Risk evaluation for production-injection recompletion and sidetrack

Oyinkepreye D. Orodu[1*] and Zhonghua Tang[2]

[1] Department of Petroleum Engineering, Covenant University, Ota PMB 1023, Ogun State, Nigeria

[2] Faculty of Environmental Studies, China University of Geosciences, Wuhan, Hubei 430074, China

**Author for corresponding. Email: preye.d.orodu@gmail.com*

(Received 12 January 2011; accepted 12 April 2011)

Abstract

Using a decision tree and stochastic parameters, expected monetary value is calculated to evaluate optimal sidetrack time. In view of placing a high confidence level on analytical approach to optimal sidetrack time for a waterflooded reservoir based on possible uncertainty of economic and reservoir parameters and probability of sidetrack success, a major assumption on a parameter in a previous study is re-evaluated. Material balance and displacement efficiency are used to re-evaluate this critical waterflood performance parameter. The change in the relative influence of the stochastic parameters to optimal sidetrack time due to re-evaluation calls for much attention with probable need to further reduce assumptions made, however insignificant the parameter may be. This change will affect the degree of acceptability of the analytical approach. The probability of success of sidetrack sums up the geological and technical uncertainties, deconvolution of these will give the analytical approach an edge.

Keywords: Sidetrack, Waterflood, Expected Monetary Value, Uncertainty

1. INTRODUCTION

The risk associated with reserves estimates decreases in the order of analog, volumetric and performance (simulation studies, material balance and decline trend analysis) techniques (Garb, 1985; Jiang *et al*., 2010). Estimation based on performance methods overlaps but with decreasing risk in estimation with time in the order of the alternatives listed in the previous sentence. In other words decline trend analysis may hold a better position for long history performance for forecasting over and above simulation studies. The weakness of simulation is highlighted in Orodu *et al*. (2009) as well as that of performance curves. But, the latter inherently reflects reservoir properties justifying its use (Erghaghi and Omoregie, 1978; Wu, 1988; Correa, 2007; Han *et al*., 2011).

Although statistical regression analysis is required to fit the decline models or other empirical models, most of these models have sound theoretical footing. Examples of these performance curves are the Arps' exponential decline by Fetkovich (1980) with decline rate represented by reservoir and fluid properties based on the constant-pressure analytical solution; Arps' harmonic and hyperbolic decline for the late stage of waterflooding by Lijek (1989); Li and Horne (2005) mechanistic model for a naturally fractured reservoir based on the imbibitions phase core flooding, having capillary and mobility ratio effects defined by theory. Yet another is the Yang's (2009) new analytical model for waterflood performance based on Buckley-Leverett frontal displacement theory for 1-D as applied to field performance with reasonable degree of curve fitting. Corrêa (2007) presented an empirical model for waterflood performance that breaks down to Arps' decline models for constant liquid rate.

Analyzing the risk associated with sidetrack (recompletion) was extensively covered by Lerche and Noeth (2001a; 2001b; 2001c) and detailed analytical solution to optimal time of sidetrack in Lerche and Mudford (2001) by optimizing expected monetary value (EMV) based on production performance following Arps' rate-time exponential decline for primary recovery. Further application to secondary recovery involving oil-production and water-injection wells sidetrack for simultaneous and sequential sidetrack operation is carried out by Orodu *et al*. (2011). The EMV is based on probable outcomes given by the risk involved.

The probable outcomes for a production well sidetrack (recompletion) are adopted from the study by Lerche and Mudford (2001) as seen in Figure 1 and further extended to cover the cases of simultaneous and sequential sidetrack of the pair of production and injection wells. The outcomes emanating from the chance nodes combine the possibilities necessitated by the combination of geological and technical uncertainties. An instance may be due to pressure depletion from unforeseen communication between the zone under production and the zone to be sidetracked into. Another reason for the outcomes may be due to uncertainty of oil-initially in place. Zone-B is the lower and more productive zone and Zone-A is the less productive zone. Injection well sidetrack outcomes are attached to Branch-B and Branch-D. Injection well sidetrack outcomes attached to Branch-B are similar to the production well sidetrack outcomes attached to Branch-A_2. For Branch-C, due to failure to produce from Zone-A there was no need to sidetrack from injection well. Similarly, for Branch-F, failure of continuous production from Zone-B and failure to produce from Zone-A closes the injection well from further operation.

