



## CIVIL & ENVIRONMENTAL ENGINEERING | REVIEW ARTICLE

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*Cogent Engineering* (2016), 3: 1216244



Received: 18 May 2016  
Accepted: 20 July 2016  
First Published: 26 July 2016

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# Estimation of longitudinal dispersion co-efficient: A review

Imokhai Theophilus Tenebe<sup>1\*</sup>, Adebajji Samuel Ogbiye<sup>1</sup>, David Olugbenga Omole<sup>1</sup> and PraiseGod Chidozie Emenike<sup>1</sup>

**Abstract:** Accurate determination of longitudinal dispersion coefficient in rivers or streams is necessary for pollution control and management. This can be achieved through tracer studies and has proven to be a reliable method for measuring pollution spread. However, tracer studies practise which is expensive, time gulping and requiring large labour input have been substituted with empirical approaches thereby reducing the applicability of the dispersion coefficient models generated. This study reviews the various models derived as well as methods associated in the collection of tracer concentration data (measurement) existing in the literature. A sustainable approach to this study was identified and research needs were also listed.

**Subjects:** Earth Sciences; Engineering & Technology; Environment & Agriculture; Environmental Studies & Management

**Keywords:** longitudinal dispersion coefficient; dispersion number; pollution; river; sustainable approach; environmental sustainability engineering



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### PUBLIC INTEREST STATEMENT

The spread (dispersion) of pollutants in rivers or stream is determined by a dispersion coefficient. This helps us to understand the rate of mixing and transport of pollutants discharged into river bodies accidentally or intentionally. However, determination of this spread is usually achieved through tracer studies which is cumbersome as time, cost and labour is involved. This review highlights the various methods used in dispersion studies with the aim of identifying sustainable approaches that may encourage the constant monitoring of pollution levels.

## 1. Introduction

The need for water cannot be over emphasized as it is required for all activities indulged by man; yet obtaining water of good quality that is safe for drinking and for other purposes globally is still a mirage. Surface water remains a vital component of the natural environment; it requires adequate protection from pollution which is fast becoming a source of concern (Duarte & Boaventura, 2008). Surface water, ground water, water quality, water quantity and ecology are all entities which are closely linked with each other (Olukanni, Adebayo, & Tenebe, 2014). The compromise of one entity affects the other, therefore, their protection should not be taken for granted. Global environmental awareness regarding pollution occurrence in rivers has become heightened (Falconer & Lin, 2003; Kashefipour & Roshanfekr, 2012). As a result, institutions (governments, research institutes, etc.) are left without any option other than to mobilize to remove these pollutants as it tends to become an issue to those that eventually use these rivers. As a result of the foregoing, the degradation caused by the discharge of pollutants into the environment can be studied by the understanding of pollutant travel pattern and dispersion in water. In addition, this should be done with great degree of accuracy for public health and safety reasons (Bárek, Velísková, Sokáč, & Fуска, 2014; Shen, Niu, Anderson, & Phanikumar, 2010). With no knowledge of assimilation capacities of streams and rivers, waste is discharged therein (Longe & Omole, 2008) owing to anthropogenic activities. This is in a bid to doing away with their waste thereby affecting the dilution strength of the water body negatively (Krantz & Kifferstein, 2007; Longe & Omole, 2008; UNESCO, 2006). However, when a pollutant is released into a river or stream, it dissolves and spreads downstream along the direction of flow. The spread of the pollutant continues while it is being consumed by downstream users unaware. The activities of polluters certainly have adverse environmental impacts such as contamination of the river body, destruction of aquatic life, endangerment of public health, and disruption of ecosystem. Water pollution interferes with the exchange of gases required for the breakdown of organics. It also introduces water related diseases, algae bloom and loss of aquatic life.

From the foregoing, it can be understood that the primary issues affecting the control and protection of the water quality is having an understanding of how these pollutants can be identified along with their spatial and temporal variations (Benedini & Tsakiris, 2013). The recovery rate of rivers or streams from contaminant load is not instantaneous as it occurs over long distances and time before dilution or mixing is complete (Chapman, 1992; Henry & Heinke, 2005; Longe & Omole, 2008). Thus, the capacity to determine the various parameters affecting contaminant transport, optimization of water use, as well as conduct holistic environmental assessment will be useful in restricting further pollution of water bodies.

Pollutant mixing and transport in water bodies has been difficult to understand in recent times because every river is peculiar and heterogeneous in nature. It can allow the movement of these pollutants at a point and still retain some due to a periodic re-emergence of a lowering or increase in velocity of the river. With the fact aforementioned, there is need for accurate scientific data on quality of river or stream within their ambience and this has been made somewhat easy to determine using tracer studies. This involves the use of tracers like sodium chloride to determine the mixing process of pollutants. In recent studies, the tracer test has been employed but not in long stretches of rivers because of the cost implications, tedious process amongst others. This has made most researchers in Sub-Saharan Africa shy away from the process as little or no studies abound in the literature. There is need for the mixing and transport of every river to be understood, this will aid in determining the pollution status so as to develop water quality control models that will aid policy makers in avoiding effects of deliberate and accidental discharges. The various use of the river both at upstream through downstream and the various actors such as human and industrial activity on this river or streams need to be checked. This is needful in order that the quality and quantity of river water would not be compromised.

