Evaluation of Rotor Design Parameters for Minimising Torque Ripple on a Synchronous Reluctance Machine using Multifactor ANOVA

M. Muteba*, and W. Doorsamy *

** Dept. of Electrical and Electronics Engineering Technology, University of Johannesburg, PO Box 17011, Doornfontein, 2023, Johannesburg, South Africa*

Abstract: This paper presents a method for evaluating, both qualitatively and quantitatively, the effects of specific rotor design parameters on the performance of a synchronous reluctance machine. The method uses multi-factor experimental design, with Analysis of Variance (ANOVA), and Finite Element Analysis (FEA) to determine the optimal rotor design parameter according to a specific objective. Using this method, two factors - rotor flux barrier pitch angle and barrier width - are selected at simultaneously varied levels for assessment with the aim of minimising the response variable, which is, the torque ripple. Results from the investigation show that the influence of the rotor flux barrier pitch angle on the torque ripple is more statistically significant than the influence of the barrier width. However, the effect of the barrier width on the actual torque is more significant.

Key words: Synchronous reluctance machine, Rotor design, ANOVA, FEA, Torque ripple.

1. INTRODUCTION

Synchronous reluctance machines (SynRMs) have high torque densities, fault-tolerant capabilities, high efficiencies, low rotor inertias and relatively uncomplicated controllability in comparison to induction machines [1-4]. Despite several advantages, one of the common problems with SynRMs is the high content of torque ripple [4]. This is due to the interaction between spatial harmonics of the electrical loading and the rotor anisotropy which causes a high torque ripple that is intolerable in most of applications [5-7]. Therefore, much emphasis has been placed on mitigating torque ripple in SynRMs. It is reported in [8] that skewing of the rotor by a stator-slot pitch can reduce the slot harmonics in the torque, but also decreases the average torque by ± 2 %. In [9] it has been shown that a reduction of in torque ripple can be achieved by means of a suitable choice of number of flux-barriers with respect to the number stator slots per pole pair. The flux-barrier ends are uniformly distributed along the air-gap (similarly to the stator slot distribution). Torque ripple reduction for SynRMs using asymmetric flux barrier has been reported in [3]. The method consists of shifting the relative position between the edge of each flux barrier and stator teeth by a certain angle. In [10], asymmetric flux barrier angles and a flipped rotor structure have been presented as an approach of torque ripple reduction without loss in the average torque.

As mentioned in above literature, there are a vast number of different rotor geometries that are possible, specifically due to the variations in flux barriers that are available. Modifications to the number, arrangement and dimensions of the flux barriers yield changes in the performance of the machine. This makes the design process quite expensive as optimisation is usually required to determine a suitable geometry for best performance.

This paper presents a method for evaluating, both qualitatively and quantitatively, the effects of specific rotor design parameters on the performance of a SynRM. Multifactor ANOVA is used together with FEA to assist with analysis and selection of optimal parameter/s during the design process. The proposed method is intended to be used in conjunction with optimisation to enhance FEA-based design.

2. BACKGROUND

The use of multi-objective optimisation algorithms together with Finite Element Method (FEM) can be time consuming, computationally expensive and does not present qualitative information regarding the influence of the input parameters on the performance variable being studied. In other words, such optimisation only yields numerical values for the design parameters. These results still need to be analysed and evaluated afterwards to assess the overall design and to perceptively stipulate tolerances required for the manufacturing process.

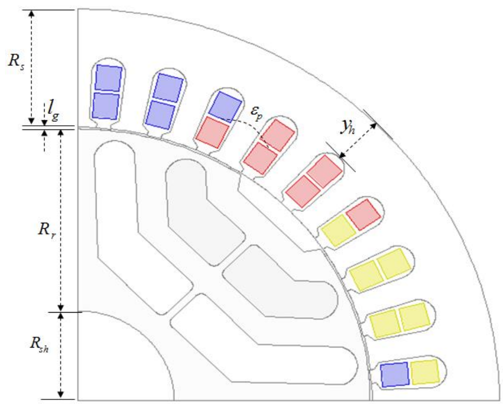
ANOVA is typically utilised in the experimental design but can offer analytical benefits to the design process. Multifactor ANOVA is used here to investigate effects of the flux barrier width and pitch angle parameters on the output torque and torque ripple of a synchronous reluctance machine.

