USING EXTREME VALUE THEORY TO MODEL INSURANCE RISK OF NIGERIA'S MOTOR INDUSTRIAL CLASS OF BUSINESS

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Abstract

Extreme losses have been recorded in Nigeria insurance companies due to motor insurance class claims; Nigeria Insurance market being a developing one requires building the confidence of the public to subscribe to their products. Nigeria's motor industrial insurance claim data for five insurance companies in a two year period is modelled in this paper with extreme value theory (EVT) to estimate the Value-at-Risk (VaR), where VaR gives estimate of the minimum amount of claims an insurance company would pay in a given period of time. The time series plot was obtained which aimed at capturing the trend of the claims over the two-year period, the mean excess plot was obtained which helped to determine threshold and the shape of the distribution in the tail area. The returns were then fitted in a Generalized Pareto model (GPD), a similar model that would have been used is the Generalized Extreme Value model (GEV) but the GPD is used in this study because it describes what happens in the tail area of the distribution and not just the maximum tail. A linear Q-Q plot reveals that parametric model fits the data well. VaR estimate was finally obtained using the extreme value method and other two methods of Historical and Gaussian at 5% confidence interval. The three methods of estimating VaR were compared and the empirical result shows that extreme VaR is most suitable to calculate VaR as compared to the Historical and Gaussian method.

Key words: Insurance, Extreme Value Theory, Value-at-Risk, GP model, peak-over-threshold,

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บทคัดย่อ

ค่าสินไหมทดแทนจำนวนมากที่ถูกบันทึกโดยบริษัทประกันภัยในประเทศไนจีเรียมาจากการเรียกร้องค่าสินไหมทดแทนจากการประกันภัยรถยนต์ ตลาดประกันภัยในประเทศไนจีเรียเป็นตลาดที่กำลังพัฒนาและจำเป็นต้องสร้างความเชื่อมั่นแก่สาธารณะเพื่อจะมาใช้บริการ ในการวิจัยครั้งนี้ใช้ข้อมูลเกี่ยวกับการเรียกร้องค่าสินไหมทดแทนของการประกันภัยรถยนต์จากบริษัทประกันภัยจำนวน 5 แห่งในประเทศไนจีเรียในช่วงระยะเวลา 2 ปีมาสร้างแบบจำลองตามทฤษฎี Extreme Value Theory เพื่อประมาณค่า Value-at-Risk (VaR) โดยค่า VaR จะแสดงมูลค่าน้อยสุดของสินไหมทดแทนที่บริษัทจะต้องจ่ายในช่วงเวลาหนึ่ง สำหรับ Time Series Plot มีเป้าหมายเพื่อแสดงแนวโน้มของการเรียกร้องค่าสินไหมทดแทนในช่วงเวลา 2 ปีดังกล่าว และ Mean Excess Plot เพื่อกำหนดเส้นแบ่งและรูปร่างของการกระจายตัวของข้อมูลในบริเวณส่วนหาง ผลที่ได้จะนำมาจัดให้เข้ากับแบบจำลอง Generalized Pareto model (GPD) ซึ่งเป็นแบบจำลองที่เหมือนกับที่ใช้ใน Generalized Extreme Value model (GEV) แต่ที่ใช้ GPD ในการศึกษาครั้งนี้เพราะสามารถอธิบายสิ่งที่เกิดบริเวณหางของการการกระจายตัวของข้อมูลได้และไม่เพียงเฉพาะของหางส่วนที่สูงสุดเท่านั้น นอกจากนี้ Linear Q-Q Plot ยังแสดงให้เห็นว่าแบบจำลองอิงพารามิเตอร์สามารถใช้ได้ดีกับข้อมูล สุดท้ายแล้วค่า VaR ถูกคำนวณโดยใช้วิธี Extreme Value รวมถึงอีกสองวิธีได้แก่ Historical และ Gaussian ที่ช่วงแห่งความเชื่อมั่นที่ระดับร้อยละ 5 หลักจากทำการเปรียบเทียบค่า VaR จากทั้งสามวิธีพบว่าค่า Extreme VaR นั้นเหมาะสมที่สุดในการใช้คำนวณ VaR เทียบกับการใช้วิธี Historical และ Gaussian

INTRODUCTION

Risk measurement and management are important activities in all forms of business and investments, risk occurs particularly in investments when behavior of the market cannot be predicted, therefore the need to estimate risk cannot be over-emphasized.

