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Application of Unified Number to Temperature Profiles of Pipe Walls and Fluids Using Mathematical Experimentation

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Abstract
The equations of energy balance and heat conductivity was queried by introducing known parameters and expanded using virtual mathematical experimentation. Distribution of temperature of the pipe wall, fluid flow and surrounding air were accounted for via mathematical expressions. A new dimensionless parameter was introduced with the aim of solving future problems in hydraulic engineering.
Keywords: dimensionless parameters, heat conductivity equations, fluid, Bessel polynomial scheme, Unified number

Introduction
The dynamics of thermal radiation through a pipe wall is important to accurately predict the fluid flow and its effects over long distances. Fluid flow along a lengthy pipe line experiences the friction losses which mitigate the rate of flow. The reality of the friction loss in laminar and turbulent fluid flow is an important phenomenon in hydraulic engineering. One of the reliable modern methods (1) of acquiring an accurate thermal analysis of the pipe wall with respect to the length of the pipe is the Rosemount 0085 Pipe Clamp Sensor shown in the figure [1a] below. Thermal signatures of fluid flow are dependent on the viscosity of the fluid (1), friction factor (2) and the temperature gradient between the pipe and fluid (3). In this paper, we shall account for the three heat transfer processes (i.e. conduction, convection and radiation) to adequately improve the theory of the work done in Ref [1].
In this paper, a two order differential equation was derived from the equations of energy balance and solved using the Bessel polynomial scheme. Previously, the Galerkin technique was used to analyze the weak singularity of the tensor kernel in fluid engineering (4). Jorge et al., (5) furthered the application of the Galerkin technique to obtain the viscous drag and capturing singular behavior of the surface tractions close to edges and corners of pipe walls. However, creeping flow of fluids within the pipe wall do not account for dynamics of thermal radiation through a pipe wall (6), hence, the Galerkin technique may not be appropriate. The validity of the Galerkin technique to analyze thermal radiation of fluids in pipes may be argued on the ground of pipe geometries which contributes to the flow properties inside the pipe (7). The pipe configurations that yields weak turbulence intensities at the exit of the pipe requires a more detailed approach which have been suggested in this text. A new dimensionless parameter (known as the unified number) was used to solve the thermal gradient between two dissimilar fluids operating at the boundaries of a separating conductive medium. The unified number was first introduced in Ref [6] to incorporate both the unified number (U) and the temperature profiles of the fluid flow to determine the volume flow rate which varies to the third power of the diameter (D). This simply means that when the diameter of the pipe is doubled, the flow rate increases by a factor 8. This idea was used to solve the shortcomings of the application of Reynolds number greater than 2000 i.e. Re>2000. Since the Reynolds number have been reported to be limited to estimate measurements of velocity and stress fields in rigid pipes (8), the unified number (U) is more appropriate for straight pipes. This hypothesis was inferred from Ref [6] as shown in the figure 1 below.

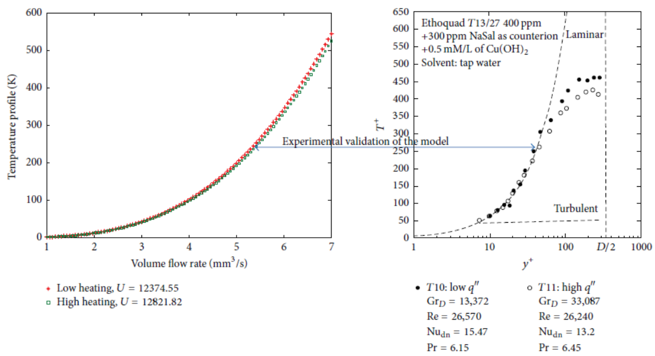

Figure 1: Theoretical Model using the unified number in laminar flow and was validated via experiments of two turbulent flow (Gasljevic et al., 2000)Ref (6)

In this paper, section two introduces the theoretical background to this study. Section three elaborates on the mathematical prospects of the unified number to fluid engineering. Section four expresses the practical application of the mathematical model-generated in section three.