3. MACHINE SPECIFICATIONS

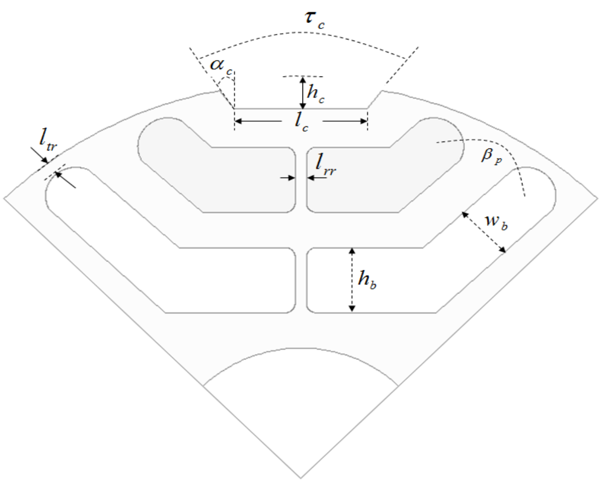
The specifications of a traditional 5.5 kW, three-phase, 50-Hz, induction machine are used to design and model the SynRM. Figures 1(a) and 1(b) show the cross-section of the basic SynRM with cut-off on the *q-axis*, while Table 1 gives the general design specifications of the machine.

Table 1: Design Specifications for SynRM

Description	Values
Stator slot Pitch_ $\boldsymbol{\alpha_s}$	10^o *mech*
Airgap length_ $\boldsymbol{l_g}$	0.45 *mm*
Barrier height_ $\boldsymbol{h_b}$	12 *mm*
Barrier width_ $\boldsymbol{w_b}$	7, 8, 9 & 10 (*mm*)
Barrier Pitch_ $\boldsymbol{\beta_p}$	10^o, 12.5^o, 15^o & 17.5^o *mech*
Cut-off angle_ $\boldsymbol{\alpha_c}$	40^o *mech*
Cut-off Pitch_ $\boldsymbol{\tau_p}$	25^o *mech*
Cut-off height_ $\boldsymbol{h_c}$	4.8 *mm*
Iron width_ $\boldsymbol{w_i}$	6.50 *mm*
Stack length	160.00 *mm*
Number of pole pairs	2
Number of stator slots	36
Rotor radius_ $\boldsymbol{R_r}$	48.50 *mm*
Stator radius_ $\boldsymbol{R_s}$	31.62 *mm*
Shaft radius_ $\boldsymbol{R_{sh}}$	24.00 *mm*
Yoke height_ $\boldsymbol{y_h}$	12.87 *mm*
Barrier end radius_ $\boldsymbol{b_{ER}}$	2.80 *mm*
Radial rib length_ $\boldsymbol{L_{rr}}$	2.00 *mm*
Tangential rib length_ $\boldsymbol{L_{tr}}$	2.00 *mm*



(a)



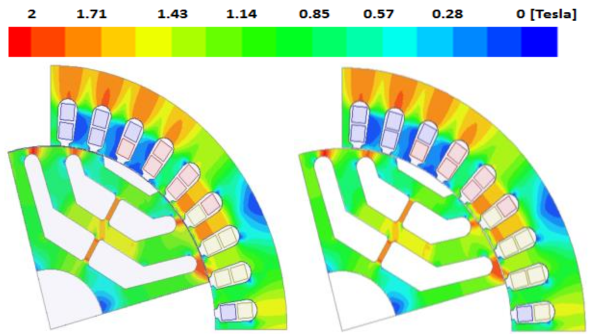
(b)

Figure 1: Cross-section of the SynRM basic model with cut-off on the *q-axis*, (a) main machine dimensions, (b) rotor design specifications

