} > 0$ is performed by the Clark and West (2007) out-of-sample MSE-adjusted statistic, where *, ** and *** indicate significance at the 10%, 5%, and 1% levels. The investor who allocates his wealth between stocks and risk-free bills at the end of each out-of-sample month is assumed to have a mean-variance preference and a relative risk aversion parameter $\gamma = 5$. $\Delta(\%)$ HA is the monthly certainty equivalent return (CER) gains, in the annualized form, for a investor who uses as risk premium forecast a combined forecasts from competing models instead of the historical average (HA) excess return. $\Delta(\%)$ EW is similarly defined with reference to EW combining forecast from individual regressive variables ($k = 1$). In bold, it is the best model of each panel.

Table B.2: Results for the out-of sample-period starting on January 2000

Pred-ictor	Equal-weighted R^2_{OOS} HA	R^2_{OOS} EW	$\Delta(\%)$-MV HA	$\Delta(\%)$-MV EW	$\Delta(\%)$-CRRA HA	$\Delta(\%)$-CRRA EW	Sparse R^2_{OOS} HA	R^2_{OOS} EW	$\Delta(\%)$-MV HA	$\Delta(\%)$-MV EW	$\Delta(\%)$-CRRA HA	$\Delta(\%)$-CRRA EW
Panel A: One-state regressive models												
$k=1$	**-0.23**	**0**	-0.62	0	**-0.77**	**0**	**-0.23**	**-0.01**	-0.29	0.34	**-1.02**	**-0.25**
$k=2$	-0.50	-0.27	-0.79	-0.17	-1.11	-0.33	-0.71	-0.48	**-0.28**	**0.34**	-1.63	-0.86
$k=3$	-0.81	-0.58	**-0.36**	**0.26**	-0.86	-0.09	-1.02	-0.79	-0.36	0.26	-1.32	-0.54
Panel B: Two-state regressive models - state variable MA(2,12)												
$k=1$	1.36*	1.59*	4.39	5.01	5.30	6.07	1.63	1.86*	5.73	6.35	6.45	7.23
$k=2$	2.00*	2.23*	6.94	7.56	7.32	8.10	2.03*	2.26*	7.09	7.71	7.40	8.17
$k=3$	**2.34***	**2.57****	**7.43**	**8.05**	**7.68**	**8.46**	**2.33***	**2.56****	**7.45**	**8.07**	**7.68**	**8.45**
Panel C: Two-state regressive models - state variable A_{10}												
$k=1$	1.83*	2.06*	5.10	5.72	5.81	6.58	1.39	1.62	4.88	5.50	5.74	6.51
$k=2$	**2.25***	**2.48****	6.68	7.30	**6.97**	**7.74**	1.99*	2.22*	**7.07**	**7.69**	**7.36**	**8.13**
$k=3$	2.18*	2.41**	**6.69**	**7.31**	6.81	7.58	**2.15***	**2.38****	6.68	7.30	6.80	7.58
Panel D: Principal component analysis												
ECON	-4.29	-4.06	-1.63	-1.00	-2.48	-1.71						
TECH	2.12*	2.35**	**6.65**	**7.27**	7.02	7.79						
ALL	**2.67****	**2.90****	6.64	7.26	**7.63**	**8.41**						

Notes: Statistical and economic results of equity premium forecasts of equal-weighted (on the left) and sparse (on the right) combination of conventional one-state predictive regressions (Panel A), two-state predictive regressions (with state variable proxied by the MA(2,12), Panel B, or A_{10}, Panel C, technical indicators) and principal component (PC) combination (Panel D). In Panels A, B and C, the candidate predictors are the set of 12 macroeconomic variables considered either in single-, two- or three-variable regressions ($k = 1, 2, 3$). The PC analysis is conducted separately for macroeconomic predictive variables (ECON), technical indicators (TECH) and all of them together (ALL). The out-of sample-period spans from January 2000 to December 2014. R^2_{OOS} is the out-of-sample R^2 given in Equation (3.12). Besides the traditional historical average (HA), the R^2_{OOS} is also evaluated by considering the equal-weighted combining (EW) forecast from individual regressive variables ($k = 1$) as benchmark. The hypothesis test H$_0$: $R^2_{OOS} \leq 0$ against $R^2_{OOS} > 0$ is performed by the Clark and West (2007) out-of-sample MSE-adjusted statistic, where *, ** and *** indicate significance at the 10%, 5%, and 1% levels. The investor who allocates his wealth between stocks and risk-free bills at the end of each out-of-sample month is assumed to have a mean-variance preference and a relative risk aversion parameter $\gamma = 5$. $\Delta(\%)$ HA is the monthly certainty equivalent return (CER) gains, in the annualized form, for a investor who uses as risk premium forecast a combined forecasts from competing models instead of the historical average (HA) excess return. $\Delta(\%)$ EW is similarly defined with reference to EW combining forecast from individual regressive variables ($k = 1$). In bold, it is the best model of each panel.