Report of Nigeria’s insurance companies reveals that; most general insurance companies pay high numbers of claims resulting from motor insurance class of business as compared to other non-life insurance. Motor insurance can be classified as private cars, commercial vehicle, motorcycle, and motor trade vehicle (vehicles without plate number). Documented claims captured in this paper include loss as a result of fire damage, accidental damage, damage as a result of extreme flooding, and theft.

Every vehicle requires a compulsory third-party policy without which is considered a serious offence for one to drive without it, taking a clue from a statement made by Serap (2009), that Nigeria had been placed as one of the countries of the world that identifies the need for motor insurance at least for the coverage of third party motor risk. (Ademunigbohun & Oreshile 2014) identified that the Nigeria’s insurance market is dominated by non-life segment, driven by mandatory third-party motor insurance. A large chunk of motor insurance is the third-party motor insurance which requires annual renewal with relatively low charges, on the other hand, comprehensive insurance require higher premium; and high claims are equally made if required, as much as hundreds of thousands and millions naira as captured in the data used in this paper.

Nigerians investors and organizations appear to be more confident in insurance companies lately as National Insurance Commission (NAICOM) being a regulatory body for insurance companies in Nigeria introduced the Market Development and Regulatory Initiative (MDRI) in 2008, which aimed at improving capacity of the industry, efficiency of the market and protection of consumers in the insurance market environment in Nigeria.

Financial risk disaster in recent decades marked by large bankruptcies of large corporations, as well as liquidations of major financial institutions; many of which are caused by failure of risk management systems and awareness on the need for the adoption of regulatory measures by countries Gaio et. al (2015).

On the importance of measuring risk, Jorion (1997) pointed out that Value-at-Risk has become benchmark tool for market risk estimation. VaR is considered to be maximum loss that an investment may experience in a particular period of time at a given confidence level. VaR measures the worst anticipated loss over a period for a given probability and under normal market conditions. It can also be said to measure the minimal anticipated loss over a period with a given probability and under exceptional market conditions (Yang 2013).

EXTREME VALUE MODEL

Extreme value is most naturally developed as a theory of large losses rather than a theory of small profits. Extreme event occur when a risk takes values from the tail of its distribution, extreme value theory is a consistent tool which attempts to provide us with the best possible estimate of the tail area of the distribution, Wainnaina & Waititu (2014).

Gray & Chan (2006) identified two major approaches to estimation VaR, the Historical Simulation (HS) and the parametric approach. The Historical approach is considered the most popular method of estimating VaR, which is just to utilize the empirical distribution of past returns on the asset of interest. For instance, if one requires the VaR for one day with an $\alpha = 5\%$ Confidence level, one takes 95% quantile from the most recent T observed daily returns. The Historical simulation method is non-parametric and takes no arbitrary assumption of the distributions of returns. Another method is the Gaussian distribution known as normal distribution method. Normal distribution cannot model financial time series data properly because it does not adequately take account of what happens at the tail area of the distribution (McNeil 1999).

Uppal J.Y (2013) pointed out that there are two ways of modelling extremes of stochastic variable using the extreme value models. One approach is to divide the sample into blocks and then obtain the maximum from each block, which is referred to as the *block maxima* method. The distribution of the block maxima can be modelled by fitting it into Generalized Extreme Value (GEV) model. The GEV model is considered to have limitations particularly because it considers values in the high extremes and leave that of low extremes, consequently waste data (Cole 2001).

An alternative and preferred approach to modelling financial time series is the peak-over-threshold (POT), it takes large observations which exceed a certain threshold u and the POT models are generally considered to be the most useful for practical applications due to their more efficient use of the (often limited) data on extreme values Reiss & Thomas (2002). The distribution of the exceedances is obtained by employing Generalized Pareto distribution (GPD). The advantage of estimating VaR using GPD method is that this method can estimate VaR outside the sampling interval (Rufino & Guia, 2011).