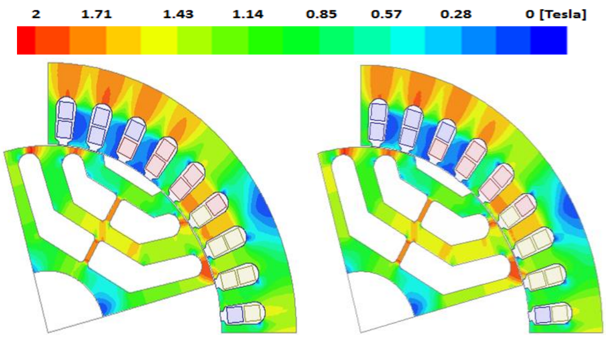
4. METHODOLOGY

2.1 Finite element model (FEM)

The FEA is carried out at constant a speed and frequency of 1500-rpm and 50-Hz respectively. The three-phase double layer lap windings are chorded by one slot and excited by 3-phase sinusoidal currents. The SynRMs were started at an initial position of $\theta = 17.5^o$ such that the phase (A) is opposite to the *d-axis*. The models were run at current space phasor angle of 45^o electric. Figure 2 shows the magnetic flux density distribution for SynRMs with $\beta_p = 15^o$ *mech* and different barrier widths, when the *d*-axis current $i_d = 8$ A and *q*-axis current $i_q = 8$ A.



(a) (b)



(c) (d)

Figure 2. Flux density distribution for the SynRM with β_p of 15^o mech, (a) $\boldsymbol{w_b}$=7 mm, (b) $\boldsymbol{w_b}$= 8 mm, (c) $\boldsymbol{w_b}$ = 9 mm, (d) $\boldsymbol{w_b}$=10mm

Figure 2 shows that the radial and tangential ribs are highly saturated in all four cases. The distribution of the flux density in the *d-axis* rotor iron parts varies with change in the barrier width. The FEA results indicate that by increasing the barrier width, the flux density in the *d-axis* rotor iron parts increases as well.

2.2 Parameter selection and Response Variables

There are numerous design configurations that are possible though adjustment of geometrical parameters of the rotor. The number of flux barriers and dimensions thereof influence the performance of the machine. The parameters selected for this study are the flux barrier width (w_b) and the pitch angle (β_p) as shown in figure 1 (b). The response variables of interest in the presented study are the torque and torque ripple. The torque ripple factor defined as the ratio of peak to peak torque value to average torque is adopted for torque ripple calculation [11], which is expressed as

$$T_{rip} = \frac{\Delta T}{T_{av}} = \frac{T_{max} - T_{min}}{T_{av}} \times 100\% \qquad (1)$$

2.2 Multifactor Analysis of Variance (ANOVA) design

ANOVA is used in experimental design to determine what factors affect a response variable. In the presented methodology, ANOVA is used to determine the significance of specific geometrical factors on each of the response variables of interest. Additionally, this method enables the use of a multifactor model where multiple factors may be varied simultaneously in order to analyse interaction effects. The multifactor model presented here consists of two factors – i.e. barrier pitch angle and barrier width. The following sets of hypotheses – i.e. null and alternative - are being testing with the presented ANOVA. The three sets of hypotheses are given as:

1. H_{01} - Means for each level of barrier width are equal.
   H_{A1} - Means for each level of barrier width race are different.
2. H_{02} - Means for each level of barrier pitch angle are equal.
   H_{A2} - Means for each level of barrier pitch angle are different.
3. H_{03} - There is no significant interaction between the barrier width and pitch angle.
   H_{A3} - There is significant interaction between the barrier width and pitch angle.

The third set of hypotheses is tested through replication of observations. Figure 2 shows the steps involved in the methodology. The levels of each of the factors are firstly selected. Four levels are selected for w_b and β_p. The first (w_b) is varied as a function of the barrier vertical height (h_b) in order to maintain the iron to insulation ratio in both *q*-and *d-axis* within an acceptable margin. The second (β_p) is varied as a function of the stator slot pitch (α_s) as the torque ripple production in SynRMs is also due mainly to slot harmonics caused by the stator slotting. The variation Δw_b is from 7 *mm* to 10 *mm* with 1 *mm* increments, and elsewhere, the variation $\Delta\beta_p$ is from 10° to 17.5° at 2.5° *mech* increments. Δw_b and $\Delta\beta_p$ are expressed as in (2) and (3) respectively.