Theoretical Background
We propose a long pipe of length AB as shown in figure [1a]. At any point within the pipe, the following assumption was made i.e. fluid temperature at point A differs to the temperature at point B

$T_{fA} < T_{fB}$ or $T_{fA} > T_{fB}$ and $T_{pA} < T_{pB}$ or $T_{pA} > T_{pB}$

Where T_{fA} the temperature of the fluid at point A is, T_{fB} is the temperature of the fluid at point B, T_{pA} is the temperature of

the pipe at point A, T_{pB} is the temperature of the pipe at point B.
The following thermal flow assumptions were applied i.e. when the fluid temperature is greater than the pipe - heat flows from the fluid to pipe. For a relatively very long pipe, we assume that the pipe and fluid are in thermal equilibrium which is represented as

$$H_p - H_f = \varepsilon \qquad (H_p > H_f, a = 0) \qquad [1]$$

H_p is the total heat energy in the pipe, H_f is the total heat energy in the fluid, ε is the heat loss/gain by the surrounding air.

$$H_f - H_p = \varepsilon \qquad (H_f > H_p, a = varies) \qquad [2]$$

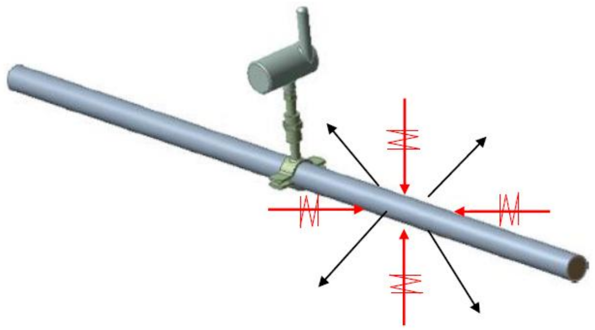

Figure2a: A clamped pipeline sensor (Ref 1)

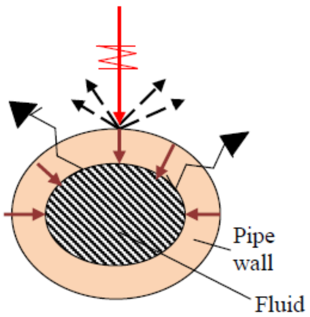

Figure2b: Cross section analysis of pipeline

The heat transfer in the pipe either as heat gain or heat loss is conductive. Therefore, the heat transfer processes can be summarized as

$$\frac{\partial u}{\partial t} = \varepsilon(T - T_\infty) + \varepsilon^*(C - C_\infty) + D\frac{\partial^2 u}{\partial x^2} \qquad [3]$$

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} \qquad [4]$$

C_p is the specific heat of at constant pressure, (Jkg^{-1}K^{-1}), C is the species concentration in the fluid, (kgm^{-3}), k is the thermal conductivity, (Wm^{-1}K^{-1}), T is the temperature profile, (K), D is the mass diffusion coefficient, (m^{2}s^{-1}), t is the time, (s), u is the velocity of the fluid, ρ is the density of the material (kgm^{-3}),

The initial and boundary conditions are

$$u = 0, \quad T = T_\infty, \quad C = C_\infty, \quad \text{for all} \;\; x,\; t \le 0$$

$$u = u_f, T = T_\infty + T_{eq}, \quad C = C_\infty + C_{eq}, \quad x = 0,\; t > 0$$

$$u \to 0, \quad T \to T_\infty, \quad C \to C_\infty, \quad x \to \infty$$

Before deriving a unified differential equation, the most significant dimensionless parameter are highlighted as

$$\theta = \frac{T_{eq}(x,y,z) - T_a}{T_f - T_a} \qquad [5]$$

Equation [5] is the dimensionless parameter for temperature which expresses the scale or magnitude of the temperature gradient of the fluid with respect to the pipe wall. The condition $H_p > H_f$ is relative because the temperature of the pipe may not necessarily be dependent on the surrounding air. To avoid complexities due to the heat source of the pipe, we assume it takes its temperature from the surrounding air. Again, this assumption is also relative because of the nature of the heat source which might be either due to weather or artificial (heat generated from machineries). Another vital factor of the condition $H_p > H_f$ is the fluid flow of very low temperatures. In this case, the pipe absorbs heat from the environment so that