According to (Nigm et al.1987), the risk of large losses and consequently large insurance claims can be modeled with Pareto, Gamma, and Lognormal distributions for deciding on deductible and premium. Fisher and Tippet (1928) developed a theory describing the limiting distribution of sample maxima and the distribution of the exceedance above a high threshold. Pickands (1975), Balkema and de Haan (1974) state the following regarding the conditional excess distribution function:

For a large class of underlying distribution functions the conditional excess distribution function $F_u(y)$ for a large value of μ , is well approximated by:

$$F_\mu(y) = G_{\beta,\xi}(y) : \mu \to \infty$$

$$G_{\beta,\xi}(y) = 1 - (1 + \xi^{\frac{y}{\beta}})^{-1/\xi} ; \xi \neq 0 \qquad (1)$$

$$= 1 - \xi^{-\frac{y}{\beta}}, \xi = 0$$

We consider modelling excess distribution; and interested in estimating the distribution function F_u of values of x above a high threshold u. The distribution F_u is called the conditional excess distribution function F of a random variable X as:

$$F_u(y) = P(X - u \leq y \mid X > u), \quad 0 \leq y \leq x_F - u \qquad (2)$$

$y = x - u$ are the right endpoint of F . We verify that F_u can be terms as F ,

$$F_u(y) = \frac{F(u+y) - F(u)}{1 - F(u)} = \frac{F(x) - F(u)}{1 - F(u)}$$
$$0 \leq X \leq u$$
$$(3)$$

From GPD, the tail index ξ gives an indication of heaviness or lightness of the tail, the larger ξ the heavier the tail. Only distributions with tail parameter $\xi \geq 0$ are fit to model financial returns.

DATA DESCRIPTION

The data used are taken from Nigeria Insurance digest publication which contains claims of ₦3.5 million and above over a period of 24 calendar months (2011 and 2012). Data of five Insurance companies for motor are collected for the analyses, which include *Leadway Asssurance Plc* with eighty five (85) observations, *Mansard Insurance plc* had one hundred and fifteen (115) observations, thirty two (32) observations were considered for *Zenith insurance Co. limited*, *Custodian & Allied Insurance limited* had forty six (46) observations while *Aiico Insurance Co. limited* were thirty six (36) observations. As at the time of this study, the latest publication was that of 2012. Detailed data can be downloaded at *http://www.nigeriainsurers.org/index.php.*

The presence of extreme values was tested in the data using the Chi-square test and presence of extreme (outliers) were found in the data set. Also Anderson Darling and Craver-Von Misses were also used to test for normality, and the data are discovered not to be normally distributed. Exploratory plots of time series, Quantile-Quantile plot and mean excess plot were made for each of company claim.

DATA ANALYSIS

Table 1: Descriptive Statistics of Motor Insurance Class of Business Claims

Descriptive				
	Mean	**Stdev**	**Skewness**	**Kurtosis**
Leadway	6.24970	9.695261	9.88351	65.16015
Aiico	6.70480	6.971299	3.709988	14.69223
Zentih	5.534433	2.38092	1.454863	0.708253
Mansard	7.985632	9.176129	4.38811	22.42858
Custodian	7.374375	10.22095	4.511361	21.79192

Source (Before descriptive Statistics): Nigeria Insurance Digest publication

The skewness shows that each data set is positively skewed or exhibits heavy tailed distribution, excess kurtosis also reveals that the data are not normally distributed

Table 2: Chi-square test for extreme Value

Company	χ^2	p-value	Extreme Value
Leadway	75.193	2.2×10^{-16}	90.321000
Aiico	24.537	7.29×10^{-7}	41.236945
Zenith	6.2359	0.01252	11.480000
Mansard	39.267	3.69×10^{-10}	65.486000
Custodian	32.889	29.70×10^{-9}	66.0000000

H_A : *There is an outlier in the data set,* $\alpha = 1\%$

The above test statistics reveals that there are extreme value(s) in the each data set, the p-value in each case reveals that H_A would be accepted in each case, we therefore conclude that extreme values are in the data sets.

Table 3: Normality tests of Motor Claims (H_0 : Normal)

	Anderson Darling	**Craver-Von Misses**
Leadway	20.2669**	4.0672**
Aiico	6.6605**	1.3276**
Zenith	3.5038**	0.6425**

Mansard	3.89510**	3.8951**
Custodian	9.9964**	2.0252**

*** indicates significance at 1%* | $p < 0.01$, $p - value = 2.2 \times 10^{-16}$ *in each case*

Anderson Darling and Craver-Von Misses also reveal that the data sets are not normally distributed.