Table B.3: Asset allocation exercise for $\gamma = 3$

	Equal-weighted						Sparse					
	Mean-variance			CRRA			Mean-variance			CRRA		
Predictor	$\Delta(\%)$ HA	$\Delta(\%)$ EW	Sharpe ratio	$\Delta(\%)$ HA	$\Delta(\%)$ EW	Sharpe ratio	$\Delta(\%)$ HA	$\Delta(\%)$ EW	Sharpe ratio	$\Delta(\%)$ HA	$\Delta(\%)$ EW	Sharpe ratio
Panel A: One-state regressive models												
$k = 1$	1.87	0	0.111	1.86	0	0.084	3.73	1.86	0.143	3.75	1.89	0.117
$k = 2$	3.83	1.96	0.145	3.73	1.87	0.117	**4.21**	**2.34**	**0.152**	**3.91**	**2.04**	**0.119**
$k = 3$	**3.98**	**2.11**	**0.149**	**3.99**	**2.12**	**0.122**	3.83	1.96	0.146	3.83	1.96	0.119
Panel B: Two-state regressive models - state variable MA(2,12)												
$k = 1$	3.02	1.14	0.130	3.44	1.58	0.112	4.84	2.96	0.165	4.98	3.11	0.142
$k = 2$	**4.72**	**2.85**	**0.163**	**4.82**	**2.96**	**0.139**	**4.90**	**3.03**	**0.167**	**5.00**	**3.14**	**0.143**
$k = 3$	4.47	2.60	0.159	4.51	2.64	0.135	4.51	2.64	0.160	4.58	2.72	0.137
Panel C: Two-state regressive models - state variable A_{10}												
$k = 1$	3.72	1.85	0.143	4.07	2.20	0.122	4.43	2.56	0.158	4.88	3.02	0.141
$k = 2$	**4.95**	**3.08**	**0.166**	**5.08**	**3.22**	**0.143**	**5.02**	**3.15**	**0.169**	**5.15**	**3.28**	**0.146**
$k = 3$	4.74	2.87	0.164	4.74	2.87	0.139	4.87	3.00	0.167	4.86	3.00	0.141
Panel D: Principal component analysis												
ECON	2.80	0.93	0.127	2.75	0.89	0.100						
TECH	3.07	1.19	0.131	3.16	1.30	0.107						
ALL	**4.49**	**2.62**	**0.160**	**4.86**	**2.99**	**0.142**						

Notes: The table summarizes the out-of-sample economic performance of equity premium forecasts of equal-weighted (on the left) and sparse (on the right) combination of conventional one-state predictive regressions (Panel A), two-state predictive regressions (with state variable proxied by the MA(2,12), Panel B, or A_{10}, Panel C, technical indicators) and principal component (PC) combination (Panel D). In Panels A, B and C, the candidate predictors are the set of 12 macroeconomic variables considered either in single-, two- or three-variable regressions ($k = 1, 2, 3$). The PC analysis is conducted separately for macroeconomic predictive variables (ECON), technical indicators (TECH) and all of them together (ALL). The out-of sample-period spans from January 1966 to December 2014. The investor who allocates his wealth between stocks and risk-free bills at the end of each month is assumed to have a mean-variance or constant relative risk aversion (CRRA) preferences and a relative risk aversion parameter $\gamma = 3$. $\Delta(\%)$ HA is the monthly certainty equivalent return (CER) gains, in the annualized form, for a investor who uses as risk premium forecast a combined forecasts from competing models instead of the historical average (HA) excess return. $\Delta(\%)$ EW is similarly defined with reference to equal-weighted combining forecast from individual regressive variables ($k = 1$). Sharpe ratio is the mean portfolio return in excess of the risk-free rate divided by the standard deviation of the excess of portfolio return. In bold, it is the best model of each panel.