From studies, it has been ascertained that the propensity of water bodies to transport or disperse substances added to it can be measured by the dispersion coefficient. This varies as the hydrodynamic and geometric properties changes (Zeng & Huai, 2014). The transportation of pollutants and

effluents which are discharged into rivers undergoes three stage of mixing (Jirka, 2004) which includes dimensional entrainment and diffusion stage in which jetty pollutants are abruptly and properly mixed in the vertical direction; pollutants moving in transverse direction and, finally, lateral mixing occurring respectively in that order (Adarsh, 2010; French, 1986; Sahay, 2013). This is also a useful tool for environmental engineers in the design of hydraulic structures like water intakes, outfalls etc. and in quantifying the extent of river or water body contamination (Fisher, List, Koh, Imberger, & Brooks, 1979; Perucca, Camporeale, & Ridolfi, 2009). Therefore, this study reviews the different methods and approaches existing in literature as it concerns tracer studies in order to suggest better sustainable approaches. Furthermore, gaps are identified for the improvement of dispersion coefficient models as well.

### 1.1. Determination of longitudinal dispersion coefficients

The release of effluents and hazardous substance into rivers or streams may not be avoided and as such, the pollutants are under mixing and transported across and along the river reach. This process may alter water quality as changes (physical, chemical and biological) are likely to occur. In that regard, the proper calculation of the mixing dynamics of the river is of the essence as it could help to know the dilution strength of the river or stream and likewise, its pollution status. This can be achieved by accurately estimating the values of dispersion coefficient ( $D$ ), which is achieved by the application of a mathematical technique termed mathematical modelling. In general, mathematical modelling involves the representation of an investigation with mathematical relationship (Hebborn, Parramore, & Stephens, 1997; Nourallah & Farzad, 2012) which is useful for real time and future predictions. However, these models may require validation and re-calibration. Table 1 itemizes some merits and demerits of mathematical models.

Although various mathematical concepts have been applied to accurately estimate  $D$  for various rivers or streams around the world, however, some very familiar approach which include the integration, moment method, empirical and tracer measurement approach. An attempt will be made to discuss these approaches.

#### 1.1.1. Integral approach

Integration approach was developed by Fischer (1968) to estimate the longitudinal dispersion (LD) coefficient values. This is in the form shown in Equation (1.1):

$$D_L = -\frac{1}{A} \int_0^B hu^1 \int_0^y \frac{1}{\xi h} \int_0^y hu^1 \partial y \partial y \partial y \tag{1.1}$$

where  $A$  = cross-sectional area,  $B$  = river width,  $h$  = local flow depth,  $u^1 = u - U$ , and is termed the deviation of local depth-averaged longitudinal velocity  $u$  from the cross sectional mean value  $U$ , coordinate in the lateral section of the river reach while  $\xi$  is the local transverse mixing coefficient. Although there have been several modifications to this equation (Agunwamba, 1997; Deng, Singh, &

**Table 1. Merits and demerits of mathematical models**

Merits	Demerits
Important factors are easily identified	Most models require large data for computation or formulation
Reduces cost & time required during experimentation	Significant errors maybe introduced resulting from lost attention as large data is required
It helps to understand better the performance of an experimental process	Models are made out of assumptions, which not hold over sometime. This may lead to re-calibration
	Variable identification for on-point modelling could be costly and time gulping

Source: Nourallah and Farzad (2012) and Novoseltsev and Novoseltseva (2014).



Bengtsson, 2001; Seo & Baek, 2004; Seo & Gadalrah, 1999; Sooky, 1969; Zeng & Huai, 2014), which result from variation in the assessment of the velocity profile of the flowing river, stream or channels under consideration which was not totally considered in the foregoing equation. Consequently, this variation in velocity profile experienced in natural channels has shown to play a very important role in pollutant spread. Nonetheless, the above equation still exists as the bed rock of other integration method that may exist in the literature and was not captured.

### 1.1.2. Empirical approach

Additionally, Fischer (1975) proposed an equation geared towards reducing the time consuming nature of the above triple integral equation in Equation (1.1), and it further became:

$$D = \frac{0.011 U^2}{Hu_*} w^2 \quad (1.2)$$

where  $U$  is the mean velocity,  $H$  is the height of the river from the surface,  $U_*$  is the shear velocity and  $w$  is the river width. Additionally, Equation (1.2) was tested using datasets obtained from straight-prismatic channels having varying aspect ratio. This equation was adopted for a long time until recently Sahay (2013) explained the reason for the significant variation between the predicted and measured values of dispersion coefficient which has been noticed by various researchers. He explained that the variation is attributed to the shear stress and inappropriate representation of the velocity across the channel sections. To mention, this has led to the infrequent use of this equation. Similarly, other empirical equations have evolved which have given rise to the empirical approaches as earlier mentioned. This approach consists of empirical equations or models developed with in-stream properties of different rivers or stream around the world. Many of such equations conform to:

$$D_L = \theta \left( \frac{B}{H} \right)^\alpha \left( \frac{U}{U_*} \right)^\beta HU \quad (1.3)$$

where  $B$  = width of channel or river,  $H$  = height of river from the surface,  $U$  = mean velocity and  $U_*$  = shear velocity of the river.