METHODS

Statistical models have been widely used in time series data analysis, the data set was fitted into a GPD Model with maximum likelihood estimate, the GPD is an alternate method to generalized extreme value (GEV) model; GP model is chosen because it is more reliable as reveal in previous studies. Statistical software from R Core team (2014) was used to carry out the analysis, fExtremes package developed by Wuertz (2013) and many others was used for Extreme VaR, PerformanceAnalytics package designed by Peterson Carl (2014) was used for Historical and Gaussian VaR. Package 'outliers' in R; developed by Lukasz Komsta (2011) was used to test for presence of extreme values and nortest package developed by Gross and Ligges (2015) was used to test for normality.

The model used in the research is generalized Pareto distribution. The cumulative density function (cdf) of a two-parameter GPD distribution is as follows:

$$G_{\xi,\beta} = 1 - \left(1 + \xi \frac{x}{\beta}\right)^{\frac{-1}{\xi}}, \xi \neq 0$$
$$G_{\xi,\beta} = 1 - \exp(-x/\beta), \xi = 0 \quad (4)$$

Where $\beta > 0, x \geq 0$ when $\xi \geq 0$ and $0 \leq x \leq -\beta/\xi$ when $\xi < 0$. ξ is the important shape parameter of the distribution and β is an additional scaling parameter. If $\xi > 0$, the $G_{\xi,\beta}$ is re-parametrized version of the ordinary Pareto distribution use for large losses, $\xi = 0$ corresponds to the exponential distribution and $\xi < 0$ is known as a Pareto of type II distribution.

The case where $\xi > 0$ is the most relevant for risk management, whereas normal distribution has moments of all orders, a heavy-tailed distribution does not possess a broad set of moments. In the case of the GPD with $\xi > 0$ we observe that $E(X^k)$ is infinite for $k \geq 1/\xi$ when $\xi = 1/2$, the GPD is a second moment distribution, when $\xi = 1/4$, the GPD has an infinite fourth moment . However, certain types of large claims data in insurance generally suggest an infinite second moment.

Maximum Likelihood Estimate is a statistical technique for estimating model parameters. It helps to determine model parameters that are most likely to characterize a given set of data. With $G_{\xi,\beta}$ for the density of the GPD, the log-likelihood may be calculated to be:

$$L(\xi, \beta; Y_j) = -N_u In\beta + \left(\frac{1}{\xi} - 1\right) \sum_{j=1}^{N_u} \left(1 - \xi \frac{Y_j}{\beta}\right), \xi \neq 0 \quad (5)$$

$$L\left(\xi,\beta;Y_j\right) = -N_u In\beta - \frac{1}{\beta}\sum_{j=1}^{N_u} Y_j,\ \xi = 0 \tag{6}$$

Adeleke et. al (2015) showed detailed derivation of the Maximum likelihood for GPD at the end came up with the equation below:

$$\hat{\xi}_{MLE} = -\left(\frac{1}{N_u}\right)\sum_{j=1}^{N_u} In\left(1-\hat{\theta}_{MLE}Y_j\right) \tag{7}$$

and $$\hat{\beta}_{MLE} = \frac{\hat{\xi}_{MLE}}{\hat{\theta}_{MLE}} \tag{8}$$

Rufino & Guia (2011) outlined methods of estimating VaR as follows:

(i) Gaussian VaR

$$VaR(\alpha) = u + (N^{-1}(1-\alpha))\beta \tag{9}$$

(ii) Historical VaR

$VaR(\alpha) = ((1-\alpha)\times N)^{th}$ observation of historical sample (10)

(iii) Extreme VaR

$$V\hat{a}R_p(\alpha) = u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{N_u}p\right)^{\hat{\xi}} - 1\right) \tag{11}$$

Where n= number of observations in the parent distributions, N_u = number of tail observations with parameters β and ξ substituted with the maximum likelihood estimates.