The probabilities associated with the branches are P_A and P_B, the probabilities of success (POS) of production from Zone-A and continuous production from Zone-B. Zone-B is the zone under exploitation prior to sidetrack. P'_A and P'_B are related to the injection well and stand for the probability of successful injection into Zone-A and that of continuous injection into Zone-B. If applied to smart wells (or intelligent wells are wells with downhole monitoring sensors and automated control valves for production optimization) then the outcomes may be tied to technical details as relates to inflow

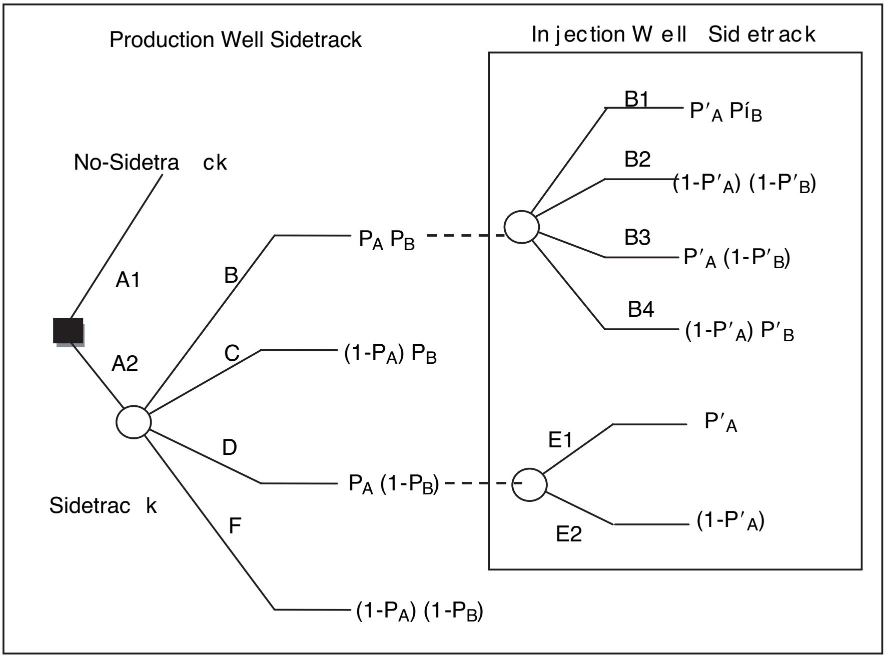

Figure 1. Production well and injection well sidetrack decision tree diagram. (Orodu *et al*., 2011).

control valve failure for known production capability from all zones thereby no geological uncertainties.

This study aims at reducing the assumptions made by Orodu *et al*. (2011), in particular, constant cumulative oil production prior to water breakthrough (Np_{BT}) at production well sidetrack time (t_R)and injection well sidetrack time (t_{RR}) for Zone-A for sequential production-injection sidetrack. The assumption affects NPV of Branches B_1, B_3 and E_1 (Fig. 1). Production from t_R to t_{RR} is under natural exponential decline for these branches before water injection at t_{RR}. The different optimal t_{RR} brought about by stochastic analysis further requires re-evaluation of Np_{BT} for Zone-A ($Np_{BT,A}$) at t_{RR} for each simulation run. Thus for large difference between t_R and t_{RR}, the former study may be unsuitable for appropriate analysis of the impact of various economic, reservoir and POS parameters on t_R and t_{RR}.

Re-evaluating $Np_{BT,A}$ due to production by exponential decline from t_R to t_{RR} makes use of material balance to obtain water saturation as in Equation (12) of Babadagli (2007) at t_{RR}. This enables computation of displacement efficiency and $Np_{BT,A}$ at t_{RR} based on remaining oil in-place and stochastic volumetric sweep efficiency at water breakthrough for Zone-A. Furthermore, stochastic $Np_{BT,A}$ is used and the effect of initial value chosen for the solution of the objective function is considered, especially for t_{RR} under the sequential sidetrack scenario. It is the intention that the reliability of the analytical approach to optimal sidetrack time is ensured to an acceptable degree.

2. RESERVOIR AND WELL DESCRIPTION

The study focus is on production from Block-Shen95 located in Liaohe basin of Damintun depression North-East China (Orodu *et al.*, 2009) within an area of 15.8km^2. The reservoir is highly heterogeneous having no edge and bottom water support. Average porosity is 17% and low permeability of 3md. The oil-bearing segment is within an interval of 5900 to 7300ft subsurface depth and situated in the Eocene. Reservoir fluid is characterized by oil gravity of 25.9 to 36.3 API, viscosity of 3.83 – 9.67 cp (mPa.s) and high wax content of 42–64%.

Full scale development started in 1988 and by 2005 there were 111 wells of which 50 were producing and 10 on injection having recovery of 8.52% based on volumetric estimate of 139MMbbl. Production data of two wells are used. One is of the most productive wells, Well-B, and the other is of the lowest productive wells, Well-A. The former will serve as production from the most productive zone (Zone-B) and the other for the less productive zone (Zone-A). Both wells are 200-400m apart. Peak production for both wells are 10,000bbl/month and 2800bbl/month respectively for Well-B and Well-A. Oil production rate for Well-B declined initially within 5yrs to economic limit and water breakthrough occurred, and then there was significant increase in oil production based on well workover. Water breakthrough for Well-A is about 6yrs. Well-B is perforated at the top and bottom of the reservoir while Well-A has a lesser perforated interval at the top. Fluid properties variations are fairly similar for both wells.

