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Subsidy for New Technology Adoption in Duopoly with Differentiated Goods under Absolute and Relative Profit Maximization

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Abstract. We present an analysis about subsidy policy for adoption of new technology in duopoly with differentiated goods under absolute and relative profit maximization. Technology itself is free, however, firms must expend fixed set-up costs to adopt new technology. There are various cases about optimal policies depending on the level of the set-up cost and whether the goods of the firms are substitutes or complements. In particular, under relative profit maximization there is a case such that the social welfare is maximized when one firm adopts new technology, but no firm adopts new technology without subsidy. Then, the government should give a subsidy to only one firm. It is a discriminatory policy. The government gives a chance to receive a subsidy to only one firm.
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1. Introduction

We present an analysis about subsidy policy for adoption of new technology in duopoly with differentiated goods under absolute and relative profit maximization. Technology itself is free, and production costs are lower with the new technology than the old technology. However, firms must expend fixed set-up costs for adoption of new technology, for example, education costs of their staffs.

Theoretical justification of relative profit maximization is mainly based on evolutionary game theoretic point of view. Schaffer (1989) demonstrates with a Darwinian model of economic natural selection that if firms have market power, profit maximizers are not necessarily the best survivors. A unilateral deviation from Cournot equilibrium decreases the profit of the deviator, but decreases the other firm's profit even more. On the condition of being better than other competitors, firms that deviate from Cournot equilibrium achieve higher payoffs than the payoffs they receive under Cournot equilibrium. He defines the finite population evolutionarily stable strategy (FPESS). It is a strategy of a player that maximizes his relative payoff. This is according to the following fact.

If there are both absolute payoff maximizing players and relative payoff maximizing players, then the latter players earn more *absolute* payoffs than the

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former players; thus relative payoff maximizing strategy is more survival than absolute payoff maximizing strategy.

We think that seeking for relative profit or utility is based on human nature. Even if a person earns big money, he is not happy enough and may be disappointed, if his brother/sister or close friend earns bigger money. On the other hand, even if he is very poor but his neighbor is poorer, he may be consoled by that fact. Also firms in an industry do not only seek to improve their own performance but also want to outperform their rival firms. TV audience-rating race and market share competition by breweries, automobile manufacturers, convenience store chains and mobile-phone carriers, especially in Japan, are examples of such behavior of firms.

For analyses of relative profit maximization please see Gibbons & Murphy (1990), Lu (2011), Matsumura, Matsushima, & Cato (2013), Satoh, & Tanaka (2013), Tanaka (2013a), Schaffer (1989), Satoh, & Tanaka (2014), Tanaka (2013b). Vega-Redondo (1997), in a framework of evolutionary game theory, showed that the equilibrium in a Cournot duopoly with a homogeneous good under relative profit maximization is equivalent to the competitive equilibrium, that is, the price of the good is equal to the marginal cost. If the goods are differentiated, however, we obtain an equilibrium which is not equivalent to the competitive equilibrium. See Satoh, & Tanaka (2014) and Tanaka (2013b).

In Hattori, & Tanaka (2014) adoption of new technology in a Cournot duopoly with differentiated goods is analyzed[1]. In this paper we analyze optimal subsidization policies about adoption of new technology by firms.

We consider the following three-stage game.

1. The first stage: The government determines the level of subsidies to the firms.
2. The second stage: The firms decide whether they adopt new technology or not.
3. The third stage: The firms determine their outputs.

Under absolute profit maximization at the sub-game perfect equilibrium after the second stage of the game one or two or no firm adopts new technology depending on the value of the set-up cost. Under relative profit maximization, if the set-up costs of firms are equal, at the sub-game perfect equilibrium two firms or no firm adopts new technology. On the other hand, if the set-up costs of firms are different, one or two or no firm adopts new technology under relative profit maximization.

The social welfare is defined to be the sum of consumers' utility minus productions costs including the set-up costs of new technology. There are various cases about optimal policies depending on the level of the set-up cost and whether the goods of the firms are substitutes or complements. Examples are as follows.

1. The social welfare is maximized when both firms adopt new technology, but only one firm adopts new technology without subsidy. Then, the government should give subsidies to the firms.
2. The social welfare is maximized when both firms adopt new technology, and both firms adopt new technology without subsidy. Then, the government should do nothing.

There are several other cases. In particular, under relative profit maximization there is the following case.

