Applied Mathematical Sciences, Vol. 8, 2014, no. 164, 8199 - 8207
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/ams.2014.410807

On Numerical Solutions of Systems of Ordinary Differential Equations by Numerical-Analytical Method

A. A. Opanuga, S. O. Edeki [*], H. I. Okagbue, G. O. Akinlabi

Department of Mathematical Sciences
Covenant University, Canaanland, Otta, Nigeria

[*]Corresponding author

A. S. Osheku

Department of Mathematics
University of Ibadan, Ibadan, Nigeria

B. Ajayi

Department of Mathematics & Statistics
Federal Polytechnic, Ado Ekiti, Nigeria

Abstract

This paper considers the solutions of systems of ordinary differential equations via a numeric-analytical method referred to differential Transforms Method (DTM). For numerical interpretation, two illustrative examples are used. The results obtained show a strong agreement with their corresponding exact solutions. The method is therefore proven to be effective and reliable, and as such, can be applied to systems of ODEs involving higher orders.

Keywords: System of equations, Differential transform, ODEs, Exact solution

1. Introduction

Various scientific and technological problems have been modeled mathematically by systems of Ordinary Differential equations. Problems like the series circuits and mechanical systems involving several springs attached in series leading to a system of differential equations as considered by Shawagfehm and Kaya [1]; such systems are further encountered in chemical, ecological, biological and engineering applications. Krishnaveni and Balachander [2] applied the ADM for the solution of singular systems of transistor circuits.

Many authors have solved system of Ordinary Differential equations using different methods and techniques- these include: Systems of first order ordinary Differential equations. Dogan [3] obtained solutions of the systems of Ordinary differential equations by combined Laplace Transform-Adomian Decomposition method (LTADM). Biazar *et al* in [4] solved a system of differential equations of first order, and of different (any) order by converting the non-order one system of ODE to that of order one, with the convergence of the associated method being addressed. Lewis in [5] applied a geometric design technique in dealing with singular system of transistor circuits. In [6], Rodrigues applied the ADM to a nonlinear reaction system of Raman type for an analytical solution. In [7], the Differential Transform Method (DTM) has been showed for exact solutions of certain class of second order differential equations, and as an iterative technique for numerical solutions of differential equations [8].

In this paper, differential transform method as a numeric-analytic technique is applied for solutions of systems of ODEs. The cases for both first and second order are considered for numerical results and interpretations. In the remaining part of the paper, section 2 deals with the preliminaries of DTM, section 3 is on numerical applications with illustrative examples, and the discussion of results while section 4 is on concluding remarks.

2. Some Fundamental Points of Differential Transform Method

An arbitrary one dimensional function $y = f(x)$ in Taylor series about a point $x = 0$ is expressed as:

$$f(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!} \left[\frac{d^k f}{dx^k} \right]_{x=0} \tag{1}$$

The differential transformation of $f(x)$ is defined:

$$F(k)=\frac{1}{k!}\left[\frac{d^k y}{dx^k}\right]_{x=0} \tag{2}$$

Then the inverse differential transform is:

$$f(x)=\sum_{k=0}^{\infty}F(k)x^k \tag{3}$$

The following theorems that can be deduced from equations (1), (2), (3)

Theorem 1: If $f(x)=\alpha p(x)\pm\beta q(x),\ \ then\ \ F(k)=\alpha P(k)\pm\beta Q(k)$

Theorem 2: If $f(x)=x^m,\ then\ \ F(\mathrm{k})=\delta(k-m)=\begin{cases}1, & k=m\\ 0, & otherwise\end{cases}$

Theorem 3: If $f(x)=e^x\ \ then\ \ F(k)=\dfrac{1}{k!}$

Theorem 4: If $f(x)=\dfrac{d^n f(x)}{dx^n},\ \ then\ \ F(k)=\dfrac{(k+n)!}{k!}Y(k+n)$

3. Numerical Applications

In this section, we apply the method for the numerical results to two considered problems (examples).