Magazine, Pathak and Pande (1988) calibrated his model using experimental data-set from the laboratory. The calibration yielded:

$$D_L = 75.86 \left( 0.4 \frac{U}{U_*} \right)^{-1.632} RU \quad (1.4)$$

where  $U$  and  $U_*$  retains their usual meanings while  $R$  is the hydraulic radius of the river.

Iwasa and Aya (1991) developed their own empirical model which has shown great consistency when compared with data sets from laboratory and rivers globally. This has made the model widely recommended by many researchers (Abderrezzak, Ata, & Zaoui, 2015; Launay et al., 2015; Zeng & Huai, 2014). The model is in the form:

$$D_L = 2HU \left( \frac{B}{H} \right)^{1.5} \quad (1.5)$$

Also, Kashefipour and Falconer (2002) developed a model using field data from 29 rivers with  $\frac{B}{H}$  values greater than 50, which however, was validated using  $\frac{B}{H}$  values greater than 10. The equation developed is in the form:

**Table 2. Constants of selected empirical equations**

Equations	$\theta$	$\alpha$	$\beta$
Iwasa and Aya (1991)	2.00	1.50	0.00
Koussis and Rodríguez-Mirasol (1998)	0.60	2.00	0.00
Seo and Cheong (1998)	5.92	1.43	0.62
Kashefipour and Falconer (2002)	10.61	0.00	1.00
Sahay and Dutta (2009)	2.00	1.25	0.96
Sahay (2013)	2.00	0.71	1.37
Etemad-Shahidi and Taghipour (2012)	14.12	0.85	0.61
Zeng and Huai (2014)	5.40	0.70	0.13

$$D_L = 10.612 \left( \frac{U}{U_*} \right) HU \quad (1.6)$$

Li, Liu and Yin (2013) used a differential evolution (DE) algorithm method to optimize hydraulic characteristic data. These data include width, depth, shear and average velocity. Sixty-five set of data was obtained from 29 rivers situated in the United States. The model generated from this method is in the form:

$$\frac{K}{HU_*} = 2.2820 \times \left( \frac{W}{H} \right)^{0.7613} \left( \frac{U}{U_*} \right)^{1.4713} \quad (1.7)$$

Additionally, Zeng and Huai (2014) calibrated a model with very high precision compared to commonly used and highly rated models, which include the model of Koussis and Rodríguez-Mirasol (1998) and Kashefipour and Falconer (2002). This model is in the form:

$$D_L = 5.4 \left( \frac{B}{H} \right)^{0.3} \left( \frac{U}{U_*} \right)^{0.13} HU \quad (1.8)$$

In contrast all empirical models are obtained with regression analysis and are a product of precise data and as such, the listed models (Equations (1.1)–(1.7)) can actively perform well when all conditions governing model generation are met. Similarly, these models may soon prove inefficient as in-stream properties begin to differ owing to climate change amongst others. Therefore, it is advised that continuous river measurement and management be practised for model modification and increased model precision accuracy. Additionally, empirical approaches are very handy tools when immediate knowledge on an accidental discharge of effluent or hazardous substance to a river is sort after.

The values of  $\theta$ ,  $\alpha$  and  $\beta$  are constants which can be obtained when regression or dimension analysis is applied. Table 2 shows a summary of some constants from widely used and recent empirical equations in literature just mentioned.

### 1.1.3. Tracer measurement approach

Tracer studies experiment have been useful and in existence *ab initio*. This technique hitherto has shown to be adequate and the basis for proper model calibrations and estimation of a particular stream or river. The method allows for direct collection of field data either *in situ* or *ex situ* (Agunwamba, 1997; Launay et al., 2015; Szeftel, Moore, & Weiler, 2011). On the other hand, tracers such as salt (Lucchetti, Latterell, Timm, & Gregersen, 2013; Marecos do Monte & Mara, 1987; Ojiako, 1988; Velísková et al., 2014 amongst others) and Rhodamine (Szeftel et al., 2011 amongst others) have been employed in the field to mimic pollutants as they are conservative in nature. Generally, these tracers have successfully been utilized as it has produced good results on the dispersion coefficient values. Additionally, it can easily be seen, detected, dissolution in water is spontaneous, very