The social welfare is maximized when one firm adopts new technology, however, no firm adopts new technology without subsidy. Then, the government should give a subsidy to only one firm. It is a discriminatory policy. The

[1] Also using a similar model Pal (2010) compared Cournot and Bertrand outcomes.

government gives a chance to receive a subsidy to only one firm. If both firms have a chance to receive a subsidy, both of them adopt new technology.

In the next section we present a model of this paper. In Section 3 we analyze the optimal subsidy policy when firms maximize their absolute profits. In Section 4 we analyze the optimal subsidy policy when firms maximize their relative profits.

2. The Model

Two firms, Firm A and B, produce differentiated goods, and consider adoption of new technology from a foreign country. Technology itself is free, however, each firm must expend a fixed set-up cost for adoption of new technology, for example, education cost of its staff. Denote the outputs of Firm A and B by x_A and x_B, the prices of their goods by p_A and p_B. The utility function of consumers is assumed to be

$$u = a(x_A + x_B) - \frac{1}{2}x_A^2 - bx_A x_B - \frac{1}{2}x_B^2,$$

where $a > 0$. If the goods of the firms are substitutes, $0 < b < 1$, and if the goods are complements, $-1 < b < 0$. From this utility function the inverse demand functions of the goods are derived as follows.

$$p_A = a - x_A - bx_B,$$

$$p_B = a - x_B - bx_A.$$

The marginal cost before adoption of new technology is $c > 0$, and the marginal cost after adoption of new technology is zero. They are common to both firms. A fixed set-up cost is $e > 0$, which is also common. We assume $a > c$ and $a > \frac{c}{1-b}$ so that the equilibrium outputs of the firms are positive under absolute and relative profit maximization.

We analyze the optimal subsidy policies of the government for adoption of new technology by the firms under absolute and relative profit maximization.

If adoption of new technology and non-adoption are indifferent for a firm, then it adopts new technology.

3. Absolute profit maximization

3.1. Case of substitutes

First we assume that the goods of the firms are substitutes. Then $0 < b < 1$. The case of complements is treated in the next subsection.

The profits of Firm A and B before adoption of new technology are

$$\pi_A = (a - x_A - bx_B)x_A - cx_A,$$

and

$$\pi_B = (a - x_B - bx_A)x_B - cx_B.$$

After adoption of new technology they are

$$\pi_A = (a - x_A - bx_B)x_A - e,$$

and

$$\pi_B = (a - x_B - bx_A)x_B - e.$$

We assume Cournot type behavior of firms.

The conditions for profit maximization in the third stage of the game when both firms adopt new technology are

$$a - 2x_A - bx_B = 0,$$

and

$$a - 2x_B - bx_A = 0.$$

The equilibrium outputs are

$$x_A = x_B = \frac{a}{2+b}.$$

The prices of the goods are

$$p_A = p_B = \frac{a}{2+b}.$$

The profits of Firm A and B, π_A and π_B, are

$$\pi_A = \pi_B = \frac{a^2}{(2+b)^2} - e.$$

The conditions for profit maximization when only Firm A adopts new technology are

$$a - 2x_A - bx_B = 0,$$

and

$$a - 2x_B - bx_A - c = 0.$$

The equilibrium outputs are

$$x_A = \frac{(2-b)a + bc}{4 - b^2},$$

and

$$x_B = \frac{(2-b)a-2c}{4-b^2}.$$

The prices of the goods are

$$p_A = \frac{(2-b)a+bc}{4-b^2},$$

and

$$p_B = \frac{(2-b)a+(2-b^2)c}{4-b^2}.$$

The profits of the firms are as follows.

$$\pi_A = \frac{[(2-b)a+bc]^2}{(4-b^2)^2} - e,$$

and

$$\pi_B = \frac{[(2-b)a-2c]^2}{(4-b^2)^2}.$$

Similarly, the profits of the firms when only Firm B adopts new technology are

$$\pi_A = \frac{[(2-b)a-2c]^2}{(4-b^2)^2},$$

and

$$\pi_B = \frac{[(2-b)a+bc]^2}{(4-b^2)^2} - e.$$

The conditions for profit maximization when no firm adopts new technology are

$$a-2x_A-bx_B-c=0,$$

and

$$a-2x_B-bx_A-c=0.$$

The equilibrium outputs are

$$x_A = x_B = \frac{a-c}{2+b}.$$

The prices of the goods are

$$p_A = p_B = \frac{a+(1+b)c}{2+b}.$$

The profits of the firms are

$$\pi_A = \pi_B = \frac{(a-c)^2}{(2+b)^2}.$$

If

$$\frac{a^2}{(2+b)^2} - e \geq \frac{[(2-b)a-2c]^2}{(4-b^2)^2},$$

the best response of each firm when the rival firm adopts new technology is adoption of new technology. Then, we have