3. OPTIMAL SIDETRACK TIME MODEL

Optimal sidetrack time for both the production (t_R) and injection (t_{RR}) wells are obtained by maximizing the EMV function based on the decision tree model (Fig. 1). EMV construction and analysis is as explained in Newendorp and Schuyler (2000) and applied by Lerche and Mudford (2001) for deterministic evaluation of production well sidetrack under primary recovery and stochastic evaluation by Orodu *et al.* (2011) for simultaneous and sequential production and injection wells sidetrack under waterflooding. Production performance schemes applied are Arps' rate-time exponential model and Yang's (2009) waterflood performance model derived from the Buckley and Leverett 1-D water displacement frontal advance equation.

3.1. Production performance models

3.1.1. Arps' rate-time exponential decline

Arps' rate-time decline models are still commonly applied for production forecasting. The exponential decline is used as it fits the history performance of the wells for the period prior to waterflooding. This model enables oil production forecast for those time intervals or periods where an injection well sidetrack has failed for both zones or during the time lag between t_R and t_{RR} for production from Zone-A. Fetkovich's (1980) theoretical derivation of the exponential equation is used. The equation is presented below for true wide open decline which represents backpressure of zero.

$$\frac{q(t)}{q_i} = e^{-\left[\frac{(q_i)_{\max}}{Np_i}\right]t} \tag{1}$$

$$\frac{(q_i)_{\max}}{Np_i} = \frac{0.00634}{\phi\mu C_t r_w^2}\left[1 \Big/ \frac{1}{2}\left[\left(\frac{r_e}{r_w}\right)^2 - 1\right]\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2}\right]\right] \tag{2}$$

The variable q_i is initial production rate, the $q(t)$ is production rate at any time, Np_i is cumulative production, t is time, ϕ is porosity, μ is viscosity, C_t is compressibility, r_w is wellbore radius and r_e is reservoir boundary radius. Equation (2) gives the decline rate with respect to physical and measurable reservoir variables.

3.1.2. Yang's (2009) waterflood performance model

Equation (3) is the equation for oil production forecast. The equation is for only performance prediction after water breakthrough and uniform production is assumed prior to breakthrough. The uniform rate is the ratio of cumulative production at water breakthrough obtained from special core analysis data to water breakthrough time. Use of Yang's model is subject to cumulative liquid production been equal to cumulative water injected based on voidage replacement ratio of 1.0. Derivation of Equation (3) from the model is due to uniform injection rate that makes possible time dependency of the model. Equation (4) gives watercut at any time.

$$q = \frac{i_w}{2}\left[1 - \sqrt{1 - 8\frac{E_v}{B}PV\left(\frac{1}{t^2 i_w - t i_w}\right)}\right] \tag{3}$$

$$f_w = 1 - \frac{1}{2}\left[1 - \sqrt{1 - 8\frac{E_v}{B}PV\left(\frac{1}{t^2 i_w - t i_w}\right)}\right] \tag{4}$$

Where q is production rate, i_w is water injection rate, E_v is volumetric sweep efficiency, PV is pore volume, B is coefficient of the straight-line function of relative permeability versus water saturation and f_w is water cut.

3.2. Optimization function

The performance models enable computation of revenue and cost incurred in the form of operational and capital expenditure to evaluate net present value (NPV).

The objective function formulation by Lerche and Mudford is followed closely using the decision tree model (Fig. 1) and production timelines of Zone-B and Zone-A (Fig. 2) as applied in Orodu *et al*. (2011). The EMV of the no production-injection well sidetrack is presented in Equation (5) and that of the sequence of production-injection well sidetrack is presented in Equation (6) as in the previous study but $Np_{BT,A}$ is modified. Details of the individual components for the path $A_2 \rightarrow B \rightarrow B_1$ that represents the NPV associated with the branches on figure 1 are in Appendix-A.

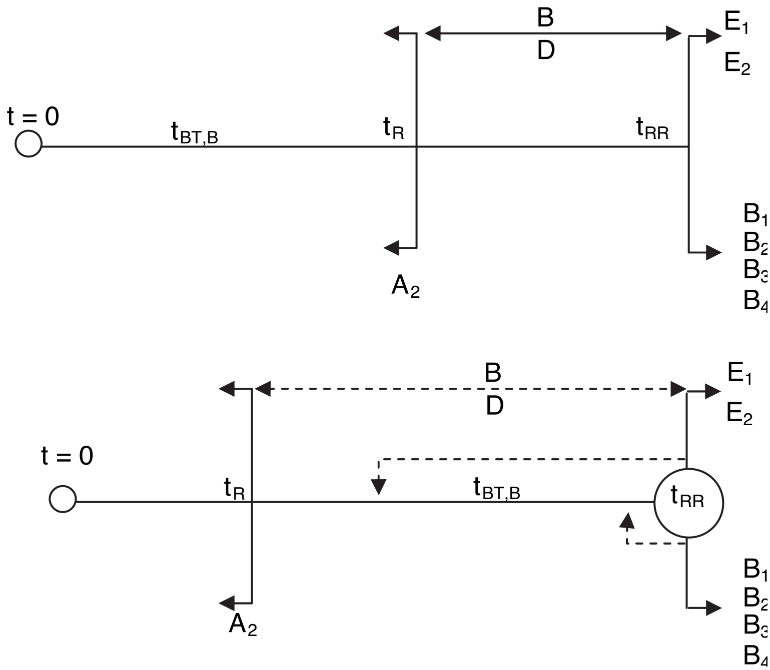

Figure 2. Production timeline for Zone-B indicating production well sidetrack time (t_R) and injection well sidetrack time (t_{RR}) scheme adopted for all branches. (A) Post water breakthrough time ($t_{BT,B}$) sidetrack of Zone-B; (B) Pre water breakthrough time ($t_{BT,B}$) sidetrack of Zone-B with the possibility of injection well sidetrack occurring before and after water breakthrough time of Zone-B.

