Hindawi Publishing Corporation
Active and Passive Electronic Components
Volume 2016, Article ID 3426713, 7 pages
http://dx.doi.org/10.1155/2016/3426713

Research Article

Computer and Hardware Modeling of Periodically Forced ϕ^6-Van der Pol Oscillator

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Received 6 June 2016; Revised 17 October 2016; Accepted 24 November 2016

Academic Editor: Jiun-Wei Horng

Numerical simulation results for the dynamics of ϕ^6-systems abound in the literature but their experimental results are yet to be known. This paper presents the chaotic dynamics of ϕ^6-Van der Pol oscillator via electronic design, simulation, and hardware implementation. The results obtained are found to be in good agreement with numerical simulation results. The condition for stability of the fixed points is also computed and the method of multiple time scale is used to investigate the dynamical behaviour of the system. Therefore, the ϕ^6-circuits which have rich dynamics and may have important applications in secure communications, random number generations, cryptography, and so forth have been practically implemented.

1. Introduction

In recent years the study of chaotic phenomena in self-excited oscillators with parametric and external-excitation has attracted much attention [1–6]. Among the 2-dimensional periodically forced oscillators, the most extensively studied examples are the Van der Pol oscillators [7, 8], Duffing oscillators [9], and Rayleigh oscillators [10]. Many works on self-excited, parametrically excited, and externally excited oscillators are also investigated in the literature. Recently, the rich dynamics of Van der Pol oscillator have been explored by theoretical analysis and numerical simulations [3, 11].

The ϕ^4-Van der Pol oscillator is a very significant classical model circuit that has been studied and even modified in some studies as reported by King et al. [12, 13], Fotsin et al. [14], and Fodjouong et al. [15]. However, with the advent of the study of chaotic motion by means of strange attractors, Poincare' map, fractal dimension, it is necessary to seek for a better understanding of nonlinear systems with higher order nonlinear terms such as the ϕ^6-Van der Pol oscillator.

A ϕ^6-Van der Pol oscillator or extended Van der Pol oscillator is the nonlinear oscillator with fifth-order nonlinear term. Its numerical simulation has been studied extensively by Siewe Siewe et al. [16, 17], Tchoukuegno et al. [18–20], and Yu and Li [21] and its synchronization behaviours for possible applications in information processing and secure communication by Njah et al. [22–24], and its application in the modeling of chemical reactions and brain activity by Chen and Dong [25] and Ott et al. [26]. In the past three decades, implementation of chaotic systems in power electronics has gone through intense development in many aspects of technology [27–29], including power devices, control methods, circuit design, computer-aided analysis, passive components, and packaging techniques. Like many areas of engineering, power electronics is mainly motivated by practical applications, and it often turns out that a particular circuit topology or system implementation has found widespread applications long before it is thoroughly analyzed and most of its subtleties are uncovered [30, 31].

The aim of this paper is to design, simulate, and implement periodically forced ϕ^6-Van der Pol oscillator by transforming the state equations into electronic circuit via analog components modeling. We also generate phase portraits for the triple-well and double-well attractors of the system. To the best of our knowledge, computer simulation with electronic software and hardware implementation of ϕ^6-Van der Pol oscillators have not been studied. The rest of the paper is organized as follows: Brief description of the conditions for stability of fixed points for unperturbed system is discussed in the next section. Section 3 explains the numerical simulation of the oscillator for at least six physically relevant situations with corresponding attractors while Section 4 explains the analog simulations with electronic software (multisim) and hardware implementation of the system. Finally, Section 5 concludes the paper.

2. Analysis of the Unperturbed System

The general form of second-order differential equation of Van der Pol systems is given as

$$\ddot{x} = \mu\left(1 - x^2\right)\dot{x} - \frac{dV(x)}{dx} + f\cos\omega t, \quad (1)$$

where $\mu < 0$ is the damping parameter and f and ω are amplitude and angular frequency, respectively, and its potential $V(x)$ can be expressed in Taylor series as

$$V(x) = \frac{1}{2}\alpha x^2 + \frac{1}{4}\beta x^4 + \frac{1}{6}\lambda x^6, \quad (2)$$

where α, β, and λ are constant parameters. The condition for stability of the fixed points of oscillator (1) for an unperturbed system can be assumed when $\mu = f = 0$. Thus, the system can therefore be expressed as a set of first-order differential equations of the form