$$e \leq \frac{4c[(2-b)a-c]}{(4-b^2)^2}.$$

If

$$\frac{[(2-b)a+bc]^2}{(4-b^2)^2} - e \geq \frac{(a-c)^2}{(2+b)^2},$$

the best response of each firm when the rival firm does not adopt new technology is adoption of new technology. Then, we have

$$e \leq \frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2}.$$

Since $\frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2} > \frac{4c[(2-b)a-c]}{(4-b^2)^2}$, we get the following lemma.

Lemma 1 *Under absolute profit maximization when the goods are substitutes, the subgame-perfect equilibria of the game after the second stage are as follows.*

1. If $e \leq \frac{4c[(2-b)a-c]}{(4-b^2)^2}$, the subgame-perfect equilibrium is a state where both firms adopt new technology.
2. If $\frac{4c[(2-b)a-c]}{(4-b^2)^2} < e \leq \frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2}$, the subgame-perfect equilibrium is a state where only one firm, A or B, adopts new technology.
3. If $e > \frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2}$, the subgame-perfect equilibrium is a state where no firm adopts new technology.

Social welfare

The social welfare is equal to the sum of the profits of the firms and the consumers' surplus. Denote the social welfare when both firms adopt new technology by W^2, that when one firm (for example, Firm A) adopts new technology by W^1 and that when no firm adopts new technology by W^0. Then, we have

$$W^2 = a(x_A + x_B) - \frac{1}{2}x_A^2 - bx_Ax_B - \frac{1}{2}x_B^2 - 2e = \frac{(3+b)a^2}{(2+b)^2} - 2e,$$

$$W^1 = ax_A + (a-c)x_B - \frac{1}{2}x_A^2 - bx_Ax_B - \frac{1}{2}x_B^2 - e$$
$$= \frac{2a(a-c)(3+b)(2-b)^2 + (12-b^2)c^2}{2(4-b^2)^2} - e,$$

and

$$W^0 = (a-c)(x_A + x_B) - \frac{1}{2}x_A^2 - bx_Ax_B - \frac{1}{2}x_B^2 = \frac{(3+b)(a-c)^2}{(2+b)^2}.$$

Let

$$e^0 = W^1 - W^0 + e = \frac{(24a - 16ab - 2ab^2 + 2ab^3 - 12c + 16bc + b^2c - 2b^3c)c}{2(4-b^2)^2},$$

and

$$e^1 = W^2 - W^1 + e = \frac{(24a - 16ab - 2ab^2 + 2ab^3 - 12c + b^2c)c}{2(4-b^2)^2}.$$

Then, if and only if $e \le e^0$, $W^1 \ge W^0$, and if and only if $e \le e^1$, $W^2 \ge W^1$. We have

$$e^0 - e^1 = \frac{(8-b^2)bc^2}{(4-b^2)^2} > 0.$$

Thus, we obtain the following lemma.

Lemma 2 *Under absolute profit maximization, when the goods are substitutes;*

1. If $e \le e^1$, W^2 is the maximum, and adoption of new technology by both firms is optimal;
2. If $e^1 < e \le e^0$, W^1 is the maximum, and adoption of new technology by one firm is optimal;
3. If $e > e^0$, W^0 is the maximum, and non-adoption of new technology is optimal.

We find

$$e^1 - \frac{4c[(2-b)a - (1-b)c]}{(4-b^2)^2} = \frac{[2a(1-b)(4-b^2) + (b^2 - 8b - 4)c]c}{2(4-b^2)^2} > 0.$$

and

$$e^1 - \frac{4c[(2-b)a-c]}{(4-b^2)^2} = \frac{(2a-2ab-c)c}{2(2-b)(2+b)} > 0.$$

These inequalities are obtained from the assumption of $a > c$ and $a > \frac{c}{1-b}$. Then, we get the following theorem.