$$NPV(SN) = -C_B - C'_B - I_B i_w \int_0^{t_{B98}} t e^{-i_D t} dt + (P_o - \alpha_B) \int_{t_{BT,B}}^{t_{B98}} q e^{-i_D t} dt + \frac{(P_o - \alpha_B)}{t_{BT,B}} Np_{BT,B} \int_0^{t_{BT,B}} t e^{-i_D t} dt - \beta (C_B + C'_B) \int_0^{t_{BT,B}} e^{-i_D t} dt \quad (5)$$

$$\begin{aligned} E(SP) = {} & P_B P_A [(B_1 + B + A_2) P'_B P'_A + (B_2 + B + A_2)(1 - P'_B)(1 - P'_A) \\ & + (B_3 + B + A_2)(1 - P'_B) P'_A + (B_4 + B + A_2) P'_B (1 - P'_A)] \\ & + P_B (1 - P_A)[C + A_2] + (1 - P_B) P_A [(E_1 + D + A_2) P'_A + (E_2 + D + A_2)(1 - P'_A)] \\ & + (1 - P_B)(1 - P_A)[A_2] \end{aligned} \quad (6)$$

Where A, A_2, B, B_2, C, D, E_1 and E_2 are branches of the decision tree (Fig. 1); P_A and P_B are probability of success of production well for Zone-A and Zone-B; P'_A and

P'_B are probability of success of injection well for Zone-A and Zone-B; C_B and C'_B are cost of production and injection wells for Zone-B; i_D is discount factor; Po is crude oil price; α is variable oil production cost; β is either fixed production cost of production or injection; t_{B98} is time to 98% watercut for Zone-B; $t_{BT,A}$ and $t_{BT,B}$ are water breakthrough time for Zone-A and Zone-B and $Np_{BT,B}$ is cumulative oil production at $t_{BT,B}$. For SP and SN, refer to figure 1.

The sidetrack time for the cases of simultaneous production-injection wells, and that of sequential production-injection wells sidetrack are obtained by the optimization of EMV of Equation (6). The equations of Appendix-A are made for sequential sidetrack but suitable for optimal simultaneous sidetrack time evaluation by making t_R and t_{RR} equal. The case for no-sidetrack is evaluated by Equation (5) to compare with the EMV of optimal sidetrack time.

Apart from EMV as a means of assessing optimal time, other methods lie on the uncertainty of possible pathways to the terminal branches. These add more credibility to the optimal time evaluation by EMV. The methods are volatility and probability of profit of EMV of the sidetrack scenario been equal to and exceeding the no-sidetrack scenario (Lerche and Noeth, 2001a). The uncertainty of the probabilities of the various outcomes, economic parameters and reservoir parameters as presented in the EMV are used to obtain optimal sidetrack time by a stochastic approach.

4. RESULTS

Additional well description information for Well-A and Well-B are water breakthrough times of 6.877 and 5 yrs from the Welge's simplified Buckley-Leverett frontal displacement theory and verified by performance history, economic production limit to 10 and 20yrs by exponential decline, 15.3 yrs production for both wells by 98% water-cut constraint and average pore volume of 2.4MMbbl for both wells. Deterministic parameters are 12% continuous discount rate, variable operating cost of \$5/bbl, fixed operating cost of 3% of capital expenditure, injection cost of \$2/bbl, injection rates of 34,700bbl/year and 52,000 bbl/year which represents injection rates to Zone-A and Zone-B respectively. The stochastic parameters are 29 in number.

4.1. Simultaneous Production-Injection Sidetrack

t_R and t_{RR} are equal for this scenario. Hence, t_R stands for simultaneous production-injection sidetrack time. The bulk of simulated optimal time (mean of 6.67, minimum of 5.00 and maximum of 9.00) is concentrated in the normal distribution and not in the spike as in previous study due to reduced (or stochastic Np_{BT} causing reduction of high Np_{BT}) Np_{BT} based on stochastic volumetric sweep efficiency. Volatility is log-normally distributed having an average of 50% of values greater than 1.0 unlike the previous case study that values tend toward zero. This actually indicates sensitivity in the analysis and the results can change with each Monte-Carlo simulation run and change in value of input variables.