RESULT AND DISCUSSION

Leadway Insurance Company

Figure 1: Plot for Leadway Insurance Company

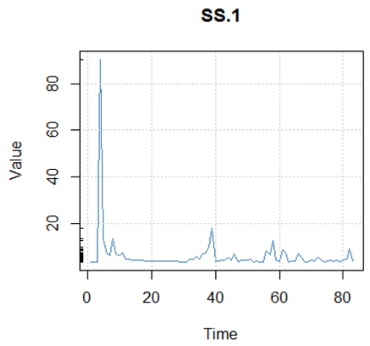

Fig 1a Time Series plot

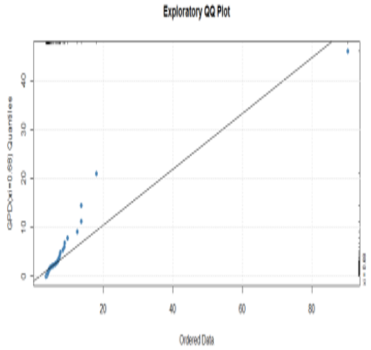

Fig 1b QQ plot

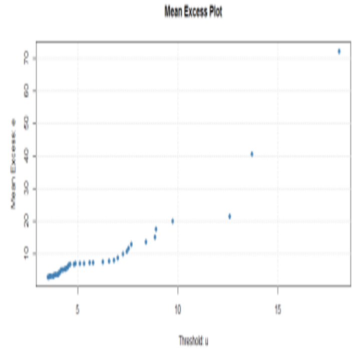

Fig 1c Mean excess plot

A threshold of 6 is chosen from the mean excess plot, and the data was fitted to a GPD model using Maximum Likelihood Estimate. The parameter estimates ξ = 0.6814555 and β =2.2753978. The shape parameter ξ is greater than 0 implying heavy tailed distribution. This can be interpreted to mean that the higher the value of the shape parameter, the higher the derived return. The distribution for the excesses shows a smooth curve meaning GDP fit was a good fit for the data. The Value at Risk (VaR) with 5% level of confidence was ₦3.538125 million, is a 1-in-20. For twenty four months period, it implies that the coming one month six day's loss for the company would exceed ₦3.538125 million. Then precautions can be taken to prevent it.

Aiico Insurance Company

Figure 2: Plot for Aiico Insurance Company

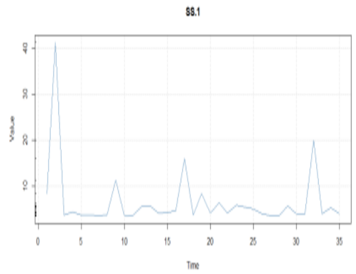

Fig 2a Time Series plot

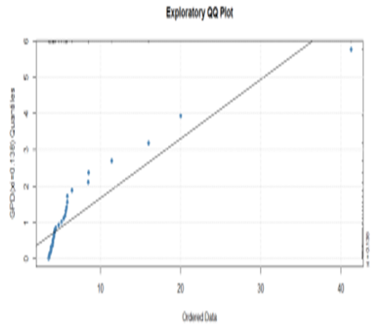

Fig 2b Q-Q plot

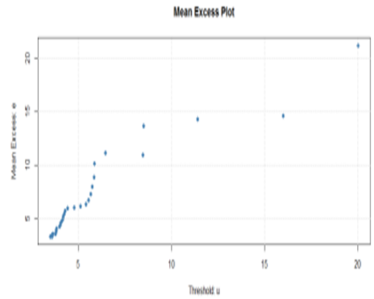

Fig 2c Mean excess plot

A threshold of 7 is chosen, and the data was fitted to a GPD model using Maximum Likelihood Estimate. The parameter estimates ξ = 0.1385216 and β =9.1854240. The shape parameter ξ is greater than 0 implying heavy tailed distribution. This can be interpreted to mean that the higher the value of the shape parameter, the higher the derived return. The distribution for the excesses shows a smooth curve meaning GDP fit was a good fit for the data. The Value at Risk (VaR) with 5% level of confidence was ₦3.6 million, is a 1-in-20. For twenty four months period, it implies that the coming one month six day’s loss for the company would exceed ₦3.6 million. Then precautions can be taken to mitigate prevent it.