Theorem 1. *Under absolute profit maximization when the goods of the firms are substitutes, the optimal policies should be as follows;*

1. If $e \leq \frac{4c[(2-b)a-c]}{(4-b^2)^2}$, W^2 is optimal and both firms adopt new technology without subsidy. The government should do nothing.

2. If $\frac{4c[(2-b)a-c]}{(4-b^2)^2} < e \leq \frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2}$, W^2 is optimal, however, one firm adopts new technology without subsidy. The government should give subsidies to the firms. The level of the subsidy to each firm is

$$e - \frac{4c[(2-b)a-c]}{(4-b^2)^2}.$$

The government must give subsidies to both firms because the best response of a firm is non-adoption when the rival firm adopts new technology without subsidy.

3. If $\frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2} < e \leq e^1$, W^2 is optimal, however, no firm adopts new technology without subsidy. The government should give subsidies to both firms. The level of the subsidy to each firm is

$$e - \frac{4c[(2-b)a-c]}{(4-b^2)^2}.$$

It is not $e - \frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2}$, and

$$e - \frac{4c[(2-b)a-c]}{(4-b^2)^2} > e - \frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2}.$$

4. If $e^1 < e \leq e^0$, W^1 is optimal, however, no firm adopts new technology without subsidy. The government should give subsidies to the firms. The level of the subsidy is

$$e - \frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2}.$$

The government give a chance to receive a subsidy to both firms, but actually gives a subsidy to one of the firms which adopts new technology. It is not a discriminatory policy. Each firm does not have an incentive to receive a subsidy when the other firm receives a subsidy and adopts new technology.

5. If $e > e^0$, W^0 is optimal and no firm adopts new technology without subsidy. The government should do nothing.

3.2. Case of complements

If the goods are complements, $-1 < b < 0$. Then, we have

$$\frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2} < \frac{4c[(2-b)a-c]}{(4-b^2)^2},$$

and

$$e^0 - e^1 < 0.$$

Let

$$\bar{e} = \frac{1}{2}(W^2 - W^0 + 2e) = \frac{(2a-c)(3+b)c}{2(2+b)^2}.$$

If and only if $e \le \bar{e}$, $W^2 \ge W^0$. We have

$$\bar{e} - e^0 = \frac{b(b^2-8)c^2}{2(2-b)^2(2+b)^2} > 0,$$

and

$$e^1 - \bar{e} = \frac{b(b^2-8)c^2}{2(2-b)^2(2+b)^2} > 0.$$

The signs of them are due to $-1 < b < 0$.

Lemma 1 is modified as follows.

Lemma 3. *Under absolute profit maximization when the goods are complements, the subgame-perfect equilibria are as follows.*

1. If $e \le \frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2}$, the subgame-perfect equilibrium is a state where both firms adopt new technology.
2. If $\frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2} < e \le \frac{4c[(2-b)a-c]}{(4-b^2)^2}$, there are two subgame-perfect equilibria. One is a state where both firms adopt new technology, and the other is a state where no firm adopts new technology.
3. If $e > \frac{4c[(2-b)a-c]}{(4-b^2)^2}$, the subgame-perfect equilibrium is a state where no firm adopts new technology.

Also Lemma 2 is modified as follows.

Lemma 4. *When the goods are complements;*

1. if $e \le \bar{e}$ ($e \le e^0$ or $e^0 < e \le \bar{e}$), W^2 is the maximum, and adoption of new technology by both firms is optimal;

2. if $e > \bar{e}$ ($\bar{e} < e \le e^1$ or $e > e^1$), W^0 is the maximum, and non-adoption of new technology is optimal.

We find, if $a > c$

$$\bar{e} - \frac{4c[(2-b)a-c]}{(4-b^2)^2} = \frac{(8a-8ab+2ab^3-2ab^2-b^3c+b^2c+8bc-4c)c}{2(2-b)^2(2+b)^2} > 0,$$

and

$$\bar{e} - \frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2} = \frac{(8a-8ab+2ab^3-2ab^2-b^3c+b^2c-4c)c}{2(2-b)^2(2+b)^2} > 0.$$

Thus, we get the following theorem.

Theorem 2. *Under absolute profit maximization when the goods of the firms are complements, the optimal policies should be as follows.*

1. If $e \le \frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2}$, W^2 is optimal and both firms adopt new technology without subsidy. The government should do nothing.