Water-cut of production from Zone-B at t_R is a multinomial distribution with mode of 0.88. Based on rank correlation of t_R with both economic and reservoir parameters and POS, the first 4 parameters of highest impact are closely similar to the previous study but not in the same order. For this study, the decreasing order of influence is

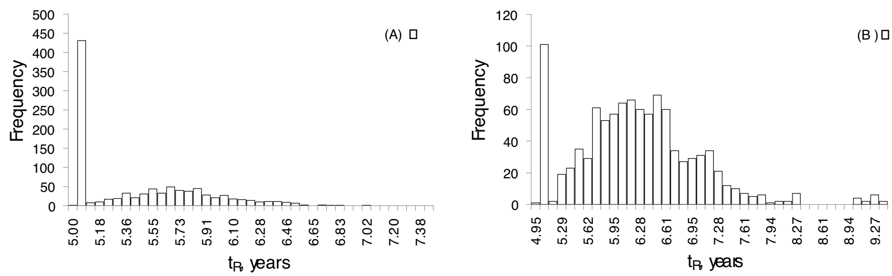

Figure 3. (A) Case-1, production well optimal time for sequential production-injection sidetrack; (B) Case-2, production well optimal time for sequential production-injection sidetrack.

sidetrack cost to Zone-A for production well, POS of Zone-A (production well), POS of Zone-A (injection well), POS of Zone-B (production well) and sidetrack cost of production well for Zone-A. EMV is likewise reduced as in the case of t_R distribution having lesser frequency of spike at 5yrs due to stochastic Np_{BT}. The mean EMV for the no-sidetrack and sidetrack scenarios are (\$0.975MM) and (\$0.129MM) respectively with probability of EMV been greater than zero of 29% and 47% respectively.

4.2. Sequential Production-Injection Sidetrack

Table 1 shows production well sidetrack time (t_R) and injection well sidetrack time (t_{RR}) for different initial values for solution of the EMV objective function. Figures 3 and 4 show the distribution of t_R and t_{RR} for initial value combinations of t_R = 5years and t_{RR} = 5years for Case-1 and t_R = 5years and t_{RR} = 11.5years for Case-2. For the latter, (t_{RR}-t_R) has 3 spikes, the spike with the highest frequency tallies with the single spike for Case-1, as the difference between t_{RR} and t_R for Case-1 is negligible. The spike frequency below 50 simulation runs out of the 1000 simulation runs tally with the initial value for the solution of t_{RR}. Case-2 show a higher frequency distribution at high water-cut compared to Case-1 for water-cut at t_R of production from Zone-B (Fig. 5). Volatility is essentially similar but cumulative density function (CDF) may be a better criteria for selection of optimal solution. The CDF of EMV for each case and that of the no-sidetrack option is shown in figure 6, obviously, the probability of obtaining a high EMV between given interval is higher for Case-2 than Case-1. Other initial values between 5.0 and 11.5years for t_{RR} do not show much appreciable and distinguishable trend on the CDF with the aid of the graph. Even the probability of EMV been greater than zero is not sufficient to choose the best initial value for the objective function. The correlation of economic and reservoir parameters and probability of success of the sidetrack operation for both cases show slight change in the order of relevance of the parameters to t_R (Fig. 7). However a significant change is the importance of crude oil price for Case-2 compared to case-1 been more relevant. Optimal sidetrack time for the injection well of Case-2 is similar to that of

Table 1. Summary statistics of simulation for sequential production-injection sidetrack.

Initial Values	*t* (years)				Branch-SP		Branch-SN	
	Minimum	Maximum	Mean	StdErr	Mean $MM	P(E(SP)>0) %	Mean $MM	P(E(SN)>0) %
$t_R = 5$	5.00	7.44	5.416	0.01	0.62	62	(0.975)	29
$t_{RR} = 5$	5.00	7.44	5.416	0.01				
$t_R = 5$	4.99	5.35	5.182	0.00	0.63	62	(0.975)	29
$t_{RR} = 5.5$	4.99	5.35	5.182	0.00				
$t_R = 5$	4.95	7.72	6.685	0.03	0.55	61	(0.975)	29
$t_{RR} = 8.5$	5.00	19.26	6.974	0.06				
$t_R = 5$	4.95	9.38	6.201	0.02	0.54	61	(0.975)	29
$t_{RR} = 11.5$	5.00	19.87	11.005	0.18				

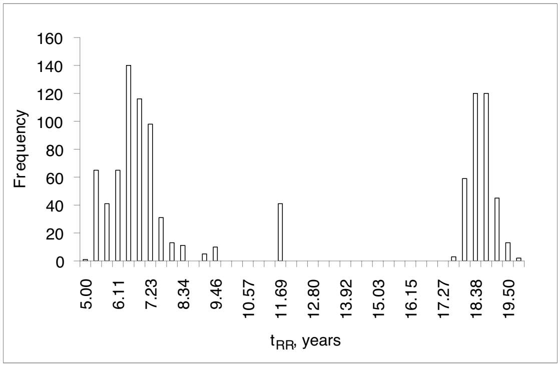

Figure 4. Case-2, injection well optimal time for sequential production-injection sidetrack.