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -\alpha x - \beta x^3 - \lambda x^5 \end{aligned} \quad (3)$$

which corresponds to an integrable Hamiltonian system with the potential function in (2) and whose associated Hamiltonian function is

$$H(x, y) = \frac{1}{2}y^2 + \frac{1}{2}\alpha x^2 + \frac{1}{4}\beta x^4 + \frac{1}{6}\lambda x^6. \quad (4)$$

From (3) and (4), we can compute for the fixed points and analyze their stabilities after some algebraic manipulations using the following conditions.

(i) For $\beta^2 < 4\lambda\alpha$, the unperturbed system has only one fixed point $(0,0)$, the centre, for which the system is single-well if $\alpha > 0$ and $\beta > 0$ and single-hump potential if $\alpha < 0$ and $\beta < 0$ as shown in Figures 1(a) and 1(b), respectively.

(ii) For $\beta^2 = 4\lambda\alpha$ with $\alpha < 0$, $\beta > 0$, and $\lambda > 0$, the unperturbed systems have three fixed (equilibrium) points, that is, one centre $(0,0)$ and two saddles connected by two heteroclinic orbits $(\pm x_2, 0)$, (5).

(iii) For $\beta^2 > 4\lambda\alpha$ with $\alpha > 0$, $\beta < 0$, and $\lambda > 0$, the unperturbed systems have five fixed points: one centre $(0,0)$, two saddles connected by two heteroclinic orbits $(\pm x_2, 0)$, and two saddle points connected to each other by one homoclinic orbit $(\pm x_1, 0)$, (5) and (6).

In previous works, Siewe Siewe et al. [16, 17] have shown that, for $\mu = 0$, system (4) is a Hamiltonian system with a pair of heteroclinic orbits defined as

$$\begin{aligned} x_{het}(t) &= \pm\frac{\sqrt{2}x_1\sinh((\gamma/2)t)}{\sqrt{-\xi + \cosh(\gamma t)}}, \\ y_{het}(t) &= \pm\frac{\sqrt{2}\gamma(\xi - 1)\cosh((\gamma/2)t)}{2(-\xi + \cosh(\gamma t))^{3/2}}, \end{aligned} \quad (5)$$

and a pair of symmetric homoclinic trajectories connecting each unstable point to itself given by

$$\begin{aligned} x_{hom}(t) &= \pm\frac{\sqrt{2}x_1\cosh((\gamma/2)t)}{\sqrt{\xi + \cosh(\gamma t)}}, \\ y_{hom}(t) &= \pm\frac{\sqrt{2}\gamma(\xi - 1)\sinh((\gamma/2)t)}{2(\xi + \cosh(\gamma t))^{3/2}}, \end{aligned} \quad (6)$$

where

$$\begin{aligned} x_1 &= \sqrt{-\frac{1}{2\lambda}\left(\beta + \sqrt{\Delta}\right)}, \\ x_2 &= \sqrt{-\frac{1}{2\lambda}\left(\beta - \sqrt{\Delta}\right)}, \\ \gamma &= x_1^2\sqrt{2\lambda(\rho^2 - 1)}, \\ \xi &= \frac{5 - 3\rho^2}{3\rho^2 - 1}, \\ \rho^2 &= \frac{\beta - \sqrt{\Delta}}{\beta + \sqrt{\Delta}}, \\ \Delta &= \beta^2 - 4\alpha\lambda. \end{aligned} \quad (7)$$

3. Numerical Simulation

The general form of the periodically forced Van der Pol oscillator model is given by the second-order nonautonomous differential equation (1), where x is the state variable, dot(s) over x denotes the derivative with respect to time t, f is the amplitude of periodic forcing, ω is the angular frequency, $\mu > 0$ is the damping parameter, and $V(x)$ is the potential. The potential for six physically relevant situations (single, double, and triple-well and hump) is shown in Figure 1. They correspond to different choices of the set of values for the parameter set α, β, and λ. For instance, the potential is double-well if ($\alpha < 0$, $\beta > 0$, and $\lambda > 0$), double-hump if ($\alpha >$