2. If $\frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2} < e \le \bar{e}$, W^2 is optimal but no firm may adopt new technology without subsidy. The government should give subsidies to both firms. The level of the subsidy to each firm is

$$e - \frac{4c[(2-b)a-(1-b)c]}{(4-b^2)^2}.$$

Both firms may adopt new technology without subsidy. However, they may not adopt. Subsidization to the firms does not reduce the social welfare.

3. If $e > \bar{e}$, W^0 is optimal and no firm adopts new technology. The government should do nothing.

4. Relative profit maximization

4.1. Case of substitutes

In this section we assume that the set-up costs of the firms may be different because the government may adopt a discriminatory policy. In such a policy the government gives a chance to receive a subsidy to only Firm A. The set-up costs of Firm A and B are denoted by e_A and e_B, and the subsidy is denoted by s. When the government gives a subsidy to only Firm A,

$$e_A = e - s,$$

and

$$e_B = e.$$

When the government gives subsidies to both firms,

$$e_A = e_B = e - s,$$

and when the government gives a subsidy to no firm,

$$e_A = e_B = e.$$

Thus, $e_A \leq e_B$ in any case.

The relative profit of each firm is defined to be the difference between its profit and the profit of its rival firm. Denote the relative profits of Firm A and B by Π_A and Π_B. When both firms adopt new technology, we have

$$\Pi_A = (a - x_A - bx_B)x_A - e_A - (a - x_B - bx_A)x_B + e_B,$$

and

$$\Pi_B = -\Pi_A = (a - x_B - bx_A)x_B - e_B - (a - x_A - bx_B)x_A + e_A.$$

The conditions for relative profit maximization are

$$a - 2x_A = 0,$$

and

$$a - 2x_B = 0.$$

The equilibrium outputs are

$$x_A = x_B = \frac{a}{2}.$$

The prices of the goods are

$$p_A = p_B = \frac{(1-b)a}{2}.$$

The absolute profits of the firms are as follows.

$$\pi_A = \frac{(1-b)a^2}{4} - e_A,$$

and

$$\pi_B = \frac{(1-b)a^2}{4} - e_B.$$

The relative profits of the firms are

$$\Pi_A = -\Pi_B = -e_A + e_B.$$

When no firm adopts new technology,

$$\Pi_A = (a - x_A - bx_B)x_A - cx_A - (a - x_B - bx_A)x_B + cx_B,$$

and

$$\Pi_B = -\Pi_A = (a - x_B - bx_A)x_B - cx_B - (a - x_A - bx_B)x_A + cx_A.$$

The conditions for relative profit maximization are

$$a - 2x_A - c = 0,$$

and

$$a - 2x_B - c = 0.$$

The equilibrium outputs are

$$x_A = x_B = \frac{a-c}{2}.$$

The prices of the goods are

$$p_A = p_B = \frac{(1-b)a + (1+b)c}{2}.$$

The absolute profits of the firms are as follows.

$$\pi_A = \pi_B = \frac{(1-b)(a-c)^2}{4}.$$

The relative profits of the firms are

$$\Pi_A = \Pi_B = 0.$$

When only Firm A adopts new technology,

$$\Pi_A = (a - x_A - bx_B)x_A - e_A - (a - x_B - bx_A)x_B + cx_B,$$

and

$$\Pi_B = -\Pi_A = (a - x_B - bx_A)x_B - cx_B - (a - x_A - bx_B)x_A + e_A.$$

The conditions for relative profit maximization are

$$a - 2x_A = 0,$$

and

$$a - 2x_B - c = 0.$$

The equilibrium outputs are

$$x_A = \frac{a}{2},$$

and

$$x_B = \frac{a-c}{2}.$$

The prices of the goods are

$$p_A = \frac{(1-b)a + bc}{2},$$

and

$$p_B = \frac{(1-b)a + c}{2}.$$

The absolute profits of the firms are

$$\pi_A = \frac{a[(1-b)a + bc]}{4} - e_A,$$

and

$$\pi_B = \frac{(a-c)[(1-b)a - c]}{4}.$$

The relative profits of the firms are

$$\Pi_A = \frac{a[(1-b)a + bc]}{4} - \frac{(a-c)[(1-b)a - c]}{4} - e_A = \frac{(2a-c)c}{4} - e_A,$$