affordable and harmless in minute concentrations. All mentioned are essential factors for the selection of tracers. Although, conducting tracer experiments is scarce, due to high labour operational cost, difficulty in embarking on the study itself (Agunwamba, 1997; Agunwamba, Ojukwu, & Omeje, 2013; Launay et al., 2015; Zeng & Huai, 2014) and also time consuming. Owing to this fact, most of the data used in literature were obtained in the 60s and 70s (Launay et al., 2015). However, in contrast, other researchers have conducted more tracer experiments (Antonopoulos, Georgiou, & Antonopoulos, 2015; Duarte & Boaventura, 2008; Piotrowski, Rowinski, & Napiorkowski, 2012; Rowiński, Guymmer, & Kwiatkowski, 2008; Tayfur & Singh, 2005). Likewise, when tracer result is obtained, it is easy to quantify the value of longitudinal or transverse dispersion (TD) coefficient by the following method:

- Moment method.
- Routing method.
- Numerical or Analytical method.

**1.1.3.1. Moment method.** This is a statistical approach used to determine both longitudinal and TD coefficient. This concept is valid so long as dispersal is taking place (Kim, Park, Jung, Lee, & Suh, 2011) i.e. not minding wake effects. This wake effect exhibited by many streams may either be as a result of improper mixing experienced at the point of injection of the tracer and the presence of transient storage (Launay et al., 2015; Rutherford, 1994). Consequently, the skewness or wake effect experienced during tracer studies has limited the efficacy of this method as it may not be avoided. The equation for this method is expressed as (Fisher et al., 1979; Kim et al., 2011; Launay et al., 2015):

$$D_L = \frac{1}{2} \frac{d\sigma_1^2}{dt} \bar{U}^2 \tag{1.9}$$

where

$$d\sigma_1^2 = \frac{\sigma_2^2(L_2) - \sigma_1^2(L_1)}{\bar{t}_2 - \bar{t}_1} \tag{1.10}$$

As  $\bar{t}_2, \bar{t}_1, L_2$  and  $L_1$  corresponds to time measured at different centroids on the concentration-time curve at various corresponding lengths while,  $\sigma_1^2$  and  $\bar{U}$  corresponds to the variance at the different time mentioned on the concentration-time curve and average velocity respectively. The same mathematical computation holds for TD except that  $\bar{U}^2$  is replaced with  $\bar{U}$ .

**1.1.3.2. Routing method.** This applies to the frozen cloud approximations for routing temporal concentration profiles obtained during tracer experiments. It is achieved at a particular section  $L_1$  and  $L_2$  (Kim et al., 2011). According to Kim et al. (2011), the value of dispersion coefficient is obtained by collecting tracer data from inlet and outlet points in a given channel. Thereafter, least square method is applied to whittle down errors and the equation is in the form:

$$C(x_2, t) = \int_{-\infty}^{\infty} \frac{\bar{U}C(x_1, t)}{\sqrt{4\pi D_L(\bar{t}_2 - \bar{t}_1)}} \exp\left(-\frac{[\bar{U}(\bar{t}_2 - \bar{t}_1 - t + \tau)]^2}{4D_L(\bar{t}_2 - \bar{t}_1)}\right) dt \tag{1.11}$$

**1.1.3.3. Numerical method.** This method unlike empirical method has a wider usability and relies more on field data. It is also employed to determine the water quality of streams and rivers (Li et al., 2013). One of such numerical equations used is the 1-D advection-dispersion equation (ADE) which is expressed as:

$$\frac{\partial C}{\partial t} + \frac{U\partial C}{\partial x} = D_L \frac{\partial^2 C}{\partial x^2} \tag{1.12}$$

where  $C$  = cross sectional average concentration;  $U$  = average velocity of channel cross section;  $t$  = time of travel of solute or tracer;  $x$  = distance in the longitudinal direction;  $D_L$  = LD coefficient.

This equation is popularly known as the Fickian's model. However, this model has disadvantages of not catering for the long tails experienced during field experiments and also depends on the attainment of dispersive equilibrium of the tracer. On the other hand, the 2-D models are void of these limitations. The models are able to cushion the effects of long tails (when sampling is elongated as a result of waiting on the river to get close to its original concentration) and tracer equilibrium (where there is even spread of tracer about the river sections) conditions where necessary by introducing more variables to the right hand side of the 1-D ADE. In Equation (1.9), the concentration values are obtained from the concentration-time curve developed during tracer studies. This equation is as follows:

$$C(x, y, t) = \frac{M}{4\pi ht \sqrt{D_L D_T}} \exp\left(-\frac{(x-ut)^2}{4D_L t} - \frac{(y-vt)^2}{4D_T t}\right) dt \quad (1.13)$$

where  $M$  represents the tracer mass, while  $u$  and  $v$  represents velocity in the  $x$  and  $y$  direction and  $D_L$ ,  $D_T$  connotes dispersion coefficients in the longitudinal and transverse respectively.