Zenith Insurance Company

Figure 3: Plot for Zenith Insurance Company

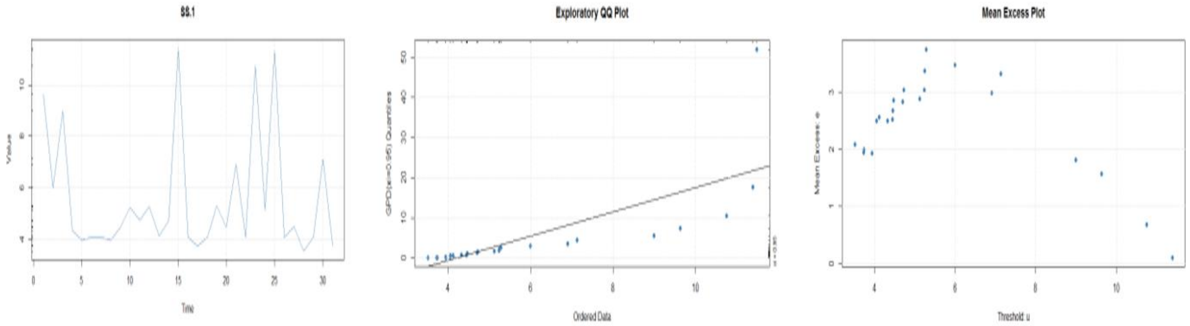

Fig 3a Time series plot **Fig 3b Q-Q plot** **Fig 3c Mean Excess plot**

A threshold of 4 is chosen, and the data was fitted to a GPD model using Maximum Likelihood Estimate. The parameter estimates $\xi = 0.9482570$ and $\beta = 0.6404502$. The shape parameter ξ is greater than 0 implying heavy tailed distribution. This can be interpreted to mean that the higher the value of the shape parameter, the higher the derived return. The distribution for the excesses shows a smooth curve meaning GDP fit was a good fit for the data. The Value at Risk (VaR) with 5% level of confidence was ₦3.726 million, is a 1-in-20. For twenty four months period, it implies that the coming one month six day’s loss for the company would exceed ₦3.726 million. Then precautions can be taken to prevent it.

Mansard Insurance Company

Figure 4: Plot for Mansard Insurance Company

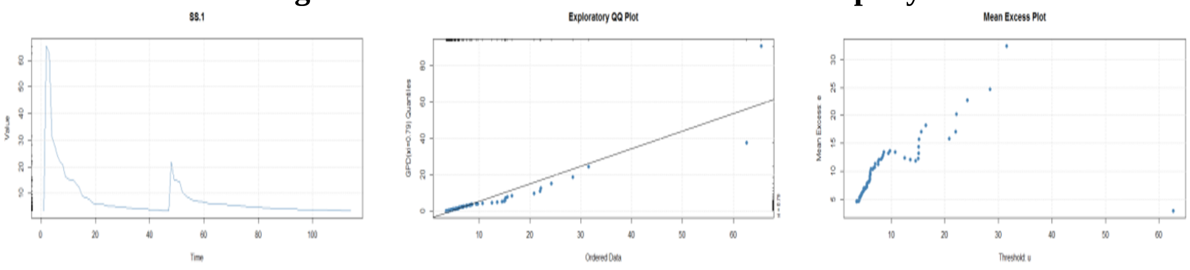

Fig 4a Time series plot Fig 4b Q-Q plot Fig 4c mean excess plot

A threshold of 5 is chosen, and the data was fitted to a GPD model using Maximum Likelihood Estimate. The parameter estimates $\xi = 0.7912678$ and $\beta = 2.4903510$. The shape parameter ξ is greater than 0 implying heavy tailed distribution. This can be interpreted to mean that the higher the value of the shape parameter, the higher the derived return. The distribution for the excesses shows a smooth curve meaning GDP fit was a good fit for the data. The Value at Risk (VaR) with 5% level of confidence was ₦3.564 million, is a 1-in-20. For twenty four months period, it implies that the coming one month six day’s loss for the company would exceed ₦3.564 million. Then precautions can be taken to prevent it.