$$H_f + H_p = \varepsilon \qquad [6]$$

The condition $H_f > H_p$ might also be relative. For clarity, we assume that the fluid temperature is dependent only on its source i.e. the machine from which it is flowing.
The other key dimensionless parameters are the Reynolds numbers which is defined as ratio between inertial and viscous forces. Therefore we shall be looking at the external airflow and for the internal fluid flow which is expressed as

$$Re_f = \frac{\rho U_f D_i}{\mu} \qquad [7]$$

ρ is the density, μ is the viscosity of the fluid flowing in the pipe, D_i is the internal diameter of the pipe and U_f is the mean velocity of that fluid. Due to the overall convective exchange of heat between the liquid and surrounding air, it is also necessary to state the Reynolds number for the air as

$$Re_a = \frac{\rho U_a D_i}{\mu} \qquad [8]$$

Another dimensionless quantity is the Prandtl number which is the ratio between momentum diffusivity and thermal diffusivity. Prandtl number of liquids varies correspondingly to the temperature of the fluid even though it is not shown in its expression

$$Pr = \frac{\mu C_p}{k} \qquad [9]$$

μ is the viscosity of fluid, C_p is specific heat at constant-pressure, and k is the thermal conductivity of the fluid. The

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other dimensionless parameter is the Nusselt number which is the ratio between total heat transfer in a convection dominated system and the estimated conductive heat transfer

$$Nu = \frac{hD_i}{k} \qquad [10]$$

D_i is the internal diameter of the pipe, k is the thermal conductivity of the fluid, h is the convective heat transfer coefficient.

Mathematical Experimentation

Solving the second order differential equation in Equation [3-5] enables the discovery of another dimensionless quantity. If equation [11] below is introduced to equation [3], equation [12] emerges with the introduction of a decelerating parameter $-a_f$ which is due to the presence of friction loss as the fluid flows through a lengthy pipeline.$\varepsilon^*(C - C_\infty)$ is negligible because the fluid is assume to have a homogenous flow.

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \times \frac{\partial x}{\partial t} \qquad [11]$$

$$-a_f \frac{\partial u}{\partial x} = D\frac{\partial^2 u}{\partial x^2} + \varepsilon(T - T_\infty)$$

$$D\frac{\partial^2 u(T_{eq})}{\partial x^2} + a_f \frac{\partial u(T_{eq})}{\partial x} + \varepsilon(T_{eq}) = 0 \qquad [12]$$

The generalized Bessel polynomial scheme was used to solve the flow rate which is dependent on the thermal equilibrium of the pipe wall and fluid. The generalized Bessel polynomial scheme is defined as follows

$$x^2 u(T_{eq})^{||} + (\alpha x + \beta)u(T_{eq})^{|} - n(n + \alpha - 1) \qquad [13]$$

$$u(T_{eq}) = \sum_{k=0}^{n} y_k{}^{(n)} T^k$$

$$y_k = \frac{\binom{n}{k}}{\binom{2n}{k}} \frac{2^k}{k!} = \frac{n!\,(2n-k)!\,2^k}{(2n)!\,(n-k)!\,k!}$$

Where $x^2 = D$, $(\alpha x + \beta) = a_f$, $n(n + \alpha - 1) = \varepsilon(T_{eq})$

Therefore,

$$\left.\begin{aligned} u_0(T_{eq}) &= 1 \\ u_1(T_{eq}) &= 1 + T_{eq} \\ u_2(T_{eq}) &= 1 + T_{eq} + \frac{T_{eq}{}^2}{3} \end{aligned}\right\} \qquad [14]$$

The dimensionless parameter are harnessed in one form as

$$\frac{PrRe}{Nu} = \frac{\rho C_p U_a}{hx} \qquad [15]$$

To cub the tendencies of bogus formular, we represented the three dimensionless parameter by a unified number (Un) and represented as $\frac{PrRe}{Nu} = Un$. The relationship between the temperature and the unified number was achieved by considering the second term of equation [14]. Higher order of the speed was avoided to reduce cumbersome mathematical representation and outrageous values. Extensively, this assumption applies to the computation of pointwise traction [(5)].