### 1.2. Model formulation of the new approach

The new approach inculcates both variation of distance and time in the model formulation which makes it seem sustainable. This is different from the existing methods whereby time is varied with distance kept constant and distance is varied and time kept constant. Interestingly, the former method has shown to underestimate dispersion coefficients (Agunwamba, 1997) in most cases while the latter approach may not be sustainable. This new approach is derived from the existing Equation (1.12) (Agunwamba, 1997):

$$C = \frac{M}{A \sqrt{4\pi Dt}} \exp - \frac{(x - \bar{ut})^2}{4Dt} \quad (1.14)$$

Furthermore, the length and time were non-dimensionalized to increase the applicability of the model, Equations (1.13) and (1.14) captures this process:

$$\xi = \frac{x}{L} \quad (1.15)$$

and

$$\tau = \frac{t}{\theta} \quad (1.16)$$

Since the dispersion equation is given as:

$$\delta = \frac{D}{\bar{u}L} \quad (1.17)$$

Therefore, Equation (1.12) can be written as:

$$\frac{C}{C_0} = \frac{1}{\sqrt{4\pi\delta\tau}} \exp - \frac{(\xi - \tau)^2}{4\delta\tau} \quad (1.18)$$

Further expansion of Equation (1.16) will result in:

$$\frac{C}{C_0} = \frac{1}{\sqrt{4\pi\delta\tau}} \exp - \frac{(1 - \tau/\xi)^2 \xi^2}{4\delta\tau} \quad (1.19)$$



The variance obtained from the concentration time graph is expressed as:

$$\sigma^2 = \int_{D_\xi} \int_{D_\tau} \psi(\xi, \tau)^2 \phi(\xi, \tau) d\tau d\xi - \left[ \int_{D_\xi} \int_{D_\tau} \psi(\xi, \tau) \phi(\xi, \tau) d\tau d\xi \right]^2 \quad (1.20)$$

where  $\psi$  depends on the variable  $\xi$  and  $\tau$  while  $\phi$  is the density of the probability at the various point  $\xi$  and  $\tau$  (Agunwamba, 1997; Brandt, 1970).

Hence,

$$\sigma^2 = \int_0^1 \int_0^\infty \left( \frac{\tau}{\xi} \right)^2 \frac{1}{\sqrt{4\pi\delta\tau}} \exp - \frac{(1 - \tau/\xi)^2 \xi^2}{4\delta\tau} d(\tau/\xi) d\xi - \left[ \int_0^1 \int_0^\infty \left( \frac{\tau}{\xi} \right) \frac{1}{\sqrt{4\pi\delta\tau}} \exp - \frac{(1 - \tau/\xi)^2 \xi^2}{4\delta\tau} d(\tau/\xi) d\xi \right]^2 \quad (1.21)$$

To mention, this approach measures the tracer in the opposite direction of flow to reduce disturbance, and also limits the collection of the tracer sample to one point before the inlet i.e.  $1 - \xi$ . All in a bid to reduce errors that may arise from sampling. Therefore, the new variance of this method will be:

$$\frac{\tau}{1 - \xi} \quad (1.22)$$

Therefore, substituting  $\tau/\xi$  with  $\frac{\tau}{1-\xi}$  into Equation (1.19), and following the procedures from Equation (1.12)–(1.17), the new model equation will be in the form:

$$\sigma^2 = \int_0^1 \int_0^\infty \left( \frac{\tau}{1 - \xi} \right)^2 \frac{1}{\sqrt{4\pi\delta\tau}} \exp - \frac{\left( \frac{\tau}{1-\xi} \right)^2}{4\delta\tau} (1 - \xi)^2 \times d\left( \frac{\tau}{1 - \xi} \right) d\xi - \left[ \int_0^1 \int_0^\infty \left( \frac{\tau}{1 - \xi} \right) \frac{1}{\sqrt{4\pi\delta\tau}} \exp - \frac{\left( 1 - \left( \frac{\tau}{1-\xi} \right) \right)^2}{4\delta\tau} (1 - \xi) d\left( \frac{\tau}{1 - \xi} \right) d\xi \right]^2 \quad (1.23)$$

Hence, the new relationship of variance with dispersion will be:

$$\sigma^2 = \int_0^1 \left[ \frac{2\delta}{1 - \xi} + \frac{12\delta^2}{(1 - \xi)^2} \right] d\xi - 4\delta^2 \left( \int_0^1 \frac{d\xi}{1 - \xi} \right)^2 \quad (1.24)$$

This equation will be used to obtain the variance from the concentration-time curve with this new method of harvesting tracer data. Further modification will yield;

$$\sigma^2 = \frac{\sum_{j=1}^M \left( \frac{\tau_j}{1 - \xi_j} \right)^2 C_j}{\sum_{j=1}^M C_j} - \left[ \frac{\sum_{j=1}^M \left( \frac{\tau_j}{1 - \xi_j} \right) C_j}{\sum_{j=1}^M C_j} \right]^2 \quad (1.25)$$

where  $C_j$  is the concentration at various points of the channel or river section during sampling. Hence the dispersion coefficient will be calculated when Equation (1.23) is equated as seen in Equation (1.22). Also, dispersion coefficient values can be achieved by precise measurements of tracer concentration as well as proper estimation leading to future prediction (Parsaie & Haghiabi, 2015). Table 3 shows the merits and demerits of the various approaches existing in the literature in line with the aforementioned.