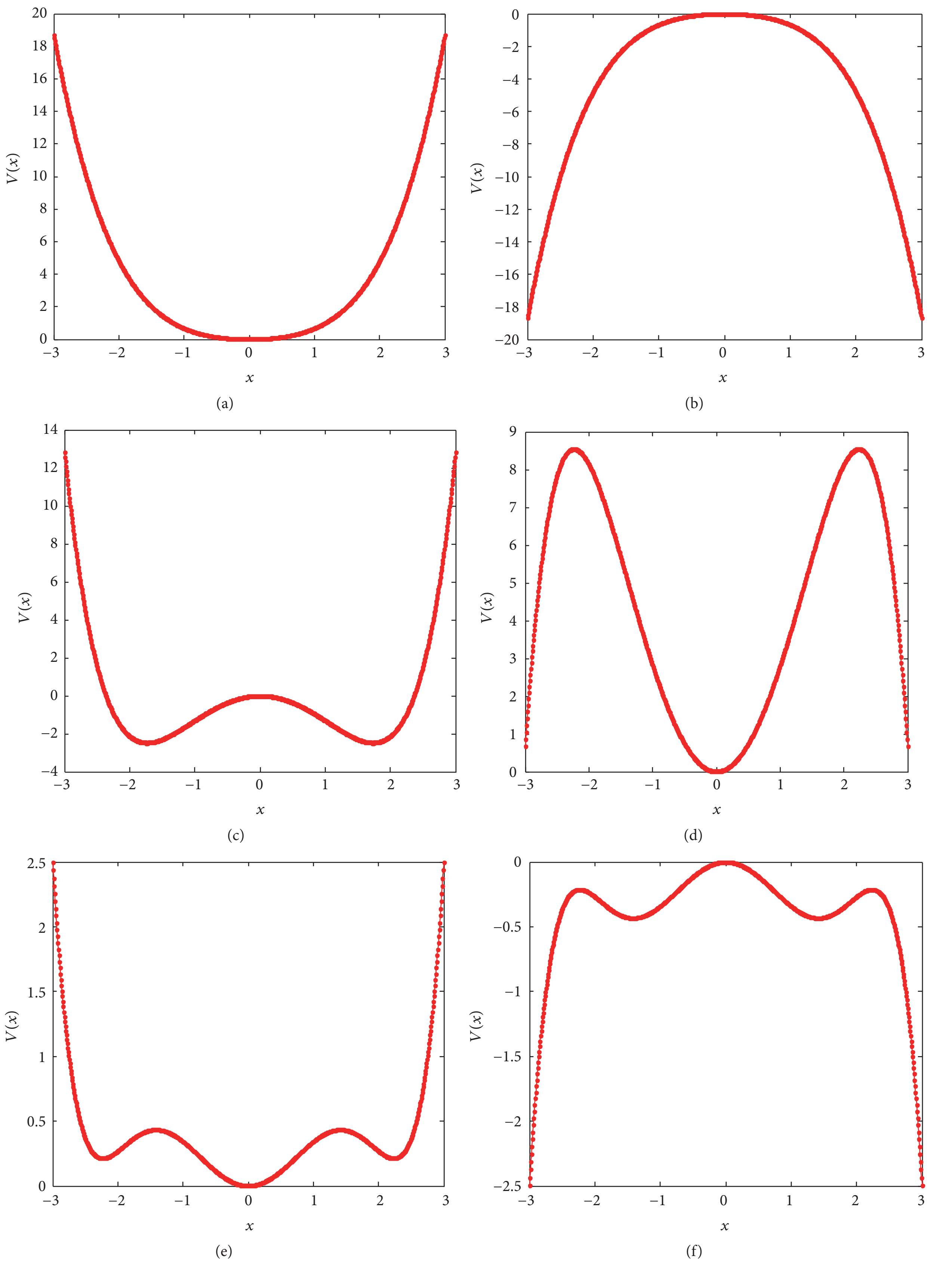

FIGURE 1: The smooth potential function, $V(x)$, of the ϕ^6-oscillators for (a) single-well: $\alpha = 0.5$ and $\beta = 0.175$, (b) single-hump: $\alpha = -0.5$ and $\beta = -0.175$, (c) double-well: $\alpha = -0.46$, $\beta = 0.7$, and $\lambda = 0.1$, (d) double-hump: $\alpha = 1.5$, $\beta = -0.7$, and $\lambda = -0.1$, (e) triple-well: $\alpha = 0.46$, $\beta = -0.7$, and $\lambda = 0.1$, and (f) triple-hump: $\alpha = -1$, $\beta = 0.7$, and $\lambda = -0.1$.