$$T_{eq} = \frac{hxUn}{\rho C_p} - 1 \qquad [16]$$

If we assume that $\frac{hxUn}{\rho C_p} \gg 1$, this concept introduces a vital dimensionless parameter for the analysis of heat exchange between two fluids possibly separated by a conducting medium (in this case, the medium is the pipe). If $x = T$, then

$$Un = \frac{\rho C_p U_a}{h}\left(\frac{T_{eq}}{T}\right)$$

$$Un = \frac{\rho C_p U_a}{h}\theta \qquad [17]$$

θ is the thermal ratio which defines the ratio of the average temperature of fluid in the pipe to the room temperature of the pipe.

Application of the Unified Number to Industrial-Piping Problems

The unified number is tested against an experimental Reynolds number and Nusselt number as shown in Figure [3a,b]. Our objective is to know the applicability of the Unified number to live problems.

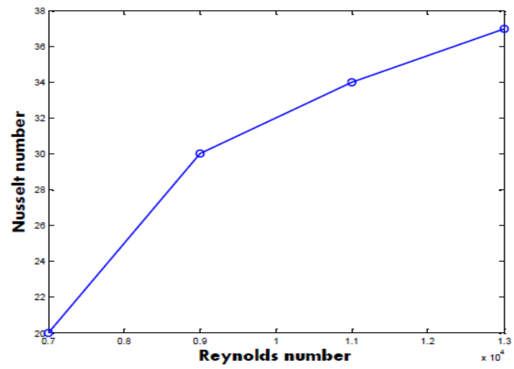

Figure 3a: experimental ratios between Nusselt and Reynolds number

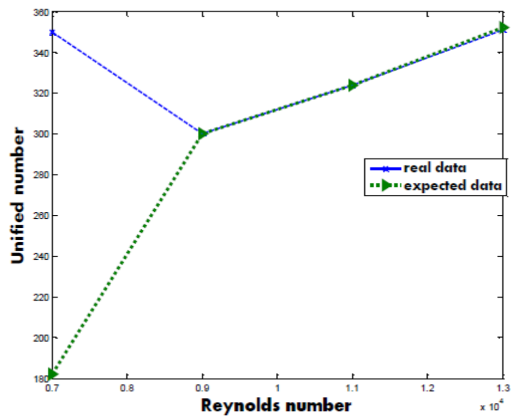

Fig 3b: experimental ratios between Unified and Reynolds number

As explained by Simone et al., (8), the effect of Reynolds number is limited for straight and reduced section pipes. When the Reynolds is far greater than 2000, it affects the stress fields as shown in figure 4. Hence, the unified number (U) effect (shown in figure 3b) enables the linearity of the stress field (see figure 4).

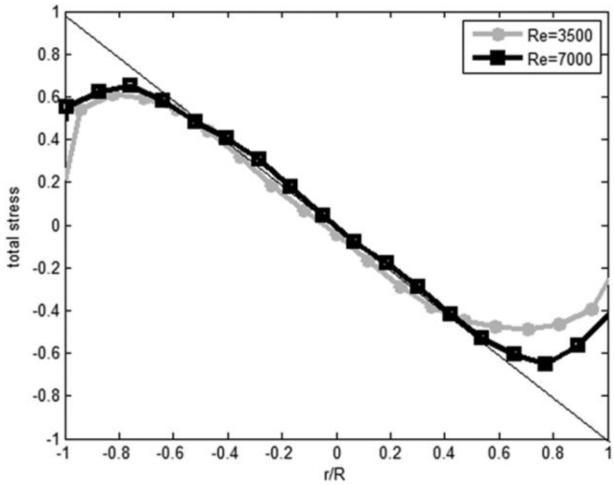

Figure 4: The linearity correction of the unified number for higher Reynolds in straight pipes. (Ref (8))

The linearity of the unified may be localized for certain fluids. The Unified number was calculated for various fluids and documented as shown in table 1 below. The fluid (liquid and gas) states were specifically treated to capture their behavioral pattern in a convective system. We believe that the thermal radiation follows a pattern-defined by the Temperature Polynomial Expansion Scheme (TPES) [10,11] for uni-layer i.e.

$$\theta_n = \frac{2}{3}^n \theta_0$$

[18]

Here n is the layer, θ_n is the temperature of the nth layer, θ_o is the fundamental temperature of the first layer. The uni-layer was applied via the homogenous flow explained in the previous section. However, the application of higher or lower order of 'n' depends on the distance between the stable regions of the pipe. For example, Donghyuk et al., [12] affirms the dependence of stable region on pipe lengths.