$$\Delta w_b = \frac{10h_b}{8} \leq w_b \geq \frac{7h_b}{8} \qquad (2)$$

$$\Delta\beta_p = \alpha_s \leq \beta_p \geq \frac{\alpha_s}{4} \qquad (3)$$

Using these parameters, 16 separate (4 barrier widths by 4 barrier pitch angles) FE models are constructed. For replication purposes, a subroutine was implemented to randomly vary the excitation by ±5% in order to mimic the randomness that is inherently achieved under experimental conditions. Each of the 16 models is simulated with a fixed current value to conduct ANOVA without replication. The models are then simulated, with randomly generated currents, to conduct ANOVA with replication. A balanced design is used resulting in 16 responses for the case without replication and 64 responses for the case with replication.

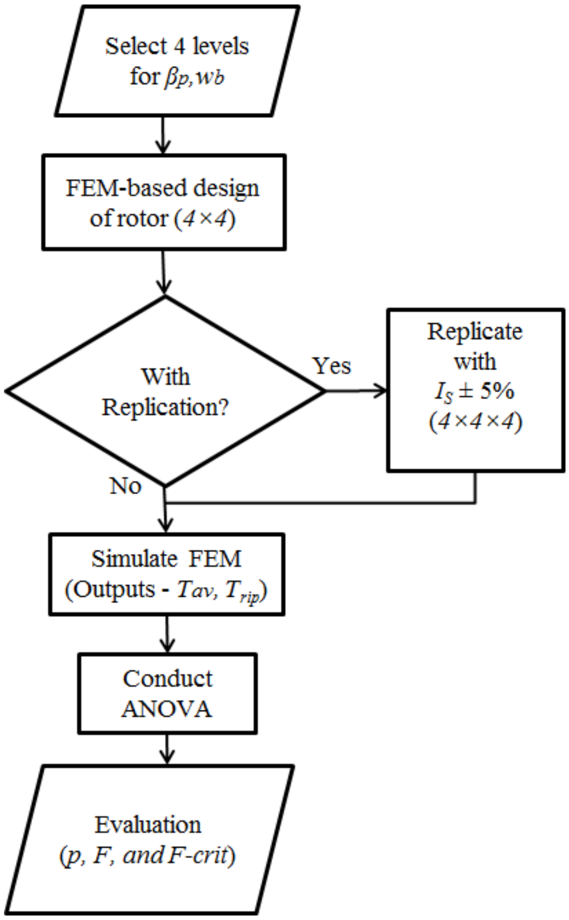


Figure 3: Overview of parameter evaluation methodology

In a two-way ANOVA model, a particular combination of levels is called a treatment or cell [12]. In this case, there are 16 cells (4 by 4 levels). The two-way ANOVA model is given by

$$Y_{ijk} = \mu + \beta_{pi} + W_{bj} + (\beta_p w_b)_{ij} + \varepsilon_{ijk} \qquad (4)$$

Where; $i = 1,..,4$ levels of factor β_p,

$j = 1,..,4$ levels of factor w_b,

Y_{ijk} is the k^{th} response at level combination ij,

μ is the grand mean,

$k = 1,..,4$ observations per cell,

β_p and w_b terms represent the main effects of each of the factors,

$(\beta_p w_b)_{ij}$ represents the interaction effect, and

ε_{ijk} is the error.

It should be noted that the interaction effects term does not feature in an additive model – i.e. without replication.

5. RESULTS

The parameter evaluation methodology given in Figure 3 is carried out twice for this investigation. The first iteration through the methodology does not utilise replication of observations for each treatment and therefore the ANOVA does not account the interaction effects between the factors.

The simulation results for the average torque and torque ripple responses for each treatment are given in Figures 4 and 5, respectively. The results indicate that the flux barrier width and pitch angle have a more noticeable effect on the torque ripple in comparison to the average torque. Furthermore, there is no apparent pattern to the behaviour of the responses relative to the treatment.