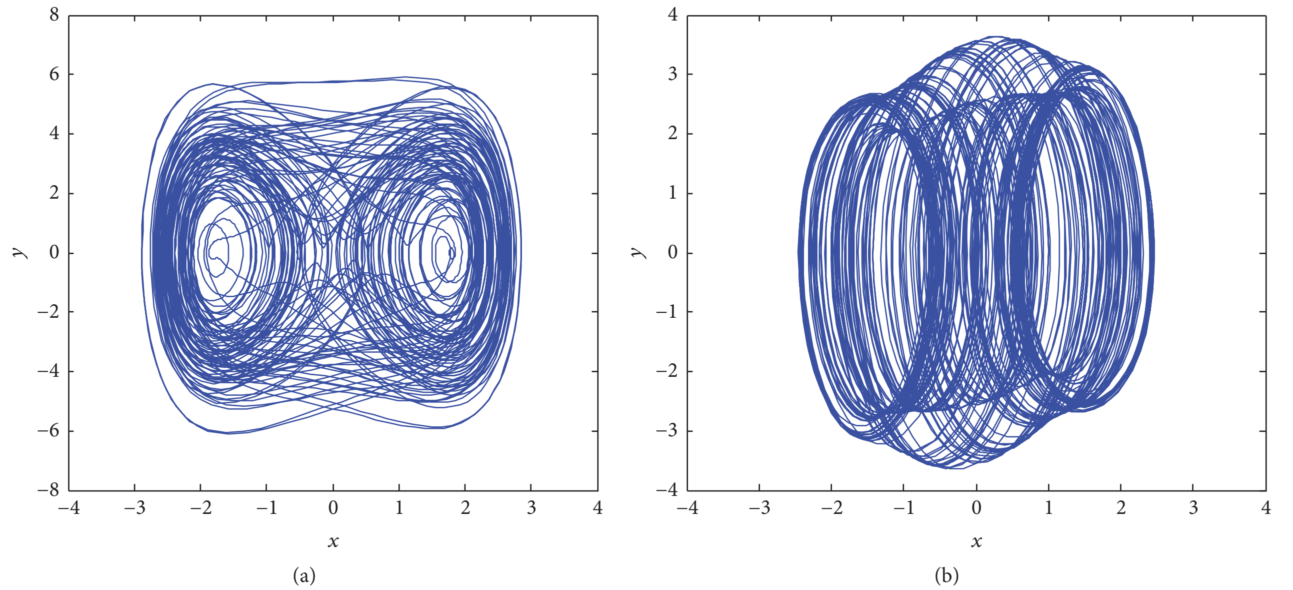

Figure 2: Phase portrait of the chaotic attractor for the ϕ^6-Van der Pol oscillator with parameters values (a) $\mu = 0.01$, $\alpha = 0.46$, $\omega = 0.86$, $f = 9.0$, $\beta = 1.0$, and $\lambda = 0.1$ for the double-well potentials and (b) $\mu = 0.4$, $\alpha = 1.0$, $\omega = 3.14$, $f = 9.0$, $\beta = -0.7$, and $\lambda = 0.1$, for the triple-well.

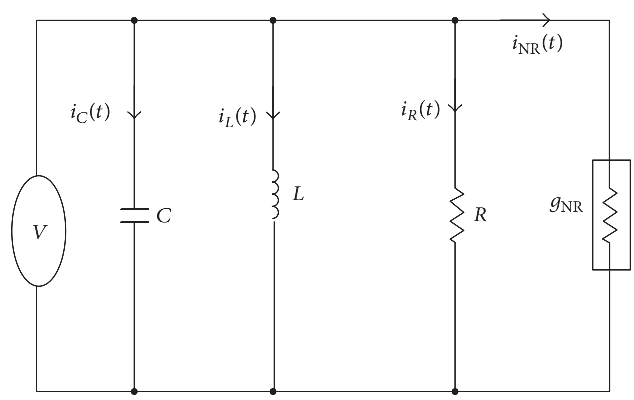

Figure 3: Schematic circuit of Van der Pol model.