Table B.4: Asset allocation exercise for $\gamma = 10$

	Equal-weighted						Sparse					
	Mean-variance			CRRA			Mean-variance			CRRA		
Pred-ictor	$\Delta(\%)$ HA	$\Delta(\%)$ EW	Sharpe ratio	$\Delta(\%)$ HA	$\Delta(\%)$ EW	Sharpe ratio	$\Delta(\%)$ HA	$\Delta(\%)$ EW	Sharpe ratio	$\Delta(\%)$ HA	$\Delta(\%)$ EW	Sharpe ratio
Panel A: One-state regressive models												
$k = 1$	1.01	0	0.089	1.01	0	0.084	1.11	0.10	0.107	1.07	0.05	0.098
$k = 2$	**1.44**	**0.43**	0.109	**1.42**	**0.41**	0.104	**1.36**	**0.35**	**0.108**	**1.28**	**0.26**	**0.102**
$k = 3$	1.34	0.33	**0.109**	1.34	0.33	**0.105**	1.25	0.24	0.106	1.17	0.16	0.100
Panel B: Two-state regressive models - state variable MA(2,12)												
$k = 1$	1.96	0.95	0.132	2.01	1.00	0.128	2.48	1.47	0.158	2.45	1.44	0.152
$k = 2$	**2.54**	**1.53**	**0.160**	**2.52**	**1.51**	**0.152**	**2.58**	**1.57**	**0.162**	**2.53**	**1.52**	**0.154**
$k = 3$	2.49	1.48	0.158	2.50	1.49	0.150	2.50	1.49	0.159	2.52	1.51	0.151
Panel C: Two-state regressive models - state variable A_{10}												
$k = 1$	2.07	1.06	0.137	2.12	1.11	0.134	2.45	1.44	0.156	2.48	1.47	0.153
$k = 2$	**2.51**	**1.50**	**0.158**	**2.50**	**1.49**	**0.152**	**2.67**	**1.66**	**0.166**	**2.65**	**1.64**	**0.159**
$k = 3$	2.35	1.34	0.151	2.36	1.34	0.144	2.40	1.39	0.153	2.39	1.38	0.146
Panel D: Principal component analysis												
ECON	-1.17	-2.18	0.094	-3.55	-4.56	0.077						
TECH	1.31	0.30	0.111	1.18	0.17	0.103						
ALL	**1.97**	**0.96**	**0.153**	**1.44**	**0.43**	**0.145**						

Notes: The table summarizes the out-of-sample economic performance of equity premium forecasts of equal-weighted (on the left) and sparse (on the right) combination of conventional one-state predictive regressions (Panel A), two-state predictive regressions (with state variable proxied by the MA(2,12), Panel B, or A_{10}, Panel C, technical indicators) and principal component (PC) combination (Panel D). The out-of sample-period spans from January 1966 to December 2014. In Panels A, B and C, the candidate predictors are the set of 12 macroeconomic variables considered either in single-, two- or three-variable regressions ($k = 1, 2, 3$). The PC analysis is conducted separately for macroeconomic predictive variables (ECON), technical indicators (TECH) and all of them together (ALL). The investor who allocates his wealth between stocks and risk-free bills at the end of each month is assumed to have a mean-variance or constant relative risk aversion (CRRA) preferences and a relative risk aversion parameter $\gamma = 10$. $\Delta(\%)$ HA is the monthly certainty equivalent return (CER) gains, in the annualized form, for a investor who uses as risk premium forecast a combined forecasts from competing models instead of the historical average (HA) excess return. $\Delta(\%)$ EW is similarly defined with reference to equal-weighted combining forecast from individual regressive variables ($k = 1$). Sharpe ratio is the mean portfolio return in excess of the risk-free rate divided by the standard deviation of the excess of portfolio return. In bold, it is the best model of each panel.