Table 1: Unified numbers for volatile liquids from laminar and turbulent flow

	Unified Number	Speed (cm/s)	θ
Water Liquid	562-805	0.2	0.3-3.0
Water Vapour	1.97-2.82	0.2	0.3-3.0
Acetone Liquid	244-349	0.2	0.18-1.8
Acetone Vapour	5-7	0.2	0.18-1.8
Methanol Liquid	252-362	0.2	0.2-2.0
Methanol vapour	13-19	0.2	0.2-2.0
Ethanol Liquid	88-126	0.2	0.2-2.0
Ethanol Vapour	2.99-4.29	0.2	0.2-2.0

The pressure gradient of fluids in the unilayer was validated via the multianalysis techniques of Ref [12] shown in figure 5 below

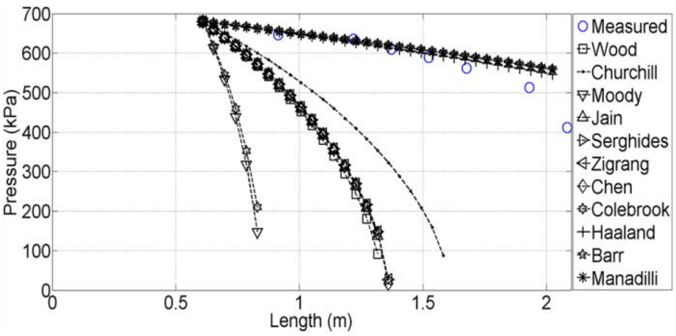

Figure 5: Pressure dependence on pipe length as expressed by Ref [7]. This is a theoretical validation of the unified number in equation [17].

Spaggiari et al.,[13] investigation on the flow mode under pressure of magnetorheological (MR) fluids further confirms the importance of the unified number to investigate the shear mode, where MR fluids were found to exhibit pressure dependency. Therefore, if water within this unified number flows through pipes made up of copper, the outcome simulated using the conservation principle of heat is summarized as

$$T_f = T_{eq} + \frac{ALhUn}{mC_pU_a} T_p \quad [19]$$

T_f is the temperature of the fluid, T_p is the temperature of the pipe, T_{eq} is the equilibrium temperature between the fluid and pipe, m is mass of water flowing through the pipe, A is the cross sectional area of the pipe. The following parameters were used for the experiment i.e. $T_p = 300K$, $h = 0.4W/mK$, $m = 30Kg$, $C_p = 381J/KgK$, $T_{eq} = 283K$, $U_a = 0.2cm/s$.The following results were obtained as shown in figure [6].