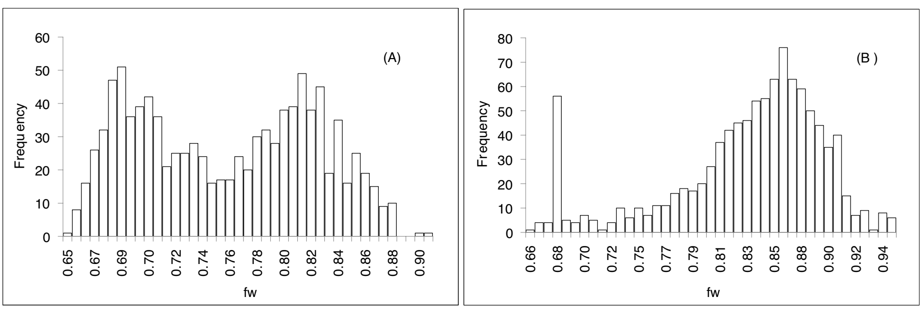

Figure 5. (A) Case-1, watercut Zone-B at t_R; (B) Case-2, watercut Zone-B at t_R.

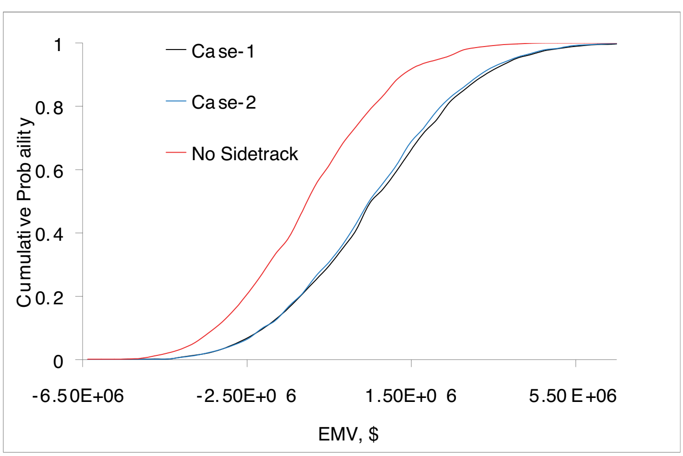

Figure 6. Cumulative probability of Case1, Case-2 and the option of no-sidetrack.

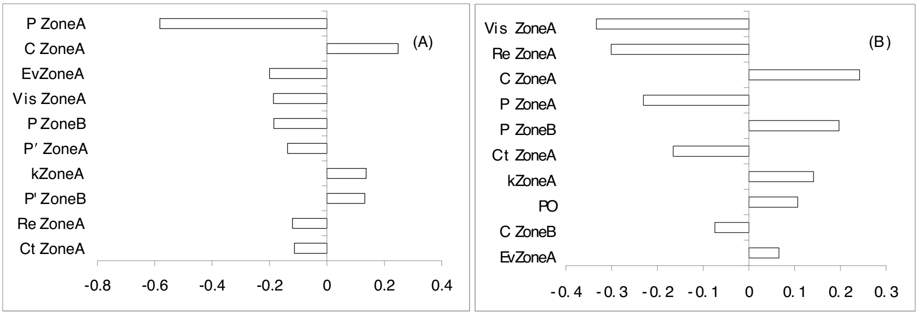

Figure 7. (A) Case-1, rank correlation of (top 10) stochastic parameters to optimal production well sidetrack time for sequential sidetrack; (B) Case-2, rank correlation of (top 10) stochastic parameters to optimal production well sidetrack time for sequential sidetrack.

simultaneous sidetrack optimal time of both the production and injection wells. This further indicates the acceptability of the initial value for solution of the EMV objective function of Case-2. In Orodu *et al*. (2011), the maximum and mean values of t_R and t_{RR} are less than that of this study due to stochastic Np_{BT} and probably to a lesser extent on the impact of re-evaluated Np_{BT} at t_{RR} for Zone-A against constant Np_{BT} at t_R and t_{RR}. Correlation of stochastic parameters to t_{RR} for Case-2, is in the order of relevance; POS for injection well (Zone-B), oil price, reservoir boundary (Zone-A), POS of production well (Zone-B) and oil viscosity (Zone-A) for the first 5 parameters of significance.

5. DISCUSSION

The significance of re-evaluating $Np_{BT,A}$ at t_{RR} due to the stochastic time difference in t_{RR} and t_R for which production by natural decline modeled by Arps' exponential decline is clearly seen. The problem here may be the availability of pressure decline data to compute water saturation based on material balance during natural decline. The assumption used here is the availability of pressure decline history for nearby wells in similar geological environment or may be based on numerical simulation result. A single regression equation may not be possible for estimating pressure from history data, hence more than 1 equation at various intervals may be suitable as deemed necessary for high accuracy. The material balance equation can be modified as required as this study neglected the impact of water production as it is minimal during early production stage.

Uniform oil production prior to water breakthrough is assumed. This holds, based on simulation study of production performance under water injection before significant water breakthrough as measured either from water-cut or water-oil-ratio (WOR). Hence uniform oil production rate is definitely applicable for steady and pseudo-steady state production. Transient state production is negligible considering the total production time.