0, $\beta < 0$, and $\lambda < 0$), triple-well if ($\alpha > 0$, $\beta < 0$, and $\lambda > 0$), and triple-hump if ($\alpha < 0$, $\beta > 0$, and $\lambda < 0$).

Van der Pol oscillator models describe various physical, electrical, mechanical, and engineering devices, for different potentials $V(x)$. The potentials $V(x)$ are approximated by finite Taylor series for the ϕ^6-chaotic oscillators as shown in (3), where α, β, and λ are constant parameters of the potential. The introduction of a new parameter λ in (2) yields ϕ^6-Van der Pol equation (9) that can be expressed as

$$\ddot{x} = \mu\left(1 - x^2\right)\dot{x} - \alpha x - \beta x^3 - \lambda x^5 + f\cos\omega t. \qquad (8)$$

Numerical simulation of (8) using fourth-order Runge-Kutta method gives the phase portraits presented in Figure 2 for double and triple scroll attractors. From the phase portraits it is obvious that the systems are chaotic for the parameter values used.

4. Design of Analog Simulator

Circuit realization of ϕ^6-Van der Pol system is shown in Figure 3 (schematic circuit) and Figure 4 (analog circuit) in which the nonlinear resistor (NR) can be modeled by using i-ϕ nonlinear characteristic given by the following relation:

$$i_{NR} = \frac{dq}{d\phi} \cdot \frac{d\phi}{d\tau} = \left(\alpha_o\phi + \beta_o\phi^3 + \delta_o\phi^5\right), \qquad (9)$$

where α_o, β_o, and δ_o are parameters of the nonlinear resistor g_{NR}, ϕ is the flux over the resistor, q is the charge, C is the capacitance, L is the inductance, R is the resistance,