Problem1: Consider the following system of ODE [9]

$$2x'(t)+y'(t)=5e^t \tag{4}$$

$$y'(t)+3x'(t)=5 \tag{5}$$

subject to

$$x(0)=y(0)=0 \tag{6}$$

with the exact solutions are:

$$x=e^t-t-1,\quad y=2t-3+3e^t \tag{7}$$

Procedure for solution 1

The differential transformation of the above system of equations with respect to their initial conditions is:

$$2(k+1)\,\mathrm{X}(k+1)+(k+1)Y(k+1)=\frac{5}{k!} \tag{8}$$

$$(k+1)\,\mathrm{Y}(k+1)-3(k+1)\,\mathrm{X}(k+1)=5\delta(k) \tag{9}$$

$$X(0)=0,\ Y(0)=0 \tag{10}$$

So, when $k=0$:

$$2X(1)+Y(1)=\frac{5}{0!} \qquad \& \qquad Y(1)-3X(1)=5\delta(0)$$

Hence, $k=0$, yields $X(1)=0, \quad Y(1)=5$

As such:

$$\text{when} \quad k=1,\ X(2)=\frac{1}{2}, \qquad Y(2)=\frac{3}{2}$$

$$\text{when} \quad k=2,\ X(3)=\frac{1}{6}, \qquad Y(3)=\frac{1}{2}$$

$$\text{when} \quad k=3,\ X(4)=\frac{1}{24}, \qquad Y(4)=\frac{1}{8}$$

Therefore, using (1)-(3), we have:

$$x(t)=\sum_{k=0}^{\infty} X(k)t^k=\frac{t^2}{2}+\frac{t^3}{6}+\frac{t^4}{24}+\cdots=\frac{t^2}{2!}+\frac{t^3}{3!}+\frac{t^4}{4!}+ \tag{11}$$

$$y(t)=\sum_{k=0}^{\infty} Y(k)t^k=5t+\frac{3t^2}{2}+\frac{t^3}{2}+\frac{t^4}{8}+\cdots \tag{12}$$

Problem 2: Consider the following system of ODE

$$x''(t)+2x=y, \tag{13}$$

$$y''(t)+2y=x \tag{14}$$

subject to the initials:

$$x(0)=4,\ y(0)=2,\ x'(0)=0,\ y'(0)=0 \tag{15}$$

with the exact solutions:

$$x(t)=3\cos t+\cos\sqrt{3}t, \quad y(t)=3\cos t-\cos\sqrt{3}t \quad [9] \tag{16}$$

Procedure for solution 2

Taking the differential transformation of the equations in the above system gives:

$$(k+1)(k+2)X(k+2)+2X(k)-Y(k)=0 \qquad (17)$$

$$(k+1)(k+2)Y(k+2)+2Y(k)-X(k)=0 \qquad (18)$$

with the corresponding initials:

$$X(0)=4, Y(0)=2, X(1)=0, Y(1)=0 \qquad (19)$$

Thus for $k=0$, we have:

$$2X(2)+2X(0)-Y(0)=0 \qquad \& \qquad 2Y(2)+2Y(0)-X(0)=0,$$

Iteratively, we obtain the following:

$$\begin{aligned}
&when\ \ k=0, \quad X(2)=-3, \quad Y(2)=0\\
&when\ \ k=1, \quad X(3)=0, \quad Y(3)=0\\
&when\ \ k=2, \quad X(4)=\frac{1}{2}, \quad Y(4)=-\frac{1}{4}\\
&when\ \ k=3, \quad X(5)=0, \quad Y(5)=0\\
&when\ \ k=4, \quad X(6)=-\frac{1}{24}, \quad Y(6)=\frac{1}{30}
\end{aligned}$$

Therefore, using (1) - (3), we have:

$$x(t)=\sum_{k=0}^{\infty}X(k)t^k=4-3t^2+\frac{t^4}{2}-\frac{t^6}{24}+\cdots \qquad (20)$$

$$y(t)=\sum_{k=0}^{\infty}Y(k)t^k=2-\frac{t^4}{4}+\frac{t^6}{30}+\cdots \qquad (21)$$

3.1 *Discussion of Results*

In this subsection, we present tables and figures for graphical representation and interpretation of our results:

Table1: for problem 1

Time, t	*x(t): exact*	*x(t): 3-term iterate (DTM)*	*Absolute error: x(t)*	*y(t): exact*	*y(t): 3-term iterate (DTM)*	*Absolute error: y(t)*
0.10	0.005171	0.005171	8.47E-08	0.515513	0.515513	2.54E-07
0.20	0.021403	0.021400	2.76E-06	1.064208	1.064200	8.27E-06
0.30	0.049859	0.049838	2.13E-05	1.649576	1.649513	6.39E-05
0.40	0.091825	0.091733	9.14E-05	2.275474	2.275200	0.000274
0.50	0.148721	0.148438	0.000284	2.946164	2.945313	0.000851
0.60	0.222119	0.221400	0.000719	3.666356	3.664200	0.002156
0.70	0.313753	0.312171	0.001582	4.441258	4.436513	0.004746
0.80	0.425541	0.422400	0.003141	5.276623	5.267200	0.009423
0.90	0.559603	0.553838	0.005766	6.178809	6.161513	0.017297
1.00	0.718282	0.708333	0.009948	7.154845	7.125000	0.029845

Table2: for problem 2

Time, t	*x(t): exact*	*x(t):3-term iterate (DTM)*	*Absolute error: x(t)*	*y(t): exact*	*y(t): 3-term iterate (DTM)*	*Absolute error: y(t)*
0.0	4.000000	4.00000	0.000000	2.000000	2.000000	0.000000
0.1	3.838725	3.97005	0.131325	2.131300	1.999975	0.131325
0.2	3.654903	3.880797	0.225894	2.225497	1.999602	0.225894
0.3	3.448763	3.734020	0.285257	2.283256	1.997999	0.285257
0.4	3.220834	3.532629	0.311796	2.305532	1.993737	0.311796
0.5	2.971934	3.280599	0.308665	2.293562	1.984896	0.308666
0.6	2.703162	2.982856	0.279694	2.248852	1.969155	0.279696
0.7	2.415885	2.645148	0.229263	2.173168	1.943897	0.229271
0.8	2.111721	2.273877	0.162156	2.068519	1.906338	0.162181
0.9	1.792522	1.875907	0.083385	1.937138	1.853690	0.083448
1.0	1.460350	1.458333	0.002020	1.781463	1.783333	0.001870