**Table 3. Merits and demerits of measuring and estimating LD in the literature**

Model	Measurement	Estimation	Cost	Time	Labour	Accuracy	Remarks
Empirical approach	Velocity, depth and width	Equation derived by regression	Not expensive	Time is reduced significantly	Less labour	Dwindling accuracy owing to fluctuating stream properties	Wide usage
Tracer method existing in the literature (constant time-variable distance method)	Concentration of tracer at different distance within specified time is required	Method of moments is used	Very expensive (more measuring device and labour is needed)	Consumes time	More labour	Accurate	Wide usage
Tracer method existing in the literature (constant time-variable distance method)	Concentration of tracer at different distance irrespective of time is required	Equation obtained by the combination of Euler-LaGrange (Agunwamba's approach)	Cost is reduced significantly (less measuring device and labour is required)	Time is reduced significantly	Less labour	Not specified	Dearth usage but showed promising results

Additionally, some research found in literature involving all types or approaches are documented below:

Agunwamba (1997) conducted a research on a laboratory scale to compare existing methods of collected tracer data. These method includes two-point sampling, where the inlet and outlet are sampled simultaneously at specific time (Levenspiel & Smith, 1957) until the tracer flows out completely. This method is also based on subjective nature of the researcher (Agunwamba, 2002). The second method of sampling is widely used in literature though modified by Agunwamba (1997). This involves sampling at different marked out stations along the channel and the value of the conductivity measured within irrespective of time. In addition, sampling is done from outlet to inlet which is the reverse in existing methods in literature. In achieving this, 60 g of salt was premixed and poured into a 14 m channel and the concentration salt was measured and time concentration graph produced. Result from both methods were compared and it showed that the former method gave underestimated values of dispersion coefficient which ranged from 0.9 to 3.3. Furthermore, the method seems to reduce cost, work and time used during tracer experiment. Consequently, this has been one of the major reasons why most researchers have resorted to laboratory and computer simulations in predicting dispersion coefficient values. This method has not been practiced in a river for validation.

Agunwamba (2002) in his study discussed about the subjective nature of sampling time and intervals indulged by various researchers during tracer studies. At will, some select intervals and periods which differs from one another. In his study, the precision, confidence and design efficiency of tracer experiment in the estimation of dispersion coefficient and velocity were investigated. This was achieved with data from a laboratory study (Agunwamba, 1997) and a field work executed in Portugal (Marecos do Monte & Mara, 1987). The study revealed that it is possible to predict proper sampling procedures before experimental process takes place and indulging in large number of experimental runs may not yield precision in experiments. Obviously, this would have led to more energy, time and money expended. Furthermore, he suggested that more time should be spent to determine the apex of the breakthrough curve (BTC) than the tail for efficient determination of dispersion coefficients.

Baek and Seo (2010) proposed a modified version of the routing procedure (RP) model. The model sort to take care of irregularities that abound in a river environment which includes presence of meanders and transient zones. The new model is a mix of channel geometry, advection equation and steam tube concept. Tracer data was obtained from a field study in natural rivers in Korea

having range of 1.3–1.9 km in length and intervals of 200–400 m. With the data collected, the new model was calibrated and when tested gave values close to other methods. The unique thing is that, the model was able to capture the irregularities such as bends and straight zones as high and low concentration values were obtained at those points when even measured.

Kim et al. (2011) explored the use of radioisotopes for the determination of dispersion coefficient in Daejong river located in south eastern Korea. The radioisotope had limited life span, therefore it was injected twice in a day (morning and evening) into the river which had width and depth range values of 18–30 and 0.2–20 m respectively. With two points marked downstream, the varying concentrations of the tracer was detected using a  $2 \times 2$  in. NaI(Tl) scintillator detector which was placed stationary at the two points downstream. The work confirmed that the concentration values that were calculated agrees with the measured when statistically compared.

Agunwamba et al. (2013) in their experiment on tracer studies compared dispersion coefficient values obtained in a natural stream by using a relatively new Euler-Lagrangian model, Fisher's model and Levenspiel and Smith models or approach. It was achieved by extracting stream data from American stream tracer analysis (ASTA) on the Humboldt river as well as performing tracer studies in a river in Ebere, Enugu state, Nigeria. 50 kg of salt was pre-mixed and then poured in the river that was 2.5 km long having spacing of 200 m. Consequently, variable distance constant time method of sampling was employed and taken to the laboratory for analysis. Computation of dispersion coefficients were done using the mentioned three approaches. Results showed that slightly increased values of dispersion were obtained from Agunwamba model than in Fisher's while larger values were obtained in Levenspiel and Smith model. Although Agunwamba's approach was observed to take for taking more time during computation than the others, it was commended for requiring less data input, labour input and cost.