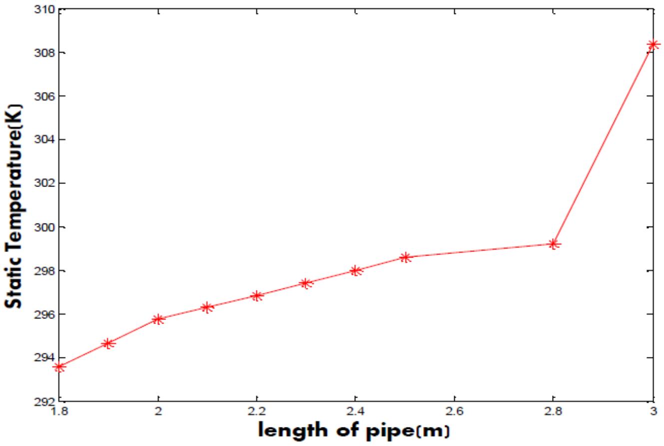

Figure 6: Centerline temperature profile of fluid

Results and Discussion

The heat loss/gain to the surrounding air dispenses via a quadratic format as expressed by the Bessel polynomial scheme. The negative nature of the quadratic equation confirmed the vanishing nature of the heat loss/gain to the surrounding air. This condition holds if the equilibrium temperature of the fluid is not thrice greater than the room temperature of the environment. The mass diffusion coefficient was also shown by the Bessel polynomial scheme to decrease by a square root magnitude as the fluid travels along a very long pipe. This process controls the pressure dependency on the length of the pipe (shown in figure 5). The Unified number governs the convective phenomena between two interacting fluids that are separated by a conducting medium [(6)]. In this case, the conducting medium is the pipe whose properties are defined by its density and specific heat capacity. The Unified number was tested against the experimental Nusselt and Reynolds number (figure [3a&b]). The unified number showed a strange convective relationship at lower Reynolds number. At higher Reynolds number, the unified numbers were normal at particular magnitude. By table 1, water was used in the experiment with a high conductive copper pipe. The static temperature of fluid was determined along the changing pipe length. This experiment is synonymous to the results gotten by Abbas *et al.* [(14)]. The temperature gradient between the pipe and fluid are independent of one another if the homogenous flow is operated at isobar at specific lengths of the pipe. This idea is opposed to the assumptions made in Ref (15,16). Therefore the unified number is a special case which sheds light on the relationship between the transverse velocity and fluid particle interaction parameter. If the thermal radiation of either the fluid or pipe is constant at the start of the pipe, the transverse velocity of the fluid initiates a non-uniform fluid particle interaction which depends on the inherent properties of the fluid.

The inverse relationship between the transverse velocity of the fluid and the fluid interacting particle established in Ref [16] may not be the outcome even in the presence of magnetic field. This theorem is affirmed in Ref [17] via the effects of half-bend angles and the presence of a nozzle on the momentum thickness and turbulence intensity at the exit plane of the curved pipes. Also, if the thermal radiation from of fluid is high, it exposes the pipeline to mimic the Bauschinger effect which is responsible for thermal gradient truncation in some heterogeneous compounds. The Bauschinger effect refers to the effects of microscopic stress distribution as a result of characteristic changes in the material's stress/strain relationship (18). Hence, the temperature distribution may align at higher length of pipe (as shown figure 6) and validated experimentally (19). The dynamics of the electronic structure of the thermally-excited fluids is hinged on the complementary stoichiometry signatures, thermal properties interacting particles. This is the idea of the unified number in solving hydraulic problems in machinery (20).

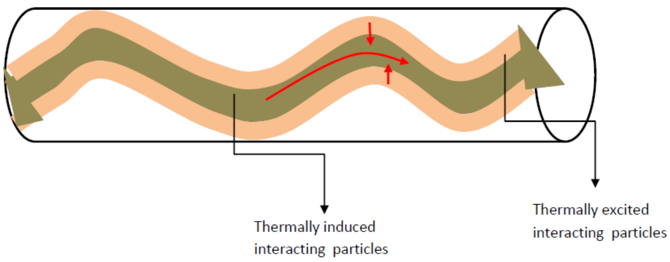

Figure 7: Thermally interacting particles on the a retarded fluid flow

Conclusion

The Bessel polynomial scheme had enabled the hypothesis of a scheme known as the Unified number which is believed to govern the convective phenomena between two interacting fluids that are separated by a conducting medium. The trivial validity of this hypothesis is the adequate inclusion of the conducting medium whose properties were defined by its density and specific heat capacity. Unified number is more effective in lower Reynolds number because of its strange convective system and can be substituted for high Reynolds number because of a lower but equivalent unified number. One of the advantages of the unified number is its ability to elaborate on the physics between the transverse velocity and fluid particle interaction parameter. If the thermal radiation of either the fluid or pipe is constant at the start of the pipe, the transverse velocity of the fluid initiates a non-uniform fluid particle interaction which depends on the inherent properties of the fluid. Hence, the solution to the abnormality noticed in the Reynolds number for straight pipes is the adequate estimation of the unified number.

Acknowledgement

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