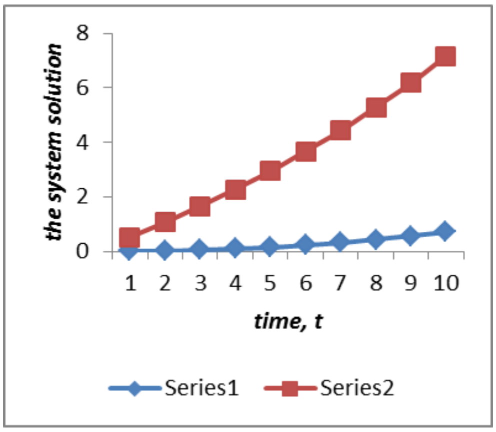

Figure1: The Exact solution: $x(t)$ & $y(t)$ (series1 & series2) –*problem1*

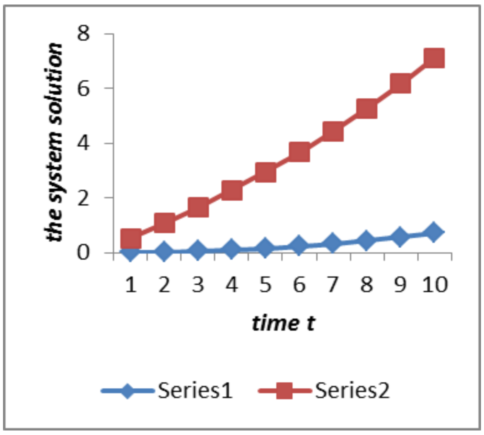

Figure2: The DTM solution: $x(t)$ & $y(t)$ (series1 & series2) –*problem1*

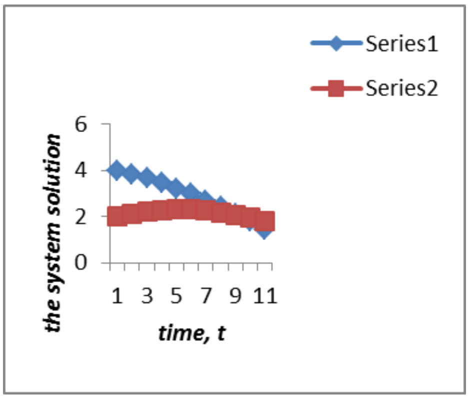

Figure3: The Exact solution: $x(t)$ & $y(t)$ (series1 & series2) –*problem 2*

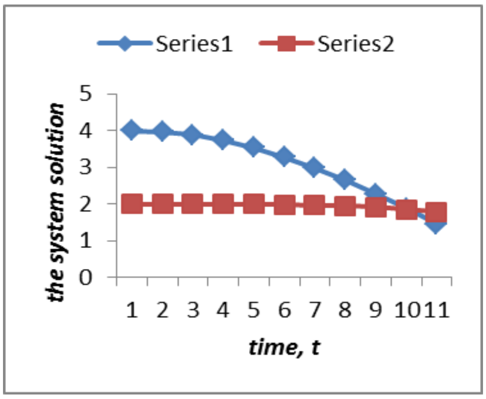

Figure4: The DTM solution: $x(t)$ & $y(t)$ (series 1 & series2 resp.)-*problem* 2

4. Concluding Remarks

In this paper, a numeric-analytical technique (DTM) has been applied to solve systems of ODEs (first and second order) with initial value conditions. The numerical results obtained revealed that the method is easy, fast, accurate, and it reduces the size of computational involvement. The graphical representations show further a strong agreement and relationship between the numerical results and the exact solutions.

Acknowledgements. The authors would like to express sincere thanks to the anonymous reviewers for their positive and constructive comments towards the improvement of the paper.

References

[1] N. Shawagfehm, D. Kaya, ‘‘Comparing Numerical Methods for the solutions of systems of Ordinary Differential equations’’, *Applied Mathematics Letters,* **17**, (2004): 323 - 328. http://dx.doi.org/10.1016/s0893-9659(04)90070-5

[2] K. Krishnaveni, S. R. Balachandar, S. K. Ayyaswamy, ‘‘Adomian's Decomposition Method for Solving Singular System of Transistor Circuits’’, *Applied Mathematical Sciences*, **6** (37), (2012): 1819 - 1826.

[3] N. Dogan, ‘‘Solution of the systems of Ordinary Differential by combined Laplace Transform-Adomian Decomposition’’ *Mathematical and Computational Applications*, **17** (3), (2012): 203 - 211.

[4] J. Biazar, E. Babolian, R. Islam, ‘‘Solution of the system of Ordinary Diffferential equations by Adomian Decomposition’’, *Applied Mathematics and Computation* **147**, (2004): 713 - 719.
http://dx.doi.org/10.1016/s0096-3003(02)00806-8

[5] F. L. Lewis, ‘‘Geometric Design Technique for observer in Singular System’’. Automatica, **26**, (1990): 411 - 415.
http://dx.doi.org/10.1016/0005-1098(90)90138-8

[6] M. M. Rodrigues, ‘‘Analysis of Adomian Series Solution to a Class of Nonlinear Ordinary Systems of Raman Type’’, *Applied Mathematics E-Notes,* **11** (2011): 50 - 60.

[7] S. O. Edeki, H. I. Okagbue, A. A. Opanuga, S. A. Adeosun, ‘‘A Semi-Analytical method for certain class of second order Ordinary Differential equations’’. *Applied Mathematics,* **5**, (2014): 2034 - 2041.
http://dx.doi.org/10.4236/am.2014.513196

[8] S. O. Edeki, A. A. Opanuga, H. I. Okagbue, ‘‘On Iterative Techniques for Numerical Solutions of Linear and Nonlinear Differential Equations’’ *J. Math. Comput. Sci.* 4 (4), (2014): 716 - 727.

[9] John Birds, ‘‘Higher Engineering Mathematics’’, *Elsevier ltd*,(2006).

Received: October 15, 2014; Published: November 21, 2014