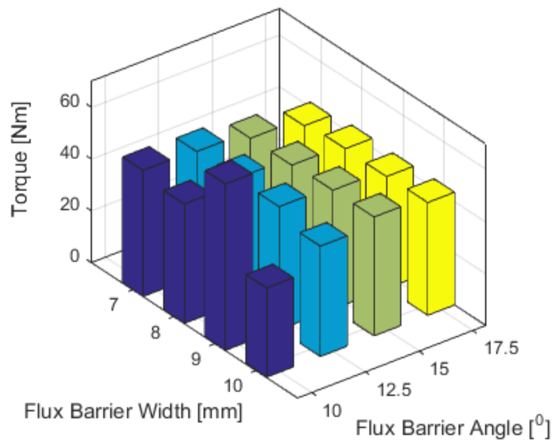


Figure 4: Average torque responses for each treatment of flux barrier width and pitch angle.

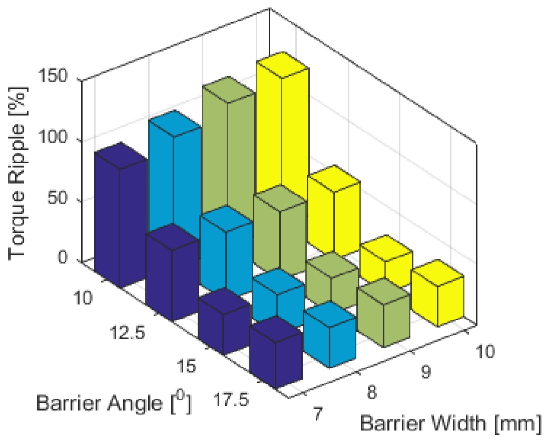


Figure 5: Torque ripple responses for each treatment of flux barrier width and pitch angle.

Thus, the two-factor ANOVA is used here in order obtain more information about the effects of these factors on the response variables. Tables 2 and 3 give summaries of the main results obtained from the ANOVA tests performed without replication for the torque and torque ripple respectively,

Where; ***SS*** – sum of squares,
df – degrees of freedom,
MS – mean squares,
F – ratio of between- and within-group variance,
p-value – probability of obtaining F-value (or more extreme) under the null hypothesis,
F-crit – Critical value of F-distribution.

Table 2: Summary of two-factor ANOVA (without replication) with torque ripple as response variable

Source of Variation	***SS***	***df***	***MS***	***F***	***p-value***	***F-crit***
Rows (w_b)	22.81	3	7.27	0.17	0.92	3.86
Columns (β_p)	16584	3	5528	127	1E-7	3.86
Error	390	9	43.44			
Total	16997	15				

Table 3: Summary of two-factor ANOVA (without replication) with torque as response variable

Source of Variation	***SS***	***df***	***MS***	***F***	***p-value***	***F-crit***
Rows (w_b)	146	3	48.75	1.35	0.31	3.86
Columns (β_p)	59	3	19.64	0.54	0.67	3.86
Error	326	9	36.22			
Total	531	15				

A significance level of 0.05, corresponding to a 95% confidence interval, was used for the ANOVA tests. The *p-value* is below the significance level for the (β_p) factor in Table 2 and below the significance level for the (w_b) factor. This implies that the aforementioned null hypothesis H_{02} may be rejected for the torque ripple and the null hypothesis H_{01} may be rejected for the torque. Simply put, the *p-values* indicate that the effects of the variation in barrier pitch angle are statistically significant on the torque ripple, and the effects of the barrier width are statistically significant on the torque. Additionally, these *p-values* also indicate that the opposite is not true – i.e. effects of modifying the barrier pitch angle and barrier width are not statistically significant on the torque and torque ripple responses respectively. The resulting *F-statistics* principally confirm these results however the *F-crit* value for the torque response is relatively larger for both factors. Figures 6 and 7 are interaction plots for the torque and torque ripple responses. Figure 7 shows more ordinal interaction (parallelism) between factors for the torque ripple response than for the torque response shown in Figure 6. This means that there is a stronger interaction effect between factors for the torque response. The second iteration of the parameter evaluation methodology is carried out with replication of observations for each treatment in order to study the interaction effects between factors. As previously mentioned, each treatment is repeated by randomly varying the current excitation. Results for the two-factor ANOVA test performed with replication are given in Tables 4 and 5 for the torque and torque ripple respectively. These results follow a similar pattern to the first iteration, however the *F-statistics* and *p-values* indicate that the effects of both factors are statistically significant in the case of both response variables. The smaller *p-values* and larger *F-statistics* indicate greater statistical significance of the interaction effect. Furthermore, the interaction effects between factors are highly significant for the torque ripple. This means that the relationship between these two factors cannot be assumed as independent when analysing or optimising the torque ripple. The effect of any possible outliers e.g. as observed in the torque in Figure 6, are also removed when using replication.