and

$$\Pi_B = -\frac{(2a-c)c}{4} + e_A.$$

By the assumption of $a > \frac{c}{1-b}$ the absolute profit of each firm is positive. If $e_A < \frac{(2a-c)c}{4}$, we have $\Pi_A > 0$ and $\Pi_B < 0$, if $e_A > \frac{(2a-c)c}{4}$, we have $\Pi_A < 0$ and $\Pi_B > 0$. When only Firm B adopts new technology, the absolute profits of the firms are

$$\pi_A = \frac{(a-c)[(1-b)a-c]}{4},$$

and

$$\pi_B = \frac{a[(1-b)a+bc]}{4} - e_B.$$

The relative profits of the firms are

$$\Pi_A = -\frac{a[(1-b)a+bc]}{4} + \frac{(a-c)[(1-b)a-c]}{4} + e_B = -\frac{(2a-c)c}{4} + e_B,$$

and

$$\Pi_B = \frac{(2a-c)c}{4} - e_B.$$

The game after the second stage is depicted as follows.

		B	
		adoption of new technology	non-adoption
A	adoption of new technology	$e_A - e_B$ $e_B - e_A$	$-\frac{(2a-c)c}{4} + e_A$ $\frac{(2a-c)c}{4} - e_A$
	non-adoption	$\frac{(2a-c)c}{4} - e_B$ $-\frac{(2a-c)c}{4} + e_B$	0 0

If $\frac{(2a-c)c}{4} - e_A \geq 0$, adoption of new technology is a dominant strategy for Firm A, and if $\frac{(2a-c)c}{4} - e_A < 0$, non-adoption is a dominant strategy for Firm A. Similarly, if $\frac{(2a-c)c}{4} - e_B \geq 0$, adoption of new technology is a dominant strategy for Firm B, and if $\frac{(2a-c)c}{4} - e_B < 0$, non-adoption is a dominant strategy for Firm B. Thus, we obtain the following lemma.

Lemma 5. *Under relative profit maximization, when the government gives a subsidy to only Firm A, we have* $e_A < e_B$ *, and the sub-game perfect equilibria are as follows.*

1. If $e_B \le \frac{(2a-c)c}{4}$, the sub-game perfect equilibrium is a state where both firms adopt new technology.
2. If $e_A < \frac{(2a-c)c}{4} \le e_B$, the sub-game perfect equilibrium is a state where only Firm A adopts new technology.
3. If $e_A > \frac{(2a-c)c}{4}$, the sub-game perfect equilibrium is a state where no firm adopts new technology.

On the other hand, if the government gives a subsidy to no firm, or gives the same subsidies to both firms, we have $e_A = e_B$, and the sub-game perfect equilibria are as follows.

1. If $e_A \le \frac{(2a-c)c}{4}$, the sub-game perfect equilibrium is a state where both firms adopt new technology.
2. If $e_A > \frac{(2a-c)c}{4}$, the sub-game perfect equilibrium is a state where no firm adopts new technology.

This lemma holds whether the goods of the firm are substitutes or complements.

Social welfare

Assume that the goods of the firms are substitutes. Denote the social welfare when both firms adopt new technology by $\tilde{W}^2$, that when one firm adopts new technology by $\tilde{W}^1$, and that when no firm adopts new technology by $\tilde{W}^0$. Then, we have

$$\tilde{W}^2 = a(x_A + x_B) - \frac{1}{2}x_A^2 - bx_Ax_B - \frac{1}{2}x_B^2 - 2e = \frac{(3-b)a^2}{4} - 2e,$$

$$\tilde{W}^1 = ax_A + (a-c)x_B - \frac{1}{2}x_A^2 - bx_Ax_B - \frac{1}{2}x_B^2 - e = \frac{3c^2 + 2abc - 6ac - 2a^2b + 6a^2}{8} - e,$$

and

$$\tilde{W}^0 = (a-c)(x_A + x_B) - \frac{1}{2}x_A^2 - bx_Ax_B - \frac{1}{2}x_B^2 = \frac{(3-b)(a-c)^2}{4}.$$

Let

$$\tilde{e}^0 = \tilde{W}^1 - \tilde{W}^0 + e = \frac{c(6a - 2ab + 2bc - 3c)}{8},$$

and

$$\tilde{e}^1 = \tilde{W}^2 - \tilde{W}^1 + e = \frac{c(6a - 2ab - 3c)}{8}.$$

Then, if and only if $e \le \tilde{e}^0$, $\tilde{W}^1 \ge \tilde{W}^0$, and if and only if $e \le \tilde{e}^1$, $\tilde{W}^2 \ge \tilde{W}^1$. We have

$$\tilde{e}^0 - \tilde{e}^1 = \frac{bc^2}{4} > 0.$$

Thus, we obtain the following lemma.