For sequential production-injection sidetrack, the optimal injection well sidetrack time is affected by the initial value chosen for the solution of the EMV objective function. This suggests probably local maximum values. The case of the simultaneous sidetrack time has three identifiable local maximum points unlike the sequential sidetrack case. But the initial value for the solution is not a problem.

The introduction of reservoir variables as it affects production performance was to study the influence of not just only economic parameters and probability of success (geological and technical uncertainties) on sidetrack time. However, how reliable is the rank correlation order or magnitude of the various parameters to optimal sidetrack time in order to narrow down on those parameters that need to be carefully evaluated? This will largely depend on the objective function representing a true reflection of subsurface flow phenomenon with respect to the analytical schemes of production performance.

A limiting case is the constant injection rate based on the waterflood performance model to enable time dependency and the restriction to constant operational scheme. These and other issues can only be resolved by the use of a numerical simulation model.

5. CONCLUSIONS

Optimal sidetrack time is evaluated for a waterflooding scenario by optimizing expected monetary value (EMV) for a pair of production-injection well. The following pertinent conclusions are drawn:

1. Cumulative density function of EMV presents a convenient means of choosing optimal initial value for the solution of the EMV objective function for optimal sidetrack time for a sequential production-injection case study based on the optimization algorithm.
2. Based on re-evaluating a major assumption, the correlation of parameter consisting of reservoir and economic parameters and probability of success to optimal sidetrack time changed. This improved the reliability of the analytical approach.
3. Uniform oil production rate prior to water breakthrough is a reasonable assumption as depicted by a numerical reservoir simulator, further upholding the analytical approach.
4. If analytical production performance schemes accurately represents subsurface flow phenomenon, this approach to sensitivity study of parameters to sidetrack time is more convenient and feasible to study the impact of geological and technical factors. However, deconvolution of the factors should be considered, to separately study the impact of each factor.

APPENDIX-A

EMV components of sequential production-injection well sidetrack ($t_{BT,B} \leq t_R \leq t_{max}$ and $t_{RR} \geq t_R$)

The EMV is designed for production well sidetrack (t_R) being greater than the water breakthrough time ($t_{BT,B}$) of Zone-B and having the necessary modifications to adapt it to t_R occurring before $t_{BT,B}$ (t_R occurring essential at any time) as seen below for the alteration of specific components of the equation under the sub-heading "For: $0 \leq t_R \leq t_{max}$ ". Refer to Fig. 2 for the limits of integration.

NPV for Branch-A_2:

$$A_2 = -C_B - C_A^R e^{-i_D t_R} - C'_B$$
$$-\beta(C_B + C'_B)\int_0^{t_R} e^{-i_D t}dt - I_B i_W \int_0^{t_R} te^{-i_D t}dt - (P_O - \alpha_B)\int_{t_{BT,B}}^{t_R} qe^{i_D t}dt \quad \text{(A.1)}$$
$$+\frac{Np_{BT,B}\left(P_O - \alpha_B\right)}{t_{BT,B}}\int_0^{t_{BT,B}} te^{-i_D t}dt$$

For: $0 \le t_R \le t_{max}$,

$$(P_O - \alpha_B)\int_{t_{BT,B}}^{t_R} qe^{-i_D t}dt = 0 \quad \text{(A.2)}$$

$$\frac{Np_{BT,B}\left(P_O - \alpha_B\right)}{t_{BT,B}}\int_0^{t_{BT,B}} te^{-i_D t}dt = \frac{Np_{BT,B}\left(P_O - \alpha_B\right)}{t_{BT,B}}\int_0^{t_R} te^{-i_D t}dt \quad \text{(A.3)}$$

NPV for Branch-B:

$$B = -\beta\left[C_B\int_{t_R}^{t_{RR}} e^{-i_D t}dt + C_A^R\int_{t_R}^{t_{RR}} e^{-i_D t}dt\right]$$
$$-I_B\int_{t_R}^{t_{RR}} i_w te^{-i_D t}dt + (P_O - \alpha_{1A})e^{-i_D t_R}\int_0^{t_{RR}-t_R} q_{o2A}e^{-(D_A+i_D)t}dt \quad \text{(A.4)}$$
$$+(P_O - \alpha_B)\int_{t_R}^{t_{RR}} qe^{-i_D t}dt$$

For: $0 \le t_R \le t_{max}$,

$$(P_O - \alpha_B)\int_{t_R}^{t_{RR}} qe^{-i_D t}dt = (P_O - \alpha_B)\int_{t_{BT,B}}^{t_{RR}} qe^{-i_D t}dt + \frac{Np_{BT,B}(P_O - \alpha_B)}{t_{BT,B}}\int_{t_R}^{t_{BT,B}} te^{-i_D t}dt \quad \text{(A.5)}$$