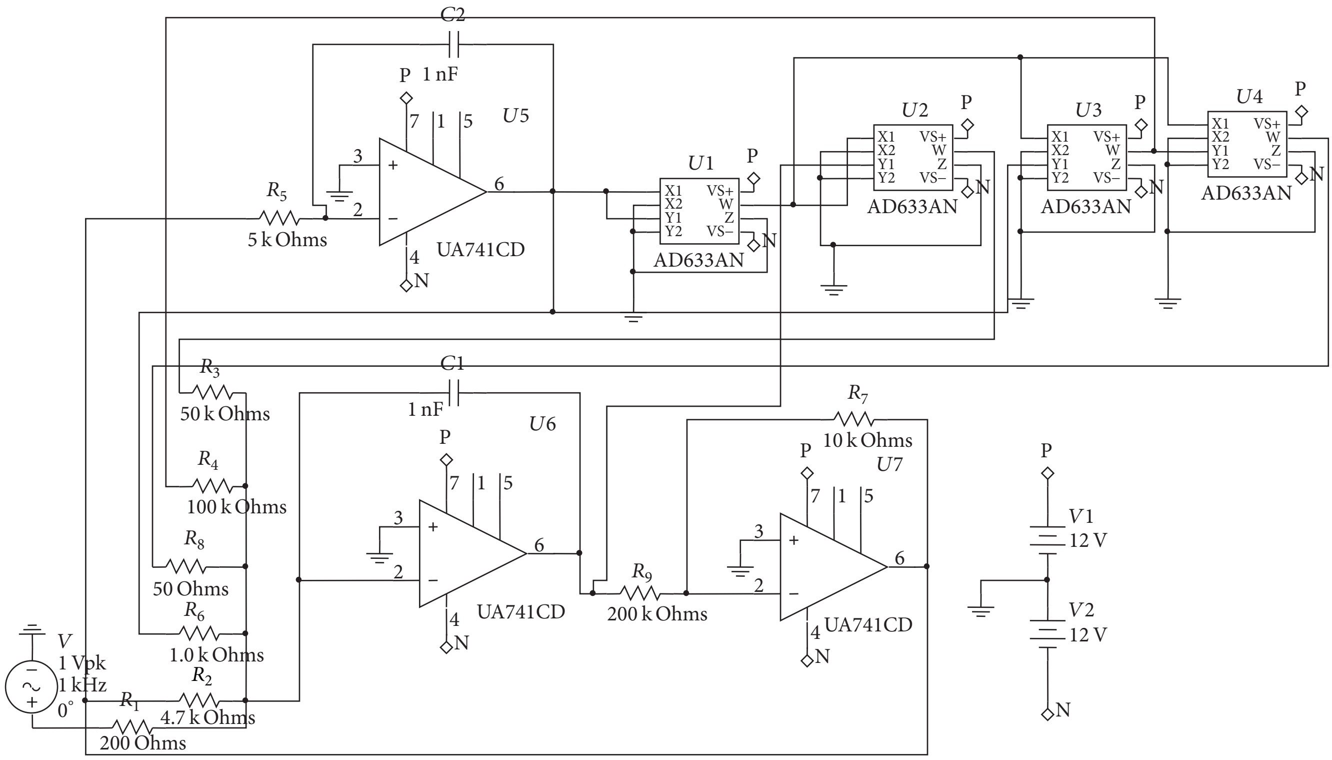

Figure 4: Analog circuit of ϕ^6-Van der Pol oscillator.

$V = E\cos\omega\tau$, and i_{NR} is the current through the nonlinear resistor. Applying Kirchoff's law to the circuit shown in Figure 3, we have

$$\begin{gathered} V_R + V_L + V_C + V_{\mathrm{NR}} = E\cos\omega\tau, \\ i_R + i_L + i_C + i_{\mathrm{NR}} = 0. \end{gathered} \tag{10}$$

Then (10) becomes

$$\begin{aligned} &\frac{d^2\phi}{dt^2} - \sqrt{\frac{L}{C}}\left[1-\phi^2\right]\frac{d\phi}{dt} + \frac{R}{L^2C}\left(\phi_o + \phi_o^3 + \phi_o^5\right) \\ &\quad = \frac{E}{\sqrt{LC}}\cos\omega t. \end{aligned} \tag{11}$$

Taking $\mu = \sqrt{L/C}$, $t = \tau/\sqrt{LC}$, $\alpha = (R/L^2C)\alpha_o$, $\beta = (R/L^2C)\beta_o$, $f = E/\sqrt{LC}$, and $\delta = (R/L^2C)\delta_o$, we can rewrite electronic equation (11) in form of flux as

$$\frac{d^2\phi}{dt^2} - \mu\left[1-\phi^2\right]\frac{d\phi}{dt} + \alpha\phi + \beta\phi^3 + \delta\phi^5 = f\cos\omega t. \tag{12}$$

The electronic analog simulator for ϕ^6-Van der Pol oscillator can be designed using the electronic multiplier, *AJ633AN*, which perform the multiplication operations. The analog circuit operates over a dynamic range of ±15 D.C. The integrators are operational amplifiers, $U5$ and $U6$ (UA741CD) with feedback capacitors, and summations of the state variables are obtained using operational amplifiers $U7$ with the help of feedback resistors and multiple input resistors. Also, the simulator output showing the dynamics when varying the input components can be viewed directly on a software oscilloscope using an appropriate time scaling. The designed circuitry realization for the oscillator consists of two channels A and B which realize the two state variables x and $\dot{x}$, respectively. The attractors were generated from ϕ^6-Van der Pol circuit of Figure 4 by varying

$$\lambda = \frac{R_9}{R_8}. \tag{13}$$

The computer simulation of ϕ^6-Van der Pol circuit on electronic software (multisim) (Figure 5(a)) and its hardware implementation on electronic breadboard (Figure 5(b)) both reproduced the numerical results in Figure 2 on the oscilloscope. Indeed, the practical implementation of the aforementioned system has application in technology such as in secure communication.