Also, Zeng and Huai (2014) obtained the LD from a set of 116 tracer data. This data was obtained from over 50 rivers sited in the United Kingdom (UK) and the United State (US). Comparisons were made as performance of ten frequently used model in literature were varied seeking precision. The rivers generally had aspect ratio between 20 and 100. Again, results from the work showed that the LD coefficient values were underestimated for any given value of aspect ratio. Zeng and Huai (2014) developed a model which is an upscale of the models commonly used for LD coefficient estimation. This new model even though may not be suitable for estimation of dispersion coefficient in a rectangular flume but is very efficient in predicting LD in streams or rivers having trapezoidal sections not having width ( $B$ )  $<15$  m and  $>259$  m. In concluding, he affirmed that LD coefficient has a strong tie with the product of depth of river ( $H$ ) and cross sectional velocity ( $U$ ) than  $HU_x$ ,  $U_x$  is the shear velocity. This is in accordance with many researchers as tracer concentrations are obtained at different measuring points as velocity variation brings about mixing process. This is against earlier method carried out by Levenspiel and Smith approach (Agunwamba, 1997).

With the relationship, level of interaction between organisms with their environment as well as the chemical status which is of prime importance to the Slovakian government, under the Water Framework Directive (WFD). This aforementioned framework needed to be actualized in 2015. Therefore, Velísková et al. (2014) conducted a research on the pollutant spread in Mala Nitra Canal. In this study, salt tracer was used to determine LD and TD coefficients in an artificial stream having aspect ratio between 12.3 and 18 m. A known concentration of salt was added into the canal and the varying conductivity values were recorded. This was achieved with conductivity meters that were placed at various measuring point downstream of the 400 m long canal at constant time interval until the background concentration returned. In addition, it is important to mention that the longitudinal and transverse sections were dosed and recorded separately which caused a variation in the dispersion coefficient values. Although the values of the dispersion coefficient were between 5 and 7.5 agreed with those obtained from Fisher et al. (1979) and Riha, Dolezal, Jandora, Oslejskova

and Ryl (2000) in laboratory flumes than in natural channels of which most rivers appear trapezoidal. In conclusion, the model generated could only be used for streams, rivers or canal having same in-stream properties with the studied canal and also having no dead zones. This makes it a bit difficult as many streams may not meet up with the latter criteria.

Abderrezzak et al. (2015) conducted a research comparing the extent to which LD empirical formula can be used in 1-D numerical modelling (Mascaret tool). This was actualized by conducting eight laboratory experimental study using Rhodamine WT as tracer. The varying concentration values were obtained at four stations downstream of a 30 m length channel using a Turner fluorimeter. Furthermore, the assessment was done comparing with Ten empirical formula existing in literature with regards the shape of their BTC's. The result showed that of the ten empirical models assessed, Iwasa and Aya (1991) with Seo and Cheong (1998) were better predictors when compared to the 1-D numerical modelling tool used by Abderrezzak et al. (2015). Therefore, their model could be employed for dispersion coefficient prediction in streams having same in-stream (width, flow, depth, meanders, amongst others) properties. On the other hand, Elder (1959), Fischer (1975) and Iwasa and Aya (1991) were better off for laboratory scale dispersion coefficient prediction. In conclusion, it was suggested that the models that performed well and models frequently used should be adjusted to fit complex geometry having transient storage conditions which are likely situations of many rivers. This will surely increase the model applicability.

Meddah, Saidane, Hadjel, and Hireche (2015) proposed the use of a 1-D Transmission Line Matrix (TLM) method for estimation of LD coefficient. This came to light as it could save time for mathematical computation and less data input. This was verified by simulating and comparing with a data-set obtained from a River Severn based in UK. Rhodamine WT was used as tracer in a 14 km long river. The result used was culled from a study carried out by Atkinson and Davis (2000). From the study, it was gathered that the model overestimated flow velocities and LD coefficients but describes well peak concentrations as time progresses. The latter is good as peak concentrations appears to be an important finding during tracer studies (Agunwamba, 2002). However, the assumptions and governing principles of the model, which are not completely true of a typical river or stream, make regular tracer data collection of streams that are prone to pollution paramount.

Parsaie and Haghiabi (2015) again calculated LD coefficient by using data obtained from Atkinson and Davis (2000). This time around, unlike Meddah et al. (2015), the data was calibrated using dispersion routing method (DRM). According to the work, it involves iterations which are not true representatives of tracer studies. This may further complicate the accuracy of dispersion coefficient determination. Furthermore, the result obtained through this method was tested against 12 known models in literature and did not yield positive result. This again affirms that conducting tracer experiment is the way forward despite the challenges.