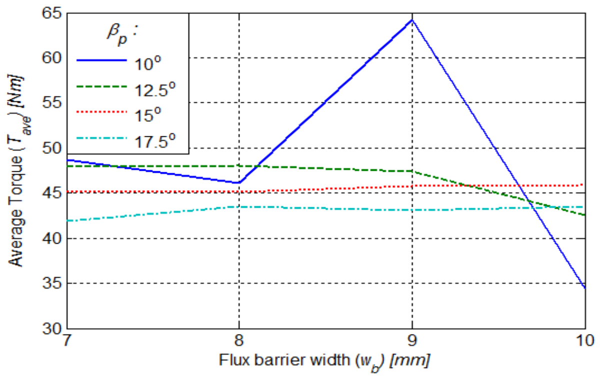


Figure 6: Interaction plot of flux barrier width and pitch angle factors for average torque response

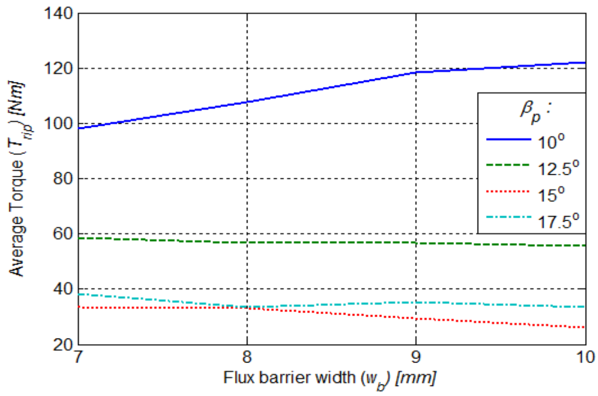


Figure 7: Interaction plot of flux barrier width and pitch angle factors for torque ripple response

Table 4: Summary of two-factor ANOVA (with replication) with torque as response variable

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F-crit***
Rows (w_b)	611	3	204	106	4E-21	2.8
Columns (β_p)	202	3	67.42	34.93	4E-12	2.8
Interaction	1317	9	146	75.81	3E-25	2
Within	92.65	48	1.93			
Total	2223	63				

Table 5: Summary of two-factor ANOVA (with replication) with torque ripple as response variable

Source of Variation	***SS***	***df***	***MS***	***F***	***P-value***	***F-crit***
Rows (w_b)	99.53	3	33.18	48.92	1E-14	2.8
Columns (β_p)	6.5E4	3	2.2E4	3.2E4	3E-79	2.8
Interaction	1685	9	187	276	3E-38	2
Within	32.55	48	0.68			
Total	6.7E4	63				

6. CONCLUSION

The ANOVA-FEA based methodology presents some key qualitative findings regarding the rotor parameters of the synchronous reluctance machine. Although there is merit in contemporary optimisation methods used during the design process – this only provides quantitative results. For practical purposes, a qualitative method is required to better understand the interaction effects of parameters. The presented methodology is intended to assist with optimisation by informing decisions regarding the practical trade-offs during the design process. Results from this specific investigation shows that the interaction effects of the barrier pitch angle and width are statistically significant for both the torque and torque ripple responses. Furthermore, the torque ripple is affected more significantly by variations in the flux barrier pitch angle, while the torque is more significantly affected by the variations in the barrier width.

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