5. Conclusion

In this paper, we have used the analog electronic software (multisim) for the electronic circuit design and simulation of ϕ^6-Van der Pol oscillator and have practically implemented the designed circuit on electronic breadboard using analog components. Our experimental results reproduced those obtained via numerical simulation and multisim simulation. Therefore the rich dynamics of extended Van der Pol oscillator which have wide application in science and technology have been practically implemented, for possible technological application such as in secure communication.

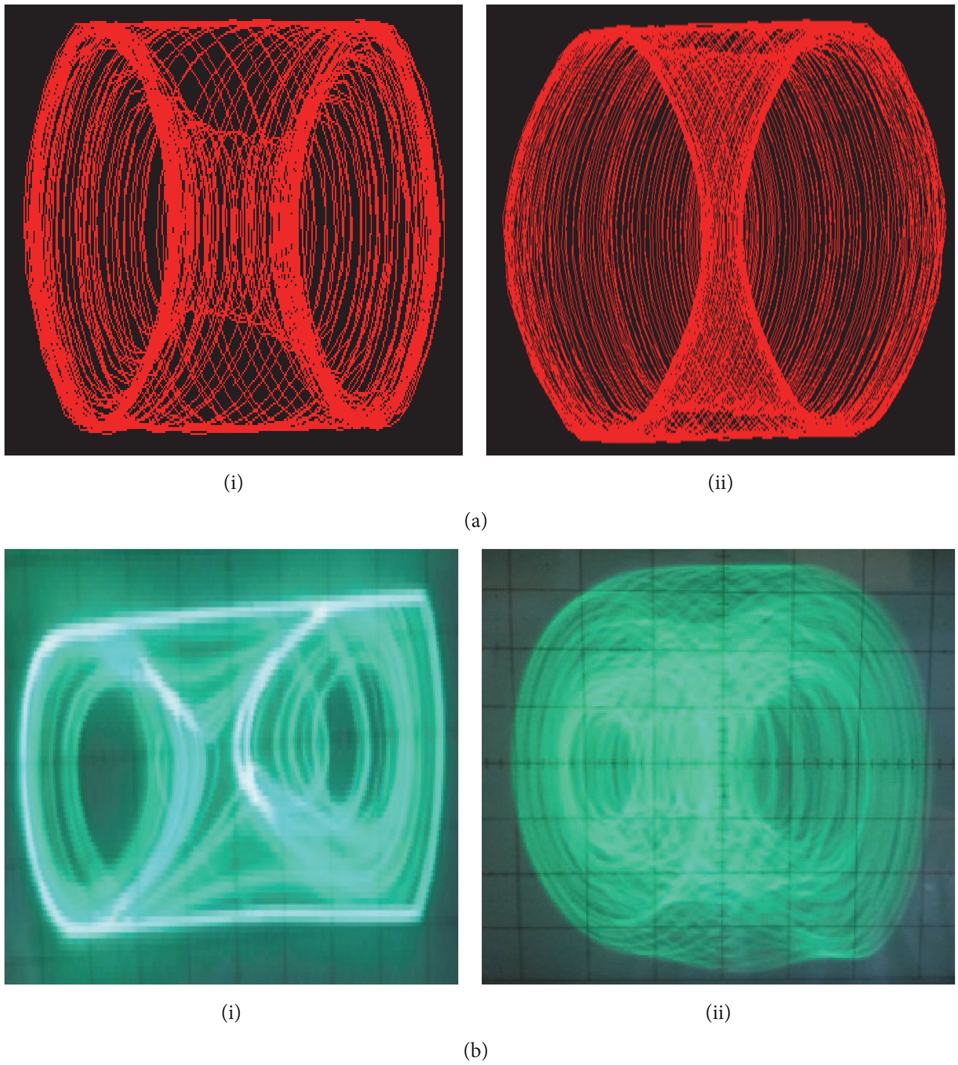

FIGURE 5: Phase portrait of ϕ^6-Van der Pol attractors for (a) computer simulation and (b) experimental implementation for (i) double-well at $R_8 = 50\,\Omega$ and $R_9 = 200\,\mathrm{k}\Omega$ and (ii) triple-well at $R_8 = 5\,\mathrm{k}\Omega$ and $R_9 = 50\,\mathrm{k}\Omega$.

Competing Interests

The authors declare that they have no competing interests.

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