Lemma 6 *Under relative profit maximization, when the goods of the firms are substitutes;*

1. If $e \le \tilde{e}^1$, $\tilde{W}^2$ is the maximum, and adoption of new technology by both firms is optimal.
2. If $\tilde{e}^1 < e \le \tilde{e}^0$, $\tilde{W}^1$ is the maximum, and adoption of new technology by one firm is optimal.
3. If $e > \tilde{e}^0$, $\tilde{W}^0$ is the maximum, and non-adoption of new technology is optimal.

We find

$$\tilde{e}^0 - \frac{(2a-c)c}{4} = \frac{(2a - 2ab + 2b - c)c}{8} > 0,$$

and

$$\tilde{e}^1 - \frac{(2a-c)c}{4} = \frac{(2a - 2ab - c)c}{8} > 0.$$

Thus, we get the following theorem.

Theorem 3 *Under relative profit maximization when the goods of the firms are substitutes, the optimal policies should be as follows.*

1. If $e \le \frac{(2a-c)c}{4}$, $\tilde{W}^2$ is optimal and both firms adopt new technology without subsidy. The government should do nothing.
2. If $\frac{(2a-c)c}{4} < e \le \tilde{e}^1$, $\tilde{W}^2$ is optimal, however, no firm adopts new technology without subsidy. The government should give subsidies to both firms. The level of the subsidy to each firm is

$$e - \frac{(2a-c)c}{4}.$$

3. If $\tilde{e}^1 < e \le \tilde{e}^0$, $\tilde{W}^1$ is optimal and no firm adopts new technology without subsidy. The government should give a subsidy to only one firm. It is a discriminatory policy. The government gives a chance to receive a subsidy to only Firm A. If both firms have a chance to receive a subsidy, they adopt new technology. The level of the subsidy to Firm A is

$$e - \frac{(2a-c)c}{4}.$$

4. If $e > \tilde{e}^0$, $\tilde{W}^0$ is optimal and no firm adopts new technology without subsidy. The government should do nothing.

4.2. Case of complements

If the goods of the firms are complements, then we have

$$\tilde{e}^0 - \tilde{e}^1 < 0.$$

Let

$$e^* = \frac{1}{2}\left(\tilde{W}^2 - \tilde{W}^0 + 2e\right) = \frac{(2a-c)(3-b)c}{8}.$$

Then,

$$\tilde{e}^1 - e^* = -\frac{bc^2}{8} > 0,$$

and

$$e^* - \tilde{e}^0 = -\frac{bc^2}{8} > 0.$$

The signs of them are due to $b < 0$.

Lemma 5 is not changed, however Lemma 6 is modified as follows;

Lemma 7. *Under relative profit maximization, when the goods of the firms are complements;*

1. If $e \le e^*$ ($e \le \tilde{e}^0$ or $\tilde{e}^0 < e \le e^*$), $\tilde{W}^2$ is the maximum, and adoption of new technology by both firms is optimal.
2. If $e > e^*$ ($e^* < e \le \tilde{e}^1$ or $e > \tilde{e}^1$), $\tilde{W}^0$ is the maximum, and non-adoption of new technology is optimal.

We find

$$e^* - \frac{(2a-c)c}{4} = \frac{(2a-c)(3-b)c}{8} - \frac{(2a-c)c}{4} = \frac{(2a-c)(1-b)c}{8} > 0.$$

Thus, we get the following theorem.

Theorem 4 *Under relative profit maximization when the goods of the firms are complements, the optimal policies should be as follows.*

1. If $e \le \frac{(2a-c)c}{4}$, $\tilde{W}^2$ is optimal and both firms adopt new technology without subsidy. The government should do nothing.
2. If $\frac{(2a-c)c}{4} < e \le e^*$, $\tilde{W}^2$ is optimal but no firm adopts new technology without subsidy. The government should give subsidies to the firms. The level of the subsidy to each firm is

$$e - \frac{(2a-c)c}{4}.$$

3. If $e > e^*$, $\tilde{W}^0$ is optimal and no firm adopts new technology without subsidy. The government should do nothing.

In the future we will study a game of subsidization for adoption of new technology between countries in an international duopoly.

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