NPV for Branch-B_1 excluding NPV of Branch-A_2 and Branch-B:

$$B_1 = -\beta\left[\left(C_B + C'_B\right)\int_{t_{RR}}^{t_{B98}} e^{-i_D t}dt + \left(C_A^R + C'_A\right)\int_{t_{RR}}^{t_{A98}} e^{-i_D t}dt\right] - C_A e^{-i_D t_{RR}}$$
$$-I_A\int_{t_{RR}}^{t_{A98}} i_w te^{-i_D t}dt - I_B^R\int_{t_{RR}}^{t_{B98}} i_w te^{-i_D t}dt + (P_O - \alpha_{1A})e^{-i_D t_{RR}}\int_{t_{BT,A}}^{t_{A98}} q_{1A}e^{-i_D t}dt \quad \text{(A.6)}$$
$$+(P_O - \alpha_B)\int_{t_{RR}}^{t_{B98}} qe^{-i_D t}dt + \frac{Np_{BT,A}(P_O - \alpha_{1A})}{t_{BT,A}}e^{-i_D t_{RR}}\int_0^{t_{BT,A}} te^{-i_D t}dt$$

$Np_{BT,A}$ in Equation (A.6) for Branch-B_1 is evaluated as explained under the introductory section of this paper.

Where A, A_2, B, B_2, C, D, E_1, E_2, C_B, C'_B, i_D, i_w, Po, α, β, $t_{BT,A}$, $t_{BT,B}$ are the same as Equation (9) and (10); C_A and C'_A are sidetrack cost of production and injection wells for Zone-A; t_{A98} and t_{B98} are time to water breakthrough for Zone-A and Zone-B; $Np_{BT,A}$ and $Np_{BT,B}$ are cumulative oil production at $t_{BT,A}$, and $t_{BT,B}$; I is variable water injection cost, t_R and t_{RR} are optimal sidetrack time for production and injection well; q is oil production rate subject to Yang's water performance model and q_o is oil production rate subject to Arps' exponential decline model.

REFERENCES

Babadagli T., 2007. Development of mature oil fields – A Review. *Journal of Petroleum Science and Engineering* **57(3-4)** , 221-246.

Correa A. C. F., 2007. Forecasting oil production in waterflood reservoirs. Paper 107992 presented at the SPE Latin American and Caribbean Petroleum Engineering Conference, Buenos Aires, Argentina, April 15–18.

Ershaghi I. and Omoregie O., 1978. A method for extrapolation of cut vs recovery curves. *Journal of Petroleum Technology* **30(2)** , 203–204.

Fetkovich M.J., 1980. Decline curve analysis using type curves. *Journal of Petroleum Technology* **32(6)** , 1065–1077.

Garb F.A., 1985. Oil and gas reserves classification, estimation and evaluation. *Journal of Petroleum Technology* **37(3)** , 373–390.

Han Y.M., Park C.-H. and Kang Joe M., 2011. Prediction of nonlinear production performance in waterflooding project using a multi-objective evolutionary algorithm. *Energy Exploration & Exploitation* **29(2)** , 129–142.

Jiang S., Wang H., Cai D.S., Guo H., Zhu G.H., Zhu X.M. and Hu X.L., 2010. The secondary porosity and permeability characteristics of Tertiary strata and their origins, Liaodong Bay Basin, China. *Energy Exploration & Exploitation* **28(4)** , 207–222.

Lerche I. and Mudford B., 2001. Risk evaluation for recompletion and sidetrack developments in oil fields, Paper 71418 presented at the SPE Annual Technical Conference and Exhibition, New Orleans, September 30 – October 2.

Lerche I. and Noeth S., 2001a. Sidetrack and recompletion risks in oil fields: I – recompletion decisions based on estimated net present value. *Energy Exploration & Exploitation* **19(1)** , 11–37.

Lerche I. and Noeth S., 2001b. Sidetrack and recompletion risks in oil fields: III – recompletion and production modelling time. *Energy Exploration & Exploitation* **19(1)** , 57–73.

Lerche, I., and Noeth, S. (2001c). Sidetrack and recompletion risks in oil fields: VII – sidetrack and production modelling with time. *Energy Exploration & Exploitation* **19(5)** , 463–478.

Li K. and Horne R. N., 2005. An analytical model for production decline-curve analysis in naturally fractured reservoirs. *SPE Reservoir Evaluation & Engineering* **8(3)** , 197–204.

Lijek S.J., 1989. Simple performance plots used in rate-time determination and waterflood analysis, Paper 19847 presented at the SPE 64th Annual Technical Conference and Exhibition, San Antonio, TX, October 8–11.

Newendorp P. and Schuyler J., 2000. Decision Analysis for Petroleum Exploration, 2nd ed. Planning Press, Colorado.

Orodu O.D., Tang, Z. and Fei Q., 2009. Application of prediction models to performance of high paraffinic content oilfields: Case study shen95 block of Jinganbao Oilfield. *Journal of Petroleum Science and Engineering* **68**, 223–234.

Orodu O.D., Ako, C.T. and Craig A.J., 2011. Risk assessment of optimal sidetrack time, *Petroleum Science and Technology* (in press).

Wu C.H., 1988. Waterflood performance projection using classical waterflood models, Paper 18597 presented at the SPE Production Technology Symposium, Hobbs, New Mexico, November 7-8.

Yang Z., 2009. A new diagnostic analysis method for waterflood performance. *SPE Reservoir Evaluation & Engineering* **12(2)** , 341–351.