Baek and Seo (2016) in their study assessed the applicability of some existing models in the determination of 2-D mixing process in rivers. It involved the collection of hydraulic and tracer experimental data-set from either via simulation of laboratory channels or from natural rivers. Various methods were used in a bid to determine the right method for calculating TD equation at transient concentration conditions and then compared with one another. The methods are 2D routing procedure (2D RP), stream-tube routing procedure (STRP) and 2-D stream-tube routing procedure (2D STRP). This method is an upscale of the existing routing procedure method existing in literature. On the other hand, it takes care of the anomalies resulting from river morphology which makes it unique. From the study, these methods were recommended for calculating 2-D mixing process with tracer data obtained. Finally, it was suggested that more tracer data should be accumulated to investigate the versatility of the proposed model.

### 3. Discussion

After going through existing literatures, it is observed that best prediction of dispersion coefficient is needful and could be achieved by tracer studies. However, this has not been possible in most cases. This is as a result of insufficient field measurement that are obtained during tracer studies and this studies will require constant monitoring of specific rivers or streams. Consequently, the following gaps in literature were identified and need to be addressed:

- (1) As far as river management or pollution control is concern, there is only one literature found where dispersion studies have been carried out in Sub-Saharan Africa. This may be related to the high cost involved in tracer studies (purchase of electrical conductivity probes, turner fluorimeters at selected points amongst others). Although this is not good news as researchers in those region have reported cases of incessant river pollution.
- (2) From studies, it is clear that conducting tracer studies could be expensive, labour intensive and time consuming. Notwithstanding, constant monitoring of river using tracer studies is inevitable. As such, a more sustainable approach to obtaining tracer data will be a global solution. One of such approach is suggested and applied by Agunwamba (1997) and Agunwamba et al. (2013). This approach is not empirical in nature as it includes the direct calculation of dimensionless dispersion coefficients values thereby making its applicability wide (Toprak, Hamidi, Kisi, & Gerger, 2013; Toprak, Şen, & Savci, 2004), This approach needs to be applied especially to rivers whose dispersion coefficient is already known to see the similarity or variance in the dispersion coefficient result obtained.
- (3) There also exist very limited studies in the literature that describes the dispersion coefficient of settle-able pollutants. This is important as the constituent of pollutants discharged varies, some may settle while others do not. This makes is difficult to use conventional dispersion equations for prediction as it was derived using soluble tracers.
- (4) On the other hand, there still exist variation in predicted and measured dispersion coefficient values. This could mean that dispersion coefficients depend on some other factors aside the already considered. Studies have captured the effect of pool volume and roughness (wall and bed) on dispersion coefficient with the studies unveiling significant findings. However, more research should be obtained at laboratory scale on vary roughness of different sizes as well as alternating the roughness arrangement at varying velocities as this could improve the applicability of laboratory models to rivers and vice versa. This aforementioned is the real scenario of rivers and streams.
- (5) The subjective nature of sampling time and number of sampling has been observed in the literature. Most if not all researchers randomly select sampling time and sequence based on experience or from other studies. No study has reported the effect of sampling time and number of sampling on dispersion coefficient.
- (6) Furthermore, other factors such as temperature, wind, turbidity and varying radius of meanders effect on dispersion need to be ascertained and have not been clearly reported.

### 4. Conclusion

From this study, it is noted that best prediction of dispersion coefficient is needful. However, insufficient field measurement that are required during tracer studies requires long sampling in specific rivers and stream makes it difficult even though necessary. Be that as it may, this study aimed at a qualitative review of some empirical models derived as well as methods associated in the collection of tracer concentration data existing in the literature. It was observed that indeed conducting tracer studies is a challenge owing to high cost, prolonged sampling time and large input. However, these shortcomings have not halted the prediction of dispersion coefficient. Instead, many researchers have resorted to other alternatives which includes the use of empirical methods (regression analysis and neural network). These methods, though very useful tool for instant calculation of dispersion coefficient especially when data are insufficient as well as difficultly in reaching sites that are troubled with accidental spills and so on, could be prone to both present and future errors. The present



errors resulting from approximations while the future errors could emerge owing to dwindling in-stream properties such as depth, length, meanders, temperature, discharge and velocity amongst others. A sustainable approach has been identified which could lead to constant pollution monitoring as time, cost and labour would be saved. However, little studies have used this approach and more studies are required for both laboratory and field scales. In addition, other research needs are listed and if accommodated will improve the estimation of dispersion coefficients and increase the prediction of the models generated.

#### Acknowledgement

The authors appreciate the efforts of Dr Ben Uche Ngene and Dr Maxwell Omeje for their support and contributions into this work. Also, the anonymous reviewers as well as the management of Covenant University are appreciated for contributing to this manuscript and creating an enabling environment for this research respectively.

#### Funding

The authors received no direct funding for this research.

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#### Citation information

Cite this article as: Estimation of longitudinal dispersion coefficient: A review, Imokhai Theophilus Tenebe, Adebanji Samuel Ogiye, David Olugbenga Omole & PraiseGod Chidozie Emenike, *Cogent Engineering* (2016), 3: 1216244.

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