Custodian Insurance Company

Figure 5: Plot for Custodian Insurance Company

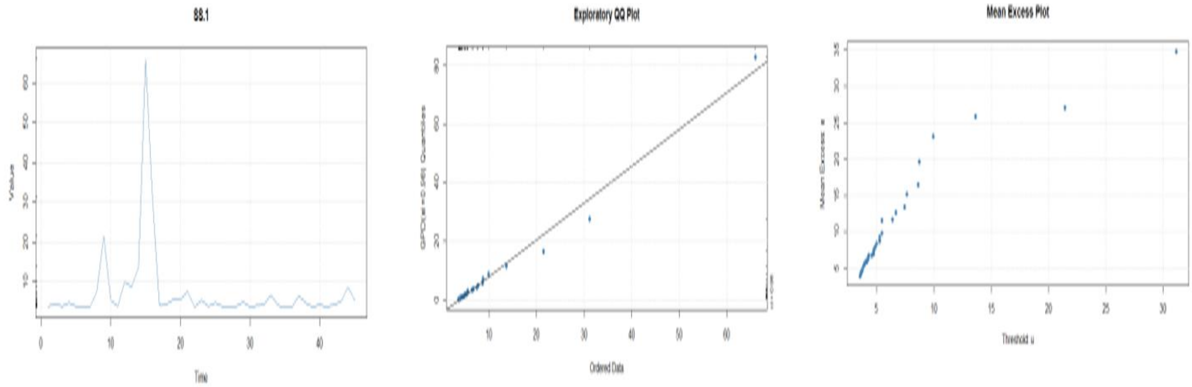

Fig 5a Time series plot Fig 5b Q-Q plot Fig 5c mean excess plot

A threshold of 4.5 is chosen, and the data was fitted to a GPD model using Maximum Likelihood Estimate. The parameter estimates ξ =0.9776848 and β = 1.6249459. The shape parameter ξ is greater than 0 implying heavy tailed distribution. This can be interpreted to mean that the higher the value of the shape parameter, the higher the derived return. The distribution for the excesses shows a smooth curve meaning GDP fit was a good fit for the data.

The Value at Risk (VaR) with 5% level of confidence was ₦3.5793 million, is a 1-in-20. For twenty four months period, it implies that the coming one month six day's loss for the company would exceed ₦3.5793 million. Then precautions can be taken to prevent it. Summary of the result from model fit for the five sets of data is as given in table 4 below.

Table 4: Value at risk, Threshold Selection and parameter of estimates

Value-at Risk estimates at α =5%				Threshold Selection and MLE Parameters		
Company	**Historical**	**Gaussian**	**Extreme VaR**	u	ξ	β
Leadway	-12.300	9.6012	3.538125	6	0.68145	2.27539
Aiico	-17.100	4.5970	3.600000	7	0.13852	9.18542
Zenith	-11.068	-1.682	3.726000	4	0.94825	0.64045
Mansard	-12.303	7.0414	4.564000	5	0.79127	2.49035
Custodian	-19.871	9.2495	3.579300	4.5	0.97768	1.62495

CONCLUSION AND RECOMMENDATIONS

Detailed analysis is carried out to model Motor industrial insurance returns of the Nigerian insurance companies. The presence of extreme values was tested using Chi-square test, Anderson Darling, Cravon Misses test and the data sets were found to contain extreme values. The empirical findings in this paper reveal that Extreme Value Theory method of calculating VaR outweighs the other two methods of estimation as EVT is known for its ability to model the tail area of the distribution much better. It is practically impossible to have negative VaR which is observed throughout in Historical Simulation method, while it is observed only in the case of Zenith Insurance Co. and that may account for low observations. The two other methods of estimation seem to perform well with high profile data, but EVT approach is able to model data irrespective of the number of observations.

Insurance companies are known to be in the business of investing premiums of many subscribers and only pay claims to those who have weighty losses from these premiums. A study such as this would help an insurance company to be able to estimate equitably reliably the amount of loss for a given number of customers within a specific time. It would also enable insurance companies to be put certain measures in place by employing adequate risk management measures so as to have minima loss and optimize profit.

The study would equally help subscribers/consumers of insurance products in Nigeria to have a good understanding of the ability of insurance companies to pay claims, as most Nigerian citizenry shun insurance products because of past defaults in paying claims. In the light of the conclusion, the following are hereby recommended:

(i) Insurance companies should always carry out thorough investigation about claims before they are paid; as some subscribers may be trying to "play smart".
(ii) All Nigeria Insurance companies should install vehicle tracking device in the vehicle of their clients so as to curb vehicle theft as claims due to vehicle theft as captured in the report is outrageous.
(iii) Consumers of insurance products should find out the integrity of any insurance company before subscribing into that company, particularly the newly established ones that are yet to have a proper registration.
(iv) Policy makers should further enforce strict law guiding claims payment so as to further gain confidence of potential clients.
(v) Nigeria government should endeavor to further provide substantial support for insurance companies such as we have in America, United Kingdom and other developed nations so as to boost the state of the economy.

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